Lecture 08/21/23

Numbers + Operations

In this class we will work a lot with numbers and operations e.g $(+, -, \times, \div, (-)^2, \sqrt{-}, ...)$. Given an expression of numbers and operations. e.g

$$(2+3) \cdot 7 \quad +2$$

we need to know in what order we perform the operations, so that everyone is on the same page.

① Parantheses
② Exponents
③ Multiplication ⎤ from left to right
   Division   ⎦
④ Addition   ⎤ from left to right.
   Subtraction ⎦

Ex! $(2+3) \cdot 7 + 2 \overset{P}{=} (5) \cdot 7 + 2$
$\overset{M}{=} 35 + 2$
$\overset{A}{=} 37$

Ex: $1 - (2+3) - 4 \div (7-5) + 6 \cdot 8$
$\overset{P}{=} 1 - 5 - 4 \div 2 + 6 \cdot 8$
~~$1 - 5 - 4 \div 2 + 48$~~
$\overset{D}{=} 1 - 5 - 2 + 6 \cdot 8$

$$\begin{aligned}
\underline{\underline{M}} &= 1 - 5 - 2 + 48 \\
\underline{\underline{S}} &= 4 - 4 - 2 + 48 \\
\underline{\underline{D}} &= -6 + 48 \\
\underline{\underline{A}} &= \boxed{42}
\end{aligned}$$

~~FRe: 2 (1-3)+2 /// (2+3)~~

Fractions:

<u>Defn</u>: A fraction is a quotient or ratio of two numbers

Ex: $\dfrac{2}{3}$ ← Numerator

↖ Denominator

~~We often write~~

┌─────────────────────────────────┐
│ A fraction is really just division │
└─────────────────────────────────┘

Ex: $\dfrac{2}{3}$ same as $2 \div 3$

Ex: $\dfrac{(2+7)}{3} = (2+7) \div 3$ → remember this trick!

Ex: $\dfrac{9-3+2}{2} - (2-3) = (9-3+2) \div 2 - (2-3)$

$\underset{P\;E\;M\;D\;A}{} \quad \begin{aligned} &= 8 \div 2 - (-1) \\ &= 4 - (-1) \\ &= 4 + 1 \\ &= \boxed{5} \end{aligned}$

Defn: A prime number is u ~~was~~ whole number greater than 1 that is divisible by only 1 and itself

Ex: 2, 3, 5, 7, 11, 13, 17, ...   are all prime

   4 is not since 2 divides it.

Every number can be written as a product of primes!

Ex: $42 = 2 \cdot 21 = 2 \cdot 7 \cdot 3$

How to reduce fractions: How can we tell if a fraction $\frac{a}{b}$ is reduced

Defn: $\frac{a}{b}$ is reduced if a and b have no common factors

① Write both numerator and denominator as a product of primes.

② ~~Cancel~~ Cancel like terms.

Ex: $\frac{72}{240} = \frac{2 \cdot 36}{24 \cdot 10} = \frac{2 \cdot 6 \cdot 6}{12 \cdot 2 \cdot 10}$

$$= \frac{2 \cdot 2 \cdot 3 \cdot 2 \cdot 3}{4 \cdot 3 \cdot 2 \cdot 10}$$

$$= \frac{2 \cdot 2 \cdot 3 \cdot 2 \cdot 3}{2 \cdot 2 \cdot 3 \cdot 2 \cdot 2 \cdot 5}$$

$$= \frac{3}{2 \cdot 5} = \boxed{\frac{3}{10}}$$

/Subtracting

<mark>Adding</mark> ↓ fractions

① $\dfrac{a}{b} + \dfrac{c}{d} = \dfrac{ad + cb}{bd}$ $\dfrac{a}{b} - \dfrac{c}{d} = \dfrac{ad - cb}{bd}$

② Ex: $\dfrac{2}{3} \diagtimes \dfrac{7}{2} = \dfrac{2 \cdot 2 + 7 \cdot 3}{3 \cdot 2}$

$= \dfrac{4 + 21}{6}$

$= \dfrac{25}{6}$

Is this reduced? Yes! 25 and 6 have no common factors.

<mark>Multiplying</mark> Fractions

$\dfrac{a}{b} \cdot \dfrac{c}{d} = \dfrac{ac}{bd}$

Ex: $\left(\dfrac{-3}{8}\right)\left(\dfrac{-4}{5}\right) = \dfrac{(-3)(-4)}{8 \cdot 5} = \dfrac{12}{40}$

$= \dfrac{3 \cdot 2 \cdot 2}{2 \cdot 2 \cdot 2 \cdot 5}$

$= \dfrac{3}{10}$

Dividing Fractions

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$$

keep-change-flip

Ex: $\dfrac{1/10}{1/2} = \dfrac{1}{10} \div \dfrac{1}{2} = \dfrac{1}{10} \cdot \dfrac{2}{1}$

$$= \frac{1 \cdot 2}{10 \cdot 1}$$

$$= \frac{2}{10}$$

$$= \boxed{\frac{1}{5}}$$

Square Roots / ~~Radicals~~

~~radical~~ ~~radical $a^{1/2}$~~

~~rad~~

How to simplify square roots

① write # as product of primes.
② If a number occurs twice remove both
from square root and place one copy outside

③ Repeat Step ② until you cannot.

Ex: $\sqrt{\dfrac{1}{\cancel{}196}} = \dfrac{\sqrt{1}}{\sqrt{196}}$

$$= \dfrac{1}{\sqrt{2 \cdot 98}}$$

$$= \dfrac{1}{\sqrt{2 \cdot 2 \cdot 49}}$$

$$= \dfrac{1}{7 \cdot 2 \sqrt{2 \cdot 2 \cdot 7 \cdot 7}}$$

$$= \dfrac{1}{7 \cdot 2 \sqrt{1}}$$

$$= \boxed{\dfrac{1}{14}}$$

Percents and Decimals

Percent to Decimal : Move decimal place in percent two spaces to left

Ex   2 %   ~ is $\boxed{.002}$

Decimal to Percent : Move Decimal place two places to the right.

Ex on 1.75   is   175 %

Percent of Something:

Ex: #9 W.S

110 L   35% S.J   .   Then 65% is
O.J.   ~~Sol elthend even~~

~~6U~~

To find 65% of 110L change 65% to a
decimal   .65   then multiply

$$110 \cdot .65 =$$

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Percent Change: Represents a change in quantity

percent change (as a decimal) = $\dfrac{\text{amount of change}}{\text{original amount}}$

Ex #11 W.S

$$\dfrac{59.99 - 43.99}{59.99}$$ $\dfrac{\text{change}}{0.9}$