1. $P(\text{losing the funding}) = (0.3) \times (0.3) \times (0.3) = 0.027$

2. $$A = \begin{pmatrix} 1 & -1 & 1 \\ -1 & 0 & 2 \\ 1 & 1 & 1 \end{pmatrix} \qquad C = \begin{pmatrix} \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} \\ -\frac{1}{\sqrt{3}} & 0 & \frac{2}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} \end{pmatrix}$$

$$C' = \begin{pmatrix} \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{6}} \end{pmatrix}$$

$$C'C = \begin{pmatrix} \frac{1}{3}+\frac{1}{3}+\frac{1}{3} & \cancel{-\frac{1}{\sqrt{3}\sqrt{2}}} - 0 + \cancel{\frac{1}{\sqrt{3}\sqrt{2}}} & \cancel{\frac{1}{\sqrt{6}\sqrt{3}} - \frac{2}{\sqrt{6}\sqrt{3}} + \frac{1}{\sqrt{6}\sqrt{3}}} \\ -\frac{1}{\sqrt{3}}\frac{1}{\sqrt{2}}\ 0 + \frac{1}{\sqrt{3}}\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}\sqrt{2}} + 0 + \frac{1}{\sqrt{2}\sqrt{2}} & \cancel{-\frac{1}{\sqrt{6}\sqrt{2}}}\ 0 + \cancel{\frac{1}{\sqrt{2}\sqrt{6}}} \\ \cancel{+\frac{1}{\sqrt{6}}\frac{1}{\sqrt{3}} - \frac{2}{\sqrt{6}}\frac{1}{\sqrt{3}} + \frac{1}{\sqrt{6}}\frac{1}{\sqrt{3}}} & \cancel{\frac{-1}{\sqrt{6}\sqrt{2}}} + 0 + \cancel{\frac{1}{\sqrt{2}\sqrt{6}}} & \frac{1}{6}+\frac{4}{6}+\frac{1}{6} \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = I$$

$$CC' = \begin{pmatrix} \frac{1}{3}+\frac{1}{2}+\frac{1}{6} & \cancel{-\frac{1}{3}} + 0 + \cancel{\frac{2}{6}} & \cancel{\frac{1}{3} - \frac{1}{2} + \frac{1}{6}} \\ -\frac{1}{3} + 0 + \frac{2}{6} & \frac{1}{3} + 0 + \frac{4}{6} & \cancel{-\frac{1}{3}} + 0 + \cancel{\frac{2}{6}} \\ \frac{1}{3} - \frac{1}{2} + \frac{1}{6} & -\frac{1}{3} + 0 + \frac{2}{6} & \frac{1}{3}+\frac{1}{2}+\frac{1}{6} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{2}{6}+\frac{3}{6}+\frac{1}{6} & 0 & 0 \\ 0 & \frac{2}{6}+\frac{4}{6} & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

3. a. let Y be the rate of return

then $E(Y) = \frac{2500}{10000} E(X_1) + \frac{3000}{10000} E(X_2) + \frac{4500}{10000} E(X_3)$

$= .25 \times .12 + .3 \times 0.04 + .45 \times 0.07$

$= 0.0735$

$V(Y) = (.25)^2 V(X_1) + .3^2 V(X_2) + .45^2 V(X_3)$

$= (.25)^2 \times 0.0196 + .3^2 \times 0.0004 + .45^2 \times 0.0064$

$= 0.002557$

b. $V(Y^*) = V(Y) + 2 \times .3 \times 0.45 \times (-0.005)$

$= 0.002557 - 0.00135$

$= 0.001207.$

c. Reduced variability => reduced risk

4. Let D = death, ND = No death or injury

$P(D|F) = \frac{1}{4}$ $P(ND|F) = 3/4$ $P(F) = 0.2$

$P(D) = \frac{1}{4} \times 0.2 + \frac{1}{3}(0.2) + \frac{1}{21}(.6) \doteq 0.1050$

$P(F|ND) = \frac{P(ND|F) * P(F)}{P(ND)} = \frac{\frac{3}{4} \times .2}{(1 - 0.1050)} = 0.1676$

5. $S = \begin{bmatrix} 1 & b \\ b & 9 \end{bmatrix}$

Positive definite matrix has all positive eigen values or if n ~~[illegible]~~ upper left determinants are positive

For a 2X2 matrix

$\Rightarrow \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$a > 0$

and $\begin{vmatrix} a & b \\ c & d \end{vmatrix} > 0.$

$\det(S) = 9 - b^2 > 0.$

$\Rightarrow b^2 < 9$

$\Rightarrow \boxed{-3 < b < 3.}$

$S = \begin{bmatrix} 2 & 4 \\ 4 & c \end{bmatrix}$

$\det(S) = 2c - 16 > 0$

$= 2c > 16$

$\Rightarrow \boxed{c > 8}$

$S = \begin{bmatrix} c & b \\ b & c \end{bmatrix}$

$\det(S) = \boxed{c^2 - b^2 > 0}$

$\boxed{\text{and } c > 0}$

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EC

let H = has HIV $\quad$ D = Drug user

R = test is positive

N = test is negative

$$P(H) = \frac{550000}{250 \text{ mil}} = 0.0022 \qquad P(R|H) = 0.99$$

$$P(N|\text{not } H) = 0.99$$

(a) $$P(H|R) = \frac{P(R|H) \times P(H)}{P(R)}$$

$$P(R) = P(R|H) \times P(H) + P(R|\text{not } H) \times P(\text{not } H)$$
$$= 0.002178 + 0.009978 = 0.012156$$

$\Rightarrow$ $P(H|R) = 0.18$

(b) $P(H|R \text{ and } D) = ?$

$$P(H \text{ and } D) = \frac{275000}{250 \text{ mil}} = 0.0011$$

$$P(\text{not } H \text{ and } D) = \frac{(10 \text{ mil} - 275000)}{250 \text{ mil}} = 0.0389$$

lets assume $P(R|H) = P(R|H \text{ and } D) = 0.99$

and $P(N|\text{not } H \text{ and } D) = 0.99$

$P(R \text{ and } D) = 0.99 \times 0.0011 + 0.01 \times 0.0389 = 0.001478$

$P(H|R \text{ and } D) = P(H \text{ and } R \text{ and } D) / P(R \text{ and } D)$

$$= \frac{P(R|H \text{ and } D) \times P(H \text{ and } D)}{P(R \text{ and } D)} = \frac{0.99 \times 0.0011}{0.001478} = 0.74$$