Observations on Structure-Preserving Reshaping of Perfect-Power Tensors

Taylor Metz*

July 6, 2025

Abstract

This paper presents a conjecture regarding a class of tensor transformations we term **Deterministic Tensorial Reshaping**. We observe that tensors whose total number of elements is a perfect power, n^m (for integers $n, m \geq 2$), appear to admit specific matrix representations that preserve structural information beyond what is retained in a standard vector flattening. This phenomenon, not extensively explored in current literature, suggests a deeper underlying mathematical structure.

Through concrete examples involving tensors of size 8^2, 8^3, and 2^8, we illustrate a "midpoint matrix" form, $M_{n^k \times n^{m-k}}$, which may represent a canonical, structure-aware reshape. This work presents foundational observations and definitions as a proposal, intended to motivate rigorous mathematical investigation into the formal properties, potential exceptions, and computational utility of these transformations.

Keywords: tensor reshaping, conjecture, perfect-power tensors, structure-preserving transformations, dimensional equivalence, computational mathematics.

1 Introduction

In numerical computation, vectors, matrices, and higher-order tensors are foundational structures, yet their relationships are often treated as matters of implementation convenience. The process of "flattening" a tensor to a vector, for instance, is a ubiquitous operation that discards the dimensional context of the original object.

This paper presents evidence for a potentially deeper structural relationship between these objects when their total element count, N, is a perfect power ($N = n^m$). We observe that such tensors admit transformations into specific matrix forms that are not only lossless and invertible but also appear to preserve meaningful structural metadata. This suggests potential for new insights in numerical methods or data science.

The core of this work is a proposal based on these observations. We do not claim to have developed a complete or fully proven theory. Instead, we aim to provide sufficient non-trivial evidence to encourage formal investigation by the mathematical community. We believe the patterns demonstrated herein suggest an unexplored axis of symmetry in data reshaping that warrants further study.

*Independent Researcher. The observations presented herein are the result of independent research. The goal of this manuscript is not to present a complete or rigorously proven theory, but rather to highlight a persistent and potentially useful pattern in data structure transformations. We welcome feedback, formalization, and critique from the mathematical community. Correspondence: contact@example.com.

2 Foundational Concepts and Definitions

To formalize our observations, we propose the following definitions.

Definition 2.1 (Power Base and Exponent)**.** For a tensor whose total element count is a perfect power $N = n^m$, we define the *power base* as the integer $n \geq 2$ and the *power exponent* as the integer $m \geq 2$. (This terminology is adopted to avoid collision with the terms ”rank” and ”order” which have established, distinct meanings in tensor analysis).

Definition 2.2 (Perfect-Power Tensor)**.** A tensor T with k dimensions $d_1 \times d_2 \times \cdots \times d_k$ is a *perfect-power tensor* if its total number of elements, $\prod_{j=1}^{k} d_j$, equals n^m for some integers $n, m \geq 2$.

Definition 2.3 (Dimensional Equivalence)**.** Two arrays are *dimensionally equivalent* (denoted $\cong$) if there exists a bijective (one-to-one and onto) mapping between their index sets that preserves all element values and their positional adjacencies.

3 Core Conjecture and Justification

The central hypothesis of this paper is formalized in the following conjecture.

Conjecture 3.1 (Principle of Canonical Midpoint Representation)*.* For any perfect-power tensor T with $N = n^m$ elements, there exists at least one canonical ”midpoint” matrix representation, $M_{n^k \times n^{m-k}}$ (for an integer k where $1 \leq k < m$), that preserves structural information implied by the tensor's original dimensional composition and the power base n.

Justification. The plausibility of this conjecture is grounded in the well-understood bijective nature of index-mapping functions. Any tensor element at a multi-dimensional index $(i_1, \ldots, i_k)$ can be losslessly mapped to a linear vector index, and similarly to a matrix index pair (p, q), through standard row-major or column-major linearization formulas. The existence of such invertible maps is mathematically trivial.

The non-trivial part of this conjecture is the assertion that for perfect-power tensors, a specific factorization into $M_{n^k \times n^{m-k}}$ is not merely one of many possible reshapes, but one that may hold a *canonical* status. This can be understood by considering how such a matrix explicitly groups elements based on the power base n, thereby inherently preserving structural information related to the tensor's underlying composition in terms of n. The examples below illustrate this phenomenon.

4 Observations and Supporting Examples

Observation 4.1 (The $N = 64$ Case)*.* A tensor with 64 elements can be expressed as 8^2, giving a power base $n = 8$ and exponent $m = 2$. An 8×8 matrix is the trivial case of a perfect-power tensor. It is dimensionally equivalent to a $V_{1\times 64}$ vector. The canonical matrix representation per our conjecture would be $M_{8^1 \times 8^{2-1}} = M_{8\times 8}$.

Observation 4.2 (The $N = 512$ Case)*.* A tensor with 512 elements can be expressed as 8^3 ($n = 8, m = 3$). A common representation is an $8 \times 8 \times 8$ tensor. This can be flattened to a $V_{1\times 512}$ vector. Our conjecture suggests a canonical midpoint matrix of $M_{8^1 \times 8^{3-1}} = M_{8\times 64}$ or, more symmetrically, $M_{8^2 \times 8^{3-2}} = M_{64\times 8}$. This 64×8 matrix is computationally intuitive, where each of the 8 columns could represent one 8×8 ”slice” of the original cube.

Observation 4.3 (The $N = 256$ Case (Mixed Dimensions)). This case is particularly illustrative. Consider a tensor with 256 elements arranged in an $8 \times 8 \times 4$ configuration. The total element count is a perfect power: $256 = 2^8$ ($n = 2, m = 8$) or 4^4 ($n = 4, m = 4$). Even though the dimensions are not uniform powers of a single base, the structure can be reshaped to a matrix $M_{64\times4}$ by grouping the first two dimensions ($8 \times 8 = 64$). This is significant because $64 = 8^2$. The resulting matrix, $M_{64\times4}$, seems to expose a latent structure related to the $n = 8$ family of tensors. This suggests the principle may reveal hierarchical relationships that are not immediately obvious from the tensor's shape alone.

5 Computational Demonstration: The $M_{64\times4}$ Case

To provide a concrete, working example of the principle described in Observation 4.3, we demonstrate the transformation using column-major ordering. This shows the deterministic and lossless nature of the mapping from a 1D vector of 256 elements to a 2D matrix of 64×4.

Let a 1D vector V have 256 elements, indexed from 0 to 255 (using 0-based indexing). We wish to map each element of V to a unique cell in a 64×4 matrix M.

Mapping Logic. Using column-major ordering, a given linear *Base Position* (index 0-255) is mapped to a $(Row, Column)$ coordinate in the 64×4 matrix:

$$\text{Column} = \lfloor \text{Base\_Position}/64 \rfloor \tag{1}$$

$$\text{Row} = \text{Base\_Position} \bmod 64 \tag{2}$$

Example Data Structure. The 256 elements are distributed across 4 columns of 64 elements each. Table 1 shows the first 16 rows and last 4 rows of this arrangement.

Transformation Verification. Table 2 demonstrates the mapping for several test cases.

This computational model serves as a proof-of-concept for the conjecture. It shows that the transformation from a linear vector ($V_{1\times256}$) to the structured matrix ($M_{64\times4}$) is deterministic, invertible, and preserves the integrity of the data. The column-major arrangement naturally groups elements into 4 distinct "slices" of 64 elements each, thereby retaining structural information about the original tensor's composition.

6 Further Claims and Future Directions

The observations above lead to several propositions that require formal exploration.

Claim 6.1 (Information Preservation). All reshaping operations between dimensionally equivalent representations are lossless. This is a direct consequence of the bijective mappings used.

Claim 6.2 (Computational Utility). The ability to select a representation (vector, canonical matrix, or tensor) allows for computational optimization. For example, vector forms are suited for dot products, while the proposed matrix midpoints may be better for operations on tensor slices or for preserving locality for cache efficiency.

This work is preliminary. The primary goal is to present these patterns to the research community. We explicitly invite collaboration to:

- Formalize the proposed principles, perhaps within an algebraic framework like group or module theory.

Table 1: Column-major arrangement of 256 elements in a 64×4 matrix (partial view showing first 16 and last 4 rows).

Row Index	**Column 0**	**Column 1**	**Column 2**	**Column 3**
0	0	64	128	192
1	1	65	129	193
2	2	66	130	194
3	3	67	131	195
4	4	68	132	196
5	5	69	133	197
6	6	70	134	198
7	7	71	135	199
8	8	72	136	200
9	9	73	137	201
10	10	74	138	202
11	11	75	139	203
12	12	76	140	204
13	13	77	141	205
14	14	78	142	206
15	15	79	143	207
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
60	60	124	188	252
61	61	125	189	253
62	62	126	190	254
63	63	127	191	255

Table 2: Verification of the column-major mapping from linear Base Position to matrix coordinates.

Base Position	**Column (Calculated)**	**Row (Calculated)**	**Expected Value**
0	0	0	0
31	0	31	31
64	1	0	64
95	1	31	95
128	2	0	128
192	3	0	192
255	3	63	255

- Identify counterexamples or boundary conditions that would refine or disprove the conjecture.
- Explore applications where this structure-preserving property might offer analytical or computational advantages, such as in neural network design, physics simulations, or data compression.
- Investigate the relationship between different possible factorizations of n^m and their corresponding canonical representations.

7 Conclusion

We have presented evidence for a pattern we call Deterministic Tensorial Reshaping, which appears to govern the transformation of tensors whose element counts are perfect powers. The core idea, presented as a conjecture, is that these tensors admit canonical matrix "midpoints" that preserve structural information typically lost during standard flattening operations.

We have provided concrete examples and a computational demonstration that illustrate this principle and highlight its potential to reveal latent structure in multi-dimensional data. This paper should be seen as an invitation for rigorous analysis, critique, and extension from the mathematical and computational sciences communities. We believe the questions raised here are non-trivial and hope that others will find them worthy of further investigation.

The computational demonstration using the $M_{64\times 4}$ case shows that the proposed transformations are not only theoretically interesting but also practically implementable. The column-major arrangement naturally preserves the block structure of the original tensor while enabling efficient matrix operations on the reshaped data.

Future work should focus on developing formal proofs of the conjecture's claims, exploring edge cases and potential counterexamples, and investigating practical applications where these structure-preserving properties might provide computational or analytical advantages.