National University

of Computer and Emerging Sciences Chiniot-Faisalabad Campus

EE1005 – Digital Logic Design
Quiz# 2

Instructor: Muhammad Adeel Tahir
Sections: BCS-2F
Time: 60 Minutes
Name: ______________________________

Roll No: ______________________________ **Total Marks: 50**

Instructions:
- Read each question carefully; marks will be deducted for not meeting the requirements.
- **Scientific calculators** are **not permitted** during the quiz.
- Marks for each question are indicated alongside the question.

Question 1: Solve the following parts of the questions carefully.	**[12 marks]**

a. Convert the sequence from 60_{10} , 61_{10} to Gray code. Show proper working or no marks will be given **(2+2=4)**

Solution: (this part can be skipped since direct answers with valid calculations can also be shown, this is only for understanding)
Conversion Steps
1. **Convert to binary:** Convert each decimal number to its standard binary representation.
   - 60 (decimal) = 111100 (binary)
   - 61 (decimal) = 111101 (binary)
   - 62 (decimal) = 111110 (binary)
   - 63 (decimal) = 111111 (binary)
2. **XOR with right-shifted version:**
   - Right-shift each binary number by one position (dropping the rightmost bit).
   - Perform an XOR operation between the original binary number and its right-shifted version.

Calculations:
- **60:**
  - 111100 (original)
  - 011110 (right-shifted)
  - **100010 (Gray code)**
- **61:**
  - 111101 (original)
  - 011110 (right-shifted)
  - **100011 (Gray code)**

The Gray code representations for the sequence 60, 61, 62, 63 are:
- 60: 100010
- 61: 100011

b. Convert the following into BCD code and add: 295 + 157 **(2)**

Solution:

0010	1001	0101
0001	0101	0111
0011	1110	1100
	0110	0110
0100	0101	0010

c. In an 8-bit two's-complement system, what decimal number does the bit pattern **10000111** represent? Show proper steps in finding the actual decimal number. **(2)**

Solution:

1. Check the Sign Bit:

- If the first bit is '1', the number is negative.
- If the first bit is '0', the number is positive.

2. If Negative:

- Flip all the bits (0s become 1s, 1s become 0s).
- Add 1 to the result.
- Put a negative sign in front of the converted decimal number.

In the given question:

1. **Sign Bit:** The first bit is '1', meaning negative.
2. **Flip Bits:** 10000111 becomes 01111000
3. **Add 1:** 01111000 + 1 = 01111001
4. **Convert to Decimal:** 01111001 equals 121
5. **Add negative sign:** The result is -121

d. One of the following bit patterns is valid BCD (binary-coded decimal), but the other one is not, Which one is not valid? For credit to be given, you must give a correct reason. **(1)**

1. **100110110100**
2. **100100111000**

1. 100110110100: If we break this down into groups of four, we get 1001 1011 0100. The second group, "1011", is not a valid BCD representation.

2. 100100111000: If we break this into groups of four, we get 1001 0011 1000. All of these groups represent valid BCD digits.

What number does the valid bit pattern from part (d) represent? Give your answer in base ten. **(1)**

938_{10}

e. The *ten-bit* Gray code for **35310** is **0111010001**. Explain briefly but precisely why it cannot be true that **0111010100** is the ten-bit Gray code for 35410 **also** calculate gray code for 35410. **(2)**

Reason:

- 0111010001 represents 353 in decimal.
- 0111010100 is not a valid Gray code for 354 in decimal because it does not satisfy the property of Gray codes, where adjacent numbers differ by only one bit.

Gray code for 35410: 0111010011

Question 2: Solve the following problems, show proper working.	[15 marks]

a. Using 10's complement. subtract 72532 - 3250. **(2)**

Solution

$$
\begin{aligned}
M &= \ 72532 \\
\text{10's complement of } N &= +\ \underline{96750} \\
\text{Sum} &= \ 169282 \\
\text{Discard end carry } 10^5 &= -\underline{100000} \\
\textit{Answer} &= \ 69282
\end{aligned}
$$

b. Given the two binary numbers **X = 1010100** and **Y = 1000011** perform the subtraction

i) X - Y and **(2)**
ii) Y - X by using 2's complements. **(2)**

Solution

(a)

$$
\begin{aligned}
X &= \ 1010100 \\
\text{2's complement of } Y &= +\ \underline{0111101} \\
\text{Sum} &= \ 10010001 \\
\text{Discard end carry } 2^7 &= -\underline{10000000} \\
\textit{Answer: } X - Y &= \ 0010001
\end{aligned}
$$

(b)

$$
\begin{aligned}
Y &= \ 1000011 \\
\text{2's complement of } X &= +\ \underline{0101100} \\
\text{Sum} &= \ 1101111
\end{aligned}
$$

There is no end carry. Therefore, the answer is $Y - X = -(\text{2's complement of } 1101111) = -0010001$.

c. Simplify the given Boolean expressions, and specify the laws used for each step within brackets where the question does not specifically mention which laws to be used. **Note**:

If the laws used are not mentioned, the question will receive zero marks even if the answer is correct. **(4)**

i) $[\mathbf{A\overline{B}(C + BD) + \overline{A}\,\overline{B}]\ C}$

Answer: B'C

Solution

Step 1: Apply the distributive law to the terms within the brackets.

$$(A\overline{B}C + A\overline{B}BD + \overline{A}\,\overline{B})C$$

Step 2: Apply rule 8 ($\overline{B}B = 0$) to the second term within the parentheses.

$$(A\overline{B}C + A \cdot 0 \cdot D + \overline{A}\,\overline{B})C$$

Step 3: Apply rule 3 ($A \cdot 0 \cdot D = 0$) to the second term within the parentheses.

$$(A\overline{B}C + 0 + \overline{A}\,\overline{B})C$$

Step 4: Apply rule 1 (drop the 0) within the parentheses.

$$(A\overline{B}C + \overline{A}\,\overline{B})C$$

Step 5: Apply the distributive law.

$$A\overline{B}CC + \overline{A}\,\overline{B}C$$

Step 6: Apply rule 7 ($CC = C$) to the first term.

$$A\overline{B}C + \overline{A}\,\overline{B}C$$

Step 7: Factor out $\overline{B}C$.

$$\overline{B}C(A + \overline{A})$$

Step 8: Apply rule 6 ($A + \overline{A} = 1$).

$$\overline{B}C \cdot 1$$

Step 9: Apply rule 4 (drop the 1).

$$\overline{B}C$$

ii) Apply Demorgan's theorem to the following expressions: **(1+1+1=3)**

Apply DeMorgan's theorems to each expression:

(a) $\overline{(\overline{A + B}) + \overline{C}}$

(b) $\overline{(\overline{A} + B) + CD}$

(c) $\overline{(A + B)\overline{C}\,\overline{D} + E + \overline{F}}$

Solution

(a) $\overline{(\overline{A + B}) + \overline{C}} = (\overline{\overline{A + B}})\overline{\overline{C}} = (A + B)C$

(b) $\overline{(\overline{A} + B) + CD} = (\overline{\overline{A} + B})\overline{CD} = (\overline{\overline{A}}\,\overline{B})(\overline{C} + \overline{D}) = A\overline{B}(\overline{C} + \overline{D})$

(c) $\overline{(A + B)\overline{C}\,\overline{D} + E + \overline{F}} = (\overline{(A + B)\overline{C}\,\overline{D}})(\overline{E + \overline{F}}) = (\overline{A}\,\overline{B} + C + D)\overline{E}F$

iii) Taking the Boolean expression of **Exclusive OR Gate as starting point**. Use any rules or laws that are applicable to **develop an expression for the exclusive-NOR gate**. **(2)**

Solution

Start by complementing the exclusive-OR expression and then applying DeMorgan's theorems as follows:

$$\overline{A\overline{B} + \overline{A}B} = (\overline{A\overline{B}})(\overline{\overline{A}B}) = (\overline{A} + \overline{\overline{B}})(\overline{\overline{A}} + \overline{B}) = (\overline{A} + B)(A + \overline{B})$$

Next, apply the distributive law and rule 8 ($A \cdot \overline{A} = 0$).

$$(\overline{A} + B)(A + \overline{B}) = \overline{A}A + \overline{A}\,\overline{B} + AB + B\overline{B} = \overline{A}\,\overline{B} + AB$$

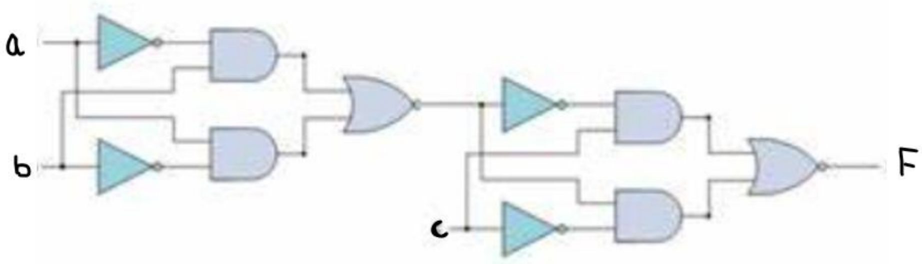
The final expression for the XNOR is $\overline{A}\,\overline{B} + AB$. Note that this expression equals 1 any time both variables are 0s or both variables are 1s.

Q3: Given the following circuit below, Solve the given parts carefully. [10]

a) Derive the Boolean expression from the following circuit diagram. Make sure you label each output carefully in neat and clean handwriting (**on the diagram)** to score the maximum marks. Write your final answer in the space provided. **(5)**

b) Draw the truth table of the derived equation. **(5)**

F = ______________________



Truth Table:

Q:4 Determine the output time diagram (waveform) for the XOR gate and for the XNOR gate, given the input waveforms, A and B, in Figure given below. (5)

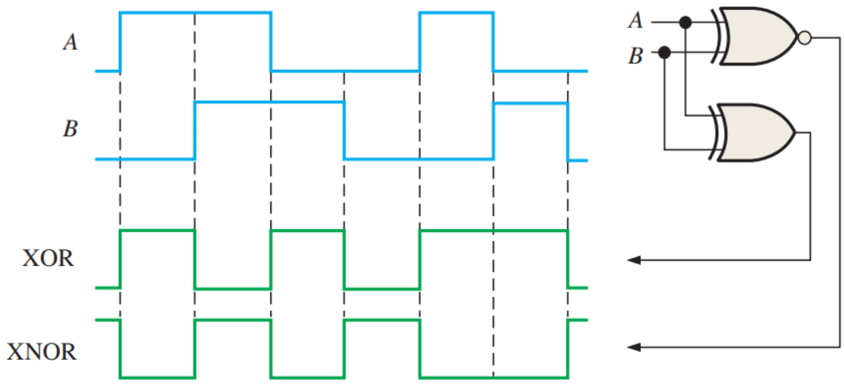


FIGURE 3–48

Solution

The output waveforms are shown in Figure 3–48. Notice that the XOR output is HIGH only when both inputs are at opposite levels. Notice that the XNOR output is HIGH only when both inputs are the same.

Q5: Convert the hexadecimal to base-7. Proper working must be shown. (2.5+2.5=5)

$(9A3.F)_{16}$

Solution:

Convert base 16 to decimal first.

Base 16 to decimal calculation:

$(9A3.F)_{16} = (9 \times 16^2) + (10 \times 16^1) + (3 \times 16^0) + (15 \times 16^{-1}) = (2467.9375)_{10}$

Convert base 10 to base 7:

(next page)

Base 16 to decimal calculation:

$(9A3.F)_{16} = (9 \times 16^2) + (10 \times 16^1) + (3 \times 16^0) + (15 \times 16^{-1}) = (2467.9375)_{10}$

Decimal to base 7 calculation:

Multiply the number with the destination base raised to the power of decimals of the result (up to 6 digits resolution):

$\text{floor}(2467.9375 \times 7^6) = 290350378$

Divide by the base to get the digits from the remainders:

Division	Quotient	Remainder (Digit)	Digit #
290350378/7	41478625	3	0
41478625/7	5925517	6	1
5925517/7	846502	3	2
846502/7	120928	6	3
120928/7	17275	3	4
17275/7	2467	6	5
2467/7	352	3	6
352/7	50	2	7
50/7	7	1	8
7/7	1	0	9
1/7	0	1	10

$= (10123.6363636363636363636364)_7$

Q:6 Choose the correct answer **[3 marks]**

1) Which of the following is a characteristic of Gray Code?

a) Only one bit changes at a time
b) It is a weighted code
c) It is a decimal to binary code
d) All of the above

Answer Key: a) Only one bit changes at a time **Explanation:** The Gray code is a binary numeral system where two successive values differ in only one bit. This is its key characteristic and what differentiates it from other binary codes.

2) What is the range of 8-bit signed binary numbers?
a) -128 to 127
b) 0 to 255
c) -256 to 255
d) -127 to 128

Answer Key: a) -128 to 127 Explanation: In an 8-bit signed binary number system, the range is from -128 (10000000 in binary) to 127 (01111111 in binary).

3) Which of the following gates is known as an inverter?
a) AND gate
b) OR gate
c) NOT gate
d) NAND gate

Answer Key: c) NOT gate **Explanation:** A NOT gate, also known as an inverter, is a logic gate that outputs true or ‘1’ if the input is false or ‘0’, and outputs false or ‘0’ if the input is true or ‘1’.