ECON 7201
Applied Econometrics
Assignment 1

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Due Date

September 18, 2025 at the start of class

Directions

Answer all questions. Submit both a PDF and Quarto file to the nexus assignment portal.

1. Git and GitHub

(a) Create a GitHub repository called **econ_3201** and connect it to RStudio.
(b) Create a new R project in this newly created directory called **assignment_1**. (Note, you do not have to click "Create git repository" as the directory is contained in a git enabled directory, i.e., **econ_3201**).
(c) Download the assignment PDF and Quarto file the **assignment_1** folder.
(d) Commit and push the changes to your **econ_3201** repository on GitHub.com.

2. LaTeX

LaTeX is useful for writing math equations and presents them in a neat and orderly way. To write in math mode, wrap your text in `$` for inline text use two `$`s for display (i.e., centered on the page). Some very useful functions include:

- Fractions:`\frac{}{}`, e.g. `$\frac{1}{2}$` gives $\frac{1}{2}$ and `$$\frac{1}{2}$$` gives:

$$\frac{1}{2}.$$

- Subscripts: `_` gives a subscript, e.g. `$x_1$` gives x_1. To include more than one term in the subscript, the items in the subscript must be enclosed by `{}`. E.g. `$x_{1,1}$` gives $x_{1,1}$. (Note that `$x_1,1$` gives $x_1, 1$)

- Exponents: `^`, e.g. `$x^2$` gives x^2. `^` can also be used for superscripts in other math functions, including summations and integrals.
- Aligned: `aligned` neatly aligns multiple lines of an equation. Align is useful when writing multiple steps to solving an equation. To use it in Quarto, write `$$\begin{aligned}...\end{aligned}`. The `&` is used to mark the point where the lines should be aligned. Use `\\` at the end of each line E.g. `$$\begin{aligned}`

```
x & = 3 + 5\\
& = 8
\end{aligned}$$
```

gives

$$\begin{aligned} x &= 3 + 5\\ &= 8 \end{aligned}$$

‘

- Summation: `\sum` gives the summation sign, i.e. $\sum$. To include subscripts, use `_` and to use superscripts use `^`, e.g. `$\sum_{i=1}^n$` gives $\sum_{i=1}^n$, which reads as the sum of i equals 1 to n.
- Integral: `\int` gives an integral, i.e. $\int$. To place a lower limit use `_` and to place an upper limit, use `^`, e.g. `$\int_{a}^{b}$` gives $\int_a^b$.
- Greek letters: `$\alpha, \beta, \gamma, \Gamma, \delta, \Delta, \epsilon, \varepsilon, \zeta, \eta,\sigma, \Sigma, \theta, \vartheta, \Theta, \iota, \kappa, \lambda, \Lambda, \mu$` gives $\alpha, \beta, \gamma, \Gamma, \delta, \Delta, \epsilon, \varepsilon, \zeta, \eta, \sigma, \Sigma, \theta, \vartheta, \Theta, \iota, \kappa, \lambda, \Lambda, \mu$. (See https://www.overleaf.com/learn/latex/List_of_Greek_letters_and_math_symbols)
- Accents: `\hat{}`, `\tilde{}`, and `\bar{}`are examples of accents in math mode. E.g. `\hat{Y}`, `\tilde{Y}`, and `\bar{Y}`gives $\hat{Y}$, $\tilde{Y}$, and $\bar{Y}$, respectively.
- Text: To include text in your equation, i.e. non italicized text, use `\text{}`, e.g. `$x=2\text{ if } y=1$` gives $x=2\text{ if } y=1$.
- Inequalities: Some mathematical expressions may be written as inequalities, rather than equations. For ‘less than’ and ‘greater than’, you can just use the symbol on your keyboard, i.e. `<` and `>`, respectively. For $\leq$, use `$\leq$` and for $\geq$, use `$\geq$`. An important note is that after writing a command, put a space after the command before writing the next term, otherwise you may get an error. E.g. To write $a \leq b$, write `$a\leq b$`, not `$a\leqb$`.

Re-write the following equations in LaTeX.

(a) $E(Y) = y_1p_1 + ... + y_kp_k = \sum_{i=1}^k y_ip_i$
(b) $\sigma_Y = Var(Y) = E[(Y - \mu_y)^2] = \sum_{i=1}^k (y_i - \mu_y)^2 p_i$
(c) $\hat{\beta} = \frac{\sum_{i=1}^n (y-y_i)(x-x_i)}{\sum_{i=1}^n (x-x_i)2}$

(d) $P(a \leq Y \leq b) = \int_a^b f_Y(y)dy$

(e) $\hat{g}(x) = \frac{\frac{1}{nh}\sum_{i=1}^{n} y_i k(\frac{x_i - x}{h})}{\frac{1}{nh}\sum_{i=1}^{n} k(\frac{x_i - x}{h})}$

3. R

3.1. Assignment

Note: When creating variables based on equation, separate each element in the equation with the appropriate arithmetic symbol. E.g., to compute $x(y-2)$ in R, you would have to type `x*(y-2)`. `x(y-2)`, with not arithmetic symbol between x and the left bracket would result in an error.

(a) In statistics, n is often used to denoted the sample size. Set the number of observations $n = 1000$.

```
n <- 1000
```

(b) Generate two random variables, $u_1 \sim U(0,1)$ and $u_2 \sim U(0,1)$ with $n/2 = 500$ observations. That is, create two variables that follow a uniform distribution between 0 and 1 that each have 500 observations. In R, we can create random uniform variables using the `runif(k,min,max)` function, where `k` is number of observations, `min` is the minimum value, and `max` is the maximum value. The default values for `min` and `max` are 0 and 1, respectively. Type `?runif` into your console to learn more.

```
u1 <- runif(n/2,0,1)
u2 <- runif(n/2,0,1)
```

(c) Generate two variables z_1 and z_2 that take on the following values:

$$z_1 = \sqrt{-2\ln(u_1)} \times \cos(2\pi u_2)$$

and

$$z_1 = \sqrt{-2\ln(u_1)} \times \sin(2\pi u_2).$$

In R, $\sqrt{}$ is computed using `sqrt()`, ln is computed using `log()`, cos is computed using `cos()`, and sin is computed using `sin()`.

```
z1 <- sqrt(-2*log(u1))*cos(2*pi*u2)
z2 <- sqrt(-2*log(u1))*sin(2*pi*u2)
```

(d) Generate a vector $z = [z_1, z_2]$

```
z <- c(z1,z2)
```

(e) Generate two variables μ (spelled mu) and σ (spelled sigma). Set $\mu = 5$ and $\sigma = 2$.

```
mu <- 5
sigma <-2
```

(f) Generate a variable $x = \mu + \sigma \times z$

```
x <- mu+sigma*z
```

(g) Calculate the mean of x, using `mean()` and the standard deviation of x using `sd()`.

```
mean(x)
```

```
[1] 5.040638
```

```
sd(x)
```

```
[1] 2.056363
```

(h) Use the following code to plot a histogram of x with the normal distribution curve.

```
hist(x,
     freq = FALSE,
     ylab = "Density",
     xlab = "$x$")

curve(dnorm(x, mean = mu, sd = sigma),
      col = "red", lwd = 2, add = TRUE)
```

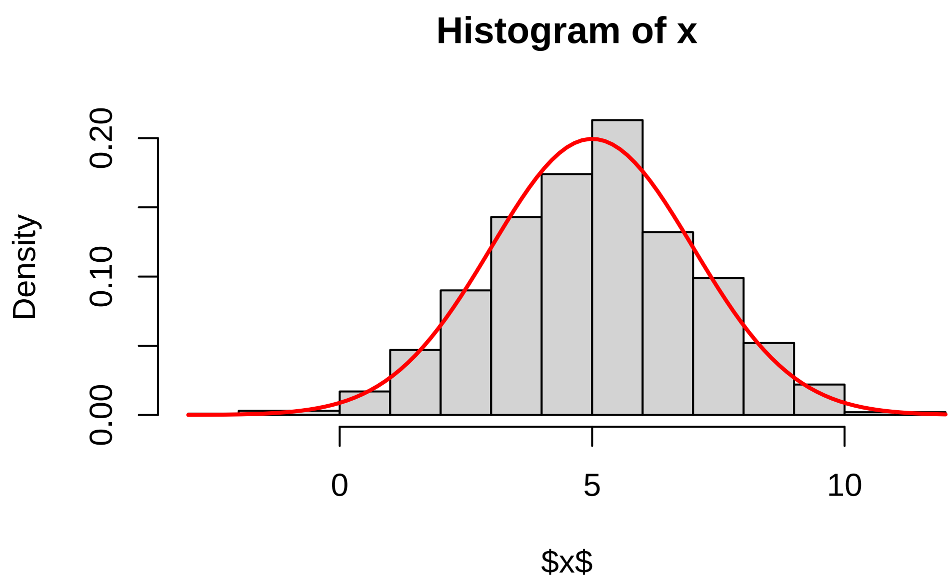


Figure 1: Histogram of x

3.2. Data frames and Indexing

A data frame in R is a table-like data structure used to store data in rows and columns, similar to a spreadsheet or a database table. It is one of the most commonly used structures for storing datasets in R.

Table 1 displays the total health expenditure by use of funds in Canada from 1975 to 2022. The data is stored in the `data.frame` called `df`.

```
cihi <- read.csv("hlthexp.csv")

df <- data.frame(Year = cihi$Year,
                 Hospitals = cihi$Hospitals,
                 Physicians = cihi$Physicians,
                 "Other Services" = cihi$Other.Institutions,
                 Dental = cihi$Other.Professionals..Dental.Services,
                 Vision = cihi$Other.Professionals..Vision.Care.Services,
                 "Other Professionals" = cihi$Other.Professionals..Other.Services,
                 check.names = FALSE)

knitr::kable(df,
             align = c(rep('c',7)),
             format.args = list(big.mark = ","))
```

Table 1: Total health expenditure by use of funds, in millions of current dollars, Canada, 1975 to 2022 (Source: CIHI National Health Expenditure Trends)

Year	Hospitals	Physicians	Other Services	Dental	Vision	Other Professionals
1,975	5,136.77	1,813.15	796.62	56.40	35.86	46.72
1,976	5,977.68	2,041.52	999.08	69.81	40.65	53.92
1,977	6,372.73	2,252.12	1,175.16	83.70	44.86	60.54
1,978	6,861.92	2,528.34	1,367.51	103.96	51.91	75.52
1,979	7,487.62	2,804.48	1,581.37	143.83	57.99	88.88
1,980	8,585.16	3,235.98	1,821.48	194.94	67.23	104.90
1,981	10,127.35	3,775.12	2,146.66	278.44	78.74	126.67
1,982	12,001.93	4,353.14	2,531.36	270.04	91.13	143.01
1,983	13,174.55	4,973.30	2,794.37	260.66	105.68	163.99
1,984	13,936.30	5,444.58	2,923.26	266.74	117.66	181.02
1,985	14,737.75	5,962.06	3,066.46	275.52	130.42	214.58
1,986	15,937.05	6,597.89	2,982.43	287.16	146.05	260.66
1,987	17,154.21	7,266.23	3,132.08	286.27	157.30	276.36
1,988	18,497.17	7,862.51	3,468.29	311.35	180.78	296.02
1,989	20,268.98	8,422.71	3,828.51	350.27	205.62	341.53
1,990	20,528.15	9,090.92	5,100.45	371.70	235.89	379.81
1,991	21,783.23	10,014.44	5,868.30	387.93	265.51	442.89
1,992	22,652.40	10,249.61	6,253.82	394.80	262.22	470.54
1,993	22,619.06	10,306.29	6,190.38	407.31	229.69	460.64
1,994	22,096.82	10,533.27	6,266.36	418.63	221.20	429.23
1,995	21,849.46	10,506.52	6,498.12	408.13	197.12	427.63
1,996	21,997.29	10,651.80	6,591.26	373.98	196.90	426.18
1,997	22,307.52	11,103.52	6,834.19	365.18	215.12	448.14
1,998	23,530.41	11,627.85	7,172.47	352.30	204.66	481.07
1,999	24,751.97	12,255.39	7,578.69	380.04	219.28	523.72
2,000	26,950.76	13,045.53	8,170.94	397.63	230.47	577.24
2,001	28,606.54	14,001.53	8,784.35	406.72	247.80	559.25
2,002	30,683.55	14,939.47	9,308.19	421.57	239.86	521.36
2,003	32,903.18	16,084.37	9,841.96	409.33	244.00	526.93
2,004	35,269.82	17,084.00	10,629.24	425.19	250.30	530.73
2,005	37,112.35	18,302.66	11,064.58	450.38	223.05	469.67
2,006	39,704.71	19,743.14	11,593.52	504.41	231.54	482.76
2,007	42,376.77	21,308.72	12,192.52	541.84	239.84	541.96
2,008	45,362.04	23,370.83	12,809.06	586.77	264.34	619.50
2,009	47,996.52	25,249.61	13,578.95	664.37	295.77	671.40
2,010	50,947.81	27,107.23	14,316.45	714.70	311.87	692.20
2,011	52,126.35	28,813.05	15,324.80	721.61	332.69	734.94

Table 1: Total health expenditure by use of funds, in millions of current dollars, Canada, 1975 to 2022 (Source: CIHI National Health Expenditure Trends)

Year	Hospitals	Physicians	Other Services	Dental	Vision	Other Professionals
2,012	53,299.96	29,801.63	15,923.80	759.13	353.62	782.67
2,013	54,954.28	31,202.28	16,386.15	762.36	358.08	730.08
2,014	56,123.22	32,490.79	16,966.03	782.00	389.71	685.88
2,015	57,352.33	33,886.08	18,313.73	821.42	430.46	1,179.18
2,016	58,168.97	35,283.98	18,809.91	875.86	461.42	1,355.90
2,017	60,356.12	36,490.87	19,665.65	918.62	484.33	1,491.51
2,018	62,896.86	37,494.64	20,548.31	961.17	517.89	1,614.12
2,019	65,034.33	38,914.04	21,446.58	1,018.36	557.19	1,729.01
2,020	67,221.53	37,288.46	23,675.08	896.76	513.22	1,711.94
2,021	69,663.71	41,479.50	25,678.66	922.86	559.07	1,906.92
2,022	73,778.17	44,195.30	28,095.86	991.82	584.06	2,047.50

(a) Determine if there are any missing values for the variable `Hospitals`.

```
is.na(df$Hospitals)
```

```
 [1] FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE
[13] FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE
[25] FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE
[37] FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE
```

(b) Add a variable called "Total Other Services" to the data frame `df`, where

$$\text{Total Other Services} = \text{Dental} + \text{Vision} + \text{Other Professional}.$$

To add a newly created variable to a data frame use the syntax `dataframe$varname <- expression`.

```
df$TotalOtherServices <- df$Dental+df$Vision+df$"Other Professionals"
```

(c) Are there any years for which Total Other Professionals

(d) Another way to add a variable to a data frame is to simply create a new data frame and append the new variable to it. Note: we can use the same data frame name. I.e., `df<-data.frame(df,newvarname = newvar)`. Add the variable "Prescription Drugs" to the `df` data frame using the append method, where presricption drugs is named "Prescribed.Drugs" in the `cihi` data.frame.

```r
df <- data.frame(df,"Prescription.Drugs"= cihi$Prescribed.Drugs)
```

(e) Using a single R command, determine the expenditure on hospitals in 1983.

```r
df$Hospitals[df$Year==1983]
```

```
[1] 13174.55
```

(f) Using a singe R command, list the expenditures by year for 2012-2022.

```r
df[df$Year>=2012 & df$Year<=2022,]
```

```
   Year Hospitals Physicians Other.Services  Dental Vision Other.Professionals
38 2012  53299.96   29801.63       15923.80  759.13 353.62              782.67
39 2013  54954.28   31202.28       16386.15  762.36 358.08              730.08
40 2014  56123.22   32490.79       16966.03  782.00 389.71              685.88
41 2015  57352.33   33886.08       18313.73  821.42 430.46             1179.18
42 2016  58168.97   35283.98       18809.91  875.86 461.42             1355.90
43 2017  60356.12   36490.87       19665.65  918.62 484.33             1491.51
44 2018  62896.86   37494.64       20548.31  961.17 517.89             1614.12
45 2019  65034.33   38914.04       21446.58 1018.36 557.19             1729.01
46 2020  67221.53   37288.46       23675.08  896.76 513.22             1711.94
47 2021  69663.71   41479.50       25678.66  922.86 559.07             1906.92
48 2022  73778.17   44195.30       28095.86  991.82 584.06             2047.50
   TotalOtherServices Prescription.Drugs
38            1895.42           12114.49
39            1850.52           12199.19
40            1857.59           12668.45
41            2431.06           13298.98
42            2693.18           13616.80
43            2894.46           13957.25
44            3093.18           14442.70
45            3304.56           14939.93
46            3121.92           15435.35
47            3388.85           16034.55
48            3623.38           17094.52
```

3.3 Other useful R commands.

Load the `mpg` dataset from the `ggplot2` package using `mpg <-ggplot2::mpg`. (Be sure to install the `gglot2` package before you start.)

```
library(ggplot2)
mpg <-ggplot2::mpg
```

(a) Subset the data to include only observations from 2008. Search `?subset` in the console.
(a) Calculate the maximum and minimum miles per gallon in city limits (`cty`). Search `?min` in the console.

```
#Creating data subset for 2008
mpg_2008 <- subset(mpg, year==2008)

max(mpg_2008$cty)
```

```
[1] 28
```

```
min(mpg_2008$cty)
```

```
[1] 9
```

(b) Estimate the average miles per gallon within city limits for cars produced in 2008 using the formula

$$\text{Average mpg} = \frac{\sum_{i=1}^{n} \text{cty}_i}{n}.$$

Recall that n is the number of observations. Search `?length` in the console.

```
#Avg_mpg
sum(mpg_2008$cty)
```

```
[1] 1954
```

```
length(mpg_2008$cty)
```

```
[1] 117
```

```r
sum(mpg_2008$cty)/length(mpg_2008$cty)
```

```
[1] 16.70085
```

(c) Estimate the average miles per gallon within city limits for cars produced in 2008 using the `mean()` function.

```r
mean(mpg_2008$cty)
```

```
[1] 16.70085
```

(d) Create a variable called `compact`, which takes a value of 1 if the vehichle is a compact and 0 otherwise. Search `?ifelse` in the console.

```r
mpg_2008$compact <- ifelse(mpg_2008$class == "compact", 1, 0)
```

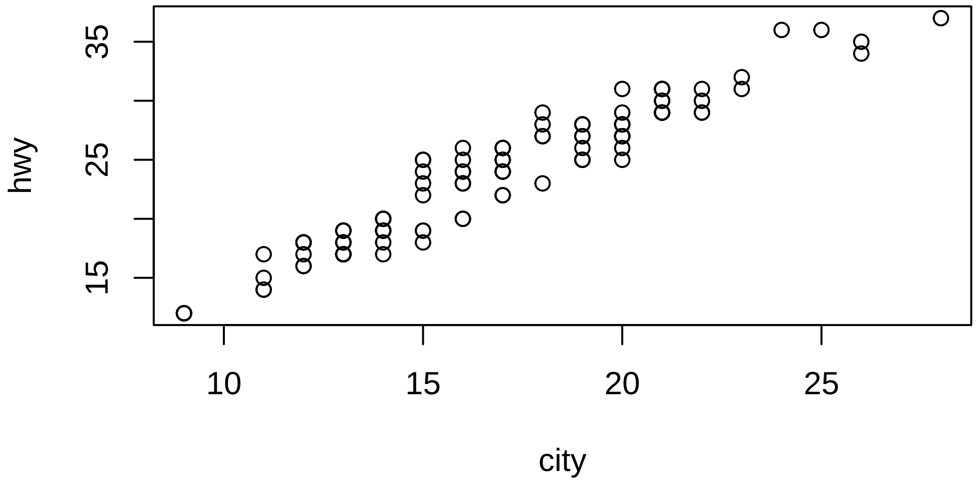
(e) Estimate the average miles per gallon within city limits for compact cars. (You may use whichever method you prefer).

```r
mean(mpg_2008$cty[mpg_2008$compact == 1])
```

```
[1] 20.54545
```

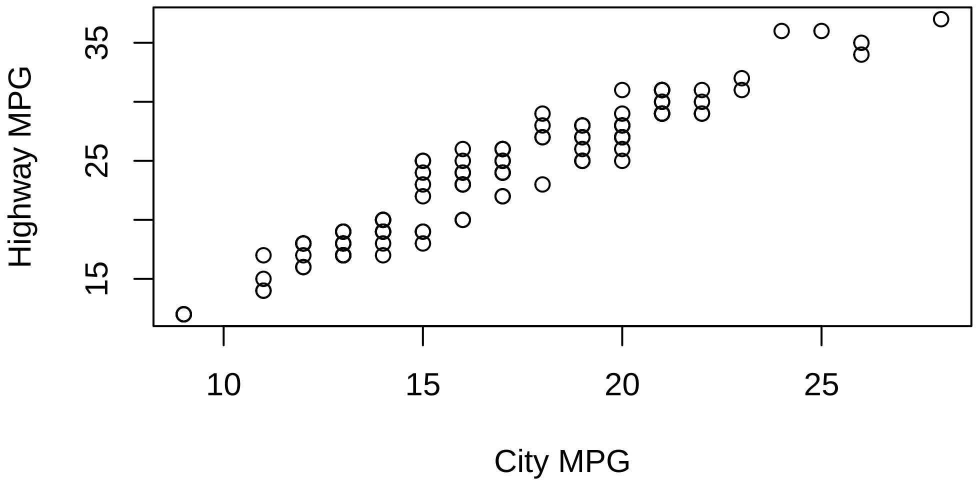
(f) Create a simple scatter plot with city mpg (`cty`) on the x-axis and highway mpg (`hwy`) on the y-axis. Search `?plot` and choose "Generic X-Y Plotting".

```r
city <- mpg_2008$cty
hwy <- mpg_2008$hwy
plot(x = city,
     y = hwy)
```



(i) Change the x-axis label using the option `xlab = "City MPG"` and change the y-axis label using the option `ylab = "Highway MPG"`.

```
plot(city, hwy,
     xlab = "City MPG",
     ylab = "Highway MPG")
```



(ii) Add the caption "City Versus Highway Fuel Efficiency (MPG)"

```
plot(city, hwy,
     xlab = "City MPG",
     ylab = "Highway MPG",
     main = "City Versus Highway Fuel Efficiency (MPG)",
     col = "green")
```

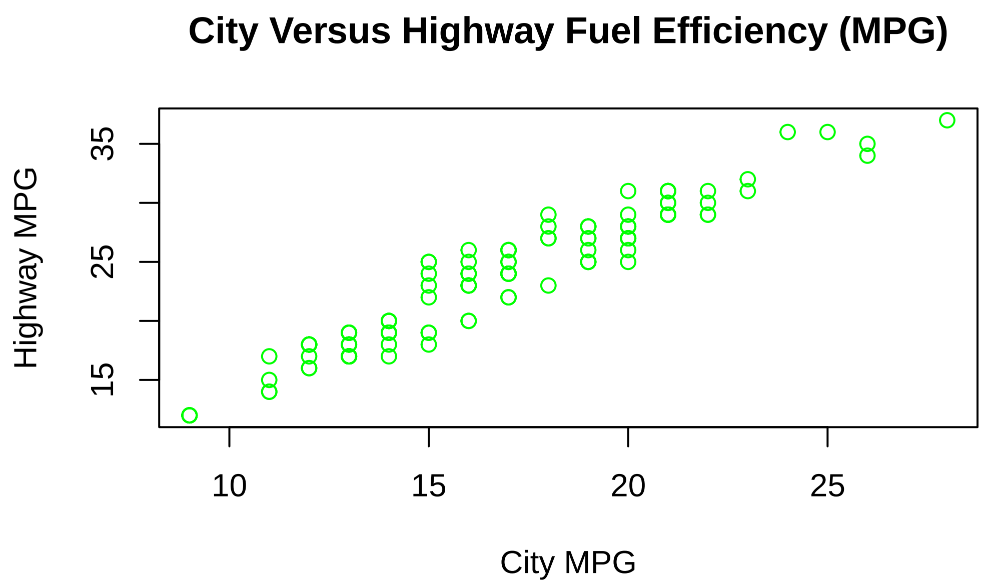


Figure 2

```
(iii) Cross reference the figure and add the text "Figure 1 shows the fuel efficiency
for city driving versus highway driving".
```

Figure 2 shows the fuel efficiency for city driving versus highway driving