Q1.

a. $\lambda = 7$ orders per hour

$\mu = 7.5$ courses cooked per hour

$$p = \frac{\lambda}{\mu} = \frac{7}{7.5} = .933$$

b. $$W_Q = \frac{\lambda}{\mu(\mu-\lambda)} = \frac{7}{7.5(7.5-7)} = 1.867 \text{ hours}$$

c. $$L = \frac{\lambda}{\mu-\lambda} = \frac{7}{7.5-7} = \frac{7}{.5} = 3.5 \text{ orders}$$

d. $$W = \frac{L}{\lambda} = \frac{3.5}{7} = .5$$

No, the average time spent in the queue is $\frac{1}{2}$ hour

e. $$.25 = \frac{L}{7} \qquad L = \frac{\lambda}{\mu-\lambda} \qquad 1.75 = \frac{7}{\mu-7}$$

$$1.75 = L \qquad 7 = 1.75\mu - 12.25$$

$$\mu = 11$$

i. Average service rate must be 11 courses per hour.

ii. New average time is 15 minutes or .25 of an hour.

Q2

a. $\lambda P_0 = \mu P_1$

$$P_1 = \frac{\lambda P_0}{2\mu} = \rho P_0$$

b. $\lambda P_0 + 2\mu P_2 = \mu P_1 + \lambda P_1$

$$P_2 = \frac{\mu P_1 + \lambda P_1 - \lambda P_0}{2\mu}$$

$$P_2 = \frac{\frac{\cancel{\mu}}{1}\cdot\frac{\lambda P_0}{\cancel{\mu}} + \frac{\lambda}{1}\cdot\frac{\lambda P_0}{\cancel{\mu}} - \lambda P_0}{2\mu}$$

$$P_2 = \frac{\lambda P_0 + \lambda^2 P_0 - \lambda P_0}{2\mu}$$

$$P_2 = \frac{\lambda^2 P_0}{2\mu} = \rho^2 P_0$$

c. $\lambda P_1 + 2\mu P_3 = \lambda P_2 + 2\mu P_2$

$$P_3 = \frac{\lambda P_2 + 2\mu P_2 - \lambda P_1}{2\mu}$$

$$P_3 = \frac{\lambda\left(\frac{\lambda^2 P_0}{\cancel{2\mu}}\right) + \cancel{2\mu}\left(\frac{\lambda^2 P_0}{\cancel{2\mu}}\right) - \lambda\cdot\left(\frac{\lambda P_0}{2\mu}\right)}{\cancel{2\mu}}$$

$$P_3 = \lambda^3 P_0 + \lambda^2 P_0 - \lambda^2 P_0$$

$$P_3 = \frac{\lambda^3 P_0}{2\mu} = \rho^3 P_0$$

d. $P_n = \frac{\lambda^n P_0}{2\mu}$

e. $1 = P_0 + P_1 + P_2 + P_3 + \ldots$

$\Rightarrow 1 = P_0 + \rho P_0 + \rho^2 P_0 + \rho^3 P_0 \ldots$

$\Rightarrow 1 = P_0(1 + \rho + \rho^2 + \rho^3 \ldots)$

$\Rightarrow P_0 = \frac{1}{1 + \rho + \rho^2 + \rho^3 \ldots}$

$\Rightarrow P_0 = \frac{1-\rho}{1+\rho}$

identify a & x

f. Probs = to 1

$1 = ax + ax^2 + ax^3 \ldots$

$1 = ax(1 + x + x^2 \ldots)$

$ax = \frac{1}{(1+x+x^3 1x^3)}$

g. $\frac{ax}{(1-x)^2}$

$1 = ax + 2ax^2 + 3ax^3 \ldots)$

$1 = ax(1 + 2x + 3x^2 \ldots)$

$ax = \frac{1}{(1+2x+3x^2 \ldots)}$

$$L = \frac{\frac{1}{1+2x+3x}}{1-x^2}$$

Q2

h. $L = \frac{2\rho}{1-\rho^2}$

$= \frac{2(.933)}{1-.1295}$

$= 14.408$

$W = \frac{14.408}{7} = 2.058$ hours

Q3

a. $n = 7$

$\hat{\mu} = \frac{1}{7}(12.1 + 15.3 + 10.5 + 9.1 + 10.7 + 12.7 + 10.8)$

$= 11.6$

$SE = \frac{s}{\sqrt{n}} = \frac{2}{\sqrt{7}} = 0.756$

$11.6 \pm 2.262 \times .756$

b. $t_{6,.975} = 2.262$

confidence interval = 9.748 to 13.452

c. We are 95% confident the true mean will be within 9.748 and 13.452 interval.

d. $t = \frac{\bar{X} - \mu_0}{SE} = \frac{11.6 - 14}{.756} = -3.17$

e. $p = 0.0193$

f. Given the p value is < 0.05 the data is not statistically significant and we reject the null hypothesis of $\mu = 14$.

Q4.

a. mean = 449.7267
conf interval = (368.29 , 531.16)

b. we are 95% confident the hourly count of people across melbourne will be between 368.29 and 531.16 people.

c.

	WF City	SCS	TAC
mean	89.14	478.37	789.43
conf int	53.279 to 125	318.88 to 637.86	638.75 to 940.11

d. The average number of people to walk past WF City each hour is 89. We are 95% confident the number of people walking by will fall within 53 to 125 people.

The same goes for Southern Cross with an average number of people per hour being 478 and we are 95% confident this number will fall between 318 to 637 people.

Finally average number of people to walk past TAC on an hourly basis is 789. We are 95% confident between 638 to 940 people will walk by TAC everyhour.

e.

	2019	2022
mean	554.62	334.83
conf int	413.03 to 696.21	267.36 to 422.3

f. The avg. hourly contacts in 2019 was 554 people. We are 95% confident the average falls betwen 413 to 696 people.
The avg. hourly contacts in 2022 is 334 people. We are 95% conf. of 267 to 422 range.

g. Yes, the average # of hourly contacts and conf int are significantly lower in 2022 than 2019. This could be from several causes - WFH, travel, etc.

```
head(pedestrians)
pedestrians<-read.csv('pedestrians.csv', header=TRUE)
head(dat)
x<-dat$Hourly_Counts
# Mean :
mean (x)
# Variance :
var (x)
# Standard error :
sd(x)/sqrt(length(x))

mean(x)+c(-1 ,1)*1.96*sd(x)/sqrt(length(x))

#Waterfront City
wc.pedestrians<-pedestrians[pedestrians$Sensor_Name == "Waterfront City", ]
wc<-wc.pedestrians$Hourly_Counts
mean(wc)
mean(wc)+c(-1 ,1)*1.96*sd(wc)/sqrt(length(wc))

#Southern Cross Station
sc.pedestrians<-pedestrians[pedestrians$Sensor_Name == "Southern Cross Station",]
sc<-sc.pedestrians$Hourly_Counts
mean(sc)
mean(sc)+c(-1 ,1)*1.96*sd(sc)/sqrt(length(sc))

#The Arts Centre
ac.pedestrians<-pedestrians[pedestrians$Sensor_Name == "The Arts Centre",]
ac<-ac.pedestrians$Hourly_Counts
mean(ac)
mean(ac)+c(-1 ,1)*1.96*sd(ac)/sqrt(length(ac))


#2019
t<-pedestrians[pedestrians$Year == 2019,]
t2<-t$Hourly_Counts
mean(t2)
mean(t2)+c(-1 ,1)*1.96*sd(t2)/sqrt(length(t2))

#2020
p<-pedestrians[pedestrians$Year == 2022,]
p2<-p$Hourly_Counts
mean(p2)
mean(p2)+c(-1 ,1)*1.96*sd(p2)/sqrt(length(p2))
```