$i^2=-1$ by means of rational angles

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Defining and clarifying each axiom and how they relate to matrix transformations and limits.

1. Define the Imaginary Unit

The imaginary unit can be understood in terms of its vector representation. It represents a 90° rotation in the complex plane:

Vector Representation: as -1 on a vector implies a 90° rotation:

$$i = \text{rotation by } 90^\circ$$

Limits: The movement can be represented as transitions from 0 to -1 or from 1 to 0.

2. Square of

The square of i is:

$$i^2 = -1$$

$$i = \sqrt-1$$

3. Cartesian Grid and Perpendicular Vectors

Given:

$$i^2 = x \cdot y$$

$$i^2 = a^2 + b^2 = c^2$$

4. Matrix Representation

You correctly identified the matrices:

Matrix :

$$M_1 = \begin{pmatrix} 0 & i \\ 1 & 0 \end{pmatrix}$$

Matrix : *Perpendicular to* :

$$M_2 = \begin{pmatrix} 0 & 1 \\ i & 0 \end{pmatrix}$$

5. Matrix Inversion and Transformation

To invert i and represent it in terms of real constants:

Inverse of:

$$M_1^{-1} = \begin{pmatrix} 0 & 1 \\ i & 0 \end{pmatrix} = M_2$$

Real Matrix Transformations *for M_{r1} and M_{r2} :*

$$M_{r1} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$M_{r2} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

6. Limits and Inversion

The transformation in the matrix can be described in terms of limits:

Limits:

\text{From } 0 \text{ to } i$\text{ and } 1 \text{ to } 0$

Inverted Matrix:

$$M_{\text{inv}} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

7. Formalizing the Derivative

Define t to capture the rate of change of i :

$$d(i^2) = \frac{d}{dt} \left(\begin{pmatrix} 0 & i(t) \\ 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} 0 & 1 \\ i(t) & 0 \end{pmatrix} \right)$$

8. Formalizing in a Matrix Context

To summarize, the formalization can be represented as:

$$d(i^2) = \frac{d}{dt} \left[\begin{pmatrix} 0 & i(t) \\ 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} 0 & 1 \\ i(t) & 0 \end{pmatrix} \right] = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

Summary

1. ***Vectors and Limits***: represents a 90° rotation on a Cartesian grid.
2. ***Matrix Representation***: Use matrices and to describe transitions.
3. ***Inversion and Real Representation***: Determine inverses and real matrix forms.
4. ***Formalizing*** : Capture the derivative of in terms of matrices and limits.