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COGS 118A: Assignment 2

1 Probability & Events

Given three events A, B, C with P(A) = ½, P(B) = ⅓, P(C) = ⅕, and P(A,B,C) = 1/30, can we determine whether A,B,C are all independent?

If they are all independent, the following conditions will be satisfied

1. $P(A, B, C) = P(A)\, P(B)\, P(C)$
2. $P(A, B) = P(A)\, P(B)$
3. $P(A, C) = P(A)\, P(C)$
4. $P(B, C) = P(B)\, P(C)$

Condition 1 is satisfied- (½)(⅓)(⅕) = 1/30
However, we do not know $P(A, B)$, $P(A, C)$, or $P(B, C)$, thus we cannot determine whether A,B, and C are independent events

2 Conditional Probability

A mammogram tests for breast cancer.
The true positive rate of a particular mammogram is 97%.
The true negative rate is 93%.
The frequency of breast cancer is .05%.

$P(test+ \mid cancer\ +) = .97$

$P(test\ - \mid cancer\ -) = .93$

$P(cancer\ +) = .0005$

$P(cancer\ -) = .9995$

2.1 What is the probability of having breast cancer, given a positive result?

$$P(cancer+ \mid test+) = \frac{P(test+ \mid cancer+)P(cancer+)}{P(test+ \mid cancer+)P(cancer+) + P(test+ \mid cancer-)P(cancer-)}$$

$$P(cancer+ \mid test+) = \frac{(.97)(.0005)}{(.97)(.0005) + (.07)(.9995)}$$

$$P(cancer+ \mid test+) \approx .006884$$

2.2 What is the probability of not having breast cancer, given a negative result?

$$P(cancer- \mid test-) = \frac{P(test- \mid cancer-)P(cancer-)}{P(test- \mid cancer-)P(cancer-) + P(test- \mid cancer+)P(cancer+)}$$

$$P(cancer- \mid test-) = \frac{(.93)(.9995)}{(.93)(.9995) + (.03)(.0005)}$$

$$P(cancer- \mid test-) \approx .9999$$

2.3 Computer precision, recall, and F-value

$$precision = P(cancer+ \mid test+) \approx 0.006884$$

$$recall = P(test+ \mid cancer+) = .97$$

$$F - value = \frac{2*precision*recall}{precision + recall} = \frac{2\,(0.006884)(.97)}{(.006884 + .97)} \approx .01367$$

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3 Communication Channel

$P(X = 3) = 3 * P(X = 1),\ P(X = 2) = 2 * P(X = 1)$

$P(X = 1) = \frac{1}{6},\ P(X = 2) = \frac{1}{3},\ P(X = 3) = \frac{1}{2}$

$P(Y = 1 \mid X = 1) = 1 - \alpha$

$P(Y = 2 \mid X = 1) = \alpha/2$

$P(Y = 3 \mid X = 1) = \alpha/2$

$P(Y = 1 \mid X = 2) = \beta/2$

$P(Y = 2 \mid X = 2) = 1 - \beta$

$P(Y = 3 \mid X = 2) = \beta/2$

$P(Y = 1 \mid X = 3) = \gamma/2$

$P(Y = 2 \mid X = 4) = \gamma/2$

$P(Y = 3 \mid X = 5) = 1 - \gamma$

3.1 $P(X = 1 \mid Y = 1) = \frac{P(Y=1 \mid X=1)\, P(X=1)}{P(Y=1 \mid X=1)\, P(X=1) + P(Y=1 \mid X=2)\, P(X=2) + P(Y=1 \mid X=3)\, P(X=3)}$

$P(X = 1 \mid Y = 1) = \frac{(1-\alpha)(1/6)}{(1-\alpha)(1/6) + (\beta/2)(1/3) + (\gamma/2)(1/2)}$

3.2 $P(X = 1, X = 1) = P(X = 1) = 1/6$

$P(X = 2, X = 1) = 0$

$P(X = 3, X = 1) = 0$

3.3 $P(X = 1 \mid X = 1) = 1$

$P(X = 2 \mid X = 1) = 0$

$P(X = 3 \mid X = 1) = 0$

4 Binary Communication System

$P(X = 0) = .4$

$P(X = 1) = .6$

$P(Y = 0 \mid X = 0) = .8$

$P(Y = 1 \mid X = 0) = .2$

$P(Y = 0 \mid X = 1) = .25$

$P(Y = 1 \mid X = 1) = .75$

$P(Y = 0,\ X = 0) = .32$

$P(Y = 1,\ X = 0) = .08$

$P(Y = 0,\ X = 1) = .15$

$P(Y = 1,\ X = 1) = .45$

Answer:

$P(Y = 0) = 0.47$

$P(Y = 1) = 0.53$

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5 Maximum Likelihood Estimation

By setting the derivatives of the log likelihood function equal to zero

$ln\, p(x\,;\ \mu, \sigma^2) \ = \ -\frac{1}{2\sigma^2}(x\ - \mu)^2\ - \frac{N}{2}\ ln\ \sigma^2 - \frac{N}{2}\ ln\ (2\pi)$

With respect to μ and σ,

$\mu_{ML}\ =\ arg\ min_{\mu}\ -\ \sum_{n=1}^{N} ln\ p\ (x_n\,;\ \mu, \sigma)$

$\sigma_{ML}\ =\ arg\ min_{\sigma}\ -\ \sum_{n=1}^{N}\ ln\ p\ (x_n;\ \mu_{ML},\ \sigma)$

Verify the following results using the maximum likelihood estimation concept:

$\mu_{ML}\ =\ \frac{1}{N}\ \sum_{n=1}^{N}\ x_n$

- Through substitution $\mu_{ML}\ =\ arg\ min_{\mu}\ \sum_{n=1}^{N}\ \frac{1}{2\sigma^2}\ (x_n - \ \mu)^2\ + \frac{1}{2}\ ln\ \sigma^2\ +\ \frac{1}{2}\ ln\ (2\pi)$
- To find $arg\ min_{\mu,}$ take the derivative and set it to 0 $\frac{-2}{2\sigma^2}\ \sum_{n=1}^{N} (x_n\ - \mu)\ =\ 0$
- $\frac{-1}{\sigma^2}\ \sum_{n=1}^{N} x_n\ + \frac{1}{\sigma^2}\ \sum_{n=1}^{N} \mu\ =\ 0$
- $\frac{1}{\sigma^2} \sum_{n=1}^{N} \mu\ =\ \frac{1}{\sigma^2}\ \sum_{n=1}^{N} x_n$
- $N\mu\ =\ \sum_{n=1}^{N} x_n$
- $\mu\ = \frac{1}{N}\ \sum_{n=1}^{N} x_n$

$\sigma_{ML}\ =\ \left[\frac{1}{N}\ \sum_{n=1}^{N}\ (x_n\ -\ \mu_{ML})^2\right]^{1/2}$

- Through substitution $\sigma_{ML}\ =\ arg\ min_{\sigma}\ \sum_{n=1}^{N}\ \frac{1}{2\sigma^2}\ (x_n - \ \mu)^2\ + \frac{1}{2}\ ln\ \sigma^2\ +\ \frac{1}{2}\ ln\ (2\pi)$
- To find $arg\ min_{\mu,}$ take the derivative and set it to $\frac{1}{2}\ \sum_{n=1}^{N} (x_n\ -\ \mu_{ML})^2(-\ 2\sigma^{-3}) + \frac{1}{2\sigma^2}\ (2\sigma) = 0$
- $\frac{1}{2}(\sum_{n=1}^{N} (x_n\ -\ \mu_{ML})^2)(\sum_{n=1}^{N} - 2\sigma^{-3}) + \frac{1}{2\sigma^2}\ (2\sigma) = 0$
- $\frac{-N}{\sigma^3}\ \sum_{n=1}^{N} (x_n\ -\ \mu_{ML})^2 + \frac{1}{\sigma}\ = 0$
- $\frac{N}{\sigma^3}\ \sum_{n=1}^{N} (x_n\ -\ \mu_{ML})^2 = \frac{1}{\sigma}$
- $\sigma^2 =\ \frac{1}{N}\ \sum_{n=1}^{N} (x_n\ -\ \mu_{ML})^2$
- $\sigma\ = \sqrt{\ }\frac{1}{N}\ \sum_{n=1}^{N} (x_n\ -\ \mu_{ML})^2$

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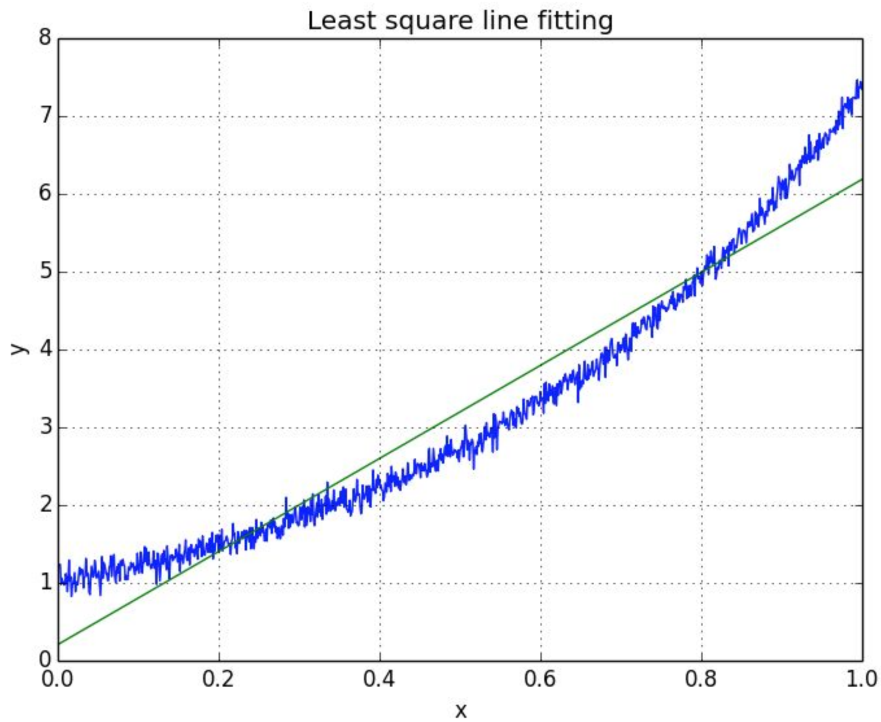
6 Least Squares Estimation




```
import scipy.io as sio
import matplotlib.pyplot as plt
import numpy as np
data = sio.loadmat('data.mat')
x = data['x'].reshape([-1, 1])
y = data['y'].reshape([-1, 1])

plt.plot(x,y)
plt.grid()

X = np.hstack((np.ones((len(x),1)),np.power(x,1)))

X_t = X.transpose((1,0))
sol = np.dot(np.linalg.inv(np.dot(X_t,X)),np.dot(X_t,y))

plt.hold(True)
plt.plot(x,sol[0]+sol[1]*x)

plt.title('Least square line fitting')
plt.xlabel('x')
plt.ylabel('y')

plt.show()
```