MATH/COSC 4340

Numerical Methods for Differential Equations

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Course Material Overview

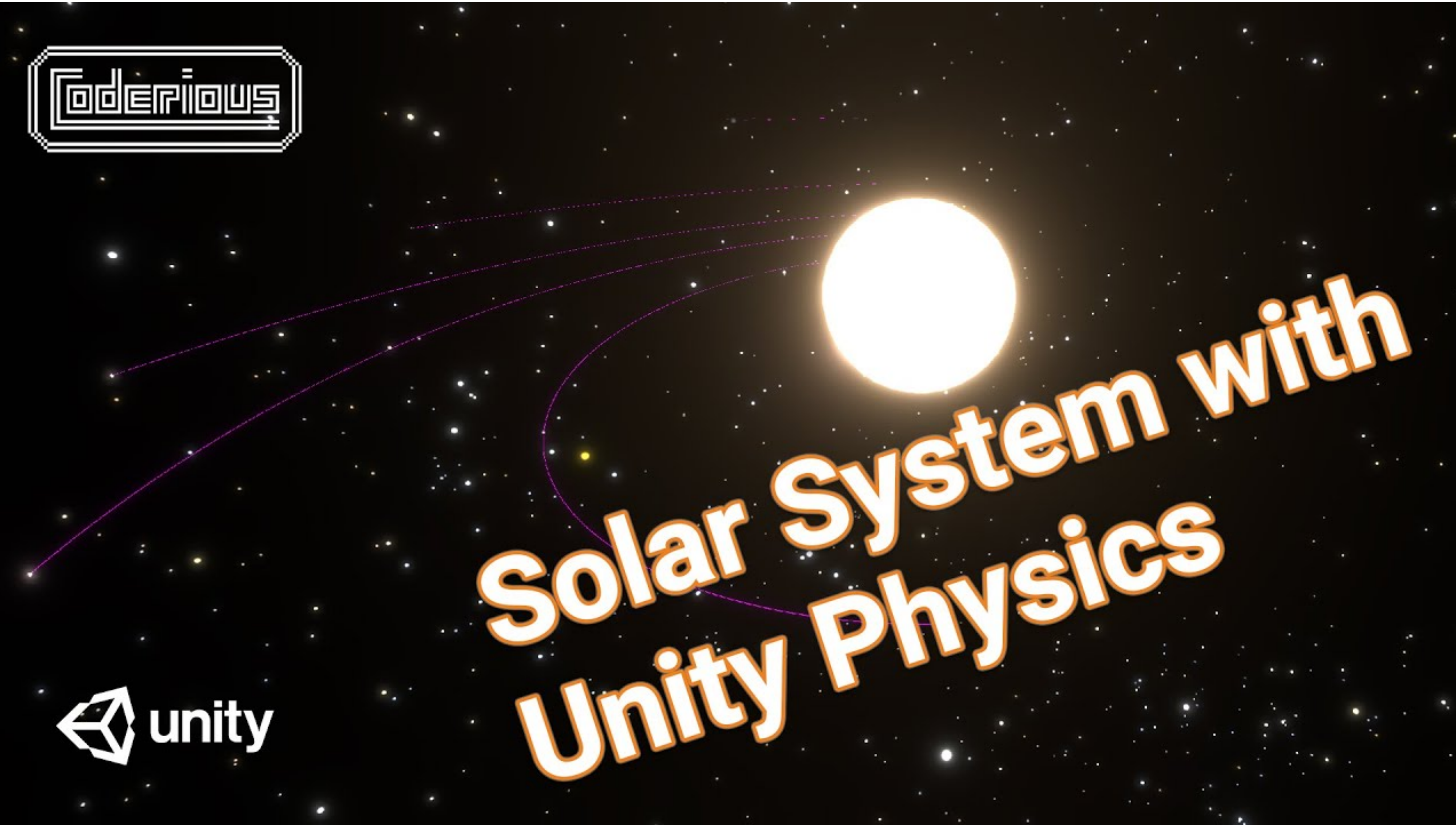
1. Ordinary Differential Equations (ODEs)
   solves for a scalar function f(t) using initial condition
   or a system, or set of equations: x(t), y(t), z(t)

2. Boundary Value Problems (BVPs)
   incorporates information about f at/along a boundary

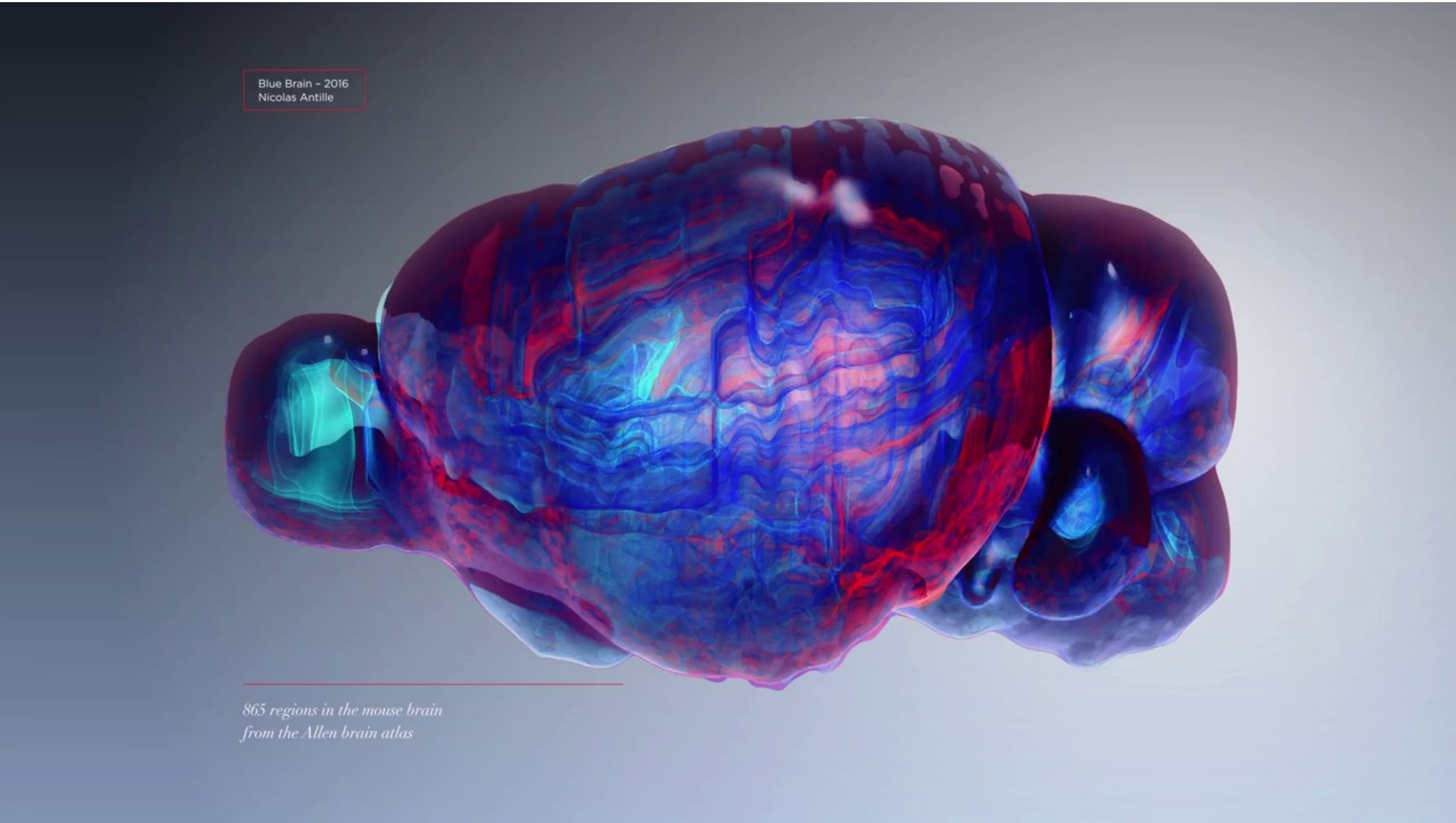
3. Numerical Linear Algebra (NLA)
   efficiently solves linear systems and computes spectral decompositions

4. Partial Ordinary Differential Equations (PDEs)
   solves a function defined over a D-dimensional space f(x,t) or f(x,y,z,t)

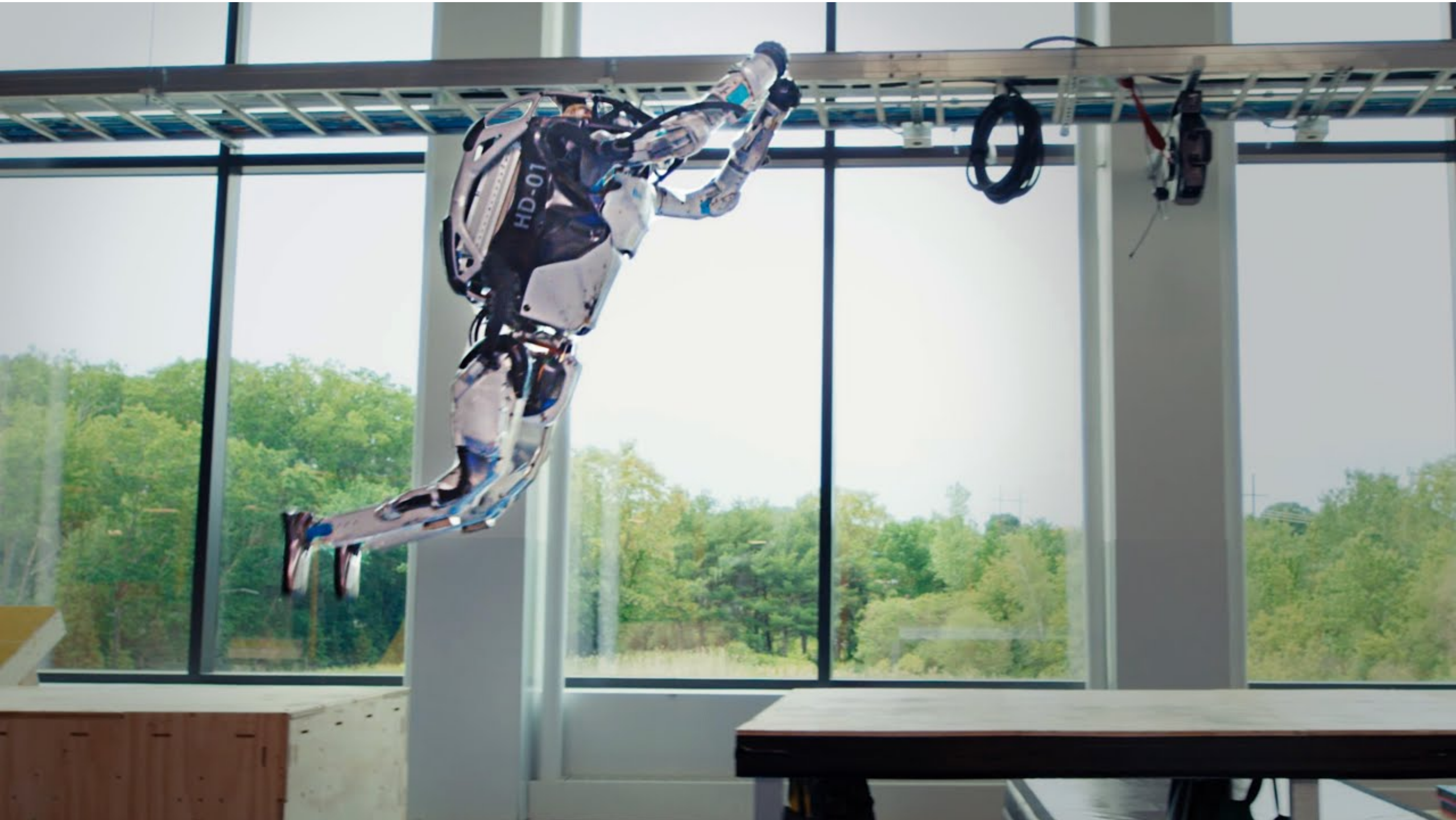
ODE Applications:
Predict Celestial Motion



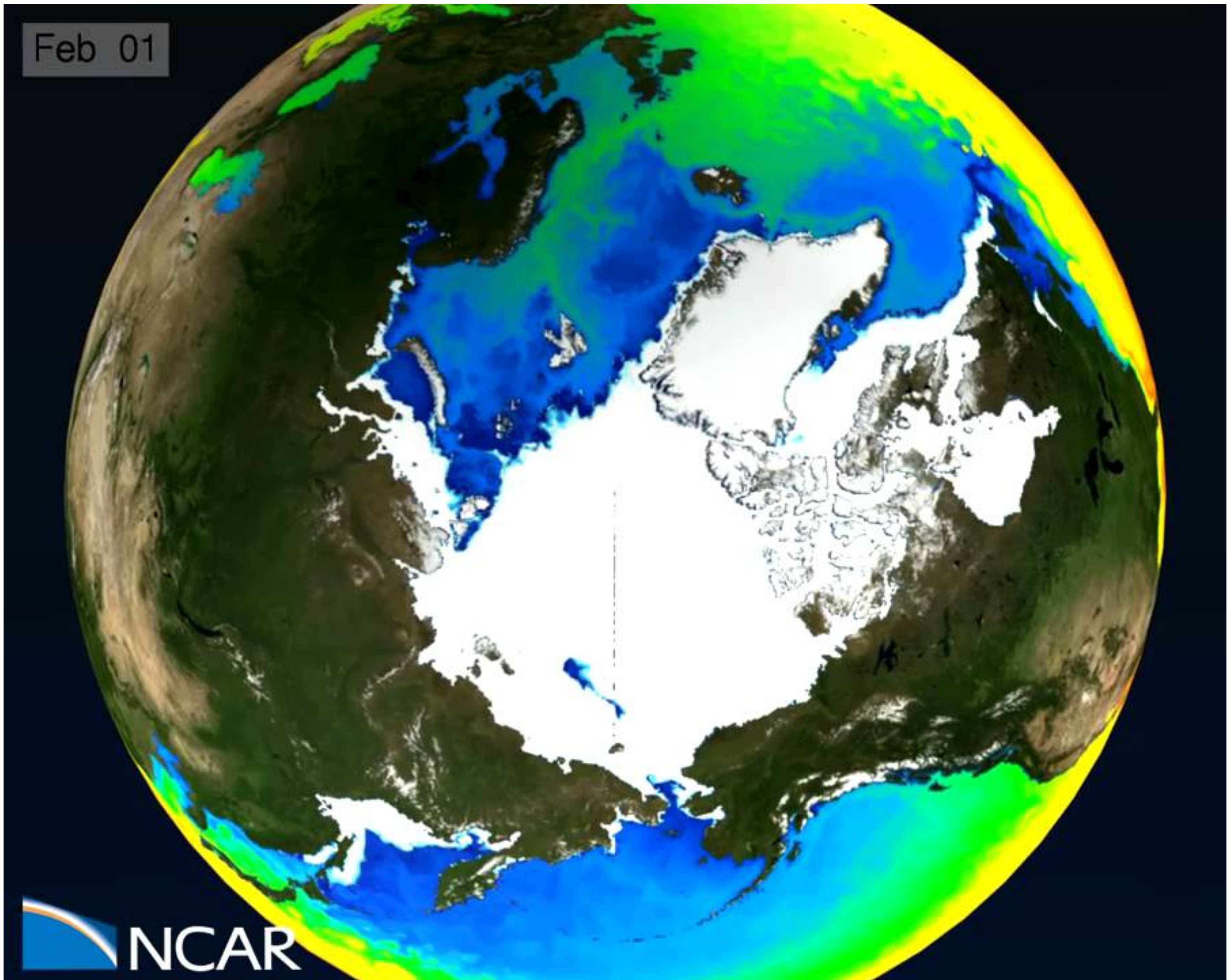
ODE Applications:
Simulate Brain Dynamics



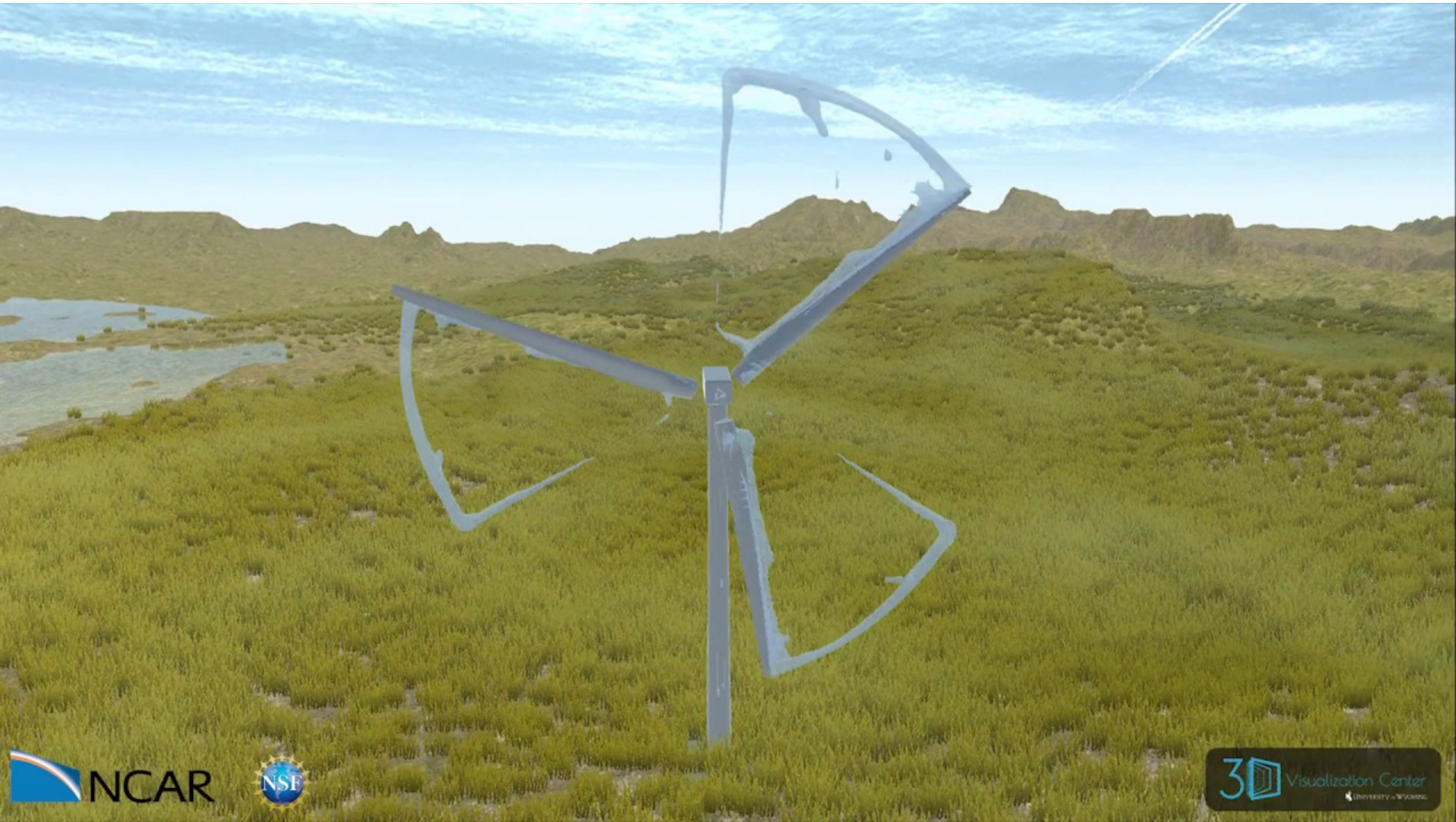
ODE Applications:
Train/Define Robot Motions



PDE Applications:
Climate Simulation and Prediction



PDE Applications:
Simulate and Optimize Wind Farms

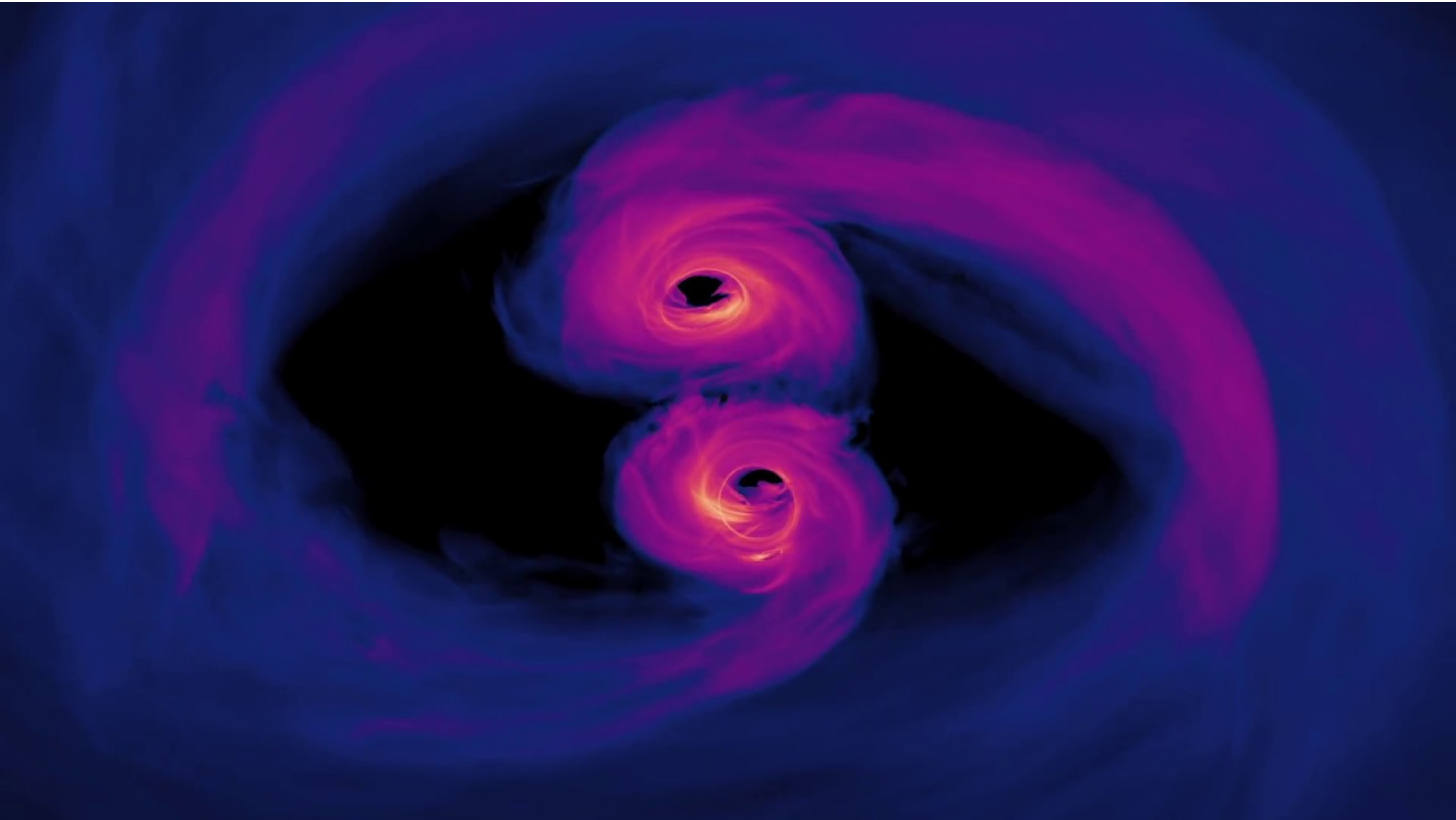


NCAR Supercomputing Facility in Cheyenne

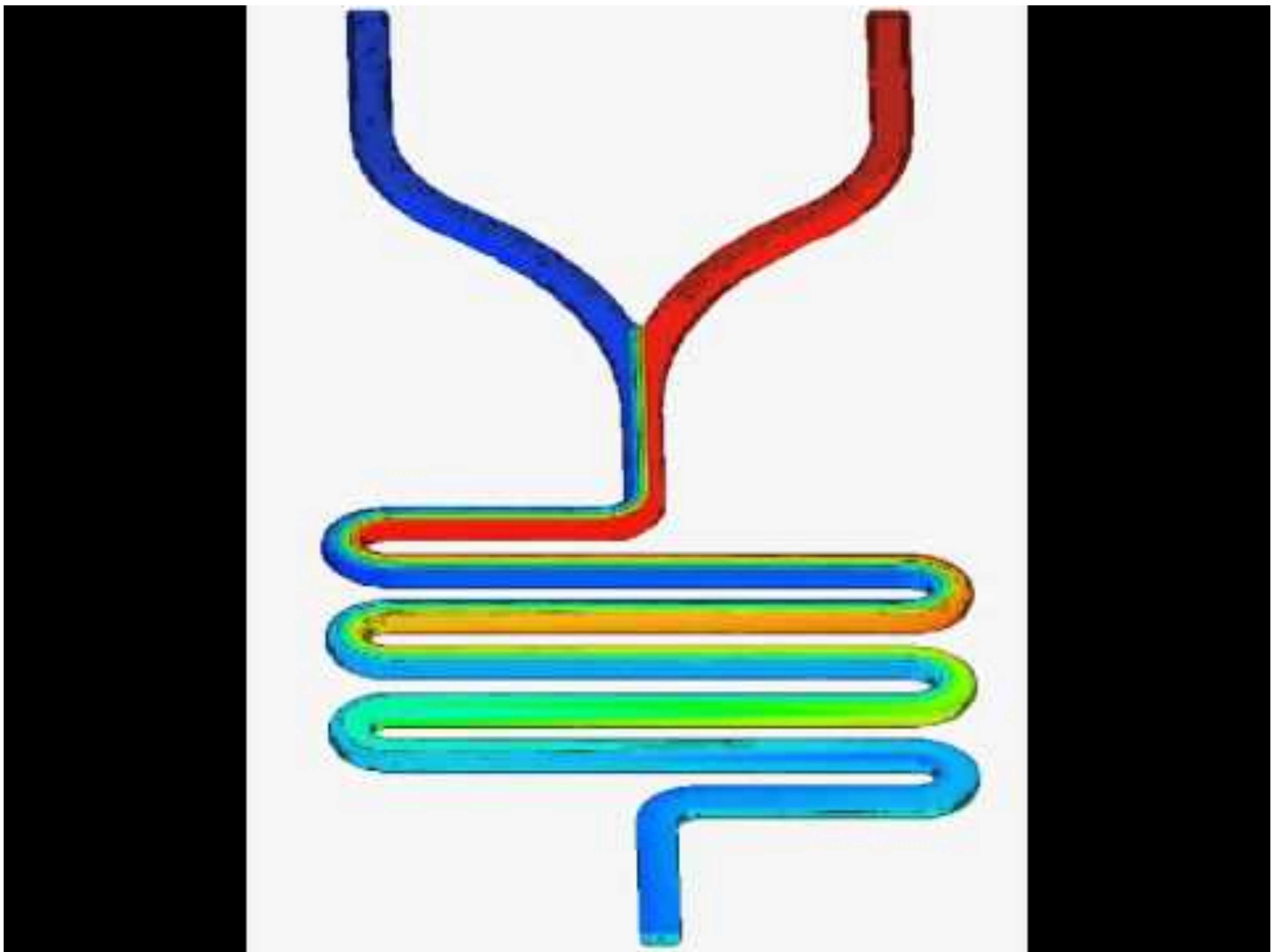
School of Computing Opportunities for Research

- Many researchers at UW are looking for research assistants with experience writing code in Python, including developing solutions to differential equations.
- Opportunities for paid summer internships and part-time positions during the semester.
- Email me if you are interested: dane.taylor@uwyo.edu

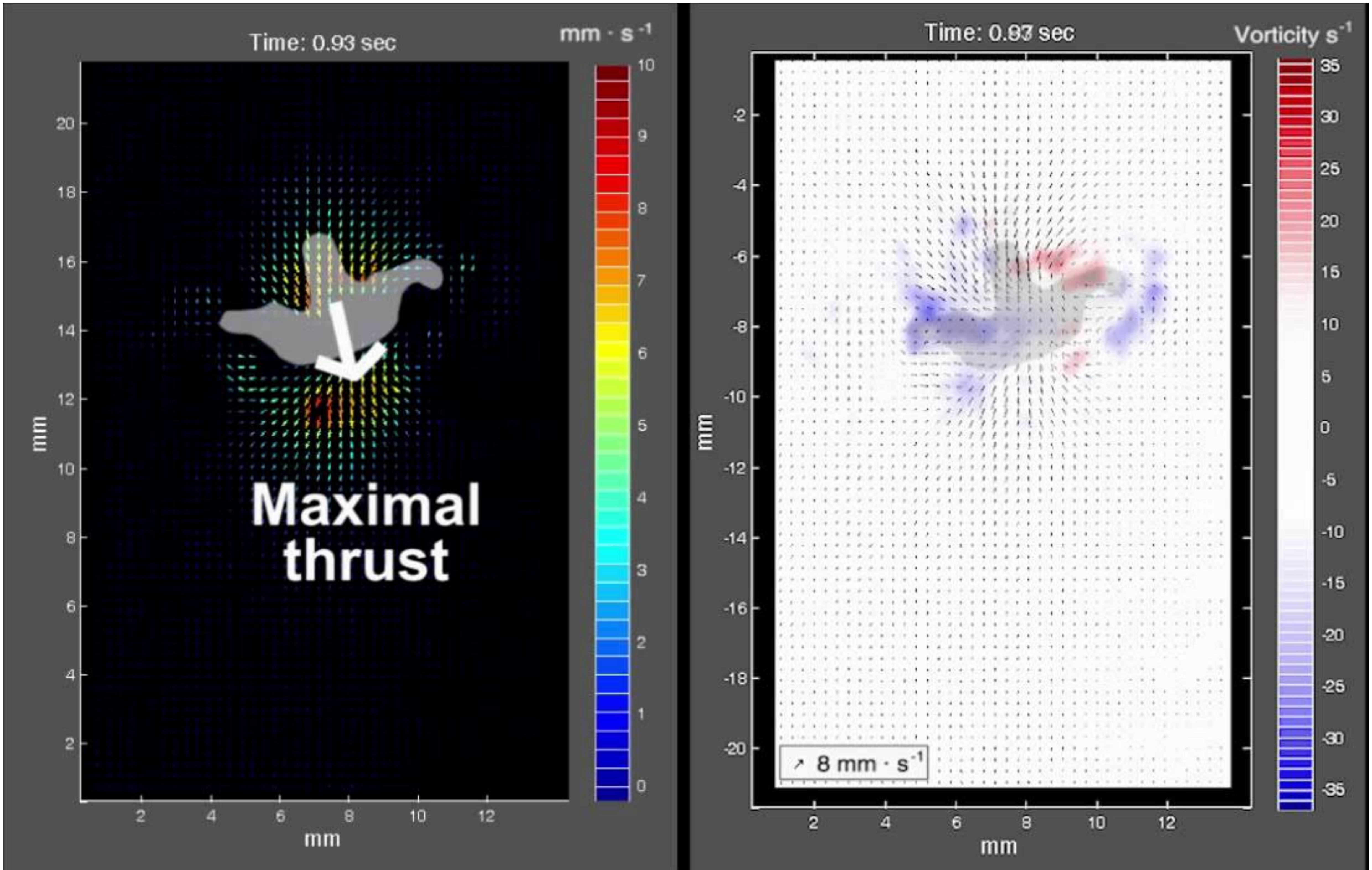
PDE Applications:
Simulate Black Holes



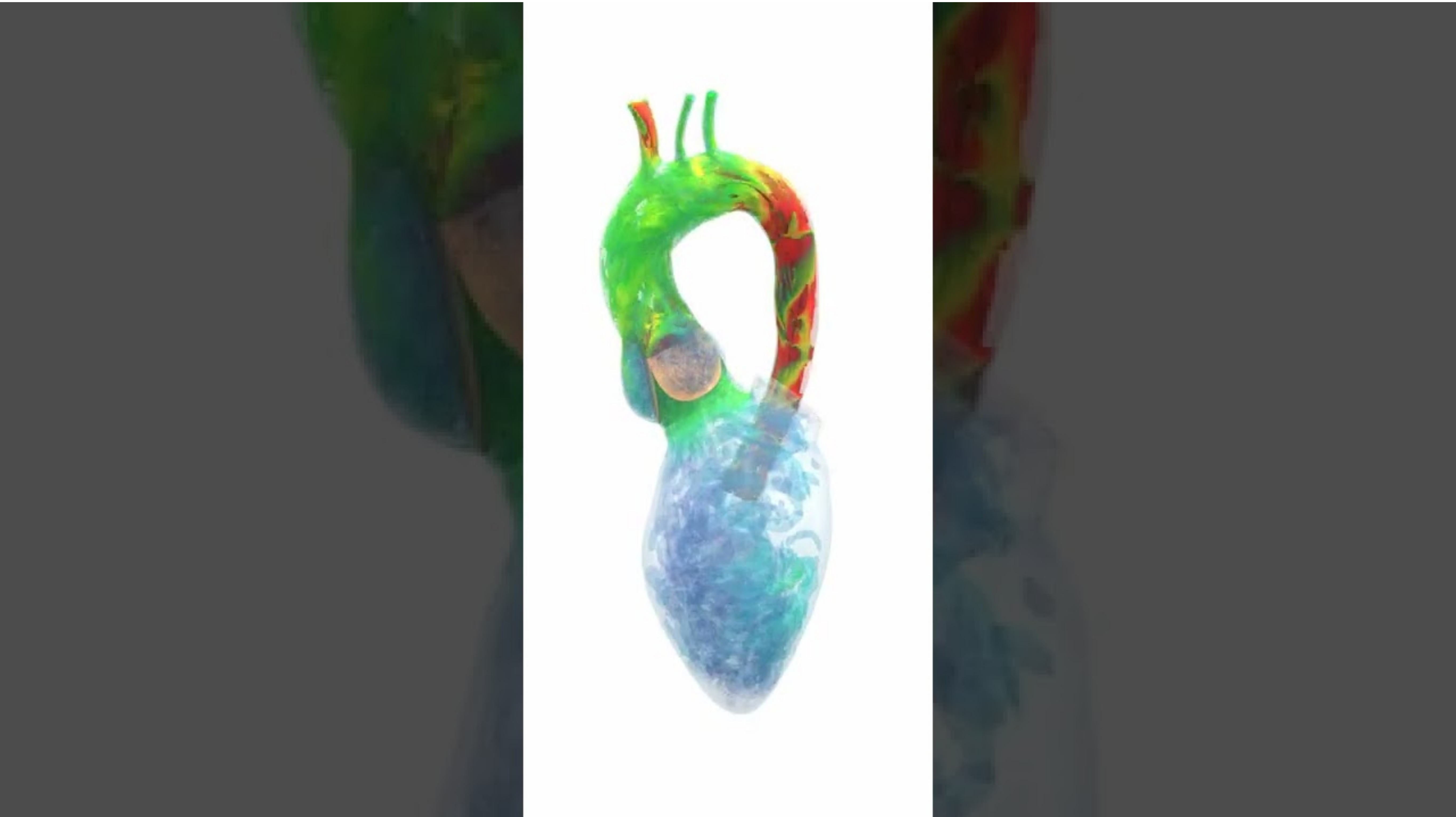
PDE Applications:
Simulate Mixing on a Microfluidic Lab



PDE Applications:
Simulate how Jellyfish Move



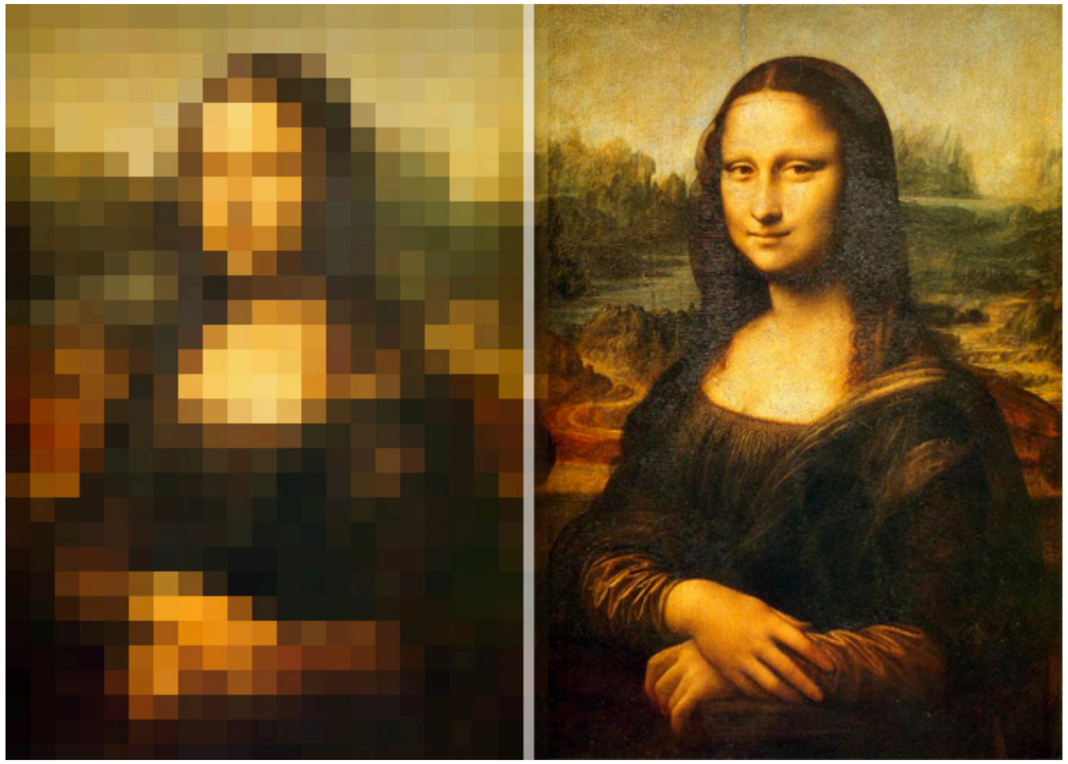
PDE Applications:
Simulate a Blood Flow in a Heart



Course Goals

1. Understand methods to simulate/solve ODEs and PDEs
2. Develop a strong mathematical understanding of error for these "numerical approximations"
3. Know how to balance approximation error and computational efficiency (accuracy vs. slow code)

high vs low resolution



approximation accuracy

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} \cdots = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

$$sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$$

$$cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$$

$$ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots = \sum_{n=0}^{\infty} \frac{(-1)^n x^{n+1}}{n+1}$$

Course Material Reisited

1. Ordinary Differential Equations (ODEs)
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3. Numerical Linear Algebra (NLA)
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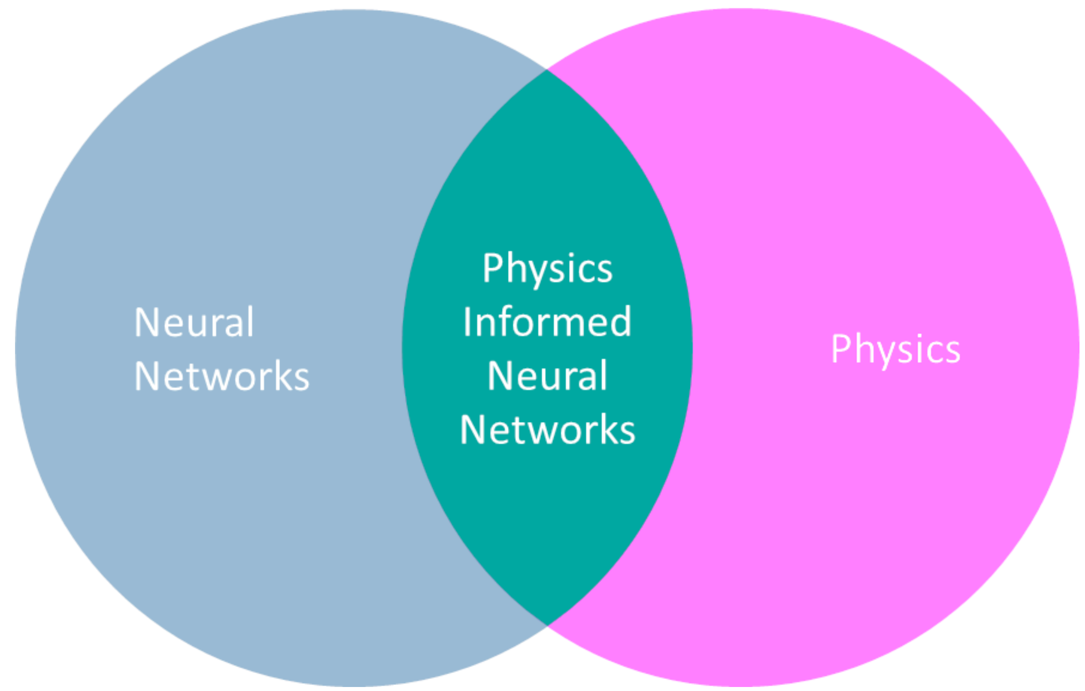
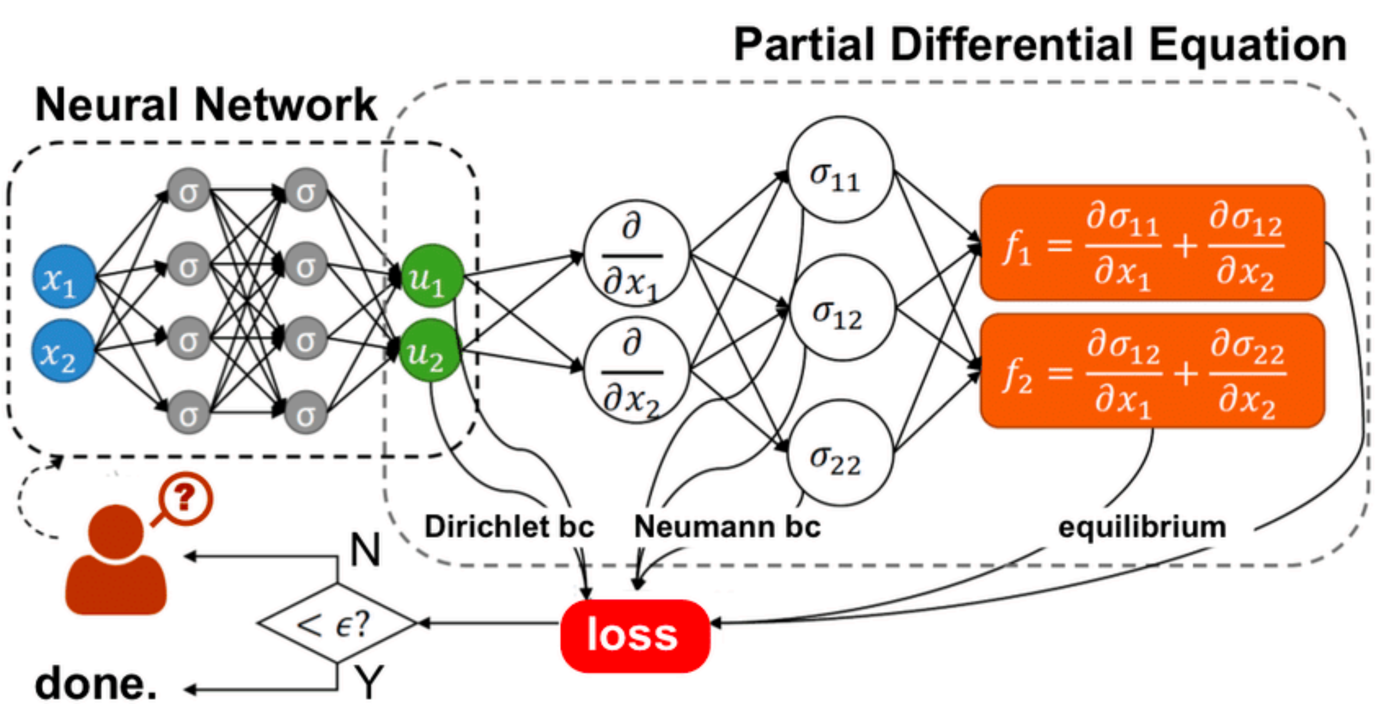
5. *time permitting:* Other topics may include
   random processes and stochastic ODEs
   data-driven modeling of ODEs/PDEs

Emerging Applications:
Learning Diff. Eqs. From Data

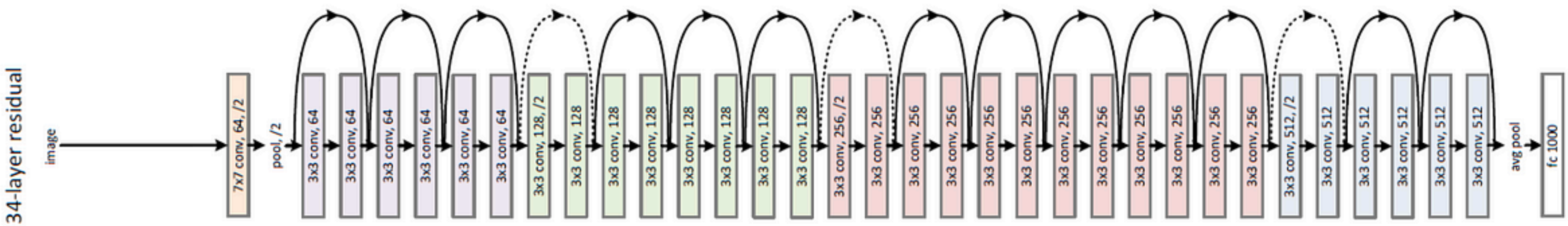
- What if we have measured data but we don't know the differential equation?





Emerging Applications: *Neural ODEs and PDEs*

We can design neural networks to “look like” numerical solutions to differential equations



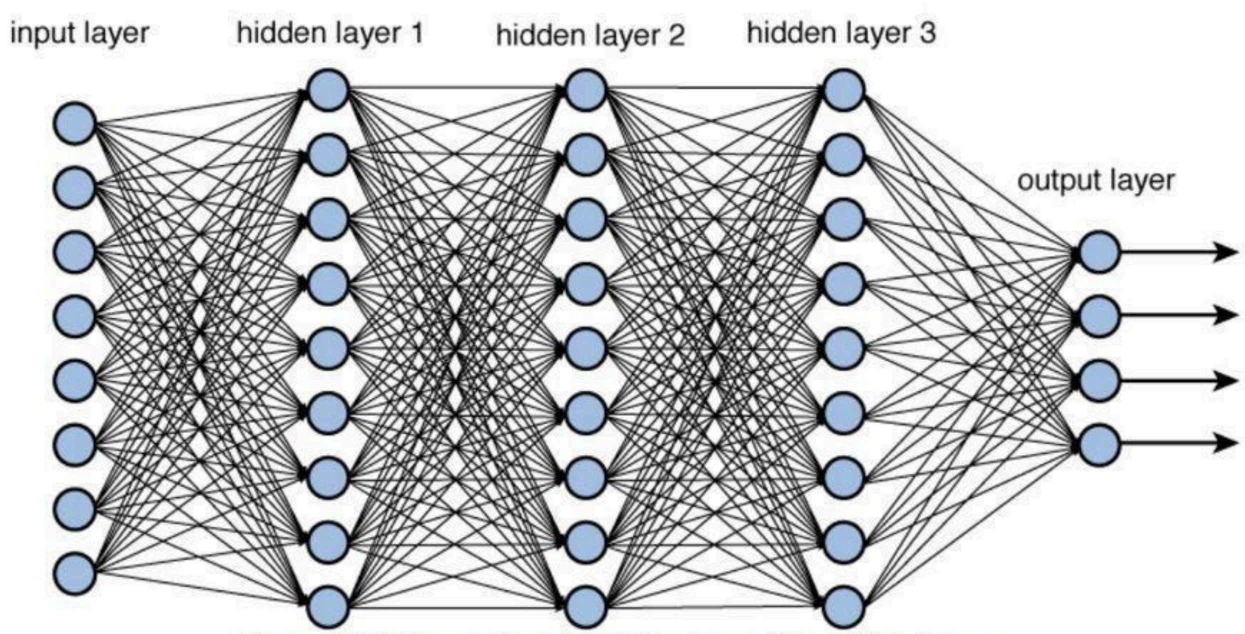
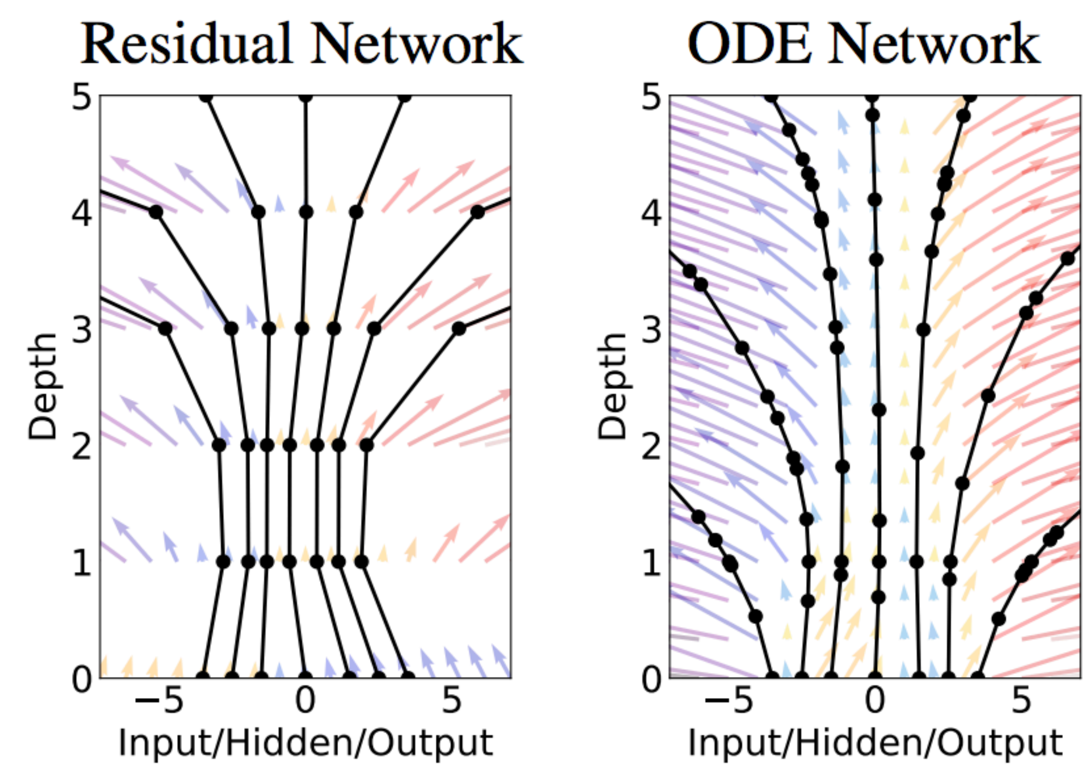


Figure 12.2 Deep network architecture with multiple layers.



Applications of ODEs

ODEs

celestial motion - https://www.youtube.com/watch?v=kUXskc76ud8
Neuroscience models - https://www.youtube.com/watch?v=bo1cYjcjV-Y&ab_channel=BlueBrainProject-EPFL
AI algorithms
robots - https://www.youtube.com/watch?v=WOwLquiFbPE&ab_channel=AntoninRaffin

PDEs

climate simulation - https://www.youtube.com/watch?v=f-mp20H2nVo&ab_channel=NCARVisLab
NCAR research - https://www.youtube.com/c/NCARVisLab
NCAR derecho - https://arc.ucar.edu/knowledge_base/74317833

Fluid dynamics

wind farm - https://www.youtube.com/watch?v=NHNL5JRQnHQ

heart valve - https://www.youtube.com/watch?v=I9NlTV72InQ&ab_channel=RiceUniversity
https://www.youtube.com/watch?v=to9mhtxOff8&ab_channel=MiguelA.Fernandez

microfluidics - https://www.youtube.com/watch?v=ZCMr1tHnGRs&ab_channel=FLOW-3D

jelly fish - https://www.youtube.com/watch?v=oPTPHDe0EPM

black hole - https://www.youtube.com/watch?v=i2u-7LMhwvE&ab_channel=NASAGoddard