ECON 0150 | Fall 2024 | Homework 3

Due: Sunday, October 13

Homework is designed to both test your knowlege and challenge you to apply familiar concepts in new applications. Answer clearly and completely. You are welcomed and encouraged to work in groups so long as your work is your own. Use the datafiles to answer the following questions. Then submit your figures and answers to Gradescope.

Q1. The employees of a company are surveyed on questions regarding their educational background (college degree or no college degree) and marital status (single or married). Of the 600 employees, 400 have college degrees, 100 are single, and 60 are single college graduates.

a. Based on the information above, what is the probability that an employee of the company is married?

A) 0.17

B) 0.33

C) 0.67

D) 0.83

Answer. Being single and being married are mutually exclusive events. So P(Single) + P(Married) = 1. Since the probability of being single is P(Single) = 100 / 600 = 1 / 6, the probability of being married is P(Married) = 5 / 6 = 0.83.

b. Based on the information above, what is the probability that an employee of the company is single or has a college degree?

A) 0.10

B) 0.67

C) 0.73

D) 0.83

Answer. Consider being "single" as one event and having a "college degree" as another event. Then this problem is asking for the probability of the union of these two events: P(Single *or* College). We can find this using the addition rule:

P(Single *or* College)

= P(Single) + P(College) - P(Single *and* College)

= 100 / 600 + 400 / 600 - 60 / 600

= 1/ 6 + 2 / 6 - 1 / 10

= 0.73

c. The probability that a new car requires engine work under warranty is 0.1, while the probability that it requires transmission work under warranty is 0.02. The probability that it requires both engine and transmission work under warranty is 0.01. What is the probability that it needs work on either the engine, the transmission, or both?

A) 0.13

B) 0.12

C) 0.11

D) 0.09

Answer. Consider "requiring engine work" as one event and "requiring transmission work" as another event. Then this problem is asking for the union of these two events: P(Engine *or* Transmission). We can find this using the addition rule:

P(Engine *or* Transmission)

= P(Engine) + P(Transmission) - P(Engine *and* Transmission)

= 0.1 + 0.02 - 0.01

= 0.11

Q2. A survey is taken among customers of a fast-food restaurant to determine preference for hamburger or chicken. Among the respondents, 75% were adults, 55% preferred hamburger, and 20% were children who preferred chicken. Use a probability grid or table to guide your answer.

a. Based on the information above, what is the probability that a randomly selected respondent prefers chicken?

A) 15%

B) 25%

C) 35%

D) 45%

Answer. Since we're asked to find P(Chicken), we'll set up Chicken and Hamburger on vertical of the probability grid. From here it's easy to see that:

P(Chicken) = 1 - P(Hamburger) = 1 - 0.55 = 0.45

b. Based on the information above, what is the probability that a randomly selected respondent is a child who prefers hamburger?

A) 5%

B) 10%

C) 15%

D) 20%

Q3. Suppose that patrons of a restaurant were asked whether they preferred water or soda. 30% said that they preferred soda, 60% of the patrons were not female, and 20% of the patrons were female who preferred water. Use a probability grid or table to guide your answer.

a. Based on the information above, what is the probability that a randomly selected patron prefers water?

A) 30%

B) 40%

C) 60%

D) 70%

b. Based on the information above, what is the probability that a randomly selected patron prefers soda and is not female?

A) 5%

B) 10%

C) 15%

D) 20%

Q4. If A and B are mutually exclusive events with $P(A) = 0.25$ and $P(B) = 0.3$, then what is the value of P(A | B)?

A) 0

B) 0.075

C) 0.25

D) 0.3

Answer. Since both events are mutually exclusive, they cannot happen at the same time and P(A *and* B) = 0. Since the conditional probability P(A | B) is the ratio of P(A *and* B) and P(B), then P(A | B) = 0.

Q5. If $P(A \text{ or } B) = \frac{2}{3}$, $P(A \text{ and } B) = \frac{1}{3}$, and $P(A) = \frac{1}{2}$, then which of the following statements is true?

A) $P(B) = \frac{5}{6}$

B) $P(A|B) = \frac{3}{6}$

C) A and B are not independent

D) All of the above

Answer. We have to check the first three options.

Option A: From the addition rule we get:

P(A *or* B) = P(A) + P(B) - P(A *and* B)

P(B)

= P(A *or* B) + P(A *and* B) - P(A)

= 2 / 3 + 1 / 3 - 1 / 2

= 1 / 2

So Option A is incorrect.

Option B: Using the conditional probability formula and the result from Option A, we get:

P(A | B)

= P(A *and* B) / P(B)

= (1 / 3) / (1 / 2)

= 2 / 3

So Option B is incorrect too.

Option C: Since P(A | B) = 2 / 3 which is not equal to P(A) = 1 / 2, then they cannot be independent. The conditional probability is not equal to the marginal probability. So Option C is incorrect too.

Q6. You are a physician meeting with a patient who has just been diagnosed with cancer. You know there are two mutually exclusive types of cancer that the patient could have: type A and type B. The probability that she has type A cancer is $\frac{1}{3}$. Type A is deadly: four patients out of five diagnosed with type A cancer die within one year. Type B is less dangerous: only one patient out of five diagnosed with type B cancer dies within one year. *Hint: Ask first, what the numbers provided in the story are really about? Then ask, how do these numbers fit with a probability grid?*

a. What is the probability that your patient has type A cancer and successfully survives it?

Answer. P(A *and* N) = 1 / 15

b. What is the probability that your patient dies within a year?

Answer. P(D) = 2 / 5

c. Suppose that your patient dies in less than one year before you learn the exact type of cancer he has. Given this sad happening, what is the probability that she had type A cancer?

Answer. P(A | D) = 2 / 3

Q7. Taylor always drinks tea after arriving at Posvar Hall in the morning, while Katherine and Jane sometimes join him. The probability that Katherine drinks tea with Taylor is $\frac{1}{4}$ and the probability that Jane drinks tea with Taylor is $\frac{3}{8}$. The probability that Taylor drinks tea by himself is $\frac{1}{2}$. *Hint: Ask first, what the numbers provided in the story are really about? Then ask, how do these numbers fit with a probability grid?*

a. What is the probability that Taylor has tea with Jane while Katherine is not there?

Answer. P(NotKatherine *and* Jane) = 1 / 4

b. If Katherine did not have tea with Taylor, what is the probability that Jane was not there either?

Answer. P(NotJane | NotKatherine) = 2 / 3

c. If Taylor had tea with Jane this morning, what is the probability that Katherine did not join them?

Answer. P(NotKatherine | Jane) = 2 / 3