CS 225

Set Theory and Logic

Fall 2020

Comparison of Set Theory and Logic

I find the similarities between set theory and logic to be strikingly accurate: both have similar identity laws, distributive laws, DeMorgan's laws, etc. To me the set-theory versions are more intuitive since you can draw venn-diagram pictures to reason about the identities, but since we have covered logic first in CS225, perhaps the logical rules are more fresh in everyone's mind. In any case, I think it is useful to compare the two.

Here is a table comparing logical symbols/operations to set theoretic ones:

In the table I will refer to two sets A and B which are both contained in some universal set $\mathbb{U}$. You can think of $\mathbb{U}$ as the domain of x.

Word	Logic Symbol	Set Theory Symbol	Set Theory Connection
Or	$\vee$	$\cup$	$\forall\ x \in \mathbb{U}$, $x \in A \cup B$ if and only if $x \in A$ **or** $x \in B$.
And	$\wedge$	$\cap$	$\forall\ x \in \mathbb{U}$, $x \in A \cap B$ if and only if $x \in A$ **and** $x \in B$.
Not	$\sim$	^{c}	$\forall\ x \in \mathbb{U}$, $x \in A^c$ if and only if x is **not** in A.
If / then	$\rightarrow$	$\subseteq$	"$A \subseteq B$" if and only if "$\forall\ x \in \mathbb{U}$, if $x \in A$ then $x \in B$."

Identities:

$\sim(\sim p) \equiv p$	Double Negation
$p \wedge \mathbb{T} \equiv p \quad p \vee \mathbb{F} \equiv p$	Identity
$p \vee \mathbb{T} \equiv \mathbb{T} \quad p \wedge \mathbb{F} \equiv \mathbb{F}$	Domination
$p \wedge p \equiv p \quad p \vee p \equiv p$	Idempotent
$p \vee \sim p \equiv \mathbb{T} \quad p \wedge \sim p \equiv \mathbb{F}$	Negation
$p \vee q \equiv q \vee p \quad p \wedge q \equiv q \wedge p$	Commutative
$(p \vee q) \vee r \equiv p \vee (q \vee r)$	Associative
$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	Associative
$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$	Distributive
$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	Distributive
$\sim(p \wedge q) \equiv \sim p \vee \sim q$	DeMorgan's
$\sim(p \vee q) \equiv \sim p \wedge \sim q$	DeMorgan's
$p \vee (p \wedge q) \equiv p$	Absorption
$p \wedge (p \vee q) \equiv p$	Absorption
$p \rightarrow q \equiv \sim q \rightarrow \sim p$	Contrapositive
$p \oplus q \equiv q \oplus p$	Contrapositive
$p \rightarrow q \equiv \sim p \vee q$	Implication
$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$	Biconditional Equivalence
$(p \wedge q) \rightarrow r \equiv p \rightarrow (q \rightarrow r)$	Exporation
$(p \rightarrow q) \wedge (p \rightarrow \sim q) \equiv \sim p$	Absurdity
$p \vee q \equiv \sim p \rightarrow q$	Alternate Implication
$p \wedge q \equiv \sim(p \rightarrow \sim q)$	Alternate Implication
$\sim(p \rightarrow q) \equiv p \wedge \sim q$	Alternate Implication
$\sim\Big(\forall\, x\ P(x)\Big) \equiv\ \exists\, x\ \sim P(x)$	DeMorgan's for Quantifiers
$\sim\Big(\exists\, x\ Q(x)\Big) \equiv\ \forall\, x\ \sim Q(x)$	DeMorgan's for Quantifiers

Sets

Symbols: $\in \notin \subseteq \subset \supseteq \supset \ \varnothing \ \cup \ \cap \ \times$

Common Sets:

$\mathbb{N} = \{0, 1, 2, 3, \dots\}$	natural numbers
$\mathbb{Z} = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$	integers ($\mathbb{Z}$ for German Zahlen, meaning "numbers")
$\mathbb{Z}^+ = \{1, 2, 3, \dots\}$	positive integers
$\mathbb{Q} = \left\{ \frac{p}{q} \;\middle|\; p \in \mathbb{Z}, q \in \mathbb{Z}, q \neq 0 \right\}$	rational numbers
$\mathbb{U} = \{*\}$	universal set

Identities:

$A \cup \varnothing = A \quad A \cap \mathbb{U} = A$	Identity
$A \cup \mathbb{U} = \mathbb{U} \quad A \cap \varnothing = \varnothing$	Domination
$A \cup A = A \quad A \cap A = A$	Idempotent
$A \cup A^c = \mathbb{U} \quad A \cap A^c = \varnothing$	Complement
$A \cup B = B \cup A \quad A \cap B = B \cap A$	Commutative
$(A \cup B) \cup C = A \cup (B \cup C)$	Associative
$(A \cap B) \cap C = A \cap (B \cap C)$	Associative
$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$	Distributive
$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$	Distributive
$(A \cup B)^c = A^c \cap B^c$	DeMorgan's
$(A \cap B)^c = A^c \cup B^c$	DeMorgan's
$A \cup (A \cap B) = A$	Absorption
$A \cap (A \cup B) = A$	Absorption

Symbols: $\geq \ \leq \ \neq \ \neg \ \sim \ \wedge \ \vee \ \oplus \ \equiv \ \to \ \leftrightarrow \ \square \ \exists \ \forall$

Sum Formulas: