

CS 225

Set Theory and Logic

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Comparison of Set Theory and Logic

I find the similarities between set theory and logic to be strikingly accurate: both have similar identity laws, distributive laws, DeMorgan's laws, etc. To me the set-theory versions are more intuitive since you can draw venn-diagram pictures to reason about the identities, but since we have covered logic first in CS225, perhaps the logical rules are more fresh in everyone's mind. In any case, I think it is useful to compare the two.

When reading through set theory notation, you may want to draw a venn diagram with sets A and B contained in some universal set \mathbb{U} to help understand the notation.

Here is a table comparing logical symbols/operations to set theoretic ones:

In the table I will refer to two sets A and B which are both contained in some universal set \mathbb{U} . You can think of \mathbb{U} as the domain of x .

Note 1: I do not claim that each of these is exactly equivalent! Just a strong relation.

Note 2: In the identities on the next page "True" corresponds to " \mathbb{U} " for the most part. In other contexts, it can be useful to think of the set A as "True" if it is not empty. So there may not be exactly one set-theory concept that corresponds exactly to "True".

Symbols in Set Theory vs. Logic

Word	Logic	Set Theory	Set Theory Connection
Or	\vee	\cup	$\forall x \in \mathbb{U}, x \in A \cup B$ if and only if $x \in A$ or $x \in B$.
And	\wedge	\cap	$\forall x \in \mathbb{U}, x \in A \cap B$ if and only if $x \in A$ and $x \in B$.
Not	\sim	c	$\forall x \in \mathbb{U}, x \in A^c$ if and only if x is not in A .
If / then	\rightarrow	\subseteq	" $A \subseteq B$ " if and only if " $\forall x \in \mathbb{U}$, if $x \in A$ then $x \in B$."
True	\mathbb{T} or \mathbb{T}	\mathbb{U} or "not empty"	" $\forall x \in \mathbb{U}, x \in \mathbb{U}$ is always true" OR " A is nonempty"
False	\mathbb{F} or \mathbb{F}	\emptyset	$\forall x \in \mathbb{U}, x$ is not in \emptyset .
Equivalent	\equiv	$=$ or "if and only if"	

Comparison of Set Theory and Logic Identities

Here is a comparison of set-theoretic versus logical identities (on the next page). Some identities (for example the biconditional) don't seem to translate well, so I have omitted the set-theory version. In the list, p, q, r are propositions, and A, B, C are sets contained in some universal set \mathbb{U} .

Identities in Set Theory vs. Logic

Logic	Set Theory	Identity
$\sim (\sim p) \equiv p$	$(A^c)^c = A$	Double Negation / Complement
$p \wedge \mathbb{T} \equiv p \quad p \vee \mathbb{F} \equiv p$	$A \cap \mathbb{U} = A \quad A \cup \emptyset = A$	Identity
$p \vee \mathbb{T} \equiv \mathbb{T} \quad p \wedge \mathbb{F} \equiv \mathbb{F}$	$A \cup \mathbb{U} = \mathbb{U} \quad A \cap \emptyset = \emptyset$	Domination
$p \wedge p \equiv p \quad p \vee p \equiv p$	$A \cap A = A \quad A \cup A = A$	Idempotent
$p \vee \sim p \equiv \mathbb{T} \quad p \wedge \sim p \equiv \mathbb{F}$	$A \cup A^c = \mathbb{U} \quad A \cap A^c = \emptyset$	Negation / Complement
$p \vee q \equiv q \vee p \quad p \wedge q \equiv q \wedge p$	$A \cup B = B \cup A \quad A \cap B = B \cap A$	Commutative
$(p \vee q) \vee r \equiv p \vee (q \vee r)$	$(A \cup B) \cup C = A \cup (B \cup C)$	Associative
$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	$(A \cap B) \cap C = A \cap (B \cap C)$	Associative
$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$	$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$	Distributive
$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$	Distributive
$\sim (p \wedge q) \equiv \sim p \vee \sim q$	$(A \cap B)^c = A^c \cup B^c$	DeMorgan's
$\sim (p \vee q) \equiv \sim p \wedge \sim q$	$(A \cup B)^c = A^c \cap B^c$	DeMorgan's
$p \vee (p \wedge q) \equiv p$	$A \cup (A \cap B) = A$	Absorption
$p \wedge (p \vee q) \equiv p$	$A \cap (A \cup B) = A$	Absorption

In the following table, the set-theoretic analogs were added by me. Please do not take my word for these—verify them yourself! I do not guarantee that the set-theoretic analogs are correct (although I believe they are).

Other Identities in Set Theory vs. Logic

Logic	Set Theory	Identity
$p \rightarrow q \equiv \sim q \rightarrow \sim p$	$A \subseteq B$ if and only if $B^c \subseteq A^c$	Contrapositive
$p \rightarrow q \equiv \sim p \vee q$	$A \subseteq B$ if and only if $A^c \cup B = \mathbb{U}$	Implication
$(p \rightarrow q) \wedge (p \rightarrow \sim q) \equiv \sim p$	If $A \subseteq B$ and $A \subseteq B^c$ then $A = \emptyset$	Absurdity