QUESTION 1: (Objective Part) Answer the following short questions. Partially cooked answers will not be considered. **[2*5 = 10 points]**

1. Negate the given Statement in English only: "**If x is prime, then x is odd or x is 2**."
   x is prime but x is not odd, and x is not 2.

2. Rephrase the following propositions in the form "p if and only if q" in English only. "**If you read the newspaper every day, you will be informed, and conversely.**"
   You will be informed if and only if you read the newspaper every day.

3. Is it true or false **($\sim p \Leftrightarrow q) \equiv (p \Leftrightarrow \sim q$)**? **Yes**

4. Write the mathematical expression for **"Either Awais or Asif, but not both are cheating."**
   $p \oplus q$

5. Define whether the following statement is true or has any counterexample. If false, please state that counterexample. $\forall y \in \mathbb{R}$, **y has a prime divisor**.
   It is false since 1 is a counterexample. Note that 1 is not a prime number, so 1 itself does not have a prime divisor.

QUESTION 2: Solve the following: [5+5 = 10 points]

a. Prove (P$\leftrightarrow$ Q) is logically equal to (P$\rightarrow$ Q) $\wedge$(Q$\rightarrow$P) with the help of truth table. **[5]**

Solution: Truth table

P	Q	P($\leftrightarrow$)Q	P$\rightarrow$Q	Q$\rightarrow$P	(P$\rightarrow$ Q) $\wedge$(Q$\rightarrow$P)
T	T	T	T	T	T
T	F	F	F	T	F
F	T	F	T	F	F
F	F	T	T	T	T

b. Prove (P$\rightarrow$ Q) $\wedge$ (Q $\rightarrow$ r) $\rightarrow$ (P $\rightarrow$ R) is logically equal to **tautology** with the help of logic laws. (Please ensure that you mention the name of every logic law you apply at each step. Failure to do so will result in a deduction of marks for the question). **[5]**

Solution:

1. (P→ Q) Λ (Q → R) → (P → R)
2. (~P V Q) Λ (Q → R) → (P→R) Equivalence Law of Implication.
3. (~P V Q) Λ (~Q V R) → (P→R) Equivalence Law of Implication.
4. (~P V Q) Λ (~Q V R) → (~P V R) Equivalence Law of Implication.
5. ~((~P V Q) Λ (~Q V R)) V (~P V R) Equivalence Law of Implication.
6. ~(~P V Q) V ~(~Q V R) V (~P V R) DeMorgan's Law
7. (~(~P) Λ ~Q) V ~(~Q V R) V (~P V R) DeMorgan's Law
8. (~(~P) Λ ~Q) V (~(~Q) Λ ~R)) V (~P V R) DeMorgan's Law
9. (P Λ ~Q) V (~(~Q) Λ ~R)) V (~P V R) Double Negation Law
10. (P ∧ ~Q) V (Q ∧ ~R) V (~P V R) Double Negation Law
11. (P ∧ ~Q) V (Q ∧ ~R) V ~P V R Equivalence Law of Disjunction
12. (P ∧ ~Q) V (Q ∧ ~R) V ~P V R Equivalence Law of Disjunction
13. (~P V P) ∧ (~P V ~Q) V (~R V R) ∧ (Q V R) Distributive Law
14. T ∧ (~P V ~Q) V (~R V R) ∧ (Q V R) Negation Law
15. T ∧ (~P V ~Q) V T ∧ (Q V R) Negation Law
16. (~P V ~Q) V T ∧ (Q V R) Identity Law
17. (~P V ~Q) V (Q V R) Identity Law
18. ~P V ~Q V Q V R Equivalence Law of Disjunction
19. ~P V T V R Negation Law
20. ~P V T Universal Bound Law
21. T Universal Bound Law

This completes the proof, demonstrating that (P→ Q) ∧ (Q → r) → (P → R) is logically equal to tautology.

QUESTION 3:

Show that the premises "If it is a weekend, then Peter will go for hiking," "If it is not a weekend, then he will watch a movie," and "If he watches a movie, then he will eat popcorn" lead to the conclusion "If peter will not go for hiking, then he will eat popcorn". Solve it using inference rules also mentioned the name of used rule for each step. **: [5 points]**

Hint: For a start convert the first premise into contrapositive.

Solution:

Let p be the proposition "If it is a weekend", q the proposition "Peter will go for hiking", r the proposition "he will watch a movie", and s the proposition "he will eat popcorn" Then the premises are $p \to q$, $\neg p \to r$, and $r \to s$. The desired conclusion is $\neg q \to s$.

	Step	Reason
1	$p \to q$	Premise
2	$\neg q \to \neg p$	The contrapositive of (1) **Why does it need to add contrapositive?**
3	$\neg p \to r$	Premise
4	$\neg q \to r$	Hypothetical syllogism using (2) and (3)
5	$r \to s$	Premise
6	$\neg q \to s$	Hypothetical syllogism using (4) and (5)

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QUESTION 4: Consider the given Input/Output Table for circuit. **: [4+6 = 10 points]**

INPUT			**OUTPUT**
x1	**x2**	**x3**	**X**
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

a. Derive Boolean Expression according to above Input/Output Table. **[4]**

Boolean Expression: (~x1 A x2 A ~x3) ∨ ((~x2 ∨ x1) A x3)

OR

(x1 A ~x2 A x3) ∨ (x1 A x2 A x3)

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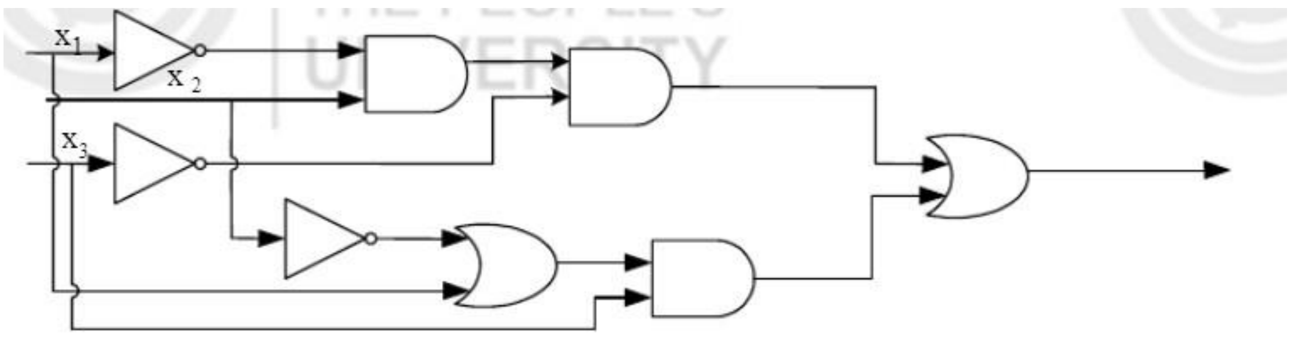
b. Now, use part **(a)** Boolean Expression to draw the Circuit Diagram. **[6]**



QUESTION 5: If x + y ≥ 2, where x and y are real numbers, then x ≥ 1 or y ≥ 1
By considering the given statement. You are required to Prove it by Contrapositive method.
[5 points]

Solution:
Let suppose.

P(x,y) = x + y ≥ 2
Q(x,y) = x ≥ 1 or y ≥ 1

The Conditional statement will be.

$$(p(x,y) \rightarrow Q(x,y))$$

Its Contrapositive will be

$$\sim Q(x,y) \rightarrow \sim P(x,y)$$

The ~Q(x,y) will be

~(x ≥ 1 or y ≥ 1) is equivalent to "x < 1 and y < 1

The ~P(x,y) will be

~(x + y ≥ 2") is equivalent to (x+ $y < 2$)

After taking the contraposition of the given statement,

if (x < 1 and y < 1) then (x+ $y < 2$)

Step 1: x < 1 and y < 1

Step 2: Adding the inequalities we get

x + y < 1 + 1
x + y < 2

The above equation shows that x + y β 2, which contradicts the statement x + y ≥ 2. Because the negation of the conclusion of the conditional statement implies the hypothesis is false, the original statement is true. Our proof by contrapositive is succeeded that if x + y ≥ 2, where x and y are real numbers, then x ≥ 1 or y ≥ 1.

QUESTION 6: Consider the following predicates in a family tree scenario: **: [15 points]**

- P(x, y): "x is the parent of y."
- Q(x, y): "x is the great-grandparent of y."
- R(x, y): "x is the sibling of y."

Let us apply these predicates to the universe of all persons, denoted as U.

a. Using the quantifiers listed above, write simple definitions for the following new predicates: **[6]**

- S(x, y): "x is a great-grandparent of y."

∀y (y ∈ U → Q(x, y))

- T(x, y): "x is an uncle or aunt of y."

∃z (R(z, x) A P(z, y))

b. In symbolic logic, formulate the following assertions using your concepts from part **(a): [6]**

- "At least one person in the set U is a great-grandparent to everyone."

∃x ∀y (y ∈ U → Q(x, y))

- "For every individual x in the set U, there exists another person y in the set U such that x is an uncle or aunt to y."

∀x ∃y (y ∈ U A T(x, y))

(c) Think of an example that shows the statement in part **(b)** that “might not always be true”? **[3]**

∃x ∈ U ∀y ∈ U Q(x, y)

GOOD LUCK 🙂