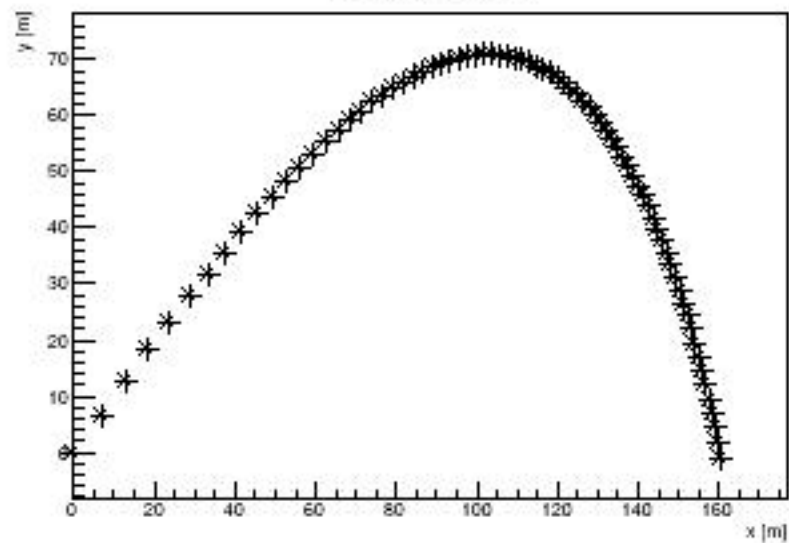
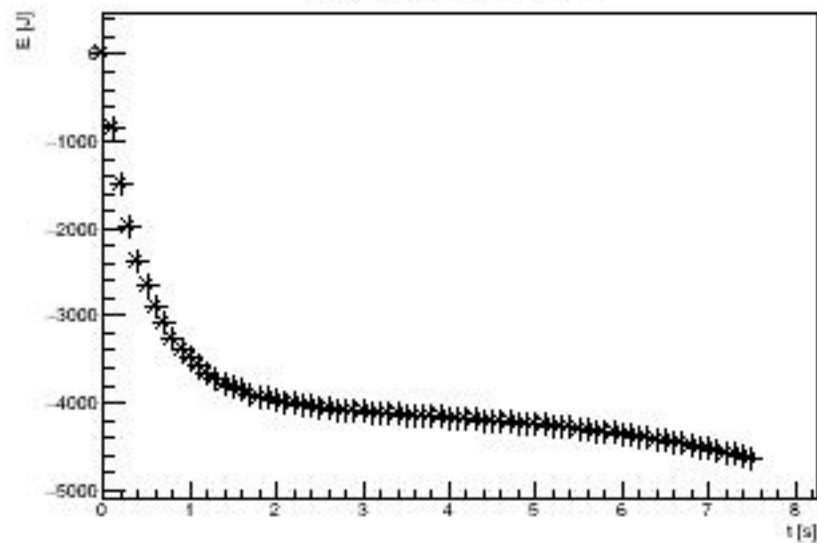


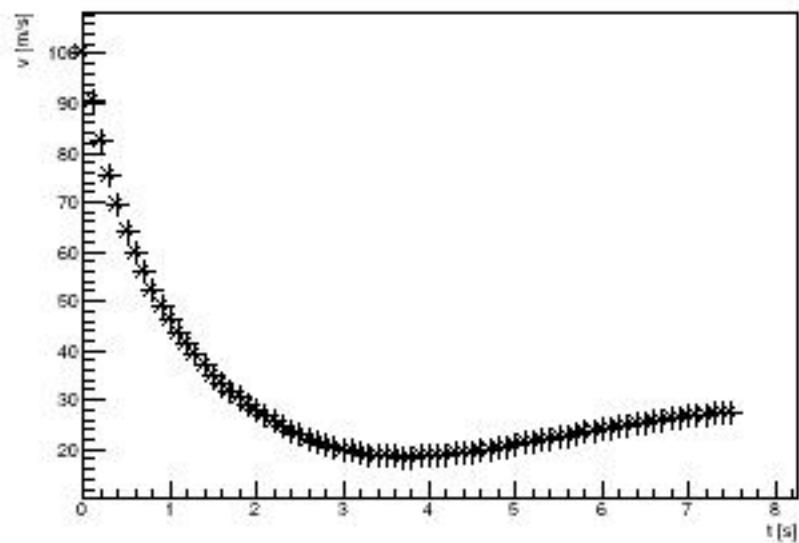
Projectile y vs x



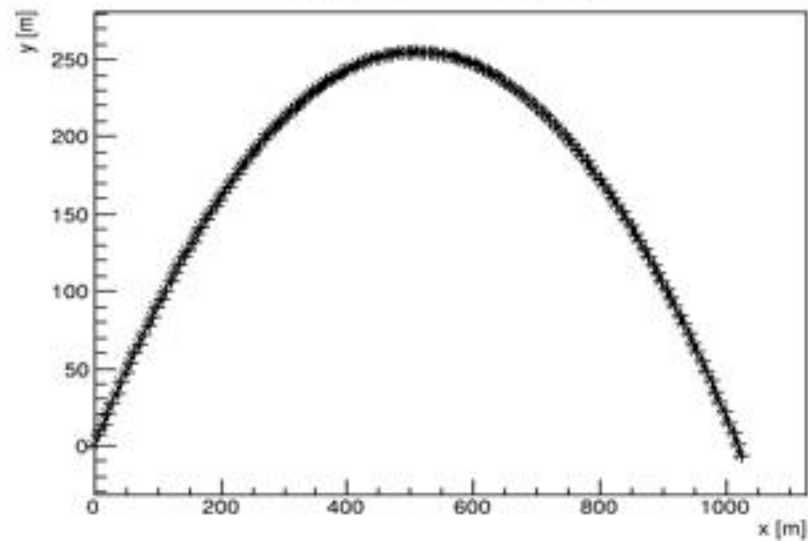
Projectile $E_{\text{tot}} - E_0$ vs t



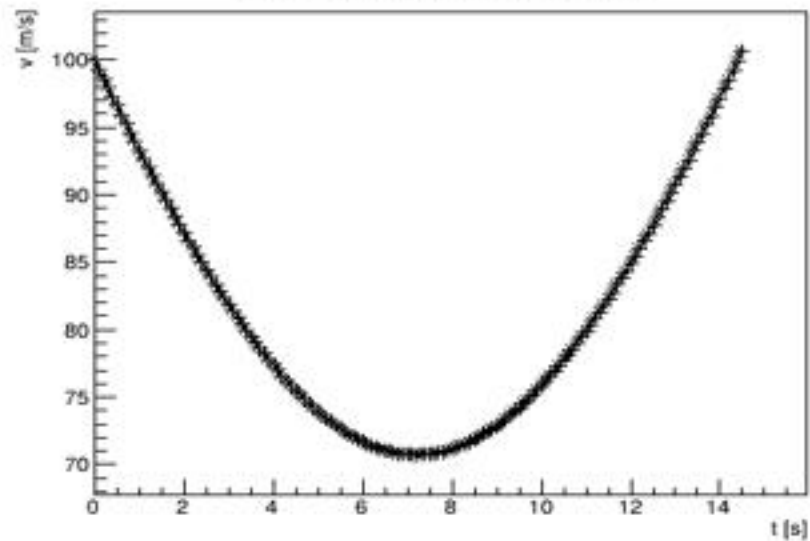
Projectile velocity vs t



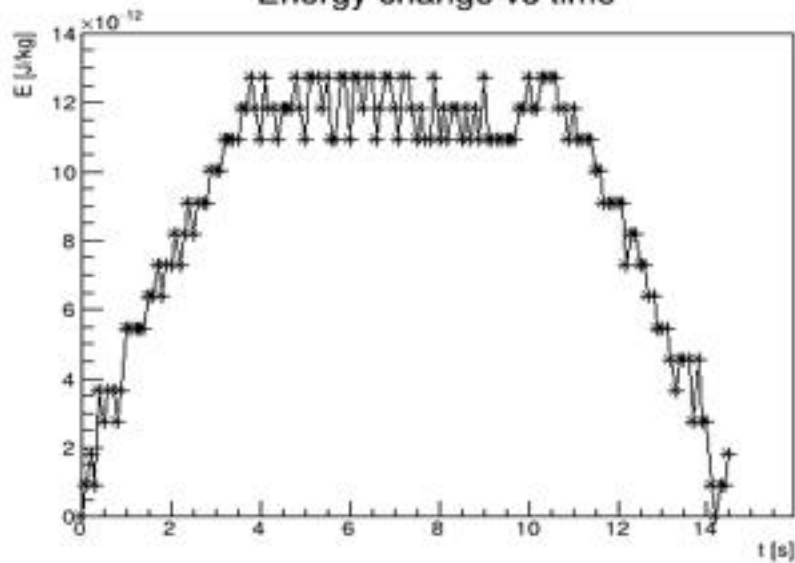
Projectile Trajectory



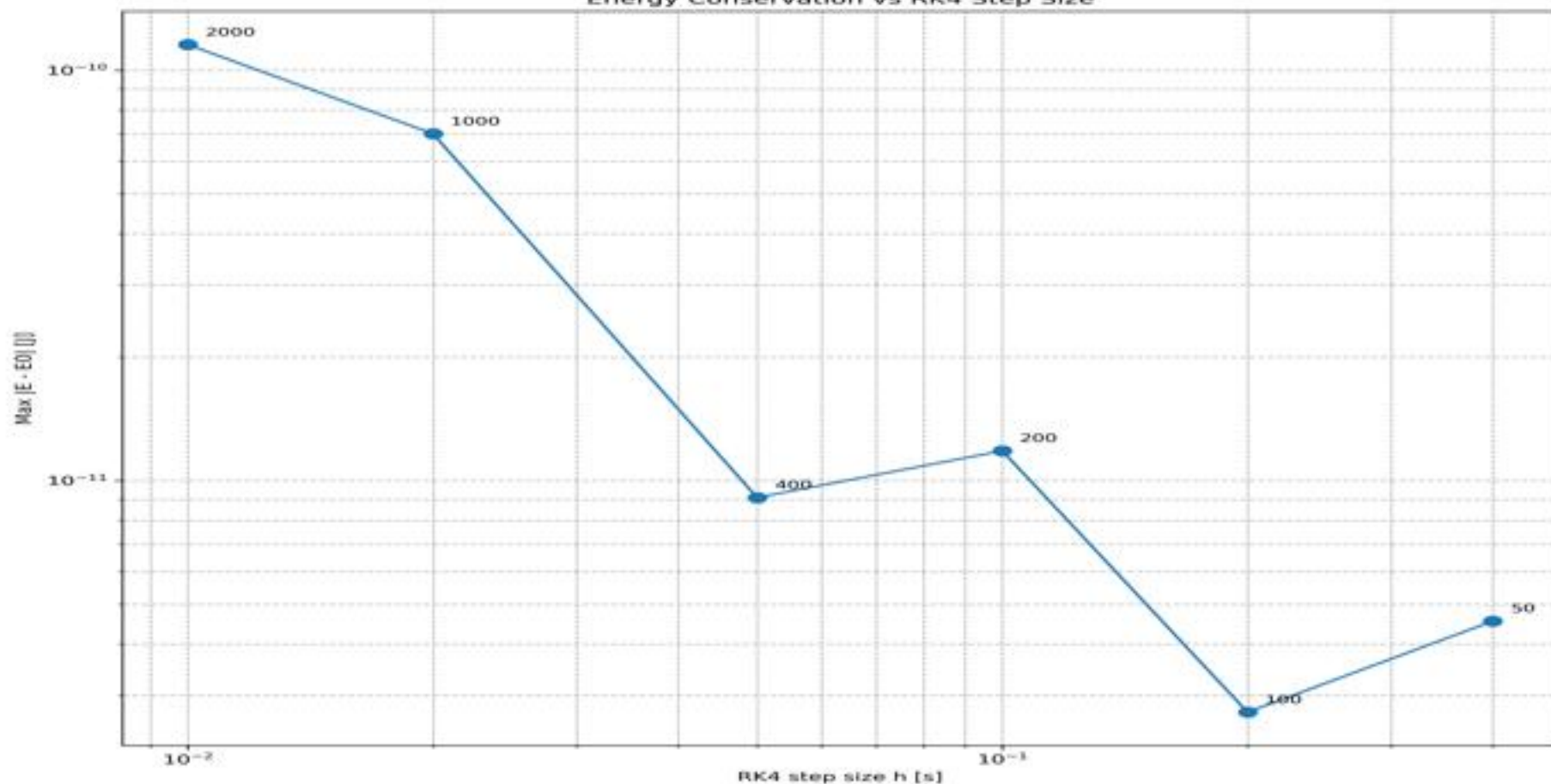
Velocity magnitude vs time



Energy change vs time



Energy Conservation vs RK4 Step Size



--- Energy Conservation Analysis ---

Step sizes ranged from 0.0100s to 0.4000s

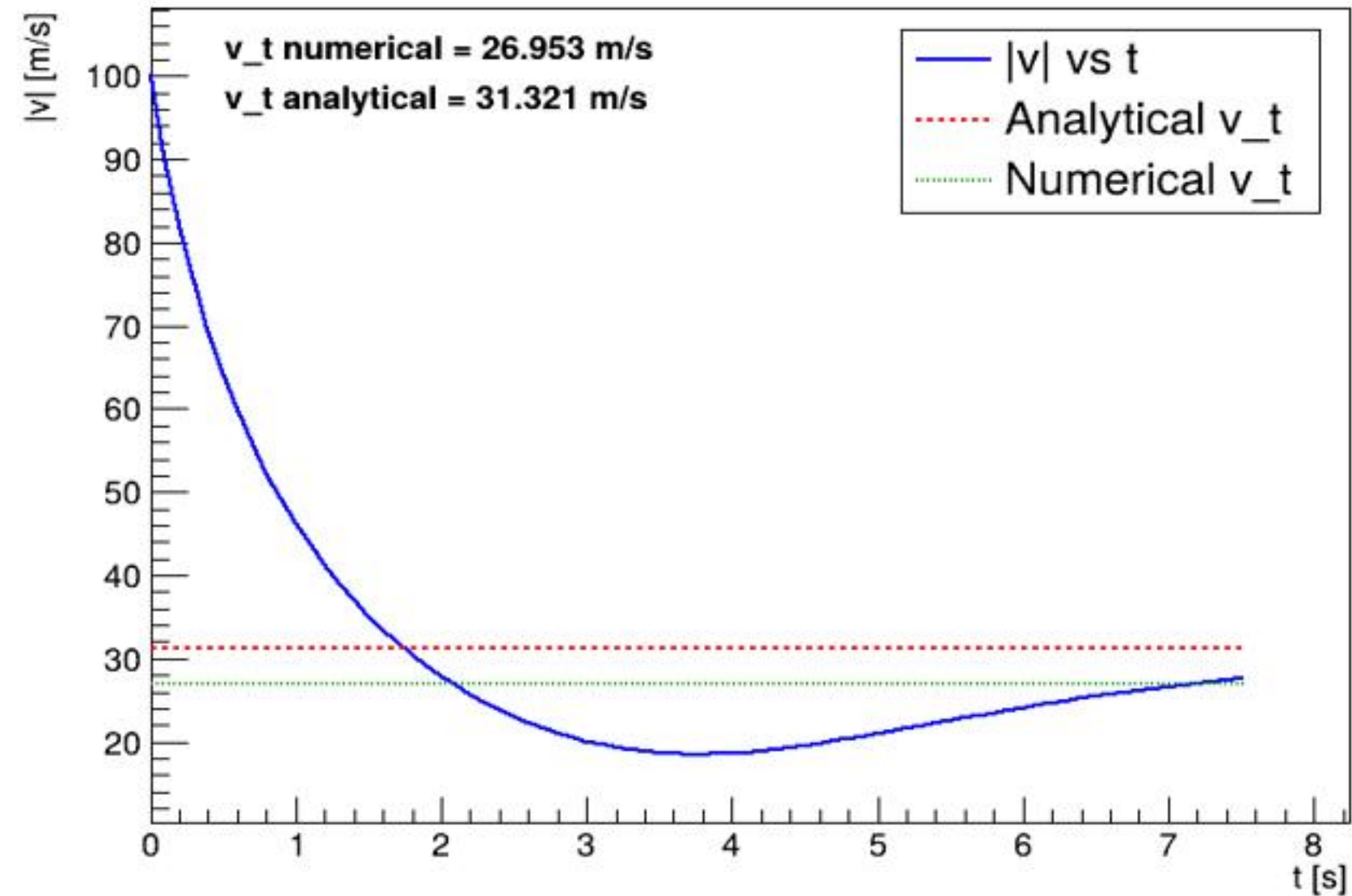
Maximum energy deviation with largest step: 0.000000 J

Maximum energy deviation with smallest step: 0.000000 J

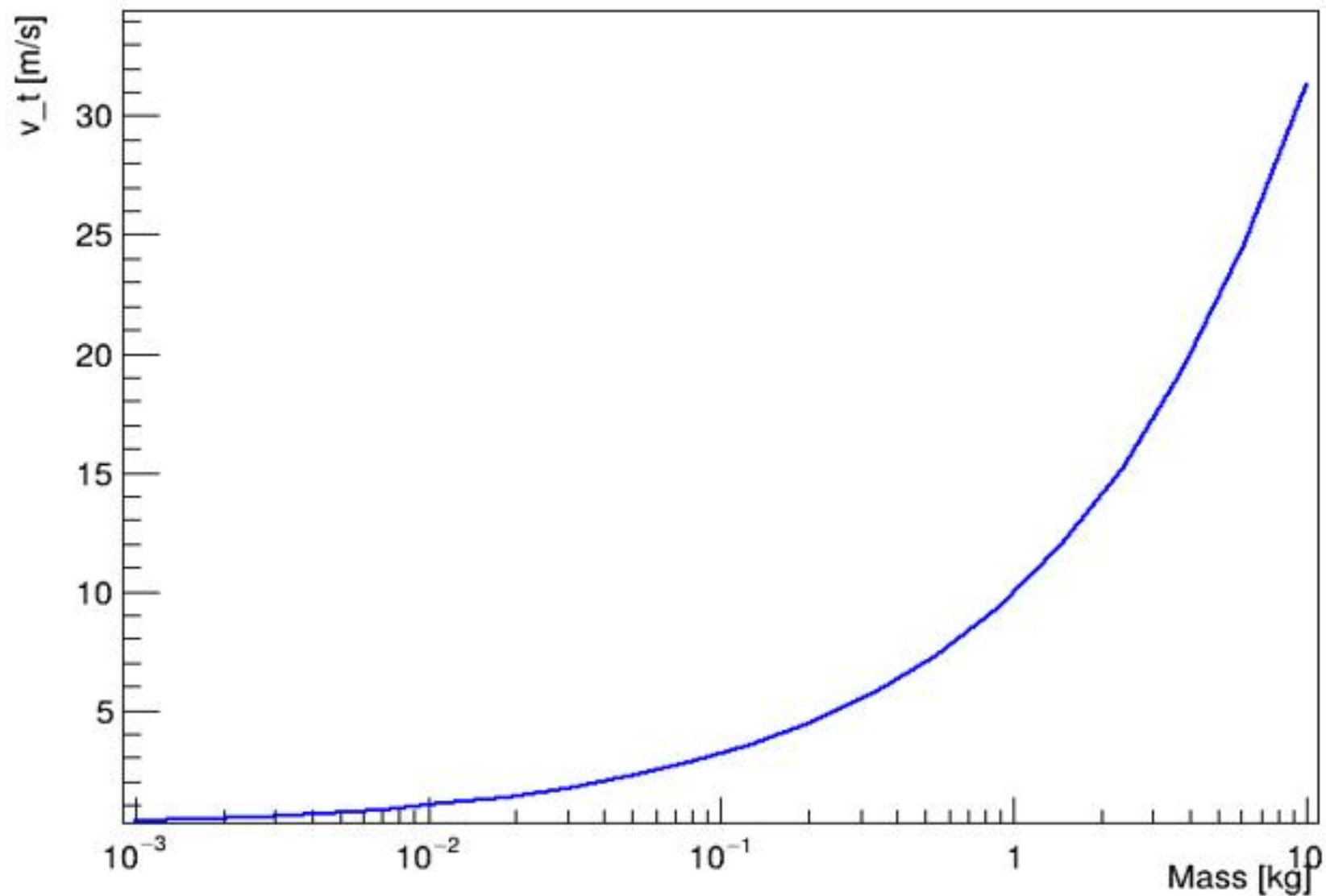
Observation:

- Energy is very well conserved for small step sizes.
- Larger step sizes produce larger numerical deviations.
- Reducing RK4 step size improves accuracy, consistent with 4th-order convergence.

Projectile speed $|v|$ vs time



Analytical Terminal Velocity vs Mass



Evaluation of the Accuracy of Projectile Motion with Air Resistance

1. Terminal Velocity Check:

- Numerical velocities asymptotically approach analytical $v_t = \sqrt{m \cdot g / k}$.

2. Step Size Convergence:

- Decreasing RK4 step size leads to consistent results.
- Confirms solver stability and 4th-order convergence.

3. Energy Considerations:

- Total mechanical energy decreases smoothly due to drag.
- No unphysical jumps or increases.

4. Directionality:

- Drag opposes motion as expected.
- Velocities reduce toward terminal value along trajectory.

5. Limiting Cases:

- No air resistance reproduces standard parabolic trajectory.
- Extreme mass limits reproduce correct scaling: $v_t \sim \sqrt{m}$.

Conclusion:

- Numerical solutions with air resistance behave as physically expected.
- Convergence, smooth energy loss, correct limiting cases, and terminal velocity agreement p