

Example Exam 2

Make judgement of each of the following statements, and circle TRUE or FALSE.

1. If two events are independent, then they are mutually exclusive.

TRUE FALSE

$$P(A \text{ and } B) = 0$$

2. If A and B are independent where $P(A) = 0.2$ and $P(B) = 0.4$, then $P(A \text{ and } B) = 0.6$.

TRUE FALSE

$$P(A \text{ and } B) = P(A)P(B)$$

3. The greater a random variable's expected value is --- in absolute terms --- the greater its variance is.

TRUE FALSE

4. Daily sales at a car dealership can be either high or low, with the probability of sales being low equal to 0.6. The dealership has two top salespeople, and at least one of them is at work every day. The probability that both top salespeople are at work is 0.7. The probability that daily sales are low and both top salespeople are at work is equal to 0.35.

$$P(H) = 0.4, P(L) = 0.6, P(ONE) = 0.3, P(BOTH) = 0.7$$

- a. What is the probability that sales are high and only one top salesperson is at work?

$$P(H \text{ and } ONE) = A$$

$$A + B = 0.4$$

$$B + D = 0.7$$

$$B = 0.7 - 0.35 = 0.35$$

$$\rightarrow A = 0.4 - 0.35 = 0.05$$

$$P(L \text{ and } BOTH) = 0.35$$

$P(H)$	A	B	0.4
	C	D = 0.35	0.6

$$0.3 \quad 0.7$$

- b. If sales were high, what is the probability that both top salespeople were at work?

$$P(BOTH|H) = \frac{B}{A+B} = \frac{0.35}{0.4}$$

- c. Suppose that the net profit is \$100K when sales are high and \$50K when sales are low. Calculate the expected net profit if both top salespeople are at work.

$$E[BOTH] = 100 \cdot P(H|BOTH) + 50 \cdot P(L|BOTH)$$

$$= 75K$$

$$P(H|BOTH) = \frac{B}{0.7} = \frac{1}{2}$$

$$P(L|BOTH) = \frac{1}{2}$$

$$P(H) = \frac{1}{2} \quad P(T) = \frac{1}{2} \quad P(R|H) = \frac{5}{8}$$

$$P(R|T) = \frac{1}{4}$$

5. I use a fair coin toss to randomly draw a ball from one of two bins. If the coin comes up head, I draw a ball from Bin 1. Otherwise, I draw it from Bin 2. Each bin has a number of balls in two different colors. Bin 1 has 5 red balls and 3 yellow balls, while Bin 2 has 1 red ball and 3 yellow balls.

a. What is the probability that a random ball I draw is yellow?

$$\begin{aligned} P(Y) &= B + D \\ &= P(H) \cdot P(Y|H) + P(T) \cdot P(Y|T) \\ &= \frac{1}{2} \cdot \frac{3}{8} + \frac{1}{2} \cdot \frac{3}{4} = \frac{3}{16} + \frac{6}{16} = \frac{9}{16} \end{aligned}$$

b. If I have drawn a yellow ball, what is the probability that I drew it from Bin 2?

$$P(T|Y) = \frac{D}{B + D} = \frac{P(T) \cdot P(Y|T)}{P(Y)} = \frac{\frac{1}{2} \cdot \frac{3}{4}}{\frac{9}{16}} = \frac{3/8}{9/16} = \frac{6}{9} = \frac{2}{3}$$

