I, taylor maurer, declane that th submitted work is original and adhres to all University policies on Academic integrity and acknowledge th consequences that may result from a violation of those rules. I have neither given nor received unauthorized assistance during th completion of this exam

10/29/2020

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Signed [illegible] 10/29/2020

Q1:

- $P(\text{success}) = 0.7 \quad P(\text{fail}) = 0.3$
- independent (constant, doesn't affect other trials)
- probability of losing funding is no successes in 1st 3 launches OR having 3 straight failures:

$$(0.3)(0.3)(0.3) = \boxed{0.0270}$$

Q2:

$$A = \begin{bmatrix} 1 & -1 & 1 \\ -1 & 0 & 2 \\ 1 & 1 & 1 \end{bmatrix}$$
$$\quad c_1 \quad c_2 \quad c_3$$

$$\|c_1\| = \sqrt{1^2 + (-1)^2 + 1^2} = \sqrt{3}$$
$$\|c_2\| = \sqrt{(-1)^2 + (1)^2} = \sqrt{2}$$
$$\|c_3\| = \sqrt{1^2 + 2^2 + 1^2} = \sqrt{6} = \sqrt{2}\cdot\sqrt{3}$$

$$C = \begin{bmatrix} 1/\sqrt{3} & -1/\sqrt{2} & 1/\sqrt{6} \\ -1/\sqrt{3} & 0 & 2/\sqrt{6} \\ 1/\sqrt{3} & 1/\sqrt{2} & 1/\sqrt{6} \end{bmatrix}$$

$$C'C = \begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ \frac{-1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{6}} \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{6}} \\ \frac{-1}{\sqrt{3}} & 0 & \frac{2}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} \end{bmatrix} =$$

$$C'C = \begin{bmatrix} \frac{1}{3} + \frac{1}{3} + \frac{1}{3}, & \frac{-1}{\sqrt{6}} + 0 + \frac{1}{\sqrt{6}}, & \frac{1}{\sqrt{3}\sqrt{6}} + \frac{-2}{\sqrt{3}\sqrt{6}} + \frac{1}{\sqrt{3}\sqrt{6}} \\ \cancel{\frac{-1}{\sqrt{6}}} + 0 + \cancel{\frac{1}{\sqrt{6}}}, & \frac{1}{2} + \frac{1}{2}, & \cancel{\frac{-1}{\sqrt{2}\sqrt{6}}} + 0 + \cancel{\frac{1}{\sqrt{2}\sqrt{6}}} \\ \cancel{\frac{1}{\sqrt{3}\sqrt{6}}} - \frac{2}{\sqrt{3}\sqrt{6}} + \cancel{\frac{1}{\sqrt{3}\sqrt{6}}}, & \cancel{\frac{-1}{\sqrt{2}\sqrt{6}}} + 0 + \cancel{\frac{1}{\sqrt{2}\sqrt{6}}}, & \frac{1}{6} + \frac{4}{6} + \frac{1}{6} \end{bmatrix}$$

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$$\boxed{C'C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I}$$

$$CC' = \begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{6}} \\ \frac{-1}{\sqrt{3}} & 0 & \frac{2}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ \frac{-1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{6}} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{3} + \frac{1}{2} + \frac{1}{6}, & \frac{-1}{3} + 0 + \frac{2}{6}, & \frac{1}{3} - \frac{1}{2} + \frac{1}{6} \\ \frac{-1}{3} + 0 + \frac{2}{6}, & \frac{1}{3} + 0 + \frac{4}{6}, & \frac{-1}{3} + 0 + \frac{2}{6} \\ \frac{1}{3} - \frac{1}{2} + \frac{1}{6}, & \frac{-1}{3} + 0 + \frac{2}{6}, & \frac{1}{3} + \frac{1}{2} + \frac{1}{6} \end{bmatrix}$$

$$\boxed{CC' = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I}$$

$$\boxed{C'C = CC' = I}$$

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Q3

rate of return = $\frac{\Delta \text{price}}{\text{initial price}} = r$

initially prices

X_1 given \$2500

X_2 given \$3000

X_3 given \$4500

All follow normal dist.

After 1 year (r)

$E(X_1) = 0.12$
$V(X_1) = 0.0196$
$E(X_2) = 0.04$
$V(X_2) = 0.0004$
$E(X_3) = 0.07$
$V(X_3) = 0.0064$

a.

$E(X) = E(X_1) + E(X_2) + E(X_3)$

X: random var assoc. w/ total rate of return

$$\boxed{E(X) = 0.12 + 0.04 + 0.07 = 0.23}$$

b) X_1 is X_2

$V(X) = V(X_1) + V(X_2) + V(X_3)$ ← b/c all independent

$V(X) = 0.0196 + 0.0004 + 0.0064$

$$\boxed{V(X) = 0.0264}$$

b.

- The non-independence b/t X_2 and X_3 does not change the expected value, thus

$E(X) = 0.23$

$V(X) = V(X_1) + V(X_2) + V(X_3) + 2\sum_{i<j}\sum \text{cov}(X_i, X_j)$

$2 \cdot \text{cov}(X_2, X_3)$
$= 2(-.005)$

$$\boxed{V(X) = 0.0264 + 2(-.005) = 0.0164}$$

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Q3

c

	independent	X_2 & X_3 non-indp.
$E(x) =$	0.23	0.23
$V(x) =$	0.0264	0.0164

- obviously the mean's dont change b/t (a) and (b) so there's no impact there. However the negative covariance of assets X_2 and X_3 decrease the total variance of the three assets. This implies your assets vary together which is better for the rate of return as long as the return is positive.

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Q4

foggy → 1/4
rainy → 1/8
sunny → 1/21

fraction of acidents that occur w/death vs.

→ 3/4
7/8
20/21

Fraction of Accidents occuring w/o death

foggy → occurs 20% of time
rainy → occurs 20% of time
sunny → occurs 60% of time

Giun it was foggy = 20%

Probability of accident occuring w/o death due to fogginess

$= 3/4 = 0.75$

$$P(\text{foggy} \mid \text{no death}) = 0.75 \cdot 0.2 = 0.15$$

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Q5

positive definite means A all eigenvalues λ are positive

a) $S = \begin{bmatrix} 1 & b \\ b & 9 \end{bmatrix}$ $-$ $|S - \lambda I| = 0$

$$\begin{vmatrix} 1-\lambda & b \\ b & 9-\lambda \end{vmatrix} = (1-\lambda)(9-\lambda) - b^2$$

$= 9 - \lambda - 9\lambda + \lambda^2 - b^2 = 0$

$= \lambda^2 - 10\lambda - b^2 + 9 = 0$

if $\lambda = 0$ $\quad -b^2 = -9$

$b^2 = 9 \quad b = |3|$

$\boxed{b > |3| \text{ to make eigenvalues positive}}$

b) $S = \begin{vmatrix} 2 & 4 \\ 4 & c \end{vmatrix}$ $\rightarrow$ $|S - \lambda I| = 0$

$(2-\lambda)(c-\lambda) - 16 = 0$

$$\begin{vmatrix} 2-\lambda & 4 \\ 4 & c-\lambda \end{vmatrix} = 2c - 2\lambda - \lambda c + \lambda^2 - 16 = 0$$

$\lambda^2 - \lambda(2+c) + 2c - 16 = 0$

if $\lambda = 0$

$2c = 16$

$c = 8$

$\boxed{c > 8 \text{ to make eigenvalues positive}}$

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Q5

c

$$S = \begin{bmatrix} c & b \\ b & c \end{bmatrix}$$

$$\begin{vmatrix} c-\lambda & b \\ b & c-\lambda \end{vmatrix} =$$

$$(c-\lambda)^2 - b^2 = 0$$

$$(c-\lambda)^2 - b^2 = 0$$

$$(c-\lambda)(c-\lambda) - b^2 = 0$$

$$c^2 - c\lambda - c\lambda + \lambda^2 - b^2 = 0$$

$$c^2 - 2c\lambda + \lambda^2 - b^2 = 0$$

$$c^2 + \lambda(\lambda - 2c\lambda) - b^2 = 0$$

$$\underset{a}{\lambda^2} - \underset{b}{2c\lambda} - \underbrace{b^2 + c^2}_{c} = 0$$

$$\lambda = \frac{2c \pm \sqrt{(-2c)^2 - 4(-b^2 + c^2)}}{2} = \frac{2c \pm \sqrt{4c^2 + 4b^2 - 4c^2}}{2}$$

$$= \frac{2c \pm \sqrt{4b^2}}{2} = \frac{2c \pm 2b}{2} = c \pm b = \lambda$$

$c + b = \lambda_1$
$c - b = \lambda_2$ } both need to be positive

for λ_1 to be positive
$c + b > 0$

for λ_2 to be positive
$c > b > 0$
this is most restrictive

$$\boxed{c > b > 0}$$

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Extra Credit

- 550,000 w/HIV
- 275,000 drug users w/i
- 250,000,000 total people
- 10,000,000 total drug users

A) 1st assumption, there are 275000 drug users w/i 550,000 people w/HIV thus percent probability of drug users w/HIV is

$\frac{275}{550} = 0.5$ ← liklihood of someone w/HIV being a drug user

2nd assumption,

$\frac{\text{10 mil drug users}}{\text{250 mil pop}} = 0.04$ ← liklihood of random person being drug users

$P\left(\text{having HIV}\right) = \frac{550{,}000}{250{,}000{,}000} = 0.0022$

$P\left(\text{correct positive test}\right) = 0.99$ (assuming ...)

$$\boxed{P\left(\text{having HIV} \mid \text{correct positive test}\right) = 0.99 \cdot 0.0022 = 0.002178}$$

the answer is surprising, but I'm assuming the person doesn't for sure have HIV, i.e. there's only a 0.0022 probability of someone having it.

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Extra Credit

B) $P\left(\begin{matrix}\text{drug user}\\\text{having HIV}\end{matrix} \middle| \begin{matrix}\text{correct}\\\text{HIV}\\\text{test}\end{matrix}\right) = \left(\frac{275000}{10,000,000}\right)(0.99)$

% of drug users with HIV

$P(\text{correct HIV test})$

$$\boxed{= 0.027225}$$