National University of Computer and Emerging Sciences

Chiniot-Faisalabad Campus

School of Computing		10 Points
Question No. 1		

a) Determine the convergence or divergence of the sequence $a_n = \frac{(-1)^{n+1}+5^{n+1}}{(-1)^n+5^n}$.

$$\lim_{n\to\infty} a_n = \frac{(-1)^n \cdot (-1)^1 + 5^n \cdot 5^1}{(-1)^n + 5^n}$$

$$\lim_{n\to\infty} a_n = \frac{(-1)^n\,[-1 + -5.5]}{(-1)^n\,[1 + -5]}$$

$$= -26$$

Using lopital rule:

$$\Rightarrow \lim_{n\to\infty} \frac{(n+1)(-1)^n + 5(n+1)5^n}{n(-1)^{n-1} + n5^{n-1}}$$

$$a_n = \frac{(-1+5)^{n+1}}{(-1+5)^n}$$

$$a_n = \frac{4^{n+1}}{4^n}$$

putting $\lim_{n\to\infty} a_n = \infty/\infty$ form

Using lopital rule

$$= \frac{(n+1)4^n}{4n}$$

$= \infty$ divergent

$\therefore$ The given sequence is divergent

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$$a_n = \frac{(-1)^{n+1}+5^{n+1}}{(-1)^n+5^n}$$

$$a_n = \frac{(-1)^n \cdot (-1)^1 + 5^n \cdot 5^1}{(-1)^n + 5^n}$$

$$a_n = \frac{\cancel{(-1)^n}\left[\frac{-1}{(-1)^n} + \frac{5^n}{(-1)^n} \cdot \frac{5}{(-1)^n}\right]}{\cancel{(-1)^n}\left[1 + \frac{(5)^n}{(-1)^n}\right]}$$

$$a_n = \frac{(1)^n - (5)^n - (5)^n}{1 - (5)^n}$$

$$a_n = \frac{\cancel{-(5)^n}\left[\left(\frac{1}{5}\right)^n + 1 + 1\right]}{\cancel{-(5)^n}\left(\frac{1}{(5)^n} - 1\right)}$$

$$a_n =$$

$a_n =$

b) Determine the convergence or divergence of the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}+5^{n+1}}{(-1)^n+5^n}$. Justify your answer in detail.

Expanding,

$$= \frac{(-1)^{1+1}+5^{1+1}}{(-1)^1+5^1} + \frac{(-1)^{2+1}+5^{2+1}}{(-1)^2+5^2} + \frac{(-1)^{3+1}+5^{3+1}}{(-1)^3+5^3} + \dots + \frac{(-1)^{n+1}+5^{n+1}}{(-1)^n+5^n}$$

$$= \frac{(-1)^2+5^2}{(-1)^1+5^1} + \frac{(-1)^3+5^3}{(-1)^2+5^2} + \frac{(-1)^4+5^4}{(-1)^3+5^3}$$

$$= \frac{1+25}{-1+5} + \frac{(-1)+125}{-1+25} + \frac{1+5^4}{-1+5^3}$$

common ratio

$$r = \frac{-1+5^3}{1+5^2} \times \frac{-1+5}{1+5^2}$$

$$= \frac{124}{26} \times \frac{4}{26}$$

$$= 124/169$$

$|r| < 1$

convergent.

$$\frac{1+5^4}{-1+5^3} \times \frac{1+5^2}{-1+5^3}$$

$$\frac{626}{124} \times \frac{26}{124}$$

$$=$$

$$15376$$

$$\frac{(-1)^{n+1+1-1}+5^{n+1+1-1}}{(-1)^{n+1-1}+5^{n+1-1}}$$

$$= \frac{(-1)^{n-1}\cdot(-1)^2+(5)^{n-1}(5)^2}{(-1)^{n-1}+(5)^{n-1}}$$

$$\frac{(-1)^{n-1}\cdot 1+(5)^{n-1}\cdot 5^2}{(-1)^{n-1}+5^{n-1}}$$

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Question No. 2	10 Points

Identify the type of series $\sum_{n=1}^{\infty} \frac{\sqrt{n+1}+\sqrt{n}}{\sqrt{n^2+n}}$ and find the sum(if possible).

Expanding.

$$= \frac{\sqrt{1+1}+\sqrt{1}}{\sqrt{(1)^2+1}} + \frac{\sqrt{2+1}+\sqrt{2}}{\sqrt{(2)^2+2}} + \frac{\sqrt{3+1}+\sqrt{3}}{\sqrt{(3)^2+3}} + \dots + \frac{\sqrt{n+1}+\sqrt{n}}{\sqrt{n^2+n}}$$

$$= \frac{\sqrt{2}+\sqrt{1}}{\sqrt{2}} + \frac{\sqrt{3}+\sqrt{2}}{\sqrt{6}} + \frac{\sqrt{4}+\sqrt{3}}{\sqrt{9+3}} + \dots$$

$$= \lim_{n\to\infty} \frac{\sqrt{n+1}+\sqrt{n}}{\sqrt{n^2+n}}$$

$$= \lim_{n\to\infty} \frac{\sqrt{\frac{n}{n^2}+\frac{1}{n^2}} + \sqrt{\frac{n}{n^2}}}{\sqrt{\frac{n^2}{n^2}+\frac{n}{n^2}}}$$

$$= \lim_{n\to\infty} \frac{\sqrt{\frac{1}{n}+\frac{1}{n^2}} + \sqrt{\frac{1}{n}}}{\sqrt{1+\frac{1}{n}}}$$

$$= \frac{\sqrt{\frac{1}{\infty}+\frac{1}{(\infty)^2}} + \sqrt{\frac{1}{\infty}}}{\sqrt{1+\frac{1}{\infty}}}$$

$$= \frac{\sqrt{0+0}+\sqrt{0}}{\sqrt{1+0}}$$

$$= \frac{0+0}{\sqrt{1}}$$

$$= 0 \quad (\text{covergent})$$

Sum is 0.

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Question No. 3	10 Points

Apply integral test to determine the convergence or divergence of the series $\sum_{n=1}^{\infty} \frac{\ln n}{\sqrt{n}}$.

$f(x) = \dfrac{\ln x}{\sqrt{x}}$

(i) $f(x)$ is positive for $[1, \infty)$ ($\sqrt{x}$ is not giving -ve value)

(ii) $f(x)$ is continuous. for $[1, \infty)$ because no value from domain gives ∞ v.

(iii) $f'(x) < 0$

$$f'(x) = \frac{(x)^{1/2} \cdot 1/x - \ln x \cdot \frac{1}{2\sqrt{x}}}{x}$$

$$= \frac{\frac{\sqrt{x}}{x} - \frac{\ln x}{2\sqrt{x}}}{x}$$

$$= \frac{\frac{1}{\sqrt{x}} - \frac{\ln x}{2\sqrt{x}}}{x}$$

$f'(x) < 0$

$$\frac{1/\sqrt{x} - \frac{\ln x}{2\sqrt{x}}}{x} < 0$$

$$= \frac{1}{\sqrt{x}} - \frac{\ln x}{2\sqrt{x}} < 0$$

$$= \frac{2 - \ln x}{2\sqrt{x}} < 0 \Rightarrow 2 - \ln x < 0$$

$$2 < \ln x$$

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Question No. 4	10 Points

Apply limit comparison test to determine the convergence or divergence of the series $\sum_{n=1}^{\infty} \frac{2^n+3^n}{3^n+4^n}$.

$a_n = \frac{2^n+3^n}{3^n+4^n}$

$b_n = \frac{3^n}{4^n} \Rightarrow \left(\frac{3}{4}\right)^n \Rightarrow \frac{3}{4} + \frac{3^2}{4^2} + \frac{3^3}{4^3} + \dots + \frac{3^n}{4^n}$

so $r = 3/4$ and $|r| < 1$ so b_n is convergent.

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \frac{\frac{2^n+3^n}{3^n+4^n}}{\frac{3^n}{4^n}}$$

$$= \frac{2^n+3^n}{3^n+4^n} \cdot \frac{4^n}{3^n}$$

$$= \frac{\cancel{3^n}\left[2^n/3^n+1\right]}{\cancel{3^n}\left[1+4^n/3^n\right]} \cdot \frac{4^n}{3^n}$$

$$= \frac{4^n\left[2^n/3^n+1\right]}{3^n\left[4^n/3^n+1\right]}$$

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$\frac{2}{3} + \frac{2^2}{3^2} + \frac{2^3}{3^3}$ $\left(\frac{2}{3}\right)^n + \left(\frac{3}{4}\right)^n$ $\frac{4}{3} + \frac{4^2}{3^2} + \frac{4^3}{3^3}$ $\frac{3}{4} + \frac{3^2}{4^2} + \frac{3^3}{4^3}$

$\frac{3}{4}$ $\frac{3^3}{4^3}$ $\frac{4^2}{3^2}$ $\frac{4^3}{3^3} \cdot \frac{3^2}{4^2}$ $\frac{4}{3}$

Question No. 5	10 Points

Apply root test and ratio test to determine whether the series $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{4^n}{(3n)^n}$ converges absolutely or not? justify your answer.

Apply root test

$$a_n = (-1)^{n+1} \frac{4^n}{(3n)^n}$$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \left| (-1)^{n+1} \frac{4^n}{(3n)^n} \right|^{1/n}$$

$$= \lim_{n\to\infty} \left[(1)^n \cdot (1)^1 \frac{4^n}{(3n)^n} \right]^{1/n}$$

$$= \lim_{n\to\infty} \left[(1)^{\cancel{n} \times \frac{1}{\cancel{n}}} (1)^{(1)/n} \frac{4^{\cancel{n} \times \frac{1}{\cancel{n}}}}{(3n)^{\cancel{n} \times \frac{1}{\cancel{n}}}} \right]$$

$$= \lim_{n\to\infty} \left[(1)^{1/n} \cdot \frac{4}{3n} \right]$$

$$= \frac{4}{3} \cdot \left[\lim_{n\to\infty} \frac{(1)^{1/n}}{n} \right]$$

$\therefore (x)^{1/n}$

Apply limit

$$= \frac{4}{3} \cdot \left[\frac{(1)^{1/\infty}}{\infty} \right]$$

$$= \frac{4}{3} \cdot \left[\frac{(1)^{1/0}}{1/0} \right]$$

$$= \frac{4}{3} \cdot \left[1/\tfrac{1}{0} \right]$$

$$= \frac{4}{3} \cdot 0$$

$$= 0$$

As $0 < 1$ So series converges absolutely.

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