Toward a Unifying Information-Theoretic Framework for Re-identification Risk Quantification

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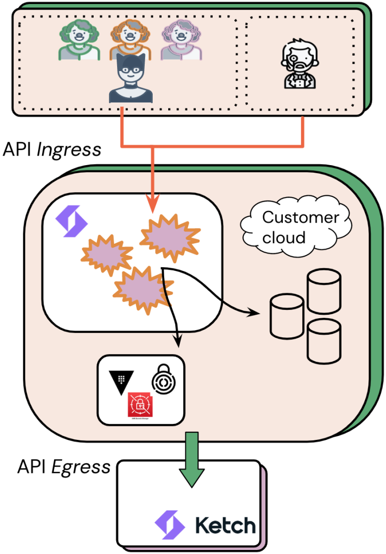
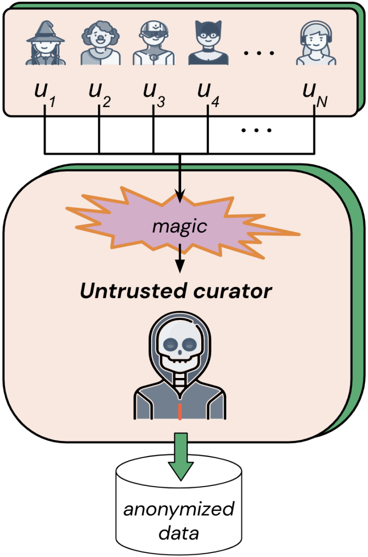
Introduction

Re-identification probabilities: elusive requisites



Re-identification probabilities: elusive requisites



The anonymization framework

$$\overset{\text{Subject}}{U \in \mathcal{U}} \xrightarrow[p_{X|U}]{} \overset{\text{Personal Data}}{X \in \mathcal{X}} \longrightarrow \boxed{\text{Anonymization mechanism } \mathcal{Q}} \xrightarrow[p^{\mathcal{Q}}_{Y|X}]{} \overset{\text{Anonymized Data}}{Y \in \mathcal{Y}}$$

The anonymization framework

Subject | Personal Data | | Anonymized Data

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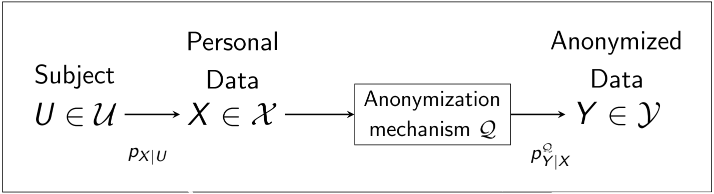
$\mathcal{U} = \{u_1, \ldots, u_N\}$

The anonymization framework

$\mathcal{X} = \{x_1, \ldots, x_N\}$



$\mathcal{U} = \{u_1, \ldots, u_N\}$

The anonymization framework

$\boxed{\mathcal{Q} : \mathcal{X} \to \mathcal{Y}}$

$\boxed{\mathcal{X} = \{x_1, \dots, x_N\}}$ $\boxed{y_i = \mathcal{Q}(x_i)}$

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The anonymization framework

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Subject | Personal Data | Anonymized Data

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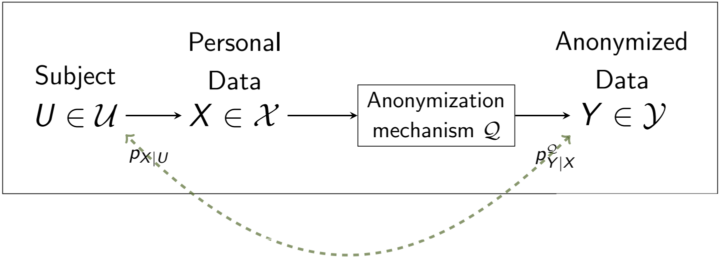
$\mathcal{Y} = \{y_1, \ldots, y_N\}$

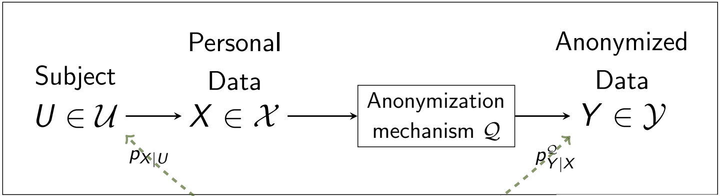
$\mathcal{U} = \{u_1, \ldots, u_N\}$

Evaluating re-identification risk



$$E_Y\left[\max_{u_i \in \mathcal{U}} P\left(U = u_i | Y\right)\right]$$

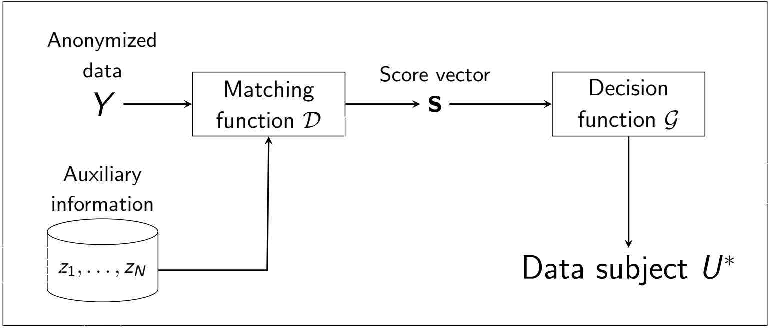
Evaluating re-identification risk



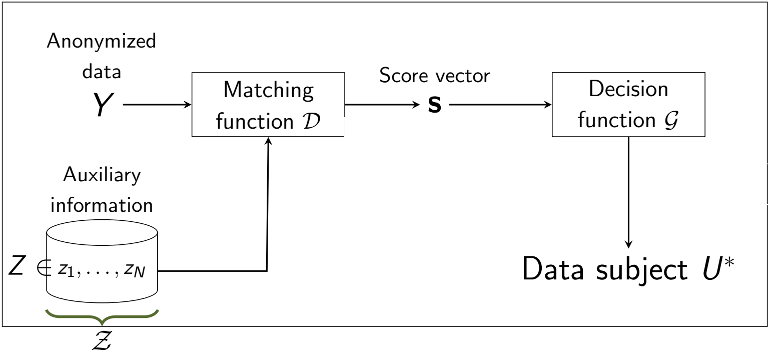
$$\text{PIE} = \boxed{I\left(U,Y\right) = \sum_{\mathcal{U}} \sum_{\mathcal{Y}} p_{U,Y}\left(u,y\right) \log \frac{p_{U,Y}\left(u,y\right)}{p_U\left(u\right) p_Y\left(y\right)}}$$

A general adversarial re-identification framework

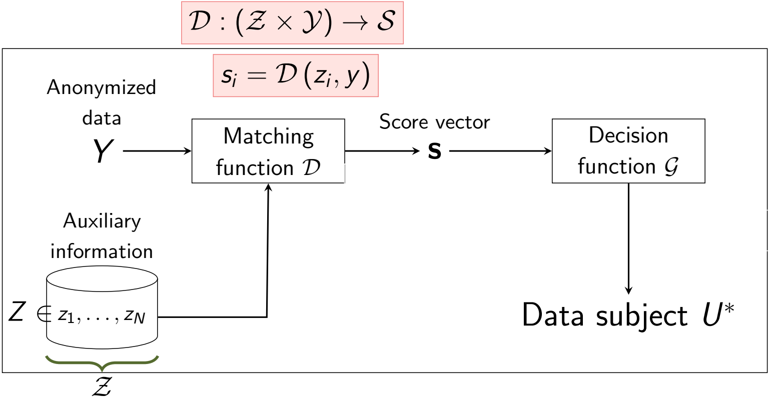
Re-identification framework



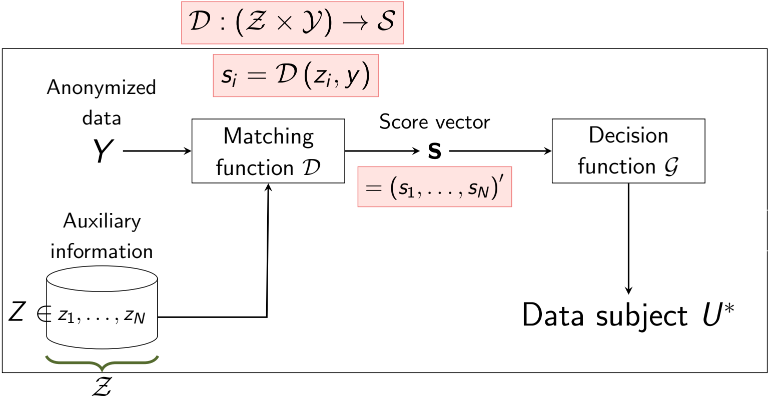
Re-identification framework



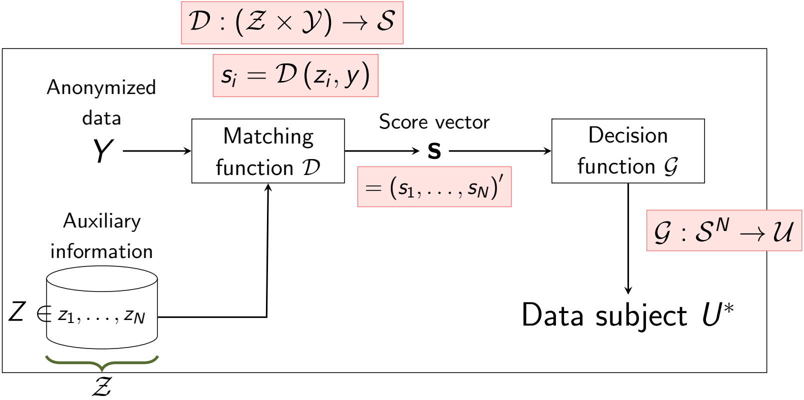
Re-identification framework



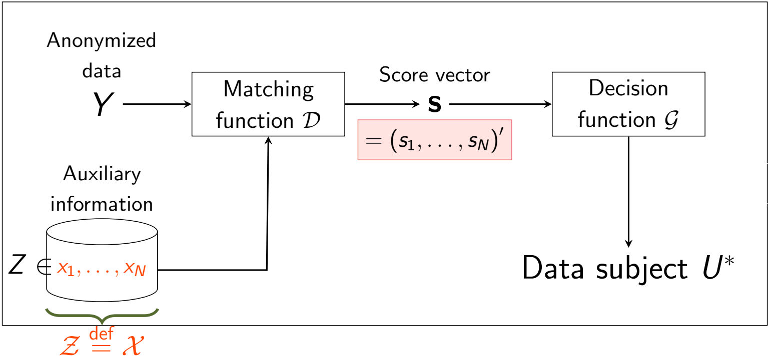
Re-identification framework



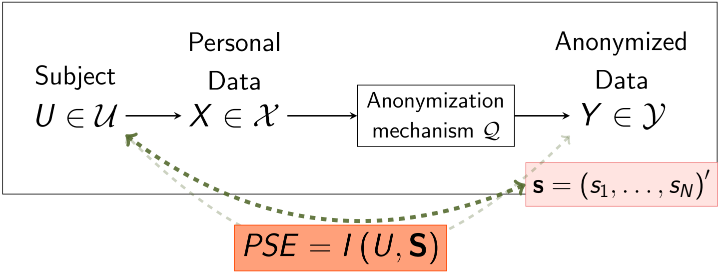
Re-identification framework



The maximum-knowledge intruder



Evaluating re-identification risk



Bounding posterior re-identification probabilities

Personal System Entropy (PSE)

The *personal system entropy* $\mathrm{PSE} = I(U, \mathbf{S})$ is a lower bound for the PIE.

Proposition

For any matching function $\mathcal{D}$ and any auxiliary information $z_1, \ldots, z_N$,

$$I(U, \mathbf{S}) \leq I(U, Y)$$

with equality if and only if U, $\mathbf{S}$, and Y form the Markov chain $U \to \mathbf{S} \to Y$ (i.e. $\mathbf{S}$ is a sufficient statistic for U).

e.g. if $\mathbf{s} = (q_1(y), \ldots, q_N(y))'$ where $q_i(y) = P(Y = y | U = u_i)$, then $\mathrm{PIE} = \mathrm{PSE}$.

Bayes re-identification error rate

Proposition (Bayes error, PIE and PSE)

$$\beta_{U|\mathbf{S}} \geq 1 - \frac{I\left(U, \mathbf{S}\right) + 1}{\log\left(1 - \beta_U\right)} \geq 1 - \frac{I\left(U, Y\right) + 1}{\log\left(1 - \beta_U\right)}.$$

where

$$\beta_U = 1 - \max_{u_i \in \mathcal{U}} P\left(U = u_i\right) = \text{Bayes error probability before observing } \mathbf{S}$$

$$\beta_{U|\mathbf{S}} = 1 - E_{\mathbf{S}}\left[\max_{u_i \in \mathcal{U}} P\left(U = u_i | \mathbf{S}\right)\right] = \text{Bayes error probability after observing } \mathbf{S}$$

Bayes re-identification error rate

Then!

$$1 - \beta_{U|\mathbf{S}} \leq \underbrace{\frac{I(U, \mathbf{S}) + 1}{\log(1 - \beta_U)}}_{\substack{\text{we can estimate this} \\ \text{using the adorable} \\ \text{result on the next slide}}} \leq \frac{I(U, Y) + 1}{\log(1 - \beta_U)}$$

When p_U is Uniform, this becomes

$$1 - \beta_{U|\mathbf{S}} \leq \frac{I(U, \mathbf{S}) + 1}{\log N} \leq \frac{I(U, Y) + 1}{\log N}$$

Estimation of the PSE

$$\text{Let } \mathcal{D}(X_i, Y_i) \sim \underbrace{f_G}_{\substack{\text{genuine}\\\text{score}\\\text{distribution}}}, \quad \mathcal{D}(X_i, Y_j) \sim \underbrace{f_I}_{\substack{\text{imposter}\\\text{score}\\\text{distribution}}} \text{ for } i \neq j.$$

Theorem (1)

Let f_G be a one-dimensional genuine score distribution and f_I be a one-dimensional imposter score distribution. Then

$$I(U, \mathbf{S}) \to D(f_G || f_I) \text{ as } N \to \infty,$$

where $D(f_G||f_I)$ denotes the Kullback-Leibler divergence of f_G from f_I.

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