

Store the original Smith and Wagner estimators and restate them in terms of estimated probability (ep) instead of n; Specify the gain estimator

```
In[21]:= mu = Simplify[enl + erl +  $\frac{-1 + enl + erl}{-1 + n}$  /. n -> 1 / ep]

gamma = Simplify[ $\frac{n (-1 + enl + erl + epl n)}{(-1 + n)^2}$  /. n -> 1 / ep]

alpha = Simplify[ $\frac{n (-1 + epl + erl + enl n)}{(-1 + n)^2}$  /. n -> 1 / ep]

gain = Simplify[ $\frac{\text{gamma}}{1 - \text{mu}}$ ]
```

$$\text{Out[21]= } \frac{enl - ep + erl}{1 - ep}$$

$$\text{Out[22]= } \frac{epl + ep (-1 + enl + erl)}{(-1 + ep)^2}$$

$$\text{Out[23]= } \frac{enl + ep (-1 + epl + erl)}{(-1 + ep)^2}$$

$$\text{Out[24]= } \frac{epl + ep (-1 + enl + erl)}{(-1 + ep) (-1 + enl + erl)}$$

Solve for implied probability when alpha =0; Substitute the value into the gamma and gain estimator

```
In[25]:= nls = Flatten[Solve[alpha == 0, ep]]
```

$$\text{Out[25]= } \left\{ ep \rightarrow -\frac{enl}{-1 + epl + erl} \right\}$$

```
In[26]:= gammaZ = Simplify[gamma /. nls]
gainZ = Simplify[gain /. nls]
```

$$\text{Out[26]= } -\frac{(enl - epl) (-1 + epl + erl)}{-1 + enl + epl + erl}$$

$$\text{Out[27]= } \frac{enl - epl}{-1 + enl + erl}$$

## Restate estimators in terms of $p$ and $\Delta$

```
In[28]:= gammaE = gamma /. ep -> p + Δ
gainE = gain /. ep -> p + Δ
Out[28]= 
$$\frac{ep_l + (-1 + en_l + er_l) (p + \Delta)}{(-1 + p + \Delta)^2}$$

Out[29]= 
$$\frac{ep_l + (-1 + en_l + er_l) (p + \Delta)}{(-1 + en_l + er_l) (-1 + p + \Delta)}$$

```

## Calculate the elasticity of the two estimators

```
In[30]:= ElasGain = Simplify[D[gainE, Δ]  $\frac{\Delta}{gainE}$ ]
Out[30]= 
$$- \frac{(-1 + en_l + ep_l + er_l) \Delta}{(-1 + p + \Delta) (ep_l + (-1 + en_l + er_l) (p + \Delta))}$$

In[31]:= ElasGamma = Simplify[D[gammaE, Δ]  $\frac{\Delta}{gammaE}$ ]
Out[31]= 
$$- \frac{\Delta (2 ep_l + en_l (1 + p + \Delta) + (-1 + er_l) (1 + p + \Delta))}{(-1 + p + \Delta) (ep_l + (-1 + en_l + er_l) (p + \Delta))}$$

```

## Calculate the ratio of the elasticities

```
In[32]:= R = FullSimplify[ $\frac{ElasGain}{ElasGamma}$ ]
Out[32]= 
$$\frac{-1 + en_l + ep_l + er_l}{2 ep_l + en_l (1 + p + \Delta) + (-1 + er_l) (1 + p + \Delta)}$$

```

## Calculate the ratio of the elasticities given $\alpha = 0$

```
In[34]:= RA = Simplify[R /. p + Δ -> ep /. nls]
Out[34]= 
$$- \frac{-1 + ep_l + er_l}{1 + en_l - 2 ep_l - er_l}$$

```