

# CS 225

## Symbols and Formulas

Fall 2020

## Logic

**Symbols:**  $\geq \leq \neq \neg \sim \wedge \vee \oplus \equiv \rightarrow \leftrightarrow \square \exists \forall$

### Identities:

$\sim(\sim p) \equiv p$	Double Negation
$p \wedge \mathbb{T} \equiv p \quad p \vee \mathbb{F} \equiv p$	Identity
$p \vee \mathbb{T} \equiv \mathbb{T} \quad p \wedge \mathbb{F} \equiv \mathbb{F}$	Domination
$p \wedge p \equiv p \quad p \vee p \equiv p$	Idempotent
$p \vee \sim p \equiv \mathbb{T} \quad p \wedge \sim p \equiv \mathbb{F}$	Negation
$p \vee q \equiv q \vee p \quad p \wedge q \equiv q \wedge p$	Commutative
$(p \vee q) \vee r \equiv p \vee (q \vee r)$	Associative
$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	Associative
$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$	Distributive
$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	Distributive
$\sim(p \wedge q) \equiv \sim p \vee \sim q$	DeMorgan's
$\sim(p \vee q) \equiv \sim p \wedge \sim q$	DeMorgan's
$p \vee (p \wedge q) \equiv p$	Absorption
$p \wedge (p \vee q) \equiv p$	Absorption
$p \rightarrow q \equiv \sim q \rightarrow \sim p$	Contrapositive
$p \oplus q \equiv q \oplus p$	Contrapositive
$p \rightarrow q \equiv \sim p \vee q$	Implication
$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$	Biconditional Equivalence
$(p \wedge q) \rightarrow r \equiv p \rightarrow (q \rightarrow r)$	Exportation
$(p \rightarrow q) \wedge (p \rightarrow \sim q) \equiv \sim p$	Absurdity
$p \vee q \equiv \sim p \rightarrow q$	Alternate Implication
$p \wedge q \equiv \sim(p \rightarrow \sim q)$	Alternate Implication
$\sim(p \rightarrow q) \equiv p \wedge \sim q$	Alternate Implication
$\sim\left(\forall x P(x)\right) \equiv \exists x \sim P(x)$	DeMorgan's for Quantifiers
$\sim\left(\exists x Q(x)\right) \equiv \forall x \sim Q(x)$	DeMorgan's for Quantifiers

A conditional statement  $p \rightarrow q$  can also be read as:

- If  $p$  then  $q$
- $p$  implies  $q$
- If  $p$ ,  $q$
- $p$  only if  $q$
- $q$  if  $p$
- $q$  unless  $\sim p$
- $q$  when  $p$
- $q$  whenever  $p$
- $q$  follows from  $p$
- $p$  is a sufficient condition for  $q$  ( $p$  is sufficient for  $q$ )
- $q$  is a necessary condition for  $p$  ( $q$  is necessary for  $p$ )

### Proofs:

- Direct: Assume  $P$  and prove  $Q$ .
- Contrapositive: Assume Not  $Q$  and prove Not  $P$ .
- Contradiction: Assume  $P$  and Not  $Q$  and prove a contradiction.
- Induction: Prove base(s), assume  $P(m)$ , prove  $P(m+1)$ .

# Sets

**Symbols:**  $\in \notin \subseteq \subset \supseteq \supset \emptyset \cup \cap \times$

**Common Sets:**

$\mathbb{N} = \{0, 1, 2, 3, \dots\}$	natural numbers
$\mathbb{Z} = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$	integers ( $\mathbb{Z}$ for German Zahlen, meaning “numbers”)
$\mathbb{Z}^+ = \{1, 2, 3, \dots\}$	positive integers
$\mathbb{Q} = \left\{ \frac{p}{q} \mid p \in \mathbb{Z}, q \in \mathbb{Z}, q \neq 0 \right\}$	rational numbers
$\mathbb{U} = \{*\}$	universal set

**Identities:**

$(A^c)^c = A$	Double Complement
$A \cup \emptyset = A \quad A \cap \mathbb{U} = A$	Identity
$A \cup \mathbb{U} = \mathbb{U} \quad A \cap \emptyset = \emptyset$	Domination
$A \cup A = A \quad A \cap A = A$	Idempotent
$A \cup A^c = \mathbb{U} \quad A \cap A^c = \emptyset$	Complement
$A \cup B = B \cup A \quad A \cap B = B \cap A$	Commutative
$(A \cup B) \cup C = A \cup (B \cup C)$	Associative
$(A \cap B) \cap C = A \cap (B \cap C)$	Associative
$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$	Distributive
$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$	Distributive
$(A \cup B)^c = A^c \cap B^c$	DeMorgan's
$(A \cap B)^c = A^c \cup B^c$	DeMorgan's
$A \cup (A \cap B) = A$	Absorption
$A \cap (A \cup B) = A$	Absorption
$\mathbb{U}^c = \emptyset \quad \emptyset^c = \mathbb{U}$	Complement of $\mathbb{U}$ , $\emptyset$
$A - B = A \cap B^c$	Set difference law

# Series and Sums

Symbols:  $\Sigma$  .

Geometric Progression

$$a_k = a \cdot r^k$$

Arithmetic Progression

$$a_k = a + d \cdot k$$

## Sum Formulas:

Formula #	Sum	Closed Form
1	$\sum_{k=0}^n ar^k \ (r \neq 0)$	$\frac{ar^{n+1} - a}{r - 1}, \ r \neq 1$
2	$\sum_{k=i}^n ar^k \ (r \neq 0)$	$\frac{ar^{n+1} - ar^i}{r - 1}, \ r \neq 1$
3	$\sum_{k=i}^n c$	$c \cdot (n - i + 1)$
4	$\sum_{k=1}^n k$	$\frac{n(n+1)}{2}$
5	$\sum_{k=1}^n k^2$	$\frac{n(n+1)(2n+1)}{6}$
6	$\sum_{k=1}^n k^3$	$\frac{n^2(n+1)^2}{4}$

# Counting

Symbols:  $\lambda$

Equations:

- Sum Rule:

if  $S$  is the union of  $m$  disjoint sets  $S_1, S_2, \dots, S_m$ , then the number of elements in  $S$  is:

$$|S| = |S_1| + |S_2| + \dots + |S_m|$$

- Product Rule:

for a sequence of  $m$  choices the total number of elements is:

$$P(m) = |P_1| \cdot |P_2| \cdot \dots \cdot |P_m|$$

- Subset Exclusion:

if  $C = A - B$  is the difference of two sets, where  $B \subseteq A$ , then the number of elements in  $C$  is:

$$|C| = |A| - |B|$$

- Inclusion/Exclusion:

when a set  $D$  to be counted is the union of non-disjoint sets  $A$  and  $B$ , the number of elements in  $D$  is:

$$|D| = |A| + |B| - |A \cap B|$$

Permutations and Combinations:

- $r$ -permutations:  $P(n, r) = \frac{n!}{(n-r)!}$

- $r$ -permutations with repetition allowed:  $n^r$

- The number of different permutations of  $n$  objects, where there are  $n_1$  indistinguishable objects of type 1, and  $n_2$  indistinguishable objects of type 2, ... and  $n_k$  indistinguishable objects of type  $k$  is  $\frac{n!}{n_1!n_2! \dots n_k!}$

- $r$ -combinations  $C(n, r) = \frac{n!}{r!(n-r)!}$

- $r$ -combinations with repetition allowed:

$$C(n+r-1, r) = \frac{(n+r-1)!}{r!(n+r-1-r)!} = \frac{(n+r-1)!}{r!(n-1)!}$$