Exam 3 | Example

Part 1. True/False (3 points; no explanation needed)

Make judgement of each of the following statements, and circle TRUE or FALSE.

1. Suppose, in testing a hypothesis about a mean, the p-value is computed to be 0.043. The null hypothesis can be rejected with a confidence level of 97%.

Claim: TRUE (FALSE) We need the p-value to be below 0.03.

2. The larger the p-value, the more likely you are to reject the null hypothesis.

Claim: TRUE (FALSE) The smaller the p-value, the less likely the null hypothesis is to be true.

3. The width of a confidence interval for the population mean depends entirely on the sample size and the value of the population standard deviation.

Claim: TRUE (FALSE) We also need the level of confidence.

Part II. Short-Answer Questions (27 points; show your work)

4. (7 points) An entrepreneur is considering the purchase of a coin-operated laundry. The current owner claims that over the past five years, the mean daily revenue was \$675. A sample of 30 days reveals a daily mean revenue of **\$645** and a standard deviation of **\$75**.

$\bar{x} = \$645$ $\quad \sigma_{\bar{x}} = \frac{s}{\sqrt{n}} = 13.7$

$s = \$75$

$n = 30$ $\quad z = \frac{|\bar{x} - \mu|}{\sigma_{\bar{x}}} = \frac{30}{13.7} = 2.19$

Suppose that you want to test the claim made by the current owner.

a) What is the null hypothesis? $H_0: \mu = \$675$.

b) Based on this sample, can you reject the null hypothesis? Justify your answer. We can reject the null at 95% confidence.

5. (20 points) *Bigbucks* is a chain of coffee shops with thousands of stores across the world. *Bigbucks* wants to introduce a new line of espresso drinks. Their marking department is big on big data and started to collect different sources of data a few months ago. Specifically, they collected data on the revenue of the new line of espresso drinks for their 36 stores in Seattle and 64 stores in Portland. In Seattle, the average revenue across those stores was **\$920** and the sample standard deviation of the revenue of those stores was **\$85**. In Portland, the average revenue across those stores was **\$1022** and the sample standard deviation of the revenue of those stores was **\$88**.

a) Construct the 95% confidence interval for the mean revenue of the new line of espresso drinks in Seattle.

b) Define the width of a confidence interval as the difference between the upper and lower bound. Suppose the average revenue and sample standard deviation of the revenues in Seattle remain unchanged. Then how many stores must *Bigbucks* have in Seattle in order for the 68% confidence interval of the mean revenue to be fifteen-dollar wide?

c) Would you claim that the mean revenue across the stores in Portland is at least **\$1000**? Justify your answer.

d) Bigbucks also wants to determine whether or not their current ad campaign is effective. They interviewed a random sample of 100 customers. Among those interviewed, 80 customers said that they have seen the campaign. Construct the 95% confidence interval for the proportion of Bigbucks customers who have seen the campaign.

SEATTLE	PORTLAND
$n=36$	$n=64$
$\bar{x}=\$920$	$\bar{x}=\$1022$
$s=\$85$	$s=\$88$

a) 95% CI in SEATTLE

$\bar{x} \pm 2 \cdot \sigma_{\bar{x}}$

$\sigma_{\bar{x}} = \frac{85}{\sqrt{36}} \approx 14.17$

$2 \cdot \sigma_{\bar{x}} = 28.3$

$[891.7, 948.3]$

b) This is the same as the range having:

$\sigma_{\bar{x}} = \frac{15}{2}$

$\hookrightarrow \frac{85}{\sqrt{n}} = \frac{15}{2}$

$\hookrightarrow \sqrt{n} = \frac{2}{15} \cdot 85 \approx 11.3$

$\hookrightarrow n \approx 128.4$

c)

$H_0: \mu = \$1000$

$\sigma_{\bar{x}} = \frac{88}{\sqrt{64}} = 11$

So how far away is μ from $\bar{x}$?

$Z = \frac{|1000-1022|}{\sigma_{\bar{x}}} = 2$

We are 95% confident they are different.

d) We use the sample proportion.

$\hat{p} = \frac{80}{100} = 0.8$

$s^2 = 0.8 \cdot 0.2 = 0.16$

$\sigma_{\hat{p}} = \frac{s}{\sqrt{n}} = \frac{\sqrt{0.16}}{10} = 0.04$

95% CI $\rightarrow 0.8 \pm 2 \cdot 0.04$

$[0.8-0.08, 0.8+0.08]$

$\hookrightarrow [0.72, 0.88]$