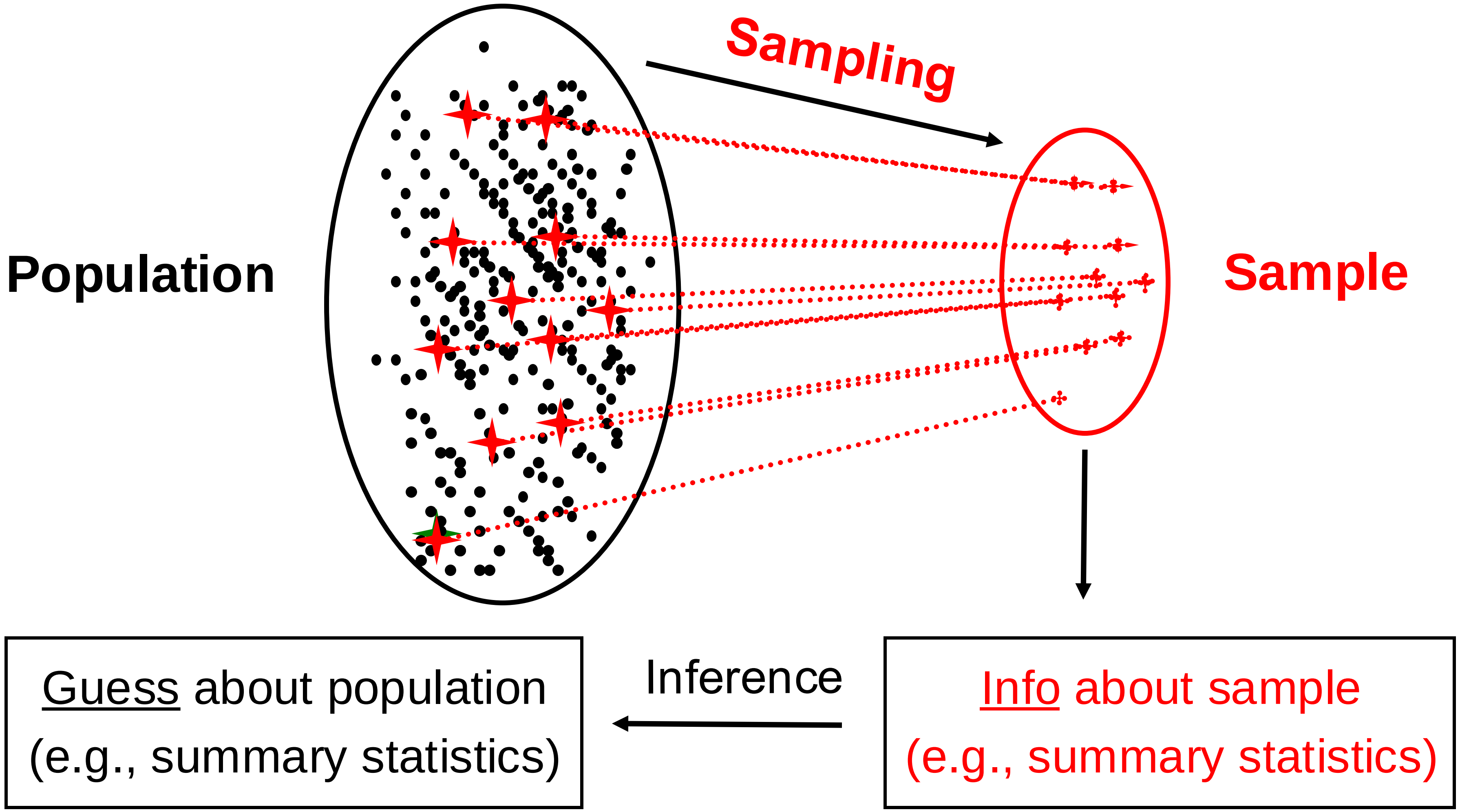
Confidence Intervals

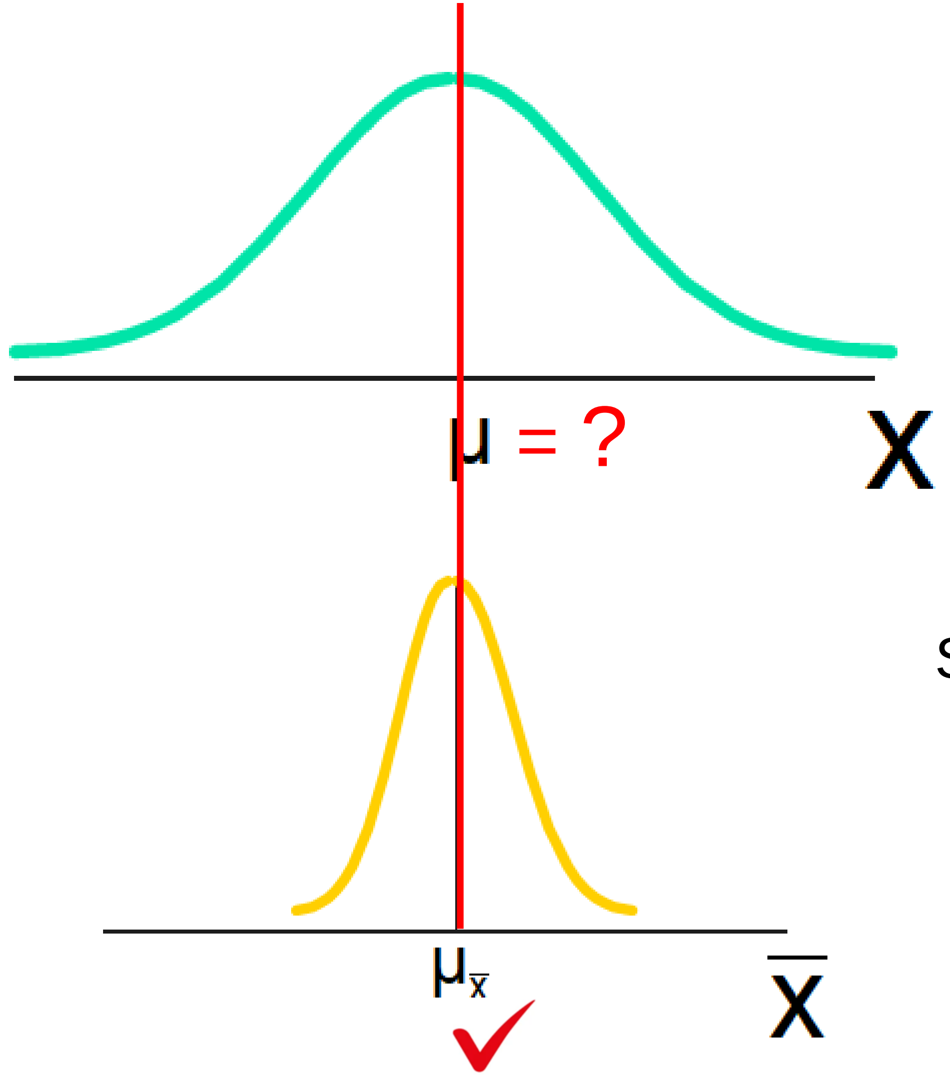
Part 3.2 Confidence Intervals and Hypothesis Testing

Be very careful!!!

But how do we learn about the population?



If Population Mean is Unknown



Population Distribution

Mean of $\bar{X}$ → μ

Sample Mean Distribution

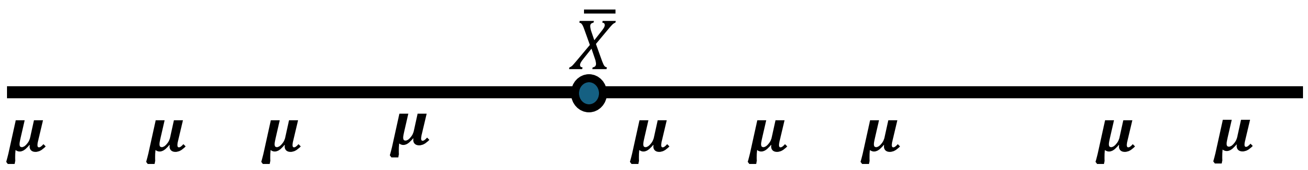
Suppose can take ALL samples of size *n*

If Population Mean is Unknown

Only one sample is available.

But we need to estimate the population parameter somehow.

Ask instead: how far is the population mean from the observed sample mean?



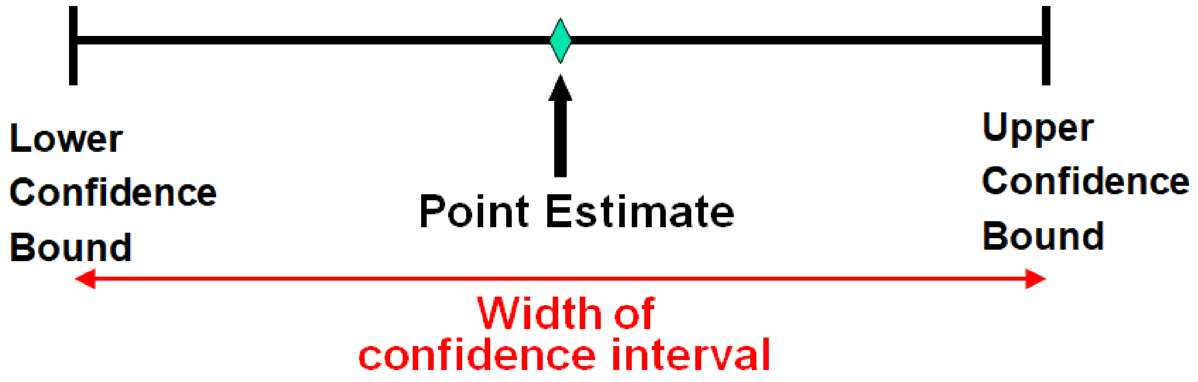
Point Estimates vs Interval Estimates

Point estimate: a single number

- This is the estimation we have done so far

Interval estimate: an interval to provide additional info on the variability of the estimate.

- How precise is it (width of the interval)?
- How confident you are (chances the interval is "correct")?



Example 5 (from last time): Vending Machine

… *X is* NOT *normally distributed and we do not know the distribution*.

For a period of 144 days, daily observations have been conducted about the number of candy bars sold from a vending machine. Using these observations:

- Mean of the sample = 258
- Standard deviation of the sample = 60

Q. For a 144-day period, find:

P(**average** number of candy bars sold > 263) =

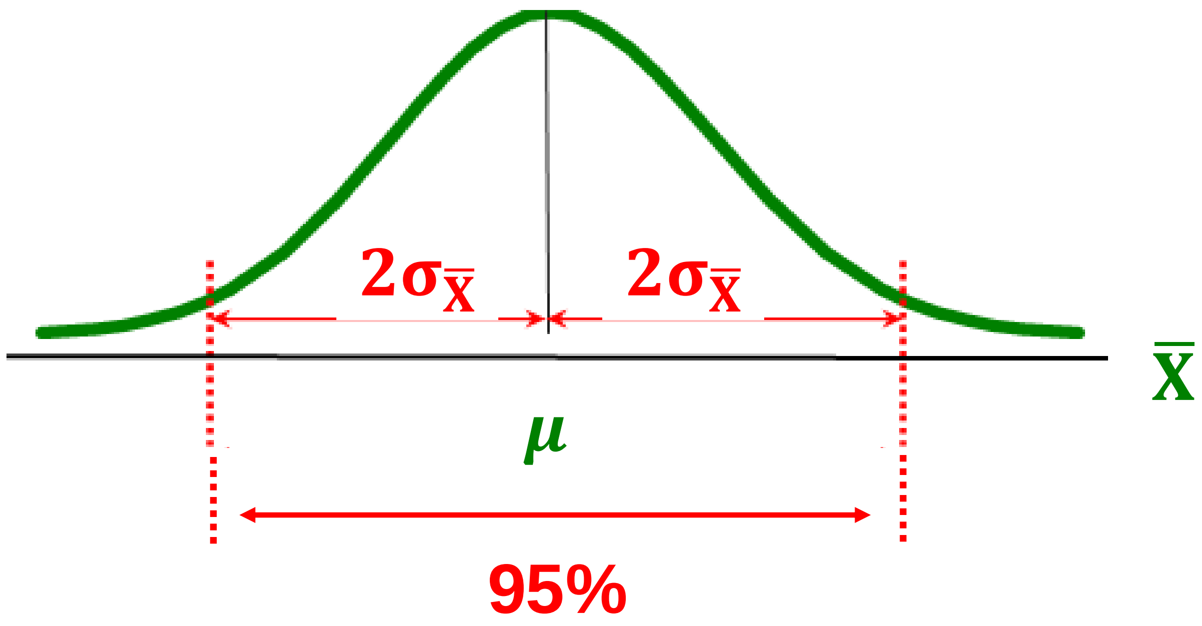
Example 5 (from last time): Vending Machine

… X *is* NOT *normally distributed and we do not know the distribution*.

$$\bar{X} \sim N(\mu_{\bar{X}}, \sigma_{\bar{X}}{}^2) = N(260, (5)^2)$$

We said 95% of all values of $\bar{X}$ were within $\mathbf{2\sigma_{\bar{X}}}$ of its center, which we claimed was centered on the population mean μ.

$$95\% = P\left(\mu - 2\sigma_{\bar{X}} < \bar{X} < \mu + 2\sigma_{\bar{X}}\right)$$



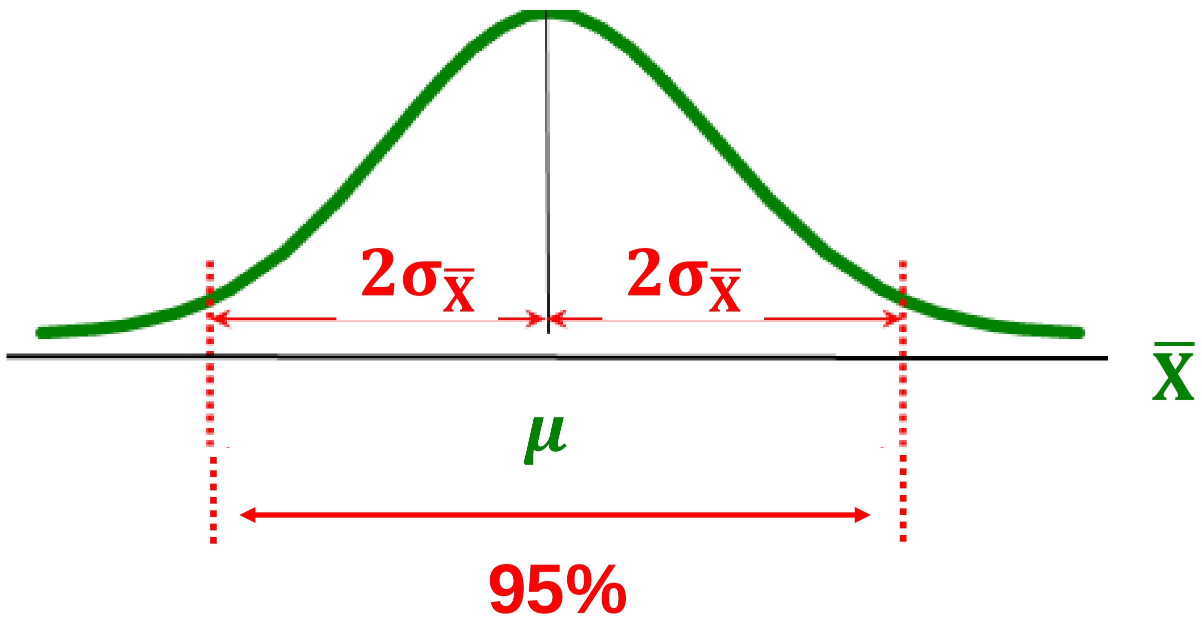
Example 5 (from last time): Vending Machine

… X *is* NOT *normally distributed and we do not know the distribution*.

If population mean μ is unknown, guess μ using what you know, the value of sample mean $\bar{X}$.

Suppose for a random sample, $\bar{X} = 258$:

$$95\% = P(\mu - 2\sigma_{\bar{X}} < \bar{X} < \mu + 2\sigma_{\bar{X}})$$



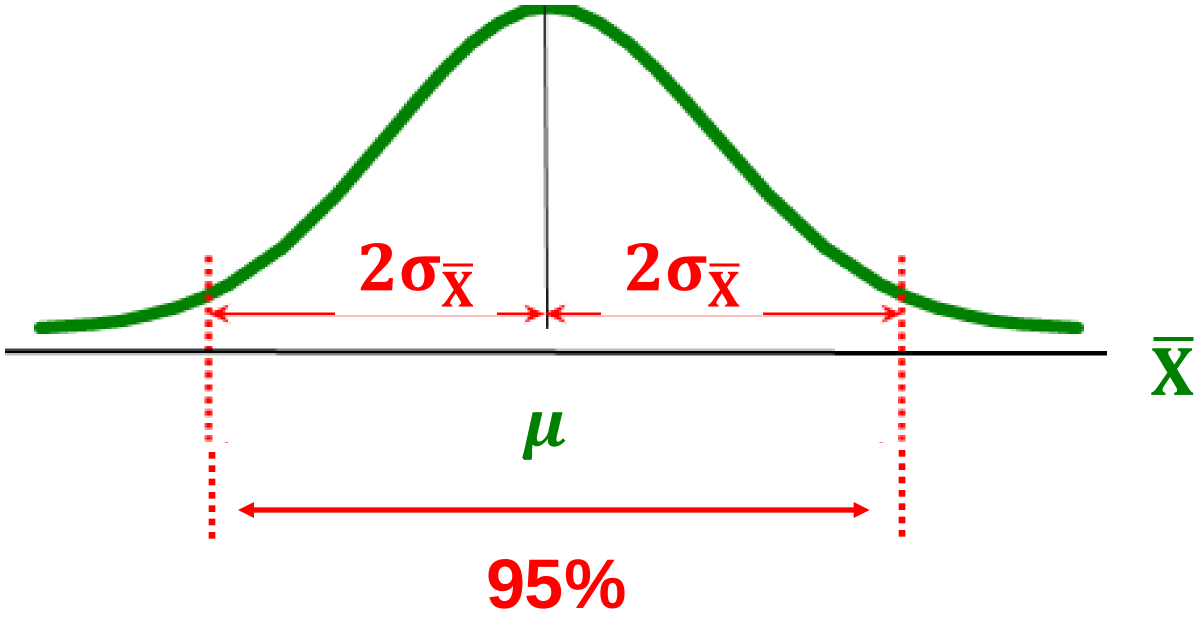
Example 5 (from last time): Vending Machine

… X is NOT *normally distributed and we do not know the distribution.*

If population mean μ is unknown, guess μ using what you know, the value of sample mean $\bar{X}$.

Suppose for a random sample, $\bar{X} = 258$:

$$95\% = P(258 - 2 \times 5 < \bar{X} < 258 + 2 \times 5) = P(248 < \bar{X} < 268)$$



Is This Interval a Good Guess of μ?

Two challenges in practice:

1. You only take one sample of size n

   → have only one interval

2. You do not know population mean μ

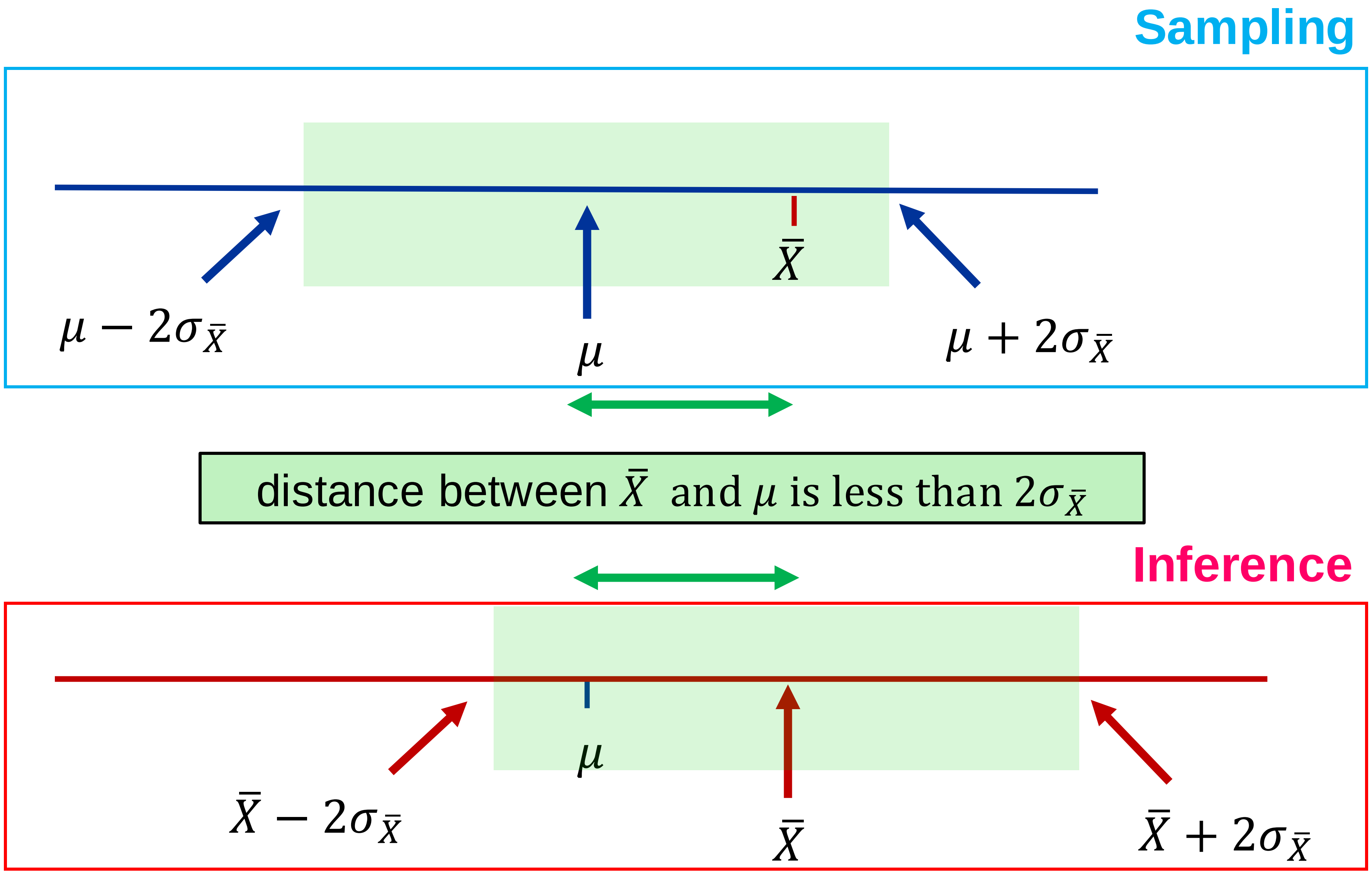
   → don't know if the interval *actually contains* μ

But! You do know that 95% of the intervals formed in this manner will contain μ.

Based on this one sample, you *actually selected* your interval such that you can be 95% confident it will contain μ.

→ 95% Confidence Interval

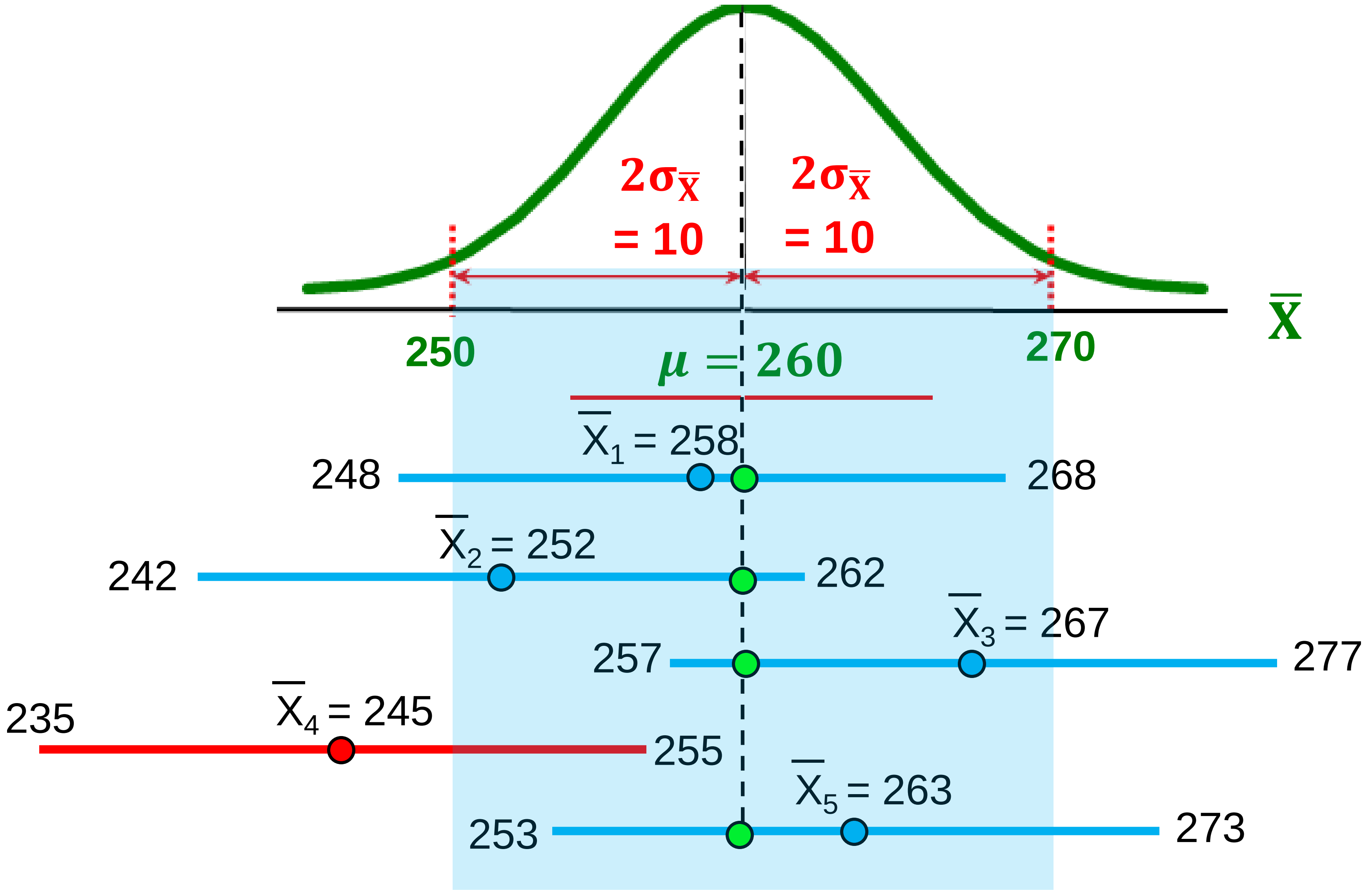
The Center Point Has Changed



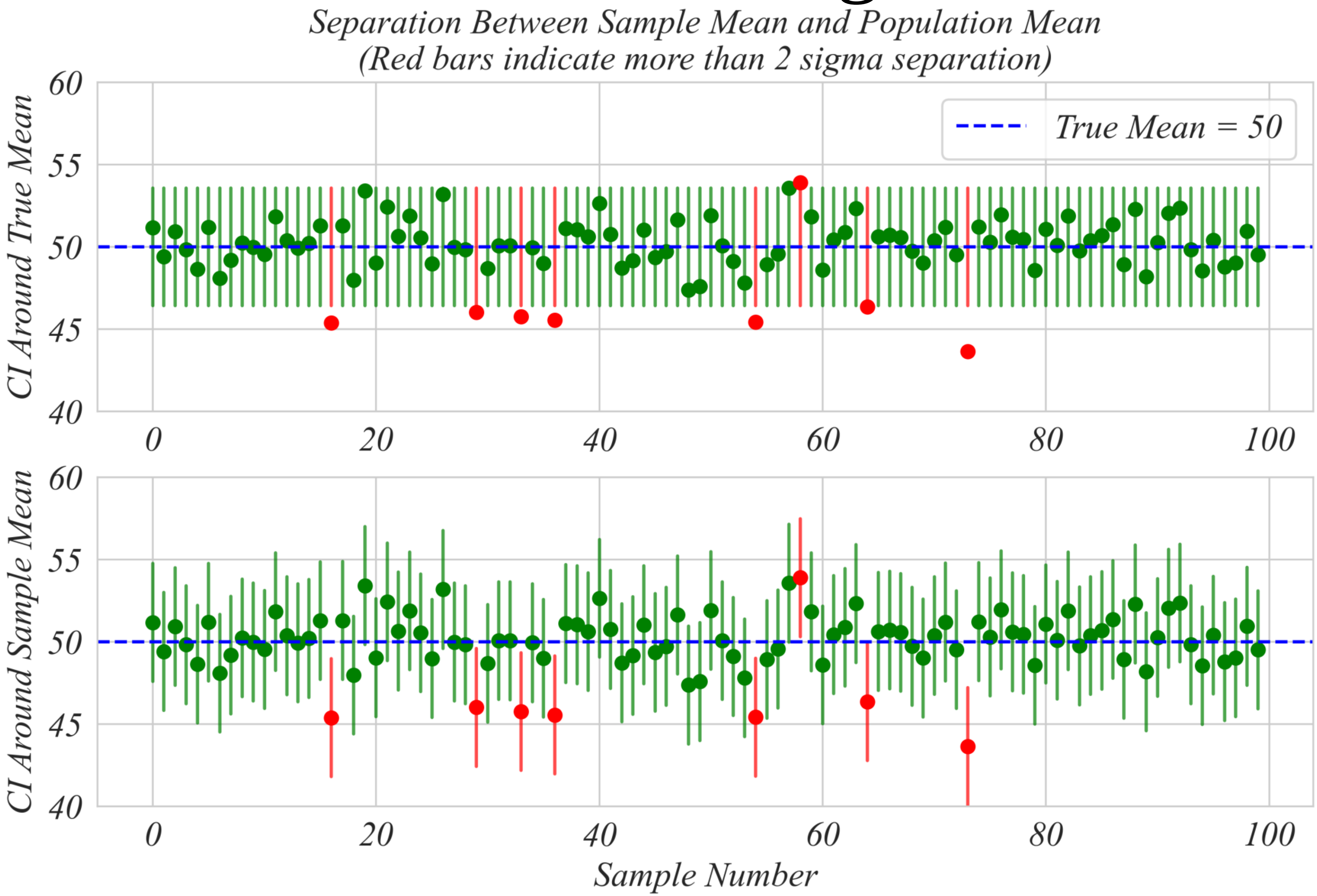
Is This Interval a Good Guess of μ?

Sample (n = 144)	Sample Mean $\bar{X}$	Lower Limit $\bar{X} - 2\sigma_{\bar{X}}$	Lower Limit $\bar{X} + 2\sigma_{\bar{X}}$	Contain Pop Mean $\mu = 260$?
1	258	248	268	Yes
2	252	242	262	Yes
3	267	257	277	Yes
4	245	235	255	No !
5	263	253	273	Yes

Is This Interval a Good Guess of μ?



The Center Point Has Changed



Estimating the Population Mean

$$\text{Interval Estimate} = \bar{X} \pm \text{Critical Value} \cdot \sigma_{\bar{X}}$$

$$= \underbrace{\bar{X}}_{\text{Point Estimate}} \pm \underbrace{\text{Critical Value} \cdot \frac{\sigma}{\sqrt{n}}}_{\text{Margin of Error}}$$

Critical Value:	= **1** if looking for 68% confidence interval
	= **2** if looking for 95% confidence interval
	= **3** if looking for 99.7% confidence interval

Estimating the Population Mean

$$\text{Interval Estimate} = \bar{X} \pm \text{Critical Value} \cdot \frac{s}{\sqrt{n}}$$

Example 1: Find Confidence Interval for μ

Working at Avis, you oversee buying mid-size cars and are considering proposals from Ford, GM and Chrysler (each offering one car model). Your key consideration: miles per gallon (mpg). You test drive 10 Ford vehicles:

- Sample mean of mpg: 31.8
- Sample variance of mpg: 2

$n = 10$

$\bar{X} = 31.8$

$S = \sqrt{2}$

Q. Based on this sample, what is the 95% confidence interval for the average mpg for a Ford fleet.

95% C.I of μ = [? , ?]

Example 1: Find Confidence Interval for μ

Working at Avis, you oversee buying mid-size cars and are considering proposals from Ford, GM and Chrysler (each offering one car model). Your key consideration: miles per gallon (mpg). You test drive 10 Ford vehicles:

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Q. Based on this sample, what is the 95% confidence interval for the average mpg for a Ford fleet.

Is the distribution (X) normal? Yes.

95% C.I

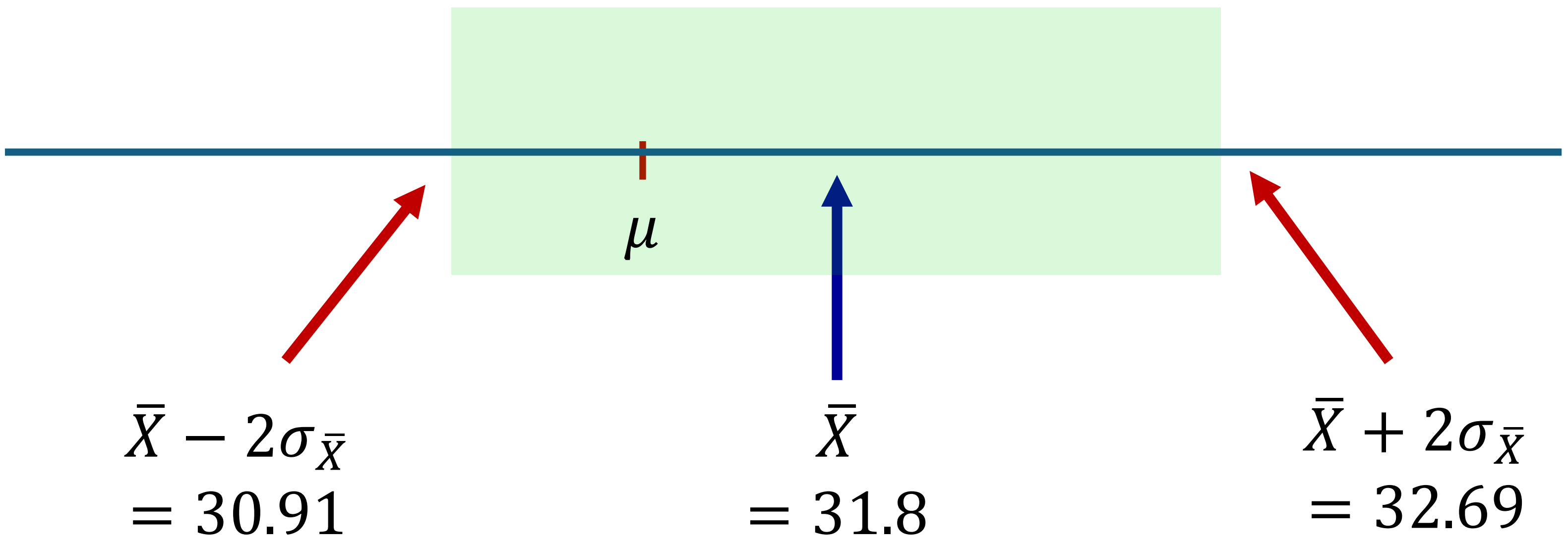
$$\bar{X} \pm 2\sigma_{\bar{X}} = 31.8 \pm 2 \times 0.447$$

$$= 31.8 \pm 0.894$$

$$= [30.91, \quad 32.69]$$

$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} \approx \frac{S}{\sqrt{n}} = \frac{\sqrt{2}}{\sqrt{10}} = 0.447$$

What Does $\bar{X}$ Suggest About μ?



Estimating the population mean

After data is collected from a sample of size *n*,

Point estimate

- $\bar{X}$ is the most likely value for μ

Interval estimate

- 68% confidence interval: $\bar{X} \pm 1 \cdot \sigma_{\bar{X}} \approx \bar{X} \pm 1 \cdot \frac{s}{\sqrt{n}}$
- 95% confidence interval: $\bar{X} \pm 2 \cdot \sigma_{\bar{X}} \approx \bar{X} \pm 2 \cdot \frac{s}{\sqrt{n}}$
- 99.7% confidence interval: $\bar{X} \pm 3 \cdot \sigma_{\bar{X}} \approx \bar{X} \pm 3 \cdot \frac{s}{\sqrt{n}}$

Example 2: Evaluate a Candidate of μ

Working at Avis, you oversee buying mid-size cars and are considering proposals from Ford, GM and Chrysler (each offering one car model). Your key consideration: miles per gallon (mpg). You test drive 10 Ford vehicles:

- Sample mean of mpg: 31.8
- Sample variance of mpg: 2

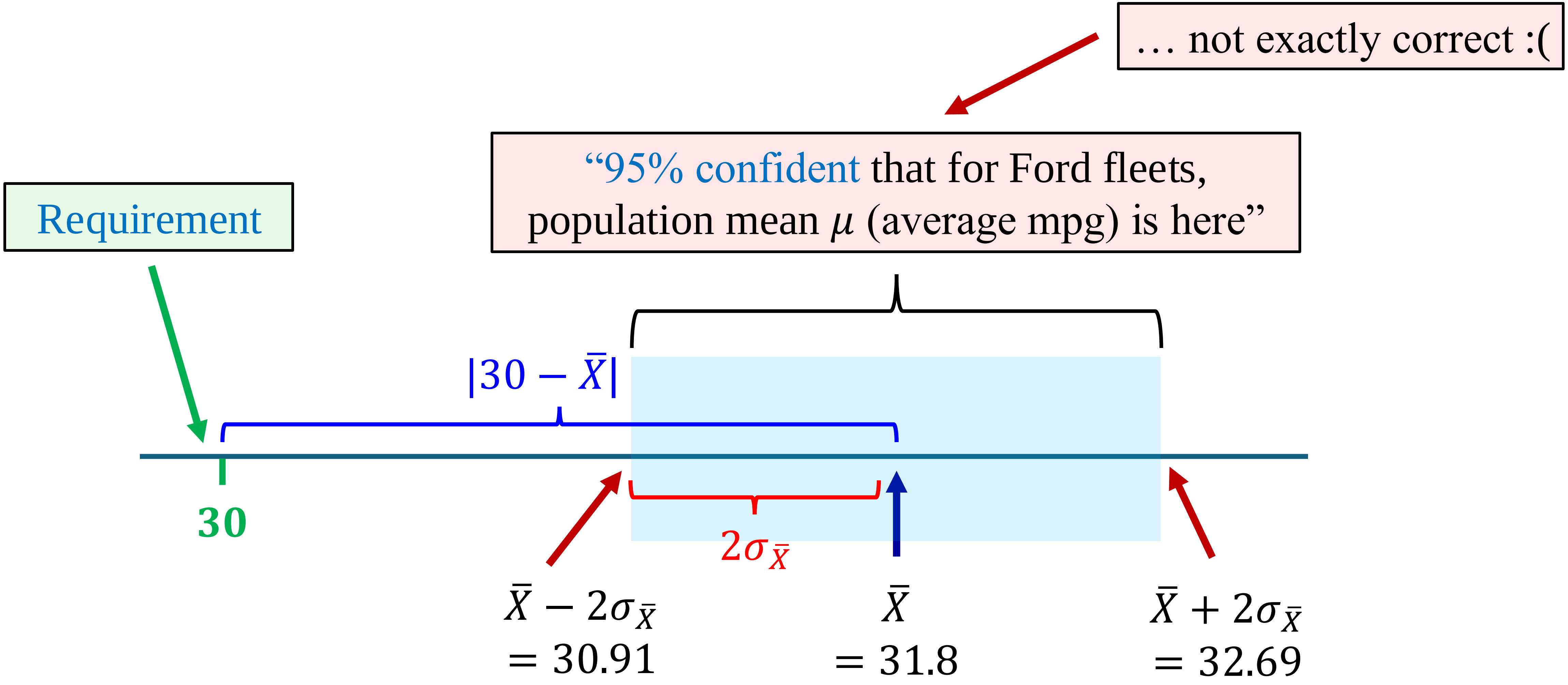
$n = 10$

$\bar{X} = 31.8$

$S = \sqrt{2}$

Q. If your manager requires the fleet to have a mpg of at least 30, will the Ford vehicles meet this requirement? Is $\mu \geq 30$?

Example 2: Evaluate a Candidate of μ



Example 2: Evaluate a Candidate of μ

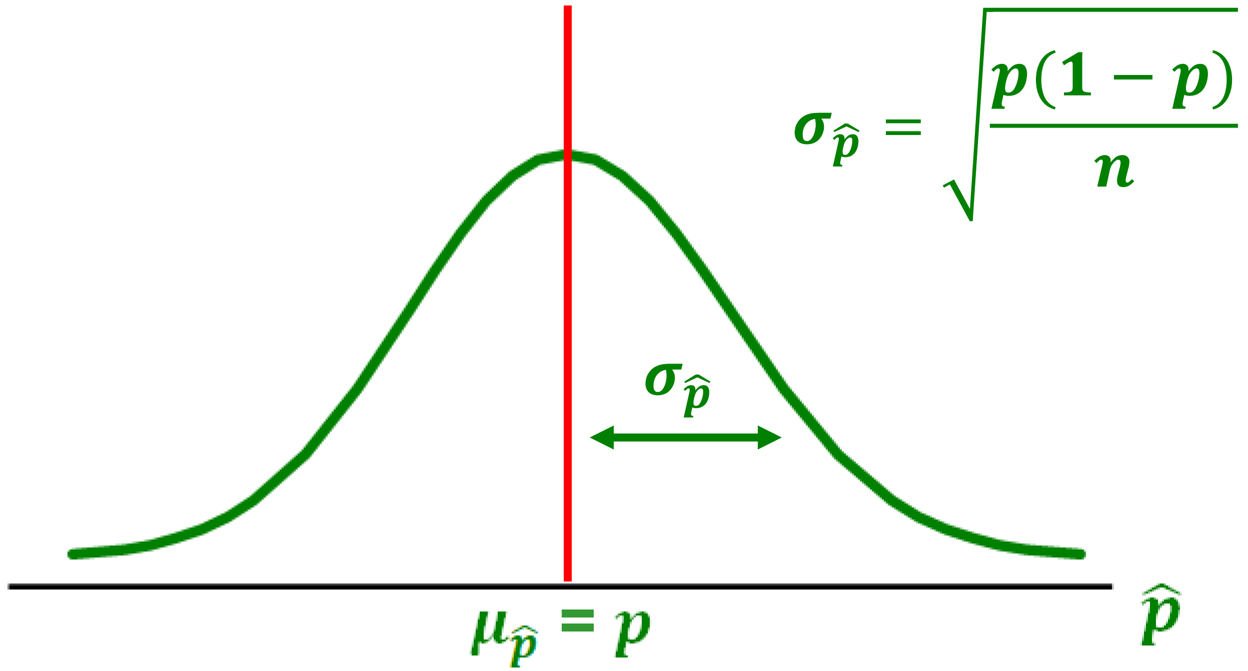
Q. Will the Ford vehicles meet the requirement: $\mu \geq 30$?

$$\left.\begin{array}{l} n = 10 \\ \bar{X} = 31.8 \\ S = \sqrt{2} \end{array}\right\} \quad \sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} \approx \frac{S}{\sqrt{n}} = \frac{\sqrt{2}}{\sqrt{10}} = 0.447$$

$$\rightarrow \quad |30 - \bar{X}| = |30 - 31.8| = 1.8 > 2\sigma_{\bar{X}}$$

Distribution of Sample Proportion ($\hat{p}$)

… works exactly the same.

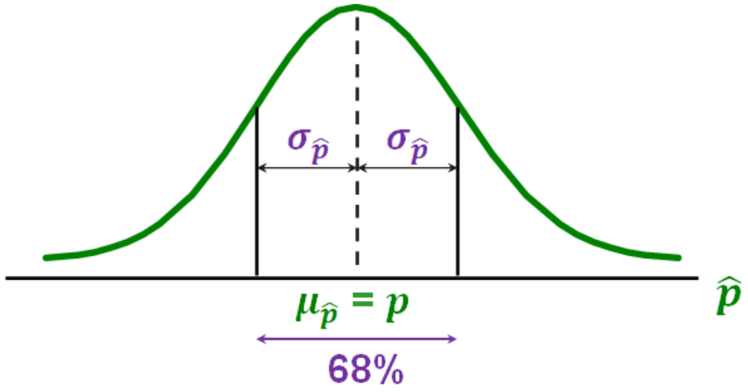


When sample is large enough → $\hat{p} \sim N\left(\mu_{\hat{p}},\ \sigma_{\hat{p}}^{2}\right) = N\left(p, \frac{p(1-p)}{n}\right)$

Empirical (1-2-3) Rule for $(\hat{p})$

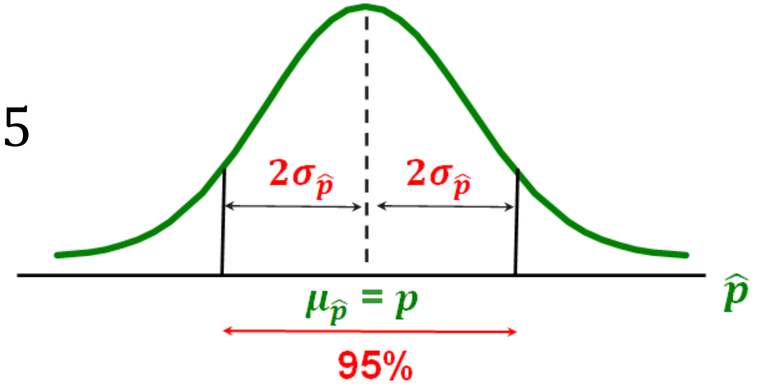
68% chance of being within **one** standard deviation of the mean

$$P\left(\mu_{\hat{p}} - \sigma_{\hat{p}} < \hat{p} < \mu_{\hat{p}} + \sigma_{\hat{p}}\right) = 0.68$$



95% chance of being within **two** standard deviations of the mean

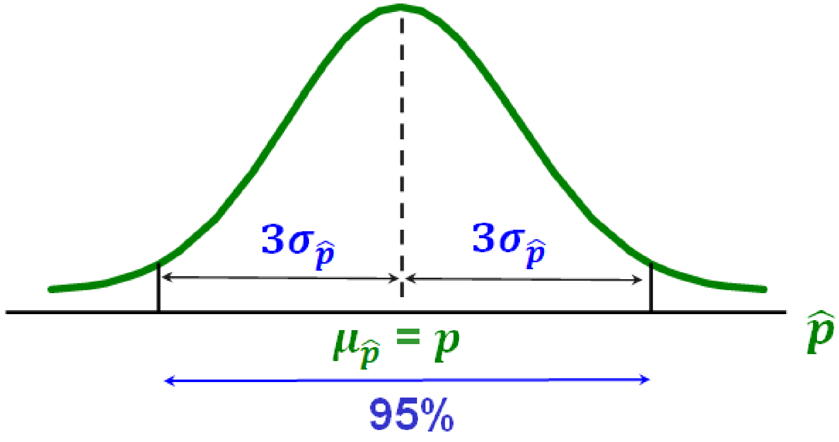
$$P\left(\mu_{\hat{p}} - 2\sigma_{\hat{p}} < \hat{p} < \mu_{\hat{p}} + 2\sigma_{\hat{p}}\right) = 0.95$$



Empirical (1-2-3) Rule for ($\hat{p}$)

99.7% chance of being within **three** standard deviation of the mean

$$P\left(\mu_{\hat{p}} - 3\sigma_{\hat{p}} < \hat{p} < \mu_{\hat{p}} + 3\sigma_{\hat{p}}\right) = 0.997$$



Estimating a Population Proportion ($\hat{p}$)

Point estimate

- $\hat{p}$ is the most likely value for p

Interval estimate

- A confidence interval around $\hat{p}$ where p is likely to be:

$$= \hat{p} \pm \text{Critical Value} \cdot \sigma_{\hat{p}}$$

$$= \hat{p} \pm \text{Critical Value} \cdot \sqrt{\frac{p(1-p)}{n}}$$

$$= \left[\hat{p} - \text{ Critical Value } \cdot \sqrt{\frac{p(1-p)}{n}}, \hat{p} + \text{ Critical Value } \cdot \sqrt{\frac{p(1-p)}{n}}\right]$$

Example 3: Find the CI for p

Q. What is a 95% confidence interval estimate for the population proportion of social media users who have purchased an item promoted by a celebrity on a social media?

$n = 1000$ $\hat{p} = 0.26$ 95% C.I of p= [?, ?]

$$\sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}$$

$$\approx \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$= \sqrt{\frac{0.26 \times (1 - 0.26)}{1000}} = 0.014$$

$$\hat{p} \pm 2\sigma_{\hat{p}}$$

$$= 0.26 \pm 2 \times 0.014$$

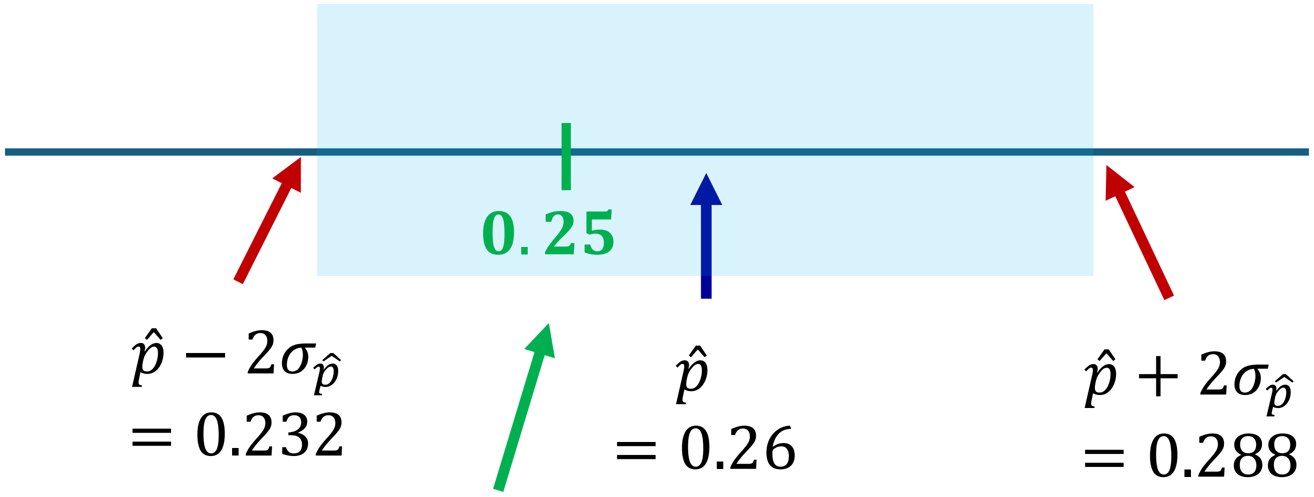
$$= 0.26 \pm 0.028$$

$$= [0.232, \ \ 0.288]$$

Example 4: Evaluate a Candidate of p

Q. Are you likely to purchase an item promoted by a celebrity on social media?

Is $p > 0.25$?	$n = 1000$	$\hat{p} = 0.26$	95% C.I of p = [0.232, 0.288]

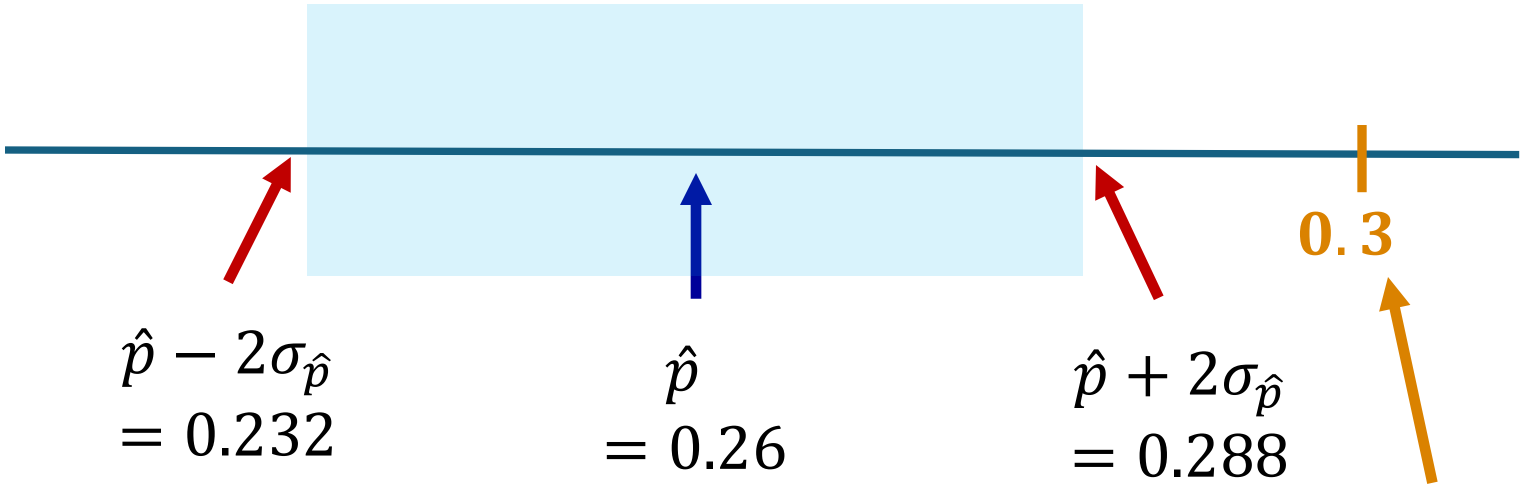


No, because the 95% interval estimate contains 0.25

Example 4: Evaluate a Candidate of p

Q. Are you likely to purchase an item promoted by a celebrity on social media?

Is $p < 0.3$? | $n = 1000$ | $\hat{p} = 0.26$ | 95% C.I of p = [0.232, 0.288]



Yes, because 0.3 is outside the 95% C.I. to the right

Hypothesis Testing

… a hypothesis is a claim (assertion) about a population parameter.

- Population mean

The mean monthly cellphone bill in this city is
$\mu = \$42$

- Population proportion

The proportion of adults in this city with cellphones is
$p = 0.68$

Hypothesis Testing: The Null Hypothesis (H_0)

… the baseline assumption about what is true.

Example: The mean diameter of a manufactured bolt is 30mm

$$\mathrm{H}_0 : \mu = 30$$

$$\mathrm{H}_0 : \mu = 30$$

$$\mathrm{H}_0 : \overline{\mathrm{X}} = 30$$

Hypothesis Testing: The Null Hypothesis (H_0)

... some examples.

The accused is innocent	(until proven guilty)
New product is no different from old one	(until proven better)
Average salary after graduation = \$46,000	(until proven different)
Average wealth is at least \$100,000	(until proven < \$100,000)

Hypothesis Testing: The Null Hypothesis (H_0)

… some examples.

Null	Alternative
Null Hypothesis: H_0	Alternative Hypothesis: H_1 or H_A
Examples • H_0: $\mu = \mu_0$ • H_0: $\mu \geq \mu_0$ • H_0: $\mu \leq \mu_0$	Examples • H_A: $\mu \neq \mu_0$ • H_A: $\mu < \mu_0$ • H_A: $\mu > \mu_0$

- H_A is the hypothesis you are gathering evidence in support of.
- H_0 is the hypothesis you would like to reject.
  - The idea: Reject H_0 only when there are lots of evidence against it.
  - A technicality: always include " = " in H_0 ($=,\ \geq,\ \leq$)

Hypothesis Testing

… how does it work in practice?

A M&M plant operation manager is looking at the efficiency of their automated production lines. According to company specification, average #(green beans) in a bag should be 4.

If population mean is not different from the desired level of 4

→ Happy for production to go on undisturbed.

If population mean is different from the desired level of 4

→ Needs to stop production and adjust the production line.

Hypothesis Testing

... how does it work in practice?

A M&M plant operation manager is looking at the efficiency of their automated production lines. According to company specification, average #(green beans) in a bag should be 4.

The population mean is 4

$H_0: \mu = 4$ → $H_A: \mu \neq 4$

Q. How to test the null hypothesis H_0?

- Randomly sample bags.
- Calculate the average number of green beans per sampled bag ($\bar{X}$).
- Compare that $\bar{X}$ to what you would expect if the null hypothesis was true.

Hypothesis Testing

… how does it work in practice?

$H_0: \mu = 4$

$H_A: \mu \neq 4$

Suppose $\bar{X} = 2.8$, significantly lower than 4.

If the null hypothesis were true:

→ Prob(get such a value as the sample mean) would be very small

→ Reject the null hypothesis

In other words, if the population mean were 4:

→ Getting a sample mean 2.8 is very unlikely.

→ Conclude that the population mean μ must not be 4.

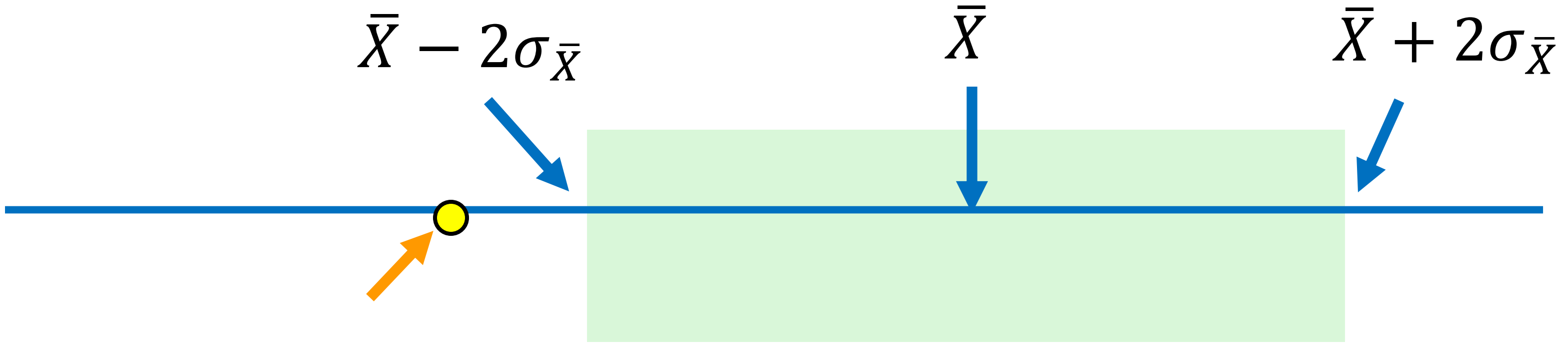
Hypothesis Testing

... how does it work in practice?

... Confidence intervals!

$H_0: \mu = 4$

$H_A: \mu \neq 4$



Unlikely for μ to be here.
So reject the null hypothesis.

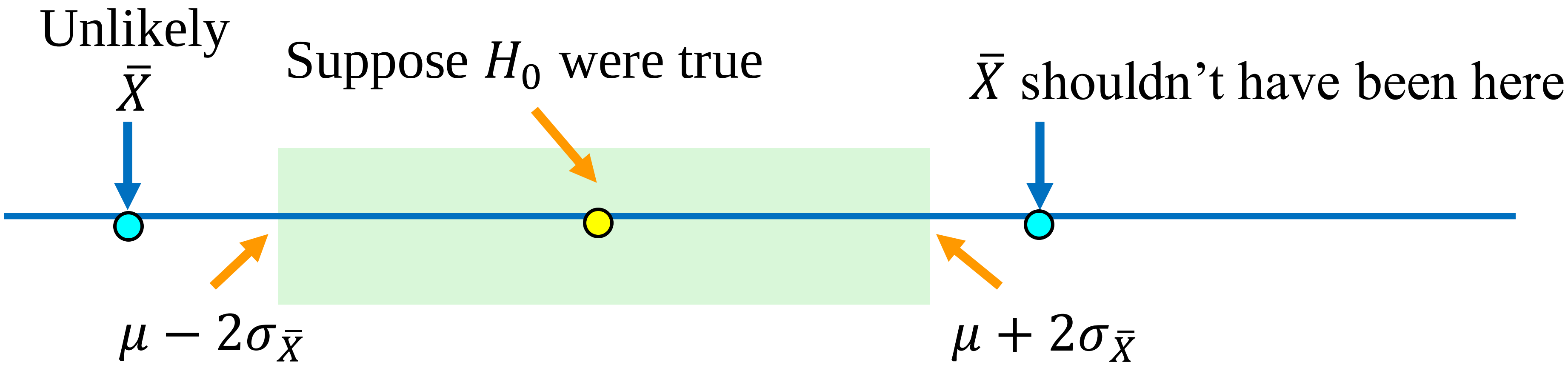
Hypothesis Testing

… how does it work in practice?

… Confidence intervals!

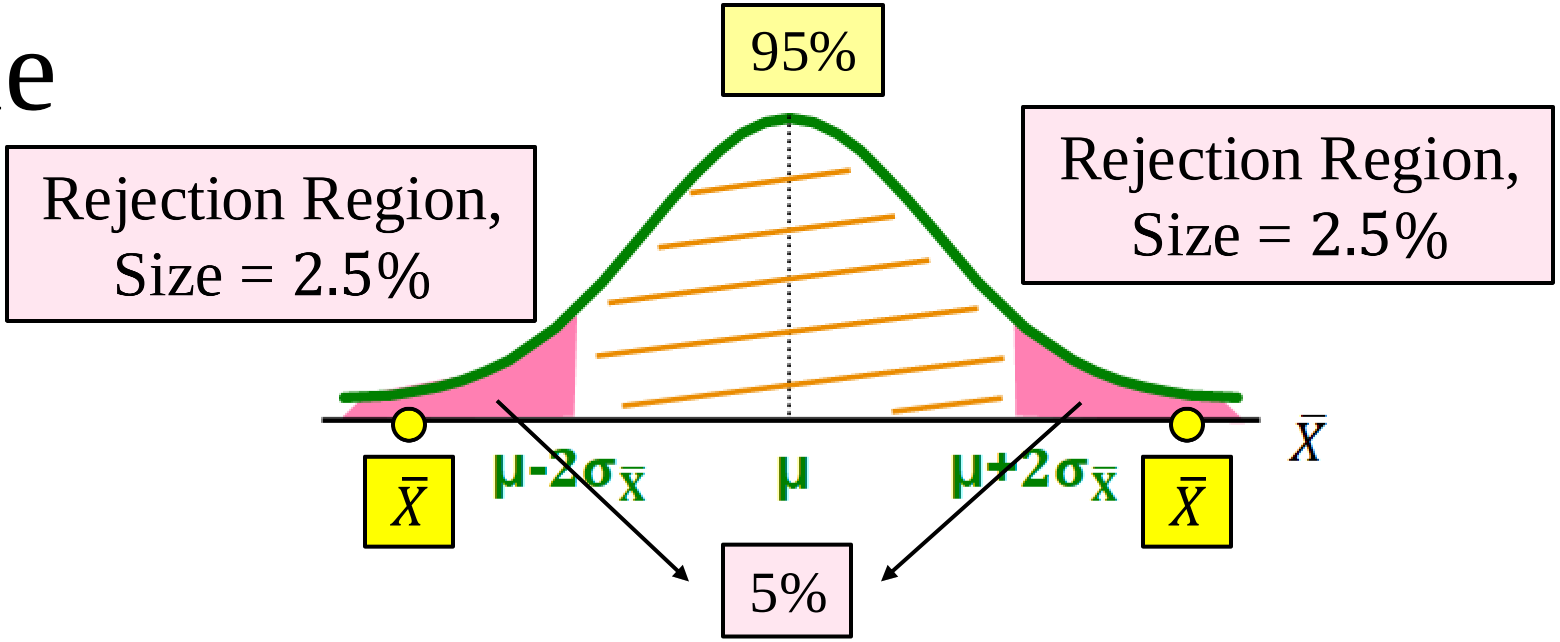
$H_0: \mu = 4$

$H_A: \mu \neq 4$



General Example

H_0: Pop. Mean = μ



5% = Size (Red Region)

= P(Observe $\bar{X}$ in Red, when $\bar{X}$ follows this distribution with mean μ)

= P(Observe $\bar{X}$ in Red, given that H_0 is true)

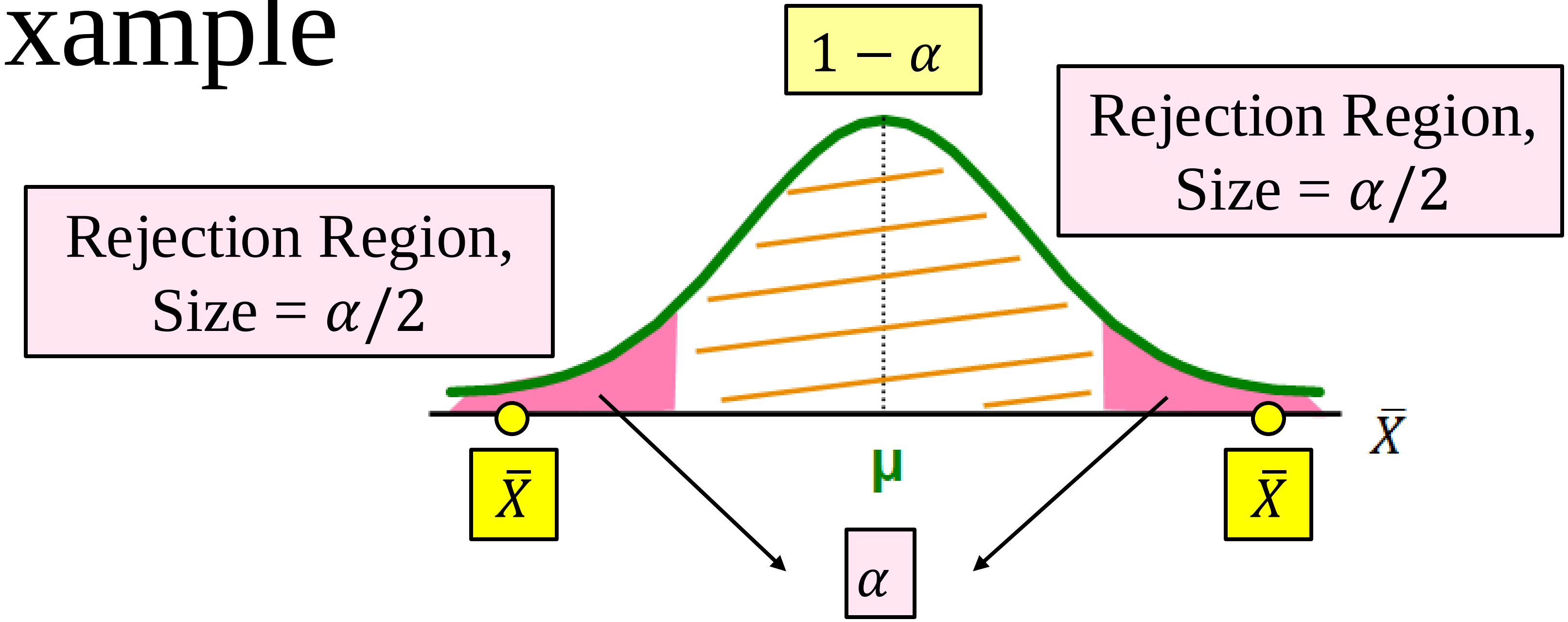
= P(Observe $\bar{X}$ in Red | H_0 is true)

= P($\bar{X}$ is too "extreme" to occur | H_0 is true)

= P(We find evidence to reject H_0 | H_0 is true)

= P(Observe $\bar{X}$ in Rejection Region | H_0 is true)

More General Example



H_0: pop. mean = μ

α = P(Observe $\bar{X}$ in Rejection Region | H_0 is true)

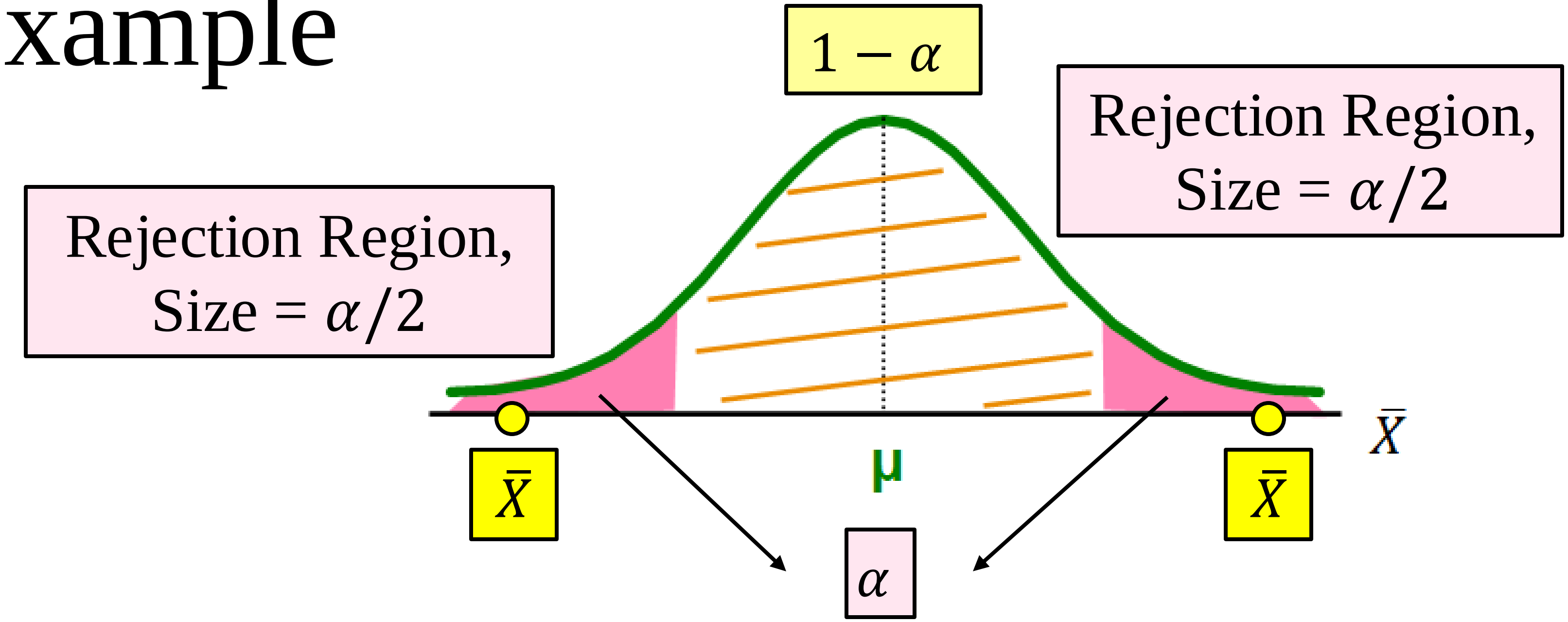
= P(Reject H_0 | H_0 is true)

= P(Incorrectly rejecting H_0 aka Type I error)

= Significance level of the hypothesis test

A situation that we dislike

More General Example



$1-\alpha$ = P(observe $\bar{X}$ outside the Rejection Region | H_0 is true)

= P(do not reject H_0 | H_0 is true)

= P(do not having a false alarm)

= confidence level of the hypothesis test

H_0: pop. mean = μ

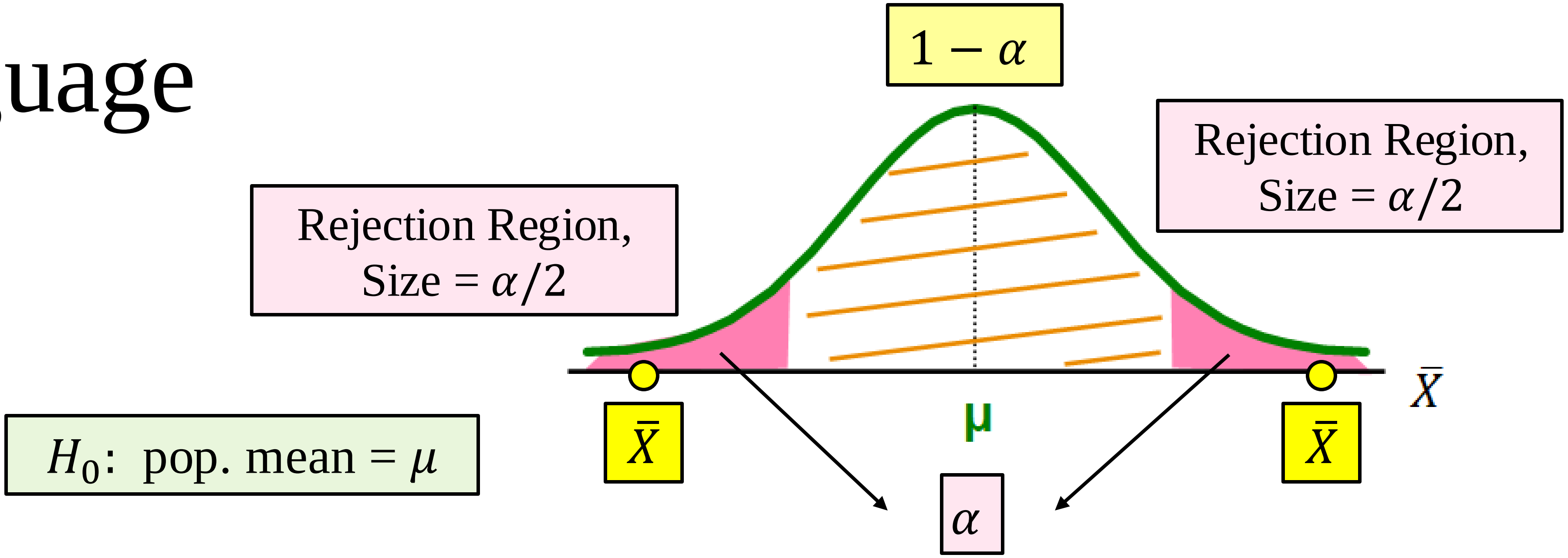
A situation that we like

$1-\alpha = 68\%$

$1-\alpha = 95\%$

$1-\alpha = 99.7\%$

Language



If $\bar{X}$ falls in the rejection region, we can reject H_0 with $100 \cdot (1-\alpha)\%$ confidence.

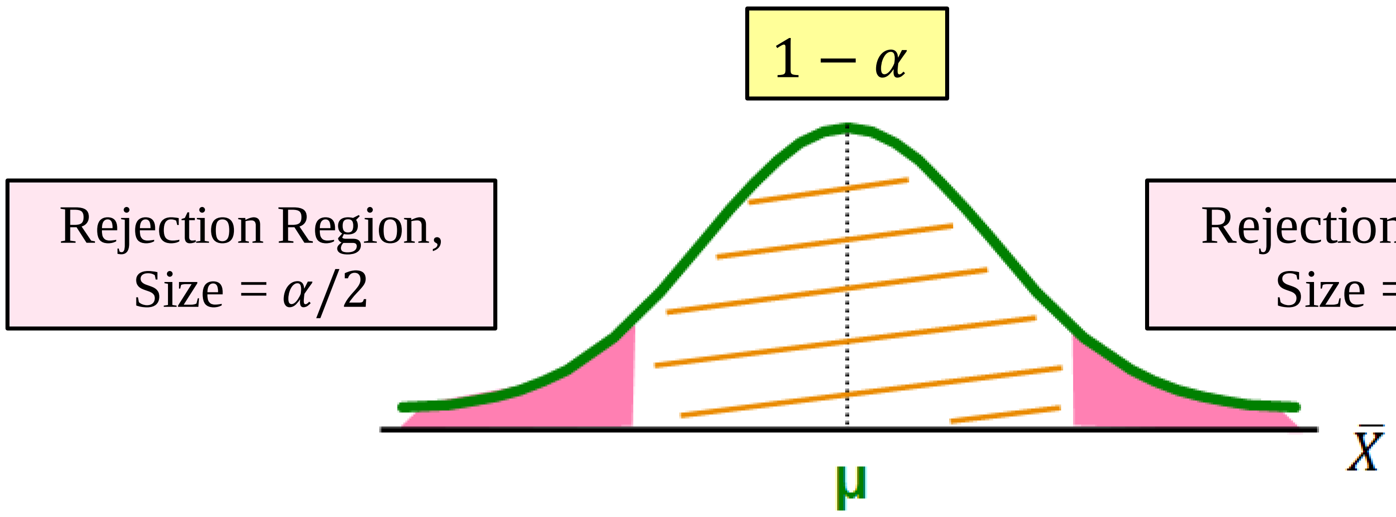
$\alpha = 0.32, \ldots .68\%$ confidence

$\alpha = 0.05, \ldots .95\%$ confidence

$\alpha = 0.003, \ldots .99.7\%$ confidence

If $\bar{X}$ falls outside the rejection region, H_0 can't be rejected with $100 \cdot (1-\alpha)\%$ confidence.

The Intuition



If $\bar{X}$ is close to the stated μ, H_0 is not rejected.

If $\bar{X}$ is far away from the stated μ, H_0 is rejected.

→ How far is "far enough" to reject H_0 ?

How Far Is Far Enough?

1. Critical Value Approach
2. p-Value Approach

Example 1: Critical Value Approach

… in 5 steps.

1. Set H_0 and H_A.
2. Identify significant level α and sample size n.
3. Use α to determine the critical values that divide the rejection and nonrejection regions.
4. Compute the test statistic (Z value).
5. Conclude:

If |test statistic| > |critical value|

→ $\bar{X}$ in the rejection region → Reject H_0

If |test statistic| < |critical value|

→ $\bar{X}$ in the nonrejection region → Can't reject H_0

Example 1: Critical Value Approach

Hawaii Tourism Authority had estimated that the average stay of Pro Bowl visitors is 10.1 days.

$H_0: \mu = 10.1$

$H_A: \mu \neq 10.1$

In 2003, they surveyed 260 Pro Bowl visitors, with an average stay of 9.1 days, and a standard deviation of 5.7 days.

Q. Can you reject their claim with 95% confidence?

Example 1: Critical Value Approach (95%)

1. $\boxed{H_0: \mu = 10.1}$ $\boxed{H_A: \mu \neq 10.1}$
2. $\alpha = 0.05$ $n = 260$
3. For $\alpha = 0.05$, the critical values are ± 2.
4. Compute the test statistics (Z value of $\bar{X}$):

$$Z = \frac{\bar{X} - \mu_{\bar{X}}}{\sigma_{\bar{X}}} = \frac{\bar{X} - \mu}{\sigma_{\bar{X}}} = \frac{9.1 - 10.1}{0.35} = -2.86$$

5. Compare test statistics and critical value:

$$|Z| = 2.86 > |\text{Critical Value}| = 2$$

Can reject the null hypothesis with 95% confidence.

$\bar{X} = 9.1$
$S = 5.7$

$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} \approx \frac{S}{\sqrt{n}} = \frac{5.7}{\sqrt{260}} = 0.35$$

Example 1: Critical Value Approach (99.7%)

1. $H_0: \mu = 10.1$ $\quad H_A: \mu \neq 10.1$
2. $\alpha = 0.003$ $\quad n = 260$
3. For $\alpha = 0.003$, the critical values are ± 3.
4. Compute the test statistics (Z value of $\bar{X}$):

$$Z = \frac{\bar{X} - \mu_{\bar{X}}}{\sigma_{\bar{X}}} = \frac{\bar{X} - \mu}{\sigma_{\bar{X}}} = \frac{9.1 - 10.1}{0.35} = -2.86$$

5. Compare test statistics and critical value:

$$|Z| = 2.86 < |\text{Critical Value}| = 3$$

Cannot reject the null hypothesis with 99.7% confidence.

$\bar{X} = 9.1$
$S = 5.7$

$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} \approx \frac{S}{\sqrt{n}} = \frac{5.7}{\sqrt{260}} = 0.35$$

Example 1: Critical Value Approach (99.7%)

If $\bar{X} = 8.6$, what do you think of the Hawaii Tourism Authority’ s claim?

Observation

If we can reject H_0 at a certain confidence level,

then we can reject H_0 at any confidence level lower than that.

The opposite does not hold.

Why? Can you show it in the graph?

Useful Facts: Hypothesis Testing

1. We need a sample mean and standard deviation of the sample mean.

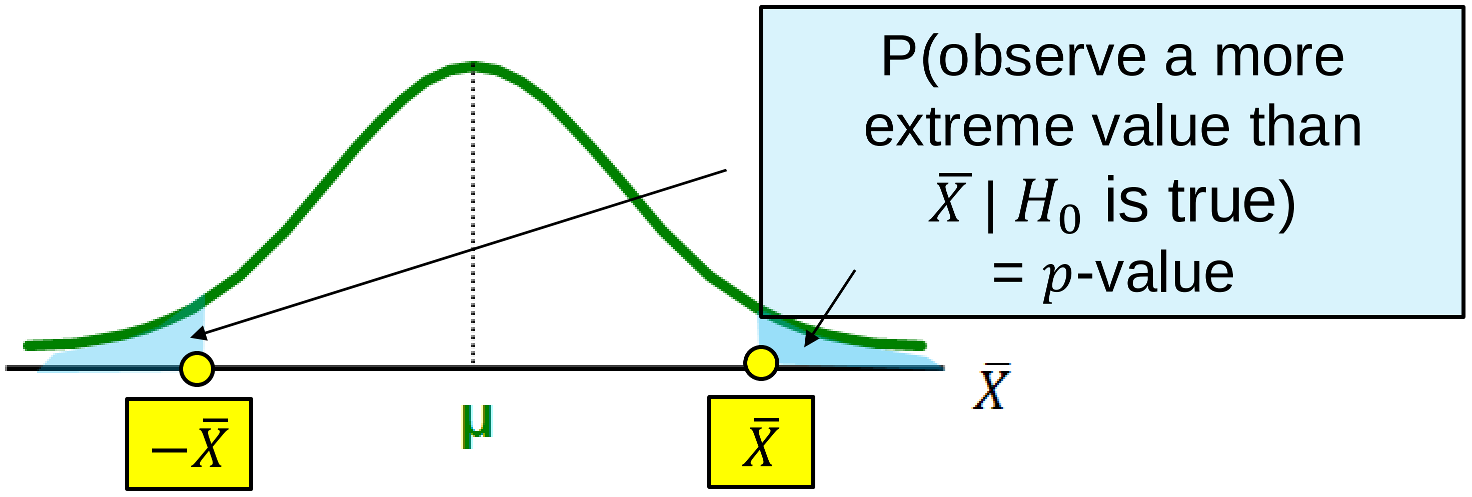
2. We construct a null and alternative hypothesis:

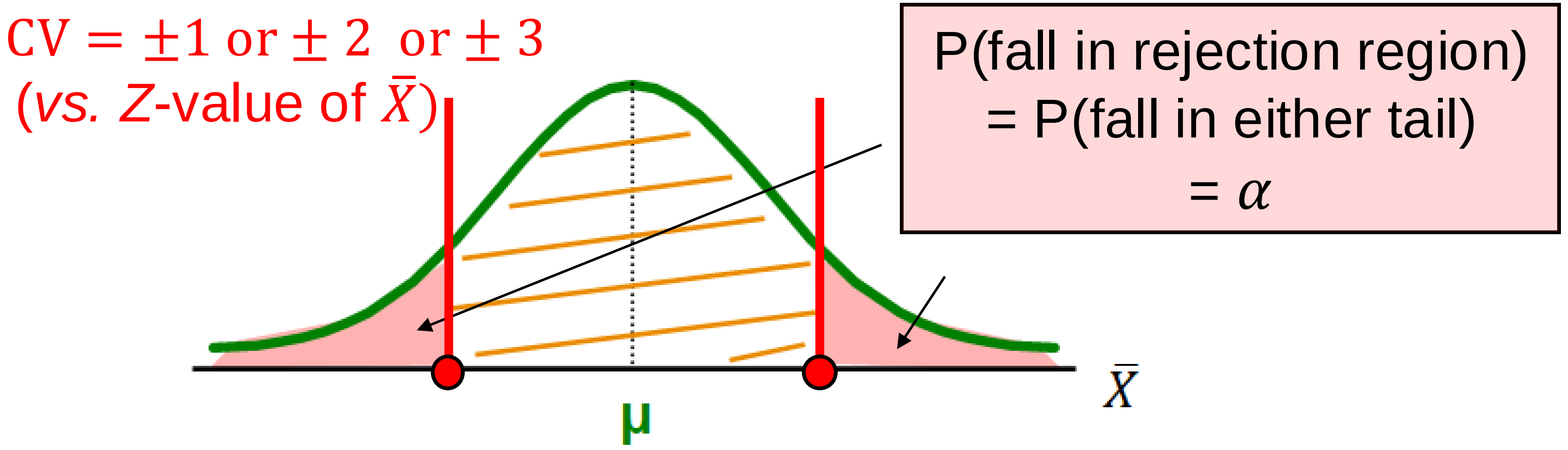
$$H_0\text{: } \mu = \mu_0 \text{ vs. } H_A\text{: } \mu \neq \mu_0$$

3. Using the critical value approach:

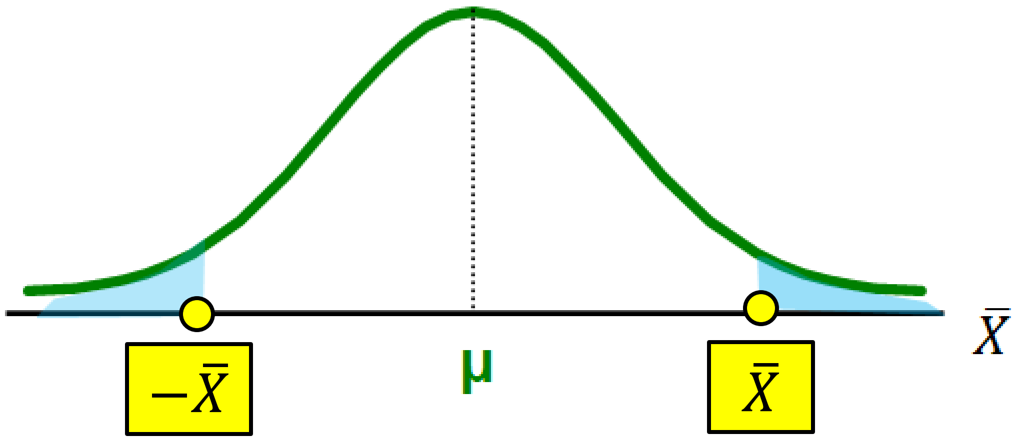
- If $\left|\text{test statistic Z} = \frac{\bar{X}-\mu}{\sigma_{\bar{X}}}\right| > |\text{critical value}|$ → Reject H_0
- If $\left|\text{test statistic Z} = \frac{\bar{X}-\mu}{\sigma_{\bar{X}}}\right| \leq |\text{critical value}|$ → Can't reject H_0

p-Value Approach to Testing





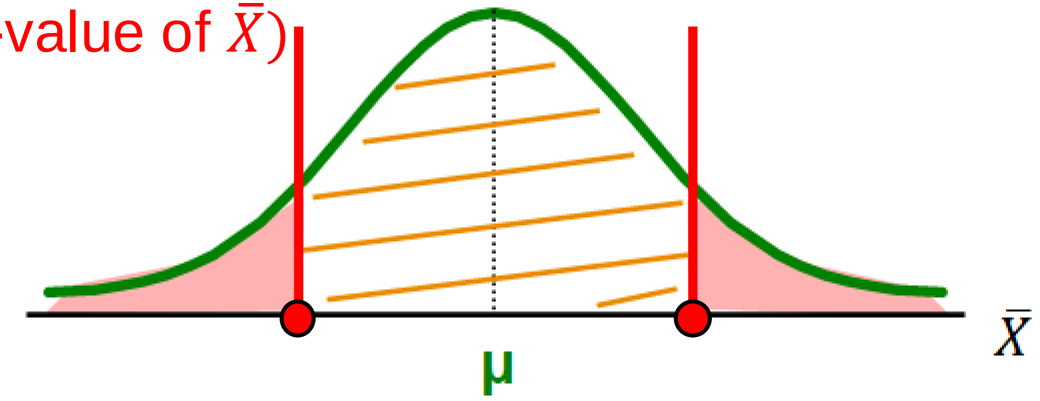
p-Value Approach to Testing



p-value =
Size of
Blue Region

$\mathrm{CV} = \pm 1 \text{ or } \pm 2 \text{ or } \pm 3$

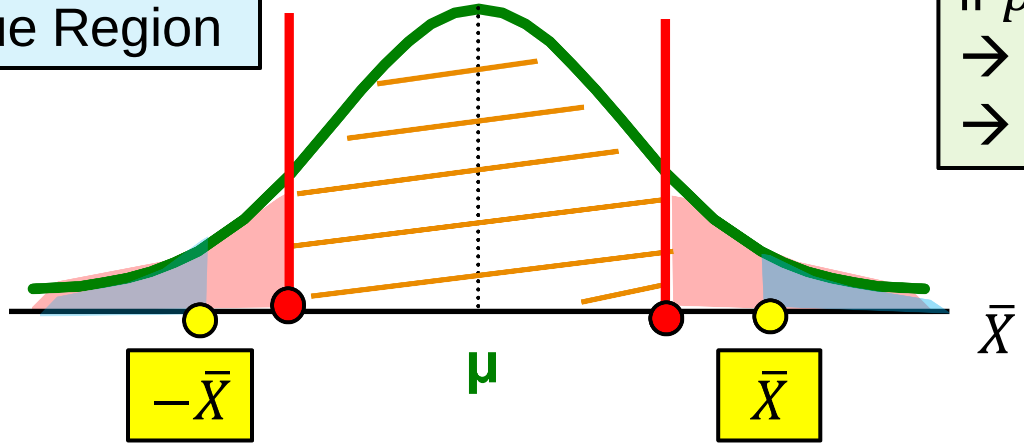
(*vs. Z*-value of $\bar{X}$)



α =
Size of
Red Region

p-Value Approach to Testing

p-value = Blue Region

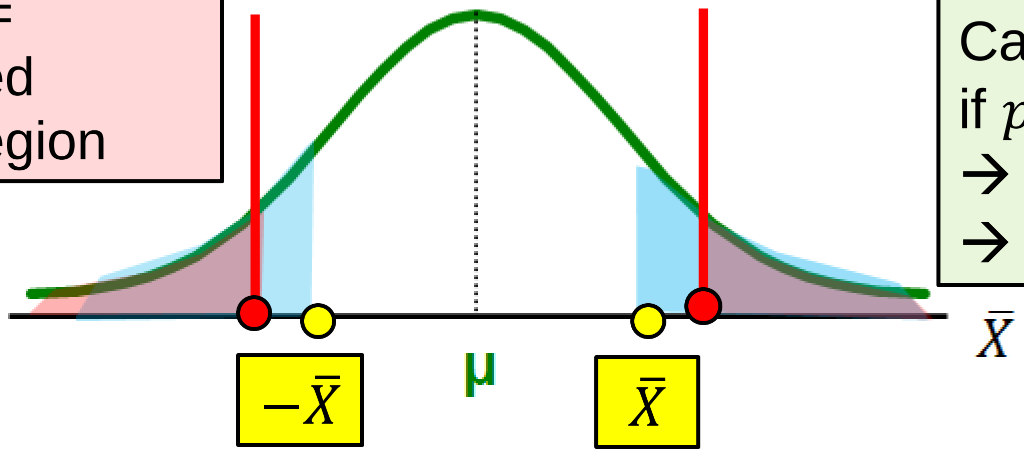


Case 1:
if p-value < α,
→ $\bar{X}$ is in rejection region
→ Reject H_0

at significance level α
(confidence level $1 - \alpha$)

Case 2:
if p-value ≥ α,
→ $\bar{X}$ is in nonrejection region
→ Can’t reject H_0

α = Red Region

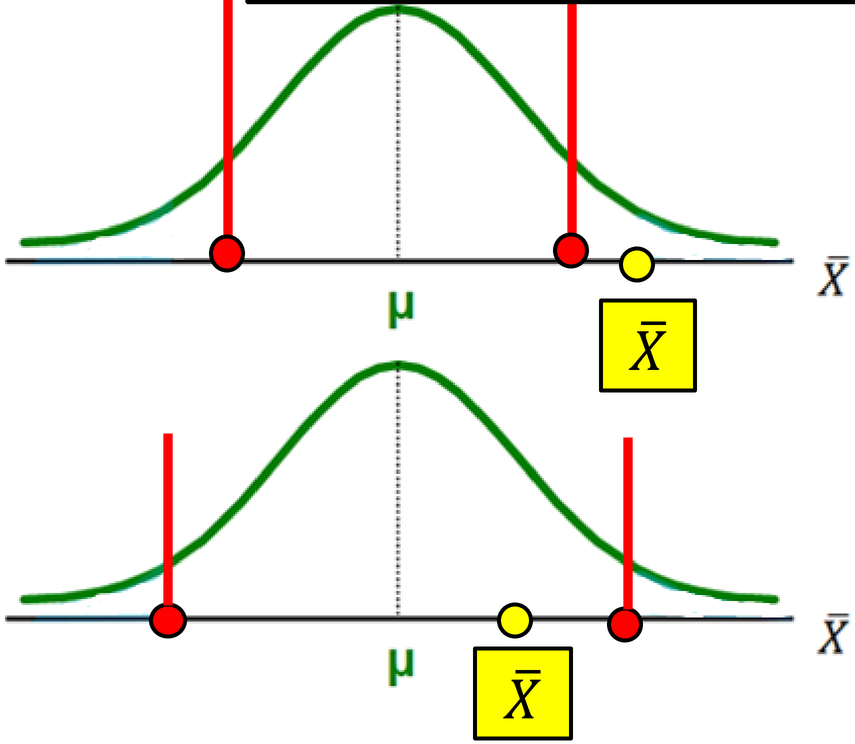


Critical Value vs p-Value

Critical value approach:

Compare numbers

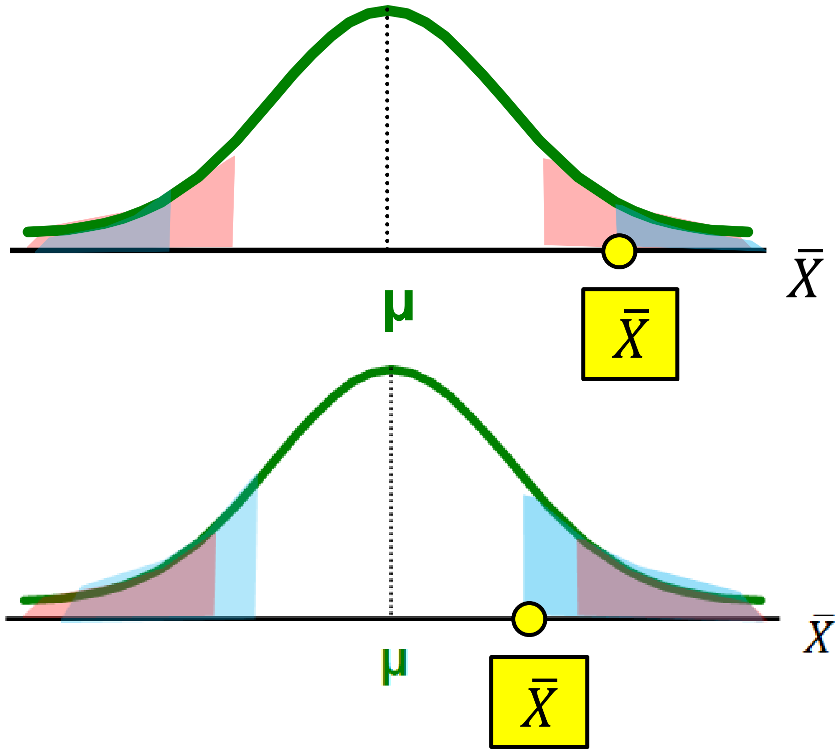
Test statistic (Z-value of $\bar{X}$)
vs. CV at ± 1 or ± 2 or ± 3



p-value approach:

Compare probabilities

p-value (size of blue region)
vs. α (size of red region)



p-Value Approach to Testing

… in 4 steps.

1. Set H_0 and H_A
2. Identify significant level α and sample size n
3. Compute the test statistic (z value) and p-value
4. Conclude:

→ $\bar{X}$ in the rejection region $\boxed{\text{If p-value} < \alpha}$

→ $\boxed{\text{Reject } H_0}$

→ $\bar{X}$ in the nonrejection region $\boxed{\text{If p-value} \geq \alpha}$

→ $\boxed{\text{Can't reject } H_0}$

Example 2: p-Value Approach

Hawaii Tourism Authority had estimated that the average stay of Pro Bowl visitors is 10.1 days.

$H_0: \mu = 10.1$

$H_A: \mu \neq 10.1$

In 2003, they surveyed 260 Pro Bowl visitors, with an average stay of 9.1 days, and a standard deviation of 5.7 days.

Q. Can you reject their claim with 95% confidence?

Example 2: p-Value Approach

1. $H_0: \mu = 10.1$ | $H_A: \mu \neq 10.1$
2. $\alpha = 0.05$ | $n = 260$
3. For $\alpha = 0.05$, the critical values are ± 2.
4. Compute the test statistics (Z value of $\bar{X}$):

$$Z = \frac{\bar{X} - \mu_{\bar{X}}}{\sigma_{\bar{X}}} = \frac{\bar{X} - \mu}{\sigma_{\bar{X}}} = \frac{9.1 - 10.1}{0.35} = -2.86$$

and the p-value.

$\bar{X} = 9.1$
$S = 5.7$

$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} \approx \frac{S}{\sqrt{n}} = \frac{5.7}{\sqrt{260}} = 0.35$$

Example 2: p-Value Approach

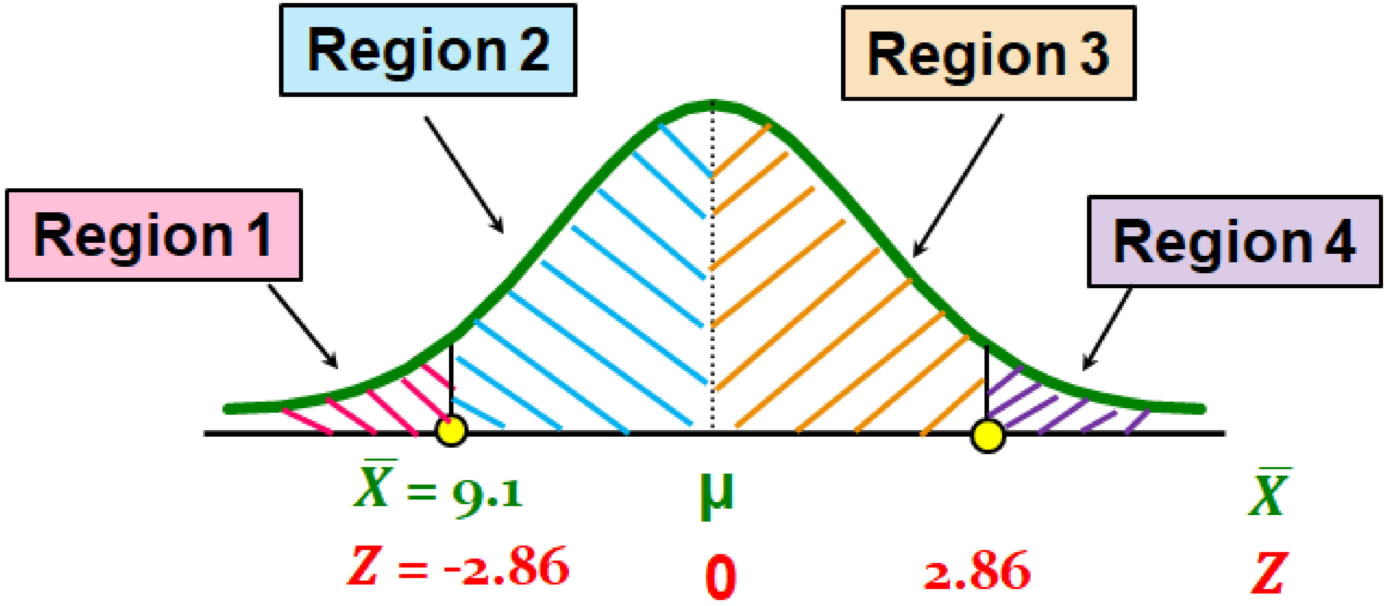
Q. Which region represents the p-value of the test statistic -2.86?

(A) Region 1

(B) Region 4

(C) Region 1 + Region 4

(D) Region 2 + Region 3



Example 2: p-Value Approach

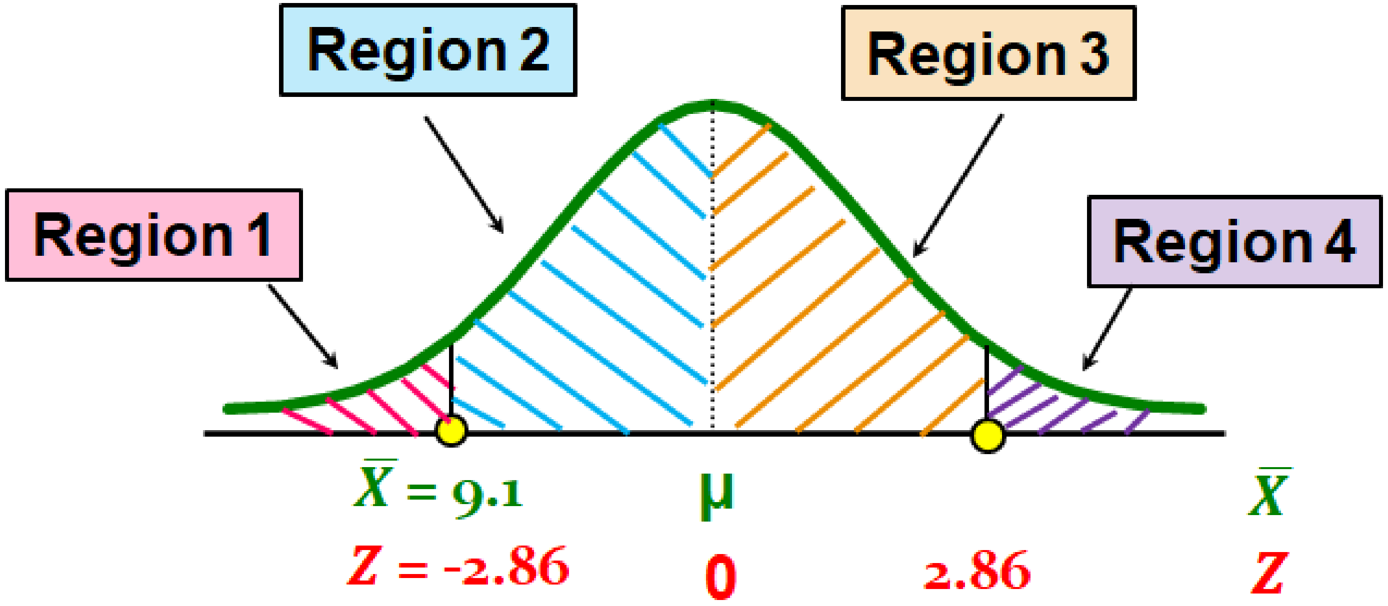
Q. Which region represents the p-value of the test statistic -2.86?

(A) Region 1

(B) Region 4

(C) Region 1 + Region 4 = 2 ×Region 1 = 2 ×P(Z < -2.86)

(D) Region 2 + Region 3



Example 2: p-Value Approach

Q. Use python to find $P(Z < -2.86)$.

Example 2: p-Value Approach

1. $H_0: \mu = 10.1$ $H_A: \mu \neq 10.1$
2. $\alpha = 0.05$ $n = 260$
3. For $\alpha = 0.05$, the critical values are ± 2.
4. Compute the test statistics (Z value of $\bar{X}$):

$$Z = \frac{\bar{X} - \mu_{\bar{X}}}{\sigma_{\bar{X}}} = \frac{\bar{X} - \mu}{\sigma_{\bar{X}}} = \frac{9.1 - 10.1}{0.35} = -2.86$$

   and the p-value.
5. Compare the test statistics and α:

   p-value = 2 × P(Z < -2.86) = 2 × 0.002
   = 0.004 < $\alpha = 0.05$

Can reject the null hypothesis with 95% confidence

$\bar{X} = 9.1$
$S = 5.7$

$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} \approx \frac{S}{\sqrt{n}} = \frac{5.7}{\sqrt{260}} = 0.35$$

How to Conclude

$\boxed{H_0: \mu = 10.1}$ $\boxed{H_A: \mu \neq 10.1}$

Statistical conclusion:

- Can't reject H_0 with 95% confidence.
- Can't reject H_0.

Practical conclusion:

- Can't reject the hypothesis that the average stay is 10.1 days
- The average stay is not significantly different from 10.1 days

***All statements work*!**

Useful Facts: Hypothesis Testing

1. We need a sample mean and standard deviation of the sample mean.

2. We construct a null and alternative hypothesis:

$$H_0\colon \mu = \mu_0 \text{ vs. } H_A\colon \mu \neq \mu_0$$

3. Using the critical value approach:
   - If $|\text{test statistic } Z = \frac{\bar{X}-\mu}{\sigma_{\bar{X}}}| > |\text{critical value}|$ → Reject H_0
   - If $|\text{test statistic } Z = \frac{\bar{X}-\mu}{\sigma_{\bar{X}}}| \leq |\text{critical value}|$ → Can't reject H_0

4. Using the p-value approach:
   - If p-value $< \alpha$ → Reject H_0
   - If p-value $\geq \alpha$ → Can't reject H_0

Example 2: p-Value Approach

1. $\boxed{H_0: \mu = 10.1}$ $\boxed{H_A: \mu \neq 10.1}$
2. $\alpha = 0.05$ $n = 260$
3. For $\alpha = 0.05$, the critical values are ± 2.
4. Compute the test statistics (Z value of $\bar{X}$):

$$Z = \frac{\bar{X} - \mu_{\bar{X}}}{\sigma_{\bar{X}}} = \frac{\bar{X} - \mu}{\sigma_{\bar{X}}} = \frac{9.1 - 10.1}{0.35} = -2.86$$

   and the p-value.
5. Compare the test statistics and α:

   p-value = 2 × P(Z < -2.86) = 2 × 0.002

   = 0.004 < $\alpha = 0.05$

Can reject the null hypothesis with 95% confidence

$$\bar{X} = 9.1$$
$$S = 5.7$$

$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} \approx \frac{S}{\sqrt{n}} = \frac{5.7}{\sqrt{260}} = 0.35$$

Example 2: p-Value Approach (extension)

1. $H_0: \mu = 10.1$ $H_A: \mu \neq 10.1$
2. $\alpha = 0.05$ $n = 260$
3. For $\alpha = 0.05$, the critical values are ± 2.
4. Compute the test statistics (Z value of $\bar{X}$):
$$Z = \frac{\bar{X} - \mu_{\bar{X}}}{\sigma_{\bar{X}}} = \frac{\bar{X} - \mu}{\sigma_{\bar{X}}} = \frac{10.5 - 10.1}{0.35} = 1.14$$
and the p-value.
5. Compare the test statistics and α:
$$\text{p-value} = 2 \times P(Z > 1.14) = 2 \times (1 - 0.87) = 2 \times 0.13$$
$$= 0.26 > \alpha = 0.05$$

Cannot reject the null hypothesis with 95% confidence

$\bar{X} = 10.5$
$S = 5.7$

$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} \approx \frac{S}{\sqrt{n}} = \frac{5.7}{\sqrt{260}} = 0.35$$

Rule of Thumb in Practice

Most Commonly Used confidence level

$$\boxed{\mathbf{1-\alpha=95\%}}$$

Most Commonly Used significance level

$$\boxed{\boldsymbol{\alpha}=\mathbf{0.05}}$$

Reject the null $(\mathrm{H_0})$ in favor of $\mathrm{H_A}$ if

$$\boxed{p\text{-}value<\alpha=0.05}$$

p-Value: What it is and what it is not

One rejects the null hypothesis H_0 when

Hypothesis Testing

$$\text{Prob}\left(\boxed{\begin{matrix}\text{Sampling data is more extreme}\\ \text{than the data you have observed}\end{matrix} \;\middle|\; \begin{matrix}H_0\\ \text{is true}\end{matrix}}\right) \leq \alpha$$

This is not

Confidence Interval

$$\text{Prob}\left(\boxed{\begin{matrix}\text{Null hypothesis } H_0\\ \text{is true}\end{matrix} \;\middle|\; \begin{matrix}\text{Observed}\\ \text{data}\end{matrix}}\right) \leq \alpha$$

We cannot interpret the p-value as the probability that the Null Hypothesis is true given our data!