Taylor Maurer, 10/1/2020, ISE201 HW2

7.2.1 *Consider the compressive strength data in Table 6.2. What proportion of the specimens exhibit compressive strength of at least 200 psi? What is the population parameter estimated here?*

Answer:

For the answer we are estimating the sample proportion. Our sample is of size 80, so $n = 80$. The number of items that belong to class $PSI \geq 200$ is 10. Therefore the estimated sample proportion is $\frac{10}{80}$ or $\frac{1}{8}$.

7.2.2 *Consider the synthetic fiber in the previous exercise.(excercise 7.2.3 not 7.2.2) How is the standard deviation of the sample mean changed when the sample size is increased from n = 6 to n = 49?*

Answer:

For n =6, the standard deviation of the sample mean is given below:

$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{3.5}{\sqrt{6}} = 1.42886 \tag{1}$$

Now for n = 49, the standard deviation of the sample mean is given below:

$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{3.5}{\sqrt{49}} = 0.5 \tag{2}$$

In conclusion, the standard deviation (and variance) decreases with increasing sample amount.

7.2.5 *A normal population has mean 100 and variance 25. How large must the random sample be if you want the standard error of the sample average to be 1.5?*

Answer:

For a normal distribution the standard error is given by:

$$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} \tag{3}$$

If we need $\sigma_{\bar{X}}$ to be 1.5, we can rearrange the equation to solve for n:

$$\begin{aligned} n &= \frac{\sigma}{\sigma_{\bar{X}}}^{2} \\ &= \frac{25}{1.5^2} \\ &= 11.11 = 12 \end{aligned} \tag{4}$$

7.2.7 *A random sample of size n1 = 16 is selected from a normal population with a mean of 75 and a standard deviation of 8. A second random sample of size n2 = 9 is taken from another normal population with mean 70 and standard deviation 12. Let X1 and X2 be the two sample means.*

1. *The probability that X1 - X2 exceeds 4*

   Answer: Since we have all the information we need we can plug in values into equation 7-4. Keep in mind to find our Z value we will be using $\bar{X}_1 - \bar{X}_2 = 4$.

$$\begin{aligned} Z &= \frac{\bar{X}_1 - \bar{X}_2 - (\mu_1 - \mu_2)}{\sqrt{\sigma_1^2/n_1 + \sigma_2^2/n_2}} \\ &= \frac{4 - (75 - 70)}{\sqrt{8^2/16 + 12^2/9}} \\ &= \frac{-1}{4.472} \\ &= -0.22361 \end{aligned} \tag{5}$$

Using the table in the back of the book we get that Z value to correspond to a probability of 0.412936. However since we want the probability of $P(\bar{X}_1 - \bar{X}_2 > 4)$ we have to subtract by one. Thus the final probability of $P(\bar{X}_1 - \bar{X}_2 > 4) = 1 - 0.41293 = 0.58707$.

2. *The probability that 3.5 leq X1 - X2 leq5.5*

Answer: So now we are going to use equation 7-4, twice one for where $\bar{X}_1 - \bar{X}_2 = 3.5$ and another where $\bar{X}_1 - \bar{X}_2 = 5.5$. Then we can subtract.

$\bar{X}_1 - \bar{X}_2 = 3.5$:

$$\begin{aligned} Z &= \frac{\bar{X}_1 - \bar{X}_2 - (\mu_1 - \mu_2)}{\sqrt{\sigma_1^2/n_1 + \sigma_2^2/n_2}} \\ &= \frac{3.5 - (75 - 70)}{\sqrt{8^2/16 + 12^2/9}} \\ &= \frac{-1.5}{4.472} \\ &= -0.33542 \end{aligned} \tag{6}$$

Now we can say $P(\bar{X}_1 - \bar{X}_2 \leq 3.5) = P(Z \leq -0.33542) = 0.370700$.
Now we repeat for $\bar{X}_1 - \bar{X}_2 = 5.5$:

$$\begin{aligned} Z &= \frac{\bar{X}_1 - \bar{X}_2 - (\mu_1 - \mu_2)}{\sqrt{\sigma_1^2/n_1 + \sigma_2^2/n_2}} \\ &= \frac{5.5 - (75 - 70)}{\sqrt{8^2/16 + 12^2/9}} \\ &= \frac{0.5}{4.472} \\ &= 0.1118 \end{aligned} \tag{7}$$

Now we can say $P(\bar{X}_1 - \bar{X}_2 \leq 5.5) = P(Z \leq 0.1118) = 0.543795$.
Now the probability that $P(3.5 \leq \bar{X}_1 - \bar{X}_2 \leq 5.5)$ is given by $P(\bar{X}_1 - \bar{X}_2 \leq 5.5) - P(\bar{X}_1 - \bar{X}_2 \leq 3.5) = 0.543795 - 0.370700 = 0.173095$.