

Lecture 09/21/23

Systems of Linear Equations

HW Due: 16 17 18
Fri Sun Wed

No quiz!

Defn: A system of linear equations is a set of 2 or more linear equations consisting of the same types of variables.

B Eg

$$\begin{cases} 2x + y = 2 \\ 4x - 2y = 0 \end{cases}$$

There may be more than two variables!

Defn: a solution to a system of linear equations is a pair of numbers $(a, b) \leftrightarrow (x, y)$ that solve both all equations in the system at the same time. It is possible to have no sols!

Prob 1: i)

$$\begin{cases} 5x - 3y = -12 \\ 2x + 3y = -9 \end{cases}$$

$(-3, -1)$ sol

$$5(-3) - 3(-1) \stackrel{?}{=} -12$$

$$-15 + 3 = -12 \quad \checkmark$$

$$2(-3) + 3(-1) \stackrel{?}{=} -9$$

$$-6 + -3 = -9 \quad \checkmark$$

Yes it is a sol.

i)

$$\begin{cases} 3x - y = -8 \\ 2x + 3y = 5 \end{cases}$$

$$3(-3) - (-1) \stackrel{?}{=} -8$$

$$2(-3) + 3(-1) \stackrel{?}{=} 5$$

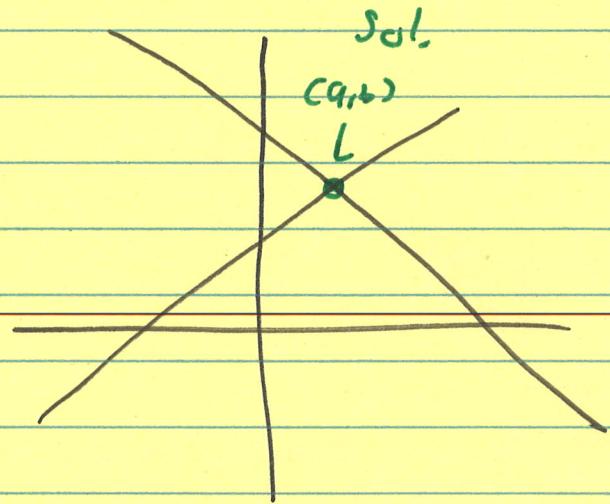
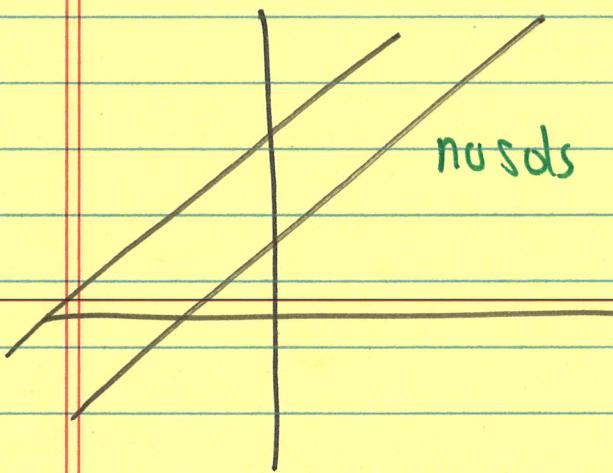
$$-9 + 1 = -8 \quad \checkmark$$

$$-6 + -3 = -8 \neq 5$$

Not a sol.

Substitution

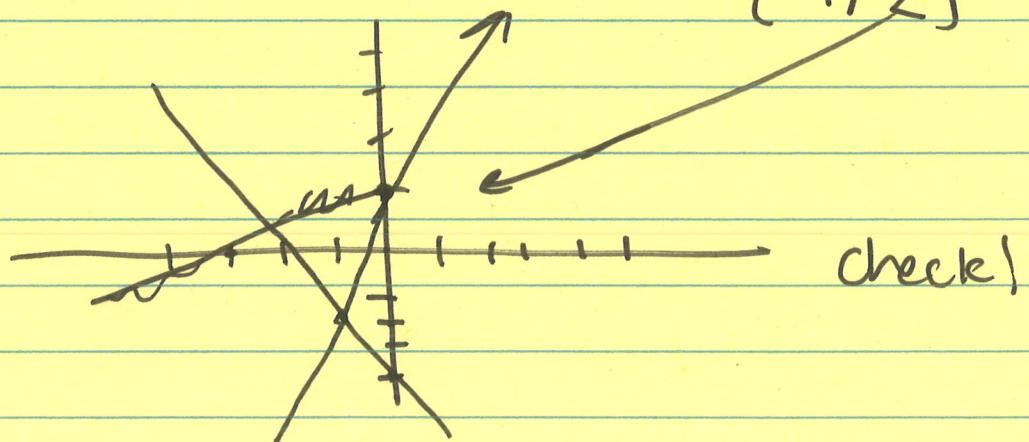
Graphing: The solution to a system of equations is their point of intersection



2a) ① Graph S-IF

$$\begin{cases} y = 1 + 3x \\ y = -4 - 2x \end{cases}$$

② Graph



Substitution

Ex : $\begin{cases} 3x+2y=16 \\ y=2x-5 \end{cases}$

$$\begin{aligned} y &= 2x - 5 \\ 3x + 2(2x-5) &= 16 \\ 3x + 4x - 10 &= 16 \\ 7x &= 26 \\ x &= \frac{26}{7} \end{aligned}$$

① Solve one equation for same variable

Choose
an easy
one to solve.

$$y = 2x - 5$$

$$y = \frac{3}{2}x - 5$$

Another option

② Substitute into equation not used in ①

~~$4x - 5y = 11$~~

~~$4x - 5\left(\frac{2}{3}x\right) = 11$~~

~~$4x - \frac{10}{3}x = 11$~~

~~$-\frac{2}{3}x = 11$~~

~~$x = -\frac{33}{2}$~~

$$2\left(\frac{2}{3}y\right) - 5y = 11$$

$$\frac{4}{3}y - 5y = 11$$

$$\frac{-11}{3}y = 11$$

$$y = \frac{11}{-33}$$

③ Solve for y when $x = \frac{2}{3}y$ with sol in ①

$$x = \frac{2}{3}y$$

$$x = \frac{2}{3}\left(-\frac{11}{3}\right) = -\frac{22}{9}$$

$$x = \frac{2}{3}(-3)$$

$$= \boxed{2}$$

$$y \neq 2x$$

$$x = \frac{2}{3}y$$

$$\frac{11}{3} = 2y$$

$$\Rightarrow$$

$$\frac{11}{3} = \frac{99}{54}$$

$$\frac{-99}{54} = y$$

$$-\frac{33}{5} + \frac{-99}{32}$$

4) write as point $(\frac{-33}{5}, \frac{-99}{32})$

$$\left(-\frac{33}{5}, -\frac{99}{32} \right)$$

$$\left(\frac{21}{5}, \frac{27}{8} \right)$$

$$\boxed{(-2, -3)}$$

Check!

Elimination:

$$\begin{cases} y - x = 3 \\ 3y + 6x = -9 \end{cases}$$

① Multiply equations by constants so that when added a variable is eliminated

$$\begin{cases} -3(y - x) = -3 \cdot 3 \\ -3y + 3x = -9 \\ 3y + 6x = -9 \end{cases}$$

Does not
change
System!
sol.

② Add equations

$$9x = -18$$

$$x = -2$$

③ Solve for remaining var. using ^{an} equation.

$$y + x = 3$$

$$y - 2 = 3$$

$$\underline{y = 1}$$

(-2, 1)

Check!