San Jose State University
Department of Industrial and Systems Engineering
ISE 201 Math Foundations for Data and Decision Science
Fall 2020 Final

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"I, (state your name), declare that the submitted work is original and adheres to all University policies on Academic Integrity and acknowledge the consequences that may result from a violation of those rules. I have neither given nor received unauthorized assistance during the completion of this exam."

Signature & Date: Taylor Maurer 12/10/2020

Instructions:

- The exam is scheduled from 4 - 8pm.
- It is an open book and open notes exam.
- Calculators are allowed.
- The exam is for 100 points total and consists of 5 questions of 25, 5, 20, 30 and 20 points respectively. There is 1 extra credit problem for 10 points.
- Please provide the details of arriving at the answers for getting the full credit. Arithmetic expressions should be fully reduced to receive full credit.
- Please include this statement on each page to get credit for the page *"I pledge that I have neither received nor given unauthorized assistance during the completion of this work." (Please sign your name)*
- Please make sure to scan all the sheets in order to get full credit, including this sheet with signature and date.

#1.

- 8 → black beans, !
- 2 → pinto beans, !

a) binomial distribution assumptions

1. trials are independent, guests decisions don't impact each other
2. there's only 2 outcomes success or failure (black beans or not)
3. the probability of success in each trial is constant, ie. the liklihood a customer prefers blackbeans is constant

b) $f(x) = \binom{6}{x} (0.8)^x (0.2)^{6-x}$

6!

c) $P(x=3) = \binom{6}{3} (0.8)^3 (0.2)^3$

6·5·4·3

$$\frac{6!}{3!(3!)} \cdot (0.8)^3 \cdot (0.2)^3$$

$= 20 \cdot 0.512 \cdot 0.008$

$\boxed{P(x=3) = 0.08192}$

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#1 cont.

$E(X) = n \cdot p = 6 \cdot 0.8 = \boxed{4.8 = E(X)}$

$V(X) = np(1-p) = 6 \cdot .8 \cdot .2 = \boxed{0.96 = V(x)}$

$\boxed{\text{Stand dev} = \sqrt{0.96} = 0.9798}$

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#2

$E(x = S.J) = 25$

$E(x = P.A.) = 33.1$

$\sqrt{V(x=SJ)} = 7$

$V = 49$

$\sqrt{V(x=PA)} = 6.2$

$V = 6.2^2$

$cov(SJ, PA) = -17.1$

$$cor(SJ, PA) = \frac{cov(SJ, PA)}{\sqrt{V(x=SJ)\,V(x=PA)}}$$

$$\boxed{cor(SJ,PA) = \frac{-17.1}{\sqrt{49 \cdot 6.2^2}} = -0.39401}$$

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#3

- First let's find $\bar{x}_T$, mean relatia frequency of any word for tale of two cities

$$\bar{x}_T = \frac{150 + 30 + 30 + 90}{4} = 75$$

do the same for long lost work $\bar{x}_L$

$$\bar{x}_L = \frac{90 + 20 + 10 + 80}{4} = 50$$

- Now find the sample variances

s_T^2 and s_L^2

$$s_T^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x}_T)^2}{n-1} = \frac{(50-75)^2 + (30-75)^2 + (30-75)^2 + (90-75)^2}{3}$$

$$s_T^2 = 3300$$

$$s_L^2 = \sum_{i=1}^{n} (x_i - \bar{x}_L)^2 = \frac{(90-50)^2 + (20-50)^2 + (10-50)^2 + (80-50)^2}{3}$$

$$s_L^2 = 1667$$

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#3 cont.

T: tale of 2 cities
L: long lost work

Now we say

$$H_0: \mu_T = \mu_L \quad \text{or} \qquad H_1: \mu_T - \mu_L \neq 0$$
$$\mu_T - \mu_L = 0$$

For this we need to find S_p, pooled estimate of σ^2, when we assume $\sigma_L^2 = \sigma_T$

$$S_p^2 = \frac{(n_1 - 1) S_L^2 + (n_2 - 1) S_T^2}{n_1 + n_2 - 2}$$

$$= \frac{3 \cdot (1667) + 3 \cdot (3300)}{6} = \frac{(1667) + (3300)}{2}$$

$$S_p^2 = 2483$$

Now we standardize to:

$$T_0 = \frac{\bar{X}_T - \bar{X}_L - (\cancel{\mu_T - \mu_L})^{\emptyset}}{S_p \sqrt{1/n_T + 1/n_L}} = \frac{75 - 50}{\sqrt{2483} \cdot \sqrt{1/4 + 1/4}} = 0.70937$$

- w/ $\alpha = 0.1$, $\alpha/2 = 0.05$ this gives $t_{0.05, 6}$, which is 1.943

- our t_0 for $v = n_1 + n_2 - 2 = 6$ gives a p-value ~~(closest to)~~ of 0.25. Since 0.25 is not larger than 1.943 (or < -1.943) we cannot reject H_0.

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#4

- data of 67 months
- data set is :

  1) Money Supply
  2) lending rate
  3) Price index
  4) Exchange

a) degrees of freedom :

\# of observations - # of explanatory independent variables - 1

$= 67 - 4 - 1 = 32$

b) Per each coefficient w/i table 4.1 I'll be following

$$\hat{\beta}_j - t_{\alpha/2, n-p} \sqrt{\hat{\sigma}^2 C_{jj}} \le \beta_j \le \hat{\beta}_j + t_{\alpha/2, n-p} \sqrt{\hat{\sigma}^2 C_{jj}}$$

- where $\sqrt{\hat{\sigma}^2 C_{jj}}$ is the standard error
- $\alpha = 0.05$, $\alpha/2 = 0.05$
- $n - p = 67 - 4 = 63$
  appr as 60
- $t_{0.025, 60} = 2.915$

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#4 cont.

b) Money Supply, β_1

$$se(\beta_1) = 0.064$$

$$0.368 - 2.915 \cdot 0.064 \leq \beta_1 \leq 0.368 + 2.915 \cdot 0.064$$

$$\boxed{0.1814 \leq \beta_1 \leq 0.5546}$$

Lending Rate, β_2 $\quad se(\beta_2) = 0.049$

$$0.005 - 2.915 \cdot 0.049 \leq \beta_2 \leq 0.005 + 2.915 \cdot 0.049$$

$$\boxed{-0.1378 \leq \beta_2 \leq 0.1478}$$

Price Index, β_3 $\quad se(\beta_3) = 0.009$

$$0.037 - 2.915 \cdot 0.009 \leq \beta_3 \leq 0.037 + 2.915 \cdot 0.009$$

$$\boxed{0.010765 \leq \beta_3 \leq 0.06323}$$

Exchange Rate, β_4 $\quad se(\beta_4) = 1.175$

$$0.268 - 2.915 \cdot 1.175 \leq \beta_4 \leq 0.268 + 2.915 \cdot 1.175$$

$$\boxed{-3.157 \leq \beta_4 \leq 3.693}$$

Money Supply + Price index have confidence intervals that don't contain zero. In the model if there's a chance that coefficient is zero it will have no contribution to the model. Money supply and price index both have fairly straight forward CIs that give indication they contribute to the model.

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#4 c.

For all tables we have R^2, but not adjusted R^2. R^2 can be increased if not-helpful variables are added to the model. To truly Asses the quality of the regression we find the $\bar{R}^2$:

$$\bar{R}^2 = 1 - (1 - R^2)\frac{n-1}{n-k-1} =$$

n: # of observations   k: # of variables/regressors

Table 4.1

$$\bar{R}^2 = 1 - (1 - 0.825)\frac{67-1}{67-4-1} = 0.81371$$

Table 4.2

$$\bar{R}^2 = 1 - (1 - 0.825)\frac{67-1}{67-3-1} = 0.81667$$

Table 4.3

$$\bar{R}^2 = 1 - (1 - 0.825)\frac{67-1}{67-2-1} = 0.819531$$

while all values are close, the best regression was actually table 4.3, where the $\bar{R}^2$ value was the highest. This is consistent w/part (b) where we said money supply and price index were most important.

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#5

a, Symmetric Matrix: is one who's transpose is equal to itself

since

$A = \begin{bmatrix} 9 & -2 \\ -2 & 6 \end{bmatrix}$ and $A' = \begin{bmatrix} 9 & -2 \\ 9 & 6 \end{bmatrix}$ this

is symmetric

b. Positive definite, so all principal - sub-matrices are positive AND symmetric

- 9 is positive
- $(9 \cdot 6) - (-2 \cdot 2) = 3$ positive

So this is positive definite

c.

$S = Q \Lambda Q^T$

$S = A$

Q : made up of eigenvectors

Λ : diagonal matrix of eigenvalues

$\lambda_1 = 10$

$\lambda_2 = 5$

$e_1 = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$ $e_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$\boxed{\Lambda = \begin{bmatrix} 10 & 0 \\ 0 & 5 \end{bmatrix}}$

$\vec{q}_1 = \begin{bmatrix} -2/\sqrt{5} \\ 1/\sqrt{5} \end{bmatrix}$ $\vec{q}_2 = \begin{bmatrix} 1/\sqrt{5} \\ 2/\sqrt{5} \end{bmatrix}$

$\boxed{Q = \begin{bmatrix} -2/\sqrt{5} & 1/\sqrt{5} \\ 1/\sqrt{5} & 2/\sqrt{5} \end{bmatrix}}$

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#5 cont.

d.

$$Q^{-1} = Q'$$

$$A^{-1} = Q \Lambda^{-1} Q^{-1}$$

since this is diag

$$\Lambda^{-1} = \begin{bmatrix} 1/\lambda_1 & 0 \\ 0 & 1/\lambda_2 \end{bmatrix} = \begin{bmatrix} 1/10 & 0 \\ 0 & 1/5 \end{bmatrix}$$

$$A^{-1} = Q \Lambda^{-1} Q^{-1}$$

$$= \begin{bmatrix} -2/\sqrt{5} & 1/\sqrt{5} \\ 1/\sqrt{5} & 2/\sqrt{5} \end{bmatrix} \begin{bmatrix} 1/10 & 0 \\ 0 & 1/5 \end{bmatrix} \begin{bmatrix} -2/\sqrt{5} & 1/\sqrt{5} \\ 1/\sqrt{5} & 2/\sqrt{5} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{-2}{\sqrt{5} \cdot 10} & \frac{1}{\sqrt{5} \cdot 5} \\ \frac{1}{\sqrt{5} \cdot 10} & \frac{2}{\sqrt{5} \cdot 5} \end{bmatrix} \begin{bmatrix} -2/\sqrt{5} & 1/\sqrt{5} \\ 1/\sqrt{5} & 2/\sqrt{5} \end{bmatrix}$$

converted to decimal!

$$\boxed{A^{-1} = \begin{bmatrix} 0.12 & 0.04 \\ 0.04 & 0.18 \end{bmatrix}}$$

e.

$$|A| = \begin{vmatrix} 9 & -2 \\ -2 & 6 \end{vmatrix} = 54 - 4 = 50$$

$$\lambda_1 \cdot \lambda_2 = 10 \cdot 5 = 50$$

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EC:

$y = X\beta + \varepsilon$ solus to $\hat{\beta} = (X'X)^{-1}X'y$

$X'X\hat{\beta} = X'y$ ← least squares setup

a) · Since $(X'X)^{-1}$ exists we know the columns of X are linearly independent

· so we should have

3x independent variables with the 4th colums for the intercept

b) we know

$(X'X)^{-1}$ is just the inverse valus for a diagonal matrix, so

$$(X'X^{-1}) = \begin{bmatrix} 8 & 0 & 0 & 0 \\ 0 & 8 & 0 & 0 \\ 0 & 0 & 8 & 0 \\ 0 & 0 & 0 & 8 \end{bmatrix}$$

So the sample size (given by top left) is 8.

c) As mentioned in (A), this is a version of a gram matrix! Since we defined it's inverse, this means the columns of X are all linearly independent.

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