Final Assignment
Summer 2024

Course Title: Differential and Integral Calculus
Course Code: MAT1101

<u>Submitted by:</u>
Student Name : Md. Tazminur Rahman Tanim

ID:242014124

Department of CSE
University of Liberal Arts Bangladesh (ULAB)

Answer to the question No: 1

(a)

(i) $\lim_{x \to -3} f(x)$ and $\lim_{x \to 0^+} f(x)$

From the graph

$\lim_{x \to -3} f(x) = 1$

As x approaches -3 from both sides the function approaches the value of 1

$\lim_{x \to 0^+} f(x) = -1$

As x approaches 0 from the positive side, the function approaches -1

(ii) $\lim_{x \to 2} f(x) = 1$

As x appraches 2 from both side the function value tends toward 1.

(iii) For $f(x)$ to be continuous at $x = 4$.

$\lim_{x \to 4^-} f(x)$ (the left hand limit) must exits

$\lim_{x \to 4^+} f(x)$ (the right hand limit) must exits.

The value of $f(4)$ must exits and equal to the limit from both sides.

From the graph,

$$\lim_{x \to 4^-} f(x) = 2$$

$$\lim_{x \to 4^+} f(x) = 2$$

and $f(4) = 2$

So, All limit and function value all agree.

So, $f(x)$ is continuous at $x=4$.

(iv) From the graph,

<u>vertical asymptote:</u> A vertical asymptote occurs where the function heads to infinity or negative infinity as x approaches a certain value. In this case, as $x \to 0^-$ $f(x)$ tends towards negative infinity, so vertical asymptote is at $x = 0$.

<u>Horizental asymptote:</u> As $x \to \infty$, $f(x)$ approaches a constant value, In this case, as $x \to \infty$, $f(x)$ approaches 1, so the horizental asymptote.

Answer to the ques. No: 1

(b) The function is

$$f(x) = \begin{cases} -2 ; & x < -1 \\ x^2 ; & -1 \le x < 1 \\ x ; & x \ge 1 \end{cases}$$

constant function for $x < -1$ where $f(x) = -2$
This a horizontal line at $y = -2$

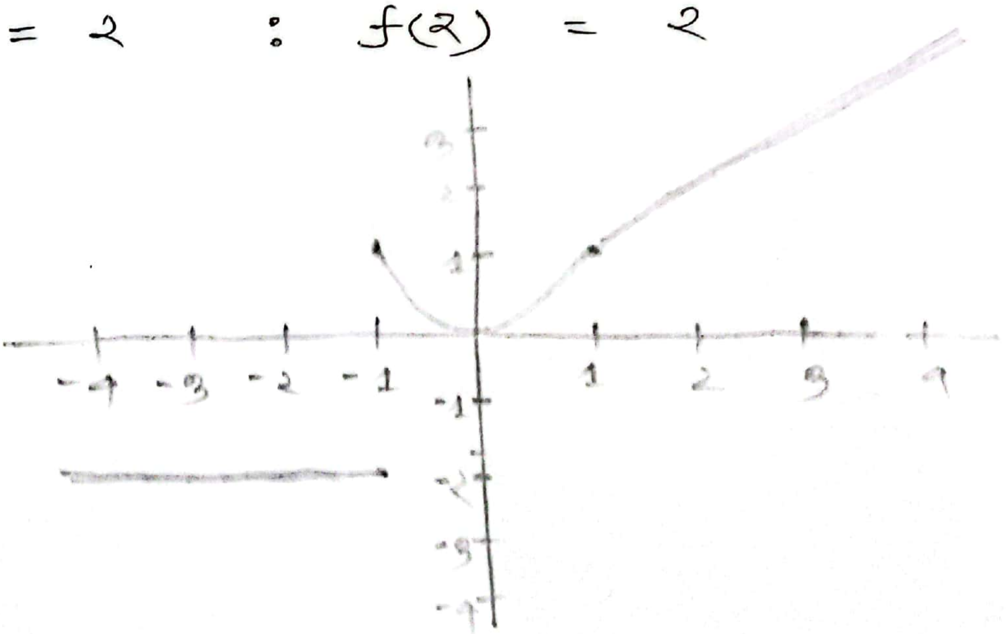
Quadratic function for $-1 \le x < 1$ where $f(x) = x^2$. This is a parabolic curve

Linear function for ~~x <~~ $x \ge 1$ where $f(x) = x$. This is a line with a slope of 1

For, $x = -2$: $f(-2) = -2$

$x = 0$: $f(0) = 0^2 = 0$

$x = 2$: $f(2) = 2$



Discontinuity:

There is a jump discontinuity at $x = -1$ because the function jump from -2 to 1.

Answer to the ques. No: 2

(i)

$$p(x) = 2x^3 - 3x^2 - 36x + 5$$

$$\Rightarrow p'(x) = 6x^2 - 6x - 36$$

$$\Rightarrow p''(x) = 12x - 6$$

Let's set $p'(x) = 0$ to find the critical points,

$$6x^2 - 6x - 36 = 0$$

$$\Rightarrow 6(x^2 - x - 6) = 0$$

$$\Rightarrow x^2 - x - 6 = 0$$

$$\Rightarrow x^2 - 3x + 2x - 6 = 0$$

~~$\Rightarrow x(x+$~~

$$\Rightarrow x(x-3) + 2(x-3) = 0$$

$$\Rightarrow (x-3)(x+2) = 0$$

$x + 2 = 0$ or, $x - 3 = 0$

$\Rightarrow x = -2$ $\quad\Rightarrow x = 3$

The critical point is $x = 3$, and $x = -2$

Ans

(ii) From (i) we got the critical points $x = 3$, and $x = -2$

$$p'(x) = 6x^2 - 6x - 36$$

Let's test with the first derivative:

$x = 3$

$p'(x) = \Big|_{3^-} = -ve < 0$

$p'(x) = \Big|_{3^+} = +ve > 0$

here,
$p'(x)$ is decreasing on $(-2 < x < 3)$

and $p'(x)$ is increasing on $(3 > x > \infty)$

For
$x = -2$

$p'(x) = \Big|_{-2^-} = +ve > 0$

$p'(x) = \Big|_{-2^+} = -ve < 0$

Here,

$p'(x)$ is increasing on $(-\infty < x < -2)$

and, $p'(x)$ is decreasing on $(-2 < x < 3)$

(iii) Relative Extrema:

For, $x = 3$

$p''(x) = 12x - 6$

$\Rightarrow p''(3) = 12 \times (3) - 6$

$\Rightarrow p''(3) = 36 - 6$

$= 30 >$

So the function has minimum value and the value is,

$p(x) = 2x^3 - 3x^2 - 36x + 5$

$\Rightarrow p(3) = 2(3)^3 - 3(3)^2 - 36 \times 2 + 5$

$= -76$

For, $x = -2$

$p''(x) = 12x - 6$

$\Rightarrow p''(x) = 12 \times (-2) - 6$

$= -30 < 0$

So the function has maximum value and the value is

$p(x) = 2x^3 - 3x^2 - 36x + 5$

$p(-2) = 2(-2)^3 - 3(-2)^2 - 36(-2) + 5$

$= 49$

$\therefore$ So, the maximum value is 49 and the minimum value is -76 Ans

Ans. to the Ques. No: 3

(i) $\int e^{2x} \sin 5x \, dx$

Formula,

$$\int e^{ax} \sin(bx) \, dx$$

$$= e^{ax} \frac{(a\sin(bx) - b\cos(bx))}{a^2 + b^2} + c$$

For,

$a = 2$ and $b = 5$,

we substitute, $\int e^{2x} \sin(5x) \, dx$

$$\Rightarrow e^{2x} \frac{(2\sin(5x) - 5\cos(5x))}{2^2 + 5^2} + c$$

$$\Rightarrow e^{2x} \frac{(2\sin(5x) - 5\cos(5x))}{29} + c$$

Ans

(ii) $\int \frac{e^{\tan^{-1}(x)}}{1+x^2}\,dx$

Here,

$\frac{1}{1+x^2}$ is the derivative of $\tan^{-1}(x)$

So, $u = \tan^{-1}(x)$

$du = \frac{1}{1+x^2}\,dx$

The integral becomes

$\int e^u\,du = e^u + e$

substitute,

$u = \tan^{-1}(x)$

$\int \frac{e^{\tan^{-1}(x)}}{1+x^2}$

$= e^{\tan^{-1}(x)} + e$ Ans

Answer to the ques. No : 3

(b) Evaluate $\int_{-6}^{5} g(x)\,dx$ where $g(x)$ is defined in the following figure :

The curve consists of 3 distinct regions,

1. $x = -6$ to $x = 0$, triangle
2. $x = 0$ to $x = 3$, semi-circle
3. $x = 3$ to $x = 5$, Triangle

Triangle from $(-6, 2)$ to $(0, 0)$
triang with base 6 unit (from -6 to 0) and height 2 unit.

$$\text{Area} = \frac{1}{2} \times \text{base} \times \text{height}$$

$$= \frac{1}{2} \times 6 \times 2$$

$$= 6$$

semi-circle from $(0, 0)$ to $(3, 0)$
semi-circle has radius of 3 (since $x=0$ to $x=3$)

The area of circle

$$\frac{1}{2}\pi r^2$$

$$= \frac{1}{2} \times \pi \times (3)^2$$

$$= \frac{9\pi}{2}$$

This semi-circle is below the x-axis. we consider the area ~~is~~ as negative,

$$-\frac{9\pi}{2}$$

Triangle from $(3,2)$ to $(5,0)$

Another triangle with a base of 2 units (from $x=3$ to $x=5$) and height 2 units.

The Area

$$= \frac{1}{2} \times \text{base} \times \text{height}$$

$$= \frac{1}{2} \times 2 \times 2$$

$$= 2$$

Total Area,

$$6 + \left(-\frac{9\pi}{2}\right) + 2$$

$$= 8 - \frac{9\pi}{2}$$

The value of $\int_{-6}^{5} g(x)$ is $8 - \frac{9\pi}{2}$

Answer to the ques. No : 4 (a)

(a) $\int$

(i) $\displaystyle\int_{-1}^{1} \frac{x^2\,dx}{\sqrt{x^3+9}}$

$9x^3 u = x^3 + 9$

$du = 3x^2\,dx$

$= \frac{1}{3}\int \frac{du}{u^{\frac{1}{2}}}$

$= \frac{1}{3}\left(2u^{\frac{1}{2}}\right)\Big|_{8}^{10}$

$= \frac{2}{3}\left(\sqrt{10} - \sqrt{8}\right)$

$= \frac{2\sqrt{2}}{3}\left(\sqrt{5} - 2\right)$

(ii) $\displaystyle\int_{x^2}^{4x^2} \frac{\sin\sqrt{x}\,dx}{\sqrt{x}}\,dx$

$x^2 u = \cos\sqrt{x}$

$du = -\sin\sqrt{x} \times \frac{1}{2} \times dx$

$= -\frac{1}{2} \cdot \frac{\sin\sqrt{x}}{\sqrt{x}} \cdot dx$

$\displaystyle\int_{x^2}^{4x^2} \frac{\sin\sqrt{x}\,dx}{\sqrt{x}}$

$= \displaystyle\int_{-1}^{1} -2\,du$

$= -2x\Big|_{-1}^{1} \quad = 2 - 2 \quad = -4$ Ans

(iii) $\int_{\ln 2}^{\ln \frac{2}{\sqrt{3}}} \frac{e^{-x} dx}{\sqrt{1-e^{-2x}}}$

$\Rightarrow \int_{\ln 2}^{\ln \frac{2}{\sqrt{3}}} \frac{e^{-x}}{\sqrt{1-e^{-2x}}} dx$

$= \int \left(-\frac{\pi}{6}\right) dx$

$\Rightarrow -\frac{\pi}{6}x + c$

(iv) $\int_0^2 f(x), dx$ where

$f(x) = \begin{cases} x, & 0 \le x \le 1 \\ x^2, & 1 < x \le 2 \end{cases}$

$\int_0^1 x. dx + \int_1^2 x^2. dx$

$= \frac{x^2}{2}\Big|_0^1 + \frac{x^3}{3}\Big|_1^2$

$\Rightarrow \frac{1}{2} + \left(\frac{8}{3} - \frac{1}{3}\right)$

$= \frac{1}{2} + \frac{7}{3}$

$\Rightarrow \frac{3}{6} + \frac{14}{6} = \frac{17}{6}$ Ans

Answer to the Ques. No : 4

4/(b)

(b) $\int_0^{\pi} 2t \sin 2t \, dt$

Let, $u = 2t$ which implies $du = 2, dt$

$dv = \sin(2t), dt$ which gives

$v = -\frac{1}{2}\cos(2t)$

The formula for Integration,

$$\int u\,dv = uv - \int v \, du$$

$$= \left[-2t \cdot \frac{1}{2}\cos(2t)\right]_0^{\pi} - \int_0^{\pi}\left(-\frac{1}{2}\cos(2t) \cdot 2dt\right)$$

$$= \left[-t \cos(2t)\right]_0^{\pi} + \int_0^{\pi} \cos(2t)\, dt$$

we evalute $-t\cos(2t)$ the limits,

$$-t\cos(2t)\Big|_0^{\pi} = -\pi\cos(2\pi) + 0.\cos(0)$$

$$= -\pi . 1 + 0$$

$$= -\pi$$

The integral of $\cos(2t)$ is,

$$\int \cos(2t)\,dt = \frac{1}{2}\sin(2t)$$

$$\left[\frac{1}{2}\sin(2t)\right]_0^{\pi} = \frac{1}{2}\sin(2\pi) - \frac{1}{2}\sin(0)$$

$$\Rightarrow 0 - 0$$

$$= 0$$

Now we combine everything,

$$= -\pi + 0$$

$$\Rightarrow -\pi$$

The value of the integral is

$-\pi$ Ans

Ans. to the question No. 5

ⓐ compute the area of the region that is enclosed between the curves,

$y^2 = 4x$ and $y = x - 3$

first curve

$y^2 = 4x$ or $x = \frac{y^2}{4}$

second curve

$y = x - 3$ or $x = y + 3$

both.

$\therefore \frac{y^2}{4} = y + 3$

Multiply both side by 4

$y^2 = 4(y+3)$

$\Rightarrow y^2 = 4y + 12$

$\Rightarrow y^2 - 4y - 12 = 0$

The quadratic formula,

$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\Rightarrow y = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-12)}}{2(1)}$

$\Rightarrow y = \frac{4 \pm 8}{2}$

$y = \frac{4+8}{2}$ and $y = \frac{4-8}{2}$

$= 6$ $= -2$

The curves intersect at $y = 6$ and $y = -2$

The Area is,

$$\text{Area} = \int_{-2}^{6} \left[(y+3) - \frac{y^2}{4} \right] dy$$

$$= \int_{-2}^{6} (y+3) - \frac{y^2}{4}) \, dy$$

$$= \int_{-2}^{6} y \, dy + \int_{-2}^{6} 3 \, dy - \int_{-2}^{6} \frac{y^2}{4} \, dy$$

$$= \left[\frac{y^2}{2} \right]_{-2}^{6} + 3y \Big|_{-2}^{6} - \frac{1}{4} \int_{-2}^{6} y^2 \, dy$$

$$\Rightarrow \left[\frac{y^2}{2} \right]_{-1}^{6} + 3y \Big|_{-2}^{6} - \frac{1}{4} \left[\frac{y^3}{3} \right]_{-2}^{6}$$

$$= \left(\frac{6^2}{2} - \frac{(-2)^2}{2} \right) + \left(3(6) - 3(-2) \right) - \frac{1}{4} \left(\frac{6^3}{3} - \frac{(-2)^3}{3} \right)$$

$$= \left(\frac{36}{2} - \frac{4}{2} \right) + (18 + 6) - \frac{1}{4} \left(\frac{216}{3} + \frac{8}{3} \right)$$

$$= 18 - 2 + 24 - \frac{1}{4} \left(72 + \frac{8}{3} \right)$$

$$= 16 + 24 - \frac{224}{12}$$

$$= 40 - \frac{56}{3}$$

$$= \frac{120}{3} - \frac{56}{3}$$

$$= \frac{64}{3}$$

$= 21.33$ square unit

The area of enclosed between curve is $\frac{64}{3}$ or 21.33 square units.

(b) From the graph

$y = x$

$y = x+5$

$y = 2$ and $y = 1$

From,

$y^2 = x$ we get $x = y^2$

$y = x+5$, $x = y-5$

The shaded region is bounded between the horizontal lines $y = -1$ and $y = 2$

The Area is given,

$$A = \int_{-1}^{2} [(y-5) - y^2] \, dy$$

$$= \int_{-1}^{2} (y-5 - y^2) \, dy$$

$$= \int_{-1}^{2} y \, dy - \int_{-1}^{2} -5 \, dy - \int_{-1}^{2} -y^2 \, dy$$

$$= \left[\frac{y^2}{2}\right]_{-1}^{2} + 5y \Big|_{-1}^{2} + \left[\frac{y^3}{3}\right]_{-1}^{2}$$

$$= \left(\frac{2^2}{2} - \frac{(-1)^2}{2}\right) + 5(2) + (5)(1) + \left(\frac{2^3}{3} - \frac{(-1)^3}{3}\right)$$

$$= \frac{4}{2} - \frac{1}{2} + 15 + \left(\frac{8}{3} - \frac{-1}{3}\right)$$

$$= \frac{3}{2} + 15 + 3$$

$$= \frac{3}{2} + 18$$

$$\Rightarrow \frac{3 - 36}{2}$$

$$= \frac{-33}{6}$$

$$= -\frac{33}{2} \text{ square unit}$$

since, Area cannot be negative, the Area is $\frac{33}{2}$ square unit.