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CS301 Spring '17
Tu/Th 10:00AM

# Project 2: Finding Roots With Numerical Methods

## PREFACE

All tables and graphs are attached. The only notable things are that all calculations run to an accuracy of 0.00001, or 0.001%. This makes the methods work a little harder and make it easier to read/interpret the data. All table data is displayed to 7 decimal points, but was calculated using a double data type with 16 decimal digit precision.

## BISECTION METHOD

The Bisection Method, while conceptually simple, does not appear to be all that effective in quickly finding roots of equations. Convergence, while guaranteed if the original points are correctly bracketing a root, is painfully slow. For function (a), it was consistently either last or second-to-last place. It regularly took two to three times more iterations to achieve the same approximate error. The same can be said for function (b). Being a bracketing method, two points must be selected that are respectively less than, and greater than your target root. This does make it easy to initially select points, especially if given a plot to visually estimate with. The user must be careful to not bracket two different roots, as it is less predictable which root will be returned. This method may be useful if it is known that roots will always be finite and easily divided (2.5, 4.75, 6.0125). Because this method always divides the space in half with each iteration, roots which are likely to fall exactly 'between' two simple numbers might be easier to find.

## NEWTON-RAPHSON METHOD

The Newton-Raphson Method utilizes the functional value of some point and its derivative. As an open method, it is not required for the selected initial point to be near a root - though it can make it significantly faster. This method is a consistently good method for calculating function (a). For roots 1, 2, and 3, it came in 1st, 3rd, and 2nd, respectively. The large advantage of the Newton-Raphson Method is its use of the derivative of the original function. By using the first few terms of the Taylor series expansion, using the derivative makes calculating the next iteration rather simple. This does have one major drawback, though - functions whose derivatives are difficult, or even impossible to calculate, make it difficult to use this method. Additionally, because the formula for the next x-value includes a division by the functional value of the derivative, if the derivative itself is zero at that point, division-by-zero means that the method is now useless - at least for the starting point picked. The Newton-Raphson method also has the tendency to break when starting near inflection points, or around local minima/maxima. Certain initial guesses that are close to one particular root can also result in a jump to a root several roots away, due to the nature of near-zero slopes (minima/maxima). It also does not make any guarantee about convergence. The accuracy of the initial guess is very important - but still will never guarantee an accurate result. In general, if a good initial guess can be made, it is worth trying the Newton-Raphson method. When conditions are right, convergence is quick.

**SECANT METHOD**

The Secant Method is a variation of the Newton-Raphson Method. The advantage of the Secant Method over Newton-Raphson is that the formula for calculating the next value does not involve taking the derivative of the function - particularly useful for those functions which have difficult to compute, or even non-existent answers. The graphs for function (a) show that the Secant Method is quite an effective method, coming in at 2nd place for two of the three roots, and 1st place for the last. On function (b), it was a close third. Still though, convergence is not guaranteed - as with Newton-Raphson. Additionally, there is still the chance of division by zero - if the two points end up with the same functional value, the resulting secant line is parallel to the x-axis, and will diverge. In summary, it's a bit more calculation-heavy than Newton-Raphson, but the lack of need to derive the function can make it a wise choice.

**MODIFIED SECANT METHOD**

Conceptually, the same as the Secant Method, but the chosen delta value is used in the formula for determining the next x-value. The downsides with using this value in the formula come down to the difficulty of choice for that value. Picking a value too small results in a lot of round off error. Picking a value too large makes the method inefficient, and sometimes even divergent. If time can be spent deciding on this value, it is useful when calculating a derivative is hard, and picking two initial points is difficult. The graph data suggests that this is true - for function (a), root 1 is wild and inefficient. For root 2, it is fastest by a large margin. For root 3, it sits somewhere in the middle of the pack.

**FALSE POSITION METHOD**

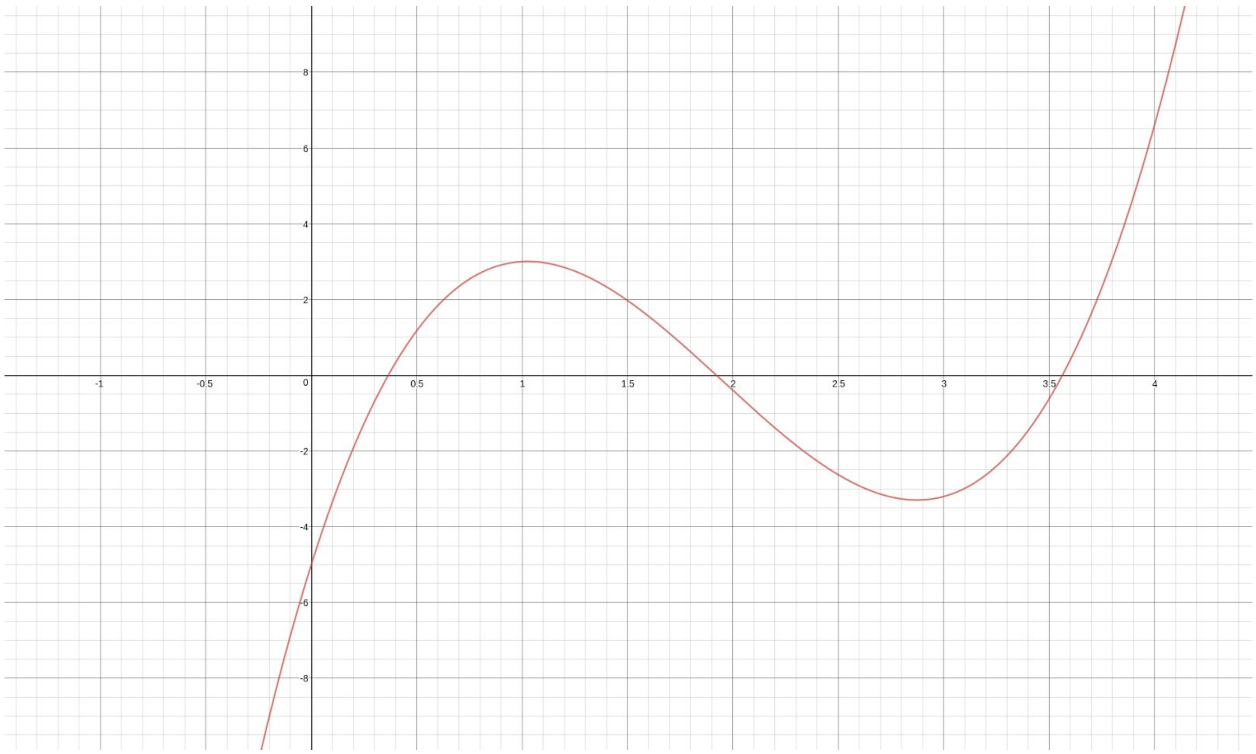
The False-Position Method is a bracketing method, not unlike the Bisection Method. Selecting two bracketing points about a root, the next point 'c' is selected by analyzing the point where the secant line intersects the x-axis. Compared to the Bisection Method, this method is usually significantly faster, and always converges to a single root. However, depending on the selection of initial points, things can be slow. One side of the bracketing points will tend to stay fixed, which can lead to poor convergence in functions that have high curvature. Even the Bisection Method is faster is many of these cases. One solution to this issue is to half the 'stuck' bound every iteration. While it does have its advantages, the graph data for function (a) suggests that it was slower than average, even being slower than Bisection in the case of root 3. That said, the single root for function (b) was found the fastest by False Position. Used carefully, false position can be quite effective.

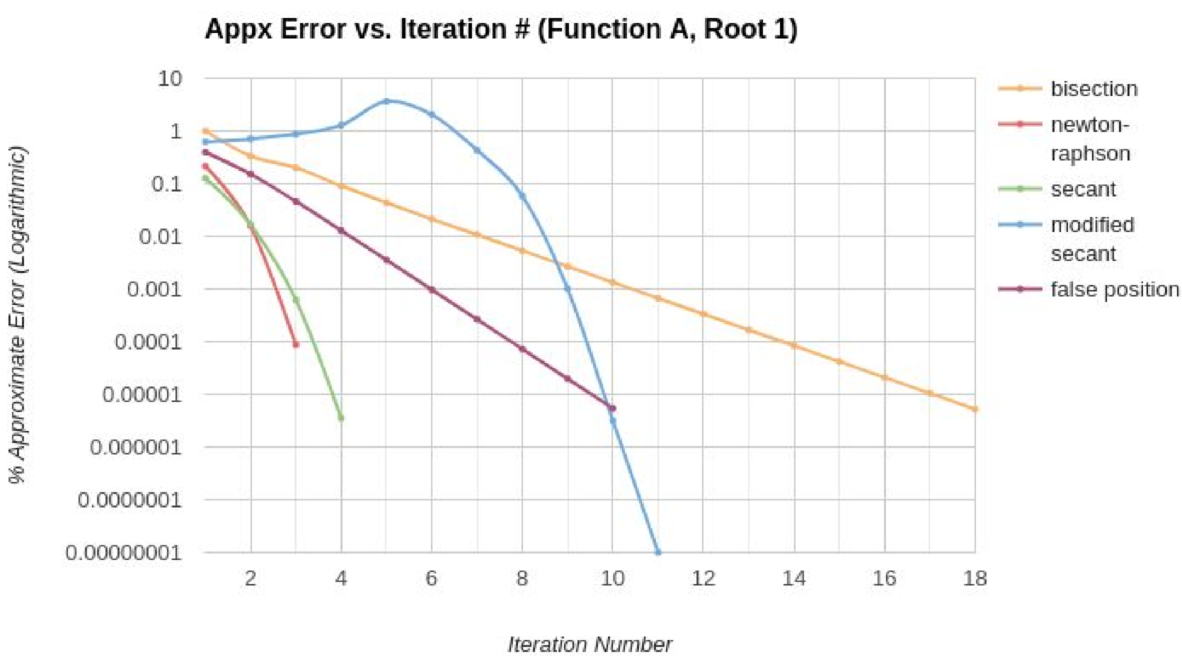
**SELECTION OF DATA TYPES**

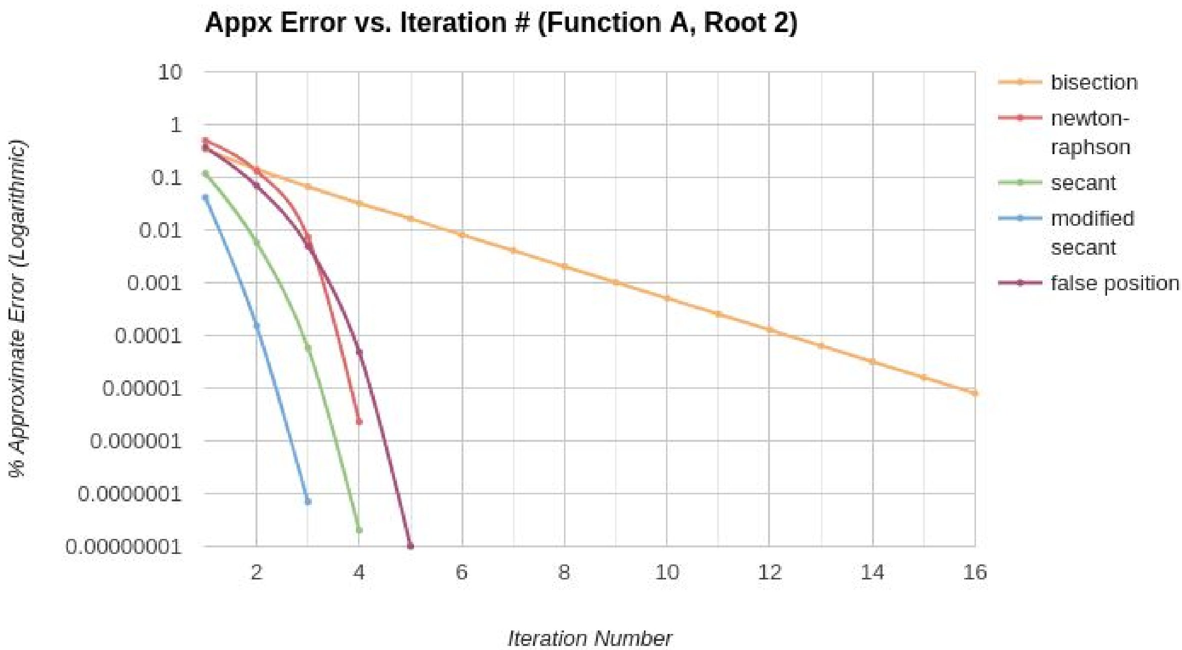
In general, it seemed important to be able to represent negative and positive values. Additionally, it needs to be floating point. Using Java, that leaves two options: float and double. Floats are 4 bytes in size, and have a 7 decimal digit precision. Doubles are 8 bytes in size, and have a 16 decimal digit precision. Given that memory usage wasn't a factor for this project, the double was selected. The more decimal precision, the less effect rounding error will have on our calculations, especially as the number of iterations increases.

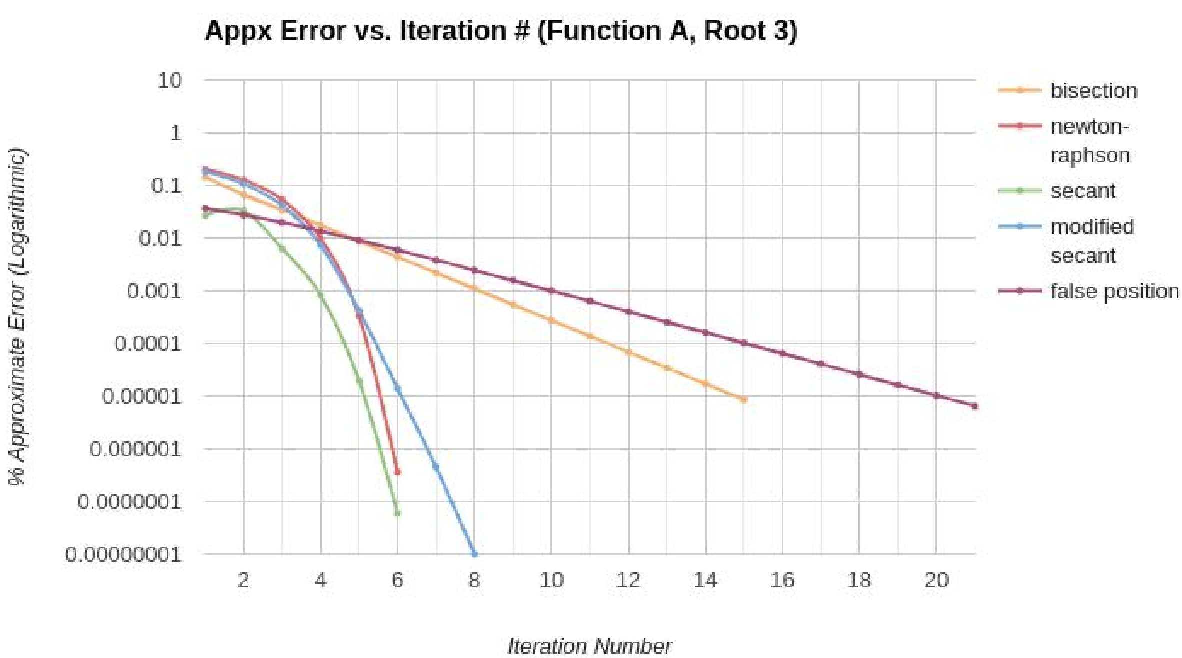
## DATA

### FUNCTION A

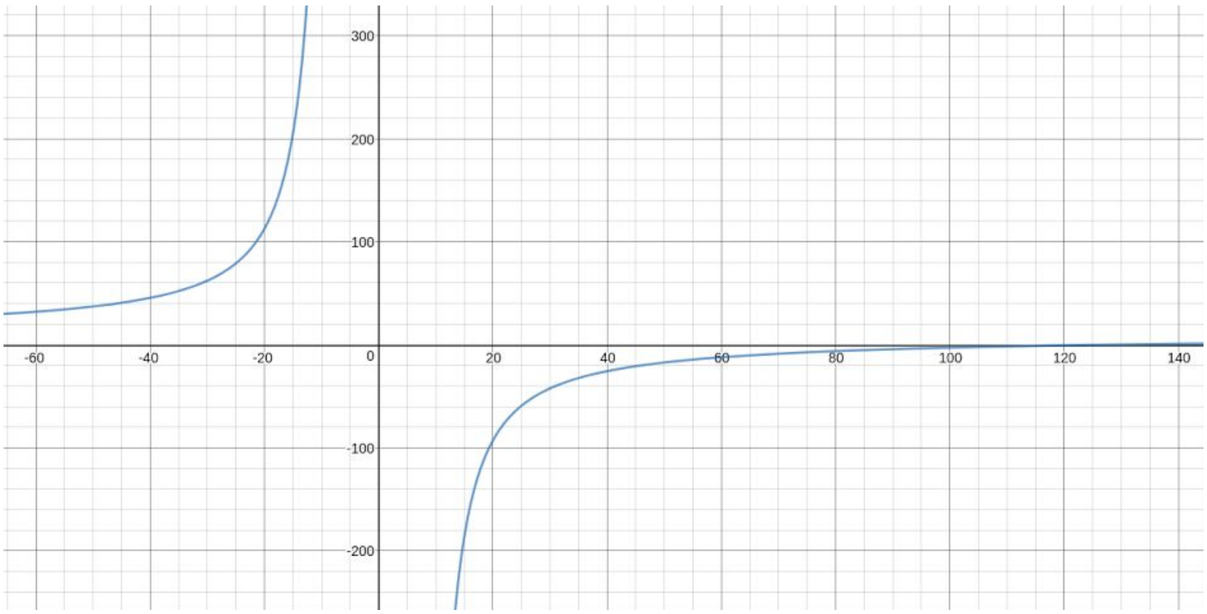




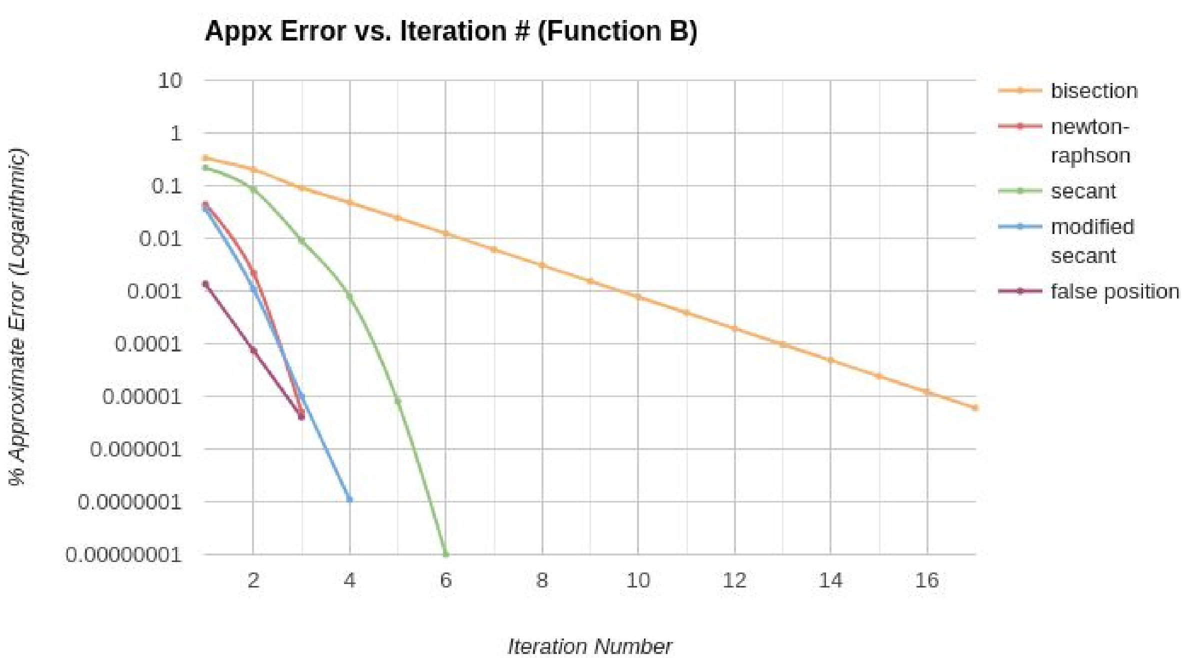

Appx Error vs. Iteration # (Function A, Root 2)
% Approximate Error (Logarithmic)
10
1
0.1
0.01
0.001
0.0001
0.00001
0.000001
0.0000001
0.00000001
2
4
6
8
10
12
14
16
Iteration Number
bisection
newton-raphson
secant
modified secant
false position


Appx Error vs. Iteration # (Function A, Root 3)
% Approximate Error (Logarithmic)
10
1
0.1
0.01
0.001
0.0001
0.00001
0.000001
0.0000001
0.00000001
2
4
6
8
10
12
14
16
18
20
Iteration Number
bisection
newton-raphson
secant
modified secant
false position

**FUNCTION B**



300
200
100
-100
-200
-60
-40
-20
0
20
40
60
80
100
120
140

Appx Error vs. Iteration # (Function B)
% Approximate Error (Logarithmic)
10
1
0.1
0.01
0.001
0.0001
0.00001
0.000001
0.0000001
0.00000001
2
4
6
8
10
12
14
16
Iteration Number
bisection
newton-raphson
secant
modified secant
false position

**BISECTION FUNCTION A, ROOT 1**

| n | a | b | c | f(a) | f(b) | f(c) | apx_err |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 1 | 0.5 | -5 | 3 | 1.175 | |
| 1 | 0 | 0.5 | 0.25 | -5 | 1.175 | -1.275 | 1 |
| 2 | 0.25 | 0.5 | 0.375 | -1.275 | 1.175 | 0.09765625 | 0.33333333 |
| 3 | 0.25 | 0.375 | 0.3125 | -1.275 | 0.09765625 | -0.55029297 | 0.2 |
| 4 | 0.3125 | 0.375 | 0.34375 | -0.55029297 | 0.09765625 | -0.21690674 | 0.09090909 |
| 5 | 0.34375 | 0.375 | 0.359375 | -0.21690674 | 0.09765625 | -0.05729523 | 0.04347826 |
| 6 | 0.359375 | 0.375 | 0.3671875 | -0.05729523 | 0.09765625 | 2.08E-02 | 0.0212766 |
| 7 | 0.359375 | 0.3671875 | 0.36328125 | -0.05729523 | 2.08E-02 | -0.01812227 | 0.01075269 |
| 8 | 0.36328125 | 0.3671875 | 0.36523438 | -0.01812227 | 2.08E-02 | 0.00135522 | 0.00534759 |
| 9 | 0.36328125 | 0.36523438 | 0.36425781 | -0.01812227 | 1.36E-03 | -0.00837445 | 2.68E-03 |
| 10 | 0.36425781 | 0.36523438 | 0.36474609 | -0.00837445 | 1.36E-03 | -0.00350735 | 1.34E-03 |
| 11 | 0.36474609 | 0.36523438 | 0.36499023 | -0.00350735 | 1.36E-03 | -0.0010755 | 6.69E-04 |
| 12 | 0.36499023 | 0.36523438 | 0.3651123 | -0.0010755 | 1.36E-03 | 1.40E-04 | 3.34E-04 |
| 13 | 0.36499023 | 0.3651123 | 0.36505127 | -1.08E-03 | 1.40E-04 | -4.68E-04 | 1.67E-04 |
| 14 | 0.36505127 | 0.3651123 | 0.36508179 | -4.68E-04 | 1.40E-04 | -1.64E-04 | 8.36E-05 |
| 15 | 0.36508179 | 0.3651123 | 0.36509705 | -1.64E-04 | 1.40E-04 | -1.19E-05 | 4.18E-05 |
| 16 | 0.36509705 | 0.3651123 | 0.36510468 | -1.19E-05 | 1.40E-04 | 6.40E-05 | 2.09E-05 |
| 17 | 0.36509705 | 0.36510468 | 0.36510086 | -1.19E-05 | 6.40E-05 | 2.61E-05 | 1.05E-05 |
| 18 | 0.36509705 | 0.36510086 | 0.36509895 | -1.19E-05 | 2.61E-05 | 7.07E-06 | 5.22E-06 |

**BISECTION FUNCTION A, ROOT 2**

| n | a | b | c | f(a) | f(b) | f(c) | apx_err |
|---|---|---|---|---|---|---|---|
| 0 | 1 | 3 | 2 | 3 | -3.2 | -0.4 | |
| 1 | 1 | 2 | 1.5 | 3 | -0.4 | 1.975 | 0.33333333 |
| 2 | 1.5 | 2 | 1.75 | 1.975 | -0.4 | 0.8625 | 0.14285714 |
| 3 | 1.75 | 2 | 1.875 | 0.8625 | -0.4 | 0.23828125 | 0.06666667 |
| 4 | 1.875 | 2 | 1.9375 | 0.23828125 | -0.4 | -0.08056641 | 0.03225806 |
| 5 | 1.875 | 1.9375 | 1.90625 | 0.23828125 | -0.08056641 | 0.07911377 | 0.01639344 |
| 6 | 1.90625 | 1.9375 | 1.921875 | 0.07911377 | -0.08056641 | -6.85E-04 | 0.00813008 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 7 | 1.90625 | 1.921875 | 1.9140625 | 0.07911377 | -6.85E-04 | 0.03922749 | 0.00408163 |
| 8 | 1.9140625 | 1.921875 | 1.91796875 | 0.03922749 | -6.85E-04 | 0.01927412 | 0.00203666 |
| 9 | 1.91796875 | 1.921875 | 1.91992188 | 0.01927412 | -6.85E-04 | 0.00929519 | 0.00101729 |
| 10 | 1.91992188 | 1.921875 | 1.92089844 | 0.00929519 | -6.85E-04 | 0.0043052 | 5.08E-04 |
| 11 | 1.92089844 | 1.921875 | 1.92138672 | 0.0043052 | -6.85E-04 | 0.00181008 | 2.54E-04 |
| 12 | 1.92138672 | 1.921875 | 1.92163086 | 0.00181008 | -6.85E-04 | 5.62E-04 | 1.27E-04 |
| 13 | 1.92163086 | 1.921875 | 1.92175293 | 5.62E-04 | -6.85E-04 | -6.13E-05 | 6.35E-05 |
| 14 | 1.92163086 | 1.92175293 | 1.92169189 | 5.62E-04 | -6.13E-05 | 2.51E-04 | 3.18E-05 |
| 15 | 1.92169189 | 1.92175293 | 1.92172241 | 2.51E-04 | -6.13E-05 | 9.46E-05 | 1.59E-05 |
| 16 | 1.92172241 | 1.92175293 | 1.92173767 | 9.46E-05 | -6.13E-05 | 1.67E-05 | 7.94E-06 |

**BISECTION FUNCTION A, ROOT 3**

| n | a | b | c | f(a) | f(b) | f(c) | apx_err |
|---|---|---|---|---|---|---|---|
| 0 | 2 | 4 | 3 | -0.4 | 6.6 | -3.2 | |
| 1 | 3 | 4 | 3.5 | -3.2 | 6.6 | -0.625 | 0.14285714 |
| 2 | 3.5 | 4 | 3.75 | -0.625 | 6.6 | 2.3125 | 0.06666667 |
| 3 | 3.5 | 3.75 | 3.625 | -0.625 | 2.3125 | 0.68671875 | 0.03448276 |
| 4 | 3.5 | 3.625 | 3.5625 | -0.625 | 0.68671875 | -0.00693359 | 0.01754386 |
| 5 | 3.5625 | 3.625 | 3.59375 | -0.00693359 | 0.68671875 | 0.33026123 | 0.00869565 |
| 6 | 3.5625 | 3.59375 | 3.578125 | -0.00693359 | 0.33026123 | 0.15927887 | 0.00436681 |
| 7 | 3.5625 | 3.578125 | 3.5703125 | -0.00693359 | 0.15927887 | 0.07557926 | 2.19E-03 |
| 8 | 3.5625 | 3.5703125 | 3.56640625 | -0.00693359 | 0.07557926 | 0.03417485 | 1.10E-03 |
| 9 | 3.5625 | 3.56640625 | 3.56445313 | -0.00693359 | 0.03417485 | 0.01358368 | 5.48E-04 |
| 10 | 3.5625 | 3.56445313 | 3.56347656 | -0.00693359 | 0.01358368 | 3.32E-03 | 2.74E-04 |
| 11 | 3.5625 | 3.56347656 | 3.56298828 | -0.00693359 | 3.32E-03 | -1.81E-03 | 1.37E-04 |
| 12 | 3.56298828 | 3.56347656 | 3.56323242 | -1.81E-03 | 3.32E-03 | 7.52E-04 | 6.85E-05 |
| 13 | 3.56298828 | 3.56323242 | 3.56311035 | -1.81E-03 | 7.52E-04 | -5.30E-04 | 3.43E-05 |
| 14 | 3.56311035 | 3.56323242 | 3.56317139 | -5.30E-04 | 7.52E-04 | 1.11E-04 | 1.71E-05 |
| 15 | 3.56311035 | 3.56317139 | 3.56314087 | -5.30E-04 | 1.11E-04 | -2.10E-04 | 8.56E-06 |

## BISECTION FUNCTION B

| n | a | b | c | f(a) | f(b) | f(c) | apx_err |
|---|---|---|---|---|---|---|---|
| 0 | 100 | 300 | 200 | -2.76259652 | 5.82367934 | 3.71738002 | |
| 1 | 100 | 200 | 150 | -2.76259652 | 3.71738002 | 1.58921983 | 0.33333333 |
| 2 | 100 | 150 | 125 | -2.76259652 | 1.58921983 | -0.13404648 | 0.2 |
| 3 | 125 | 150 | 137.5 | -0.13404648 | 1.58921983 | 0.80847301 | 0.09090909 |
| 4 | 125 | 137.5 | 131.25 | -0.13404648 | 0.80847301 | 0.36045351 | 0.04761905 |
| 5 | 125 | 131.25 | 128.125 | -0.13404648 | 0.36045351 | 0.11945895 | 0.02439024 |
| 6 | 125 | 128.125 | 126.5625 | -0.13404648 | 0.11945895 | -0.00566914 | 0.01234568 |
| 7 | 126.5625 | 128.125 | 127.34375 | -0.00566914 | 0.11945895 | 0.05729322 | 0.00613497 |
| 8 | 126.5625 | 127.34375 | 126.953125 | -0.00566914 | 0.05729322 | 0.02591258 | 0.00307692 |
| 9 | 126.5625 | 126.953125 | 126.7578125 | -0.00566914 | 0.02591258 | 0.01014698 | 1.54E-03 |
| 10 | 126.5625 | 126.7578125 | 126.6601563 | -0.00566914 | 0.01014698 | 0.00224525 | 7.71E-04 |
| 11 | 126.5625 | 126.6601563 | 126.6113281 | -0.00566914 | 0.00224525 | -1.71E-03 | 3.86E-04 |
| 12 | 126.6113281 | 126.6601563 | 126.6357422 | -0.00171036 | 2.25E-03 | 2.68E-04 | 1.93E-04 |
| 13 | 126.6113281 | 126.6357422 | 126.6235352 | -1.71E-03 | 2.68E-04 | -7.21E-04 | 9.64E-05 |
| 14 | 126.6235352 | 126.6357422 | 126.6296387 | -7.21E-04 | 2.68E-04 | -2.27E-04 | 4.82E-05 |
| 15 | 126.6296387 | 126.6357422 | 126.6326904 | -2.27E-04 | 2.68E-04 | 2.06E-05 | 2.41E-05 |
| 16 | 126.6296387 | 126.6326904 | 126.6311646 | -2.27E-04 | 2.06E-05 | -1.03E-04 | 1.21E-05 |
| 17 | 126.6311646 | 126.6326904 | 126.6319275 | -1.03E-04 | 2.06E-05 | -4.12E-05 | 6.02E-06 |

## NEWTON-RAPHSON FUNCTION A, ROOT 1

| n | xi | xi+1 | f(xi) | f(xi+1) | apx_err |
|---|---|---|---|---|---|
| 0 | 0 | 0.28248588 | -5 | -0.88855599 | |
| 1 | 0.28248588 | 0.35929331 | -0.88855599 | -0.05811758 | 0.21377362 |
| 2 | 0.35929331 | 0.36506634 | -0.05811758 | -3.18E-04 | 0.01581364 |
| 3 | 0.36506634 | 0.36509824 | -3.18E-04 | -1.00E-08 | 8.74E-05 |
| 4 | 0.36509824 | 0.36509824 | -1.00E-08 | 0 | 0 |

## NEWTON-RAPHSON FUNCTION A, ROOT 2

| n | xi | xi+1 | f(xi) | f(xi+1) | apx_err |
|---|---|---|---|---|---|
| 0 | 1.25 | 2.51436782 | 2.75 | -2.67172663 | |
| 1 | 2.51436782 | 1.68047836 | -2.67172663 | 1.19494606 | 0.49622148 |
| 2 | 1.68047836 | 1.93585518 | 1.19494606 | -7.22E-02 | 0.13191938 |
| 3 | 1.93585518 | 1.92174534 | -7.22E-02 | -2.25E-05 | 7.34E-03 |
| 4 | 1.92174534 | 1.92174093 | -2.25E-05 | 0 | 2.29E-06 |

## NEWTON-RAPHSON FUNCTION A, ROOT 3

| n | xi | xi+1 | f(xi) | f(xi+1) | apx_err |
|---|---|---|---|---|---|
| 0 | 3 | 5.13333333 | -3.2 | 48.09007407 | |
| 1 | 5.13333333 | 4.26975006 | 48.09007407 | 12.95624356 | 0.20225617 |
| 2 | 4.26975006 | 3.79293448 | 12.95624356 | 2.94760312 | 0.12571152 |
| 3 | 3.79293448 | 3.59981929 | 2.94760312 | 0.39797274 | 0.0536458 |
| 4 | 3.59981929 | 3.56433803 | 0.39797274 | 0.0123726 | 0.00995451 |
| 5 | 3.56433803 | 3.5631621 | 0.0123726 | 1.34E-05 | 3.30E-04 |
| 6 | 3.5631621 | 3.56316082 | 1.34E-05 | 0 | 3.60E-07 |

## NEWTON-RAPHSON FUNCTION B

| n | xi | xi+1 | f(xi) | f(xi+1) | apx_err |
|---|---|---|---|---|---|
| 0 | 100 | 120.7836401 | -2.76259652 | -0.49772027 | |
| 1 | 120.7836401 | 126.351685 | -0.49772027 | -0.0227976 | 0.04406783 |
| 2 | 126.351685 | 126.6317898 | -0.0227976 | -5.24E-05 | 0.00221196 |
| 3 | 126.6317898 | 126.632436 | -5.24E-05 | 0 | 5.10E-06 |

### SECANT FUNCTION A, ROOT 1

| n | xi-1 | xi | xi+1 | f(xi-1) | f(xi) | f(xi+1) | apx_err |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0.5 | 0.4048583 | -5 | 1.175 | 0.38096291 | |
| 1 | 0.5 | 0.4048583 | 0.35921124 | 1.175 | 0.38096291 | -0.05894382 | 0.12707581 |
| 2 | 0.4048583 | 0.35921124 | 0.36532756 | 0.38096291 | -0.05894382 | 0.00228272 | 0.01674203 |
| 3 | 0.35921124 | 0.36532756 | 0.36509953 | -0.05894382 | 0.00228272 | 1.28E-05 | 6.25E-04 |
| 4 | 0.36532756 | 0.36509953 | 0.36509824 | 0.00228272 | 1.28E-05 | 0 | 3.52E-06 |
| 5 | 0.36509953 | 0.36509824 | 0.36509824 | 1.28E-05 | 0 | 0 | 0 |
| 6 | 0.36509824 | 0.36509824 | 0.36509824 | 0 | 0 | 0 | 0 |
| 7 | 0.36509824 | 0.36509824 | 0.36509824 | 0 | 0 | 0 | 0 |
| 8 | 0.36509824 | 0.36509824 | 0 | 0 | 0 | 0 | 0 |

### SECANT FUNCTION A, ROOT 2

| n | xi-1 | xi | xi+1 | f(xi-1) | f(xi) | f(xi+1) | apx_err |
|---|---|---|---|---|---|---|---|
| 0 | 1.25 | 1.5 | 2.13709677 | 2.75 | 1.975 | -1.08840128 | |
| 1 | 1.5 | 2.13709677 | 1.91074153 | 1.975 | -1.08840128 | 0.05618606 | 0.11846461 |
| 2 | 2.13709677 | 1.91074153 | 1.92185297 | -1.08840128 | 0.05618606 | -5.73E-04 | 0.00578163 |
| 3 | 1.91074153 | 1.92185297 | 1.92174089 | 0.05618606 | -5.73E-04 | 2.40E-07 | 5.83E-05 |
| 4 | 1.92185297 | 1.92174089 | 1.92174093 | -5.73E-04 | 2.40E-07 | 0 | 2.00E-08 |
| 5 | 1.92174089 | 1.92174093 | 1.92174093 | 2.40E-07 | 0 | 0 | 0 |
| 6 | 1.92174093 | 1.92174093 | 1.92174093 | 0 | 0 | 0 | 0 |
| 7 | 1.92174093 | 1.92174093 | 1.92174093 | 0 | 0 | 0 | 0 |
| 8 | 1.92174093 | 1.92174093 | 0 | 0 | 0 | 0 | 0 |

### SECANT FUNCTION A, ROOT 3

| n | xi-1 | xi | xi+1 | f(xi-1) | f(xi) | f(xi+1) | apx_err |
|---|---|---|---|---|---|---|---|
| 0 | 4 | 5 | 3.80813953 | 6.6 | 41 | 3.1822481 | |
| 1 | 5 | 3.80813953 | 3.70784812 | 41 | 3.1822481 | 1.72771381 | 0.02704841 |
| 2 | 3.80813953 | 3.70784812 | 3.58872075 | 3.1822481 | 1.72771381 | 0.27470339 | 0.03319494 |
| 3 | 3.70784812 | 3.58872075 | 3.56619875 | 1.72771381 | 0.27470339 | 0.03198373 | 0.00631541 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 4 | 3.58872075 | 3.56619875 | 3.56323097 | 0.27470339 | 0.03198373 | 7.37E-04 | 8.33E-04 |
| 5 | 3.56619875 | 3.56323097 | 3.56316102 | 0.03198373 | 7.37E-04 | 2.06E-06 | 1.96E-05 |
| 6 | 3.56323097 | 3.56316102 | 3.56316082 | 7.37E-04 | 2.06E-06 | 0 | 6.00E-08 |
| 7 | 3.56316102 | 3.56316082 | 3.56316082 | 2.06E-06 | 0 | 0 | 0 |
| 8 | 3.56316082 | 3.56316082 | 3.56316082 | 0 | 0 | 0 | 0 |
| 9 | 3.56316082 | 3.56316082 | 0 | 0 | 0 | 0 | 0 |

## SECANT FUNCTION B

| n | xi-1 | xi | xi+1 | f(xi-1) | f(xi) | f(xi+1) | apx_err |
|---|---|---|---|---|---|---|---|
| 0 | 100 | 200 | 142.6328167 | -2.76259652 | 3.71738002 | 1.14612496 | |
| 1 | 200 | 142.6328167 | 117.061662 | 3.71738002 | 1.14612496 | -0.84146317 | 0.21844175 |
| 2 | 142.6328167 | 117.061662 | 127.8874385 | 1.14612496 | -0.84146317 | 0.10063905 | 0.08465082 |
| 3 | 117.061662 | 127.8874385 | 126.7309866 | -0.84146317 | 0.10063905 | 0.00797766 | 0.00912525 |
| 4 | 127.8874385 | 126.7309866 | 126.6314222 | 0.10063905 | 0.00797766 | -8.21E-05 | 7.86E-04 |
| 5 | 126.7309866 | 126.6314222 | 126.6324369 | 0.00797766 | -8.21E-05 | 7.00E-08 | 8.01E-06 |
| 6 | 126.6314222 | 126.6324369 | 126.632436 | -8.21E-05 | 7.00E-08 | 0 | 1.00E-08 |
| 7 | 126.6324369 | 126.632436 | 126.632436 | 7.00E-08 | 0 | 0 | 0 |
| 8 | 126.632436 | 126.632436 | 126.632436 | 0 | 0 | 0 | 0 |
| 9 | 126.632436 | 126.632436 | 0 | 0 | 0 | 0 | 0 |

## MODIFIED SECANT FUNCTION A, ROOT 1

| n | xi | xi+1 | f(xi) | f(xi+1) | apx_err |
|---|---|---|---|---|---|
| 0 | 1 | -11.33552632 | 3 | -4622.118244 | |
| 1 | -11.33552632 | -6.98725303 | -4622.118244 | -1382.147535 | 0.62231513 |
| 2 | -6.98725303 | -4.09505649 | -1382.147535 | -411.0295048 | 0.70626536 |
| 3 | -4.09505649 | -2.18887596 | -411.0295048 | -120.774475 | 0.87084904 |
| 4 | -2.18887596 | -0.95913473 | -120.774475 | -34.50466797 | 1.28213605 |
| 5 | -0.95913473 | -0.2062671 | -34.50466797 | -9.16626894 | 3.64996473 |
| 6 | -0.2062671 | 0.19561028 | -9.16626894 | -1.9704102 | 2.05447986 |
| 7 | 0.19561028 | 0.34340896 | -1.9704102 | -0.22044279 | 0.4303868 |
| 8 | 0.34340896 | 0.364731 | -0.22044279 | -0.00365771 | 0.05845963 |

| | | | | | |
|---|---|---|---|---|---|
| 9 | 0.364731 | 0.3650994 | -0.00365771 | 1.15E-05 | 0.00100902 |
| 10 | 0.3650994 | 0.36509824 | 1.15E-05 | -4.00E-08 | 3.17E-06 |
| 11 | 0.36509824 | 0.36509824 | -4.00E-08 | 0 | 1.00E-08 |
| 12 | 0.36509824 | 0.36509824 | 0 | 0 | 0 |
| 13 | 0.36509824 | 0.36509824 | 0 | 0 | 0 |
| 14 | 0.36509824 | 0.36509824 | 0 | 0 | 0 |
| 15 | 0.36509824 | 0.36509824 | 0 | 0 | 0 |
| 16 | 0.36509824 | 0.36509824 | 0 | 0 | 0 |

**MODIFIED SECANT FUNCTION A, ROOT 2**

| n | xi | xi+1 | f(xi) | f(xi+1) | apx_err |
|---|---|---|---|---|---|
| 0 | 1.5 | 2.00126267 | 1.975 | -0.40643916 | |
| 1 | 2.00126267 | 1.92144785 | -0.40643916 | 0.0014977 | 0.0415389 |
| 2 | 1.92144785 | 1.92174079 | 0.0014977 | 7.30E-07 | 1.52E-04 |
| 3 | 1.92174079 | 1.92174093 | 7.30E-07 | 0 | 7.00E-08 |
| 4 | 1.92174093 | 1.92174093 | 0 | 0 | 0 |
| 5 | 1.92174093 | 1.92174093 | 0 | 0 | 0 |
| 6 | 1.92174093 | 1.92174093 | 0 | 0 | 0 |

**MODIFIED SECANT FUNCTION A, ROOT 3**

| n | xi | xi+1 | f(xi) | f(xi+1) | apx_err |
|---|---|---|---|---|---|
| 0 | 3 | 4.89259522 | -3.2 | 35.76320461 | |
| 1 | 4.89259522 | 4.14294942 | 35.76320461 | 9.73047031 | 0.18094496 |
| 2 | 4.14294942 | 3.74231596 | 9.73047031 | 2.20306285 | 0.10705495 |
| 3 | 3.74231596 | 3.59105474 | 2.20306285 | 0.30042492 | 0.04212167 |
| 4 | 3.59105474 | 3.56470059 | 0.30042492 | 0.01618853 | 0.00739309 |
| 5 | 3.56470059 | 3.56321219 | 0.01618853 | 5.39E-04 | 4.18E-04 |
| 6 | 3.56321219 | 3.56316247 | 5.39E-04 | 1.73E-05 | 1.40E-05 |
| 7 | 3.56316247 | 3.56316088 | 1.73E-05 | 5.50E-07 | 4.50E-07 |
| 8 | 3.56316088 | 3.56316083 | 5.50E-07 | 2.00E-08 | 1.00E-08 |
| 9 | 3.56316083 | 3.56316082 | 2.00E-08 | 0 | 0 |

| | | | | | |
|---|---|---|---|---|---|
| 10 | 3.56316082 | 3.56316082 | 0 | 0 | 0 |
| 11 | 3.56316082 | 3.56316082 | 0 | 0 | 0 |
| 12 | 3.56316082 | 3.56316082 | 0 | 0 | 0 |
| 13 | 3.56316082 | 3.56316082 | 0 | 0 | 0 |

**MODIFIED SECANT FUNCTION B**

| | | | | | |
|---|---|---|---|---|---|
| 0 | 150 | 121.8859616 | 1.58921983 | -0.40011317 | |
| 1 | 121.8859616 | 126.4950638 | -0.40011317 | -0.0111418 | 0.03643701 |
| 2 | 126.4950638 | 126.6337058 | -0.0111418 | 1.03E-04 | 0.00109483 |
| 3 | 126.6337058 | 126.6324229 | 1.03E-04 | -1.07E-06 | 1.01E-05 |
| 4 | 126.6324229 | 126.6324362 | -1.07E-06 | 1.00E-08 | 1.10E-07 |
| 5 | 126.6324362 | 126.632436 | 1.00E-08 | 0 | 0 |
| 6 | 126.632436 | 126.632436 | 0 | 0 | 0 |
| 7 | 126.632436 | 126.632436 | 0 | 0 | 0 |
| 8 | 126.632436 | 126.632436 | 0 | 0 | 0 |

**FALSE POSITION FUNCTION A, ROOT 1**

| n | a | b | c | f(a) | f(b) | f(c) | apx_err |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 1 | 0.625 | -5 | 3 | 1.98046875 | |
| 1 | 0 | 0.625 | 0.44767767 | -5 | 1.98046875 | 0.75847872 | 0.39609375 |
| 2 | 0 | 0.44767767 | 0.38871175 | -5 | 0.75847872 | 0.22983138 | 0.15169574 |
| 3 | 0 | 0.38871175 | 0.37162934 | -5 | 0.22983138 | 0.06462164 | 0.04596628 |
| 4 | 0 | 0.37162934 | 0.36688756 | -5 | 0.06462164 | 0.01778488 | 0.01292433 |
| 5 | 0 | 0.36688756 | 0.36558718 | -5 | 0.01778488 | 0.00486578 | 0.00355698 |
| 6 | 0 | 0.36558718 | 0.36523175 | -5 | 0.00486578 | 0.00132908 | 9.73E-04 |
| 7 | 0 | 0.36523175 | 0.36513469 | -5 | 0.00132908 | 3.63E-04 | 2.66E-04 |
| 8 | 0 | 0.36513469 | 0.36510819 | -5 | 3.63E-04 | 9.91E-05 | 7.26E-05 |
| 9 | 0 | 0.36510819 | 0.36510096 | -5 | 9.91E-05 | 2.70E-05 | 1.98E-05 |
| 10 | 0 | 0.36510096 | 0.36509898 | -5 | 2.70E-05 | 7.38E-06 | 5.41E-06 |

## FALSE POSITION FUNCTION A, ROOT 2

| n | a | b | c | f(a) | f(b) | f(c) | apx_err |
|---|---|---|---|---|---|---|---|
| 0 | 1 | 1.5 | 2.46341463 | 3 | 1.975 | -2.49994922 | |
| 1 | 1 | 2.46341463 | 1.79823353 | 3 | -2.49994922 | 0.62479418 | 0.36990807 |
| 2 | 1.79823353 | 2.46341463 | 1.93123687 | 0.62479418 | -2.49994922 | -0.04853978 | 0.06886951 |
| 3 | 1.79823353 | 1.93123687 | 1.92164883 | 0.62479418 | -0.04853978 | 4.71E-04 | 0.00498949 |
| 4 | 1.92164883 | 1.93123687 | 1.92174091 | 4.71E-04 | -0.04853978 | 1.30E-07 | 4.79E-05 |
| 5 | 1.92174091 | 1.93123687 | 1.92174093 | 1.30E-07 | -0.04853978 | 0 | 1.00E-08 |

## FALSE POSITION FUNCTION A, ROOT 3

| n | a | b | c | f(a) | f(b) | f(c) | apx_err |
|---|---|---|---|---|---|---|---|
| 0 | 3 | 5 | 3.14479638 | -3.2 | 41 | -2.84464808 | |
| 1 | 3.14479638 | 5 | 3.2651623 | -2.84464808 | 41 | -2.32200928 | 0.03686369 |
| 2 | 3.2651623 | 5 | 3.35814758 | -2.32200928 | 41 | -1.76280062 | 0.02768946 |
| 3 | 3.35814758 | 5 | 3.42582927 | -1.76280062 | 41 | -1.26444181 | 0.01975629 |
| 4 | 3.42582927 | 5 | 3.47292435 | -1.26444181 | 41 | -0.87002551 | 0.01356064 |
| 5 | 3.47292435 | 5 | 3.50465575 | -0.87002551 | 41 | -0.58149969 | 0.00905407 |
| 6 | 3.50465575 | 5 | 3.52556751 | -0.58149969 | 41 | -0.38110933 | 0.00593146 |
| 7 | 3.52556751 | 5 | 3.53914665 | -0.38110933 | 41 | -0.24656425 | 0.00383684 |
| 8 | 3.53914665 | 5 | 3.54787936 | -0.24656425 | 41 | -0.15818278 | 0.00246139 |
| 9 | 3.54787936 | 5 | 3.55346028 | -0.15818278 | 41 | -0.10093439 | 0.00157056 |
| 10 | 3.55346028 | 5 | 3.55701265 | -0.10093439 | 41 | -0.06418263 | 9.99E-04 |
| 11 | 3.55701265 | 5 | 3.55926801 | -0.06418263 | 41 | -0.04072301 | 6.34E-04 |
| 12 | 3.55926801 | 5 | 3.56069759 | -0.04072301 | 41 | -0.02580211 | 4.01E-04 |
| 13 | 3.56069759 | 5 | 3.5616028 | -0.02580211 | 41 | -0.01633375 | 2.54E-04 |
| 14 | 3.5616028 | 5 | 3.56217561 | -0.01633375 | 41 | -0.01033411 | 1.61E-04 |
| 15 | 3.56217561 | 5 | 3.56253792 | -0.01033411 | 41 | -0.00653591 | 1.02E-04 |
| 16 | 3.56253792 | 5 | 3.56276704 | -0.00653591 | 41 | -0.00413277 | 6.43E-05 |
| 17 | 3.56276704 | 5 | 3.56291189 | -0.00413277 | 41 | -0.00261285 | 4.07E-05 |
| 18 | 3.56291189 | 5 | 3.56300347 | -0.00261285 | 41 | -0.00165177 | 2.57E-05 |
| 19 | 3.56300347 | 5 | 3.56306136 | -0.00165177 | 41 | -0.00104414 | 1.63E-05 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 20 | 3.56306136 | 5 | 3.56309796 | -0.00104414 | 41 | -6.60E-04 | 1.03E-05 |
| 21 | 3.56309796 | 5 | 3.56312109 | -6.60E-04 | 41 | -4.17E-04 | 6.49E-06 |

**FALSE POSITION FUNCTION B**

| n | a | b | c | f(a) | f(b) | f(c) | apx_err |
|---|---|---|---|---|---|---|---|
| 0 | 120 | 130 | 126.8156039 | -0.5682456 | 0.26549651 | 0.01481715 | |
| 1 | 120 | 126.8156039 | 126.6424016 | -0.5682456 | 0.01481715 | 8.07E-04 | 0.00136765 |
| 2 | 120 | 126.6424016 | 126.6329782 | -0.5682456 | 8.07E-04 | 4.39E-05 | 7.44E-05 |
| 3 | 120 | 126.6329782 | 126.6324655 | -0.5682456 | 4.39E-05 | 2.39E-06 | 4.05E-06 |