EE1005 Digital Logic Design

Wednesday, March 16, 2022

Course Instructor

Dr. Adil Zulfiqar, Engr. Umer Farooq and Engr. Muhammad Sajid Iqbal

Serial No:

1st Mid Term Exam

Total Time: 1 Hour

Total Marks: 40

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Signature of Invigilator

Roll No	Section	Signature

DO NOT OPEN THE QUESTION BOOK OR START UNTIL INSTRUCTED.

Instructions:

1. Verify at the start of the exam that you have a total of four (4) questions printed on six (6) pages including this title page.
2. Attempt all questions on the question-book and in the given order.
3. The exam is closed books, closed notes. Please see that the area in your threshold is free of any material classified as 'useful in the paper' or else there may a charge of cheating.
4. **The use of Scientific Calculator is not allowed.**
5. Read the questions carefully for clarity of context and understanding of meaning and make assumptions wherever required, for neither the invigilator will address your queries, nor the teacher/examiner will come to the examination hall for any assistance.
6. Fit in all your answers in the provided space. You may use extra space on the last page if required. If you do so, clearly mark question/part number on that page to avoid confusion.
7. Use only your own stationery and calculator. If you do not have your own calculator, use manual calculations.
8. Use only permanent ink-pens. Only the questions attempted with permanent ink-pens will be considered. Any part of paper done in lead pencil cannot be claimed for checking/rechecking.

	Q-1	Q-2	Q-3	Q-4	Total
Total Marks	5	5	20	10	**40**
Marks Obtained					

Vetted By: ____________________________ **Vetter Signature:** _____________________

University Answer Sheet Required: **No** X **Yes** ☐

Question Number 1 **(2 + 2 + 1 = 5 Marks)**

Given the two numbers

$A = (11101110)_2$ $B = (10111000)_2$

Find A + B if:

i. Both the numbers are signed and represented in 2's complement form.
ii. Both the numbers are unsigned and there is no limitation on number of bits.
iii. In both cases verify that your answers are correct by carrying the same calculations in decimal.

<table>
<tr><th>Numbers are Signed</th><th>Numbers are Unsigned</th></tr>
<tr><td>

$$\begin{array}{r|ccccccccc} \textbf{Carry} & 1 & 1 & 1 & 1 & 1 & & & & \\ \hline \textbf{A} & & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 \\ \textbf{B} & & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ \hline \textbf{A+B} & 1 & 1 & 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ \hline \end{array}$$

After ignoring the end carry

$A + B = (10100110)_2$

</td><td>

$$\begin{array}{r|ccccccccc} \textbf{Carry} & 1 & 1 & 1 & 1 & 1 & & & & \\ \hline \textbf{A} & & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 \\ \textbf{B} & & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ \hline \textbf{A+B} & 1 & 1 & 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ \hline \end{array}$$

In this case end carry will not be ignored

$A + B = (110100110)_2$

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<tr><th>Verification</th><th>Verification</th></tr>
<tr><td>

$A = (11101110)_2$
$B = (10111000)_2$

Both in A and B the MSB is 1, so both the numbers are negative.

2's complement of A is 00010010
2's complement of B is 01001000

$A = 16 + 2 = (18)_{10}$
$B = 64 + 8 = (72)_{10}$

Thus $A = (-18)_{10}$ and $B = (-72)_{10}$

$A + B = (-18) + (-72) = (-90)_{10}$

Now MSB in A+B is also 1, so
2's complement of A + B is 01011010

$A + B = 64 + 16 + 8 + 2 = 90$

Thus $A + B = (-90)_{10}$

Hence Proved.

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$A = (11101110)_2$
$B = (10111000)_2$

Converting to decimal

$A = 128 + 64 + 32 + 8 + 4 + 2 = (238)_{10}$
$B = 128 + 32 + 16 + 8 = (184)_{10}$

$A + B = 238 + 184 = (422)_{10}$

$A + B = (110100110)_2$

$A + B = 256 + 128 + 32 + 4 + 2 = (422)_{10}$

Hence Proved.

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Question Number 2 **(1.5 + 1 + 2.5 = 5 Marks)**

i. Write the gray codes for the following numbers.

a) $(55)_{10}$ b) $(00101101)_2$

Part (a)

By using the repeated division method
$(55)_{10} = (110111)_2$

Binary bit	Gray Bit	
1		1
1	1 + 1	0
0	0 + 1	1
1	1 + 0	1
1	1 + 1	0
1	1 + 1	0

Thus, the gray code of $(55)_{10}$ is 101100

Part (b)

Binary bit	Gray Bit	
0		0
0	0 + 0	0
1	1 + 0	1
0	0 + 1	1
1	1 + 0	1
1	1 + 1	0
0	0 + 1	1
1	1 + 0	1

Thus, the gray code of $(00101101)_2$ is 00111011

ii. Formulate a weighted code system for the given decimal digits by using 6, 4, 2, -3 weights.

Decimal Number	BCD Code	6, 4, 2, -3 Code
0	0000	0000
1	0001	0101
2	0010	0010
3	0011	1001 OR 0111
4	0100	0100
5	0101	1011
6	0110	1000 OR 0110
7	0111	1101
8	1000	1010
9	1001	1111

Question Number 3 **(3 + 3 + 6 + 6 + 2 = 20 Marks)**

Consider the following Boolean function given as sum of minterms.

$$F(w, x, y, z) = \Sigma(2, 3, 12, 13, 14, 15)$$

i. Write the Boolean expression in the form of inputs w, x, y, and z.
ii. By considering the fact that there are no limitations to inputs of a gate, identify the total number of gates and inputs to each gate required to implement the function F.
iii. Use Boolean algebra to simplify the function.
iv. Construct the truth table and logic diagram of the simplified function.
v. By considering the fact that there are no limitations to inputs of a gate, identify the total number of gates and inputs to each gate required to implement the simplified function F.

i. F = w'x'yz' + w'x'yz + wxy'z' + wxy'z + wxyz' + wxyz

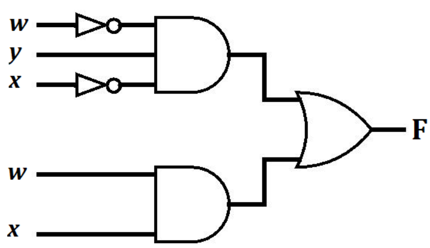
ii. We need 6 AND gates (each gate having 4 inputs), 1 OR gate with 6 inputs and 4 NOT gates to implement this function.

iii.
F = w'x'yz' + w'x'yz + wxy'z' + wxy'z + wxyz' + wxyz
F = w'x'y(z' + z) + wxy'(z' + z) + wxy(z' + z)
F = w'x'y + wxy' + wxy
F = w'x'y + wx(y + y')
F = w'x'y + wx

iv. Truth table of simplified function is

w	*x*	*y*	*z*	*w'*	*x'*	*w'x'y*	*wx*	*F*
0	0	0	0	1	1	0	0	0
0	0	0	1	1	1	0	0	0
0	0	1	0	1	1	1	0	1
0	0	1	1	1	1	1	0	1
0	1	0	0	1	0	0	0	0
0	1	0	1	1	0	0	0	0
0	1	1	0	1	0	0	0	0
0	1	1	1	1	0	0	0	0
1	0	0	0	0	1	0	0	0
1	0	0	1	0	1	0	0	0
1	0	1	0	0	1	0	0	0
1	0	1	1	0	1	0	0	0
1	1	0	0	0	0	0	1	1
1	1	0	1	0	0	0	1	1
1	1	1	0	0	0	0	1	1
1	1	1	1	0	0	0	1	1

Logic Diagram



v. We need 2 AND gates with 3 and 2 inputs respectively, 1 OR gate with 2 inputs and 2 NOT gates to implement the simplified function.

Question Number 4 **(4 + 6 = 10 Marks)**

i. Simplify the following Boolean Function by using K – Map.

$$F(w, x, y, z) = x'z + w'xy' + w(x'y + xy')$$

$F(w, x, y, z) = x'z + w'xy' + w(x'y + xy')$

$F(w, x, y, z) = x'z + w'xy' + wx'y + wxy'$

After completing the minterms we will get

$$F(w, x, y, z) = \Sigma(1, 3, 4, 5, 9, 10, 11, 12, 13)$$

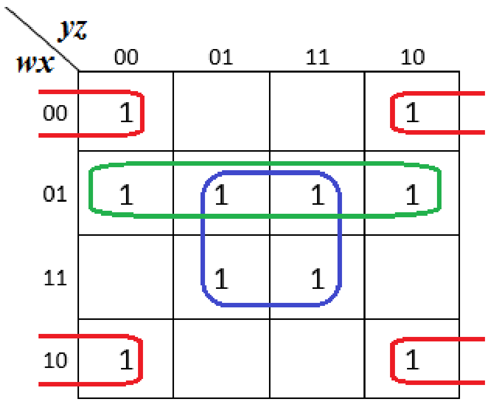
wx \ yz	00	01	11	10
00		1	1	
01	1	1		
11	1	1		
10		1	1	1

$$F = xy' + x'z + wx'y$$

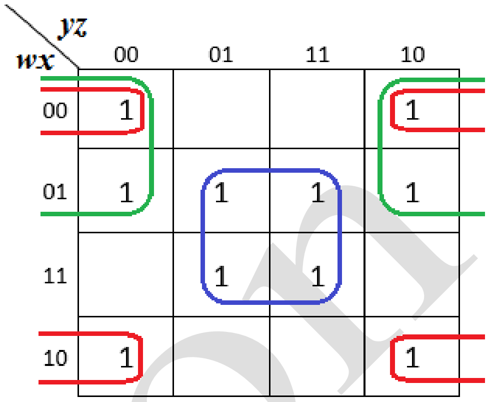
ii. Use the k-map to obtain all the prime implicants and essentials in the following function.

$$F(w, x, y, z) = \Sigma(0, 2, 4, 5, 6, 7, 8, 10, 13, 15)$$

The possible solutions are:



$$F = x'z' + xz + w'x$$



$$F = x'z' + xz + w'z'$$

From the above solutions we can notice that $x'z'$ and xz are essential prime implicants. All other terms are prime implicants.