Probability and Statistics

Part 2.1 Probability, Joint Probability, Conditional Probability

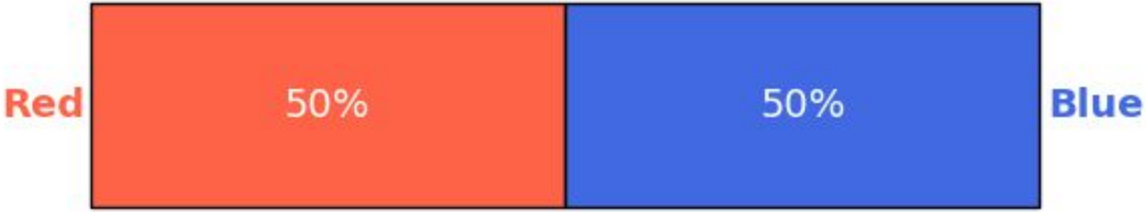
Taylor Weidman

Probability Theory

Q. What probability might we place on the red team winning if we know nothing about the game?

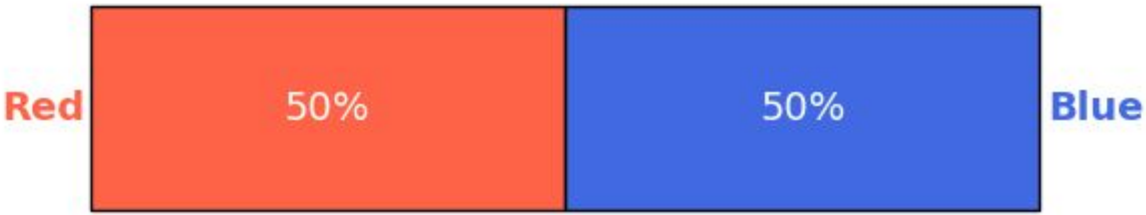
Probability Theory

Q. What probability might we place on the red team winning if we know nothing about the game?



Probability Theory

Q. What might be different if we knew the blue team is the current European champion?



Probability Theory

Q. What might be different if we knew the blue team is the current European champion?

Probability Theory

Q. What if the game were recorded and you had already seen the final score?

Probability Theory

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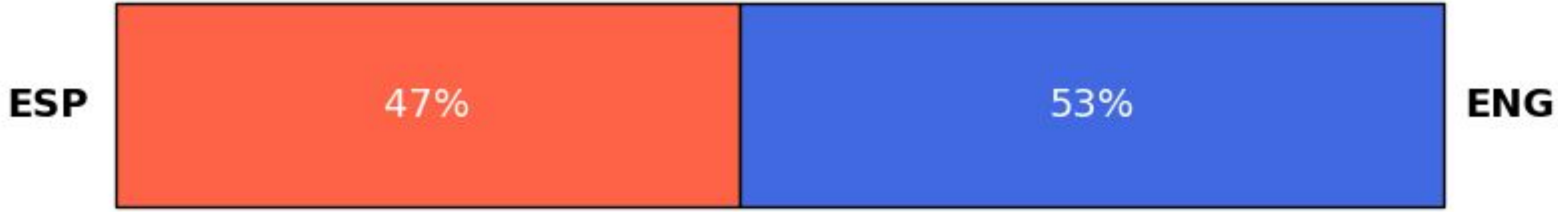
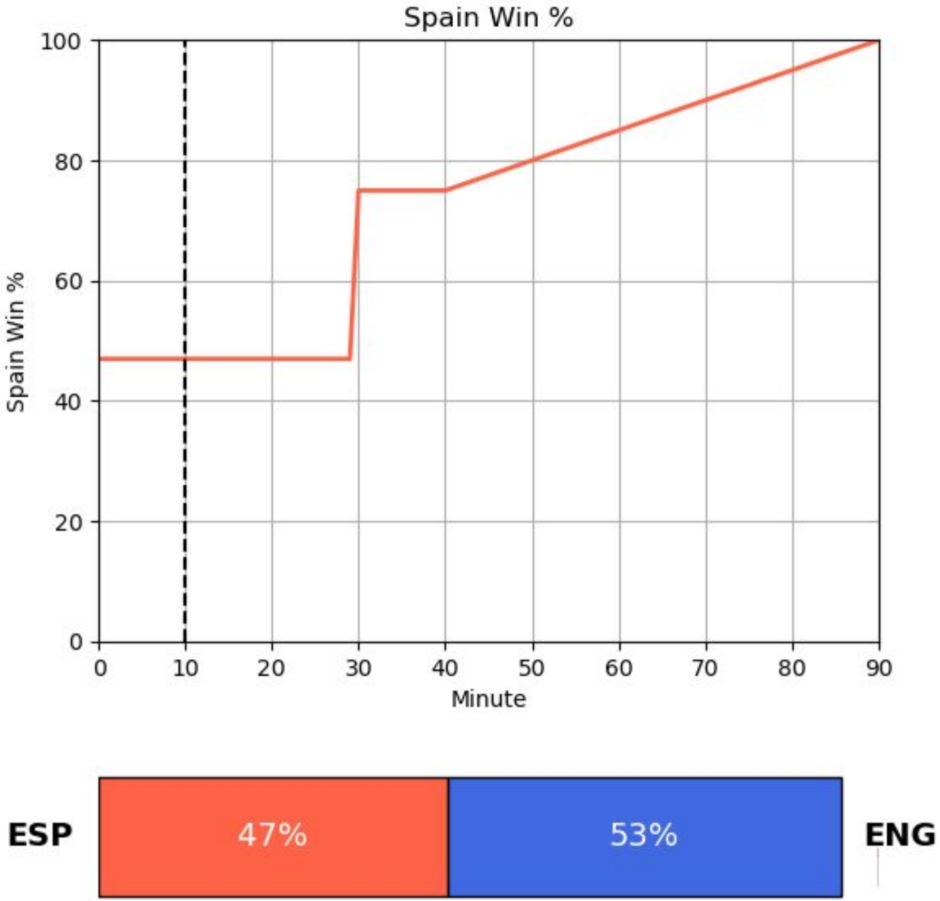
2023 Women's World Cup Final

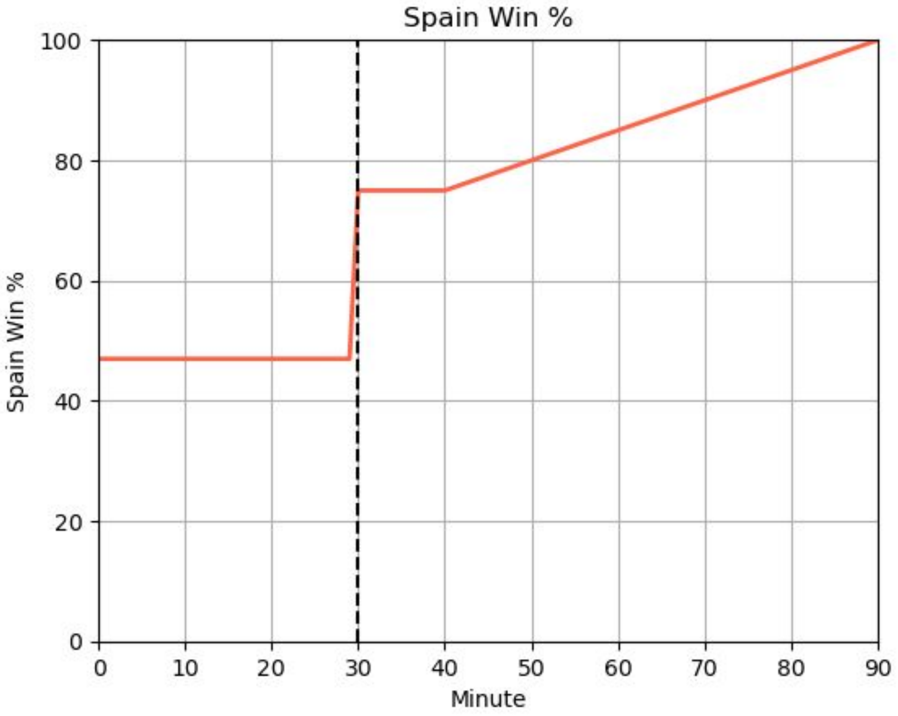
Predictions going into the game…



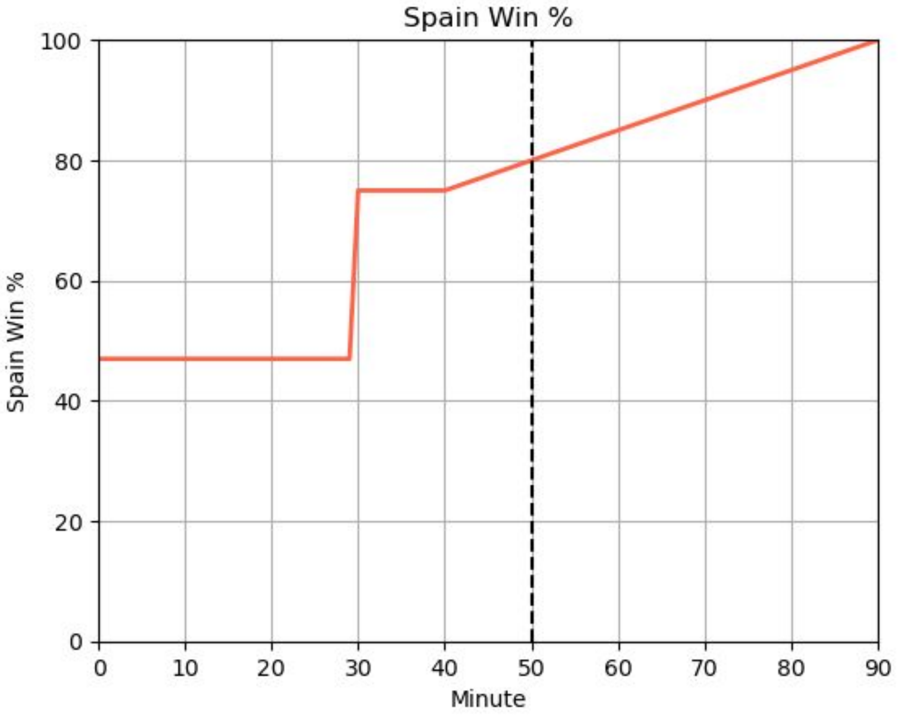
2023 Women’s World Cup Final



2023 Women's World Cup Final



2023 Women’s World Cup Final



2023 Women’s World Cup Final

Q. If we were to run a simulation 1000 times with Spain winning 400 times, what’s the probability of Spain winning the match?

2023 Women’s World Cup Final

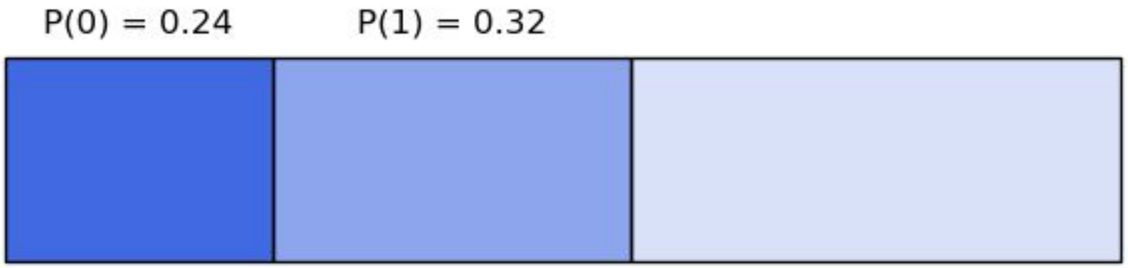
Q. If we were to run a simulation 1000 times with Spain winning 400 times, what’s the probability of Spain winning the match?

A. 40%!

English Premier League Matches (2013 - 2023)

Q. How many games have a home team score of 0?

Q. How many games have a home team score of 1?

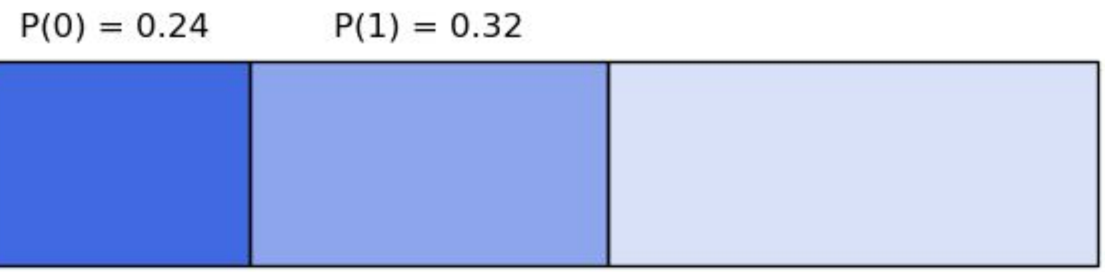


English Premier League Matches (2013 - 2023)

Q. How many games have a home team score of 0?

Q. How many games have a home team score of 1?

Q. How many games have a home team score of 2 or more?



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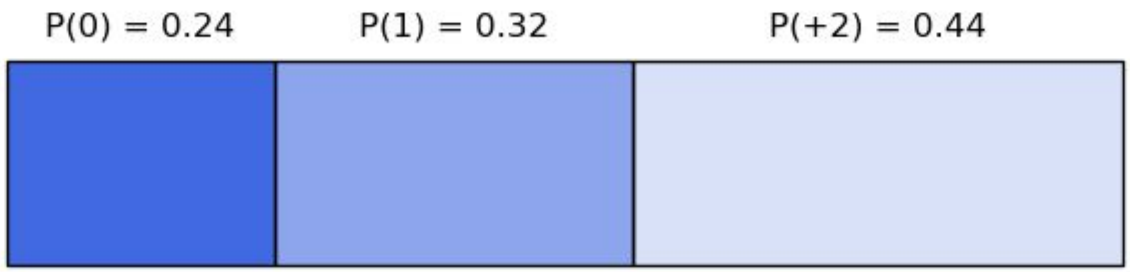
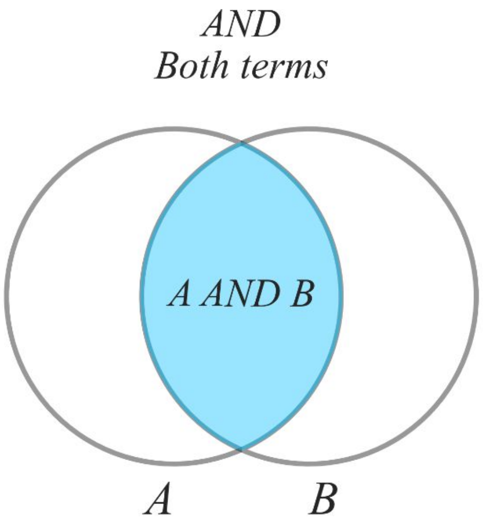


Excel Exercise: English Premier League Matches

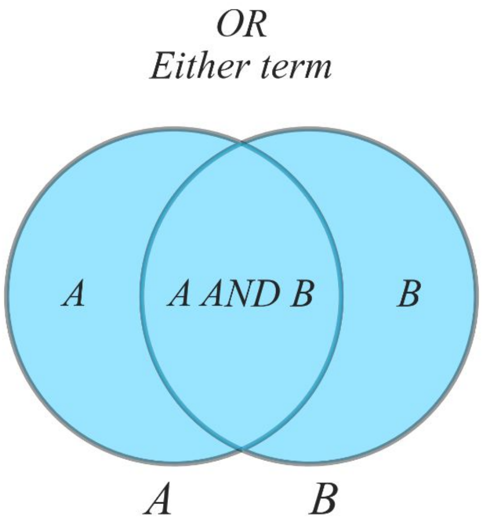
- Data: Part_2_1_Premier_League_Matches.csv
- Filter to find how often the home team scores: 0, 1, 2+.
- Filter to find how often the away team scores: 0.

Joint Probability

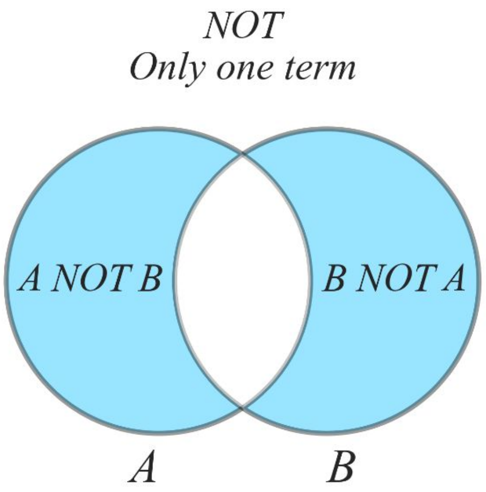
… is the probability of two (or more) events happening.



P(A AND B)



P(A OR B)



P(A NOT B) P(B NOT A)

English Premier League Matches (2013 - 2023)

$P(H0) = 0.24$, $P(A0) = 0.33$

Q. How often does at least one team not score?

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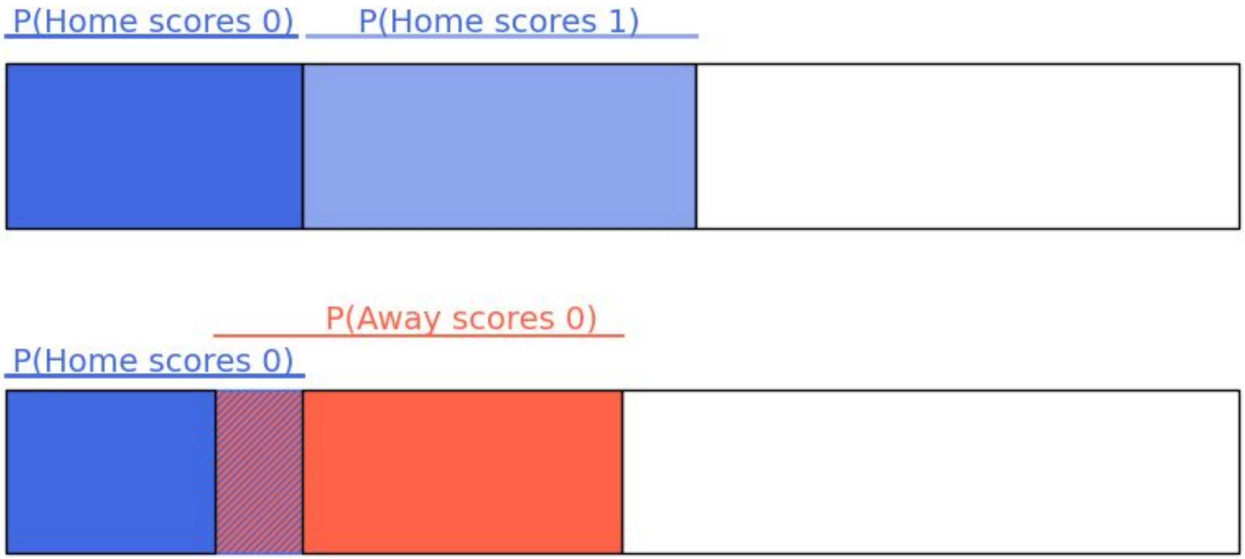

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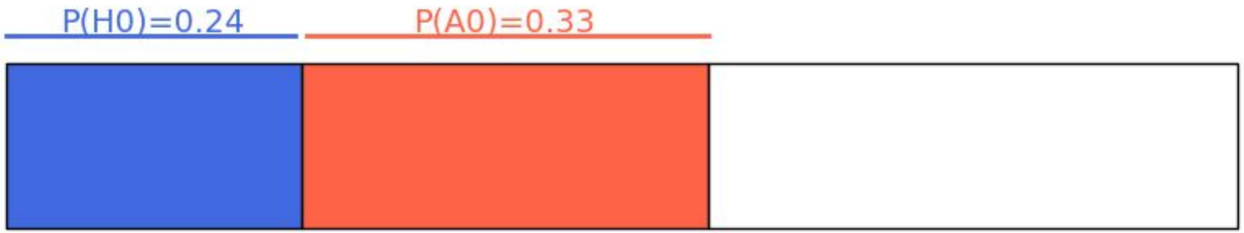
Incorrect



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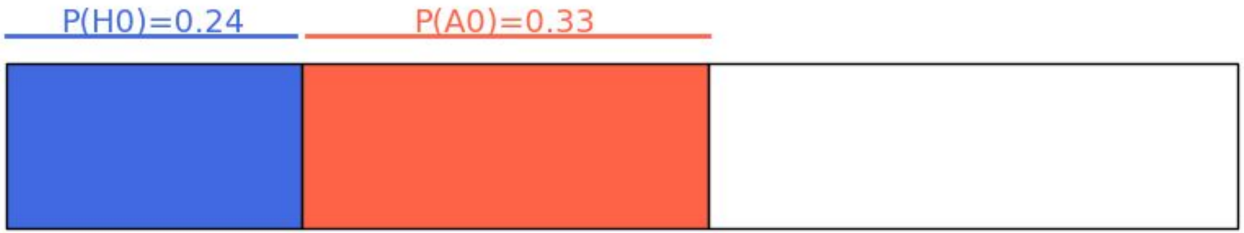

We need to know how often neither team scores.

English Premier League Matches (2013 - 2023)

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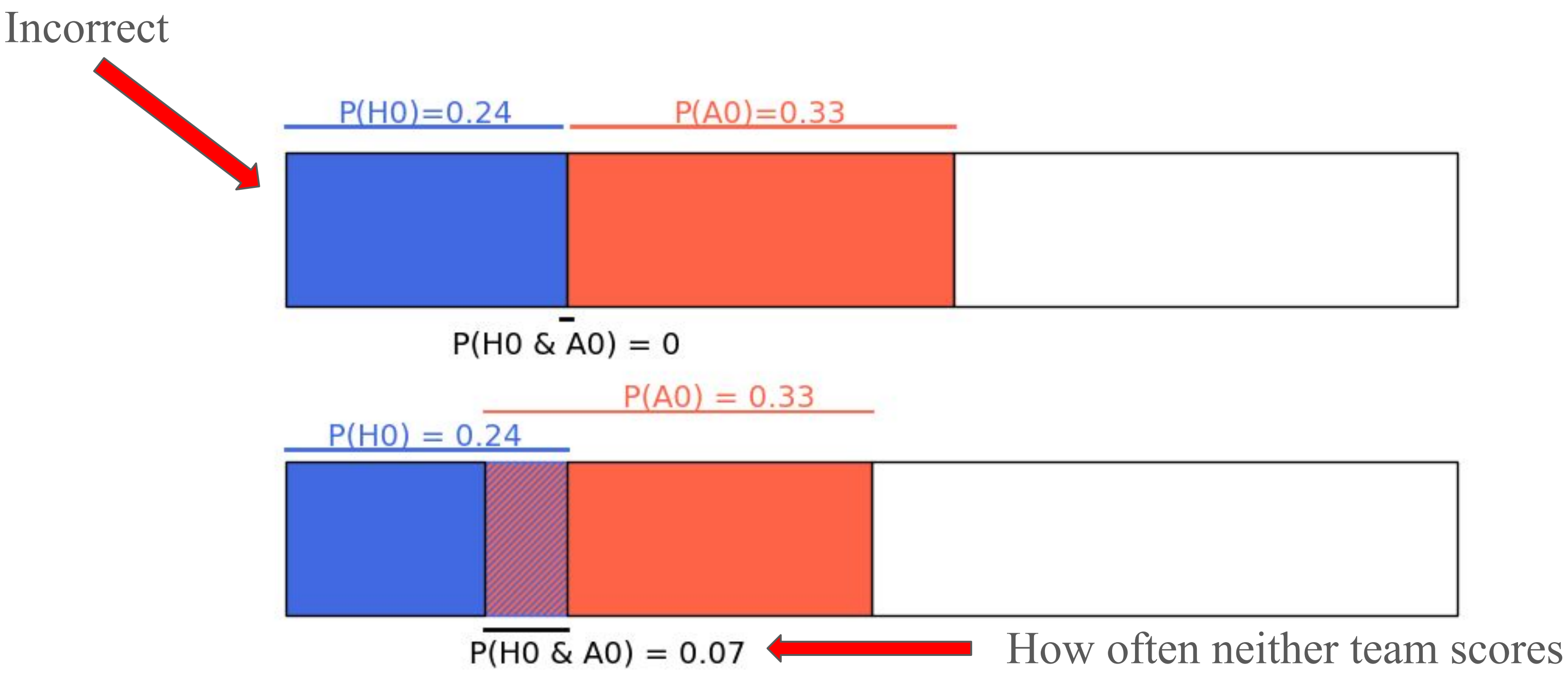


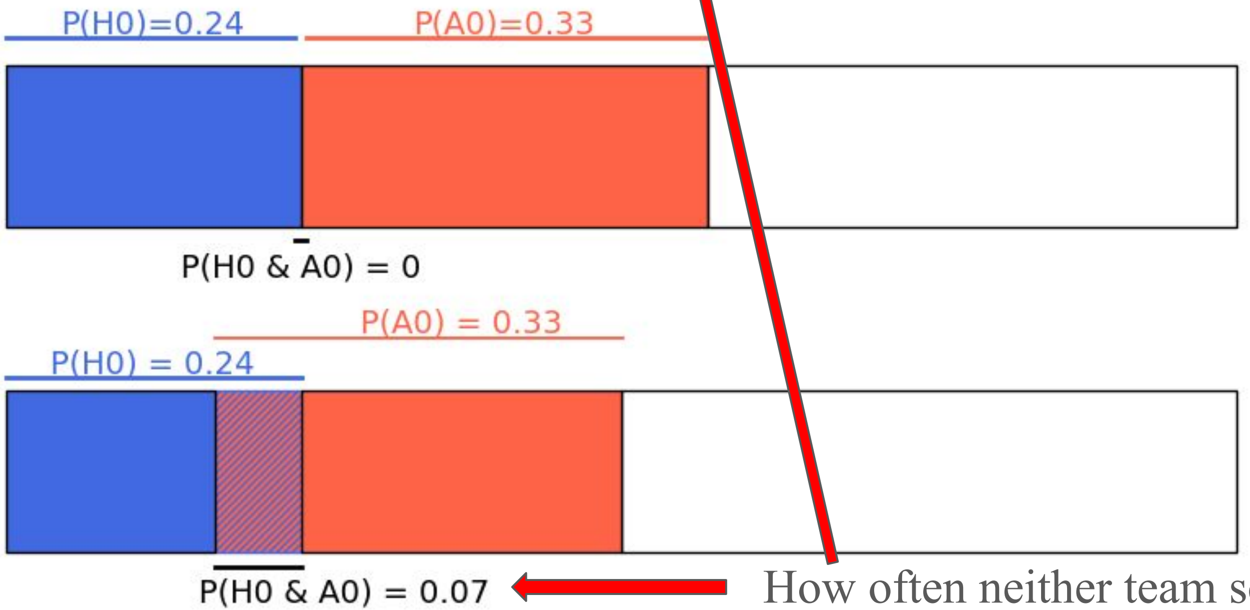
English Premier League Matches (2013 - 2023)

P(H0) = 0.24, P(A0) = 0.33

Q. How often does at least one team not score?

P(H0) + P(A0) - P(H0 AND A0) = 0.50

Incorrect

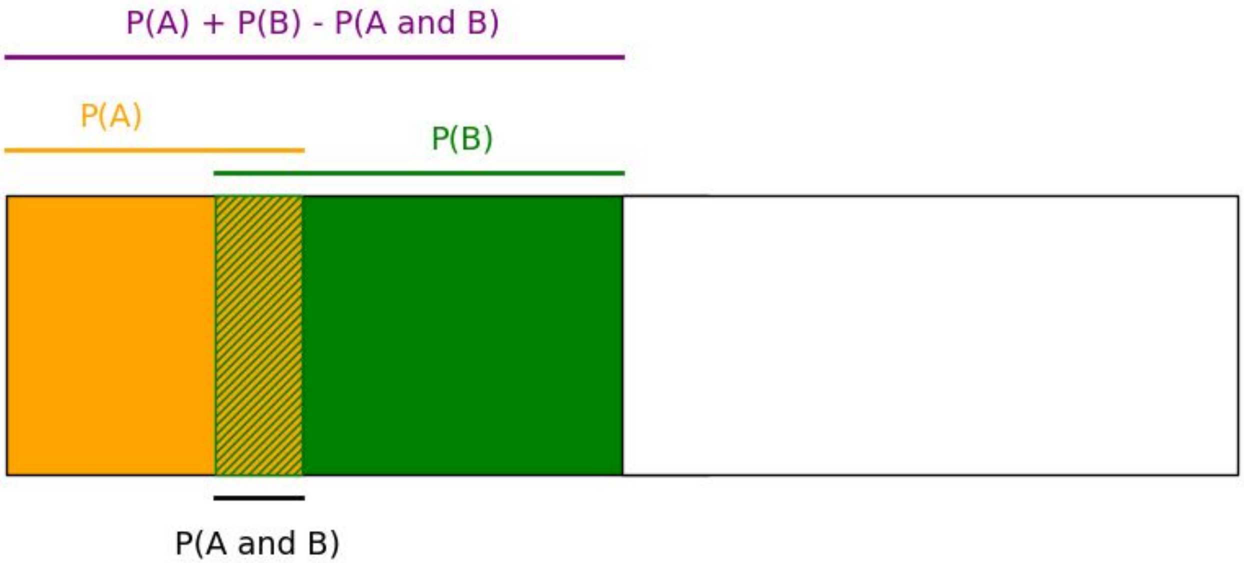


How often neither team scores

General Rule

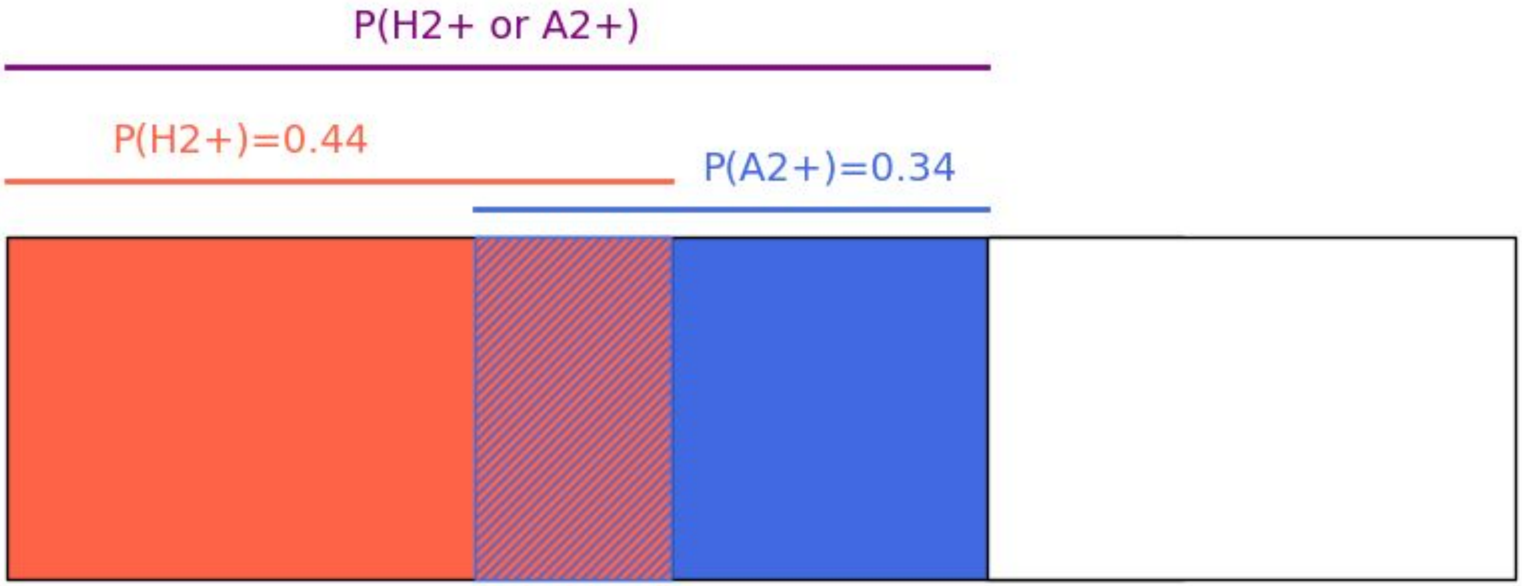
P(A OR B) = P(A) + P(B) - P(A AND B)

- Joint probability: P(A AND B)
- When both events can happen at once, subtract the overlap: P(A AND B)
- When both events are ***disjoint*** (cannot happen at the same time): P(A AND B) = 0



English Premier League Matches (2013 - 2023)

Q. What’s the probability that at least one team scores twice or more?



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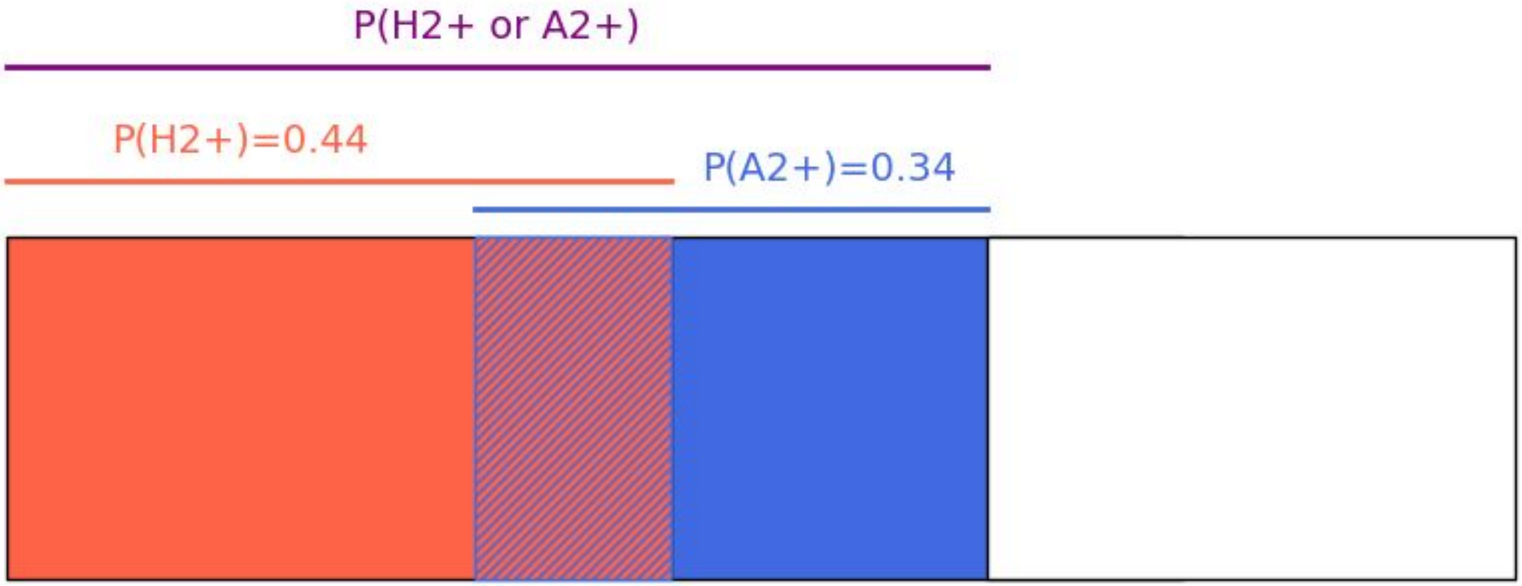
Q. What’s the probability that at least one team scores twice or more?

A. We need to know the probability *both* teams score twice or more.



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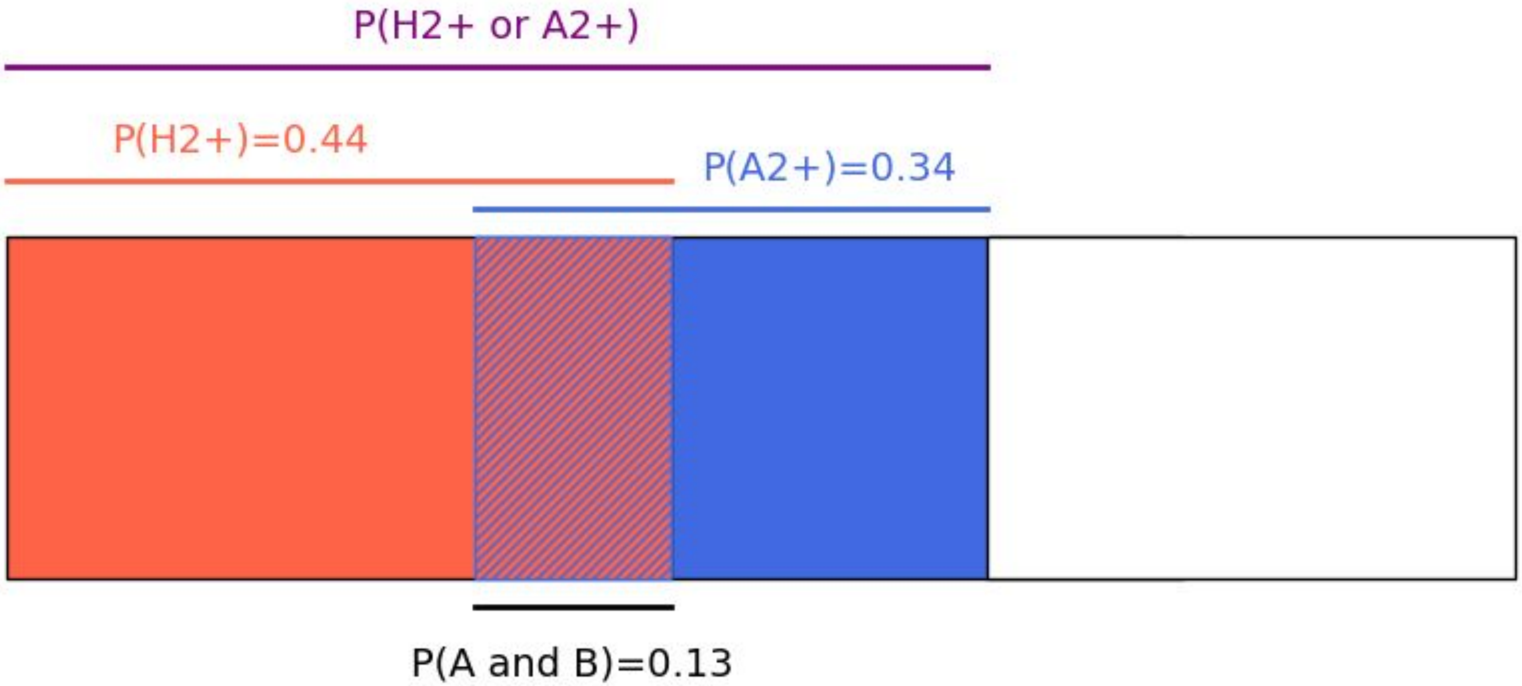

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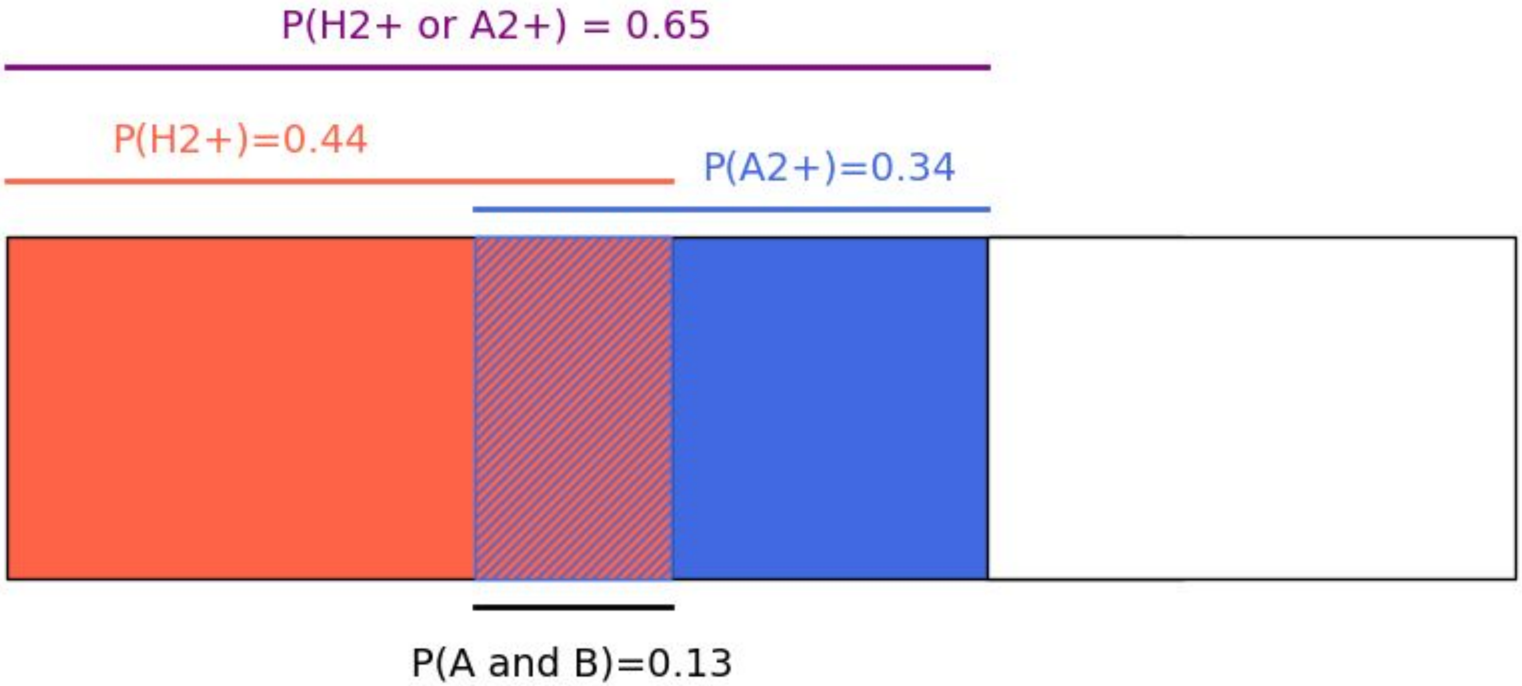

Conditional Probability

… is the probability of an event ***given*** that another event has already occurred.

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Q1. What's the probability of the home team winning?

Q2. What's the probability the away team is leading at the half?

Conditional Probability

… is the probability of an event ***given*** that another event has already occurred.

Q3. What's the probability the away team lead at the half but the home team wins?

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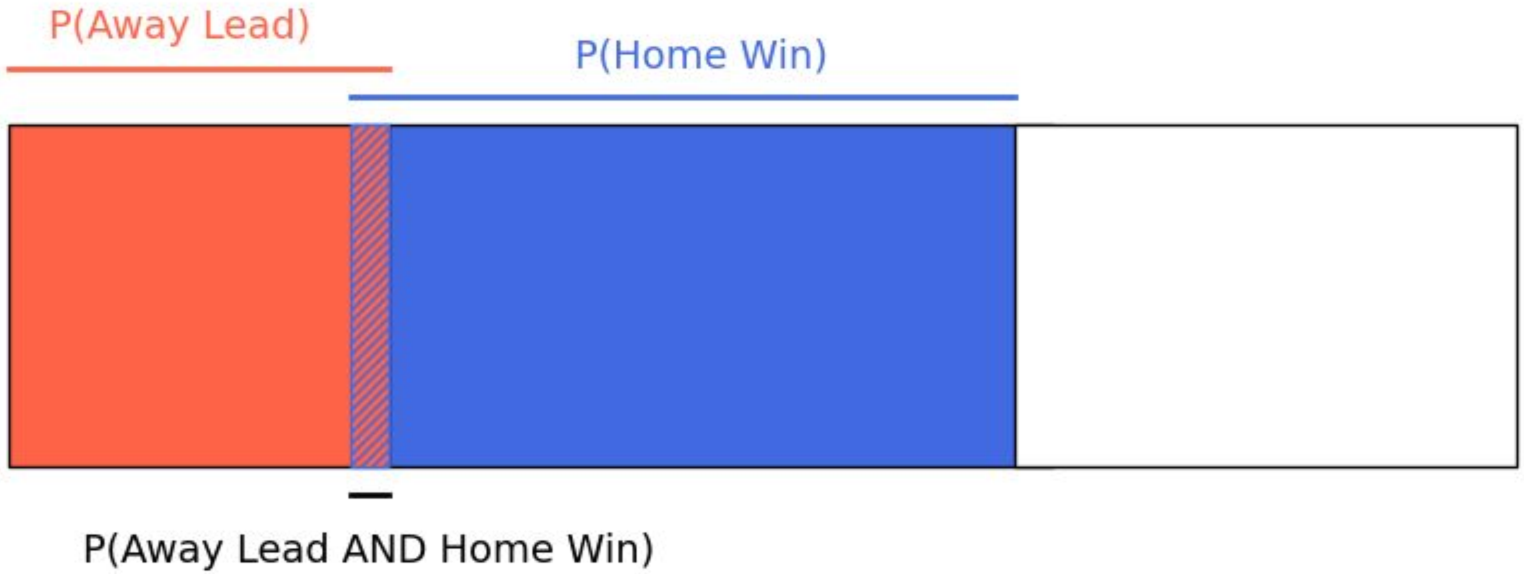
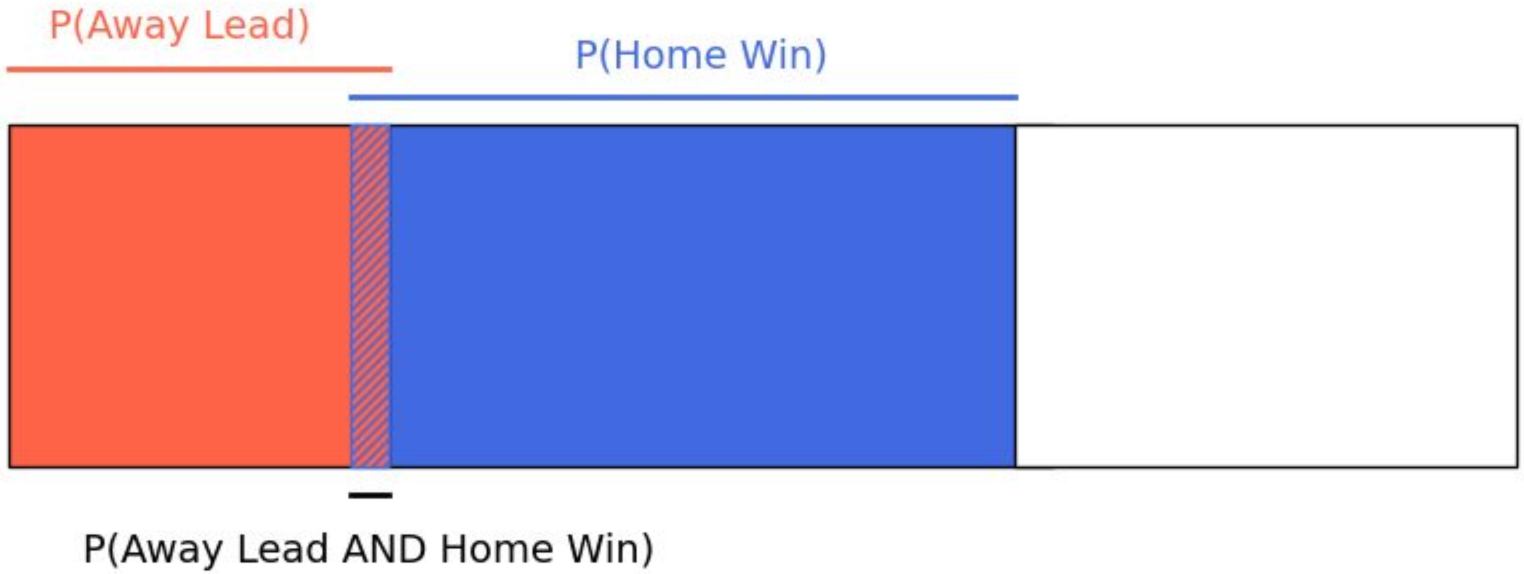


Conditional Probability

… is the probability of an event ***given*** that another event has already occurred.

Q3. What's the probability the away team lead at the half but the home team wins?

Q4. What's the probability the home team wins ***given*** the away team lead at the half?



Conditional Probability

… is the probability of an event ***given*** that another event has already occurred.

Q4. What's the probability the home team wins ***given*** the away team lead at the half?

Conditional Probability: Summary

… is the probability of an event ***given*** that another event has already occurred.

- $P(A|B)$: probability of A happening given that B happens
- $P(A|B) = P(A \text{ and } B) / P(B)$
- $P(A \text{ and } B) = P(B)\, P(A|B)$

Excel Exercise: Tied At Halftime

- Data: Part_2_2_Premier_League_Matches.csv
- What are the chances of a home win when the scores are tied at the half?
    - P(Home Win | Halftime Draw)?

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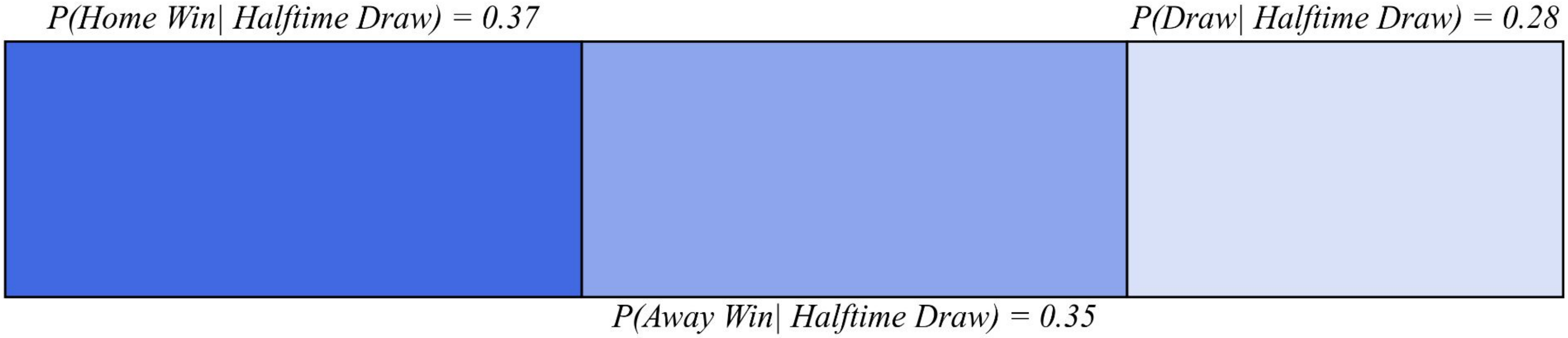

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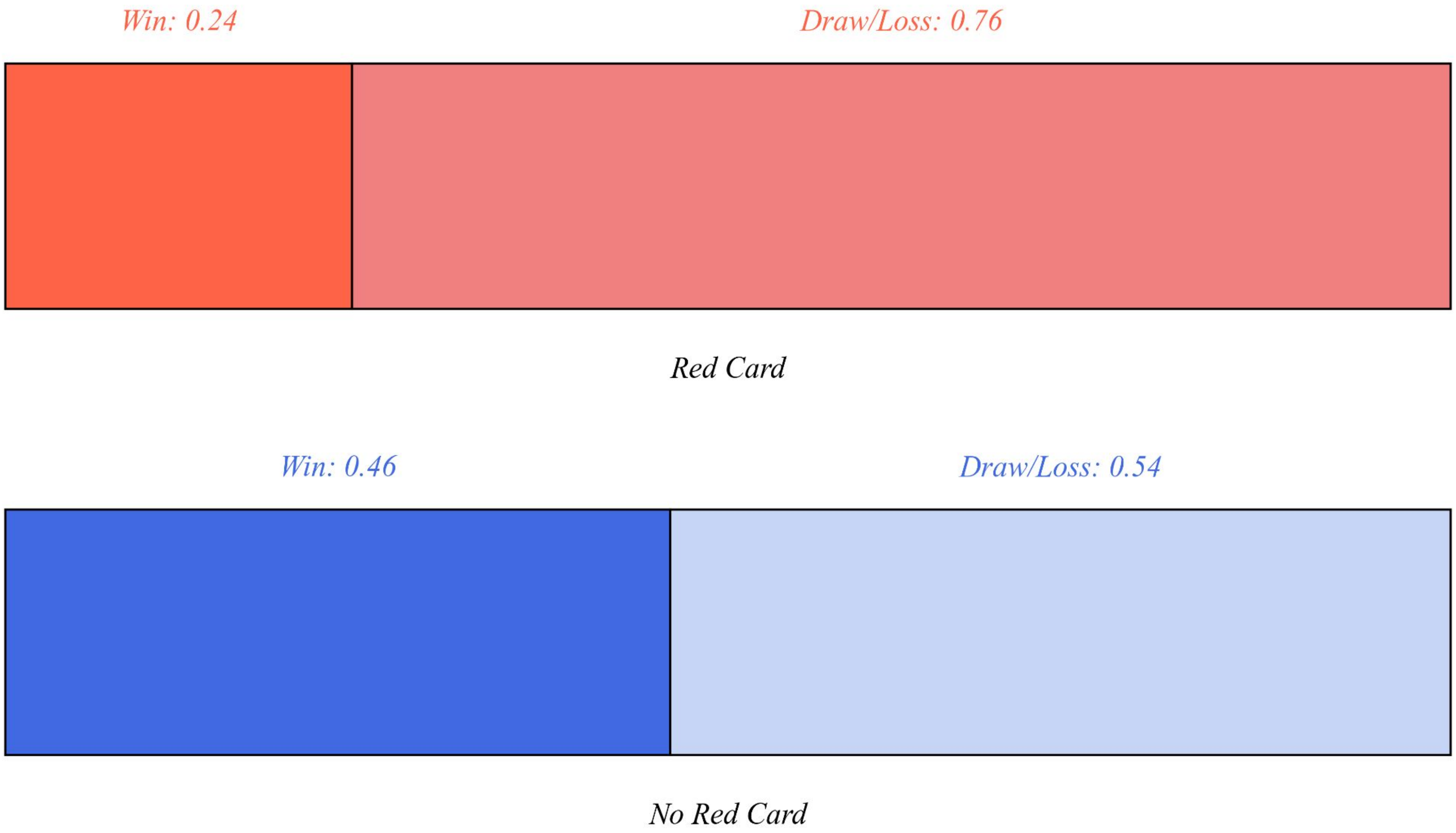

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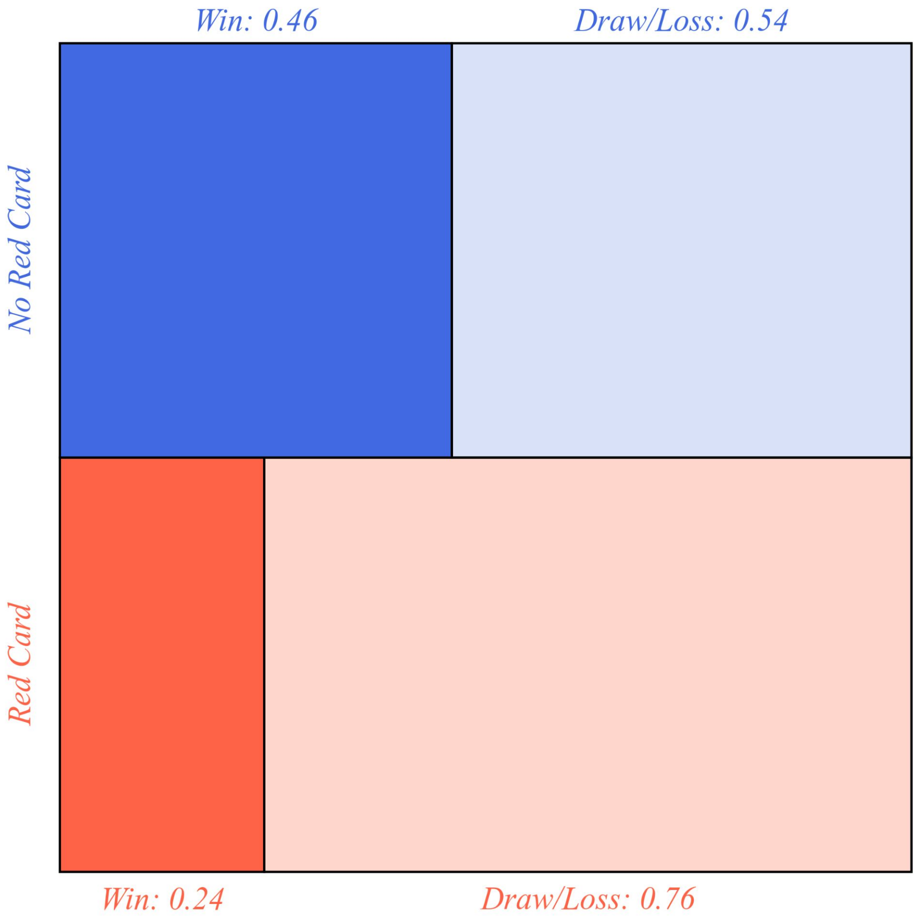

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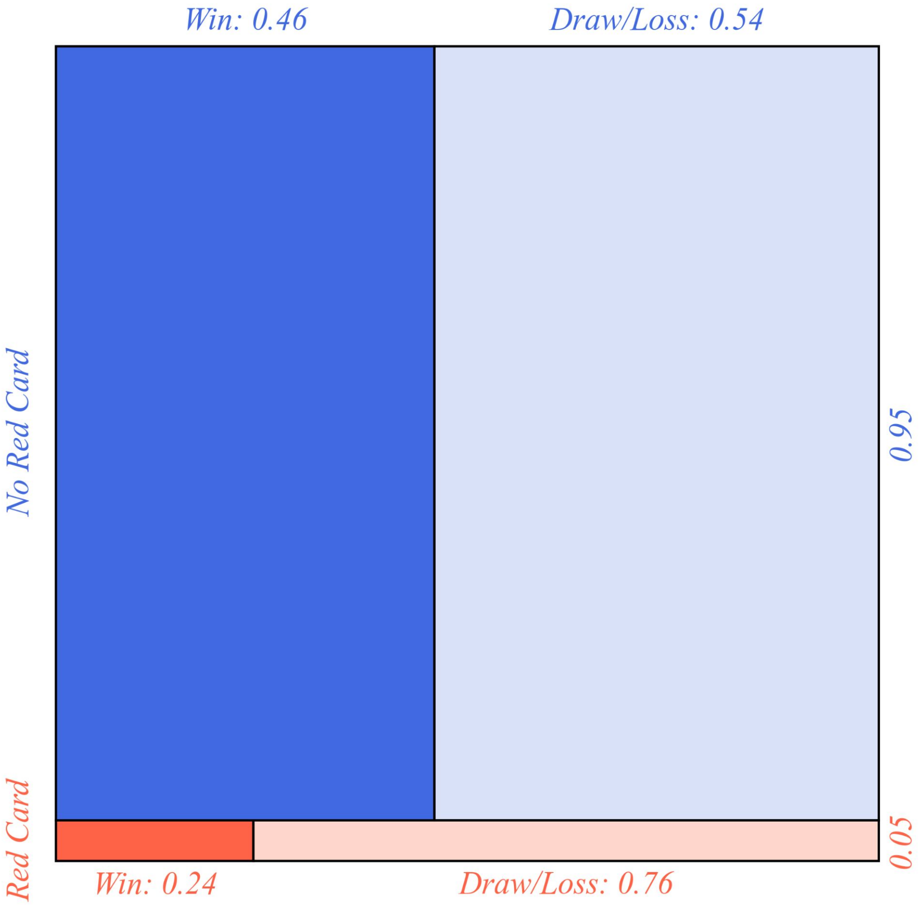


Law of Total Probability

… the probability of an event is the sum of two mutually exclusive parts:

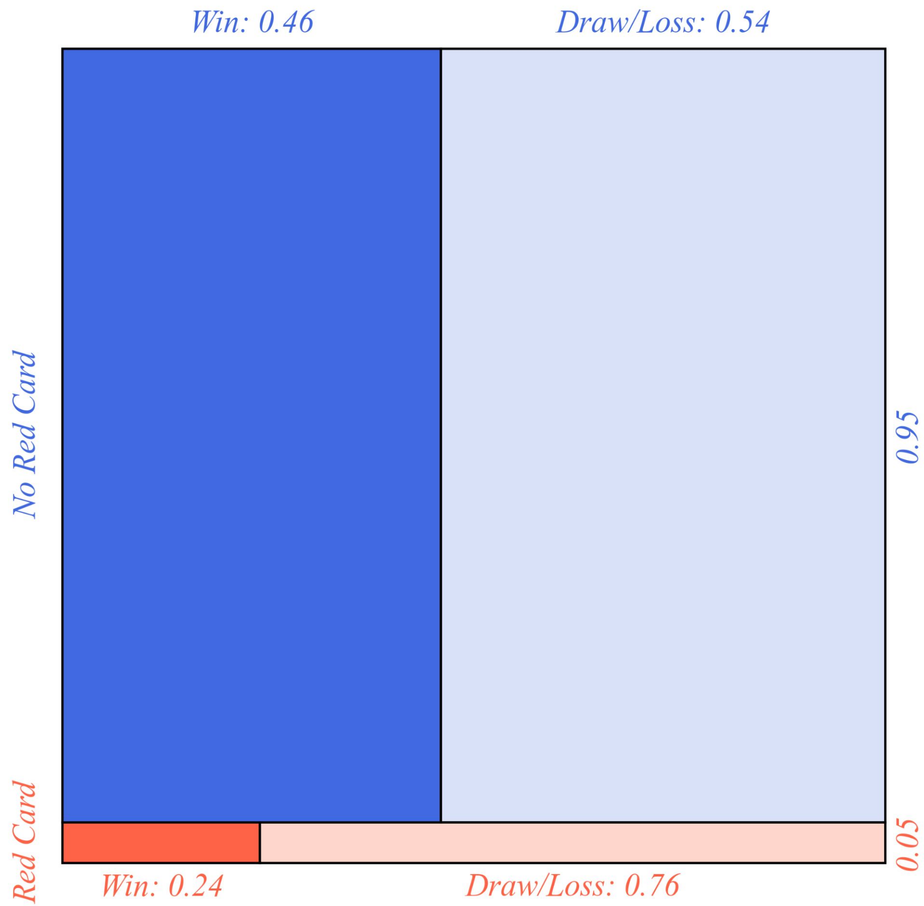
$$P(A) = P(A \text{ and } B) + P(A \text{ and NOT } B)$$

Probability Grid

Q. What is the probability of a win?

P(Win)

=



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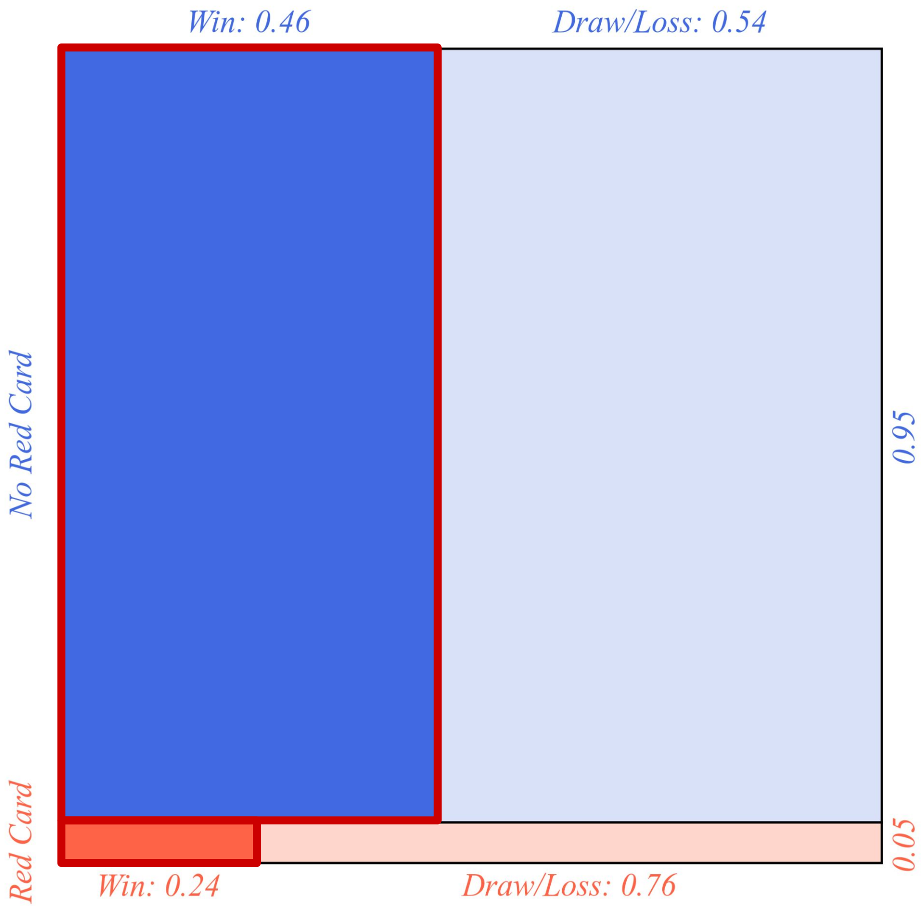


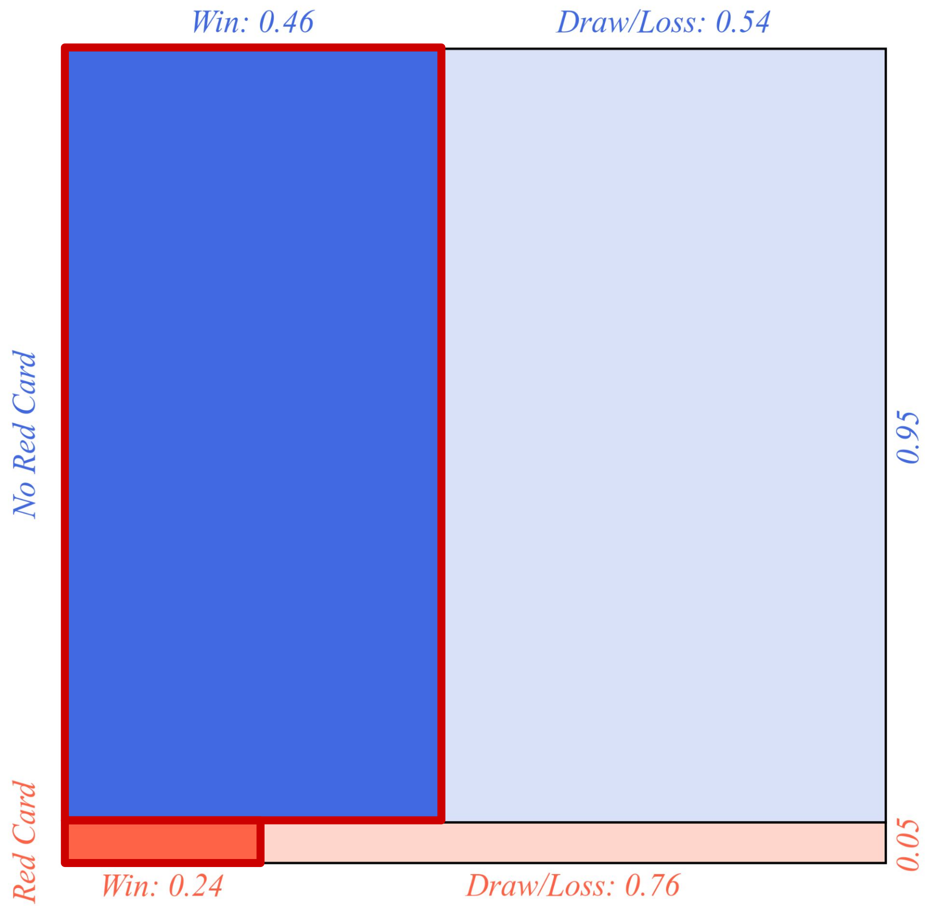
Probability Grid

Q. What is the probability of a win?

P(Win)

$= P(\text{NoRed})\, P(\text{Win} \mid \text{NoRed})$

$+ P(\text{Red})\, P(\text{Win} \mid \text{Red})$



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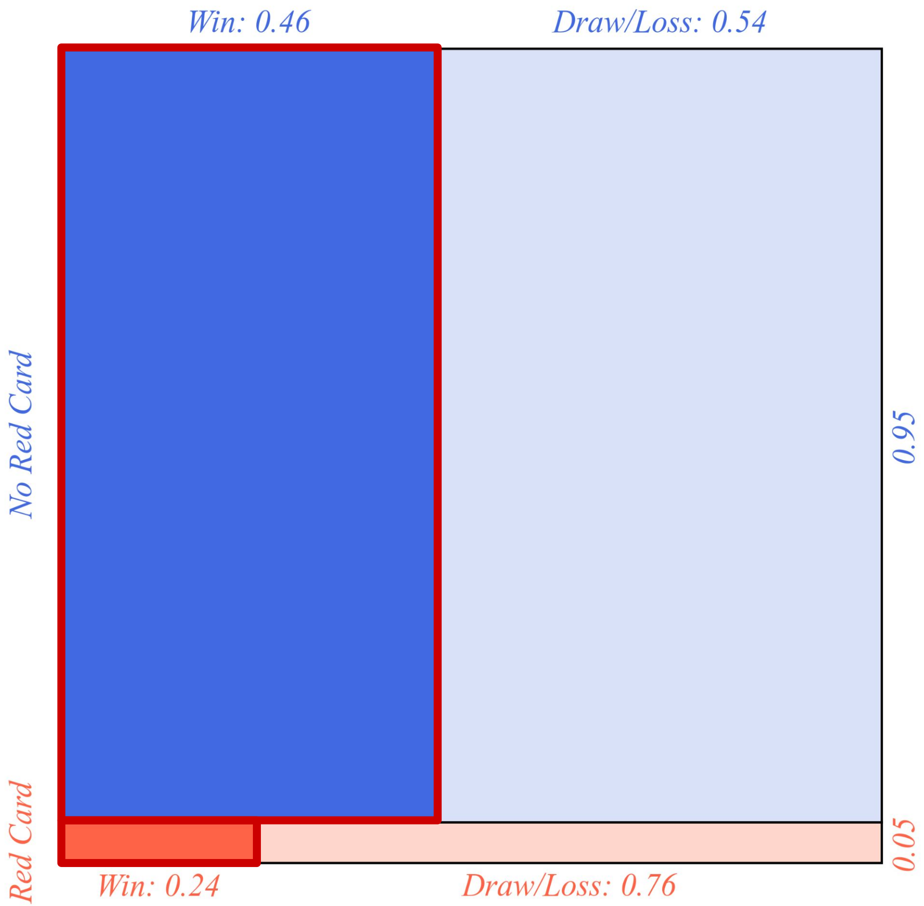
$= 0.05 \cdot 0.24 + 0.95 \cdot 0.46$



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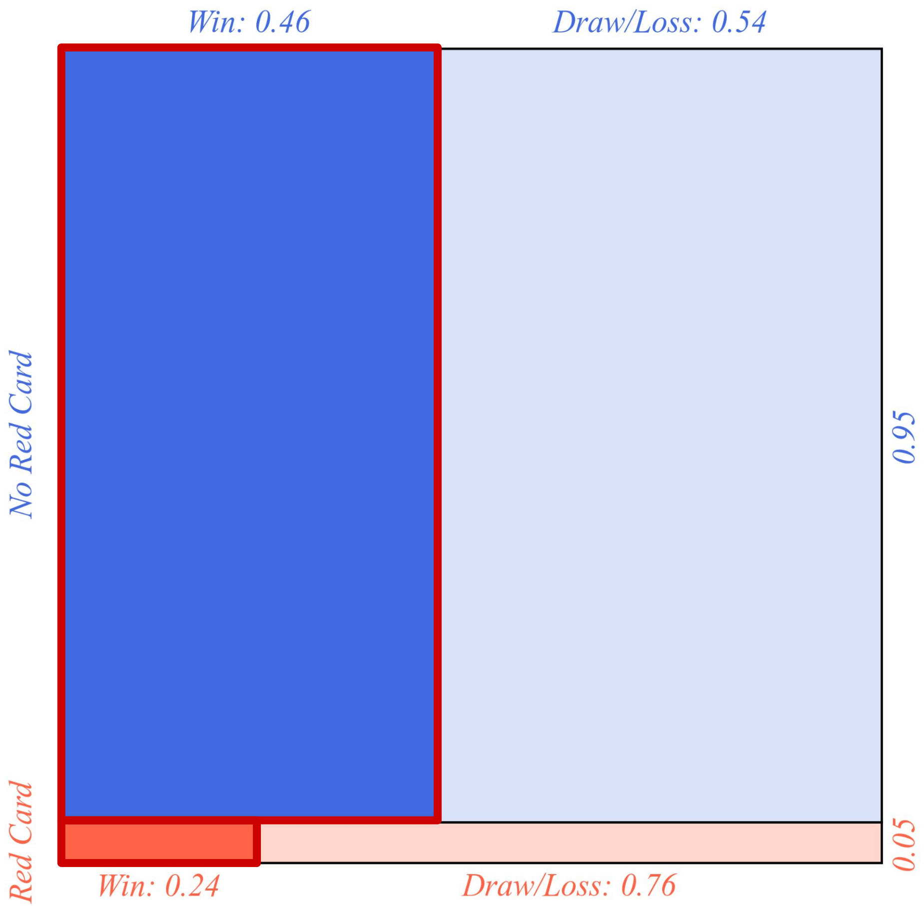
$$=$$

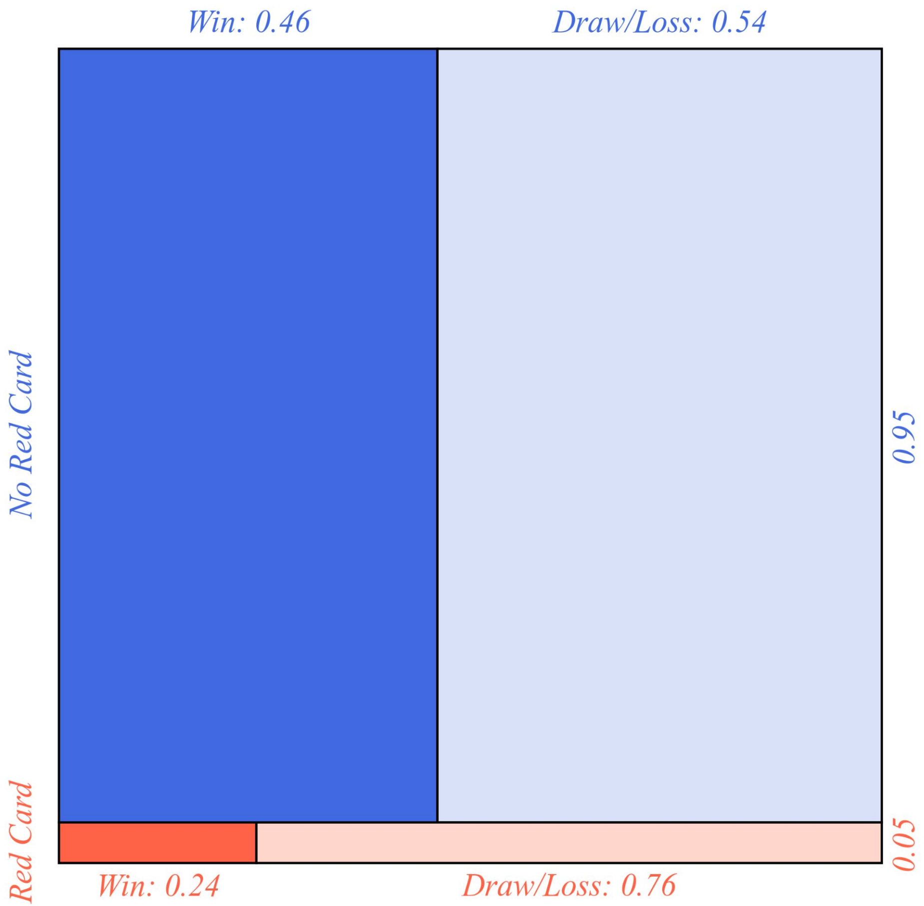


Probability Grid

Q. What is the probability of winning with a red card?

P(Win and Red)

=



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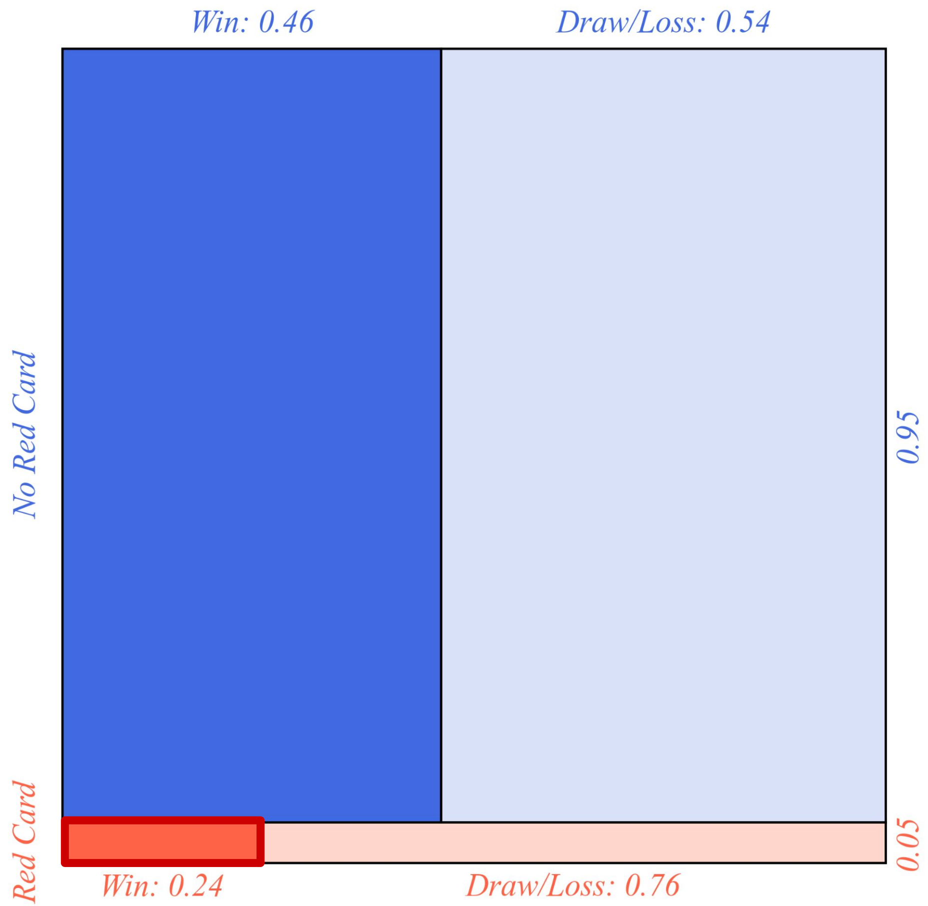

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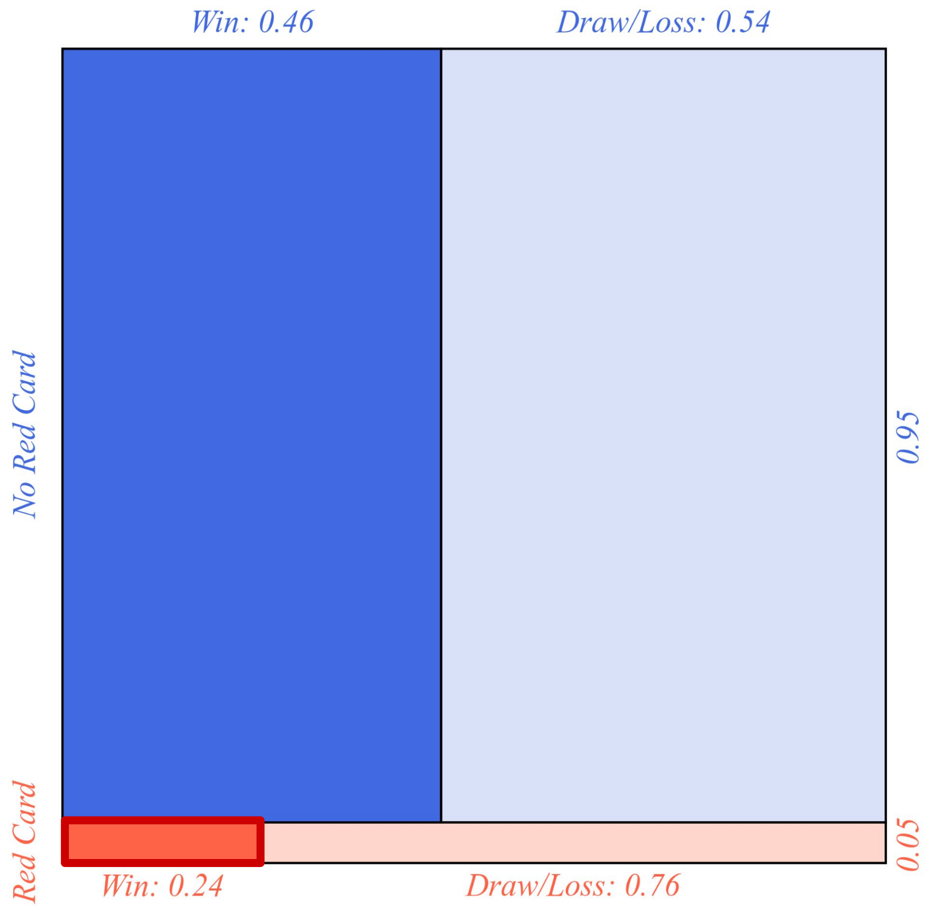

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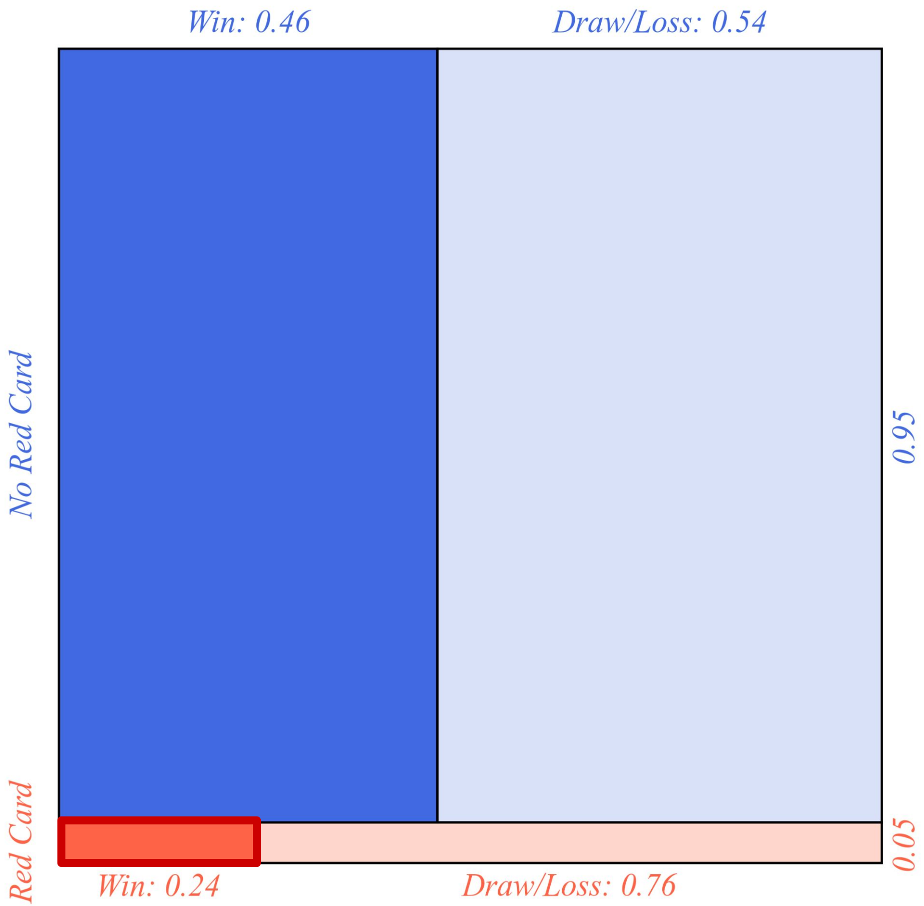

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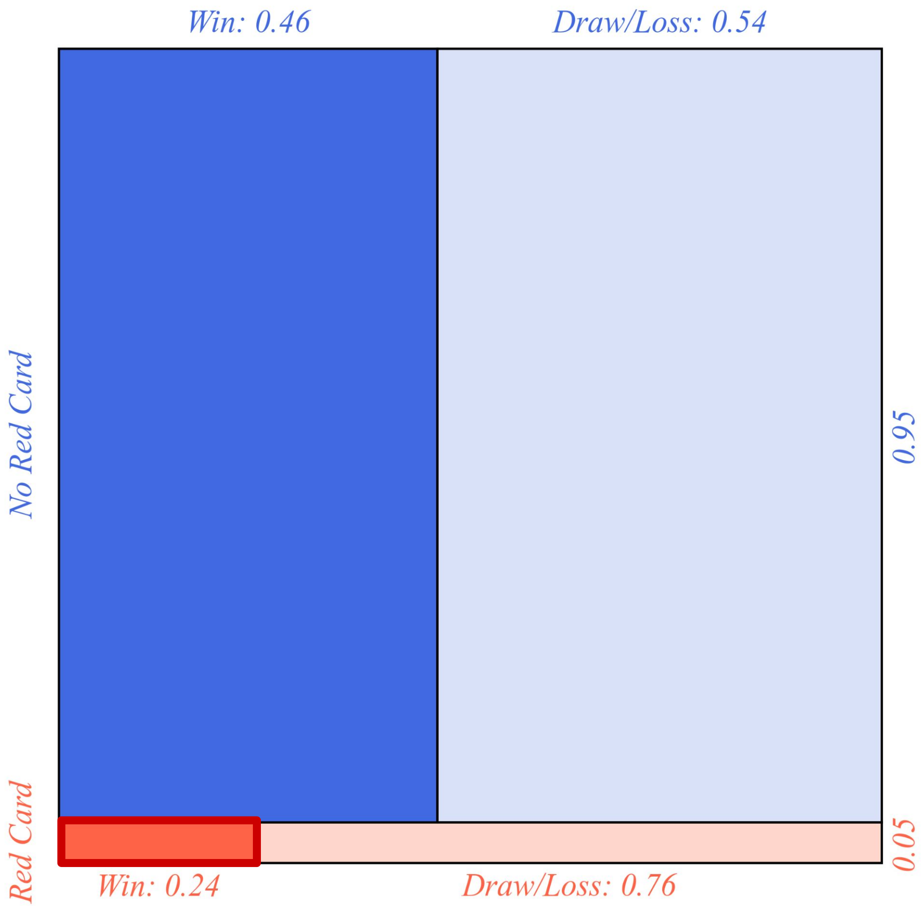
$=$

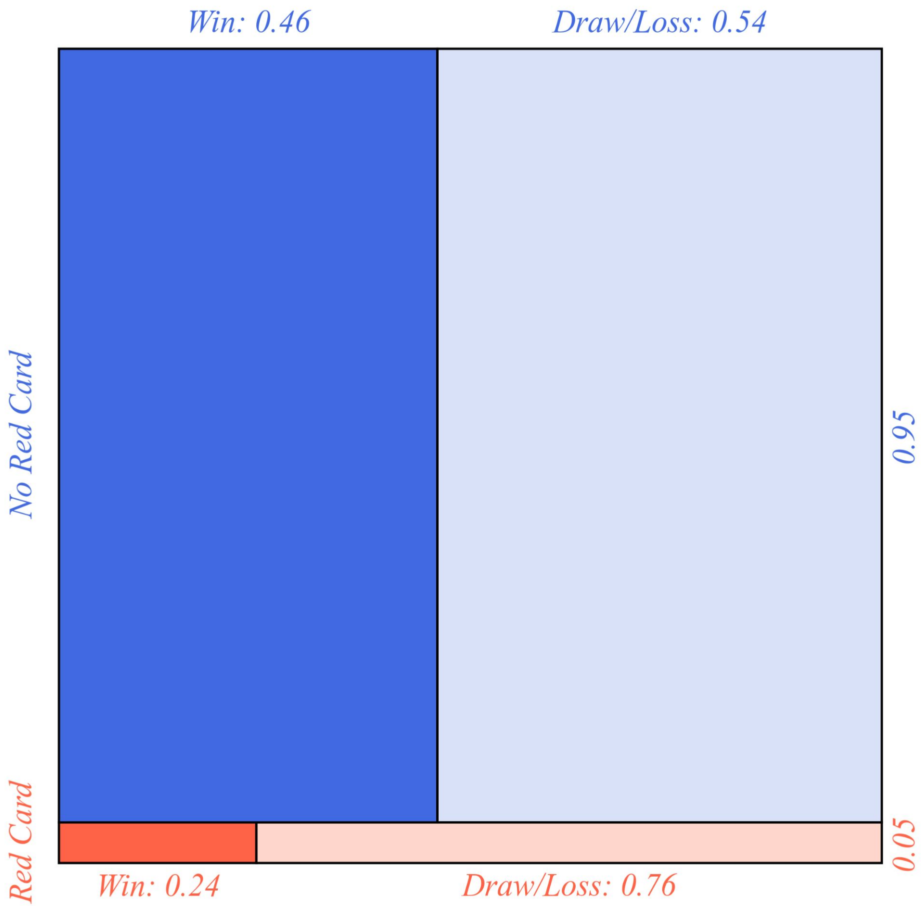


Probability Grid

Q. What is the probability of a win if the team received a red card?

P(Win | Red)

=



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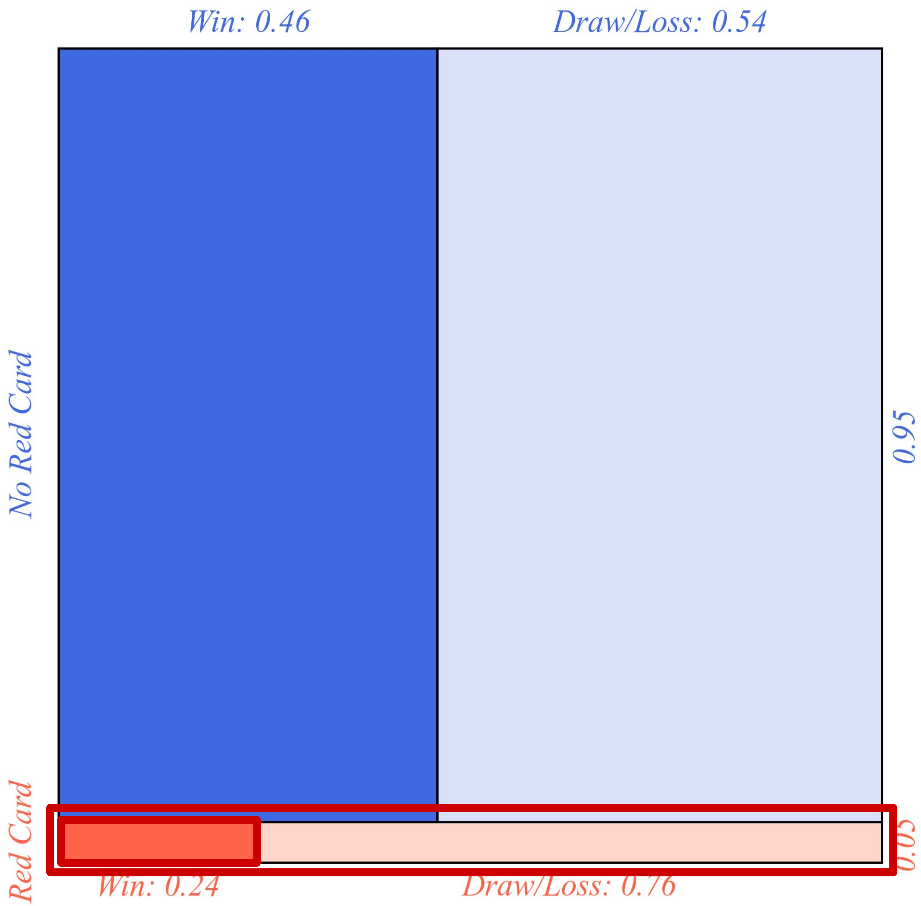
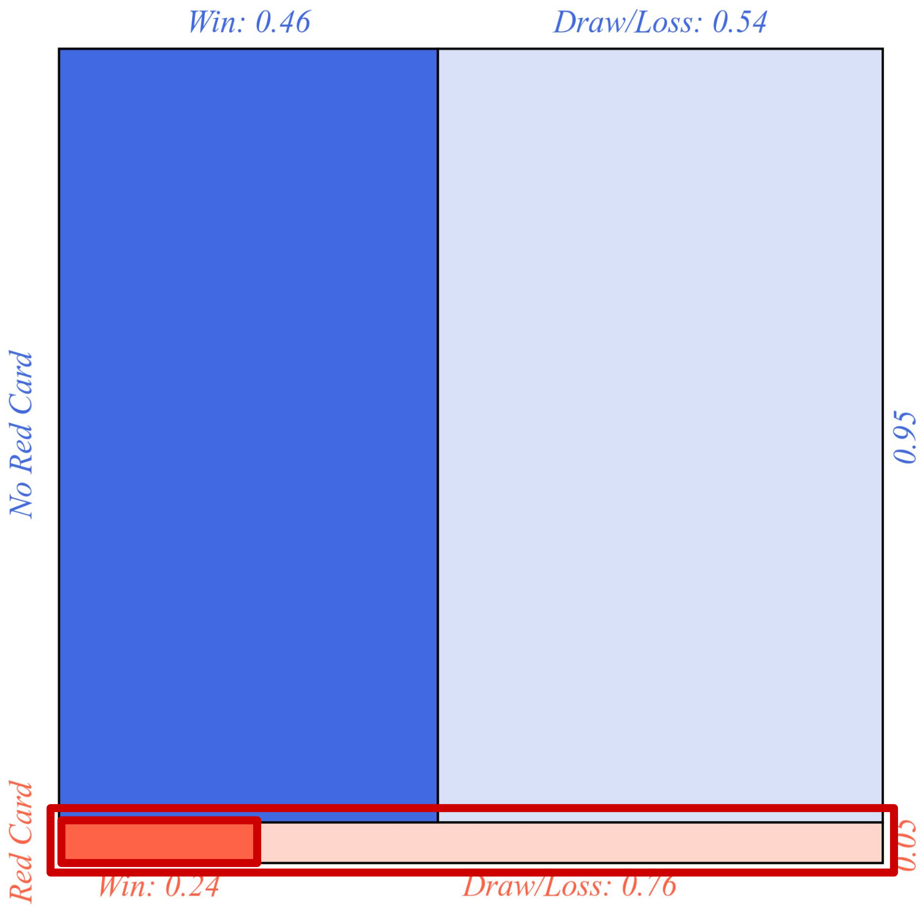


Probability Grid

Q. What is the probability of a win if the team received a red card?

P(Win | Red)

= P(Win and Red) / P(Red)



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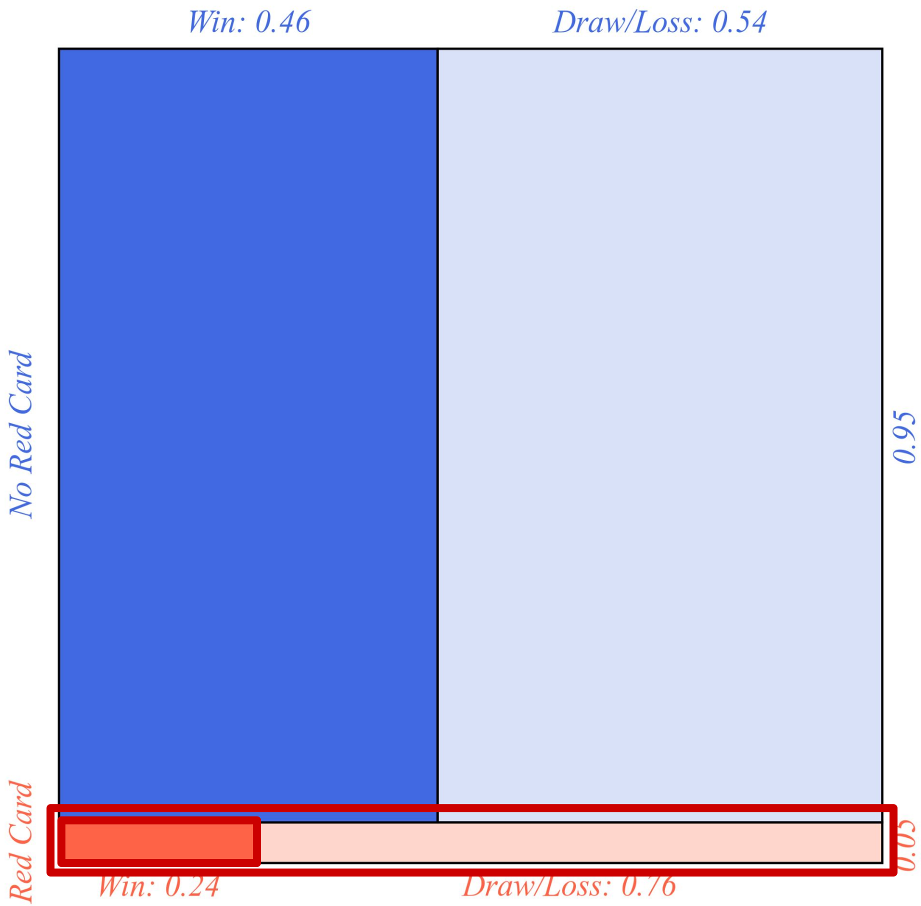

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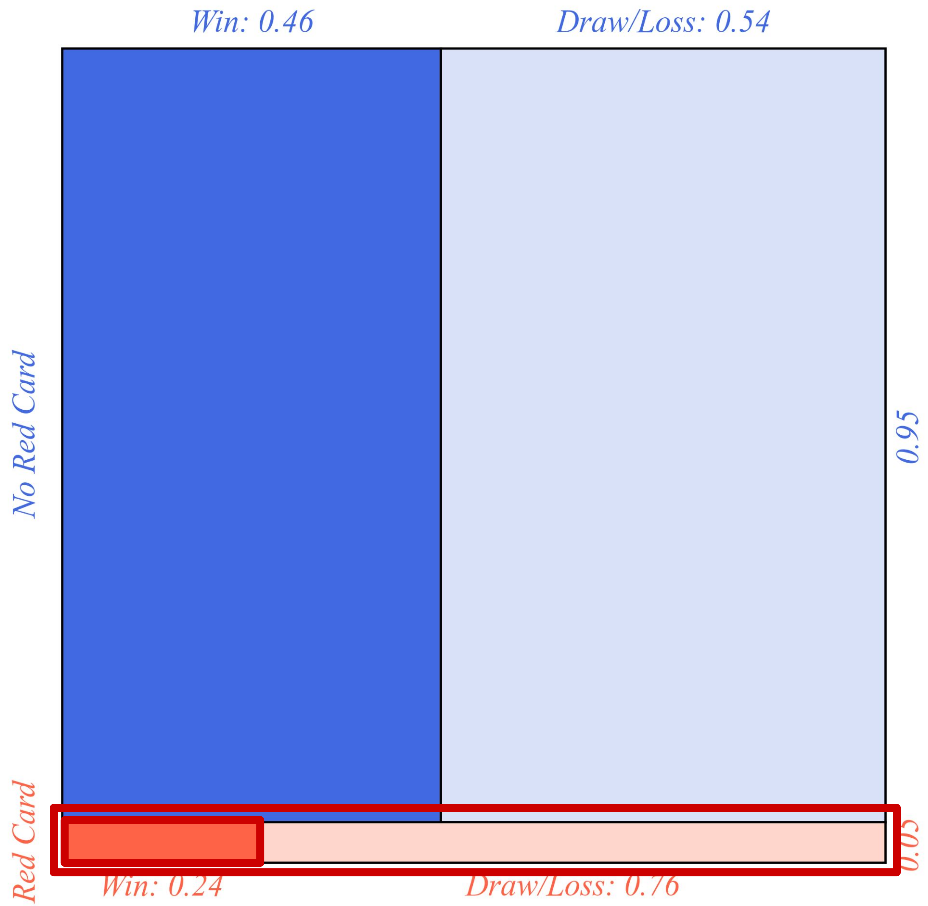
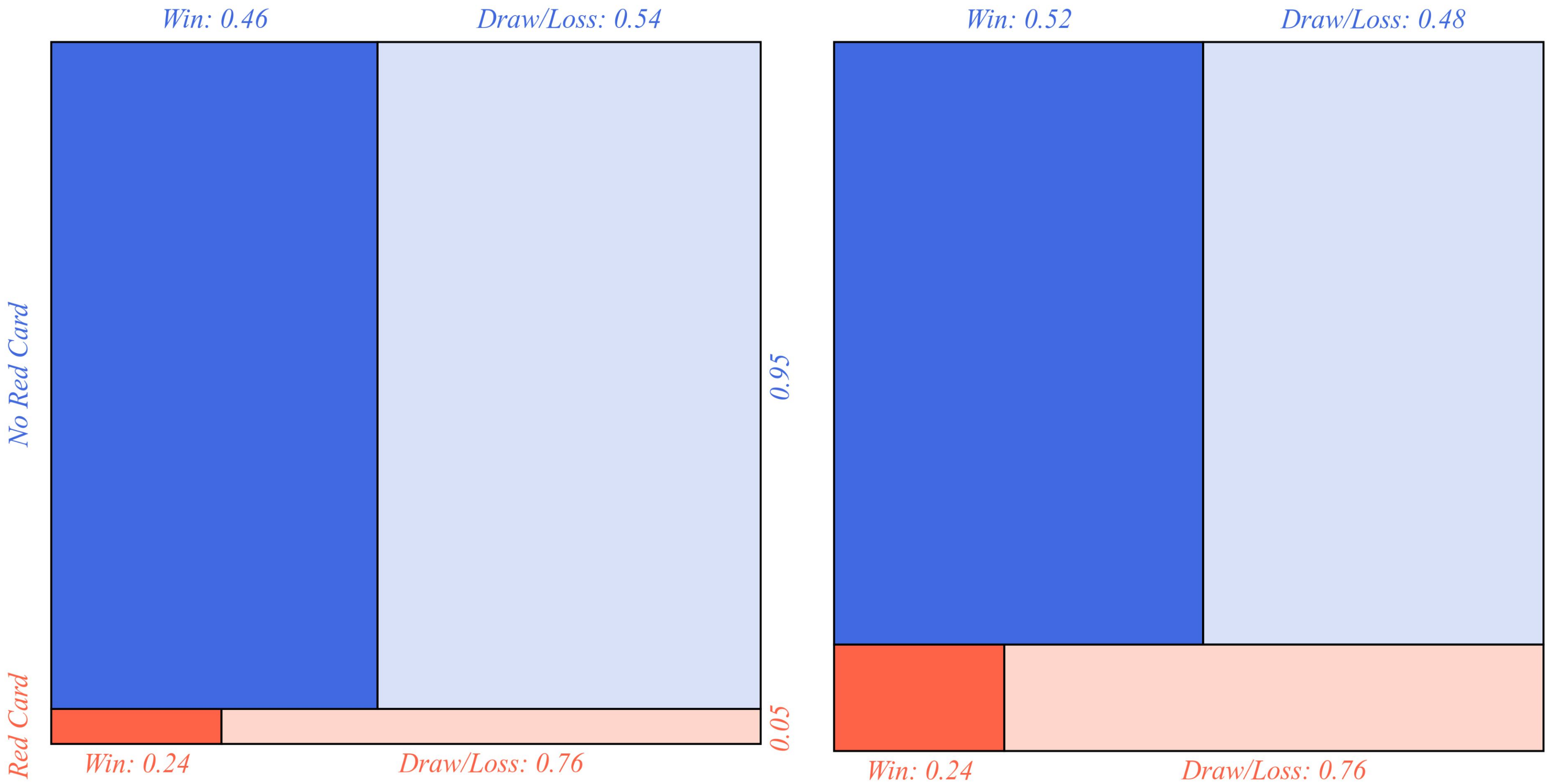
= 0.05 · 0.24 / 0.05

= 0.24



Probability Grid

Adding a bad-tempered player changes the probability grid.

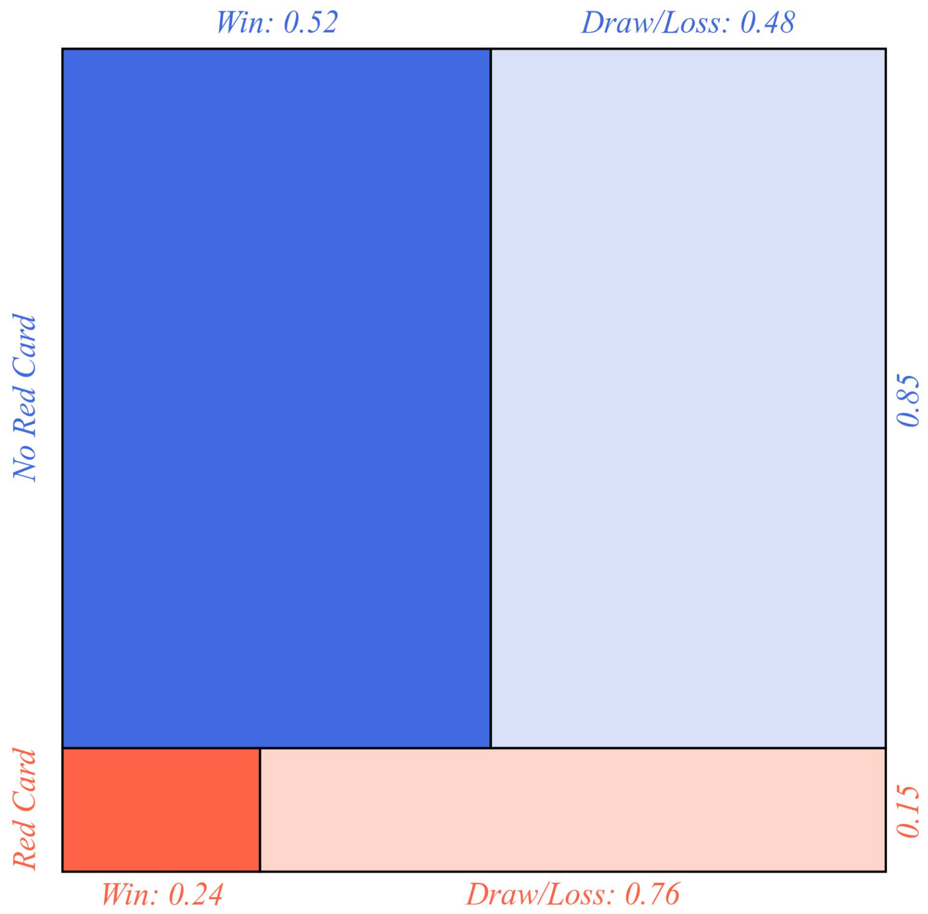


Probability Grid

Q. How does the new player change probability of a win?

P(Win)

=



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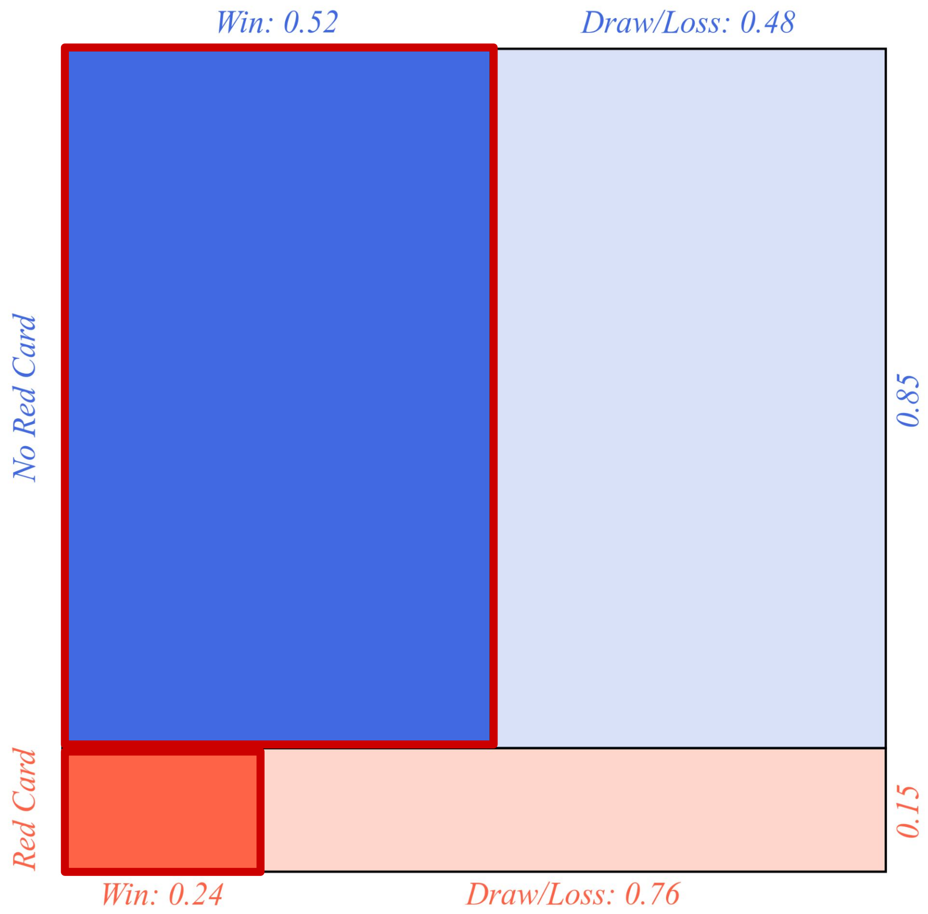


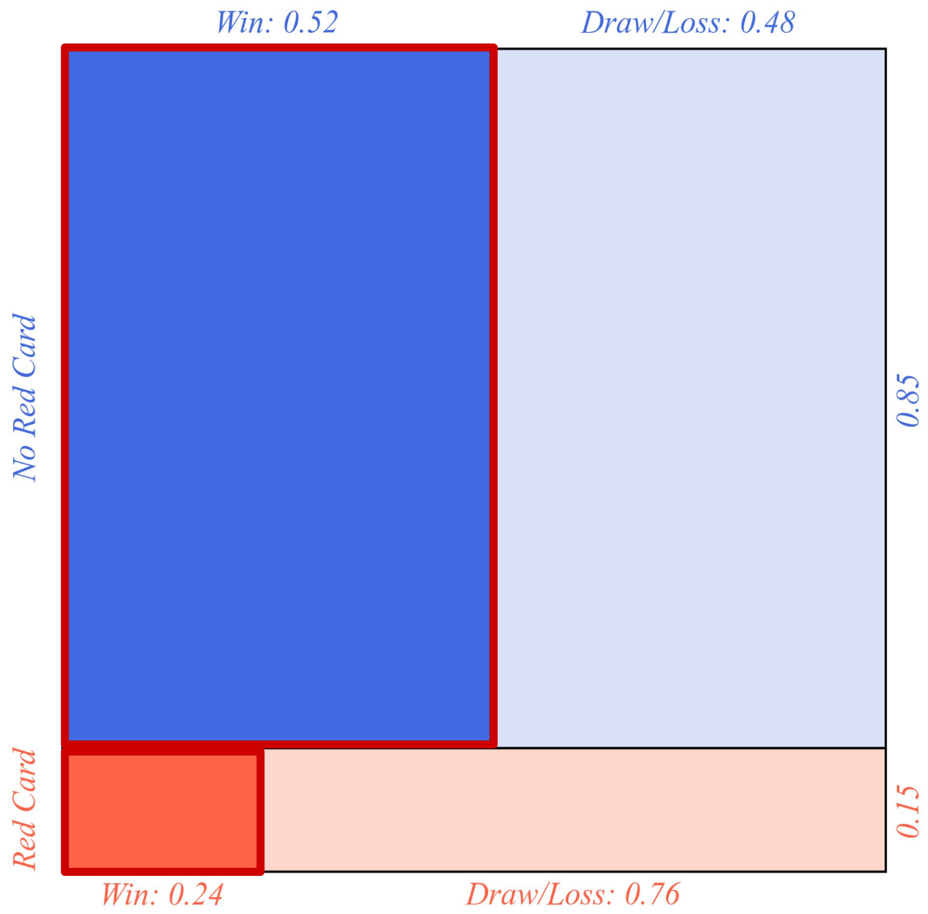
Probability Grid

Q. How does the new player change probability of a win?

P(Win)

$= P(NoRed)\, P(Win \mid NoRed)$

$+ P(Red)\, P(Win \mid Red)$



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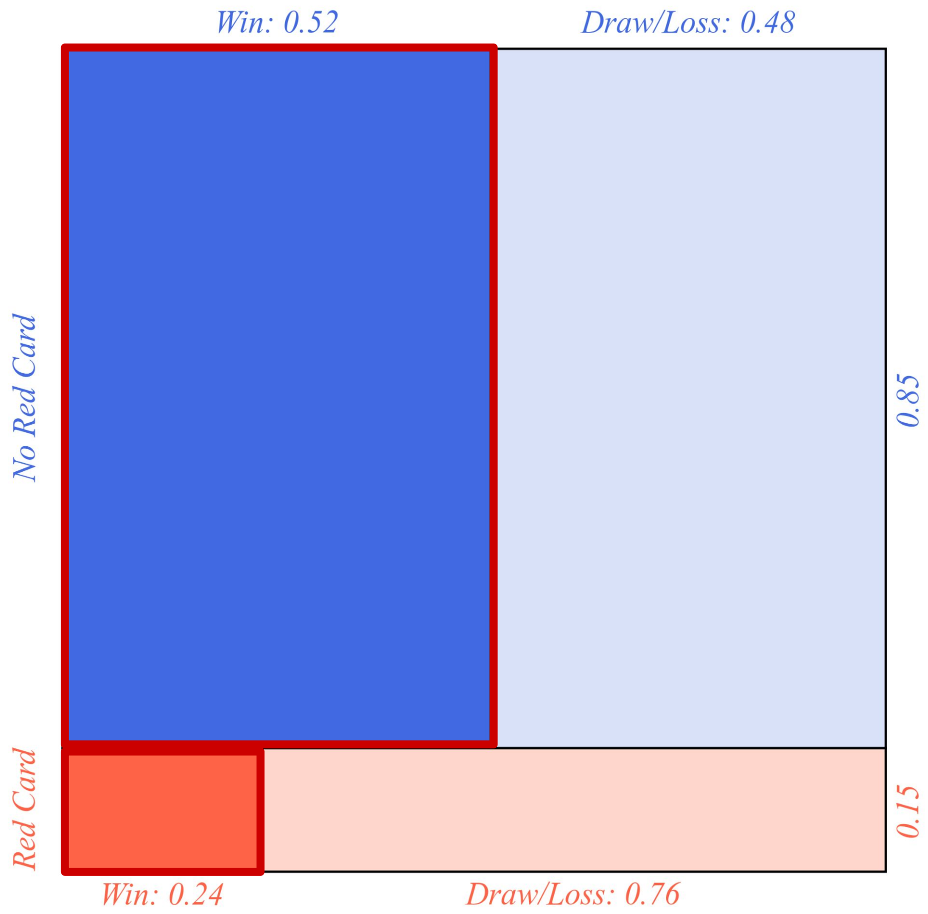

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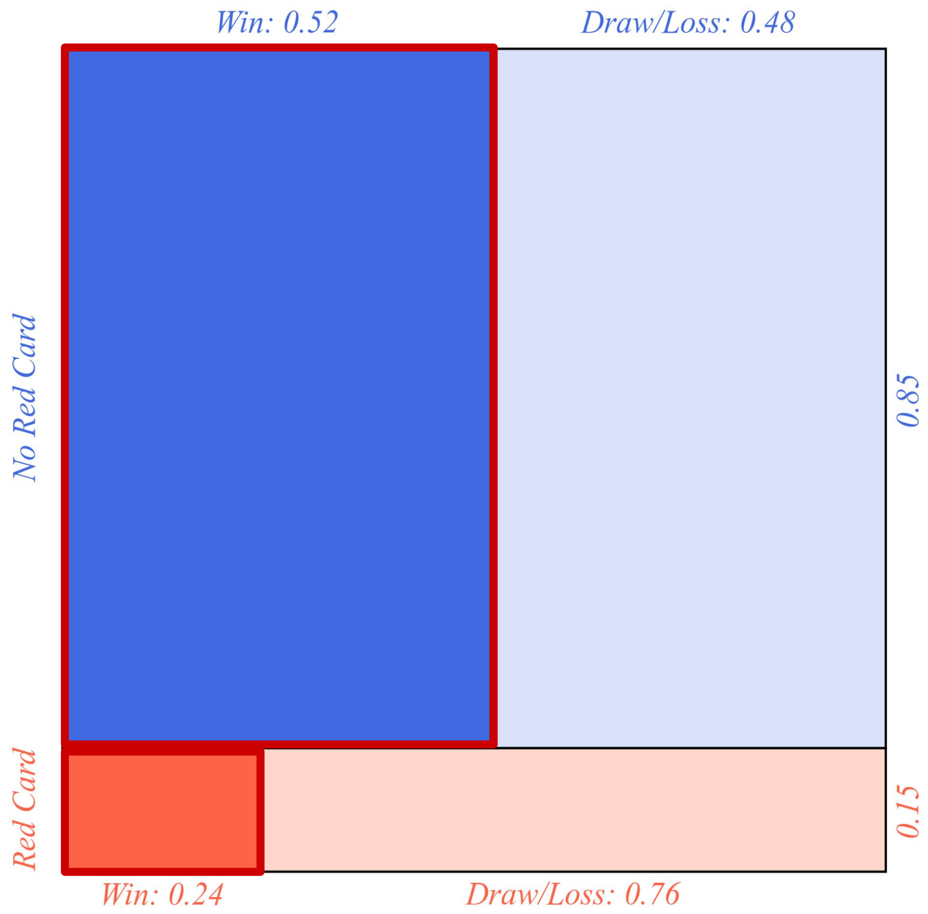
$$=$$



Probability Grid

Q. How does the new player change probability of a win?

P(Win)

$$= P(\text{NoRed})\, P(\text{Win} \mid \text{NoRed})$$

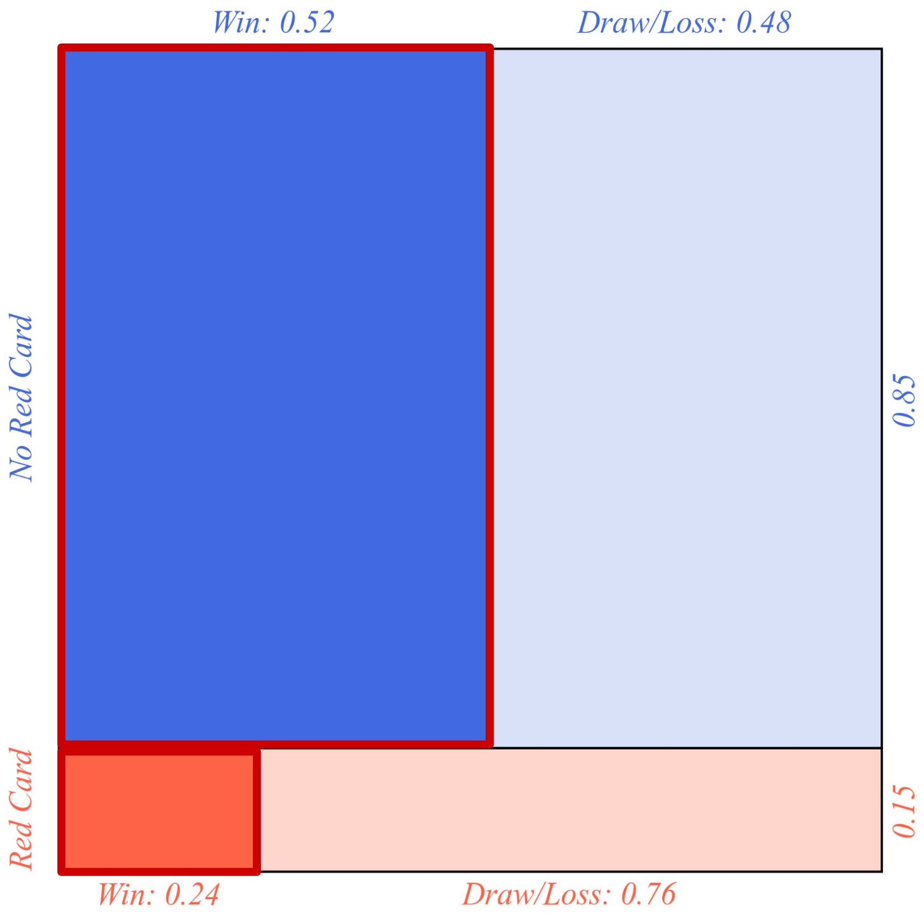
$$+ P(\text{Red})\, P(\text{Win} \mid \text{Red})$$

$$= 0.15 \cdot 0.24 + 0.85 \cdot 0.52$$

$$=$$

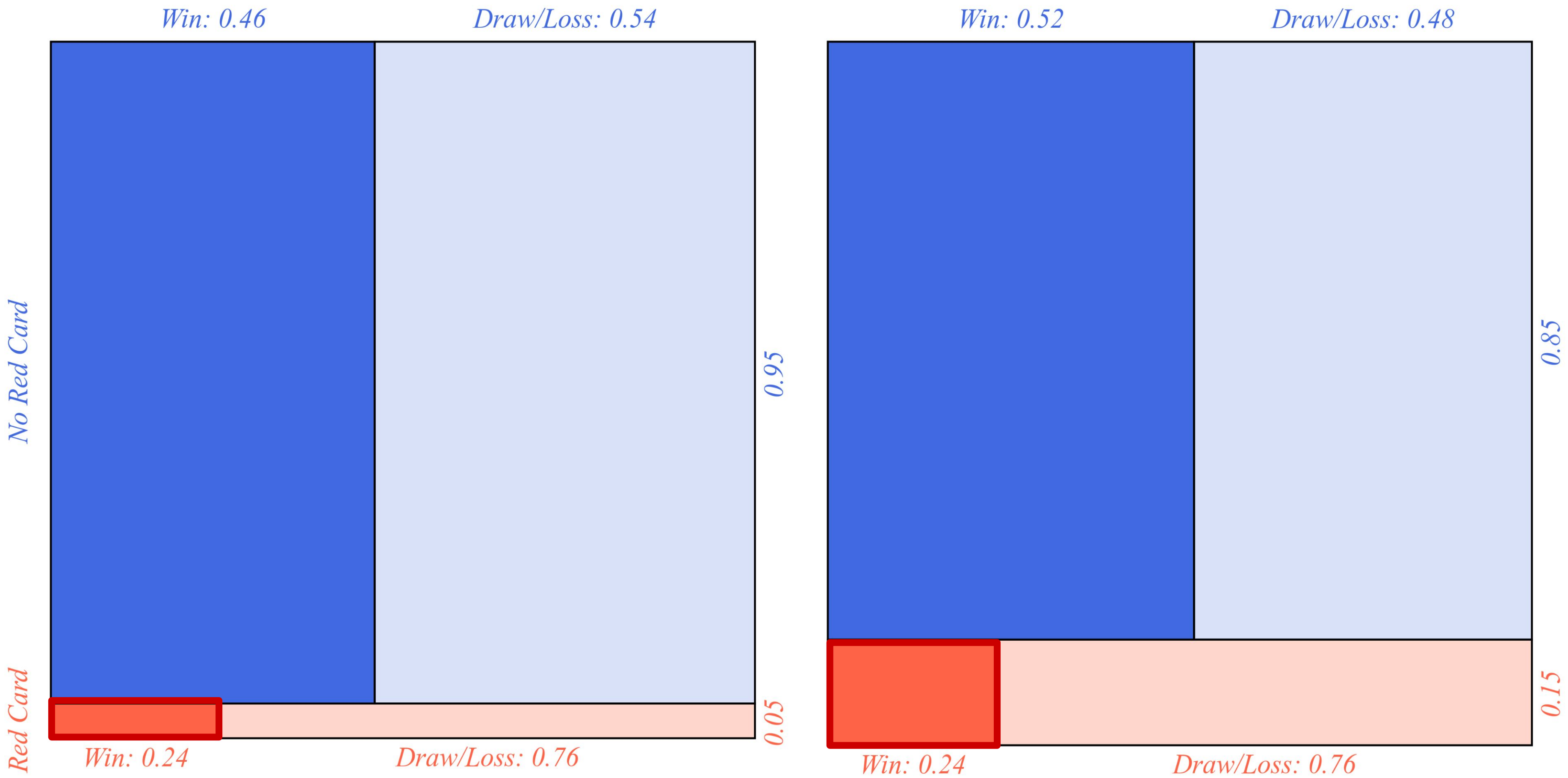
From Before:

$$P(Win) = 0.05 \cdot 0.24 + 0.95 \cdot 0.46$$



Probability Grid

Q. How does the new player change the probability of a win *and* a red card?

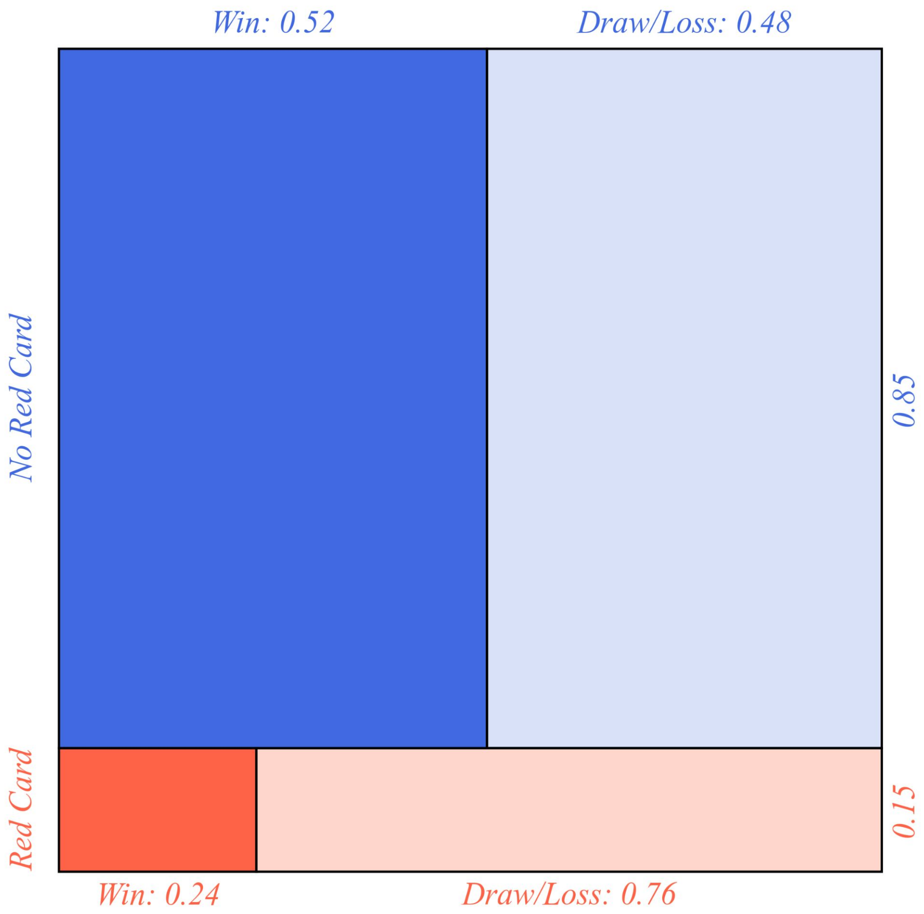


Probability Grid

Q. How does the new player change the probability of a win *and* a red card?

P(Win and Red)

=



Probability Grid

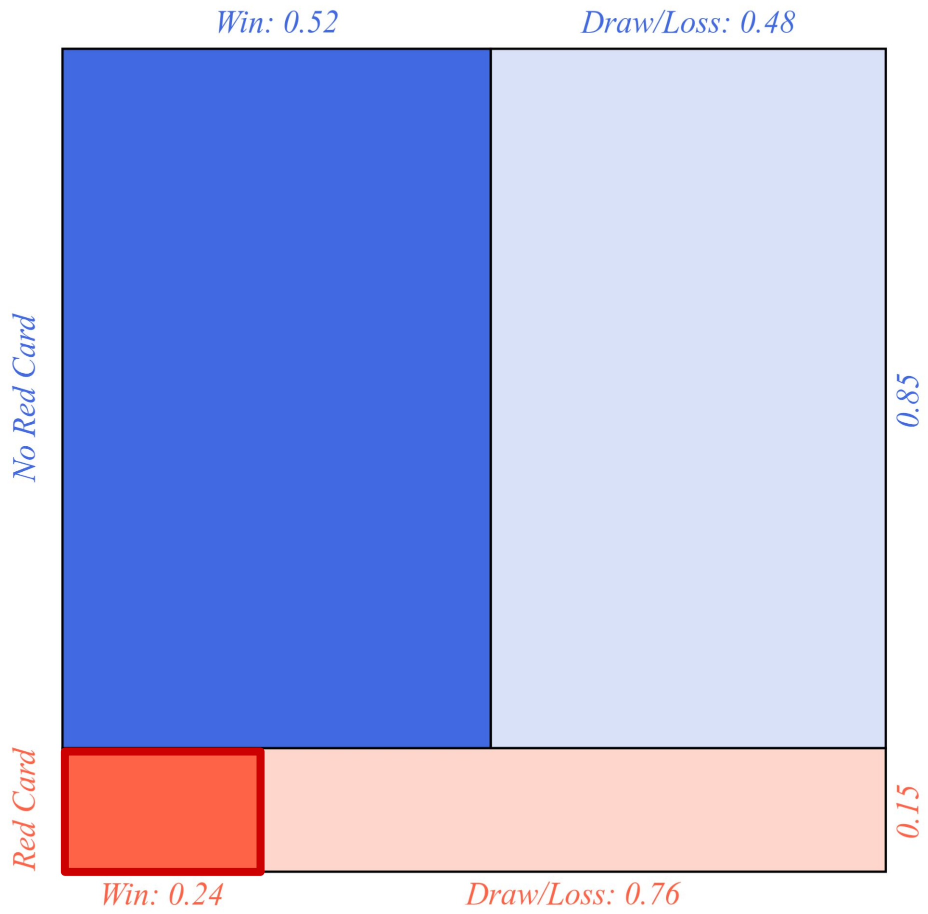
Q. How does the new player change the probability of a win *and* a red card?

P(Win and Red)

$= P(\text{Red})\, P(\text{Win} \mid \text{Red})$

$= 0.15 \cdot 0.24$

$=$



From Before:

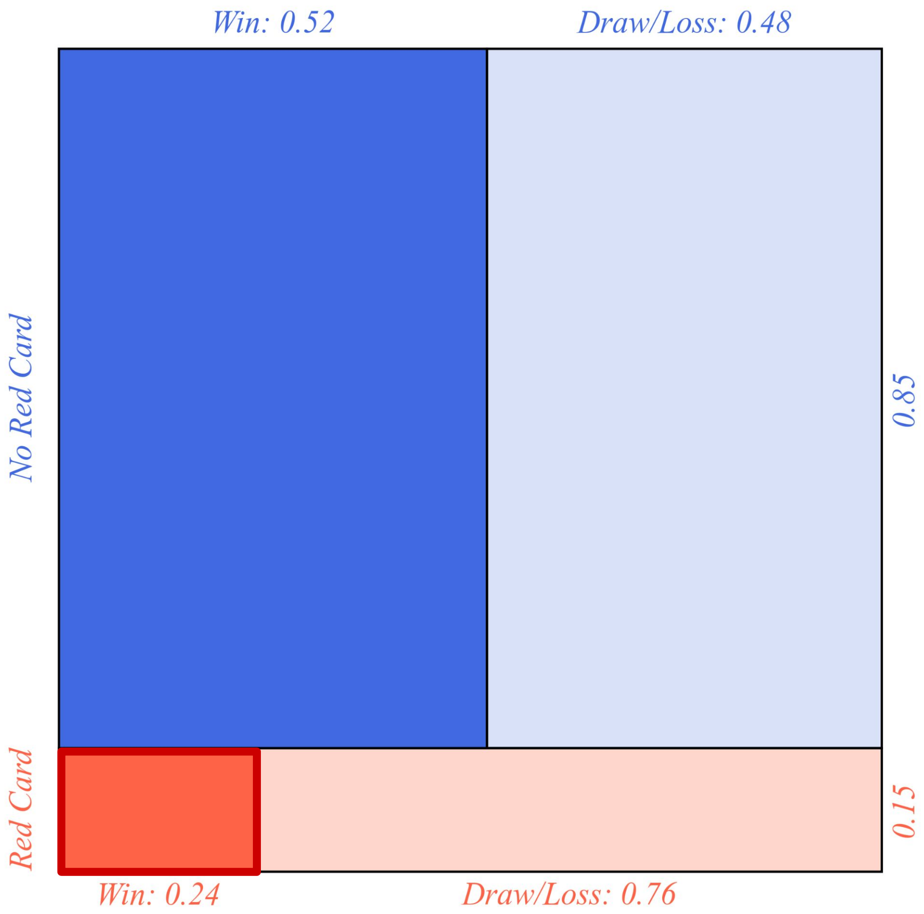
$P(Win) = 0.05 \cdot 0.24$

Probability Grid

Q. How does the new player change the probability of a win *and* a red card?

P(Win and Red)

=

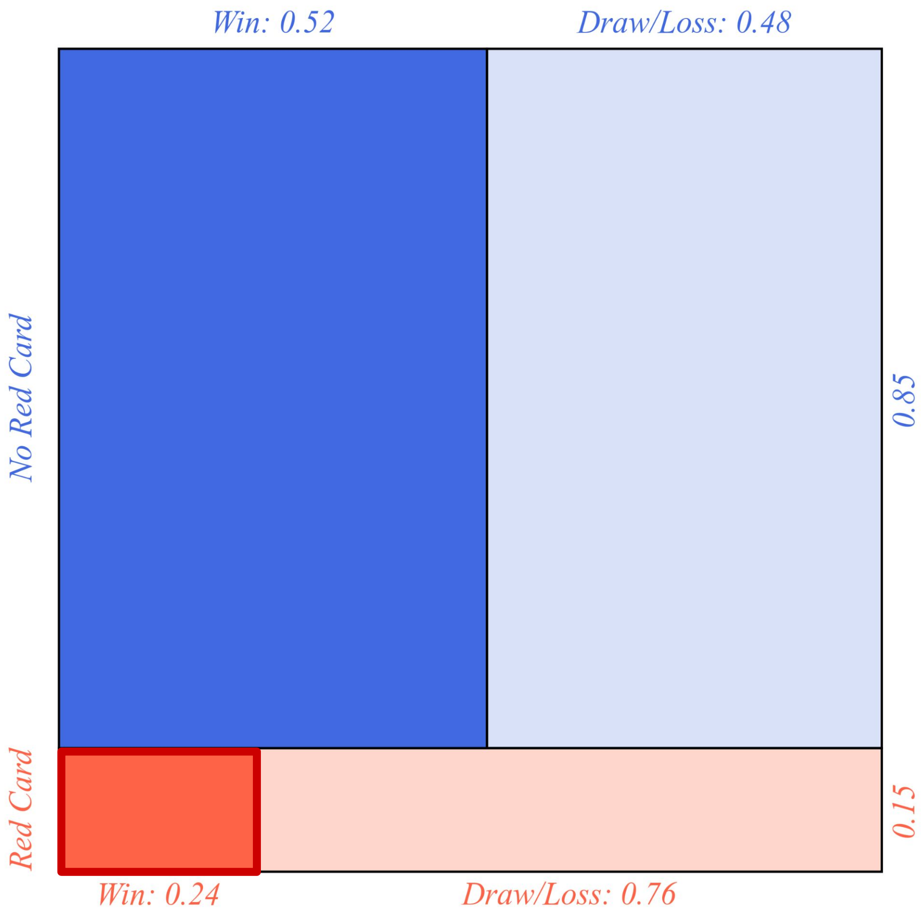


Probability Grid

Q. How does the new player change the probability of a win *and* a red card?

P(Win and Red)

= P(Red) P(Win | Red)



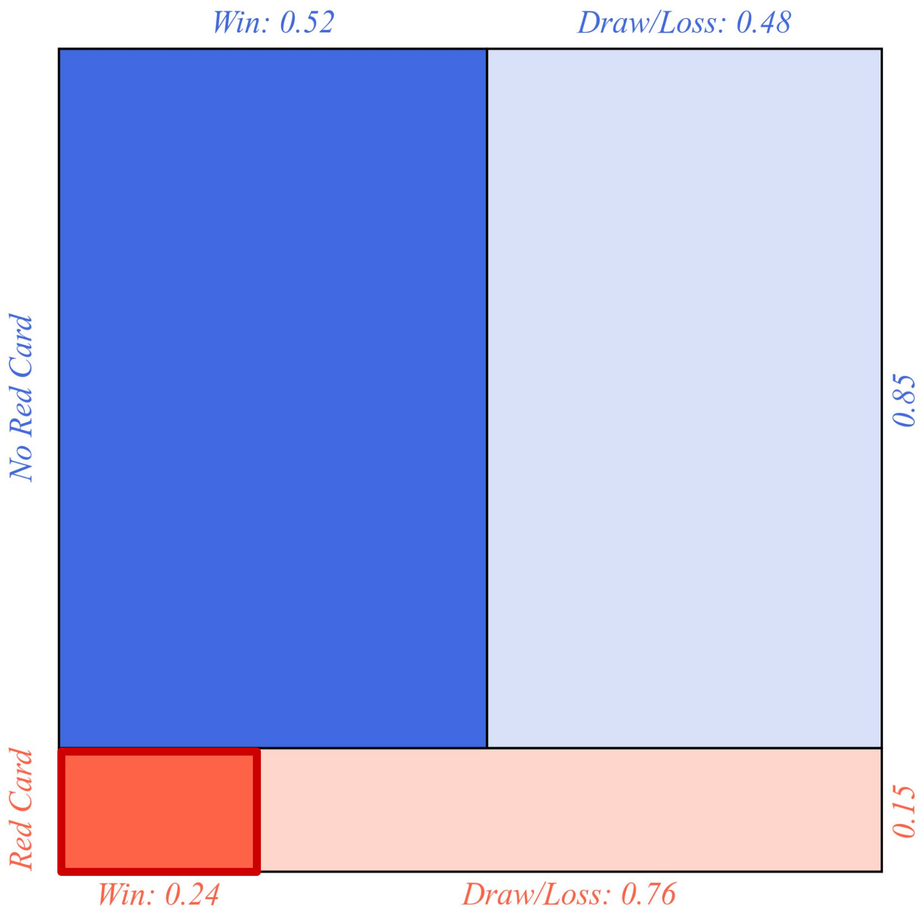
Probability Grid

Q. How does the new player change the probability of a win *and* a red card?

P(Win and Red)

$$= P(\text{Red})\, P(\text{Win} \mid \text{Red})$$

$$= 0.15 \cdot 0.24$$



Probability Grid

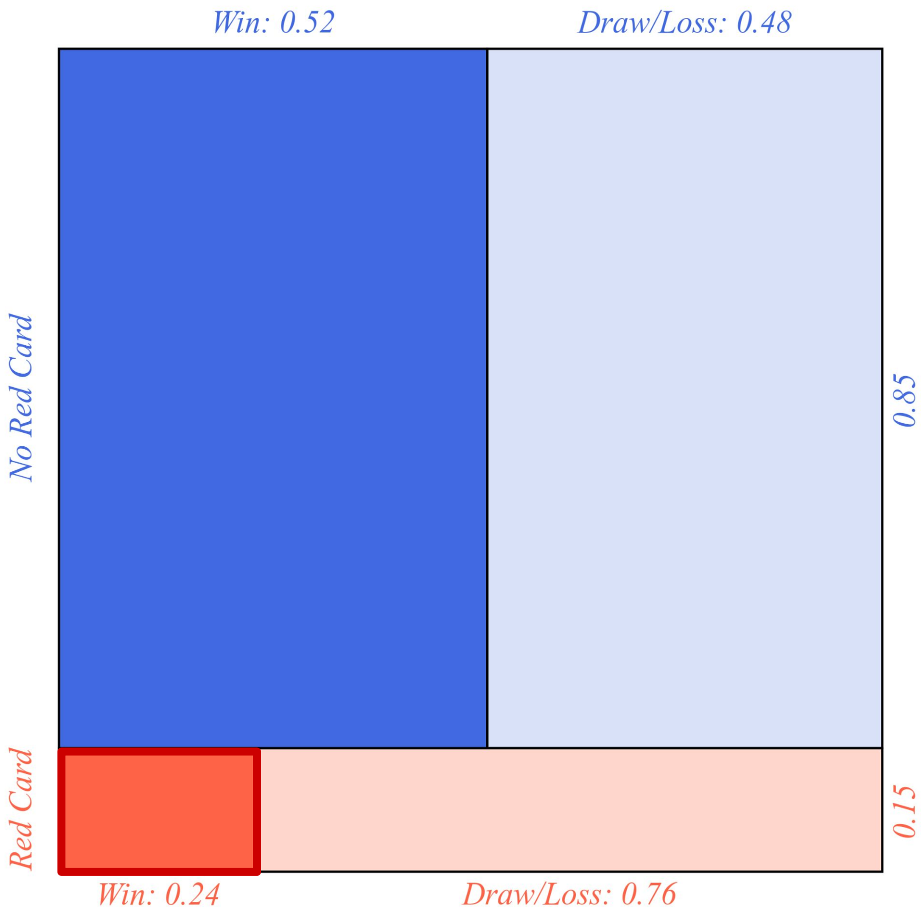
Q. How does the new player change the probability of a win *and* a red card?

P(Win and Red)

$= P(\text{Red})\ P(\text{Win} \mid \text{Red})$

$= 0.15 \cdot 0.24$

$=$



Probability Grid

Q. How does the new player change the probability of a win *and* a red card?

P(Win and Red)

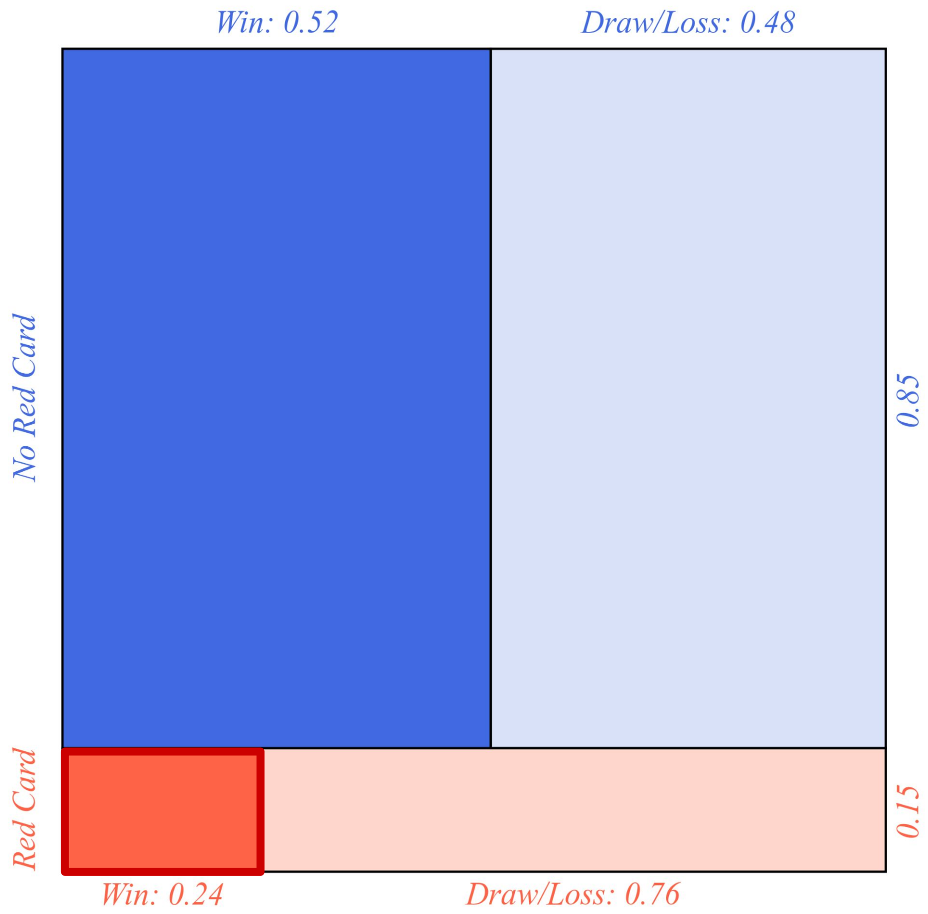
$= P(\text{Red})\, P(\text{Win} \mid \text{Red})$

$= 0.15 \cdot 0.24$

$=$

From Before:

$P(Win) = 0.05 \cdot 0.24$

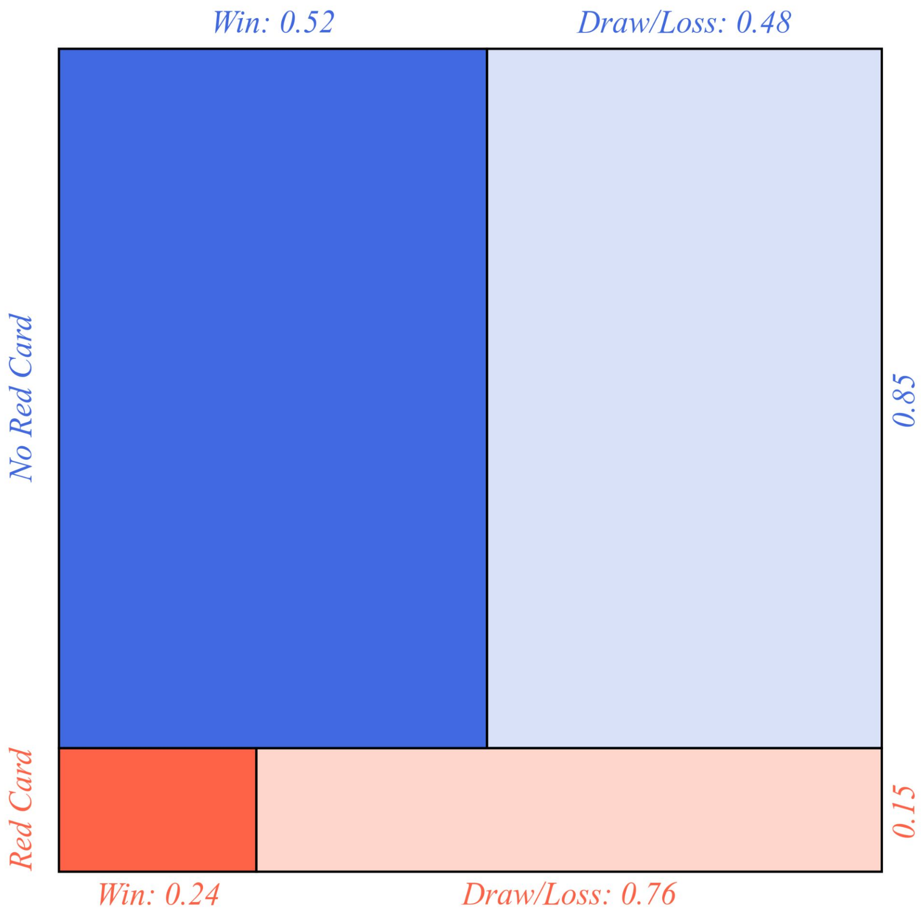


Probability Grid

Q. How does the new player change the probability of a win *given* a red card?

P(Win | Red)

=

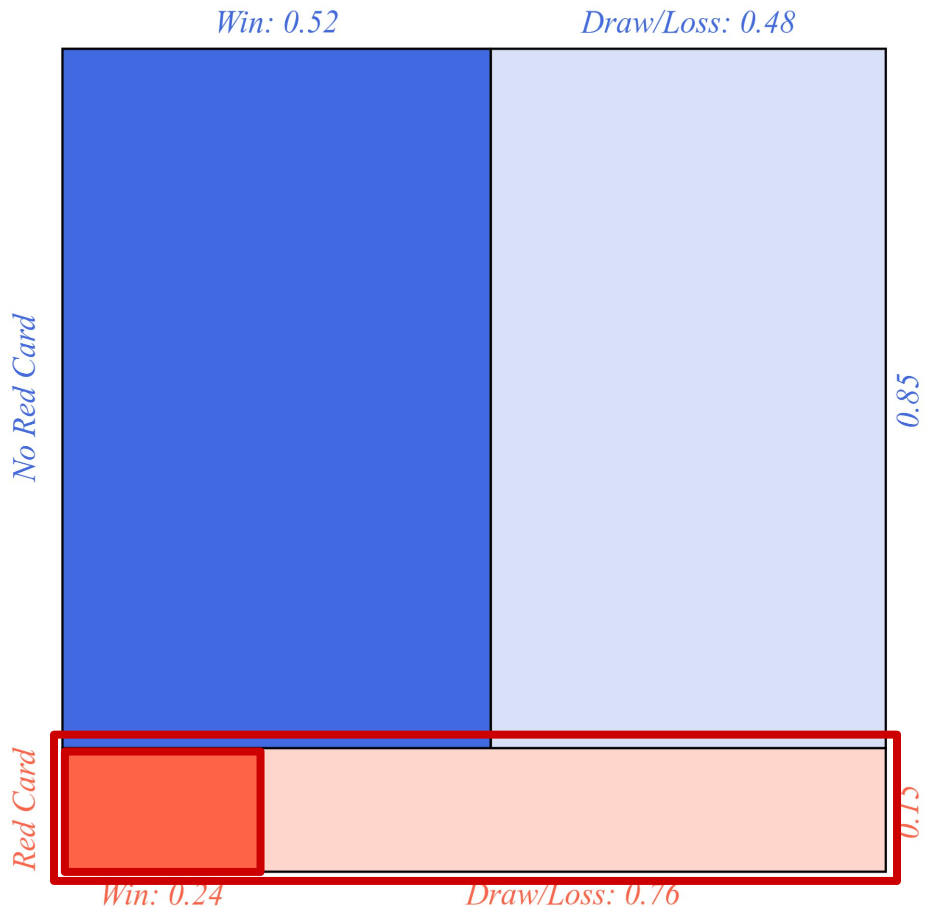


Probability Grid

Q. How does the new player change the probability of a win *given* a red card?

P(Win | Red)

=

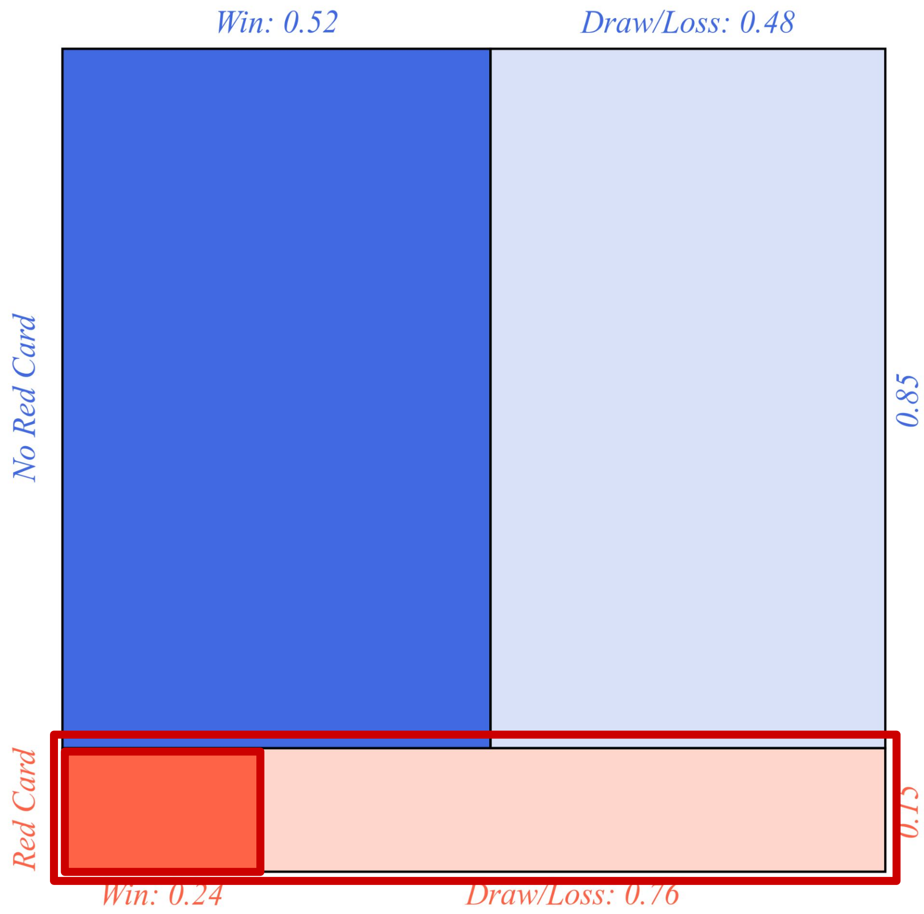


Probability Grid

Q. How does the new player change the probability of a win *given* a red card?

P(Win | Red)

= P(Win and Red) / P(Red)



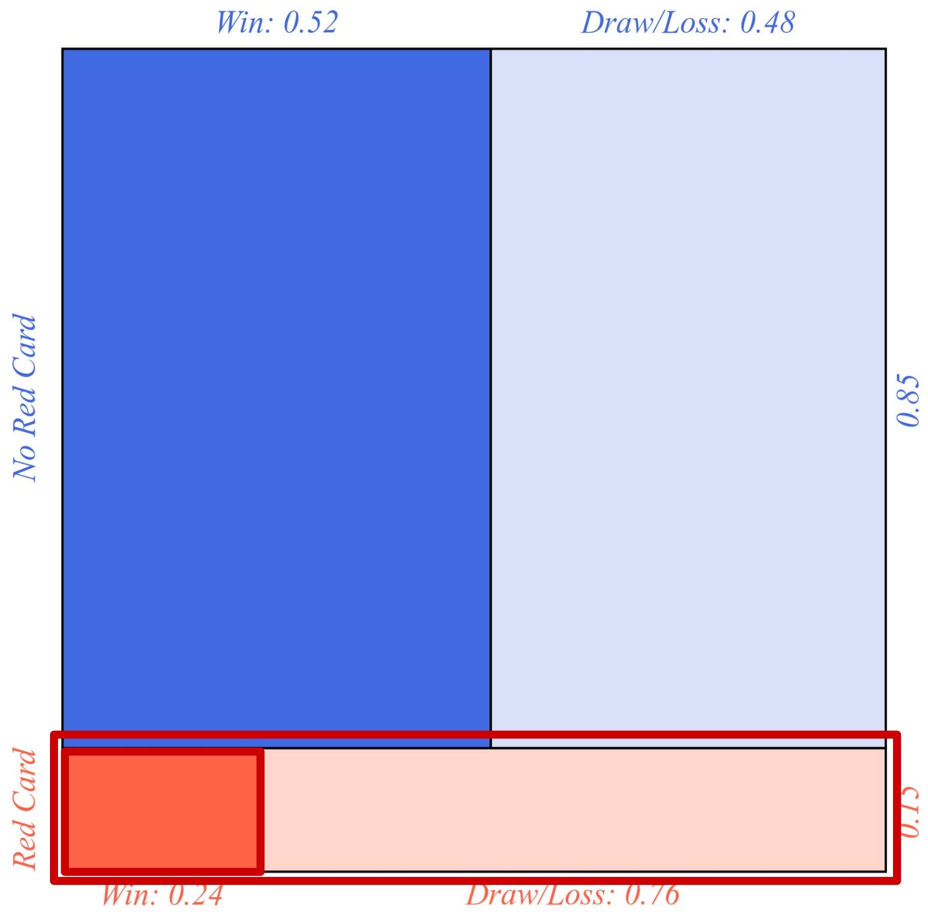
Probability Grid

Q. How does the new player change the probability of a win *given* a red card?

P(Win | Red)

= P(Win and Red) / P(Red)

= 0.15 · 0.24 / 0.15



Probability Grid

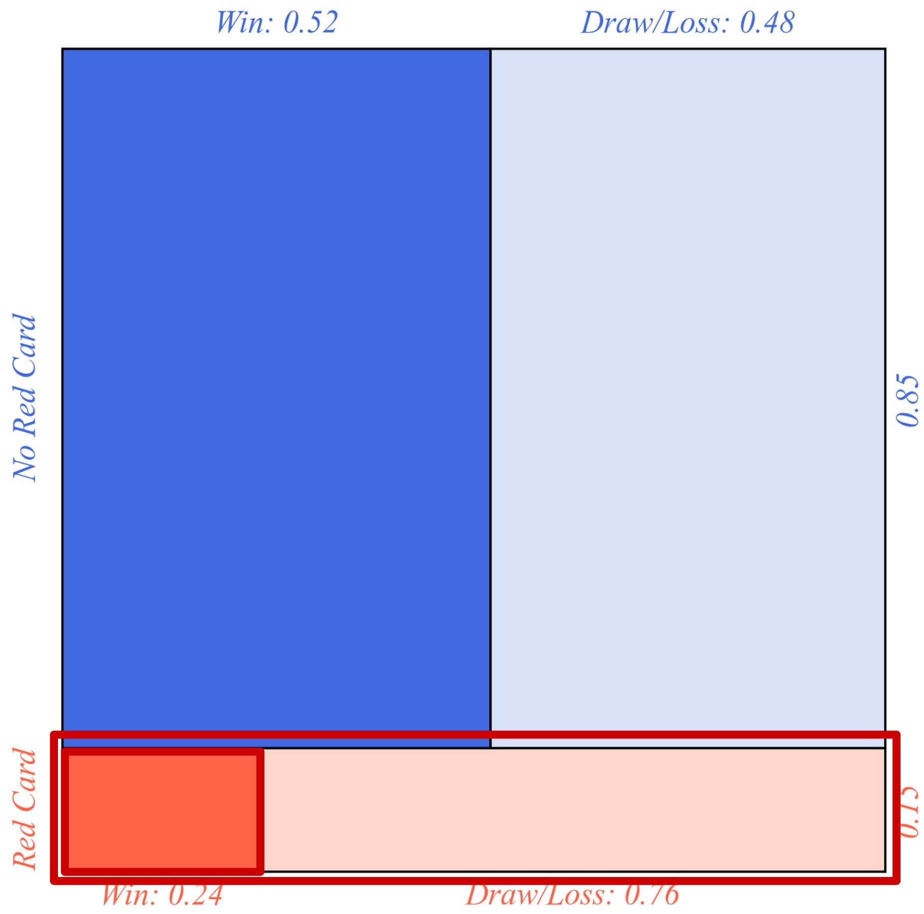
Q. How does the new player change the probability of a win *given* a red card?

P(Win | Red)

= P(Win and Red) / P(Red)

$= 0.15 \cdot 0.24 / 0.15$

$= 0.24$



Probability Grid

Q. How does the new player change the probability of a win *given* a red card?

P(Win | Red)

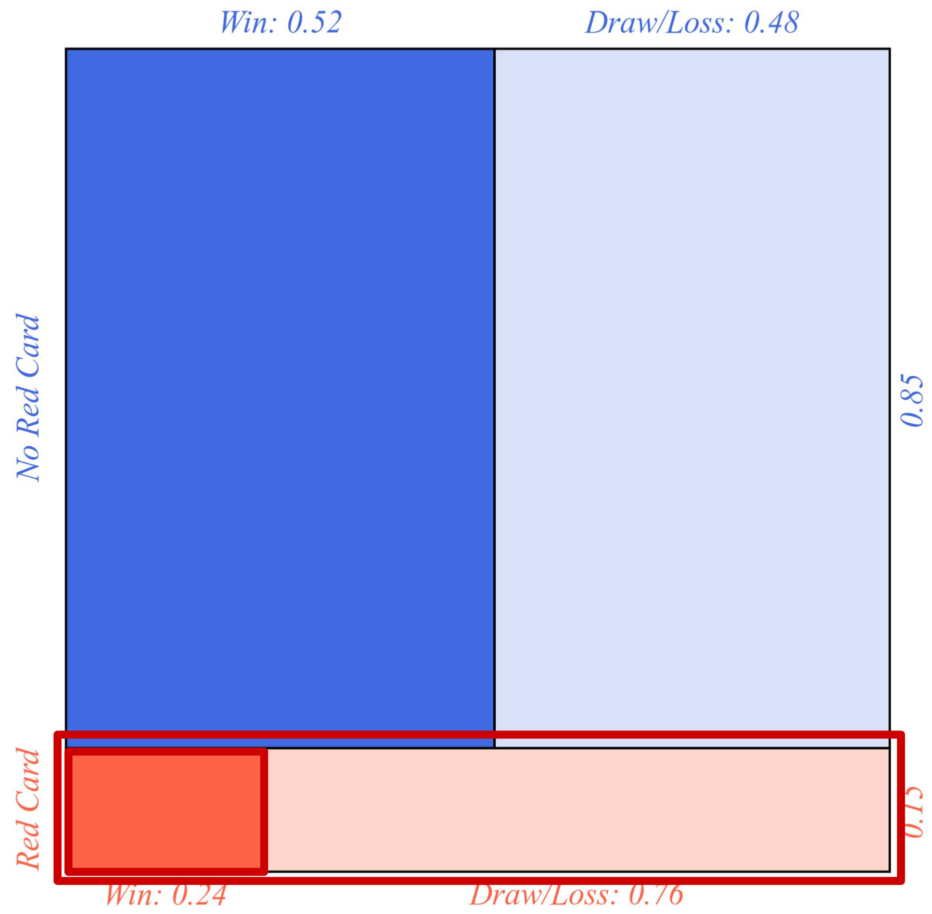
= P(Win and Red) / P(Red)

= 0.15 · 0.24 / 0.15

= 0.24

From Before:

P(Win) = 0.24



Bayes' Theorem

… but how do we update probabilities with new information?

Bayes' Theorem

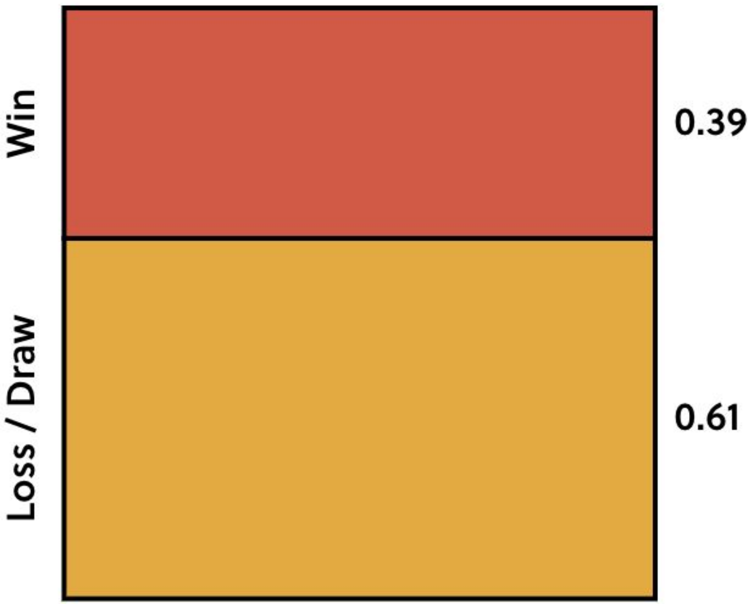
… how to update probabilities.

$$P(\text{Win} \mid \text{Predicted Win}) = P(\text{Win}) \frac{P(\text{Predicted Win} \mid \text{Win})}{P(\text{Predicted Win})}$$

Let's make this equation feel intuitive!

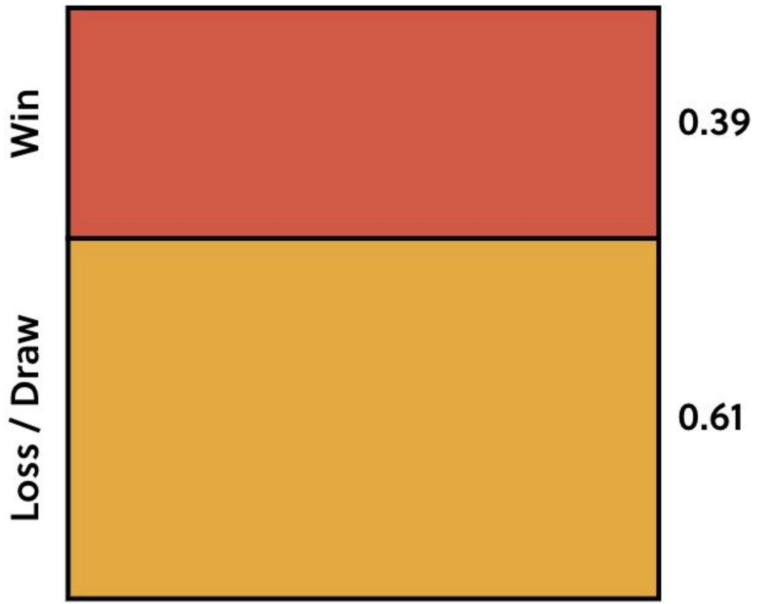
AFC Richmond’s Record

Probabilities of winning/not winning last season.



AFC Richmond’s Record

Q. If the captain predicts a win next game, how might we update our estimate?



AFC Richmond's Record

Q. If the captain predicts a win next game, how might we update our estimate?

A. It depends on his track record!

AFC Richmond’s Record

Q. If the captain has correctly predicted every win, how might we update our estimate?

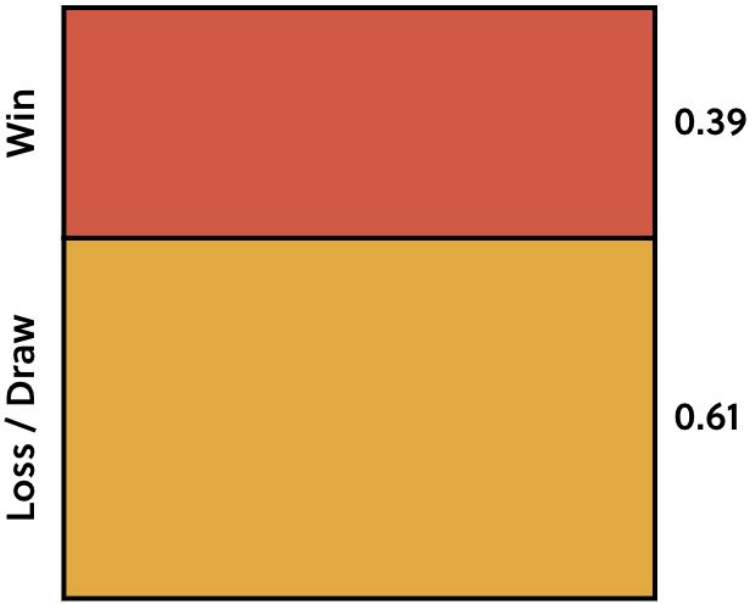
AFC Richmond's Record

Q. If the captain has correctly predicted every win, how might we update our estimate?
… what if he predicted a win in every game?

A. His track record does not provide us with any new information!

AFC Richmond’s Record

Q. How should we update our prediction based on the groundskeeper’s predictions?



AFC Richmond's Record

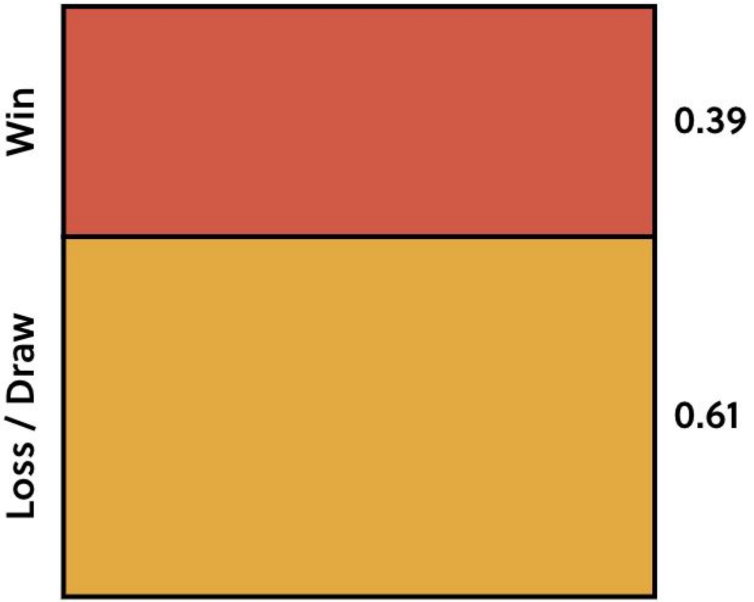
Update our prediction based on the groundskeeper's predictions:

$$P(\text{PredictWin} \mid \text{Win}) = 0.4$$

$$P(\text{PredictWin}) = 0.2$$

Q. What is the probability groundskeeper predicted a win *given* that the team won?

$$P(\text{PredictWin} \mid \text{Win}) = 0.4$$



AFC Richmond's Record

Update our prediction based on the groundskeeper's predictions:

$$P(\text{PredictWin} \mid \text{Win}) = 0.4$$

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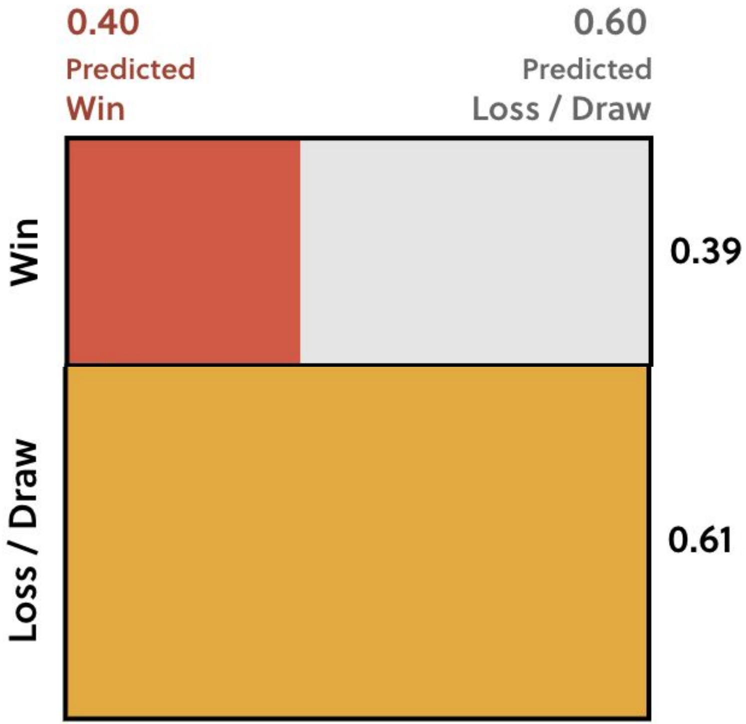
Q. What is the probability groundskeeper predicted a win *given* that the team won?

$$P(\text{PredictWin} \mid \text{Win}) = 0.4$$



AFC Richmond's Record

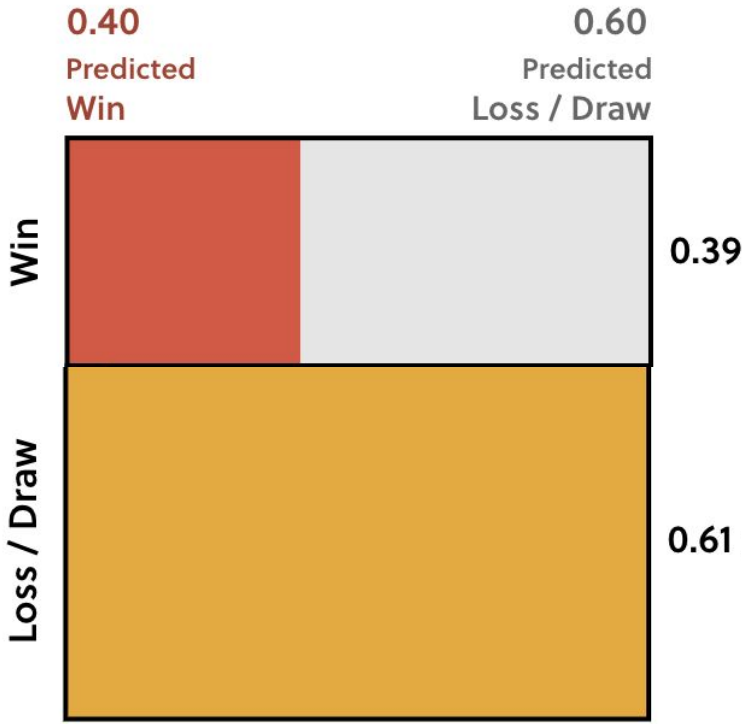
Update our prediction based on the groundskeeper's predictions:

$$P(\text{PredictWin} \mid \text{Win}) = 0.4$$

$$P(\text{PredictWin}) = 0.2$$

Q. What is the probability the groundskeeper predicted a win *given* that the team didn't win (correct win predict)?

$$P(\text{PredictWin} \mid \text{NotWin}) = ?$$



AFC Richmond's Record

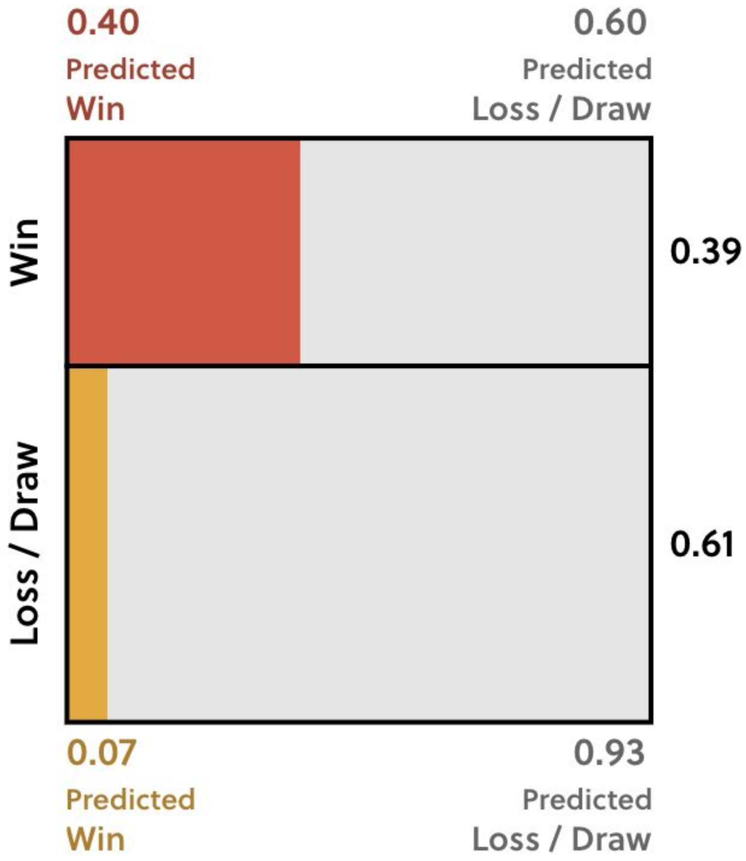
Update our prediction based on the groundskeeper's predictions:

$$P(\text{PredictWin} \mid \text{Win}) = 0.4$$

$$P(\text{PredictWin}) = 0.2$$

Q. What is the probability the groundskeeper predicted a win *given* that the team didn't win?

$$0.2 = P(\text{PredictWin}|\text{NotWin}) \cdot 0.61 + 0.4 \cdot 0.39$$



AFC Richmond's Record

Update our prediction based on the groundskeeper's predictions:

$$P(\text{PredictWin} \mid \text{Win}) = 0.4$$

$$P(\text{PredictWin}) = 0.2$$

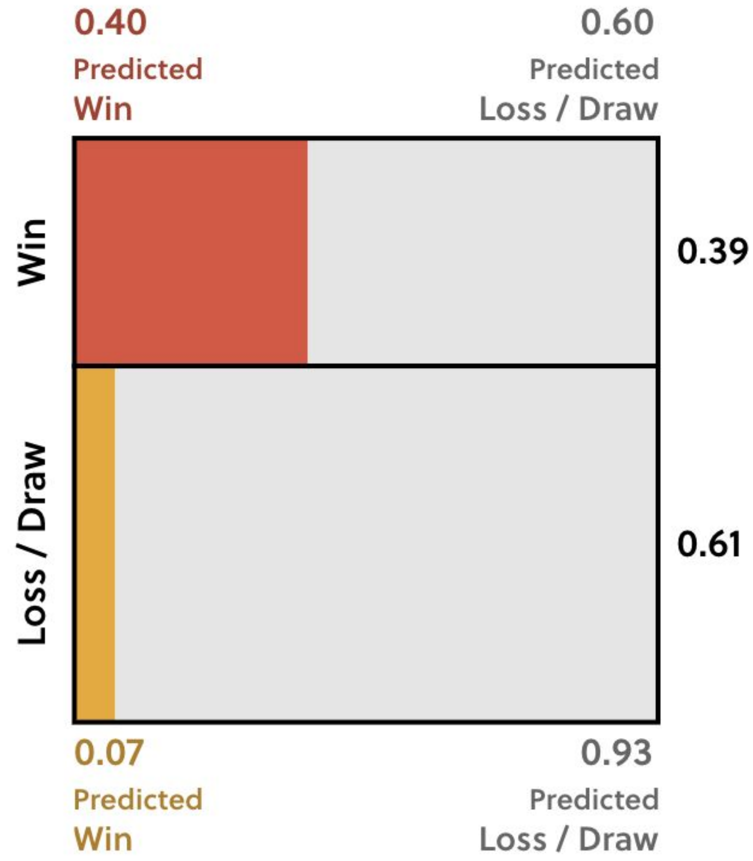
Q. What is the probability the groundskeeper predicted a win *given* that the team didn't win?

$$P(\text{PredictWin}) = 0.2$$

$$= P(\text{PredictWin} \mid \text{NotWin}) \cdot P(\text{NotWin})$$

$$+ P(\text{PredictWin} \mid \text{Win}) \cdot P(\text{Win})$$

$$= P(\text{PredictWin} \mid \text{NotWin}) \cdot 0.61 + 0.4 \cdot 0.39$$



AFC Richmond's Record

Update our prediction based on the groundskeeper's predictions:

$$P(\text{PredictWin} \mid \text{Win}) = 0.4$$

$$P(\text{PredictWin}) = 0.2$$

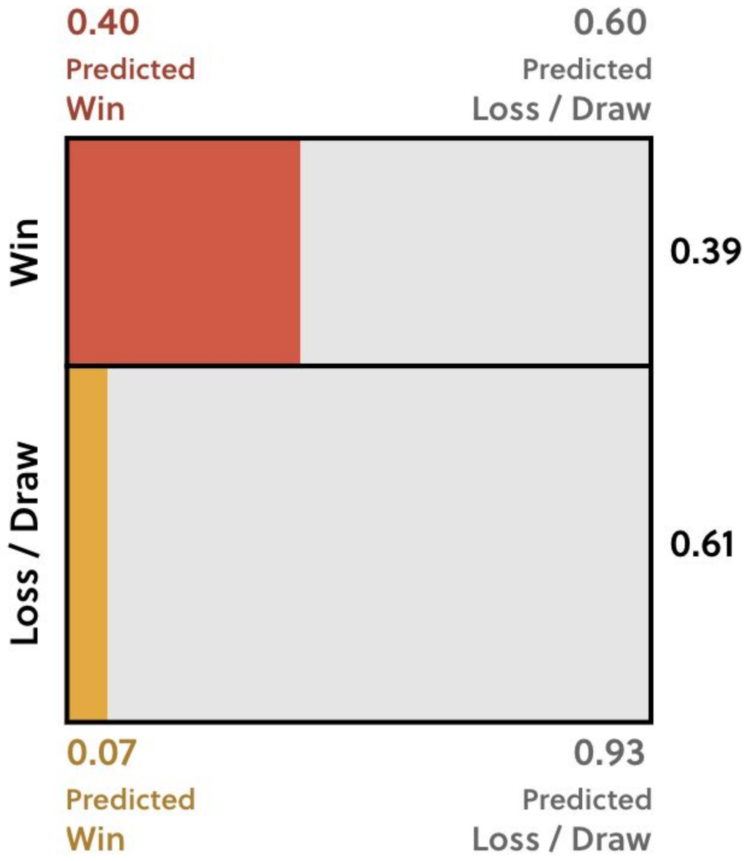
Q. What is the probability the groundskeeper predicted a win *given* that the team didn't win?

$$0.2 = P(\text{PredictWin}|\text{NotWin}) \cdot 0.61 + 0.4 \cdot 0.39$$

$$P(\text{PredictWin}|\text{NotWin})$$

$$= (0.2 - 0.4 \cdot 0.39) / 0.61$$

$$= 0.072$$



AFC Richmond's Record

Update our prediction based on the groundskeeper's predictions:

$P(\text{PredictWin} \mid \text{Win}) = 0.4$

$P(\text{PredictWin}) = 0.2$

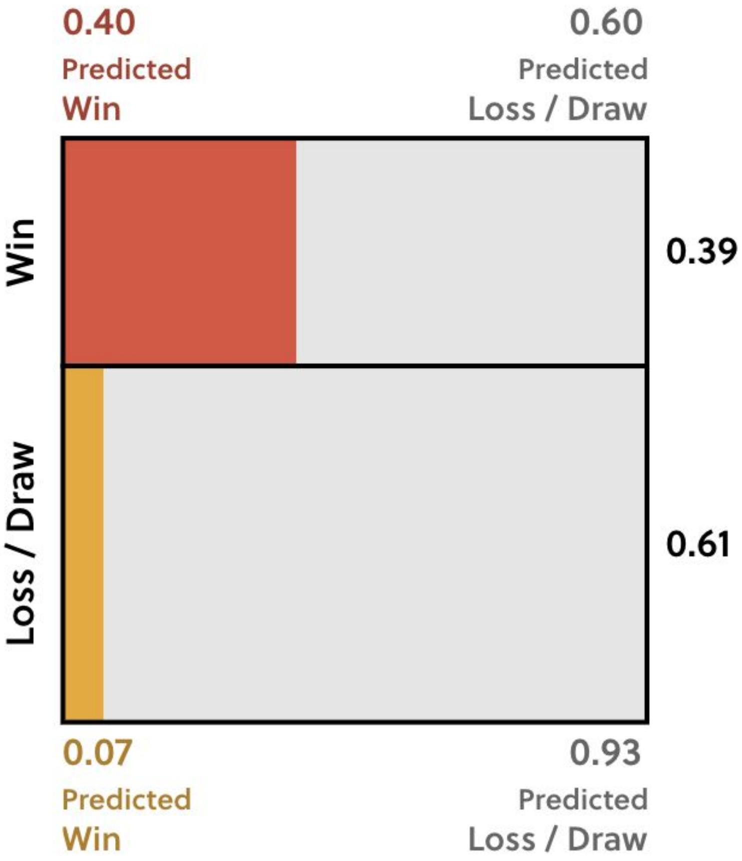
Q. What is the probability of a predicted win *and* a win?

$P(\text{PredictWin and Win})$

$= P(\text{Win}) \cdot P(\text{PredictWin} \mid \text{Win})$

$= 0.39 \cdot 0.4$

$=$



AFC Richmond's Record

Update our prediction based on the groundskeeper's predictions:

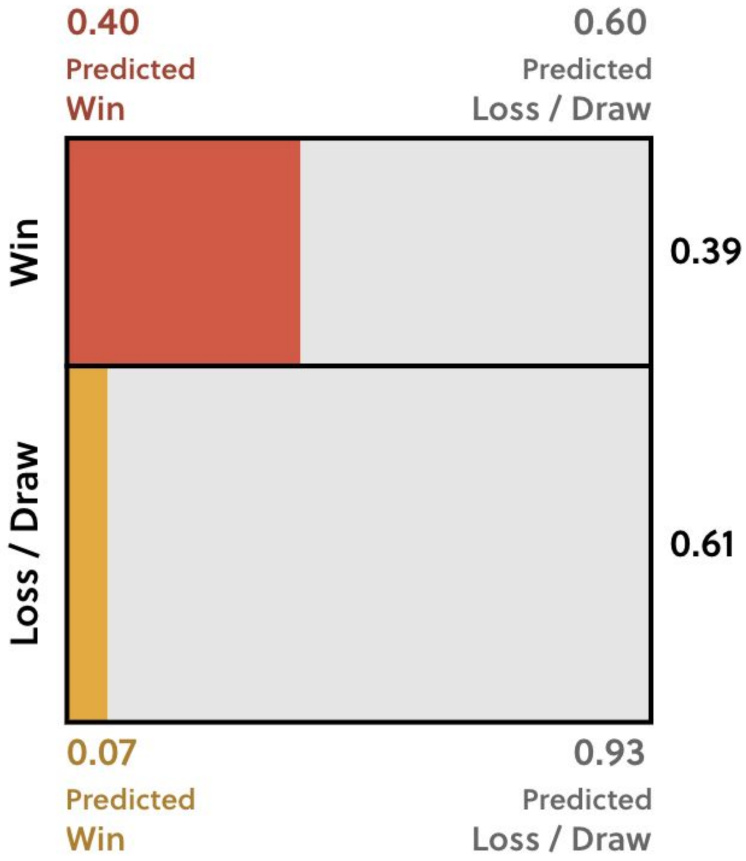
$$P(\text{PredictWin} \mid \text{Win}) = 0.4$$

$$P(\text{PredictWin}) = 0.2$$

Q. What is the probability of a win *given* a predicted win?

$$P(\text{Win} \mid \text{PredictWin})$$

$$=$$



AFC Richmond's Record

Update our prediction based on the groundskeeper's predictions:

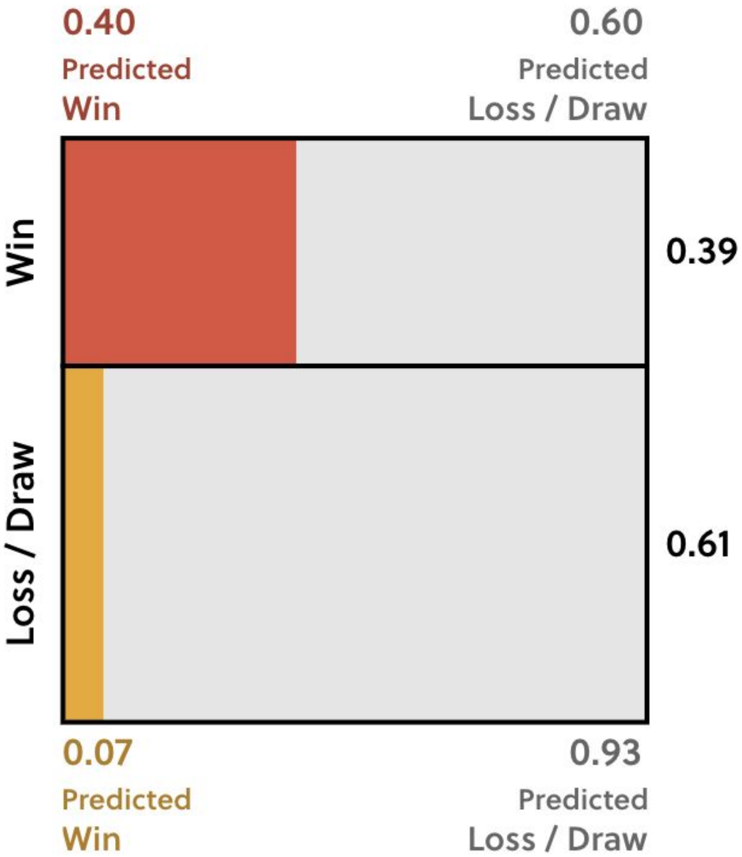
$P(\text{PredictWin} \mid \text{Win}) = 0.4$

$P(\text{PredictWin}) = 0.2$

Q. What is the probability of a win *given* a predicted win?

$P(\text{Win} \mid \text{PredictWin})$

= "of the times we predicted a win, how many have we won?"



AFC Richmond's Record

Update our prediction based on the groundskeeper's predictions:

$$P(\text{PredictWin} \mid \text{Win}) = 0.4$$

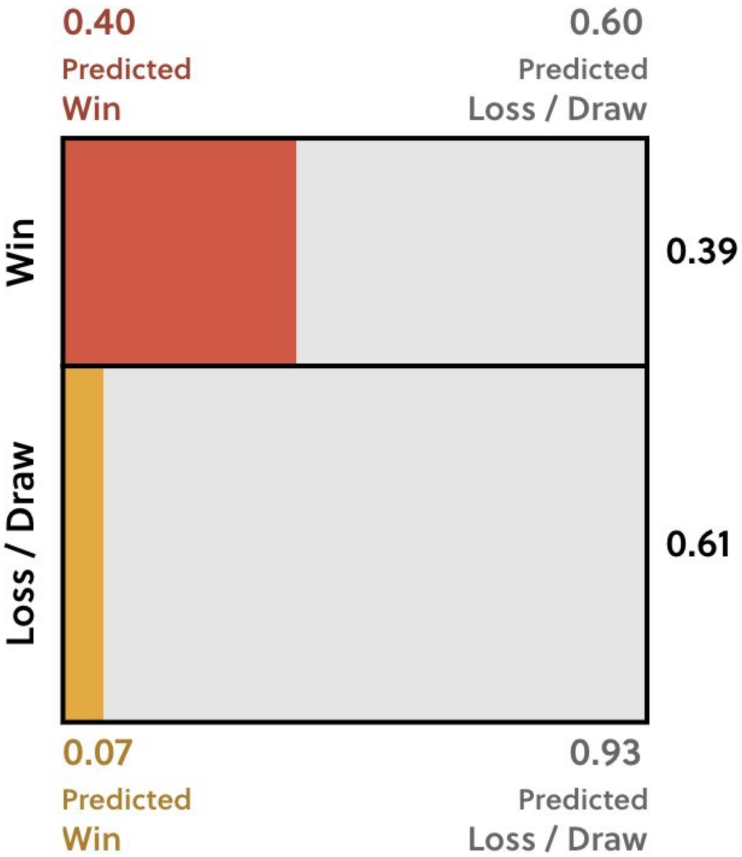
$$P(\text{PredictWin}) = 0.2$$

Q. What is the probability of a win *given* a predicted win?

$$P(\text{Win} \mid \text{PredictWin})$$

= "of the times we predicted a win, how many have we won?"

= Red / (Red + Yellow)



AFC Richmond's Record

Update our prediction based on the groundskeeper's predictions:

$P(\text{PredictWin} \mid \text{Win}) = 0.4$

$P(\text{PredictWin}) = 0.2$

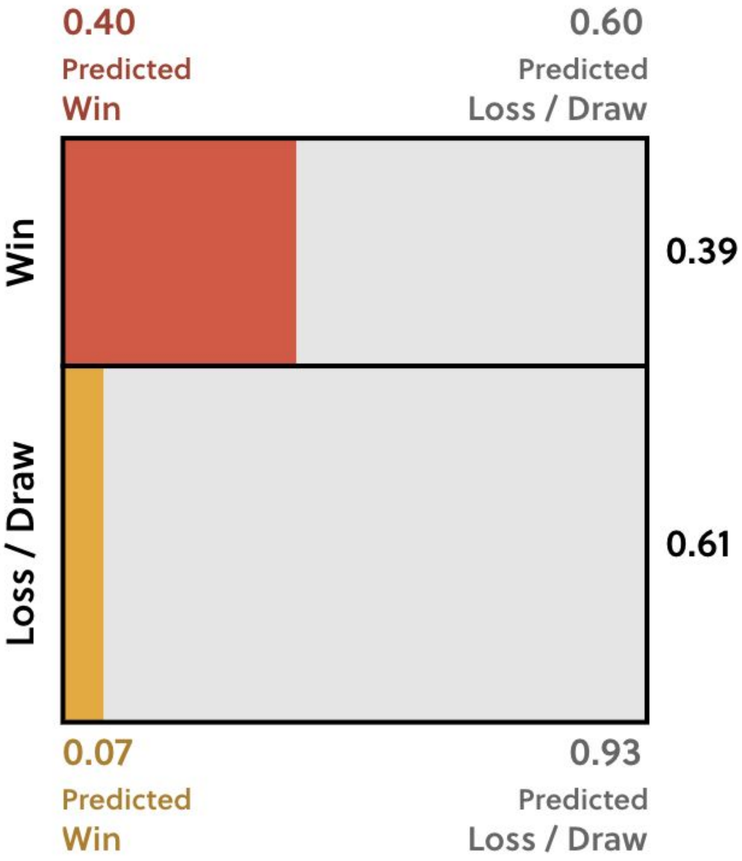
Q. What is the probability of a win *given* a predicted win?

$P(\text{Win} \mid \text{PredictWin})$

= "of the times we predicted a win, how many have we won?"

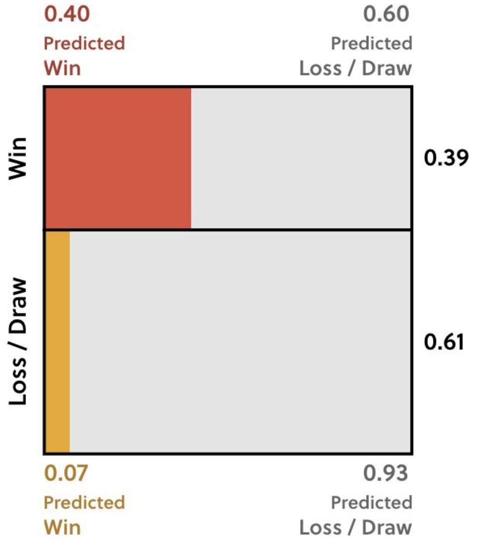
= Red / (Red + Yellow)

$= P(\text{Win}) \cdot P(\text{PredictWin} \mid \text{Win}) \,/\, P(\text{PredictWin})$



Bayes' Theorem

… how to update probabilities.

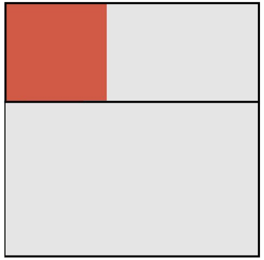


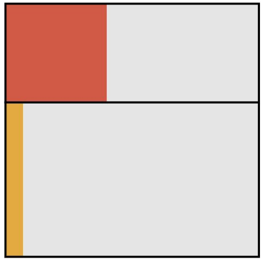
$$P(\text{Win} \mid \text{Predicted Win})$$

$$= \frac{P(\text{Win}) \cdot P(\text{Predicted Win} \mid \text{Win})}{P(\text{Predicted Win})}$$

$$= \frac{P(\text{Predicted Win } \mathit{AND} \text{ Win})}{P(\text{Predicted Win})}$$

=





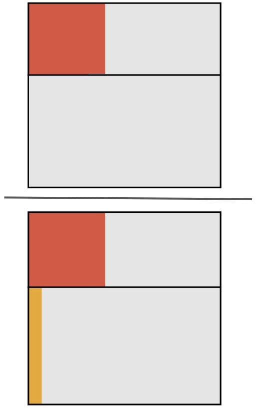
Bayes' Theorem: Intuition

$P(H \mid E)$

$$= \frac{P(H) \cdot P(E \mid H)}{P(E)}$$

$$= \frac{P(E \textit{ and } H)}{P(E)}$$

$=$



Marbles Example

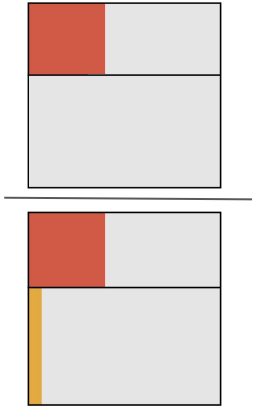
Green = 22
White = 24
Dot = 11
Green Dot = 3

Bayes' Theorem: Intuition

$$P(H \mid E)$$

$$= \frac{P(H) \cdot P(E \mid H)}{P(E)}$$

$$= \frac{P(E \textit{ and } H)}{P(E)}$$

$=$ 

Marbles Example

Green = 22
White = 24
Dot = 11
Green Dot = 3

P(Green) = 22 / 46

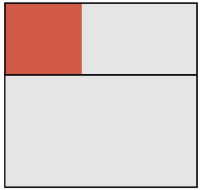
Bayes' Theorem: Intuition

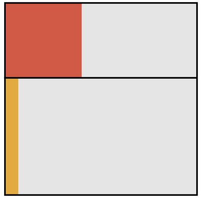
$$P(H \mid E)$$

$$= \frac{P(H) \cdot P(E \mid H)}{P(E)}$$

$$= \frac{P(E \textit{ and } H)}{P(E)}$$

=





Marbles Example

Green = 22
White = 24
Dot = 11
Green Dot = 3

P(Green) = 22 / 46
P(Dot) = 11 / 46

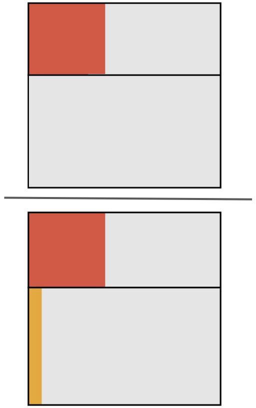
Bayes' Theorem: Intuition

$$P(H \mid E)$$

$$= \frac{P(H) \cdot P(E \mid H)}{P(E)}$$

$$= \frac{P(E \textit{ and } H)}{P(E)}$$

=



Marbles Example

Green = 22
White = 24
Dot = 11
Green Dot = 3

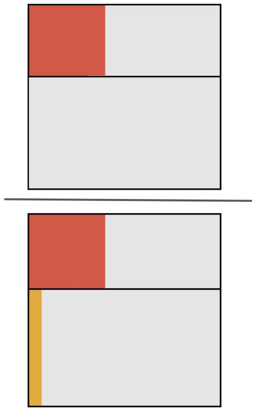
P(Green) = 22 / 46
P(Dot) = 11 / 46
P(Green *and* Dot) = 3 / 46

Bayes' Theorem: Intuition

$P(H \mid E)$

$$= \frac{P(H) \cdot P(E \mid H)}{P(E)}$$

$$= \frac{P(E \textit{ and } H)}{P(E)}$$

$=$ 

Marbles Example

Green = 22
White = 24
Dot = 11
Green Dot = 3

P(Green) = 22 / 46
P(Dot) = 11 / 46
P(Green *and* Dot) = 3 / 46

Q. If we get a White marble, what is the probability it has a dot?

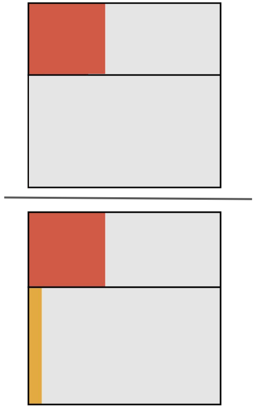
Bayes' Theorem: Intuition

$$P(H \mid E)$$

$$= \frac{P(H) \cdot P(E \mid H)}{P(E)}$$

$$= \frac{P(E \textit{ and } H)}{P(E)}$$

$=$



Marbles Example

Green = 22
White = 24
Dot = 11
Green Dot = 3

P(Green) = 22 / 46
P(Dot) = 11 / 46
P(Green *and* Dot) = 3 / 46

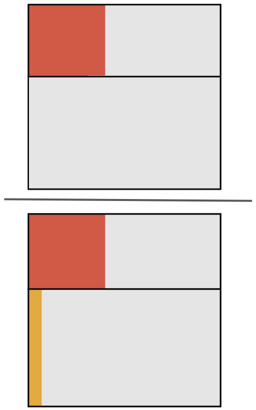
Q. If we get a White marble, what is the probability it has a dot?

P(Dot | White)
= P(White *and* Dot) / P(White)

Bayes' Theorem: Intuition

$$P(H \mid E) = \frac{P(H) \cdot P(E \mid H)}{P(E)} = \frac{P(E \textit{ and } H)}{P(E)}$$

=



Marbles Example

Green = 22
White = 24
Dot = 11
Green Dot = 3

P(Green) = 22 / 46
P(Dot) = 11 / 46
P(Green *and* Dot) = 3 / 46

Q. If we get a White marble, what is the probability it has a dot?

P(Dot | White)
= P(White *and* Dot) / P(White)
= (8 / 46) / (24 / 46) = 8 / 24

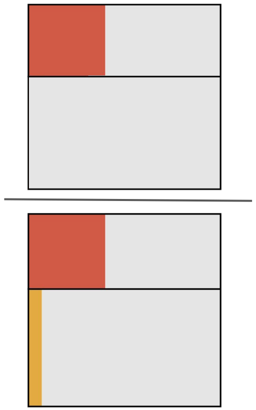
Bayes' Theorem: Intuition

$P(H \mid E)$

$$= \frac{P(H) \cdot P(E \mid H)}{P(E)}$$

$$= \frac{P(E \textit{ and } H)}{P(E)}$$

=



Excel Example

Green = 22
White = 24
Dot = 11
Green Dot = 3

$P(\text{Green}) = 22 / 46$
$P(\text{Dot}) = 11 / 46$
$P(\text{Green } \textit{and} \text{ Dot}) = 3 / 46$

Q. If we get a White marble, what is the probability it has a dot?

$P(\text{Dot} \mid \text{White})$

$$= P(\text{White } \textit{and} \text{ Dot}) / P(\text{White})$$
$$= (6 / 46) / (24 / 46) = 6 / 24$$

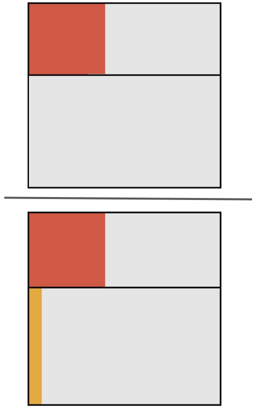
Bayes' Theorem

$P(H \mid E)$

$$= \frac{P(H) \cdot P(E \mid H)}{P(E)}$$

$$= \frac{P(E \text{ } \textit{and} \text{ } H)}{P(E)}$$

=



Coin Example: Compute P(H2 | E)

$E = \{W, W, W\}$

H1 = Coin A (fair coin)

H2 = Coin B (biased coin)

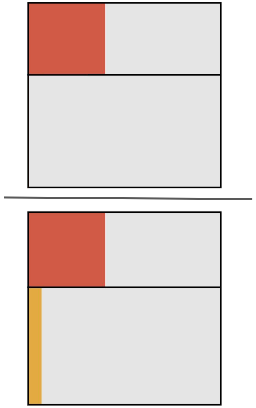
Bayes' Theorem

$$P(H \mid E)$$

$$= \frac{P(H) \cdot P(E \mid H)}{P(E)}$$

$$= \frac{P(E \textit{ and } H)}{P(E)}$$

=



Coin Example: Compute P(H2 | E)

E = {W, W, W}

H1 = Coin A (fair coin)

H2 = Coin B (biased coin)

P(H2): the likelihood of hypothesis H2

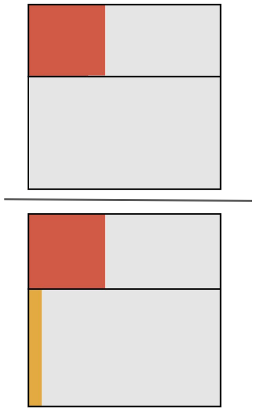
Bayes' Theorem

$P(H \mid E)$

$$= \frac{P(H) \cdot P(E \mid H)}{P(E)}$$

$$= \frac{P(E \textit{ and } H)}{P(E)}$$

=



Coin Example: Compute P(H2 | E)

E = {W, W, W}

H1 = Coin A (fair coin)

H2 = Coin B (biased coin)

P(H2): the likelihood of hypothesis H2

= 1 / 2

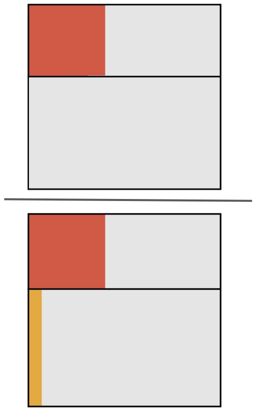
Bayes' Theorem

$$P(H \mid E)$$

$$= \frac{P(H) \cdot P(E \mid H)}{P(E)}$$

$$= \frac{P(E \textit{ and } H)}{P(E)}$$

$=$



Coin Example: Compute P(H2 | E)

E = {W, W, W}

H1 = Coin A (fair coin)

H2 = Coin B (biased coin)

P(H2): the likelihood of hypothesis H2

$= 1 / 2$

P(E | H1): the likelihood of observing E given H1

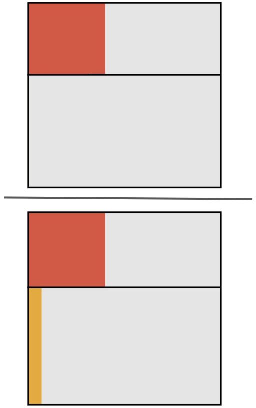
Bayes' Theorem

$$P(H \mid E)$$

$$= \frac{P(H) \cdot P(E \mid H)}{P(E)}$$

$$= \frac{P(E \textit{ and } H)}{P(E)}$$

=



Coin Example: Compute P(H2 | E)

E = {W, W, W}

H1 = Coin A (fair coin)

H2 = Coin B (biased coin)

P(H2): the likelihood of hypothesis H2

$= 1 / 2$

P(E | H1): the likelihood of observing E given H1

$= (1 / 2) \cdot (1 / 2) \cdot (1 / 2) = 1 / 8$

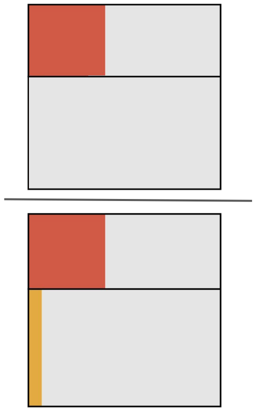
Bayes' Theorem

$P(H \mid E)$

$$= \frac{P(H) \cdot P(E \mid H)}{P(E)}$$

$$= \frac{P(E \textit{ and } H)}{P(E)}$$

=



Coin Example: Compute P(H2 | E)

E = {W, W, W}

H1 = Coin A (fair coin)

H2 = Coin B (biased coin)

P(H2): the likelihood of hypothesis H2

$= 1 / 2$

P(E | H1): the likelihood of observing E given H1

$= (1 / 2) \cdot (1 / 2) \cdot (1 / 2) = 1 / 8$

P(E | H2): the likelihood of observing E given H2

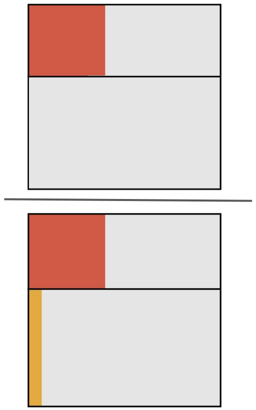
Bayes' Theorem

$$P(H \mid E)$$

$$= \frac{P(H) \cdot P(E \mid H)}{P(E)}$$

$$= \frac{P(E \textit{ and } H)}{P(E)}$$

=



Coin Example: Compute P(H2 | E)

E = {W, W, W}

H1 = Coin A (fair coin)

H2 = Coin B (biased coin)

P(H2): the likelihood of hypothesis H2

$= 1 / 2$

P(E | H1): the likelihood of observing E given H1

$= (1 / 2) \cdot (1 / 2) \cdot (1 / 2) = 1 / 8$

P(E | H2): the likelihood of observing E given H2

$= 1 \cdot 1 \cdot 1 = 1$

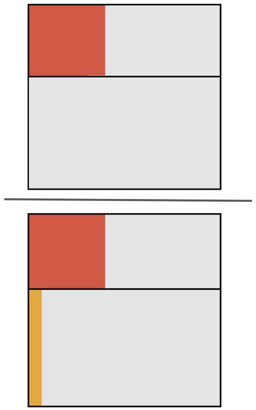
Bayes' Theorem

$P(H \mid E)$

$$= \frac{P(H) \cdot P(E \mid H)}{P(E)}$$

$$= \frac{P(E \textit{ and } H)}{P(E)}$$

=



Coin Example: Compute P(H2 | E)

E = {W, W, W}

H1 = Coin A (fair coin)

H2 = Coin B (biased coin)

P(H2): the likelihood of hypothesis H2

$= 1 / 2$

P(E | H1): the likelihood of observing E given H1

$= (1 / 2) \cdot (1 / 2) \cdot (1 / 2) = 1 / 8$

P(E | H2): the likelihood of observing E given H2

$= 1 \cdot 1 \cdot 1 = 1$

P(E): the total probability of observing E

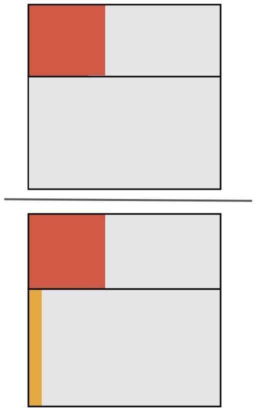
Bayes' Theorem

$P(H \mid E)$

$= \frac{P(H) \cdot P(E \mid H)}{P(E)}$

$= \frac{P(E \textit{ and } H)}{P(E)}$

=



Coin Example: Compute P(H2 | E)

$E = \{W, W, W\}$

$H1$ = Coin A (fair coin)

$H2$ = Coin B (biased coin)

$P(H2)$: the likelihood of hypothesis H2

$= 1 / 2$

$P(E \mid H1)$: the likelihood of observing E given H1

$= (1 / 2) \cdot (1 / 2) \cdot (1 / 2) = 1 / 8$

$P(E \mid H2)$: the likelihood of observing E given H2

$= 1 \cdot 1 \cdot 1 = 1$

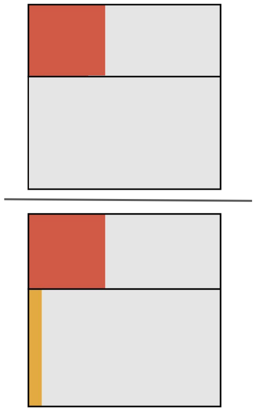
$P(E)$: the total probability of observing E

$= P(E \mid H1) \cdot P(H1) + P(E \mid H2) \cdot P(H2)$

Bayes' Theorem

$$P(H \mid E) = \frac{P(H) \cdot P(E \mid H)}{P(E)} = \frac{P(E \textit{ and } H)}{P(E)}$$

=



Coin Example: Compute P(H2 | E)

$E = \{W, W, W\}$

$H1$ = Coin A (fair coin)

$H2$ = Coin B (biased coin)

$P(H2)$: the likelihood of hypothesis $H2$

$$= 1 / 2$$

$P(E \mid H1)$: the likelihood of observing E given $H1$

$$= (1 / 2) \cdot (1 / 2) \cdot (1 / 2) = 1 / 8$$

$P(E \mid H2)$: the likelihood of observing E given $H2$

$$= 1 \cdot 1 \cdot 1 = 1$$

$P(E)$: the total probability of observing E

$$= P(E \mid H1) \cdot P(H1) + P(E \mid H2) \cdot P(H2)$$

$$= (1) \cdot (1 / 2) + (1 / 8) \cdot (1 / 2) = 9 / 16$$

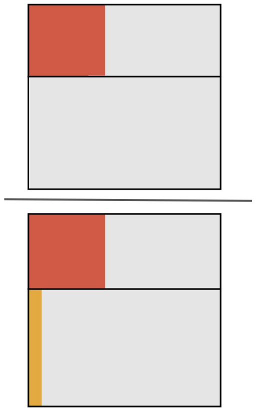
Bayes’ Theorem

$$P(H \mid E)$$

$$= \frac{P(H) \cdot P(E \mid H)}{P(E)}$$

$$= \frac{P(E \textit{ and } H)}{P(E)}$$

=



Coin Example: Compute P(H2 | E)

$E = \{W, W, W\}$

$H1$ = Coin A (fair coin)

$H2$ = Coin B (biased coin)

$P(H2)$: the likelihood of hypothesis H2

$= 1 / 2$

$P(E \mid H1)$: the likelihood of observing E given H1

$= (1 / 2) \cdot (1 / 2) \cdot (1 / 2) = 1 / 8$

$P(E \mid H2)$: the likelihood of observing E given H2

$= 1 \cdot 1 \cdot 1 = 1$

$P(E)$: the total probability of observing E

$= P(E \mid H1) \cdot P(H1) + P(E \mid H2) \cdot P(H2)$

$= (1) \cdot (1 / 2) + (1 / 8) \cdot (1 / 2) = 9 / 16$

$P(H2 \mid E)$: the posterior probability of H2 given E

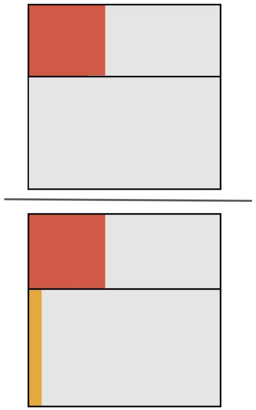
Bayes' Theorem

$$P(H \mid E)$$

$$= \frac{P(H) \cdot P(E \mid H)}{P(E)}$$

$$= \frac{P(E \textit{ and } H)}{P(E)}$$

=



Coin Example: Compute P(H2 | E)

E = {W, W, W}

H1 = Coin A (fair coin)

H2 = Coin B (biased coin)

P(H2): the likelihood of hypothesis H2

$= 1 / 2$

P(E | H1): the likelihood of observing E given H1

$= (1 / 2) \cdot (1 / 2) \cdot (1 / 2) = 1 / 8$

P(E | H2): the likelihood of observing E given H2

$= 1 \cdot 1 \cdot 1 = 1$

P(E): the total probability of observing E

$= P(E \mid H1) \cdot P(H1) + P(E \mid H2) \cdot P(H2)$

$= (1) \cdot (1 / 2) + (1 / 8) \cdot (1 / 2) = 9 / 16$

P(H2 | E): the posterior probability of H2 given E

$= P(H2 \textit{ and } E) / P(E)$

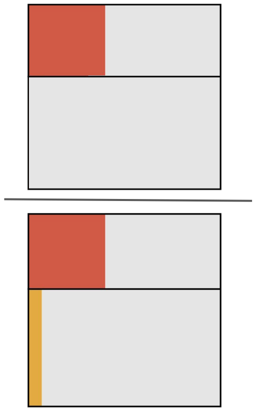
Bayes' Theorem

$P(H \mid E)$

$$= \frac{P(H) \cdot P(E \mid H)}{P(E)}$$

$$= \frac{P(E \textit{ and } H)}{P(E)}$$

=



Coin Example: Compute P(H2 | E)

$E = \{W, W, W\}$

H1 = Coin A (fair coin)

H2 = Coin B (biased coin)

P(H2): the likelihood of hypothesis H2

$= 1 / 2$

P(E | H1): the likelihood of observing E given H1

$= (1 / 2) \cdot (1 / 2) \cdot (1 / 2) = 1 / 8$

P(E | H2): the likelihood of observing E given H2

$= 1 \cdot 1 \cdot 1 = 1$

P(E): the total probability of observing E

$= P(E \mid H1) \cdot P(H1) + P(E \mid H2) \cdot P(H2)$

$= (1) \cdot (1 / 2) + (1 / 8) \cdot (1 / 2) = 9 / 16$

P(H2 | E): the posterior probability of H2 given E

$= P(H2 \textit{ and } E) / P(E)$

$= P(H2) \cdot P(E \mid H2) / P(E)$

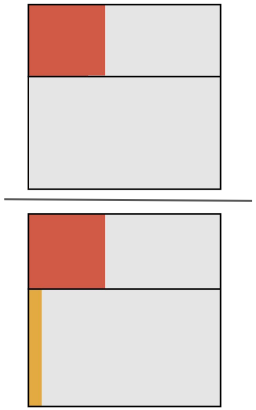
Bayes' Theorem

$P(H \mid E)$

$$= \frac{P(H) \cdot P(E \mid H)}{P(E)}$$

$$= \frac{P(E \textit{ and } H)}{P(E)}$$

=



Coin Example: Compute P(H2 | E)

$E = \{W, W, W\}$

$H1$ = Coin A (fair coin)

$H2$ = Coin B (biased coin)

$P(H2)$: the likelihood of hypothesis H2

$= 1 / 2$

$P(E \mid H1)$: the likelihood of observing E given H1

$= (1 / 2) \cdot (1 / 2) \cdot (1 / 2) = 1 / 8$

$P(E \mid H2)$: the likelihood of observing E given H2

$= 1 \cdot 1 \cdot 1 = 1$

$P(E)$: the total probability of observing E

$= P(E \mid H1) \cdot P(H1) + P(E \mid H2) \cdot P(H2)$

$= (1) \cdot (1 / 2) + (1 / 8) \cdot (1 / 2) = 9 / 16$

$P(H2 \mid E)$: the posterior probability of H2 given E

$= P(H2 \textit{ and } E) / P(E)$

$= P(H2) \cdot P(E \mid H2) / P(E)$

$= (1 / 2) \cdot (1) / (9 / 16) = 8 / 9$