

CS211 Discrete Structure

Tuesday, September 27, 2022

Course Instructor

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Serial No:

1st Mid Term Exam

Total Time: 1 Hour

Total Marks: 60

Signature of Invigilator

Roll No

Section

Signature

DO NOT OPEN THE QUESTION BOOK OR START UNTIL INSTRUCTED.

Instructions:

1. Verify at the start of the exam that you have a total of four (04) questions printed on six (06) pages including this title page.
2. Attempt all questions on the question-book and in the given order.
3. The exam is closed books, closed notes. Please see that the area in your threshold is free of any material classified as 'useful in the paper' or else there may a charge of cheating.
4. Read the questions carefully for clarity of context and understanding of meaning and make assumptions wherever required, for neither the invigilator will address your queries, nor the teacher/examiner will come to the examination hall for any assistance.
5. Fit in all your answers in the provided space. You may use extra space on the last page if required. If you do so, clearly mark question/part number on that page to avoid confusion.
6. Use only your own stationery and calculator. If you do not have your own calculator, use manual calculations.
7. Use only permanent ink-pens. Only the questions attempted with permanent ink-pens will be considered. Any part of paper done in lead pencil cannot be claimed for checking/rechecking.

	Q-1	Q-2	Q-3	Q-4	Total
Total Marks	20	10	15	15	60
Marks Obtained					

Vetted By: _____ Vetter Signature: _____

University Answer Sheet Required: No ☐ Yes ☐

National University of Computer and Emerging Sciences

Department of **Computer Science**

Chiniot-Faisalabad Campus

Question 1: Provide the solution of the following parts a, b, c, and d

[04+06+06+4= 20 Points]

a. Which of the following is a negation for “All FAST university students are not a blonde”? More than one answer may be correct.

- a. There is a FAST university student who is a not blonde.
- b. All FAST university students are not a blonde.
- c. No FAST university students are blonde.
- d. Some FAST university students are not blonde.
- e. No blonde people are FAST university students.

Answer: **a and d.**

b. [06 points]: Look into the following argument and answer the following questions with solid reason.

If $\sqrt{2} > \frac{3}{2}$ then $(\sqrt{2})^2 > (\frac{3}{2})^2$. We know that $\sqrt{2} > \frac{3}{2}$. Therefore, $(\sqrt{2})^2 = 2 > (\frac{3}{2})^2 = \frac{9}{4}$.

1. Is the argument valid, if yes, according to which inferencing rule?

The argument is valid: modus ponens inference rule.

2. Does the conclusion must be true, if not Why? Specify the condition.

We cannot conclude that the conclusion is true, since one of its premises, $\sqrt{2} > 3/2$, is false.

3. What is wrong?

Indeed, in this case the conclusion is false, since $2 \nprec 9/4 = 2.25$.

c. [03*2 = 06 Points]: Write Converse, Inverse, and Contrapositive of the following statement: (06 Marks)

If the weather is good, I go for a walk. $P \rightarrow Q$

a. Converse:

If I go for a walk, the weather is good. $Q \rightarrow P$

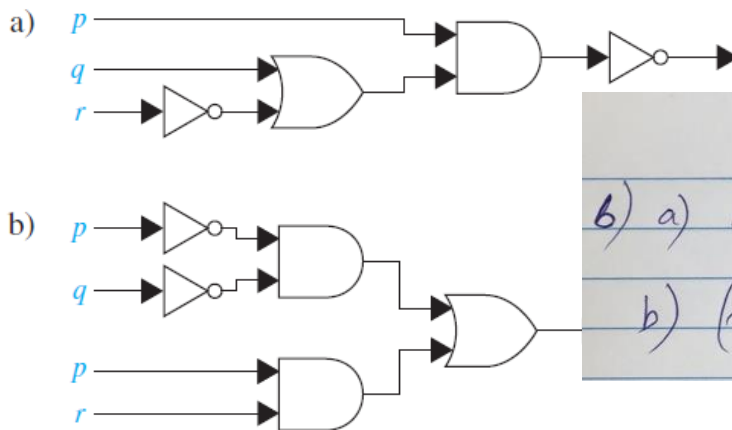
b. Inverse:

If the weather is not good, I do not go for a walk. $\sim P \rightarrow \sim Q$

c. Contrapositive:

if I do not go for a walk, the weather is not good. $\sim Q \rightarrow \sim P$

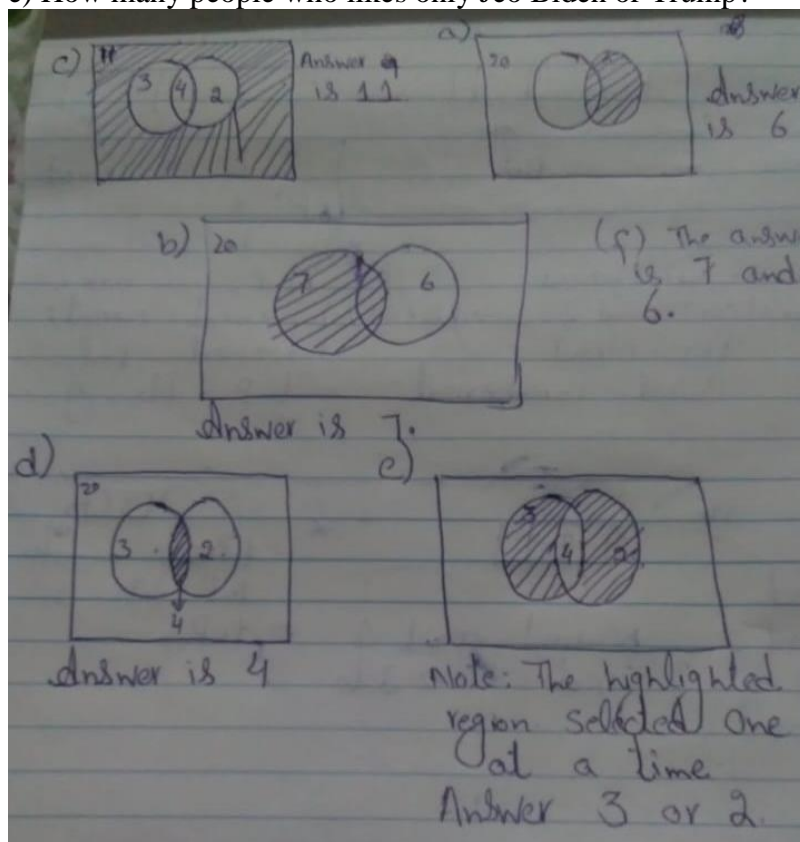
- d. Find the output of each of these combinatorial circuits in terms of Logical Operators.



b) a) $\sim(p \wedge (q \vee \sim r))$
 b) $(\sim p \wedge \sim q) \vee (p \wedge r)$

Question 2: In 2020 US election, 20 million people cast the vote for recent presidential election as per their likes. 7 million likes Jeo Biden, 6 million likes Trump and 4 million likes both Trump and Jeo Biden as presidential Candidate. Show by the Ven Diagram. [10 Points]

- How many people who likes only Trump?
- How many people who likes only Jeo Biden?
- How many people who likes neither Jeo Biden nor Trump?
- How many people who likes both Jeo Biden and Trump?
- How many people who likes only Jeo Biden or Trump?



Question 3: Provide the solution of the following parts a and b [08+07= 15 Points]

- a. In a survey of university students, 64 had taken mathematics course, 94 had taken chemistry course, 58 had taken physics course, 26 had taken mathematics and physics, 26 had taken mathematics and chemistry, 22 had taken chemistry and physics course, and 14 had taken all the three courses. Find how many had taken one course only. [Note: Use venn diagram]

Step 1 :

Venn diagram related to the information given in the question:

Step 2 :

From the venn diagram above, we have

Number of students who had taken only math = 24

Number of students who had taken only chemistry = 60

Number of students who had taken only physics = 22

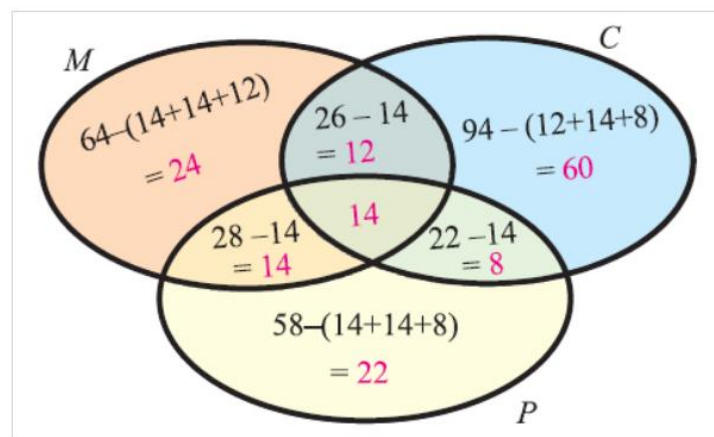
Step 3 :

Total Number of students who had taken only one course :

$$= 24 + 60 + 22$$

$$= 106$$

Hence, the total number of students who had taken only one course is 106.



b. Use logical equivalences or rules of inference to determine whether the following argument is valid.

$$(i) \quad \neg p \wedge q$$

$$(ii) \quad r \rightarrow p$$

$$(ii) \quad \neg r \rightarrow s$$

$$(iii) \quad s \rightarrow t$$

conclusion t

From the following premisses

1. $\neg p \wedge q$
2. $r \rightarrow p$
3. $\neg r \rightarrow s$
4. $s \rightarrow t$

Show that t is a conclusion.

Modus Ponens	Modus Tollens
$\frac{p \quad p \rightarrow q}{\therefore q}$	$\frac{\neg q \quad p \rightarrow q}{\therefore \neg p}$
Simplification	
$\frac{p \wedge q}{\therefore p}$	

Rules

Step	Reason
1. $\neg p \wedge q$	From premise #1
2. $\neg p$	Using the Simplification Rule on (step #1)
3. $r \rightarrow p$	From premise #2
4. $\neg r$	Using the Modus Tollens Rule on (step #2) & (step #3)
5. $\neg r \rightarrow s$	From premise #3
6. s	Using the Modus Ponens Rule on (step #4) & (step #5)
7. $s \rightarrow t$	From premise #4
8. t	Using the Modus Ponens Rule on (step #6) & (step #7)

Question 4: Provide the solution of the following parts a, b and c [05+05+05= 15 Points]

a. Prove the logical equivalence by using logic laws.

$$\neg(p \vee (\neg p \wedge q)) \equiv \neg p \wedge \neg q$$

$\neg(p \vee (\neg p \wedge q)) \equiv \neg p \wedge \neg(\neg p \wedge q)$	Using De Morgan's law
$\equiv \neg p \wedge (\neg(\neg p) \vee \neg q)$	Using De Morgan's law
$\equiv \neg p \wedge (p \vee \neg q)$	Using Double negation law
$\equiv (\neg p \wedge p) \vee (\neg p \wedge \neg q)$	Using Distributive law
$\equiv F \vee (\neg p \wedge \neg q)$	Using Negation Law
$\equiv \neg p \wedge \neg q$	Using Identity Law

b. Using truth table, prove the following statement?

$$\sim(p \oplus q) \equiv p \leftrightarrow q$$

p	q	$p \oplus q$	$\sim(p \oplus q)$	$p \leftrightarrow q$
T	T	F	T	T
T	F	T	F	F
F	T	T	F	F
F	F	F	T	T

c. Use the logical equivalences above to show that $\neg(p \vee \neg(p \wedge q))$ is a contradiction or tautology.

Solution.

$$\begin{aligned} & \neg(p \vee \neg(p \wedge q)) \\ \Leftrightarrow & \neg p \wedge \neg(\neg(p \wedge q)) && \text{De Morgan's Law} \\ \Leftrightarrow & \neg p \wedge (p \wedge q) && \text{Double Negation Law} \\ \Leftrightarrow & (\neg p \wedge p) \wedge q && \text{Associative Law} \\ \Leftrightarrow & F \wedge q && \text{Contradiction} \\ \Leftrightarrow & F && \text{Domination Law and Commutative Law} \end{aligned}$$