ECONOMICS 150: Quantitative Methods for Economics
Class 6

- Catch up with Class 5 unfinished material
- What Have We Learned So Far
- Practice Problems for the Test

Summary Statistics for Random Variables

Types of summary measures for random variables

- Measures of Location: **Expected Value**
- Measures of Dispersion, or Spread: **Variance** and **Standard Deviation**

- They use information about the probability distribution of the random variable

Expected Value of a Random Variable

$$E(X) = \mu_X = \sum_{\substack{\text{all possible}\\ \text{values of } x}} x \times P(X = x)$$

- Multiply each value by its probability, and then sum everything up
  - This is a "smart" average in that each outcome is weighted by its likelihood

- A measure of the <u>center</u> of the distribution

  (weighted average of the outcomes where the weights are given by the probabilities)

- The "expected value" is sometimes called the "mean"

- Note: The **<u>expected value</u>** does not need to be a possible outcome.

Example of Expected Value

Example:

X can have 5 different \$ values:

X=1

X=2

X=3

X=4

X=5

the probabilities are:

P(X=1)=0.12

P(X=2)=0.4

P(X=3)=0.35

P(X=4)=0.03

P(X=5)=0.1

$$E(X) = \mu_X = \sum_{\substack{\text{all possible}\\ \text{values of } x}} x \times P(X = x)$$

The expected value of X is:

$$\mu_X = 1 \times (0.12) + 2 \times (0.4) + 3 \times (0.35)$$
$$+ \cancel{3}\,4 \times (0.03) + 5 \times (0.1)$$

$$\mu_X = 2.59$$

Variance of a Random Variable

$$Var(X) = \sigma_X^2 = \sum_{all\ possible\ x} (x - \mu_X)^2 \times P(X = x)$$

- Multiply the square of the difference between each value and the expected value by its probability, and then sum everything up
  - Same as the expected value of X's squared deviation from its own expected value: $\sigma_X^2 = E[(X - \mu_X)^2]$

- This is a measure of the spread of the distribution (relative to the expected value)

  (Finance: measure of risk)

- There is an alternative formula for computing variance:

$$\sigma_X^2 = E(X^2) - [E(X)]^2 = \mu_{X^2} - (\mu_X)^2$$

Example of Variance

Example:

X can have 5 different \$ values:

X=1

X=2

X=3

X=4

X=5

the probabilities are:

P(X=1)=0.12

P(X=2)=0.4

P(X=3)=0.35

P(X=4)=0.03

P(X=5)=0.1

$$E(X) = \mu_X = \sum_{\substack{all\ possible\\ values\ of\ x}} (x - \mu_X)^2 \times P(X = x)$$

The variance of X is:

$$\sigma_X^2 = (1 - 2.59)^2 \times (0.12) + (2 - 2.59)^2 \times (0.4) + (3 - 2.59)^2 \times (0.35) + (4 - 2.59)^2 \times (0.03) + (5 - 2.59)^2 \times (0.1)$$

$$\sigma_X^2 = 1.14$$

What are the units of the variance?

Standard Deviation of a Random Variable

$$StdDev(x) = \sigma_X = \sqrt{\sigma_X^2} = \sqrt{Var(X)}$$

- A measure of the spread of the distribution in the **original units**

Example:

- X measured in \$
- μ_X measured in \$
- σ_X^2 measured in $\$^2$
- σ_X measured in \$

Example: a bet with a rich person

"We'll toss a coin once: If it is heads, you get \$10 million.
If it is tails, you'll have to pay me \$1 million"

What are expected value, variance, and standard deviation of the consequences of this coin toss on your wealth?

$P(X=10)=\frac{1}{2}$

$P(X=-1)=\frac{1}{2}$

Let X denote the change in your wealth (in \$millions) after the coin toss.

$X=10$ $X=-1$

- The expected value of X:

$$\mu_X = ½(10) + ½(-1) = 4.5.$$

- However, you are not guaranteed \$4.5 million.
  - 50% chance to get \$10 million $\Rightarrow$ \$5.5 million more than expected
  - 50% chance to lose \$1 million $\Rightarrow$ \$5.5 million less than expected

$$\sigma_X^2 = (-1-4.5)^2 P(X=-1) + (10-4.5)^2 P(X=10)$$
$$= (-5.5)^2(.5) + (5.5)^2(.5) = 30.25 \ (\$million)^2$$

- Thus, $\sigma_X = 5.5 \ (\$million)$

Top Hat Attendance Code for Today

5142

REMINDER: Attendance is based on code + location

Location tracking must be enabled on your device for you to be correctly marked present

Announcement

In-Class Test 1 is **next class**

- Covers everything so far (Classes 1 to 6, Practice Problem Sets 1, 2, and 3):
  - Basic probability: probability, conditional probability, independence, trees, tables, …
  - Random variables: distributions, expected value, standard deviation, variance, …
- A sample test 1 (with solutions) is available on Canvas
  - The actual test will look very similar to the sample test (maybe a touch easier)
  - Take it in "test mode"
    - Give yourself only 30 minutes to work on it since the time you read it
    - Make sure you do not discuss it with anyone beforehand
  - I will post the answer key by the weekend
- Monday's review session will go over the Sample Test in detail (**4pm in WWPH 5201**)
  - TA office hours: Thursday 2-3pm and Friday noon-1pm in WWPH 4515

Important test norms: 30 minutes, 20 points

- start right at the beginning of class (be here a few minutes early)
- open notes and any other printed material you may want to bring
- bring your own **calculator** (smartphones in airplane mode are equivalent to calculators)
- can use computer/tablet in airplane mode to look at handouts, but no typing is allowed
- no communication allowed

Summary of topics so far

- Summary Statistics:
  - Mean, Median, Mode, Standard Deviation, Variance
- Basic Probability:
  - Union, Intersection, Mutually Exclusive Events, Collectively Exhaustive Events
- Conditional Probability
  - Conditional Probability Formula, Independence
- Tools for Problems
  - Probability Tables, Probability Trees
- Random Variables
  - Expected Value, Variance, and Standard Deviation

Now let's do some problems!

Coffee Problem: PPS 1, #2a

Alex and Riley meet at Freshbrew for a drink every morning. Alex orders coffee 85% of the time, they both order coffee 80% of the time. Rarely, 5% of the time, neither orders coffee.

a. What is the probability that at least one of them orders coffee?

SUMMARIZE INFO

Start by describing the relevant events and the information:

A is the event Alex orders coffee **R** is the event that Riley orders coffee.

We know that:

$$P(\mathbf{A})=0.85 \qquad P(\mathbf{A}\cap\mathbf{R})=0.8 \qquad P(\mathbf{A}^C\cap\mathbf{R}^C)=0.05$$

or, using a table,

	R	**R^C**	
A	0.8		0.85
A^C		0.05	
			1

We are looking for $P(\mathbf{A}\cup\mathbf{R})$

$$P(\mathbf{A}\cup\mathbf{R})=1-P(\mathbf{A}^C\cap\mathbf{R}^C)=1-0.05=0.95$$

Coffee Problem: PPS 1, #2b

b. What is the probability that Riley orders coffee?

	R	R^C	
A	0.8		0.85
A^C		0.05	
			1

	R	R^C	
A	0.8		0.85
A^C	0.1	0.05	0.15
			1

	R	R^C	
A	0.8		0.85
A^C	0.1	0.05	0.15
	0.9		1

We need to find P(**R**)

From the probability table: $P(\mathbf{A^C} \cap \mathbf{R}) = 0.1$

Thus: $P(\mathbf{R}) = P(\mathbf{A} \cap \mathbf{R}) + P(\mathbf{A^C} \cap \mathbf{R}) = 0.8 + 0.1 = 0.9$

Stereo Equipment: PPS2, #2a

The owner of a stereo equipment has found that once a person enters the store and asks about stereo equipment, there is a .15 probability that a receiver will be purchased, a .10 probability that speakers will be purchased, and a .20 probability that either speakers or a receiver or both will be purchased. If a customer purchases speakers, what is the probability that a receiver will not be purchased?

Start by describing the relevant events and the information:

R is the event buys a receiver **S** is the event buys speakers

We know that: P(**R**)=0.15 P(**S**)=0.10 P(**R**∪**S**)=0.20

or

	S	S^C	
R			0.15
R^C	0.05	0.80	0.85
	0.10		1

We are looking for $P(R^C|S) = \frac{P(R^C \cap S)}{P(S)}$

From last week class, we know $P(R^C \cap S) = 0.05$

Thus: $P(R^C|S) = \frac{0.05}{0.10} = \frac{1}{2}$

Oil Wildcatter: PPS 3 #4 (first part)

An oil wildcatter owns drilling rights at two widely separated locations. After consulting a geologist, he feels that at each location the odds against discovering oil if a well is drilled are 9 to 1. A well costs $100,000 to drill, and this is a total loss if no oil is found. On the other hand, if oil is discovered, rights to the oil can be sold for $1,600,000. The wildcatter has $100,000 available for drilling expenses.
Find the expected value and standard deviation of the wildcatter's profit.

X = profit = revenue − cost

$$X = \begin{cases} -1 & \text{if no oil} \\ 16-1 & \text{if oil} \end{cases} \qquad \begin{matrix} P(X=-1) = & 9 \cdot P \\ P(X=15) = & P \\ & \overline{\quad 1 \quad} \end{matrix}$$

$9P + P = 1$

Oil Wildcatter: PPS 3 #4a

Odds against discovering oil if a well is drilled are 9 to 1.
A well costs $100,000 to drill.
If oil is discovered, rights can be sold for $1,600,000.
The wildcatter has $100,000 available for drilling expenses.
Find the expected value and standard deviation of the wildcatter's profit.

If no oil, profit equal -1. If oil, profit equals 16-1 = 15 in 100,000$

Probability of no oil is nine times the probability of oil:

If p is the probability of oil, this means $9p + p = 1$ (probabilities must sum to 1)

Expected profit $= (-1) \cdot 0.9 + (15) \cdot 0.1 = 0.6M$

Variance: $= (-1 - 0.6)^2 \cdot 0.9 + (15 - 0.6)^2 \cdot 0.1 = 23.04$

Standard Deviation: $\sqrt{23.04} = 4.8M$

Sharphotos: PPS 3 #2

Belinda Shar has assessed the following probabilities distribution for the number of packages that might be purchased by a parent.

Number of Packages	X=0	X=1	X=2	X=3
Probability	0.3	0.4	0.2	0.1

What are the expected number of packages to be purchased by each parent and the standard deviation for the number of packages to be purchased by each parent?

Suppose all of the packages are to be priced at the same level. How much should they be priced if they want to break even in expectation? Assume that the production costs are \$3.00 per photo, and remember that the sitting charge is \$0.99.

Sharphotos: PPS 3 #2

Belinda Shar has assessed the following probabilities distribution for the number of packages that might be purchased by a parent.

Number of Packages	X=0	X=1	X=2	X=3
Probability	0.3	0.4	0.2	0.1

Expected value: $0 \cdot 0.3 + 1 \cdot 0.4 + 2 \cdot 0.2 + 3 \cdot 0.1 = 1.1$

Variance: $(0 - 1.1)^2 \cdot 0.3 + (1 - 1.1)^2 \cdot 0.4 + (2 - 1.1)^2 \cdot 0.2 + (3 - 1.1)^2 \cdot 0.1 = 0.89$

Standard Deviation: $\sqrt{0.89} = 0.9434$

How much should packages be priced if Sharphotos wants to break even in expectation? Production costs are \$3.00 per photo, and the sitting charge is \$0.99.

Expected profit = Expected revenue – expected cost

= Expected number sold ·price + sitting charge – cost of package

= $1.1 \cdot p + 0.99 - 9$

Break even means zero profit, and thus $p = 7.281$

Select Exercises from Sample Test 1

- If time allows we will solve some problems the true/false from the sample test

Sample Test, TRUE/FALSE

If A and B are independent, then $P(A|B)$ must equal $P(B|A)$ F

For any two events A and B: $P(A \cup B)^C = P(A^C \cap B^C)$ T

The greater is a random variable's expected value, the greater is its variance F

$X = \begin{cases} 25 \\ 25 \end{cases}$ $\mu_X = 25$ $\sigma_X^2 = 0$ $Y = \begin{cases} 0 \\ 2 \end{cases}$

Sample Test, TRUE/FALSE

If A and B are independent, then $P(A|B)$ must equal $P(B|A)$

FALSE: If A and B are independent, $P(A \cap B) = P(A)P(B)$; using the conditional probability formula

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A)P(B)}{P(B)} = P(A) \quad \text{and} \quad P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{P(A)P(B)}{P(A)} = P(B)$$

For any two events A and B: $P(A \cup B)^C = P(A^C \cap B^C)$

TRUE: Draw a Venn diagram

The greater is a random variable's expected value, the greater is its variance

FALSE: Let X= 50 for all outcomes; it has expected value 50 and variance 0

Let Y have range between 0 and 10; it has expected value between 0 and 10, but variance strictly larger than 0

Next time

- **In-Class Test 1**
- Gathering information
- Sampling