CS1005-Discrete Structures

Saturday, November 05, 2022 (Solution)

Course Instructors

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Serial No:

2nd Mid Term Exam

Total Time: 1 Hour

Total Marks: 50

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Signature of Invigilator

Roll No	Section	Signature

DO NOT OPEN THE QUESTION BOOK OR START UNTIL INSTRUCTED.

Instructions:

1. Verify at the start of the exam that you have a total of XXX questions printed on XXX pages including this title page.
2. Attempt all questions on the question-book and in the given order.
3. The exam is closed books, closed notes. Please see that the area in your threshold is free of any material classified as 'useful in the paper' or else there may a charge of cheating.
4. Read the questions carefully for clarity of context and understanding of the meaning and make assumptions wherever required, for neither the invigilator will address your queries, nor the teacher/examiner will come to the examination hall for any assistance.
5. Fit in all your answers in the provided space. You may use extra space on the last page if required. If you do so, clearly mark the question/part number on that page to avoid confusion.
6. Use only your own stationery and calculator. If you do not have your own calculator, use manual calculations.
7. Use only permanent ink-pens. Only the questions attempted with permanent ink-pens will be considered. Any part of the paper done in lead pencil cannot be claimed for checking/rechecking.

	Q-1	Q-2	Q-3	Q-4	Q-5	Q-6	Total
Total Marks	9	9	8	8	8	8	50
Marks Obtained							

Vetted By: ______________________________ **Vetter Signature:** ______________________

University Answer Sheet Required: No ☐ Yes ☐

Question # 1: [9 Points]:

The probability of a car repair being on time is 0.40. The probability of a car repair being satisfactory is 0.50. The probability that a car repair is neither satisfactory nor on time is 0.25. What is the probability of a repair being satisfactory and on time?

Solution:

Question 7:- $P(A) = 0.40$, $P(B) = 0.50$, $P((A \cup B)^c) = 0.25$.

$P(A \cup B) = 1 - P((A \cup B)^c) = 1 - 0.25 = 0.75$

B = Satisfactory

A = on time.

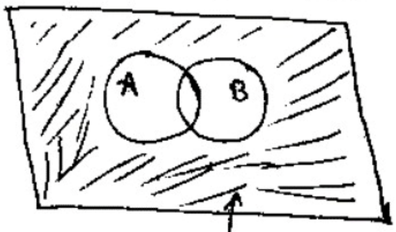
Since we know that

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

$\Rightarrow$ $P(A \cap B) = P(A) + P(B) - P(A \cup B)$.

$= 0.40 + 0.50 - 0.75 =$

$P(A \cap B) = 0.90 - 0.75 = 0.15$



$P((A \cup B)^c)$

Question # 2: [9 points]:

Question # 2: We have a bag containing 4 yellow, 5 green and 6 orange candies. We draw two candies without replacement. Find the probability of getting both candies green.

Solution:

Example 1: We have a bag containing 4 yellow, 5 green and 6 orange candies. We draw two candies without replacement. Find the probability of getting both candies green.

Solution: Clearly n(S) = 4 + 5 + 6 = 15

Let A be the event of drawing a green candy first.

Then P (A) = 5/15

Now since we are not replacing back, thus, number of green candies left in bag now is 4 and total number of candies is 14.

Let B be the vent of drawing a green candy again.

Then P (B|A) = 4/14.

Thus the probability of getting both candies green = P (A & B)

= P (A) * P (B|A)

= 5/15 * 4/14

= 1/3 * 2/7 = 2/21

Question # 3: [8 points]:

Define R={ (x,y) ∈ ZxZ: x-y is divisible by 3}. Prove that this relation has partial order relation.

Solution:

Reflexive:

Let a ∈ Z, a-a=0/3=0,
so (a,a) ∈ R is reflexive.

Symmetric:

Let a,b ∈ Z such that (a,b) ∈ R. So, a-b is divisible by 3.
a-b=3k [for some integer k]
if (a.b) ∈ R then, (b,a) ∈R.
b-a=-3k=> -(a-b)=3(-k) [-(a-b)=> -a+b=>b-a]
which is divisible by 3. So (b,a) ∈ R. So it is symmetric.

Transitive:

Let a,b ,c ∈ Z such that (a,b) ∈ R and (b,c) ∈ R
if (a.b) ∈ R and (b,c) ∈ R then, (a,c) ∈R.
So, a-b and b-c are divisible by 3.

- ⇨ a-b=3k & b-c=3s [for some integer k & s]
- ⇨ a-c= (a-b) +(b-c)
- ⇨ = 3k + 3s= 3(k+s)
- ⇨ a-c=3t

which is divisible by 3. So it has transitive property as well. It is not a partial order relation.

Question # 4: [8 Points]

Given A = {2,3,4,5,6} and B = {4,16,23,46}, a ∈ A, b ∈ B, find the set of all ordered pairs of A and B.
Define a relation R ={(a,b) | a ∈ A, b ∈ B and (a,b) ∈ AxB | a^2 < b}?
Define a relation S ={(a,b) | a ∈ A, b ∈ B and (a,b) ∈ AxB | a^2 /b}?

Solution:

AxB= {(2,4), (2,16), (2,23), (2,46), (3,4), (3,16), (3,23), (3,46), (4,4), (4,16), (4,23), (4,46), (5,4), (5,16), (5,23), (5,46), (6,4), (6,16), (6,23), (6,46)}

Define a relation R ={(a,b) | a ∈ A, b ∈ B and (a,b) ∈ AxB | a^2 < b}?

As 2^2 < 16,23 and 46, 3^2 < 16,23 and 46, 4^2 < 23 and 46. We have the set of ordered pairs such that a^2 < b is:

R= {(2, 16), (2, 23),(2,46), (3, 16), (3, 23),(3,46), (4, 23),(4,46),(5,46),(6,46)}

Define a relation S ={(a,b) | a ∈ A, b ∈ B and (a,b) ∈ AxB | a^2/b}?

S= {(2,4), (4,4), (4,16), (6,4)}

Question # 5: [5+3=8]

a) Consider the set A={1,2,3} with the relations R={(1,1),(1,3),(2,1), (2,2),(3,1)} and S={(1,2),(2,1), (2,2),(3,3)}. Calculate the composition R∘S in matrix form. Finaly convert the matrix into listing method (e.g. order pair).

Solution.

The relations R and S are represented by the following matrices:

$$M_R = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \quad M_S = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

To find the composition of relations $R \circ S$, we multiply the matrices M_S and M_R :

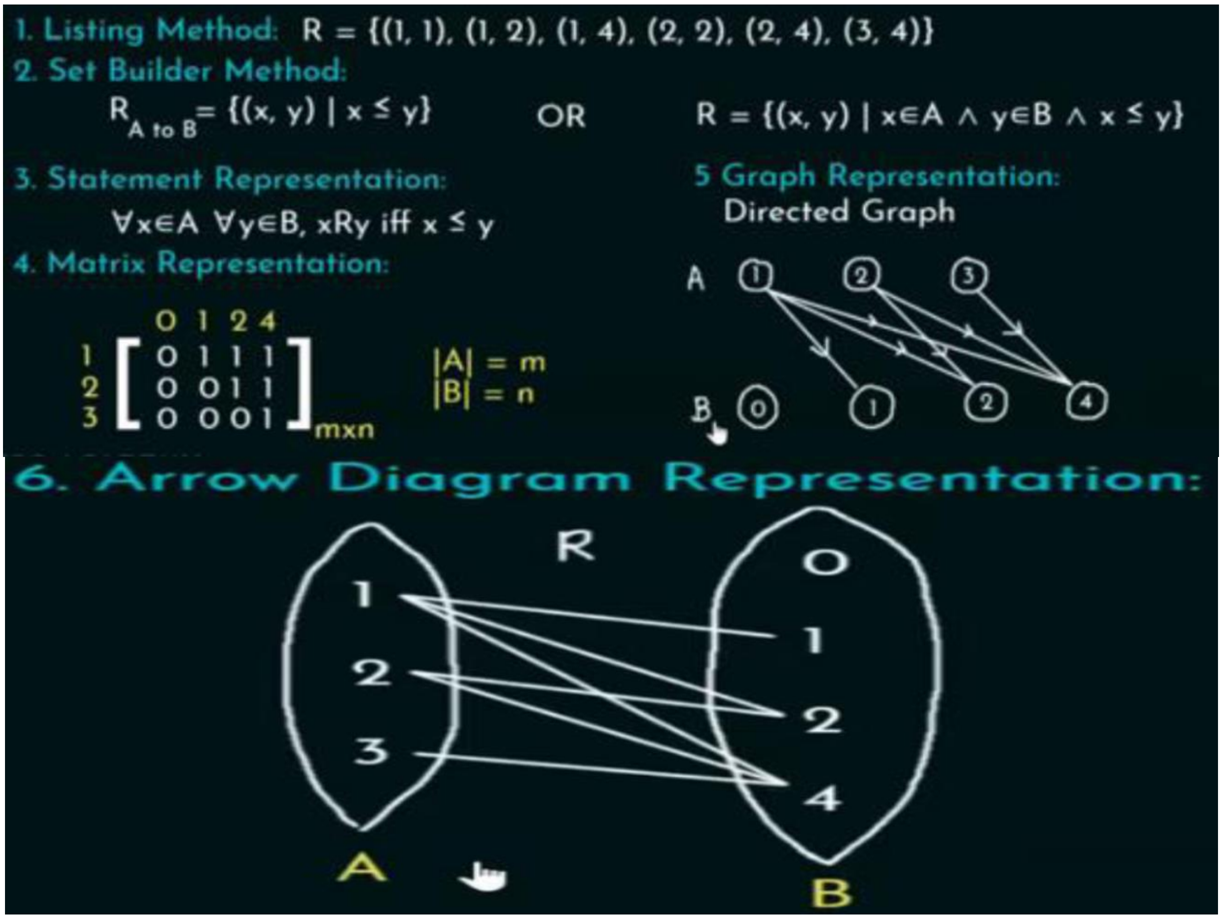
$$M_{R \circ S} = M_S \times M_R = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0+1+0 & 0+1+0 & 0+0+0 \\ 1+1+0 & 0+1+0 & 1+0+0 \\ 0+0+1 & 0+0+0 & 0+0+0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}.$$

We used here the Boolean algebra when making the addition and multiplication operations.

Hence, the composition of relations $R \circ S$ is given by

$$R \circ S = \{(1,1),(1,2),(2,1),(2,2),(2,3),(3,1)\}.$$

b) Let say we have two sets A and B A={1,2,3} and B={0,1,2,4}, there are multiple ways to represent a relation. Let say we want to represent a relation R which consist of all order pairs (x,y) where xεA and yεB and x≤y. Show all representaitons of given relation R.



Question # 6: [5+3=8 Points]

a) Let R be the relation on the set of ordered pairs of positive integers such that ((a,b), (c,d)) ε R if and only if ad=bc. Show that R is an equivalence relation.

Solution: A is the set of ordered pairs of positive integers.

$A = \{(1,1), (1,2), (1,3), \ldots\}$

$R = \{((a,b),(c,d)) \mid ad = bc\}$

(i) Reflexivity:

Original - $\forall a \in A\ ((a,a) \in R)$

New - $\forall (a,b) \in A\ (((a,b),(a,b)) \in R)$

$((a,b),(a,b)) \in R$ means $ab = ba$ which is true.

Therefore, R is reflexive.

(ii) Symmetry:

Original - $\forall a,b \in A\ ((a,b) \in R \rightarrow (b,a) \in R)$

New - $\forall (a,b),(c,d) \in A\ (((a,b),(c,d)) \in R \rightarrow ((c,d),(a,b)) \in R)$

$((a,b),(c,d)) \in R$ means $ad = bc \equiv da = cb \equiv cb = da$

Therefore, $((c,d),(a,b)) \in R$

Hence, R is symmetric.

(iii) Transitivity:

Original - $\forall a,b,c \in A\ (((a,b) \in R \land (b,c) \in R) \rightarrow (a,c) \in R)$

New - $\forall (a,b),(c,d),(e,f) \in A\ ((((a,b),(c,d)) \in R \land ((c,d),(e,f)) \in R) \rightarrow ((a,b),(e,f)) \in R)$

$((a,b),(c,d)) \in R$ means $ad = bc$ — 1

$((c,d),(e,f)) \in R$ means $cf = de$ — 2

From 1 $a = \frac{bc}{d}$ — 3 From 2 $f = \frac{de}{c}$ — 4

multiply 3 and 4

$af = \frac{b\cancel{c}}{\cancel{d}} \cdot \frac{\cancel{d}e}{\cancel{c}} = \boxed{be}$

$af = be$ $((a,b),(e,f)) \in R$

Therefore, R is transitive.

Hence, R is an equivalence relation.

b) Answer the following by considering congruence relation where notation is a ≡ b (mod m) (Yes/Not)?

Question	Yes/Not	Reason why or why not?
15 ≡ 3 (mod 12)	Yes	15-3 is divisible by 12
10 ≡ -2 (mod 12)	Yes	10-(-2) is divisible by 12
10 ≡ 2 (mod 6)	No	10-2 is not divisible by 6
2 ≡ -3 (mod 15)	No	2-(-3) is not divisible by 15
39 ≡ 3 (mod 9)	Yes	39-3 is divisible by 9
39 ≡ 12 (mod 9)	Yes	39-12 is divisible by 9