ECONOMICS 150: Quantitative Methods for Economics

Class 3

- Introduction to Probability
- Events
- Probability Tables

Announcements

- Online Quiz 3 is due at 9am before next class (January 20)

- Zoom videos are available on Canvas
  - Follow the link to Panopto Video on Canvas.
  - Helpful if something during the lecture is unclear as you can go back and a "replay it".
  - Not helpful as a substitute for attendance: being in class is necessary to follow what happens (that is why attendance is compulsory).

Roadmap: where are we

REGRESSION ANALYSIS

CORRELATION

ESTIMATION

CONFIDENCE INTERVALS

HYPOTHESIS TESTING

SAMPLING

CENTRAL LIMITT THEOREM

NORMAL DISTRIBUTION

OTHER DISTRIBUTIONS

RANDOM VARIABLES

PROBABILITY THEORY

Probabilities: How Likely?

Starbucks ad on Facebook is viewed by a 20 year old man
Price of GOOGL at closing today will be above $2,790
Snow in Pittsburgh on 12/25/2022
President will be re-elected in 2024

- Probabilities are a measure of uncertainty
  - They may be derived from data, or expert opinions, or be a subjective measure of likelihood of events
- Probabilities quantify uncertainty
  - They give a clear and precise answer to the question "How likely?"
- Quantifying uncertainty is important in many settings
  - Insurance, finance, marketing, energy, policy changes, etc

Why Quantify Uncertainty With Numbers?
Because words can be ambiguous

- What probability would you assign to the following verbal statements?
  - "Usually"
  - "Possible"
  - "Somewhat likely"
  - "Probably"

Interpretations of uncertainty terms

(data from past students)

Statement	# of responses	Mean	Range
Usually	187	0.77	0.15 - 0.99
Probably	188	0.71	0.01 - 0.99
Somewhat likely	187	0.59	0.20 - 0.92
Possible	178	0.37	0.01 - 0.99

Vocabulary

Examples of Experiments:

- Die roll
  - What is the random outcome?
    - Top face of the die
- Online advertisement on Facebook
  - What is the random outcome?
    - The pair "viewer demographics, viewer clicks the ad"

Language and Definitions: Sample Space, Events

- **Sample space**: denoted as S and corresponds to all possible outcomes of the experiment
  - Examples:
    - die roll: $S = \{1,2,3,4,5,6\}$
    - ads on Facebook, $S = \{\text{User Clicks on Ad, User Does Not Click on Ad}\}$

- **Empty Set:** denoted as $\emptyset$ and corresponds to no outcomes

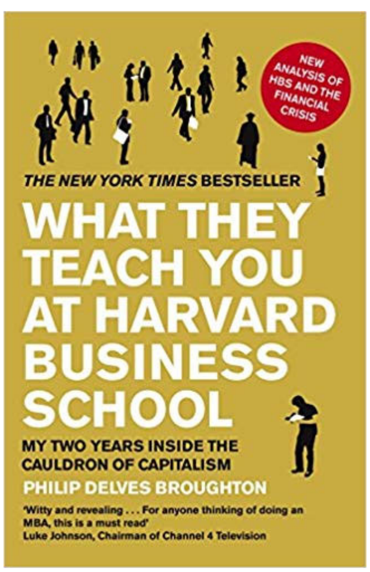
- **Event:** $A \subseteq S$ is a subset of the sample space (collects some of the possible outcomes)
  - Examples:
    - die roll: top face is an even number, $A = \{2,4,6\}$
    - ads on Facebook: viewer of the ad is a woman, $A = \{\text{Woman}\}$

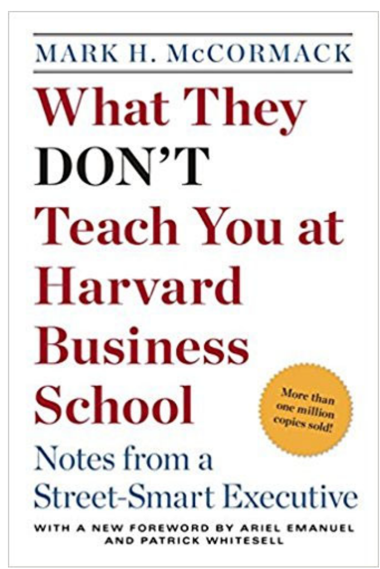
Language and Definitions: Complement

- **Complement:** $\mathrm{A^c}$ means "A does not occur" (sometimes also written as $\bar{A}$)
  - Examples:
    - die roll: if $\mathrm{A} = \{2,4,6\}$, then $\mathrm{A^c} = \{1,3,5\}$
    - ads on Facebook: if $\mathrm{A} = \{\text{Woman}\}$, then $\mathrm{A^c} = \{\text{not a Woman}\}$

- Taken together, an event and its complement describe the whole sample space

Complements at the Bookstore





Probability

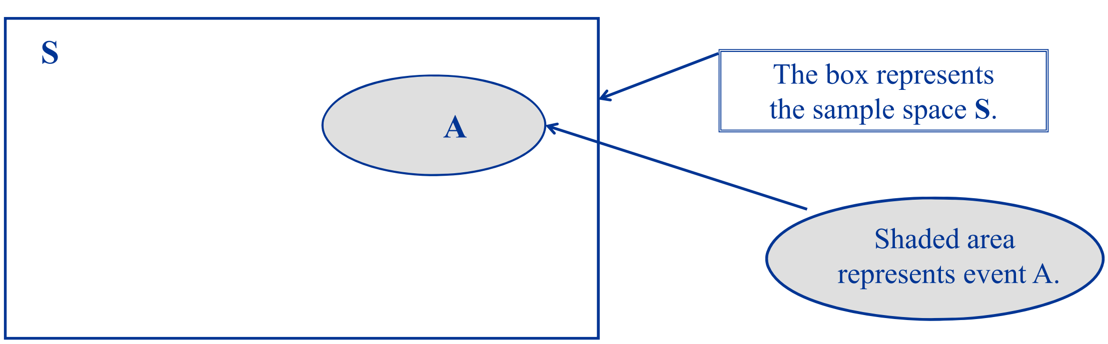
The probability of an event is a number between zero and one:

$$0 \leq P(A) \leq 1$$

- $P(S) = 1$ (something must happen)　　$P(\emptyset) = 0$ (nothing cannot happen)
- The <u>probability of the complement</u>: $\boxed{P(A^c) = 1 - P(A)}$
- Examples:
  - Die Roll: if $A = \{2,4,6\}$, then $P(\{2,4,6\}) = 1/2$ and $P(A^c) = P(\{1,3,5\}) = 1/2$
  - Ads on Facebook: if $A = \{\text{Woman}\}$, then… (need data)

VENN DIAGRAMS

- Can be used for graphical depiction of the sample space and events



The probability of an event, P(A), corresponds to its area in the diagram.
(Thus, the area of the box is equal to one because P(S)=1)

Venn diagram examples: complement

$\rightarrow A^c$

$$P(A) + P(A^c) = 1$$
$$= P(S)$$

Combining Events

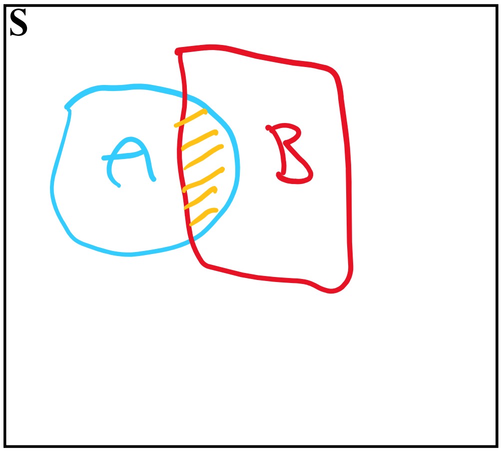
The **Intersection** of events collects outcomes that are in all of them

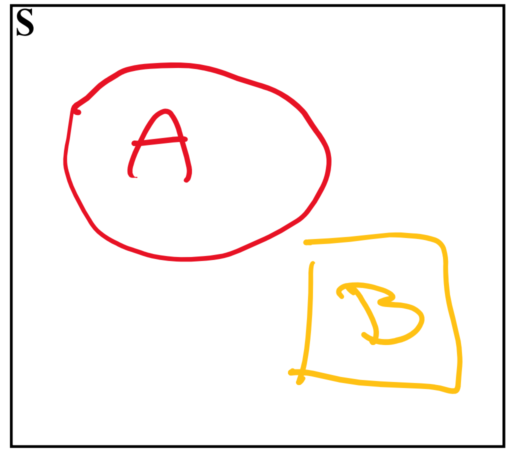
- The intersection of two events A and B is denoted A ∩ B → intersection
  - A ∩ B means both A and B occur

$A = \{2, 4, 6\}$ $B = \{1, 2, 3\}$

$A \cap B = \{2\}$

Venn diagram examples: intersection





$A \cap B = $ ////

$A \cap B = \emptyset$

Combining Events

The **Union** of events of events collects outcomes that are in <u>any</u> of them

- The union of two events A and B is denoted A ∪ B
  - A ∪ B means that A <u>or</u> B (or both) occurs

$A = \{2, 4, 6\}$

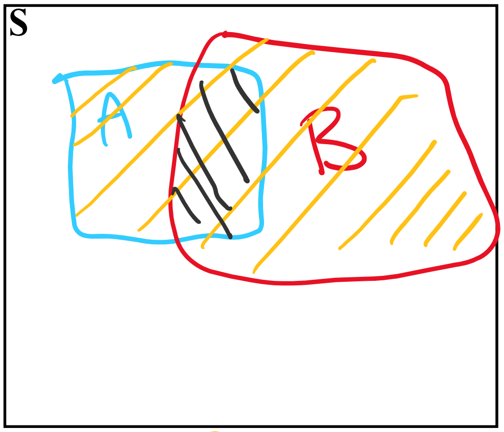
$B = \{1, 2, 3\}$

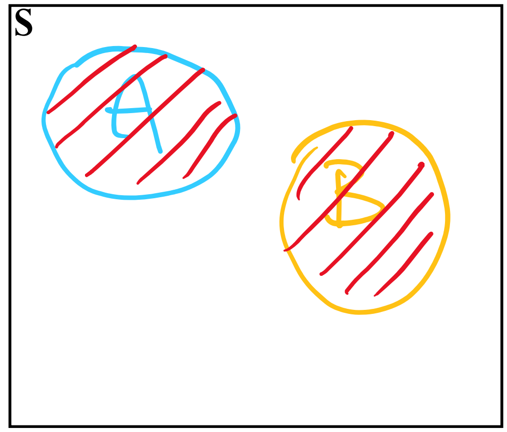
$A \cup B = \{1, 2, 3, 4, 6\}$

Probability of the union formula:

$$\mathbf{P(A \cup \boldsymbol{B}) = P(\boldsymbol{A}) + P(\boldsymbol{B}) - P(A \cap \boldsymbol{B})}$$

Venn diagram examples: union





→ $A \cup B$

→ $A \cap B$

→ $A \cup B$

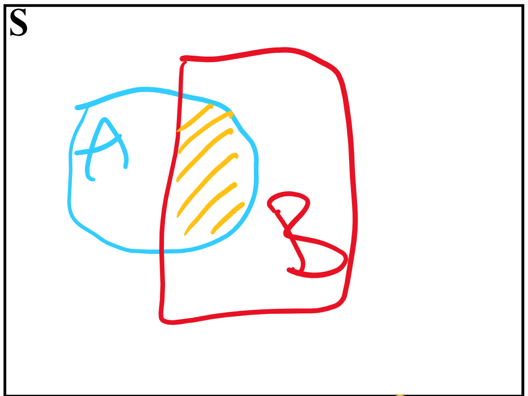
$$\boxed{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$$

More Language

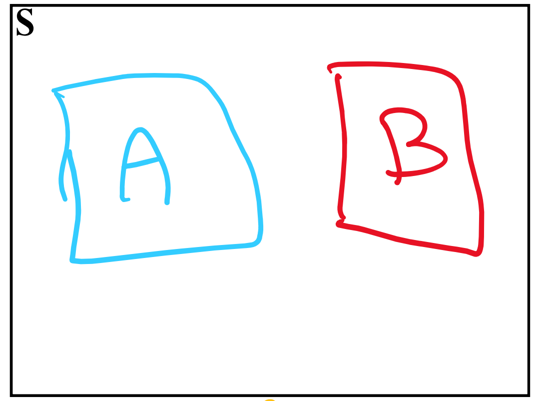
- Events are **mutually exclusive** if their intersection is the empty set
  - The events A and B are mutually exclusive if $A \cap B = \emptyset$

- Events are **collectively exhaustive** if their union equals the sample space;
  - The events A, B, C are collectively exhaustive if $A \cup B \cup C = S$

Venn diagram examples: mutually exclusive

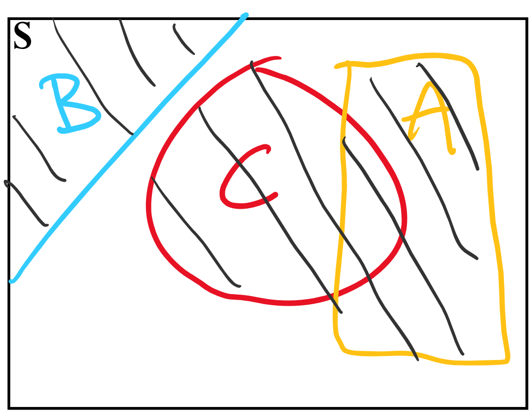


not mutually exclusive
$A \cap B =$ ////

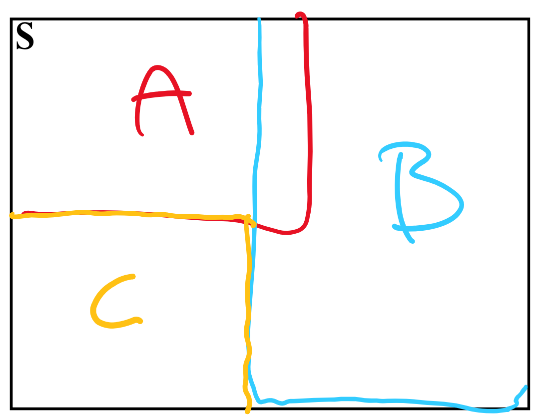


mutually exclusive
$A \cap B = \emptyset$

Venn diagram examples: collectively exhaustive



/// → $A \cup B \cup C \neq S$
not collectively exhaustive



$A \cup B \cup C = S$
COLLECTIVELY EXHAUSTIVE

Probabilities When Combining Events

- When events are mutually exclusive, the probability of their intersection is zero
  - If $\mathbf{A} \cap \boldsymbol{B} = \emptyset$ then $\mathbf{P}(\mathbf{A} \cap \boldsymbol{B}) = \mathbf{P}(\emptyset) = \mathbf{0}$

$P(A \cup B) = P(A) + P(B)$

- When events are collectively exhaustive, the probability of their union is one
  - If $\mathbf{A} \cup \mathbf{B} \cup \mathbf{C} = \mathbf{S}$ then $\mathbf{P}(\boldsymbol{A} \cup \boldsymbol{B} \cup \boldsymbol{C}) = \boldsymbol{P}(\boldsymbol{S}) = 1$

Example: an Event and Its Complement

$A \cap A^c = \emptyset$

M.E.

$A \cup A^c = S$

C.E.

A and A^c are mutually exclusive and collectively exhaustive

Example: A=Even and B=Low

$A = \{2, 4, 6\}$ $B = \{1, 2, 3\}$

$A \cap B = \{2\}$ $A \cup B = \{1, 2, 3, 4, 6\}$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$= 1/2 + 1/2 - \frac{1}{6}$$
$$= \frac{5}{6}$$

Top Hat Attendance Code for Today

8371

Probability Tables

Useful tool for organizing info about probabilities

Columns are labeled by mutually exclusive and collectively exhaustive events

Rows are labeled by mutually exclusive and collectively exhaustive events

	B	B^c	
A	$P(A \cap B)$	$P(A \cap B^c)$	$P(A)$
A^c	$P(A^c \cap B)$	$P(A^c \cap B^c)$	$P(A^c)$
	$P(B)$	$P(B^c)$	1

This entry is always 1 because it corresponds to P(S)

- The entries in the interior cells correspond to probabilities of the intersection events.
- The marginal entries correspond to probabilities of the corresponding events and are always equal to the sum of the interior entries in the corresponding row or column.

Probability Tables

Useful tool for organizing information about probabilities

	B	B^c	
A	$P(A \cap B)$	$P(A \cap B^c)$	$P(A)$
A^c	$P(A^c \cap B)$	$P(A^c \cap B^c)$	$P(A^c)$
	$P(B)$	$P(B^c)$	1

$= P(A^c \cap B) + P(A^c \cap B^c)$

- To construct the table we use the property:

$$P(A) = P(A \cap B) + P(A \cap B^c)$$

  - This is a fundamental property sometimes called the **total probability rule**.

Why Are Probability Tables Useful?

Because entries must "add up" by column and by row we can use a table to figure out entries which are not known to begin with

	B	B^c	
A	$P(A \cap B)$	$P(A \cap B^c)$	$P(A)$
A^c	$P(A^c \cap B)$	$P(A^c \cap B^c)$	$P(A^c)$
	$P(B)$	$P(B^c)$	1

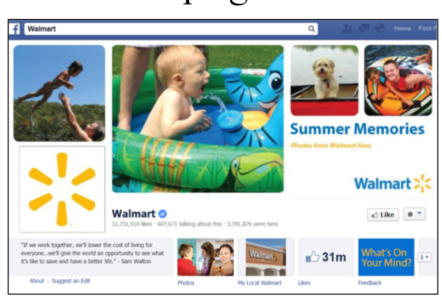
Walmart Ads on Facebook

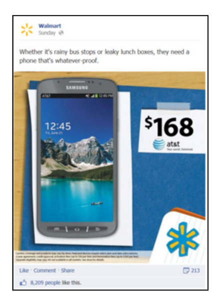
Walmart
Save money. Live better.

Walmart launches a new untargeted advertising campaign with a massive presence on social media:

- budgets $12 million for ads on Facebook
- buys 6 billion impressions for a 12-month campaign

What does Walmart want?





Walmart Ads on Facebook

Walmart obtained the following information about ads displayed on Facebook:

- P(Click) = 0.02
- P(any gender, age ≤ 24) = 0.34, P(Female, age > 24) = 0.40,
- P(any gender, age ≤ 24, No Click) = 0.3366,
- P(not female, age > 24, No Click) = 0.2587

$\}$ intersection

Calculate the probability that the next ad will be shown to a woman above 24 and she clicks on it.

- Events to consider:

  A_1 = {any gender, age ≤ 24},

  A_2 = {Female, age > 24}, A_3 = {not female, age > 24},

  B_1 = {Click}, B_2 = {No Click}

Walmart Ads on Facebook: Build a Table

Given Information

- P(Click) = 0.02
- P(any gender, age ≤ 24) = 0.34
- P(Female, age > 24) = 0.40
- P(any gender, age ≤ 24, No Click) = 0.3366
- P(not female, age > 24, No Click) = 0.2587

List the basic events

Click no Click

Female age > 24 not female age > 24 any gender age ≤ 24

	click	no click	
F >24	0.0153	0.3847	0.40
not F >24		0.2587	
any ≤24		0.3366	0.34
	0.02	0.98	1

Class 3: useful facts

- Fundamental probability facts:

  for any event A:

$$0 \leq P(A) \leq 1$$

$$P(A^c) = 1 - P(A)$$

  for two events A and B:

$$P(A) = P(A \cap B) + P(A \cap B^c)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

- Problem solving: Probability tables
  - Particularly useful when you have information about the probability of intersection-like events (you need to know some of the entries).

Next Time

- Conditional Probability
- Independence
- Probability Trees