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Generation of arbitrary intensity profiles by dynamic jaws or multileaf collimators

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An algorithm, which calculates the motions of the collimator jaws required to generate a given arbitrary intensity profile, is presented. The intensity profile is assumed to be piecewise linear, i.e., to consist of segments of straight lines. The jaws move unidirectionally and continuously with variable speed during radiation delivery. During each segment, at least one of the jaws is set to move at the maximum permissible speed. The algorithm is equally applicable for multileaf collimators (MLC), where the transmission through the collimator leaves is taken into account. Examples are presented for different intensity profiles with varying degrees of complexity. Typically, the calculation takes less than 10 ms on a VAX 8550 computer.

Key words: dynamic collimation, intensity modulation, independent jaws, multileaf collimators

I. INTRODUCTION

The aim of conformal radiotherapy is to deliver a prescribed dose which conforms to the target volume, while keeping doses to the surrounding tissues below tolerance.[1] This is a difficult task, since sensitive normal tissue structures are frequently in close proximity to the target volume. Moreover, scatter of the primary radiation results in a dose distribution which is not localized.[2–4] The use of multiple complex radiation fields will generally result in a better dose distribution, which has been postulated to improve local control.[5–7] Presently, the number of fields, as well as the complexity of each field, is limited by time constraints to both plan and deliver multiple radiation fields. Traditionally, field modulation has been performed by cerrobend blocking, wedges, and compensators. Recent advances in computer and accelerator technology have prompted several researchers to explore the use of computer-controlled collimator jaws or leaves,[8,9] which permit the intensity to be modulated through the use of a narrow slit field[10] or through the dynamic motion of the jaws or leaves.[11–13] (Strictly speaking, it is the fluence distribution that is modulated by the motion of jaws and leaves, but the term "intensity modulation" has appeared in the literature and, therefore, is used in this paper.)

The use of a physical compensator has several drawbacks: (a) it is labor-intensive in fabrication, (b) it requires re-entry into the treatment room between fields, and (c) it generates scattered radiation which, due to its proximity to the patient, increases the skin dose and the dose outside the field.[2–4] Dynamic intensity modulation, on the other hand, does not have these drawbacks. One implementation of this method employs an elementary slit which scans the target volume at an appropriate speed in order to generate the required intensity distribution.[14] A drawback of this technique is its inefficient use of the beam output, since only a small area is irradiated at any one time, due to the narrow opening of the slit. A second implementation is to use dynamic collimation of the jaws or leaves to deliver intensity modulated fields.

Two techniques of using dynamic collimation have been proposed:[11–13] (a) the "close-in" technique, where the jaws start at the opposite ends of the field and move towards a central point, and (b) the "sliding window" technique, where both jaws start at the same end and move unidirectionally, at different speeds, to the other end. In the "close-in" technique the jaw motions result in the generation of a single-maximum profile, the slope of which is limited by the mechanical speed of the jaws and the dose rate. If the slope of the desired intensity profile exceeds the constraints placed by the mechanical jaw speeds and the dose rate, then the profile cannot be generated. In this case, the profile must be approximated by a series of discrete steps. For an arbitrary intensity distribution, with several maxima and minima, this technique splits the profile into a series of single-maximum subprofiles, generated in sequence with the beam switched off during jaw repositioning. For a multileaf collimator, however, the different pairs of leaves are generally required to generate different intensity profiles, with correspondingly different beam-on and -off times, thus rendering the "close-in" technique inappropriate. The "sliding window" technique, on the other hand, has none of these drawbacks, except that a longer beam-on time may be required. The overall treatment time of the "sliding window" technique, however, is usually shorter than that of the "close-in" technique.

Convery and Rosenbloom[12] have previously reported a version of the "sliding window" algorithm in which the generation of an arbitrary intensity distribution is treated as a minimization problem in linear programming. As in the work of Convery and Rosenbloom, in this paper the intensity profile, which has been determined by the inverse treatment planning process taking all relevant factors into account, is represented by a piecewise linear function, i.e., consisting of segments of straight lines. The algorithm presented here takes a segment-by-segment approach, in which the beam-on time for each segment is minimized. This is achieved by setting at least one of the jaws to move at the maximum allowed speed during each segment. In Sec. II the algorithm is developed for use on independent jaws and multileaf collimators, assuming that there is infinite acceleration and that

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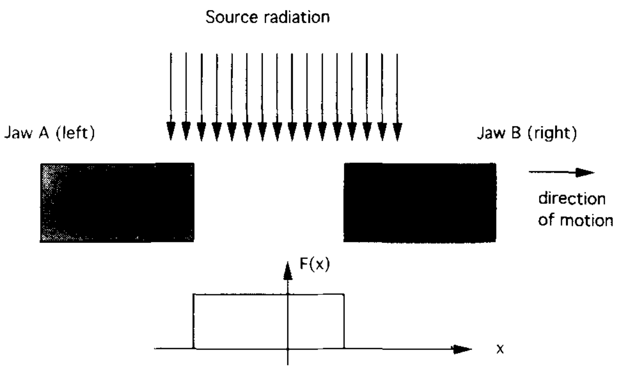

FIG. 1. Schematic showing the basic geometry of the problem. The radiation is assumed to be perpendicular to the direction of motion of the jaws and edge effects are ignored.

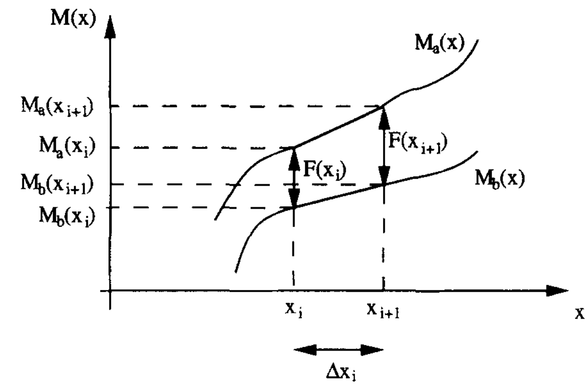

FIG. 2. The relationship between $F(x)$, $M_a(x)$, and $M_b(x)$ at the points x_i and x_{i+1}. Note that both $M_a(x)$ and $M_b(x)$ are piecewise linear.

there is no penumbra, collimator scatter, or leakage between adjacent MLC leaves. In Sec. III four example profiles are presented. In one example the resultant pattern of jaw motions is compared to that obtained by Convery and Rosenbloom and good agreement is obtained. However, the algorithm presented here requires only a few milliseconds compared to the 1–10 minutes reported by Convery and Rosenbloom. Finally, in Sec. IV certain features of the algorithm, the assumptions built into it, and some practical considerations are discussed.

II. METHOD

The intensity profile to be delivered is considered to be piecewise linear, i.e., consisting of segments of straight lines. The algorithm strives to achieve maximum efficiency in *each* of the segments, so that overall the most efficient pattern of dynamic motion is obtained.

The algorithm consists of two main parts: (a) the basic equations that determine the motion of the jaws, which are described in Sec. II A below, and (b) the start and finish positions of the jaws, which are described in Sec. II B. The extension to multileaf collimators is described in Sec. II C. The equations are first described assuming there is no transmission through the jaws or the leaves. Whereas this may be a good assumption for dynamic jaws, where the transmission is typically of the order of 0.5%, for MLCs this assumption may not hold. A magnitude of 1%–2% transmission through the leaves has been observed.[15] Incorporation of transmission into the algorithm is discussed in Sec. II D.

A. Basic equations

A diagram of the basic geometry is given in Fig. 1. The left jaw is labeled A and the right jaw is labeled B. Both jaws move in the positive x-direction. The radiation source is assumed to be perpendicular to the line of motion of the jaws. Let $F(x)$ be the field intensity (in monitor units MU) to be delivered, which is assumed to have been previously determined by optimization or inverse procedures.[16–22] The functions $M_a(x)$ and $M_b(x)$ describe the cumulative beam-on time (in MU) of jaws A and B at the position x, respectively. Both $M_a(x)$ and $M_b(x)$ are monotonically increasing functions since they represent beam-on time.

Each jaw is assumed to totally block the beam below it and edge effects (penumbra, etc.) are ignored. Under these conditions the field intensity produced by jaws A and B, when each is at $x=x_0$, is described by the Heaviside step function:

$$F_a(x)=\begin{cases} 0 & \text{if } x<x_0, \\ M/2 & \text{if } x=x_0, \\ M & \text{if } x>x_0, \end{cases}$$

and

$$F_b(x)=\begin{cases} 0 & \text{if } x>x_0, \\ M/2 & \text{if } x=x_0, \\ M & \text{if } x<x_0, \end{cases}$$

where M is the field intensity corresponding to the beam-on time.

Then the intensity profile satisfies

$$F(x)=M_a(x)-M_b(x), \tag{1a}$$

i.e., the irradiation of point x starts when the right jaw unblocks it and finishes when the left jaw blocks it.

In practice $F(x)$ is a piecewise linear function and is discretized at N points, so the above formula becomes

$$F(x_i)=M_a(x_i)-M_b(x_i), \quad \text{where } i=1,\ldots,N. \tag{1b}$$

Equation (1b) is one of the two equations needed to calculate M_a and M_b at any one point. The second equation is obtained by demanding that at least one of the jaws moves with the maximum allowed speed between any two successive points. Starting, therefore, with the initial positions of the two jaws, determined as explained below, one can proceed segment by segment to calculate the motions of the jaws needed to produce the desired intensity profile.

Assume now that both M_a and M_b are known at the ith position, i.e., $x=x_i$. Then (see Appendix A) at the $(i+1)$th position M_a and M_b are given by (see Fig. 2) the following.

If $F(x_{i+1})\geqslant F(x_i)$, then

$$M_b(x_{i+1})=M_b(x_i)+\frac{\Delta x_i}{V_{\max}}$$

and (2a)

$$M_a(x_{i+1})=M_b(x_{i+1})+F(x_{i+1}).$$

Otherwise,

$$M_a(x_{i+1})=M_a(x_i)+\frac{\Delta x_i}{V_{\max}}$$

and (2b)

$$M_b(x_{i+1})=M_a(x_{i+1})-F(x_{i+1}),$$

where $V_{\max}$ is the maximum jaw speed, defined in units of cm/MU, and Δx_i is the distance to be traveled in the ith segment, $x_{i+1}-x_i$.

To maximize the efficiency of treatment delivery, the total beam-on time must be minimized. This is achieved by the algorithm through forcing, at each segment, at least one of the jaws to move at the maximum allowed speed, $V_{\max}$. The mathematical proof is given in Appendix B. The total beam-on time is measured in MU. The correspondence between beam-on time in MU and treatment time in seconds is discussed in Sec. IV.

B. Start and finish positions of the jaws

Depending on the particular profile to be generated, the start and finish positions of the jaws can be selected to further reduce the total beam-on time.

Clearly the start position of jaw A is the leftmost point x_1, and the finish position of jaw B is the rightmost point x_N. In some cases the start position of jaw B may not be x_1. If the profile $F(x)$ to be generated is initially increasing at a sufficiently high rate, i.e., if it satisfies the condition $F(x_{i+1})-F(x_i)>\Delta x_i/V_{\max}$, then jaw B will optimally start at x_m, where $1<m<N$ and m is the first point that does not satisfy this condition. The initial portion of $F(x)$ is then generated by jaw A alone, i.e., $M_a(x_i)=F(x_i)$ for $i=1,\ldots,m$. Although counterintuitive, the slow-varying profiles are more difficult to generate than the fast-varying ones. This is because the jaw takes a finite amount of time $\Delta t(=\Delta x/V_{\max})$ to move from one point to the next, so the radiation difference between these two points must be at least Δt. If it is not, then one jaw alone cannot generate this particular segment of the profile.

In the case of a single pair of jaws a similar situation exists for the finish position of jaw A. If $F(x)$ decreases sufficiently rapidly in the region $x_K<x<x_N$ for some k, then jaw A can stop at x_K while jaw B alone generates $F(x_i)$ for $k<i<N$.

C. Multileaf collimators

Assume that a MLC has P pairs of leaves. Let $F_p(x)$ be the desired intensity profile for the pth pair of leaves, and $M_{p,a}(x)$ and $M_{p,b}(x)$ describe the motion of the left and right leaves of the pth pair, respectively. Then the set of equations presented in Sec. II A is applicable to each pair of leaves.

The start position of each B_p leaf can be determined the same way as that for jaw B, as described in Sec. II B. To determine the finish position of each A_p leaf one must account for the difference in beam-on times due to the generation of different profiles by the pairs of leaves. For example, if the first pair of leaves finishes delivering its intensity profile before the second pair, then the leaves of the first pair cannot remain at their *optimized*, unclosed, finish positions while the beam is still on. One solution is to force both leaves to finish at the rightmost point, effectively closing the gap between them. Since, in practice, the leaves may have a small gap between the ends, to reduce the leakage through the gap, the machine can be instructed to move the leaves out of the field. Another possibility is to reduce the maximum allowed speed of all but the slowest pair of leaves such as to force the total beam-on times of all leaf pairs to be the same. However, since the maximum allowed speed also determines the start position of B leaf, for a given profile there is no guarantee that by varying $V_{\max}$ a particular value of the total beam-on time will be obtained. This is illustrated by example 2 in Sec. III. A third possibility is to change $V_{\max}$ in *only the very last segment* of all but the slowest pair of leaves, so as to make all the beam-on times equal. This method is simple and easy to implement.

D. Transmission through the leaves

Multileaf collimator leaves generally exhibit a higher transmission than jaws, typically 1%–2%.[15] Since the beam-on time may be significantly greater for dynamically modulated fields compared to fields using conventional beam modifiers, it is necessary to take into consideration the transmission of radiation through the leaves. This is achieved by substitution of the desired intensity profile by one which has been modified to account for transmission.

If $F(x_i)$ is the desired intensity at point x_i, $f(x_i)$ the modified intensity which represents the actual time interval during which point x_i is not covered by either leaf, T_{tot} the total beam-on time, and τ the transmission factor, then (see Fig. 3)

$$\begin{aligned}F(x_i)&=[T_{\text{tot}}-f(x_i)]*\tau+f(x_i)\\&=T_{\text{tot}}*\tau+(1-\tau)*f(x_i),\quad i=1,\ldots,N.\end{aligned}\tag{3}$$

The desired intensity at the point x_i is the sum of the direct exposure plus the transmitted radiation. If T_{tot} is known, then, since τ is known, the above equation can be solved for $f(x_i)$ and the modified profile used in the algorithm.

In the case of a MLC, where the leaves are forced to close-in on the side of leaf B, the total beam-on time is given by (see Appendix C)

$$T_{\text{tot}}=\frac{(x_N-x_m)}{V_{\max}}+f(x_m)+\sum_{\substack{i=m\\ f(x_{i+1})>f(x_i)}}^{N-1}[f(x_{i+1})-f(x_i)],\tag{4}$$

where x_m is the starting position of leaf B. Solving Eq. (3) for $f(x_i)$ and substituting we obtain

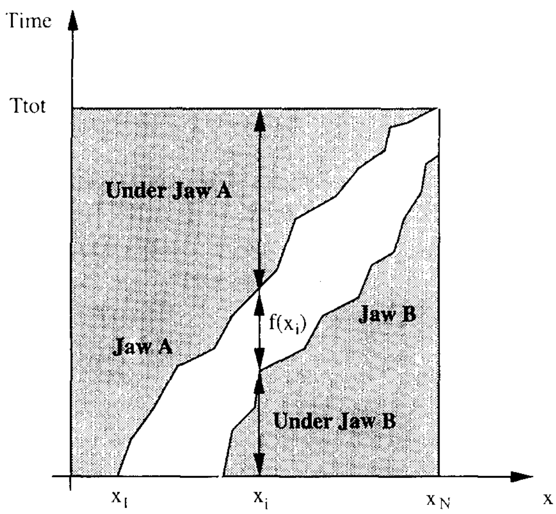

FIG. 3. Directly exposed (unshaded) and covered (shaded) regions during the motion of the jaws.

$$T_{\text{tot}}=(1-\tau)\times\frac{(x_N-x_m)}{V_{\max}}+F(x_m)+\sum_{\substack{i=m\\F(x_{i+1})>F(x_i)}}^{N-1}[F(x_{i+1})-F(x_i)], \quad (5)$$

where the $F(x_i)$'s are the desired intensity profile to be generated and τ is the transmission factor.

Thus, as soon as the starting position of leaf B is found, i.e., the point x_m, the total beam-on time can be calculated. Then the algorithm, as described in Sec. II, can be employed not with the desired dose profile as input, but with the modified transmission profile, i.e., the $f(x_i)$'s, which are given by

$$f(x_i)=\frac{F(x_i)-T_{\text{tot}}\times\tau}{1-\tau}. \quad (6)$$

The path of motions, M_a and M_b, calculated according to Eq. (2) based on the modified intensity profile $f(x)$ will generate the desired profile $F(x)$ when leaf transmission is taken into account.

Note that when $\tau=0$, Eq. (6) reduces to $f(x_i)=F(x_i)$.

III. RESULTS

The algorithm is applied to four intensity profiles. The first two are simple linear profiles to illustrate the basic principles of the algorithm. The third compares the results of our algorithm with previously published results.[12] The fourth demonstrates the effect of transmission through the leaves. In the first two examples the mechanical speed of the jaws is taken to be 1 cm/s.

A. Example 1: Simple wedge profile

The algorithm is applied to a simple wedge profile, shown in Fig. 4(a), defined only at the endpoints as follows:

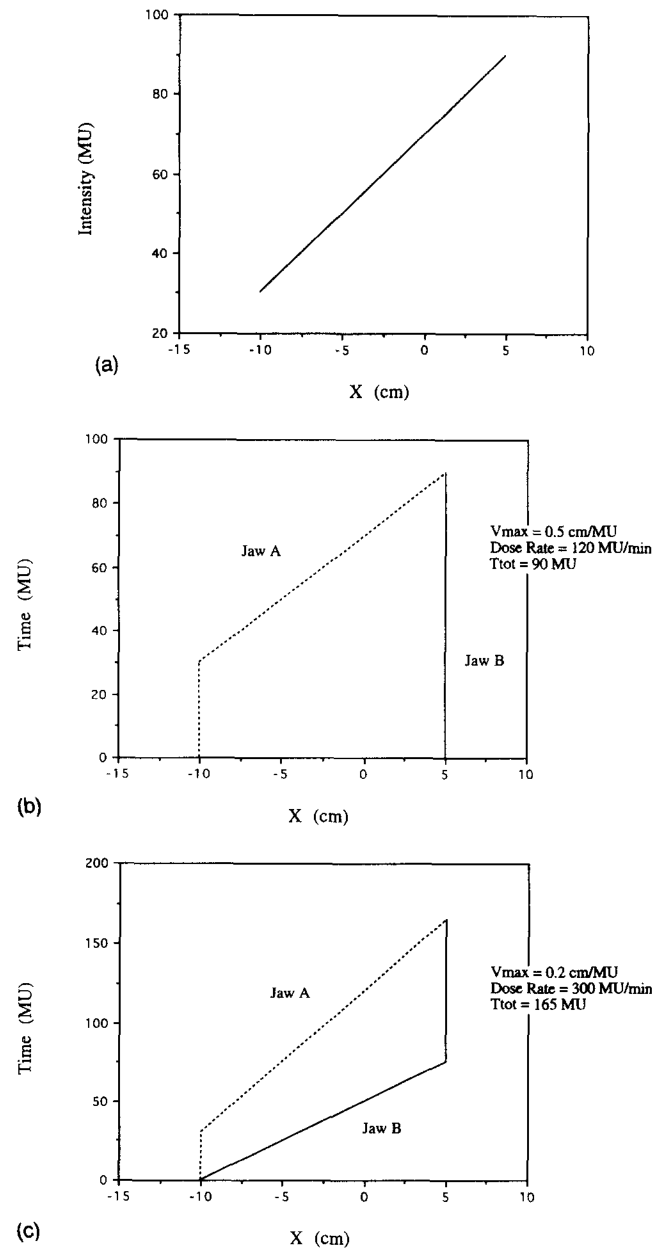

FIG. 4. (a) Wedge profile to be generated; (b) and (c) the beam-on time as a function of jaw positions for high (0.5 cm/MU) and low (0.2 cm/MU) maximum allowed speeds, respectively.

$$x_1=-10 \text{ cm}, \quad F(x_1)=30 \text{ MU},$$

$$x_2=5 \text{ cm}, \quad F(x_2)=90 \text{ MU}.$$

The slope $dF(x)/dx$ of the profile is constant and equal to 4 MU/cm. The critical speed, V_{cr}, defined as the inverse of the slope, is 0.25 cm/MU. If the machine permits jaw speeds greater than V_{cr}, then the condition is satisfied and jaw B remains stationary at point x_2, while jaw A alone generates the desired intensity profile. This is illustrated in Fig. 4(b) for a maximum allowed speed of 0.5 cm/MU.

If, on the other hand, the maximum speed is less than V_{cr}, as shown in Fig. 4(c), where $V_{\max}$ is 0.2 cm/MU, then both jaws are required to generate the profile. Both jaws A and B start at the point x_1; then jaw B moves to the point x_2 at the maximum speed, while jaw A moves at an appropriate,

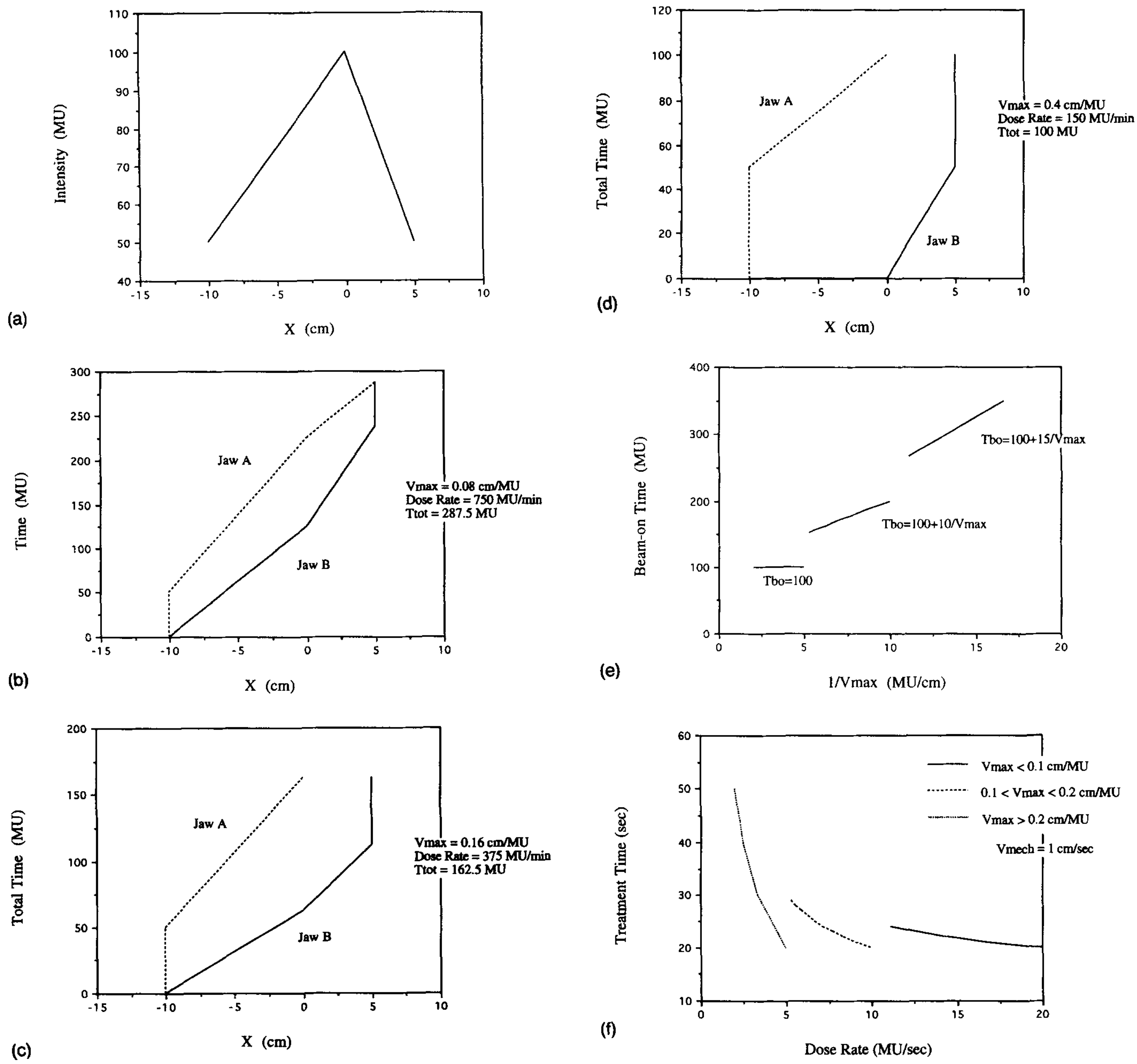

FIG. 5. (a) A triangular profile to be generated; (b)–(d) beam-on time as a function of jaw positions for low (0.08 cm/MU), intermediate (0.16 cm/MU), and high (0.4 cm/MU) maximum allowed speeds, respectively; (e) total beam-on time as a function of maximum allowed speed; (f) total treatment time as a function of the dose rate.

slower rate so as to generate the desired profile. The total beam-on times for the higher and lower maximum speeds are 90.0 and 165.0 MU, respectively, illustrating that higher maximum speeds result in shorter beam-on times.

As already stated in Sec. II, the algorithm takes a segment-by-segment approach to calculate $M_a(x)$ and $M_b(x)$ at each point x_i at which $F(x)$ has been defined; therefore, one would expect no change in the total beam-on time for a given profile, if that profile is overdetermined. For example, the simple wedge profile of Fig. 4(a) need only be defined at the endpoints and it should make no difference in the total beam-on time if more points are used to define it. This has been verified by recalculating the profile with nine additional intermediate points. The results of these calculations for both beam-on times and jaw motions were identical, as expected, for both higher and lower speeds.

B. Example 2: Single-maximum profile

In Fig. 5(a) the profile to be generated is a simple triangle, defined at three points as follows:

$$x_1 = -10 \text{ cm} \quad F(x_1) = 50 \text{ MU},$$

$$x_2 = 0 \text{ cm} \quad F(x_2) = 100 \text{ MU},$$

$$x_3 = 5 \text{ cm} \quad F(x_3) = 50 \text{ MU}.$$

TABLE I. Start position of jaw B, finish position of jaw A, and total beam-on time corresponding to different values of the maximum speed for the single-maximum profile of Fig. 5(a). The dose rate corresponds to a mechanical jaw speed of 1 cm/s.

V_{max} (cm/MU)	Dose rate (MU/min)	Start position of jaw B	Finish position of jaw A	Total beam-on time (MU)
0.08	750	x_1	x_3	287.5
0.16	375	x_1	x_2	162.5
0.40	150	x_2	x_2	100.0

Thus, there are two critical speeds, $V_{cr}^1=0.2$ cm/MU and $V_{cr}^2=0.1$ cm/MU, corresponding to the slope of the profile between the points x_1, x_2 and x_2, x_3, respectively.

Figures 5(b)–5(d) show the required jaw motions in order to generate the desired intensity profile, when the maximum allowed speed is less than V_{cr}^2, between V_{cr}^2 and V_{cr}^1, and greater than V_{cr}^1, respectively. The results are summarized in Table I.

When the maximum allowed speed is 0.08 cm/MU, i.e., less than both V_{cr}^2 and V_{cr}^1, both jaws have to be used in each segment in order to generate the desired intensity profile. The separation between the jaws remains small during the entire treatment, so the total beam-on time is long. When the maximum allowed speed is 0.16 cm/MU, i.e., between V_{cr}^2 and V_{cr}^1, both jaws are needed to generate the first segment of the profile, i.e., from x_1 to x_2. In the second segment, i.e., from x_2 to x_3, jaw B alone is capable of generating the desired intensity profile, so jaw A stops at x_2 while jaw B continues to x_3. Finally, when the maximum allowed speed is 0.4 cm/MU, i.e., greater than both V_{cr}^2 and V_{cr}^1, jaw A alone is capable of generating the desired intensity profile from x_1 to x_2, and jaw B alone is capable of generating the desired intensity profile from x_2 to x_3. Therefore, the motion of jaw A is restricted between x_1 and x_2, while the motion of jaw B is restricted between x_2 and x_3.

The two critical values of the speed effectively trisect the range of possible speeds as shown in Fig. 5(e). For speeds greater than V_{cr}^1, the total beam-on time is constant and equal to 100 MU, which is the desired intensity at the point x_2. The total beam-on time cannot be less than the number of MU required at point x_2, regardless of the value of V_{max}. For intermediate speeds, i.e., between V_{cr}^2 and V_{cr}^1, and low speeds, i.e., less than V_{cr}^2, Eq. (C2) of Appendix C is applicable, with $k=2$ and $k=3$, respectively ($m=1$).

C. Example 3: Multiple maxima and minima profile

We compare our algorithm with that of Convery and Rosenbloom[12] by using the example of Fig. 3 from their paper. Figures 6(a) and 6(b) show the desired intensity profile and the calculated jaw motions. In Fig. 6(c) the axes are switched to facilitate comparison with the results of Convery and Rosenbloom. Both methods are in close agreement for the resulting jaw motions and the total beam-on time. The advantage of the algorithm described in this paper is its speed. Convery and Rosenbloom state typical calculation times of 1–10 min on a DEC VaxStation 3100, compared to this algorithm which takes less than 10 ms on a VAX 8550. Even for a multileaf collimator with, say, 26 leaf pairs, the total calculation time is less than 0.3 s.

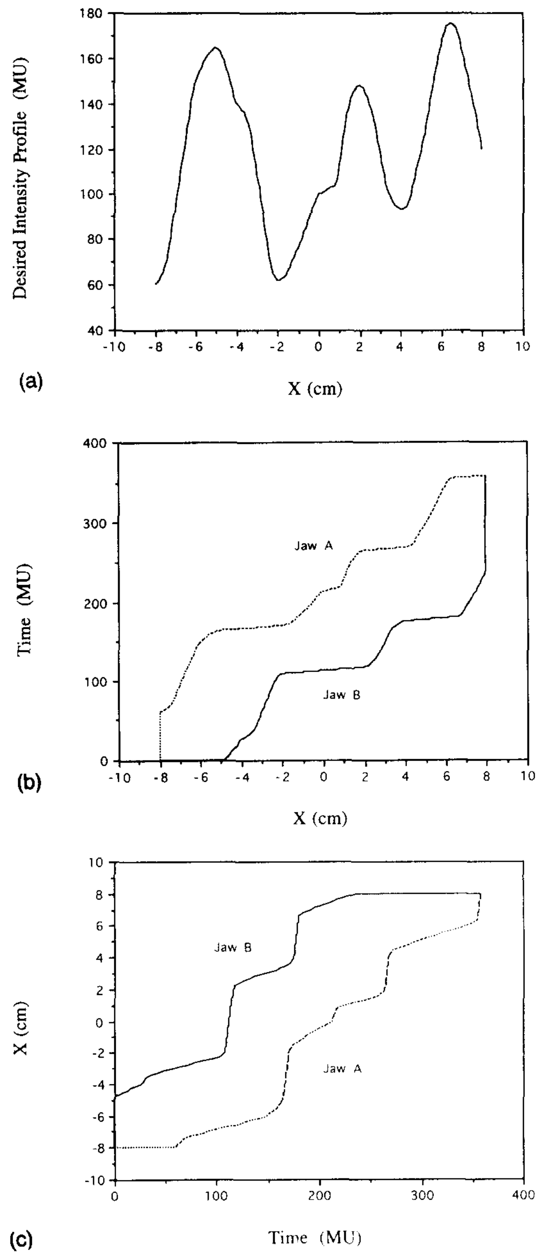

FIG. 6. (a) A profile with multiple maxima and minima similar to the one in Fig. 3(a) of Convery and Rosenbloom;[12] (b) total beam-on time as a function of jaw positions ($V_{max}=5$ mm/MU); and (c) jaw positions as functions of time.

D. Example 4: Transmission through the leaves

In this example, we show the effect of radiation transmission through the leaves. In Fig. 7(a) the solid line shows the desired intensity profile, $F(x)$. If the transmission τ were not taken into account in the determination of the leaf motions, the resultant profile for $\tau=1\%$ would be higher than the desired one, as shown by the dashed line. The dotted line *below* the solid one shows the modified intensity profile, $f(x)$. This is the profile that the leaves should be instructed to generate in order to achieve the desired intensity profile.

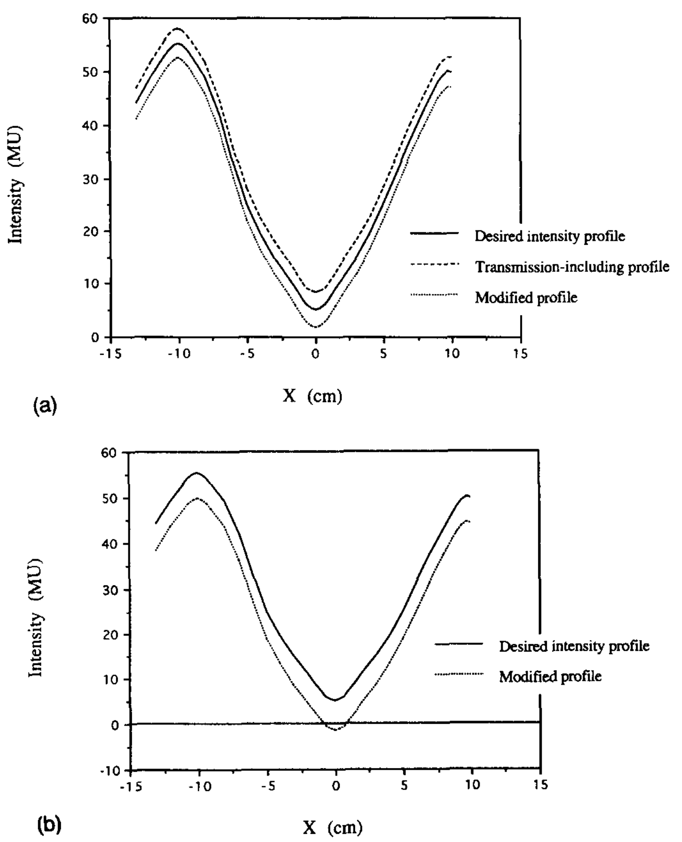

FIG. 7. The effect of transmission through the MLC leaves: (a) desired intensity profile (solid line), transmission-including profile (dashed line) and modified profile (dotted line) assuming $\tau=1\%$; (b) same desired profile (solid line) results in modified profile (dotted line) with negative segments if $\tau=2\%$.

An obvious consequence of leaf transmission is that, ceteris paribus, some profiles may not be achievable. This is because it is not possible to deliver intensities in the profile less than the background transmitted intensity, which is proportional to the total beam-on time. This is illustrated in Fig. 7(b). The solid line represents the same desired intensity profile as in Fig. 7(a). If the transmission coefficient were 2%, then the dotted line, which represents the modified intensity profile that the leaves should be instructed to generate, becomes negative around $x=0$. In this example the background transmission near $x=0$ is 6.5 MU, which exceeds the desired intensity by about 1 MU.

One way to solve this problem, assuming of course that the transmission coefficient cannot be reduced further and that the desired intensity profile cannot be redesigned, is to set all negative intensities equal to zero. Although this means that these points will be overexposed, in some cases this may be acceptable. Another solution is to increase the maximum allowed speed by reducing the dose rate.

IV. DISCUSSION

This paper presents an algorithm which minimizes the total beam-on time, expressed in MU, to generate arbitrary intensity profiles using dynamic collimation. In Secs. IV A and IV B certain features and results of the algorithm are discussed. In Secs. IV C and IV D the assumptions made in developing the algorithm are considered. Lastly, in Secs. IV E and IV F some practical considerations are given.

A. Slowly vs rapidly varying profiles

For a given profile $F(x)$, the jaw motions are determined by the maximum allowed speed $V_{\max}$. If the maximum speed is increased, a shorter beam-on time results. The maximum allowed speed is specified as the ratio of the mechanical speed to the dose rate, in units of cm/MU. Assuming that the mechanical constraints of jaw motion have been fully exploited, a greater $V_{\max}$ is equivalent to a lower dose rate and a greater jaw separation.

Consider, for example, two successive points x_i and x_{i+1}, spaced 1 cm apart, and assume that x_{i+1} is specified to receive 10 MU more than x_i. If the mechanical speed of the jaws is 1 cm/s, then jaw A takes 1 s to move from x_i to x_{i+1}. If the dose rate is greater than 10 MU/s, then the amount of radiation delivered to x_{i+1} while jaw A is moving from x_i to x_{i+1} will be more than 10 MU. Therefore, jaw A alone cannot deliver this segment of the profile. In this case it is necessary to reduce the jaw separation using jaw B. This lowers the efficiency and requires longer beam-on time to generate this segment. If, on the other hand, the dose rate is less than 10 MU/s, then jaw A alone can deliver this segment.

It can be shown that the total beam-on time is inversely related to $V_{\max}$ or, equivalently, linearly related to the dose rate (see Appendix C).

B. Beam-on time vs treatment time

In the foregoing discussion, the beam-on time is expressed in MU. However, minimizing the total beam-on time is not necessarily equivalent to minimizing the treatment time measured in seconds. This is because an increase in the dose rate does not necessarily imply a decrease in the treatment time. This is shown, for example, in Fig. 5(f) where the treatment time is 20 s at a dose rate of 5 MU/s, but 26 s at a dose rate of 6.25 MU/s. In this example as the dose rate tends to infinity the treatment time asymptotically approaches a minimum of 15 s. However, in general, the optimal dose rate may not be infinite.

As described earlier, each segment of the desired intensity profile defines a critical speed, in cm/MU. The corresponding critical dose rate (MU/s) is defined as the ratio of the mechanical speed (cm/s) to the critical speed (cm/MU). In example 2 the critical speeds are 0.2 and 0.1 cm/MU, and the corresponding critical dose rates are 5 and 10 MU/s, assuming that the mechanical speed is 1 cm/s [see Fig. 5(f)]. The entire range of possible dose rates is divided into regions by the critical dose rates. It can be shown rigorously that within any region, increasing the dose rate reduces the treatment time in seconds. However, this may not be true between distinct regions. Thus, the optimum dose rate will depend on each individual profile and can be found by trial and error. Since the algorithm is very fast, even an intensity profile defined, say, by 161 points, with 160 critical speeds, requires only a few seconds to calculate the optimal dose rate.

In general, there is a tradeoff between minimizing the total treatment time in seconds and minimizing the total beam-on time in MU. The choice of one or the other depends upon other considerations, such as leaf transmission and the increase in dose to critical organs versus the ability of the patient to remain still for the duration of the treatment.

C. Finite acceleration

In setting the speeds of the jaws or leaves in Eqs. (2a) and (2b), it was assumed that acceleration and deceleration were infinite. One might then expect that the fact that acceleration is really finite would cause the actually generated profile to differ from the desired one. However, comparisons on our machine (Scanditronics MM50) between the desired profile, film measurements, and calculations showed no appreciable difference. For a more thorough discussion of this issue the reader is referred to Svensson *et al.*[13]

D. Collimator scatter and penumbra

As stated in the Introduction, collimator scatter and penumbra effects are ignored in the development of this algorithm. When these effects are taken into account, the in-air fluence becomes a function of the relative positions of the jaws or leaves. Equations (1) and (2) then become integral equations, so to determine the paths of the jaws one then has to solve a set of coupled integral equations, which will, in general, have to be solved iteratively. This increases the complexity of the algorithm unnecessarily, since these effects can be accounted for in a much simpler way. In particular, for a given desired profile one can begin by calculating the jaw motions assuming that there is no collimator scatter or penumbra effects. Once these motions have been obtained, one can then calculate the actually generated fluence distribution, this time including scatter and penumbra. Then the input profile, which was initially the same as the desired one, can be modified, i.e., reduced at points of overdosing and increased at points of underdosing, a new set of jaw motions can be calculated, and the process repeated until the actually generated profile sufficiently matches the desired one. Using measured data of collimator scatter (i.e., the variation of output in air with field size) for a machine currently in use (Varian 2100C), our trials showed that agreement to within 1% is usually obtained in a few such iterations.

E. Jaw/leaf mechanics and design

An important practical consideration is the actual design of the treatment machine, especially the mechanics of motion of the jaws or leaves. In some machines the jaws either move at a single speed or remain stationary. In such a case the calculated motions will have to be approximated by a series of moving and stationary steps. Another limitation may be the range of leaf travel that may place constraints on the field size. Leakage between adjacent leaves is also of concern. Underdosing in the region under the boundary between adjacent leaves, which depending on the profile to be generated can be as high as 10%, has been observed in some MLC designs. This cannot be taken into account by any algorithm such as the one presented in this paper; however, by carefully designing the treatment plan one may be able to diffuse this effect. Furthermore, future improvements in MLC design may reduce or even eliminate altogether this effect, as well as minimize transmission and increase leaf speed which, in turn, may further reduce the beam-on time and/or the treatment time.

F. Record and verify

In an actual implementation of any algorithm for dynamic collimation, such as the one presented in this paper, one must be able to record and verify the true positions of the jaws or leaves. This is important in order to not only more accurately assess the effects of a particular plan, but also to ensure that there are no positional errors, such as a miscalibrated jaw or leaf, or one that simply gets stuck. The driving software, which controls the jaw/leaf positions, is one such mechanism but one would like to have independent confirmation. The portal imaging devices available on some therapy machines are a good independent record-and-verify system, which can also record the dose. This feature may prove to be especially useful in cases where the dose rate has to be changed between segments.

V. CONCLUSIONS

A segment-by-segment algorithm to generate any arbitrary intensity profile has been presented. The algorithm permits the jaws to move at different speeds, unidirectionally as they sweep across the treatment field. The total beam-on time is minimized by requiring that, in each segment, at least one of the jaws moves at the maximum allowed speed. The algorithm is applicable to both dynamic jaws and multileaf collimators. In the latter case it takes into account transmission through the MLC leaves. For a MLC, the overall beam-on time is determined by the treatment time of the single most complicated profile that the individual pairs of leaves have to generate. Finally, the results of the algorithm compare favorably with previously published ones,[12] but the calculations require typically only 10 ms on a VAX 8550.

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APPENDIX A: DERIVATION OF BASIC EQUATIONS

To derive the paths M_a and M_b at x_{i+1} the algorithm assumes that these are known at x_i. Then it examines the profile to see which jaw moves faster between x_i and x_{i+1}, and sets the speed of that jaw equal to the maximum allowed speed. The speed of the other jaw is set so that the desired intensity at x_{i+1} is obtained.

Assume $M_a(x_i)$ and $M_b(x_i)$ are known. At $x=x_i$ and $x=x_{i+1}$ we have

$$F(x_i)=M_a(x_i)-M_b(x_i)$$

and

$$F(x_{i+1})=M_a(x_{i+1})-M_b(x_{i+1}).$$

The speed of jaw A between x_i and x_{i+1} is

$$V_a(x_{i+1})=\frac{\Delta x_i}{M_a(x_{i+1})-M_a(x_i)},$$

where

$$\Delta x_i=x_{i+1}-x_i.$$

The speed of jaw B is

$$V_b(x_{i+1})=\frac{\Delta x_i}{M_b(x_{i+1})-M_b(x_i)}$$
$$=\frac{\Delta x_i}{M_a(x_{i+1})-M_a(x_i)-F(x_{i+1})+F(x_i)}.$$

If $F(x_{i+1})\geqslant F(x_i)$, i.e., the profile is increasing between x_i and x_{i+1}, then $V_b\geqslant V_a$.

Jaw B is set to maximum speed, $V_b=V_{\max}$; then solving for $M_b(x_{i+1})$ and $M_a(x_{i+1})$ we obtain

$$M_b(x_{i+1})=M_b(x_i)+\Delta x_i/V_{\max}$$

and

$$M_a(x_{i+1})=M_b(x_{i+1})+F(x_{i+1}).$$

If $F(x_{i+1})<F(x_i)$, i.e., the profile is decreasing between x_i and x_{i+1}, then $V_a=V_{\max}$, which results in

$$M_a(x_{i+1})=M_a(x_i)+\Delta x_i/V_{\max}$$

and

$$M_b(x_{i+1})=M_a(x_{i+1})-F(x_{i+1}).$$

APPENDIX B: PROOF OF EFFICIENCY

To prove the efficiency of the algorithm, we will show that, for a particular desired intensity profile, the total beam-on time determined by this algorithm is less than or equal to the total beam-on time of any other path of jaw or leaf motions that generate the same intensity profile. The proof is given here for the MLC case; for the case of a single pair of dynamic jaws only the indices in the sums need be changed.

Let $M_{p,a}(x)$ and $M_{p,b}(x)$ be the paths of a particular pair of leaves A and B, respectively, determined by the algorithm presented in the main text of this paper, and $M_{p,a}'(x)$ and $M_{p,b}'(x)$ be any other set of paths that generates the same intensity profile. Then, dropping for simplicity the index p, we have

$$F(x_i)=M_a(x_i)-M_b(x_i)=M_a'(x_i)-M_b'(x_i)\quad \text{for all } i=1,\ldots,N.$$

Consider the segment-to-segment differences $M_a(x_{i+1})-M_a(x_i)$ and $M_a'(x_{i+1})-M_a'(x_i)$. There are two cases to examine.

(a) If $F(x_{i+1})<F(x_i)$, then

$$\left.\begin{array}{l} M_a(x_{i+1})-M_a(x_i)=\Delta x_i/V_{\max} \\ M_a'(x_{i+1})-M_a'(x_i)=\Delta x_i/V_a' \\ V_{\max}\geqslant V_a' \end{array}\right\}$$
$$\Rightarrow M_a(x_{i+1})-M_a(x_i)\leqslant M_a'(x_{i+1})-M_a'(x_i),$$

where $\Delta x_i=x_{i+1}-x_i$ and V_a' is the speed of leaf A at the primed path.

(b) If $F(x_{i+1})\geqslant F(x_i)$, then, using Eq. (2a),

$$M_a(x_{i+1})=M_b(x_{i+1})+F(x_{i+1})$$
$$=M_b(x_i)+\Delta x_i/V_{\max}+F(x_{i+1})$$
$$=M_a(x_i)-F(x_i)+\Delta x_i/V_{\max}+F(x_{i+1})$$

or

$$\left.\begin{array}{l} M_a(x_{i+1})-M_a(x_i)=\Delta x_i/V_{\max}+F(x_{i+1})-F(x_i) \\ M_b'(x_{i+1})-M_b'(x_i)=\Delta x_i/V_b' \\ V_{\max}\geqslant V_b' \end{array}\right\}$$
$$\Rightarrow M_a(x_{i+1})-M_a(x_i)\leqslant M_b'(x_{i+1})-M_b'(x_i)+F(x_{i+1})-F(x_i)$$
$$\Rightarrow M_a(x_{i+1})-M_a(x_i)\leqslant M_a'(x_{i+1})-M_a'(x_i).$$

Therefore, $M_a(x_{i+1})-M_a(x_i)\leqslant M_a'(x_{i+1})-M_a'(x_i)$ for all $i=1,\ldots,N-1$.

The total beam-on time can be written as a telescoping series in terms of these segment-to-segment differences:

$$M_a(x_N)=\sum_{i=1}^{N-1}[M_a(x_{i+1})-M_a(x_i)]+M_a(x_1).$$

Using the fact that $M_a(x_1)=F(x_1)=M_a'(x_1)$ and the inequalities derived previously, the total beam-on time becomes

$$M_a(x_N)\leqslant\sum_{i=1}^{N-1}[M_a'(x_{i+1})-M_a'(x_i)]+M_a'(x_1)$$
$$\leqslant M_a'(x_N).$$

Therefore, the total beam-on time determined by this algorithm is less than or equal to that of any other set of leaf motions that generate the same intensity profile.

APPENDIX C: DERIVATION OF TOTAL BEAM-ON TIME

For a MLC where the leaves are forced to close-in at the end, the total beam-on time is simply $M_a(x_N)$, since leaf A is the slower of the two.

Here $M_a(x_N)$ can be calculated by noticing that, if x_m is the starting position of leaf B, then $M_a(x_m)=F(x_m)$ and subsequently, for $i=m,\ldots,N-1$, the set of equations (2) is applicable. Therefore, so one can consider the differences $M_a(x_{i+1})-M_a(x_i)$ for $i=m,\ldots,N-1$ and write

$$M_a(x_N)=F(x_m)+\sum_{i=m}^{N-1}[M_a(x_{i+1})-M_a(x_i)].$$

If $F(x_{i+1})\geqslant F(x_i)$ we have, using Eq. (2a),

$$M_a(x_{i+1})=M_b(x_{i+1})+F(x_{i+1})$$
$$=M_b(x_i)+\Delta x_i/V_{\max}+F(x_{i+1})$$
$$=M_a(x_i)-F(x_i)+\Delta x_i/V_{\max}+F(x_{i+1}).$$

Hence, $M_a(x_{i+1})-M_a(x_i)=\Delta x_i/V_{\max}+F(x_{i+1})-F(x_i)$.

If, on the other hand, $F(x_{i+1})<F(x_i)$, then by Eq. (2b)

$$M_a(x_{i+1})-M_a(x_i)=\Delta x_i/V_{\max}.$$

Therefore,

$$M_a(x_{i+1})-M_a(x_i)=\begin{cases}\dfrac{\Delta x_i}{V_{\max}}+F(x_{i+1})-F(x_i) & \text{if } F(x_{i+1})\geqslant F(x_i),\\ \dfrac{\Delta x_i}{V_{\max}} & \text{if } F(x_{i+1})<F(x_i).\end{cases}$$

The total beam-on time is then

$$M_a(x_N)=F(x_m)+\begin{cases}\sum_{i=m}^{N-1}\left[\dfrac{\Delta x_i}{V_{\max}}+F(x_{i+1})-F(x_i)\right]\\ \quad\text{such that } F(x_{i+1})\geqslant F(x_i),\\ \sum_{i=m}^{N-1}\dfrac{\Delta x_i}{V_{\max}}\quad\text{such that } F(x_{i+1})<F(x_i).\end{cases}$$

The sums containing the $\Delta x_i/V_{\max}$ terms can be collected to yield

$$\sum_{\substack{i=m\\F(x_{i+1})\geqslant F(x_i)}}^{N-1}\frac{\Delta x_i}{V_{\max}}+\sum_{\substack{i=m\\F(x_{i+1})<F(x_i)}}^{N-1}\frac{\Delta x_i}{V_{\max}}=\sum_{i=m}^{N-1}\frac{\Delta x_i}{V_{\max}}=\frac{x_N-x_m}{V_{\max}}.$$

Therefore, the total beam-on time is

$$M_a(x_N)=\frac{x_N-x_m}{V_{\max}}+F(x_m)+\sum_{\substack{i=m\\F(x_{i+1})\geqslant F(x_i)}}^{N-1}[F(x_{i+1})-F(x_i)].\tag{C1}$$

This expression can easily be generalized to the case of a single pair of jaws. In particular, the time it takes jaw A to complete its path is

$$M_a(x_k)=\frac{x_k-x_m}{V_{\max}}+F(x_m)+\sum_{\substack{i=m\\F(x_{i+1})\geqslant F(x_i)}}^{k-1}[F(x_{i+1})-F(x_i)],\tag{C2}$$

where x_k is the finishing position of jaw A.

One can similarly derive the expression for the time it takes jaw B to complete its path:

$$M_b(x_N)=\frac{x_k-x_m}{V_{\max}}+F(x_k)-F(x_N)+\sum_{\substack{i=m\\F(x_{i+1})\leqslant F(x_i)}}^{k-1}[F(x_i)-F(x_{i+1})].\tag{C3}$$

It follows by subtraction that

$$M_a(x_k)-M_b(x_N)=F(x_N),$$

so that jaw A is *always* the slower of the two.

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