Taylor Maurer, 11/8/2020, ISE201 HW3

7.2.1 *Consider the compressive strength data in Table 6.2. What proportion of the specimens exhibit compressive strength of at least 200 psi? What is the population parameter estimated here?*

Answer:

For the answer we are estimating the sample proportion. Our sample is of size 80, so $n = 80$. The number of items that belong to class $PSI \geq 200$ is 10. Therefore the estimated sample proportion is $\frac{10}{80}$ or $\frac{1}{8}$.

7.2.2 *Consider the synthetic fiber in the previous exercise.(excercise 7.2.3 not 7.2.2) How is the standard deviation of the sample mean changed when the sample size is increased from n = 6 to n = 49?*

Answer:

For n =6, the standard deviation of the sample mean is given below:

$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{3.5}{\sqrt{6}} = 1.42886 \tag{1}$$

Now for n = 49, the standard deviation of the sample mean is given below:

$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{3.5}{\sqrt{49}} = 0.5 \tag{2}$$

In conclusion, the standard deviation (and variance) decreases with increasing sample amount.

7.2.5 *A normal population has mean 100 and variance 25. How large must the random sample be if you want the standard error of the sample average to be 1.5?*

Answer:

For a normal distribution the standard error is given by:

$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} \tag{3}$$

If we need $\sigma_{\bar{X}}$ to be 1.5, we can rearrange the equation to solve for n:

$$\begin{aligned} n &= \frac{\sigma}{\sigma_{\bar{X}}}^2 \\ &= \frac{25}{1.5^2} \\ &= 11.11 = 12 \end{aligned} \tag{4}$$

7.2.7 *A random sample of size n1 = 16 is selected from a normal population with a mean of 75 and a standard deviation of 8. A second random sample of size n2 = 9 is taken from another normal population with mean 70 and standard deviation 12. Let X1 and X2 be the two sample means.*

1. *The probability that X1 - X2 exceeds 4*

   Answer: Since we have all the information we need we can plug in values into equation 7-4. Keep in mind to find our Z value we will be using $\bar{X}_1 - \bar{X}_2 = 4$.

$$\begin{aligned} Z &= \frac{\bar{X}_1 - \bar{X}_2 - (\mu_1 - \mu_2)}{\sqrt{\sigma_1^2/n_1 + \sigma_2^2/n_2}} \\ &= \frac{4 - (75 - 70)}{\sqrt{8^2/16 + 12^2/9}} \\ &= \frac{-1}{4.472} \\ &= -0.22361 \end{aligned} \tag{5}$$

Using the table in the back of the book we get that Z value to correspond to a probability of 0.412936. However since we want the probability of $P(\bar{X}_1 - \bar{X}_2 > 4)$ we have to subtract by one. Thus the final probability of $P(\bar{X}_1 - \bar{X}_2 > 4) = 1 - 0.41293 = 0.58707$.

2. *The probability that 3.5 leq X1 - X2 leq5.5*

   Answer: So now we are going to use equation 7-4, twice one for where $\bar{X}_1 - \bar{X}_2 = 3.5$ and another where $\bar{X}_1 - \bar{X}_2 = 5.5$. Then we can subtract.

   $\bar{X}_1 - \bar{X}_2 = 3.5$:

$$\begin{aligned} Z &= \frac{\bar{X}_1 - \bar{X}_2 - (\mu_1 - \mu_2)}{\sqrt{\sigma_1^2/n_1 + \sigma_2^2/n_2}} \\ &= \frac{3.5 - (75 - 70)}{\sqrt{8^2/16 + 12^2/9}} \\ &= \frac{-1.5}{4.472} \\ &= -0.33542 \end{aligned} \tag{6}$$

   Now we can say $P(\bar{X}_1 - \bar{X}_2 \leq 3.5) = P(Z \leq -0.33542) = 0.370700$.
   Now we repeat for $\bar{X}_1 - \bar{X}_2 = 5.5$:

$$\begin{aligned} Z &= \frac{\bar{X}_1 - \bar{X}_2 - (\mu_1 - \mu_2)}{\sqrt{\sigma_1^2/n_1 + \sigma_2^2/n_2}} \\ &= \frac{5.5 - (75 - 70)}{\sqrt{8^2/16 + 12^2/9}} \\ &= \frac{0.5}{4.472} \\ &= 0.1118 \end{aligned} \tag{7}$$

   Now we can say $P(\bar{X}_1 - \bar{X}_2 \leq 5.5) = P(Z \leq 0.1118) = 0.543795$.
   Now the probability that $P(3.5 \leq \bar{X}_1 - \bar{X}_2 \leq 5.5)$ is given by $P(\bar{X}_1 - \bar{X}_2 \leq 5.5) - P(\bar{X}_1 - \bar{X}_2 \leq 3.5) = 0.543795 - 0.370700 = 0.173095$.

7.3.7 *Data on the oxide thickness of semiconductor wafers are as follows: 425, 431, 416, 419, 421, 436, 418, 410, 431, 433, 423, 426, 410, 435, 436, 428, 411, 426, 409, 437, 422, 428, 413, 416.*

1. *Calculate a point estimate of the mean oxide thickness for all wafers in the population.*
Answer: I will use the sample mean as the point estimation. This is found by adding all the values within the random sample and dividing by the total amount.

$$\frac{10160}{24} = 423.33 \tag{8}$$

2. *Calculate a point estimate of the standard deviation of oxide thickness for all wafers in the population.*
Answer: I will use the variance/standard deviation estimator for this. This is done by finding the variance of the random sample, and then taking the square root.

$$\begin{aligned} S^2 &= \frac{\sum_{i=1}^{n}(X_i - \bar{x})}{n-1} \\ &= \frac{\sum_{i=1}^{n=24}(X_i - 423.33)}{23} \\ &= 82.493 \end{aligned} \tag{9}$$

Then to find the standard deviation, we take the square root: $S = \sqrt{82.493} = 9.0826$.

3. *Calculate the standard error of the point estimate from part (a).*
Answer: Since we were not given σ of the population we need to utilize the calculated S (from part b) to find the standard error:

$$\begin{aligned} \hat{\sigma}_{\hat{\Theta}} &= \frac{S}{\sqrt{n}} \\ \hat{\sigma}_{\hat{\Theta}} &= \frac{9.0826}{\sqrt{24}} \\ &= 1.85398 \end{aligned} \tag{10}$$

4. *Calculate a point estimate of the median oxide thickness for all wafers in the population.*
Answer: I will use the median estimator. So just taking the median of the random sample.

Organized from largest to smallest:
409 410 410 411 413 416 416 418 419 421 422 423 425 426 426 428 428 431 431 433 435 436 436 437
Then performing the median calculation:
~~409~~ ~~410~~ ~~410~~ ~~411~~ ~~413~~ ~~416~~ ~~416~~ ~~418~~ ~~419~~ ~~421~~ ~~422~~ ~~423~~ Implied 424 ~~425~~ ~~426~~ ~~426~~ ~~428~~ ~~428~~ ~~431~~ ~~431~~ ~~433~~ ~~435~~ ~~436~~ ~~436~~ ~~437~~
Since there are even numbers within the sample, the median is going to end up being between the last two pairs of 423 and 425. Thus the median is 424.

5. *Calculate a point estimate of the proportion of wafers in the population that have oxide thickness of more than 430 angstroms.*
Answer: For this calculation we will use the proportional estimator. This is simply just counting the proportion of samples that have a thickness greater than 430 within the random sample. Here is our list again (sorted):
~~409~~ ~~410~~ ~~410~~ ~~411~~ ~~413~~ ~~416~~ ~~416~~ ~~418~~ ~~419~~ ~~421~~ ~~422~~ ~~423~~ ~~425~~ ~~426~~ ~~426~~ ~~428~~ ~~428~~ 431 431 433 435 436 436 437
As we can see there are 7 samples of the 24 that are $>$ 430. Thus the proportion estimation is $\frac{7}{24} = 0.2916$.