

San Jose State University
ISE 201 Math Foundations for Data and Decision Science
Fall 2020 Midterm

Name: _____

"I, (state your name), declare that the submitted work is original and adheres to all University policies on Academic Integrity and acknowledge the consequences that may result from a violation of those rules. I have neither given nor received unauthorized assistance during the completion of this exam."

Signature & Date: _____

Instructions:

- The exam is scheduled from 4 - 6pm.
- It is an open book and open notes exam.
- Calculators are allowed.
- The exam is for 100 points total and consists of 5 questions of 10, 25, 25, 20, and 20 points respectively. There is 1 extra credit problem for 20 points.
- Please provide the details of arriving at the answers for getting the full credit. Arithmetic expressions should be fully reduced to receive full credit.
- Please include this statement on each page to get credit for the page
"I pledge that I have neither received nor given unauthorized assistance during the completion of this work." (Please sign your name)
- Please make sure to scan all the sheets in order to get full credit, including this sheet with signature and date.

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Question 1 (10 points)

A certain satellite launch has a 0.7 probability of success. The probability remains constant at 0.7 for repeated launches and the success or failure of one launch does not depend on the outcome of other launches. It is feared that funding for the satellite program will be cut off if no success occurs within 3 trials. What is the probability of losing the funding?

Question 2 (25 points)

The columns of the following matrix are mutually orthogonal:

$$A = \begin{pmatrix} 1 & -1 & 1 \\ -1 & 0 & 2 \\ 1 & 1 & 1 \end{pmatrix}$$

- a) Normalize the columns of A by dividing each column by its length; denote the resulting matrix by C
- b) Show that

$$C' C = C' C = I$$

Question 3 (25 points)

The rate of return of an asset is the change in price divided by the initial price (denoted as r). Suppose that \$10,000 is used to purchase shares in three stocks with rates of returns X_1, X_2, X_3 . Initially, \$2500, \$3000, and \$4500 are allocated to each one, respectively. After one year, the distribution of the rate of return for each is normally distributed with the following parameters: $E(X_1) = 0.12$, $V(X_1) = 0.0196$, $E(X_2) = 0.04$, $V(X_2) = 0.0004$, $E(X_3) = 0.07$, $V(X_3) = 0.0064$.

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- (a) Assume that these rates of return are independent. Determine the mean and variance of the rate of return after one year for the entire investment of \$10,000.
- (b) Assume that X_1 is independent of X_2 and X_3 but that the covariance between X_1 and X_3 is -0.005 . Repeat part (a).
- (c) Compare the means and variances obtained in parts (a) and (b) and comment on any benefits from negative covariances between the assets.

Question 4 (20 points)

Accidents on highways are one of the main causes of death or injury in developing countries and the weather conditions have an impact on the rates of death and injury. In foggy, rainy, and sunny conditions, $1/4$, $1/8$, and $1/21$ of the accidents result in death, respectively. Sunny conditions occur 60% of the time, while rainy and foggy conditions each occur 20% of the time. Given that an accident without deaths occurred, what is the conditional probability that it was foggy at the time?

Question 5 (20 points)

For which numbers b and c are these matrices positive definite?

$$a) \ S = \begin{bmatrix} 1 & b \\ b & 9 \end{bmatrix} , \quad b) \ S = \begin{bmatrix} 2 & 4 \\ 4 & c \end{bmatrix} , \quad c) \ S = \begin{bmatrix} c & b \\ b & c \end{bmatrix}$$

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Extra Credit (20 points)

There are 550,000 people in the US infected with HIV. Of these people, 275,000 are drug users, and the rest are not drug users. The total population of the US is 250 million. There are 10 million drug users in the US. The standard blood test for HIV infection is not always accurate. The probability that someone who is infected with HIV will test positive for HIV is 0.99. The probability that someone who is not infected with HIV will test negative for HIV is also 0.99. Answer the following questions, clearly stating any assumptions that you need to make.

- A. Suppose after a randomly chosen person takes the standard blood test for HIV, and the outcome of the test is positive. What is the probability that this person is infected with HIV? Is your answer surprising?
- B. Suppose that a randomly chosen drug user takes the standard blood test for HIV and the outcome of the test is positive. What is the probability that this person is infected with HIV?