BRITISH PHYSICS OLYMPIAD

2010 AS Competition Paper

Mark Scheme and Notes for Teachers

Thank you for entering the 2010 AS Competition. Please note the following information:

Before the test

- It is intended that the paper should be taken on Friday 19th March. However, if this is not possible, any date during the period 15th –26thth March will be acceptable.

During the test

- The paper lasts one hour.
- Candidates may use any calculator and should write their answers directly on the exam script.

After the test

- Teachers should mark their students' scripts. The mark scheme begins overleaf.
- Medals are awarded in the following manner

Award	Gold	Silver	Bronze	Participation
Mark Range	50 – 38	37 – 31	30 – 20	19 – 0

- Free certificates can be claimed for participating students. To order certificates please go to http://www.physics.ox.ac.uk/olympiad/Enter.html

- The scripts of any Gold Medalists (those scoring 38 or above) should be sent to the address below for consideration in the national competition. Please ensure the student's name and school is written clearly. Scripts must arrive by Monday 29th March to be considered for a national prize.

BPhO Office, Clarendon Laboratory, Parks Road, Oxford, OX1 3PU

- Five outstanding Gold Medalists, together with their teachers will be invited to the BPhO Presentation Ceremony at The Royal Society in London on Thursday 29th April 2010.

If you have any further questions please contact us on schools.liaison@physics.ox.ac.uk

Marking

The mark scheme is prescriptive, but markers must make some allowances for alternative answers.

A value quoted at the end of a section must have the units included. Candidates lose a mark the first time that they fail to include a unit, but not on subsequent occasions except where it is a specific part of the question.

Significant figures are related to the number of figures given in the question. A single mark is lost the first time that there is a gross inconsistency (more than 2 sf out) in the final answer to a section.

Ecf: this is allowed in numerical sections provided that unreasonable answers are not being obtained. Ecf can not be carried through for more than one section after the first mistake (e.g. a mistake in section (d) can be carried through into section (e) but not then used in section (f)).

Section A: Multiple Choice

1. **D**
2. **B**
3. **B**
4. **D**
5. **D**

6. **A**
7. **B**
8. **A**
9. **C**
10. **A**

There is 1 mark for each correct answer, maximum 10 marks available.

Reasoning

1. As B slides along, *mg* at the centre of mass is closer to the "pivot" at B. The moments about B are such that the anticlockwise moment provide by *mg* is balanced by the clockwise moment provided by F at A. But the distance ratios are not 2:1 as before.

2. Constant acceleration so the speed increases linearly with time. But the distance fallen increases as t^2 ($s = ½\,gt^2$). So the average speed occurs at half the time taken to pass the window, which is before it has covered half the height of the window.

3. To halve the pressure takes 20 s, ¼ takes a further 20 s and then to 1/8 takes another 20 s. This is 60 s in total; 2½ times the volume will take 150 seconds.

4. Substitute the units for m and v into $(½)mv^2$: $1\text{ m x }(1\text{ m s}^{-1})^2 = 10^3\text{ g x }(10^2\text{ cm s}^{-1})^2$

   $\text{I J} = 10^7\text{ erg}$

5. A larger component of mg is supported by T_R than T_L.

6. At constant velocity there is no resultant force on the soup in the bowl (Newton's 1st Law) and the soup would be level. As the train decelerates the soup's inertia keeps it moving forwards so it rises up due to the shape of the bowl. It stops rising when the weight of liquid is equal to the vertical component of the reaction force of the bowl.

7. Energy of the electron = 8×10^{-8} J. Snail mass is about 0.035 kg speed 2/1000 m s^{-1} so energy ~7×10^{-8} J. Estimate $\frac{1}{2} mv^2$ for the other quantities. The oxygen molecule is several thousand times more massive than the electron at rest, but then the electron travels close to the speed of light. The oxygen molecule does not ionise other molecules which would require several eV of energy.

8. The mercury levels on the two sides balance. Pressures above the mercury on each side must be the same. The pressure is greater at the bottom of the mercury than at the surface, due to the weight of the mercury.

9. The mass of 1 m^3 of water would be 1000 kg, and a gas is typically 1/1000 th the density of a liquid.

10. The plate expands uniformly. A small square of metal in the hole would expand as a square, so the hole remains square, or the sides of the hole would all expand by the same amount.

Section B: Written Answers

11. An archaeologist at an excavation discovers a crown that looks like gold. It has a mass of 546 g and a volume of 34.6 cm³. However, chemical analysis shows that the bar consists of a mixture of gold and silver. Unfortunately the analysis is unable to give the proportions without removing a sample. The problem is to find the mass of gold in the crown. We assume that the volume of the crown is equal to the initial volumes of gold and silver of which it is composed.

Density of gold, $\rho_g = 19.3\ \text{g cm}^{-3}$
Density of silver, $\rho_s = 10.5\ \text{g cm}^{-3}$

a) Write down an equation relating the masses m_g and m_s of gold and silver in the crown and an equation for the corresponding volumes V_g and V_s.

$m_s + m_g = 546$ g ✓

$V_s + V_g = 34.6$ cm³ ✓ [2]

b) Write down an equation for the total mass in terms of the volumes V_g, V_s and densities ρ_g, ρ_s of the gold and silver in the crown.

$\rho_s V_s + \rho_g V_g = 546$ ✓

[1]

c) In the equation from (b) substitute for V_s and then substitute for V_g to obtain a relation between ρ_g, ρ_s and m_g.

$\rho_s(34.6 - V_g) + \rho_g V_g = 546$ ✓

$\rho_s(34.6 - m_g/\rho_g) + m_g = 546$ ✓

$m_g\,(1 - \rho_s/\rho_g) = 546 - 34.6\,\rho_s$ or similar forms of results ✓ [3]

d) Determine the value of the mass of gold, m_g.

$m_g = 401$ g $= 400$ g ✓

[1]

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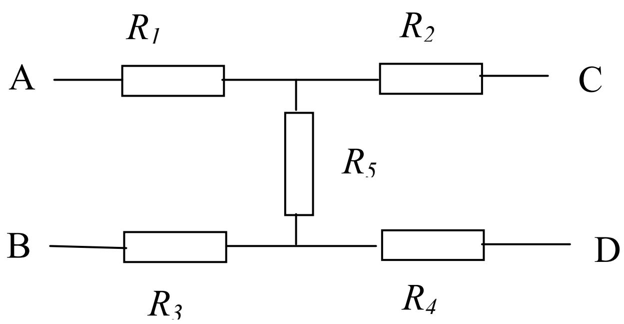
12. A combination of resistors shown below represents a pair of transmission lines with a fault in the insulation between them. The wires have a uniform resistance, but do not have the same resistance as each other. The following procedure is used to find the value of the resistance R_5.



A potential difference of 1.5 V is connected in turn across various points in the arrangement.

With 1.5 V applied across terminals AC a current of 37.5 mA flows

With 1.5 V applied across terminals BD a current of 25 mA flows

With 1.5 V applied across terminals AB a current of 30 mA flows

With 1.5 V applied across terminals CD a current of 15 mA flows

a) Write down four equations relating the potential difference, the resistor values and the currents.

______AC $R_1 + R_2 = 1.5\text{ V} / 37.5\text{ mA}$ $R_1 + R_2 = 40\ \Omega$____________(✓)___

______BD $R_3 + R_4 = 1.5\text{ V} / 25\text{ mA}$ $R_3 + R_4 = 60\ \Omega$____________(✓)___

______AB $R_1 + R_5 + R_3 = 1.5\text{ V} / 30\text{ mA}$ $R_1 + R_5 + R_3 = 50\ \Omega$ ______(✓)___

______CD $R_2 + R_5 + R_4 = 1.5\text{ V} / 15\text{ mA}$ $R_2 + R_5 + R_4 = 100\ \Omega$________(✓)___ [4]

One mark off for each mistaken result

b) Determine the value of resistor R_5.

______For R_5 add: AB + CD gives $R_1 + R_5 + R_3 + R_2 + R_5 + R_4 = 50 + 100\ \Omega$____✓_____

for reasonable attempt at solution

______And using AC and BD we obtain $40\ \Omega + 60\ \Omega + 2\ R_5 = 150\ \Omega$________________

______So that $R_5 = 25\ \Omega$____✓__ [2]

Reasonable ecf allowed

c) If the ends C and D are connected together, what would be the resistance measured between A and B?

______For CD connected together, we have $R_2 + R_4$ in parallel with 25 Ω________✓ mark for idea if numbers wrong

______Using CD, ($R_2 + R_5 + R_4$ = 100 Ω) we find $R_2 + R_4$ is 75 Ω____________________

______75 Ω // 25 Ω is 18.75 Ω____✓_____

______And this is in series with $R_1 + R_3$ which from AB is 25 Ω_____________________

______Resulting in 25 Ω +18.75 Ω which is 44 Ω___________________________✓___ [3]

d) If the length AC (and also BD) is 60 metres of resistive wire, how far from A (or C) does the fault occur?

_An intuitive guess that the fault is ¼ of the distance from A or C (15 metres)____✓_____

__Some justification gains the second mark_$R_1 + R_3$ = 25 Ω___$R_2 + R_4$ = 75 Ω___✓_____

______Or even further with

$$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$

$$So\ R_1 + \frac{R_1 R_4}{R_2} = 25$$

$$R_1\left(1 + \frac{R_4}{R_2}\right) = 25$$

similarly

$$R_2\left(1 + \frac{R_4}{R_2}\right) = 75$$

$$\frac{R_1}{R_2} = 1/3 = \frac{15\,metres}{45\,metres}$$

or

$$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$

$$So\ \frac{R_2 R_3}{R_4} + R_3 = 25$$

$$R_3\left(\frac{R_2}{R_4} + 1\right) = 25$$

similarly

$$R_4\left(\frac{R_2}{R_4} + 1\right) = 75$$

$$\frac{R_3}{R_4} = 1/3 = \frac{15\,metres}{45\,metres}$$

(ratio is important, not the 15 m)

_______________ [2]

/11

13. Waves on the open sea, known as gravity waves in order to distinguish them from ripples on a pond, have a speed v that depends upon the wavelength λ and the depth of the sea, h.

In **deep water**, $h >> \lambda$ and the speed v is independent of h, but does depend upon λ.

$$v = \sqrt{\frac{g\lambda}{2\pi}}$$

In **shallow water**, $h << \lambda$, and the speed v is independent of λ, but does depend upon h.

$$v = \sqrt{gh}$$

a) For a ship in **deep water**, the motion of the ship creates a wave such that the faster the speed the longer the wavelength. At some speed, known as the hull speed, v_{hull}, the wavelength becomes equal to the length of the ship L, as shown below. It is then very difficult for the ship to increase its speed as it has to climb the wave at the bow.

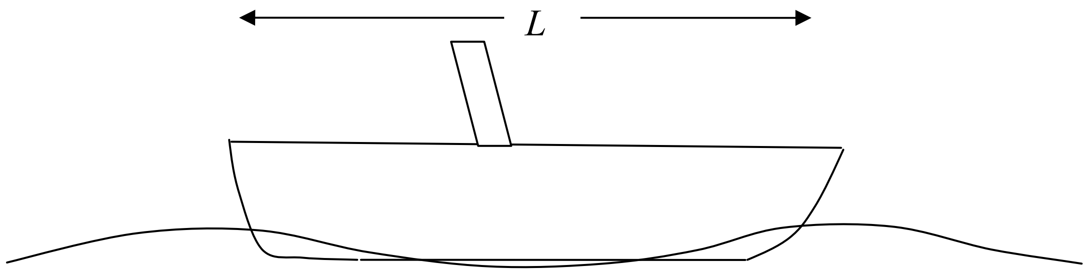


Figure 1

Show that $v_{hull} = 1.2\ L^{½}$

______ $v = \sqrt{\frac{g\lambda}{2\pi}}$ and with $\lambda = L$____

___________ $\sqrt{\frac{g}{2\pi}} = 1.25 = 1.2$______________________________ ✓ __________ [1]

b) The formula $v_{hull} = 1.2\ L^{½}$ only works when L is measured in metres. Explain why.

___________we have substituted a specific unit dependent value for g___✓___________

___________which is in metres________________owtte______________✓__________ [2]

c) Show that for deep water waves, $v = \frac{g}{2\pi}T$ where T is the period of the wave.

$v = \sqrt{\frac{g\lambda}{2\pi}}$ so $v^2 = \frac{g}{2\pi}\frac{v}{f}$ and hence $v = \frac{gT}{2\pi}$ ✓

[1]

d) A Tsunami (a wave produced as the result of an earthquake) on the ocean has an immense wavelength of 80 km (so the **shallow water** situation applies). Calculate the speed of the wave when the depth of the ocean is 4.7 km, and also when it enters the coastal shallows where the depth is 10 m.

$v = \sqrt{gh}$

for h = 4.7 km then v = 215 m s^{-1} = 2.2 x 10^2 m s^{-1} ✓

for h = 10 m then v = 9.9 m s^{-1} ✓ [2]

e) The power P associated with a Tsunami wave progressing across the ocean is proportional to the speed of the wave, v (the speed of energy flow), and the square of the amplitude A. The power flowing past a point is constant (otherwise energy would accumulate). Show that for the Tsunami, A is proportional to $h^{-1/4}$.

$P \propto v A^2$ and if the power is constant then $v \propto 1/A^2$ ✓

But $v = \sqrt{gh}$ so $\sqrt{h} \propto A^{-2}$ and hence $h^{\frac{1}{4}} \propto \frac{1}{A}$ ✓

A proportional to h to power (-1/4) [2]

f) If the amplitude of the wave is 35 cm on the open ocean where the depth is 4.7 km, calculate the amplitude of the wave when the depth of the water is 10 metres.

$A = kh^{-\frac{1}{4}}$ So $\frac{A}{0.35} = \left(\frac{10}{4700}\right)^{-\frac{1}{4}}$ A = 1.6 m ✓

(or $A_1^4 h_1 = A_2^4 h_2$ so $(0.35)^4$ x 4700 = A_2^4 x 10) [1]

g) If the distant from the source of the Tsunami is only a few thousand kilometres then the Earth can be considered as a flat surface. However, if the distance from the source is very great then the curvature of the surface of the Earth will focus the waves. The intensity of the wave varies as $\dfrac{1}{\sin\left(\dfrac{r}{R}\right)}$ where r is the distance from the source and R is the radius of the Earth. At what distance from the source will the wave intensity begin to increase due to focusing?

$$R = 6{,}400 \text{ km}$$

Note that in $\sin(r/R)$ the term r/R will give the angle in radians.

_When the factor $\sin(r/R) = 1$ then the intensity factor changes from the intensity decreasing to the intensity increasing.____

______This is when $\pi/2 = r/R$______________________________________✓__________

______At $r = R\pi/2$_______________ $R = 10{,}053$ km $= 1.0 \times 10^4$ km____✓__________ [2]

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14. Light travelling at speed c behaves both as a particle (a photon) and a wave. As a particle, the particle energy, E, is given by $E = hf$ where h is Planck's constant and f is the frequency of light. As a wave it is described by a frequency f and wavelength λ.

In figure 2 below, a thick walled insulating sphere, of internal radius 20 cm, behaves like the inside of a furnace. Negligible heat escapes through the walls when it is hot, but after a long period of time the 24 W heater at the centre has warmed the internal wall to such an extent that it is at equilibrium, having reached the same temperature as the heater. There is a small observation hole in the wall of the furnace through which radiation escapes.

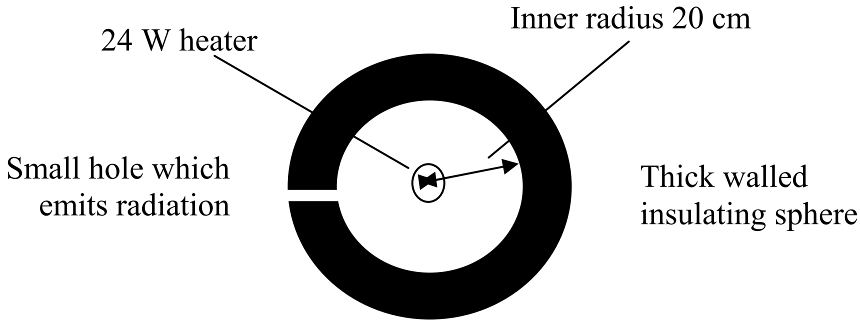

Figure 2

speed of light $c = 3.0 \times 10^{8}$ m s^{-1}

volume of a sphere $= \dfrac{4}{3}\pi r^3$

a) When the final temperature is achieved, someone looking through the observation hole can see the heater and one eighth of the surface area of the furnace. How much power is emitted from the observation hole? Explain your answer.

24 W ✓ in equilibrium/temperature would change/due to increased or decreased energy

in the furnace owtte ✓ [2]

b) If the observation hole is closed for a short time, what change will occur inside the furnace?

The energy increases ✓

The temperature will rise ✓ [2]

c) A photon emitted from the heater is absorbed by the wall. Determine the time between emission and absorption of a photon.

$t = r/c = 6.7 \times 10^{-10}$ s ✓

[1]

d) If the heater emits radiation for the time period obtained in part (c), calculate the amount of this energy.

E = power x time

$E = 6.7 \times 10^{-10} \times 24 = 1.6 \times 10^{-8}$ J ✓ ecf [1] [1]

e) Calculate the number n of photons emitted in this time if we assume that they have the average wavelength of 2900×10^{-9} m.

Planck's constant $h = 6.6 \times 10^{-34}$ Js

n = energy/ (hc/λ)

$= 1.6 \times 10^{-8} \times 2900 \times 10^{-9} \div (6.6 \times 10^{-34} \times 3 \times 10^{8})$ ✓ for frequency

$= 2.3 \times 10^{11}$ photons ✓✓ ecf [3]

f) In fact there are several orders of magnitude more photons in the furnace because the walls, with their large surface area, also radiate. The photons exert a pressure P on the furnace walls, which is given by $P = \frac{1}{3}U$, where U is the energy per unit volume of the radiation in the furnace. When it is at a steady temperature, the total radiation energy in the furnace is 2.5×10^{-5} J. Calculate the pressure P on the walls.

Pressure = $(1/3) \times 2.5 \times 10^{-5} \div ((4/3) \times \pi \times 0.2^{3})$

$= 2.5 \times 10^{-4}$ Pa ✓

[1]

g) If we used the energy emitted by the heater in a short time as in part (d), estimate how many orders of magnitude greater is the pressure of radiation in the furnace than that pressure which corresponds to the amount of energy in part (d).

(not rigorous) ratio = $10^{-5} / 10^{-8}$ ecf

$= 10^{+3}$ or three orders, owtte ✓ [1]

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