PHYSICS ADMISSIONS TEST
Wednesday, 30 October 2019

Time allowed: 2 hours

For candidates applying to Physics, Physics and Philosophy, Engineering, or Materials Science

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Total 24 questions [100 Marks]

Answers should be written on the question sheet in the spaces provided, and you are encouraged to show your working. You should attempt as many questions as you can.

No tables, or formula sheets may be used.

Answers should be given exactly and in simplest terms unless indicated otherwise.

Indicate multiple-choice answers by circling the best answer. Partial credit may be given for correct workings in multiple choice questions.

The numbers in the margin indicate the marks expected to be assigned to each question. You are advised to divide your time according to the marks available.

You may take the gravitational field strength on the surface of Earth to be $g \approx 10\,\mathrm{m\,s^{-2}}$

Do NOT turn over until told that you may do so.

1. What is the next number in the sequence? -972, 324, -108, 36, -12 [2]

A	B	C	D	E
-4	-3	3	4	9

Divide by -3

2. Which values of x and y solve the following equations simultaneously: [2]

$$\log x + 2\log y = \log 32$$
$$\log x - \log y = -\log 2$$

A	B	C	D	E
$x = 2$ $y = 4$	$x = -2$ $y = -4$	$x = 2$ $y = -4$	$x = -2$ $y = 4$	no solution exists

$\log x + \log(y^2) = \log 32$

$\log(xy^2) = \log 32$

$xy^2 = 32$

$\log\left(\frac{x}{y}\right) = \log\left(\frac{1}{2}\right)$

$\frac{x}{y} = \frac{1}{2}$

$y = 2x$

$\Rightarrow \quad x(2x)^2 = 32$

$4x^3 = 32$

$x = 2 \qquad y = 4$

3. Consider a system of many interacting particles. Let each particle have a potential energy $V(r)$ with respect to any other particle, where $V(r) \propto r^n$ where r is the distance to another particle and n is an integer. For such systems the Virial Theorem relates the time averaged total kinetic energy of all particles $\langle T_{tot} \rangle$ to the time averaged total potential energy $\langle V_{tot} \rangle$ as follows:

$$2\langle T_{tot} \rangle = n\langle V_{tot} \rangle$$

If the particles in our system interact only via gravity, what is the time averaged total energy $\langle E_{tot} \rangle$ of the system? [2]

A	B	C	D	E
$\langle E_{tot} \rangle = 0$	$\langle E_{tot} \rangle = 2\langle V_{tot} \rangle$	$\langle E_{tot} \rangle = \langle V_{tot} \rangle/2$	$\langle E_{tot} \rangle = -\langle V_{tot} \rangle$	$\langle E_{tot} \rangle = -2\langle V_{tot} \rangle$

$V(r) \propto r^{-1}$ for gravity

$\Rightarrow 2\langle T_{tot} \rangle = -\langle V_{tot} \rangle$

$\langle E_{tot} \rangle = \langle T_{tot} \rangle + \langle V_{tot} \rangle$

$= \dfrac{-\langle V_{tot} \rangle}{2} + \langle V_{tot} \rangle$

$= \dfrac{\langle V_{tot} \rangle}{2}$

4. The acceleration g due to gravity on a spherical planet in any universe is given by:

$$g = \frac{GM}{R^2}$$

where M is the mass, R the radius of the planet and G is the gravitational constant in that planet's universe.

In a different universe the gravitational constant is G' and has twice the value of the gravitational constant in our Universe G.

Find the ratio $\frac{g_{planet}}{g_{Earth}}$ for a planet in the different universe which has half the radius and twice the density of the Earth. [2]

A	B	C	D	E
$\frac{g_{\text{planet}}}{g_{\text{Earth}}} = 2$	$\frac{g_{\text{planet}}}{g_{\text{Earth}}} = 1$	$\frac{g_{\text{planet}}}{g_{\text{Earth}}} = \frac{1}{2}$	$\frac{g_{\text{planet}}}{g_{\text{Earth}}} = 4$	$\frac{g_{\text{planet}}}{g_{\text{Earth}}} = \frac{1}{4}$

$G' = 2G,\ R' = R/2,\ \rho' = 2\rho$

$m' = \rho' V' = 2\rho \frac{4}{3}\pi (R')^3 = 2\rho \cdot \frac{4}{3}\pi \left(\frac{R}{2}\right)^3 = \frac{1}{4}\rho \frac{4}{3}\pi R^3$

$= \frac{M}{4}$

$g' = \frac{G'M'}{(R')^2} = \frac{2G \cdot M/4}{(R/2)^2} = 2G \cdot \frac{m}{4} \cdot \frac{4}{R^2} = 2g$

5. In which range of α does the following equation have real solutions? [2]

$$\sec^2\theta + \alpha\tan\theta = 0$$

A	B	C	D	E
$\alpha \leq -2$ or $\alpha \geq 2$	$\alpha \leq -2$	$\alpha \geq 2$	$\alpha \geq -0$	$\alpha \leq 0$

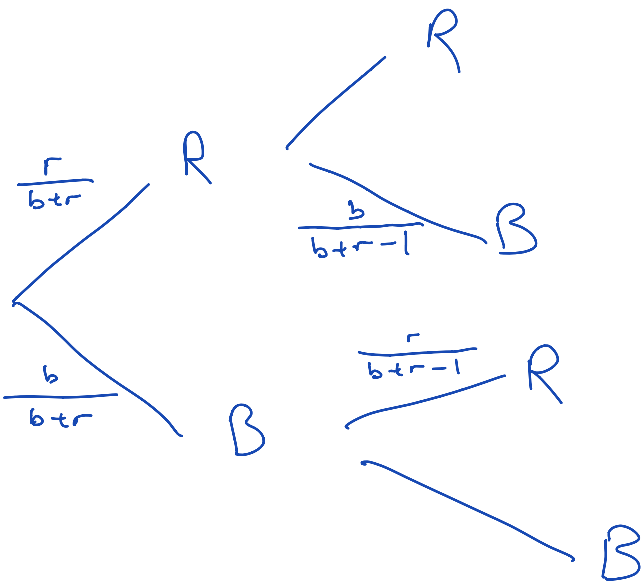
$\tan^2\theta + 1 + \alpha\tan\theta = 0$

Discriminant $\geqslant 0$: $\alpha^2 - 4 \geqslant 0$

$\alpha \geqslant 2$ or $\alpha \leq -2$

6. A bag contains b blue balls and r red balls. If two balls are picked at random and removed from the bag, what is the probability P that they are different colours? [2]

A	B	C	D	E
$\frac{2br}{(b+r)(b+r-1)}$	$\frac{br}{(b+r)(b+r-1)}$	$\frac{br}{(b+r)^2}$	$\frac{2br}{(b+r)^2}$	$2br$



$$\left(\frac{r}{b+r}\right)\left(\frac{b}{b+r-1}\right)+\left(\frac{b}{b+r}\right)\left(\frac{r}{b+r-1}\right) = \frac{2br}{(b+r)(b+r-1)}$$

7. We wish to represent integer numbers by using our ten fingers. A finger is assumed to be either stretched out or curled up. How many different integers can we represent with our fingers?

[2]

A	**B**	**C**	**D**	**E**
10	512	1000	20	1024

$2^{10} = 1024$

8. Without explicit calculation state which integrals are non-zero.

$$I_1 = \int_{-3\pi}^{3\pi} x^2 \sin(x) dx \quad = 0 \quad \text{odd} \tag{1}$$

$$I_2 = \int_{-\infty}^{\infty} e^{-x^2} dx \quad \neq 0 \quad \text{even} \tag{2}$$

$$I_3 = \int_{3\pi/2}^{-3\pi/2} \cos^2(x) dx \quad \neq 0 \quad \text{even} \tag{3}$$

$$I_4 = \int_{-\infty}^{\infty} x e^{-x^2} dx \quad = 0 \quad \text{odd} \tag{4}$$

[2]

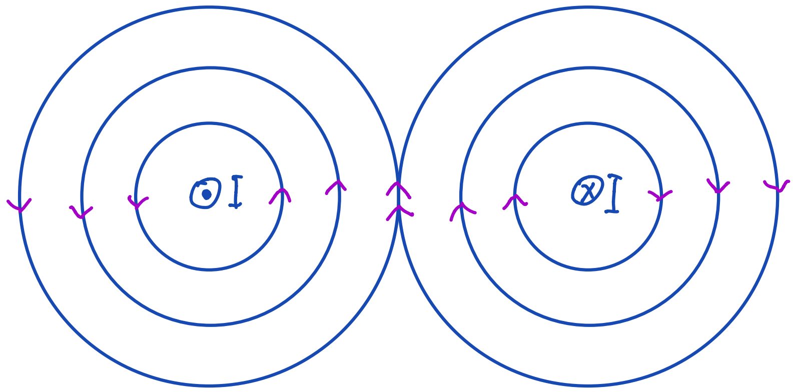
A	**B**	**C**	**D**	**E**
2 and 3	1 and 4	1 and 3	2 and 4	all

9. A long, thin, straight wire carrying an electric current I causes a magnetic field of flux density B at a perpendicular distance r from the wire. The magnitude of this flux density is given by the following relation:

$$B = \frac{\alpha I}{r}$$

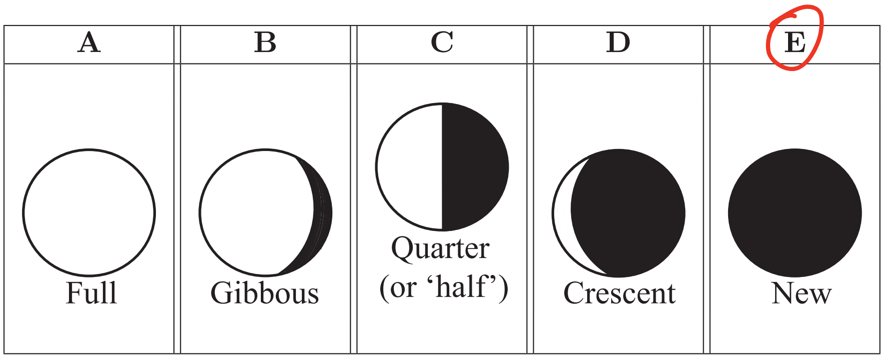
where α is a constant. The magnetic field points circumferentially around the wire. A second, identical wire is placed parallel to the first one at a distance D. Find the current I_2 that has to flow in the second wire if the flux density at a line half way between and parallel to the wires is to double, compared to the flux density from only one wire at current I. [2]

A	**B**	**C**	**D**	**E**
$I_2 = I$	$I_2 = 2I$	$I_2 = -2I$	$I_2 = -I$	$I_2 = -I/2$



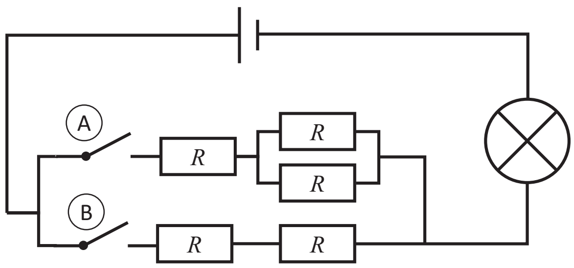
10. When the phase of the Moon as seen from the Earth is Full, what phase of the Earth is seen by an observer on the Moon?

[2]



The symbols above show phases of the Earth as seen from the Moon

11. In the circuit shown below all resistors have the same resistance R and the light bulb has a fixed resistance. You wish to change the state of the switches so that the brightness of the bulb increases from its minimum to its maximum. Which sequence of switch states will achieve this? [2]



	A	B	C	D	E
1. 2. 3.	both closed only A closed only B closed	both closed only B closed only A closed	only B closed only A closed both closed	only A closed only B closed both closed	all states give the same brightness

Brightness $\propto (V_{bulb})^2/R$

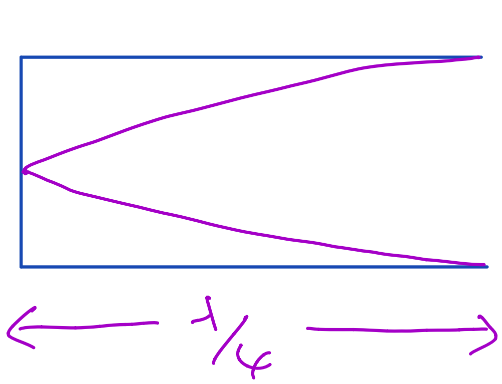
Only A: $R_T = \frac{3R}{2}$

Only B: $R_T = 2R \Rightarrow$ lowest V_{bulb}

Both: $R_T = \frac{3R/2 \times 2R}{7R/2} = \frac{6R}{7} \Rightarrow$ highest V_{bulb}

12. An organ pipe is open at one end and closed at the other. The lowest note you can play on this pipe has frequency f_{min}. If the speed of sound in the pipe is v, what is the length L of the pipe? [2]

A	B	C	D	E
$L = \frac{v}{2f_{min}}$	$L = \frac{v}{4f_{min}}$	$L = \frac{v}{f_{min}}$	$L = \frac{2v}{f_{min}}$	$L = \frac{4v}{f_{min}}$



$v = f\lambda$

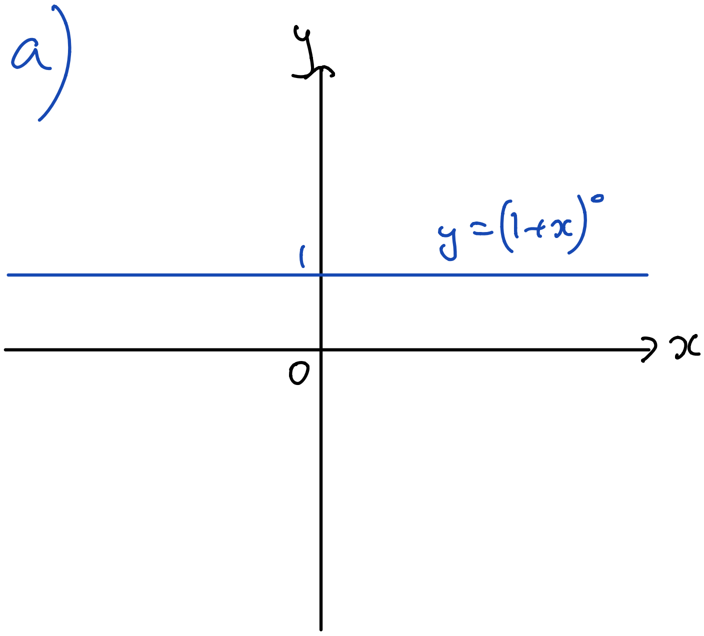
$L = \frac{\lambda}{4} = \frac{v}{4f}$

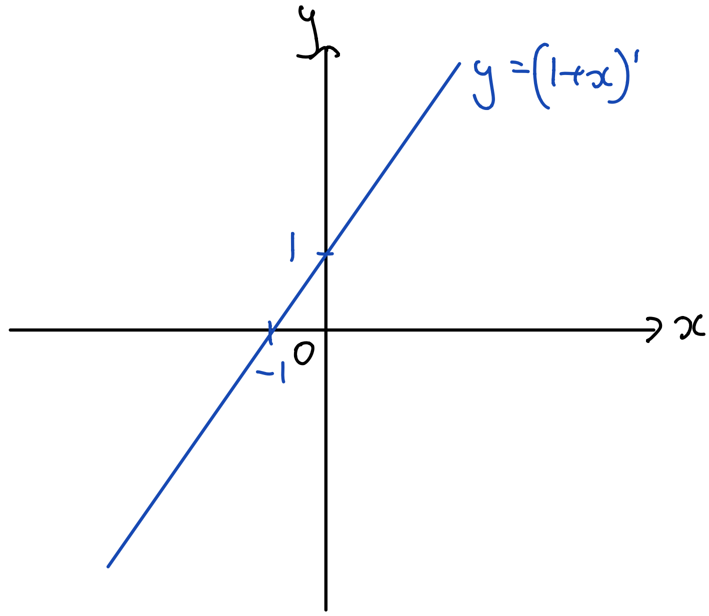
13.

(a) Sketch the graphs of $y = (1 + x)^n$ for integer values of n from 0 to 3, each on a separate set of axes. Which point(s) are common to all the graphs ? [4]

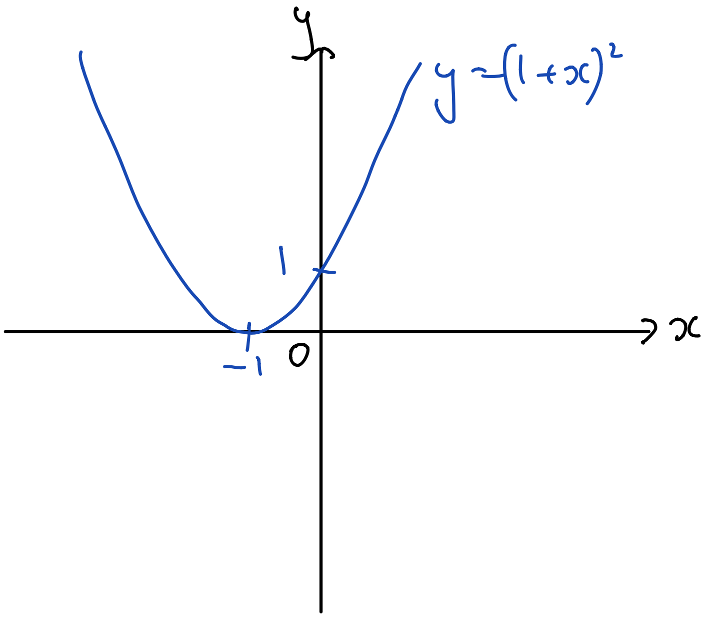
(b) Describe two of the further features common to the graphs for integer $n > 1$. [2]

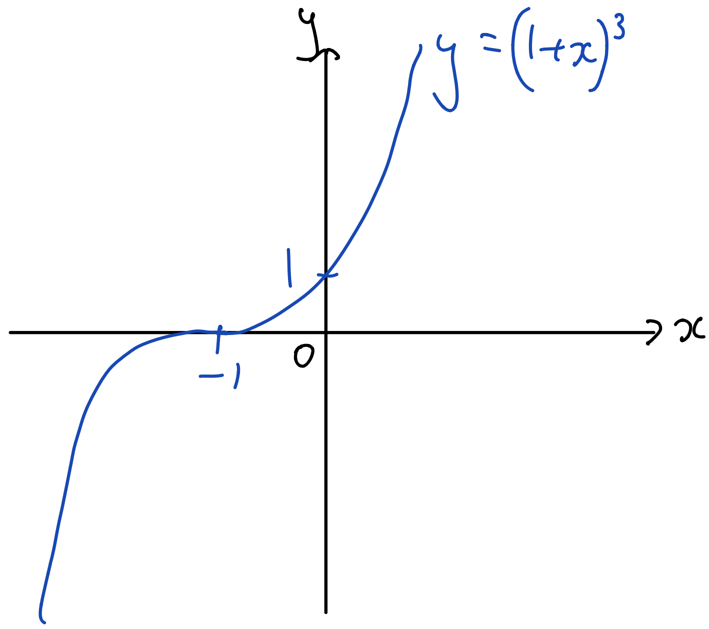
a)





Common point: $(0, 1)$





b) • $(-1, 0)$ is a common point

• $y \to \infty$ as $x \to \infty$

14. A radioactive sample contains two isotopes, A and B. Isotope A decays to isotope B with a half life of $t_{1/2}$. Isotope B is stable.

(a) The number of atoms of A left after a time t is given by:

$$N_A(t) = N_{A0}\, e^{-\lambda t}$$

where N_{A0} is the initial number of atoms of A. Derive an expression for λ in terms of $t_{1/2}$. [2]

(b) Initially the number of B atoms in the sample is $N_B(t=0) = N_{B0}$. Let $N_B(t)$ be the time dependent number of B atoms in the sample. Write down an expression for $N_B(t)$ in terms of λ, N_{B0}, N_{A0} and t. [1]

(c) At the start there are x times as many A atoms in the sample as there are B atoms. How long does it take until this ratio is reversed? [3]

a) At half life, $\frac{N_{A0}}{2} = N_{A0} e^{-\lambda t_{1/2}}$

$$\ln\left(\frac{1}{2}\right) = -\lambda t_{1/2}$$

$$\underline{\lambda = \frac{\ln 2}{t_{1/2}}}$$

b) $N_B(t) = N_{B0} + N_{A0} - N_A(t)$

$$\underline{= N_{B0} + N_{A0}\left(1 - e^{-\lambda t}\right)}$$

c) $N_{A0} = x N_{B0}$

When reversed, $N_B(t) = x N_A(t)$

$$N_{B0} + N_{A0}\left(1 - e^{-\lambda t}\right) = x N_{A0} e^{-\lambda t}$$

$$\frac{1}{x} + 1 - e^{-\lambda t} = x e^{-\lambda t}$$

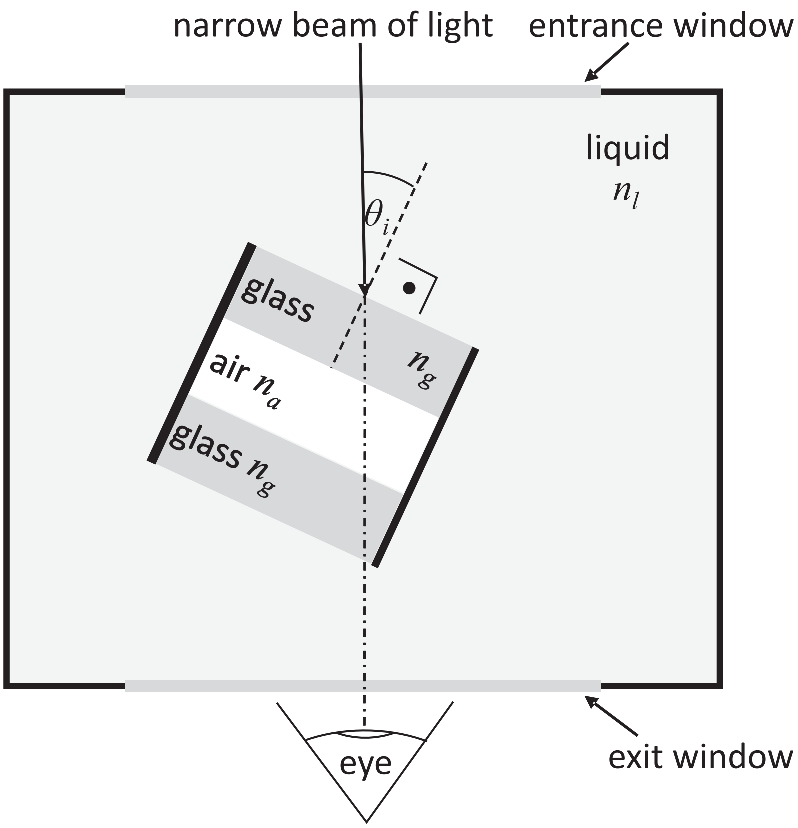
$$\frac{1}{x} + 1 = e^{-\lambda t}(x+1)$$

$$e^{-\lambda t} = \frac{x+1}{x} \times \frac{1}{x+1}$$

$$-\lambda t = \ln\left(\frac{1}{x}\right)$$

$$\underline{t = \frac{\ln x}{\lambda}}$$

15. The diagram below shows an air-cell refractometer.



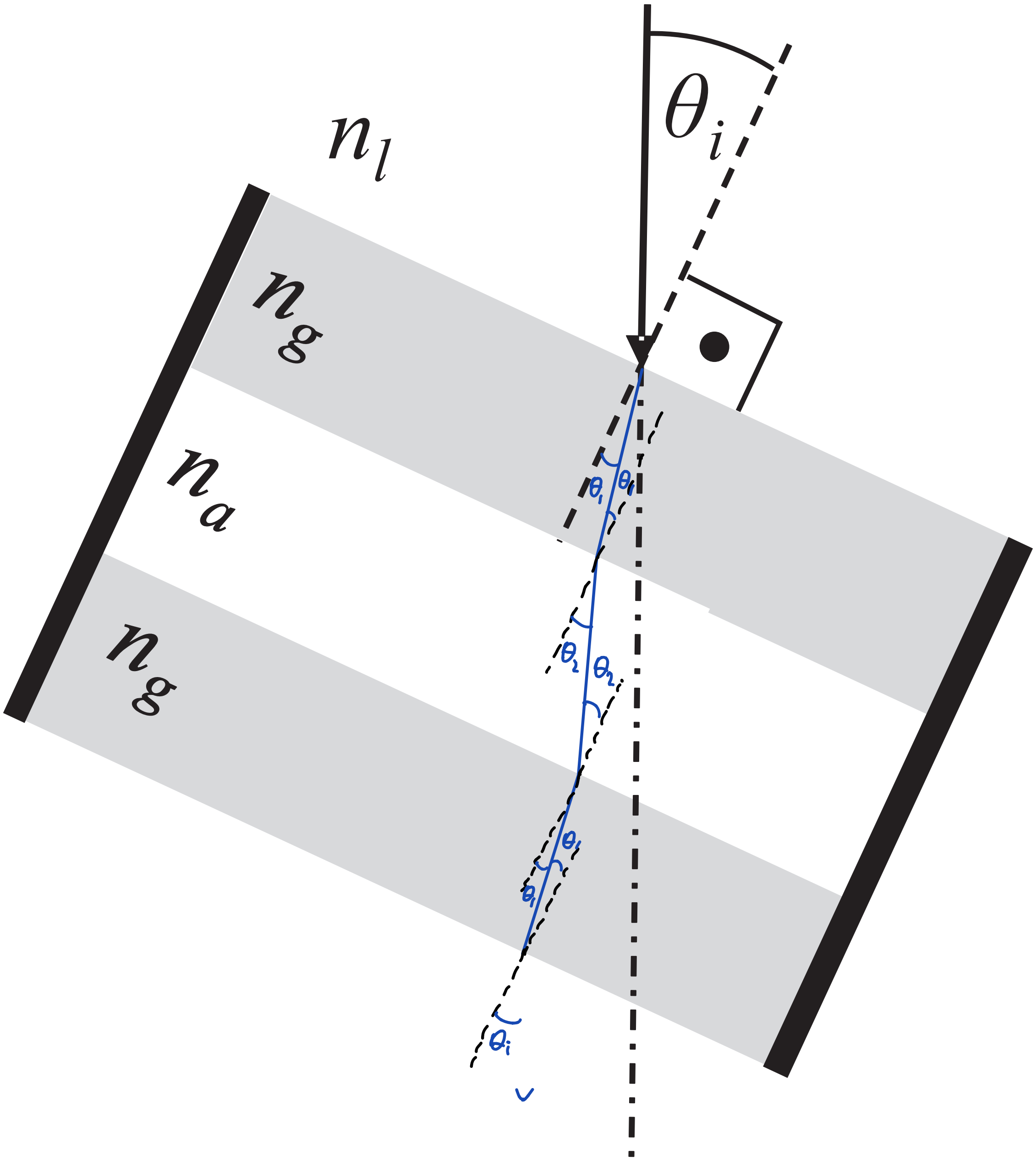
A narrow beam of light enters the entrance window at normal incidence and you can observe the light leaving the exit window by eye. The outer box is filled with a liquid of unknown refractive index n_l. The glass of the air cell has refractive index n_g and the cell is filled with air of refractive index n_a. The air-cell in the liquid filled vessel makes angle θ_i with respect to the incoming beam of light, as shown in the diagram. This angle can be precisely adjusted and measured.

(a) On the diagram provided on the next page, draw the refracted path of the beam through the air cell. For this diagram you should assume $n_g > n_l > n_a$. [3]

(b) Describe qualitatively what you will observe at the exit window as you increase θ_i from zero and hence explain how this instrument could be used to determine the refractive index of the liquid n_l in the chamber. Find the relation between n_l and a special value of θ_i. [3]

(c) Suggest with reasons, a way to modify the apparatus or its use to improve the measurement. [2]

b) Light observed moves towards the edge of the air cell until it is no longer seen because of total internal reflection at the glass-air boundary



$n_g \sin\theta_c = n_a \sin 90$
$= 1$

$n_l \sin\theta_i = n_g \sin\theta_c$

$n_l = \dfrac{1}{\sin\theta_i}$

To find n_l, measure the angle θ_i when the light is no longer observed at the exit window

c) Use gas with higher refractive index than air. This increases θ_i at TIR, which can be measured with less percentage error

16. The energy levels of the electron in a hydrogen atom are characterised by a quantum number n:

$$E_n = -\frac{hcR}{n^2}$$

where h is Planck's constant, c is the speed of light and R is the Rydberg constant.

(a) State a formula that relates the wavelength of light λ to h, c and E which is the energy of a photon. [1]

(b) Let p and q be the quantum numbers of the upper and lower energy levels of an electron transition in hydrogen. Find a formula that relates the wavelength of light emitted in such transitions to p and q. [2]

(c) For each of the three hydrogen emission line sets shown in the table below, identify the quantum number of its lower energy level q . Each set (column) of five emission lines has the same lower energy level. The first column shows the quantum number of the upper energy level q relative to the lower level.

p	Set-A λ [nm]	Set-B λ [nm]	Set-C λ [nm]
$q+1$	121.57	4051	7460
$q+2$	102.57	2625	4654
$q+3$	97.254	2166	3741
$q+4$	94.974	1944	3297
$q+5$	93.780	1817	3039

You may assume that $p < 6$ and $R = 10973731.6\,\mathrm{m}^{-1}$ [4]

a) $E = \frac{hc}{\lambda}$

b) $\frac{hc}{\lambda} = E_p - E_q$

$= \frac{-hcR}{p^2} + \frac{hcR}{q^2}$

$\frac{1}{\lambda} = R\frac{(p^2-q^2)}{p^2q^2}$

$\lambda R = \frac{p^2q^2}{p^2-q^2}$

c) For $p = q+1$,

set	A	B	C
λR	1.33	44.5	81.9

q	1	2	3	4	5
$\frac{p^2q^2}{p^2-q^2}$	1.33	7.2	20.5	44.4	81.8

$\therefore$ For A, $q = 1$

For B, $q = 4$

For C, $q = 5$

17.

(a) Find an expression for the angle θ for which the grey area A_g is f times the area of the outer square A_S. Your expression for θ should take the form:

$$\theta = B - \frac{C(f)x^2}{x^2 - 1}$$

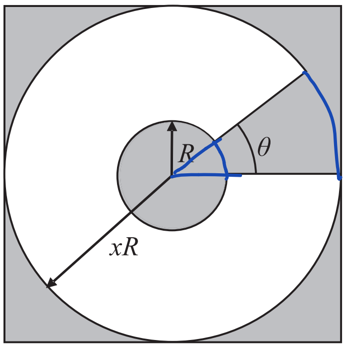
where B is a constant and $C(f)$ is a function of f. State B and $C(f)$ explicitly. You may assume that $x > 1$, $f < 1$ and $f > 0$. The value of xR indicates the radius of the outer circle. [6]

(b) Find the numerical value for θ to five significant figures when $x = 3$ and $f = 1/2$. [1]

Square, $A_S = (2xR)^2 = 4x^2R^2$

Small circle $= \pi R^2$

Big circle $= \pi(xR)^2 = \pi x^2 R^2$



White area = fraction of (big circle − small circle)

$$= \left(\frac{2\pi - \theta}{2\pi}\right)\left(\pi x^2 R^2 - \pi R^2\right)$$

Grey area, A_g = Square − white area

$$= 4x^2R^2 - \left(1 - \frac{\theta}{2\pi}\right)(x^2 - 1)\pi R^2$$

$$A_g = fA_S \Rightarrow 4x^2\cancel{R^2} - \pi\cancel{R^2}(x^2-1) + \frac{\theta}{2}\cancel{R^2}(x^2-1) = 4fx^2\cancel{R^2}$$

$$\theta(x^2-1) = 8fx^2 - 8x^2 + 2\pi(x^2-1)$$

$$\theta = 2\pi - \frac{8(1-f)x^2}{x^2-1}$$

$B = 2\pi$, $C(f) = 8(1-f)$

b) $\theta = 2\pi - \dfrac{8\left(1-\frac{1}{2}\right)(3)^2}{(3)^2 - 1} = 2\pi - \dfrac{9}{2} = 1.7832$

18. Solve the following equation for x:

$$\frac{e^x + 9}{e^{-x} + 5} = 2$$

[3]

$$e^x + 9 = 2e^{-x} + 10$$

$$(e^x)^2 - e^x - 2 = 0$$

$$(e^x - 2)(e^x + 1) = 0$$

$$e^x = 2 \quad \text{or} \quad e^x = -1$$

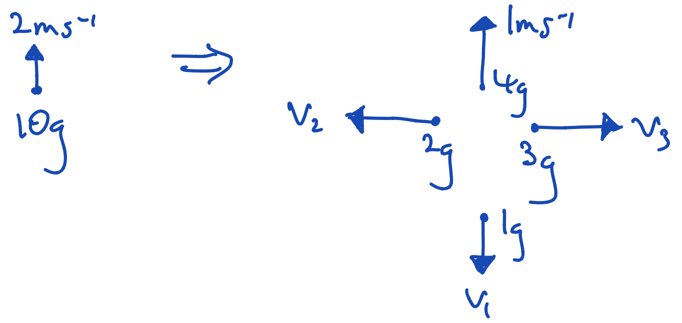
$$\underline{x = \ln 2}$$

19. A firework rocket is launched vertically. At the moment of explosion it is moving with a vertical speed of $v_0 = 2\ \mathrm{m\,s^{-1}}$ upwards. The explosion releases an energy of $E_{exp} = 1$ J and the rocket bursts into four pieces with masses of $m_1 = 1$ g, $m_2 = 2$ g, $m_3 = 3$ g and $m_4 = 4$ g. The piece with mass m_4 moves vertically upwards with a speed of $v_4 = 1\ \mathrm{m\,s^{-1}}$. The pieces of mass m_3 and m_2 move horizontally and the piece of mass m_1 moves vertically. All velocities and directions in this question are given relative to the ground and your answer should do the same.

(a) Obtain the speeds of all the pieces after the explosion. [5]

(b) Higher speed pieces can be obtained if the directions of movement of the pieces are different from those in part (a). Under which choice of directions would the maximum speed of one of the pieces be achieved? [3]

a)



$\uparrow$ CoM: $10(2) = 4(1) - (1)v_1$

$v_1 = 16\ \mathrm{ms^{-1}}$

$\rightarrow$ CoM: $2v_2 + 3v_3 = 0$

$v_2 = -\dfrac{3v_3}{2}$ *

CoE: $\frac{1}{2}(0.01)(2)^2 + 1 = \frac{1}{2}(0.001)(16)^2 + \frac{1}{2}(0.002)v_2^2 + \frac{1}{2}(0.003)v_3^2 + \frac{1}{2}(0.004)(1)^2$

$1.02 - 0.128 - 0.002 = 0.001v_2^2 + 0.0015v_3^2$

$890 = v_2^2 + \frac{3}{2}v_3^2$

$= \left(-\frac{3v_3}{2}\right)^2 + \frac{3}{2}v_3^2$ *

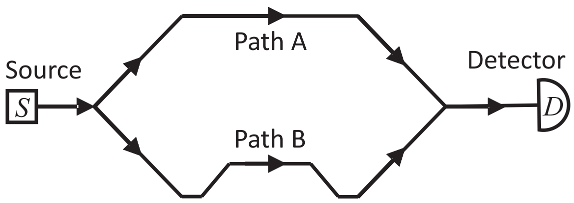
$= \dfrac{15v_3^2}{4}$

$v_3 = 15.4\ \mathrm{ms^{-1}}$

* $\Rightarrow v_2 = -\dfrac{3(15.4)}{2} = -23.1\ \mathrm{ms^{-1}}$

b) m_1 upwards; m_2, m_3 and m_4 downwards
m_1 has the lowest mass so for a certain momentum, it can have the highest speed. Total momentum in the vertical direction needs to be conserved so sending all the others downwards gives the highest momentum for m_1

20. The diagram below shows an interferometer with two paths (Path A and Path B) which a wave can take from its source S to a detector D.



The lengths of the paths differ by an amount L which can change with time. The intensity at the detector I is measured and varies as a function of L as follows:

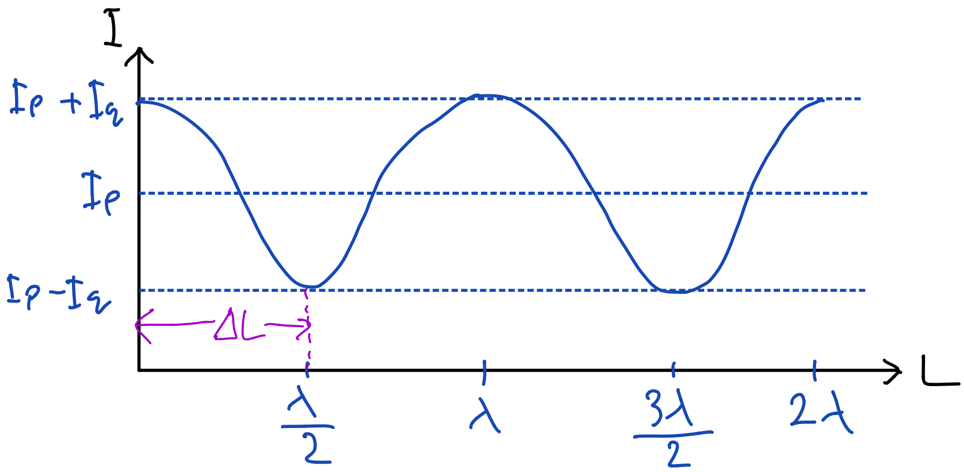
$$I = I_p + I_q \cos(kL)$$

In the above k is the wavenumber of the wave which relates to the wavelength λ via $k = 2\pi/\lambda$. I_p and I_q are constants.

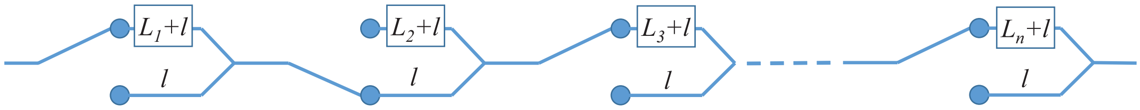
(a) Sketch the intensity as a function of L in the range from 0 to 2λ. Label both axes and identify I_p and I_q in the sketch. [3]

We wish to use the interferometer to measure how the path length difference L changes with time by measuring the intensity at the detector as a function of time. The change in path length difference is ΔL.

(b) Indicate on your sketch the biggest ΔL you can infer unambiguously from a measurement of intensity. [2]



21. You wish to build an adjustable delay line using electrical switches as shown in the diagram below.



Its purpose is to adjust the delay of an electrical signal through the delay line by switching different amounts of delay into the signal path.

The delay line should use the minimum amount of switches.

The delay line should have a minimal delay of L_{min} and a maximal delay of $L_{max} \leq L_{min} + \Delta L$. We refer to ΔL as the delay range.

The delay should be adjustable in increments of δL so that the line can achieve an evenly spaced set of delays between L_{min} and L_{max} with a resolution of δL.

For all segments in the line you have to determine a common, small delay l which is active when its switch is in the lower position.

You further have to determine a larger, individual delay of $L_i + l \gg l$ for each segment which is active when its switch is in the upper position.

For a line with n segments to satisfy the demands on the minimum number of switches, minimum delay L_{min}, delay range ΔL and delay resolution δL:

(a) Find a value for l in terms of L_{min} and n. [1]

$L_{min} = nl$

$l = \dfrac{L_{min}}{n}$

(b) Find the delays of each segment L_i in terms of δL. [3]

(c) Find the minimum necessary n in terms of ΔL and δL [3]

b) Use a 'binary' system: 1, 2, 4, 8 ...

Multiply each by δL, since that's the resolution

$\Rightarrow \delta L, 2\delta L, 4\delta L, \ldots, (2^{i-1})\delta L$

$L_i = (2^{i-1})\delta L$

c) $\Delta L = (1 + 2 + 4 + \ldots 2^{n-1})\delta L$

$= (2^n - 1)\delta L$

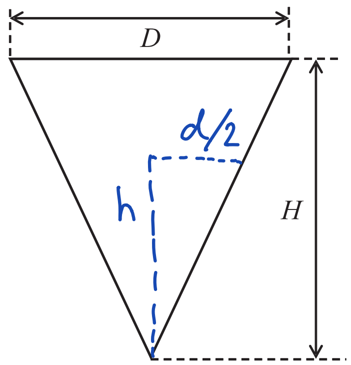
$2^n = \dfrac{\Delta L}{\delta L} + 1$

$n = \log_2\left(\dfrac{\Delta L}{\delta L} + 1\right)$ This needs to be rounded up to next integer

22. A conical cup has dimensions as shown in the diagram of its cross-section below. The cup can hold a maximum volume V when filled to its full depth H. Find an expression for the depth h to which you have to fill the cup so that it contains a volume of liquid equal to $V/2$. Your expression for h should only depend on the dimensions of the cup. [4]

$$V = \frac{1}{3}\pi\left(\frac{D}{2}\right)^2 H$$

$$v = \frac{1}{3}\pi\left(\frac{d}{2}\right)^2 h$$



$$\frac{D/2}{H} = \frac{d/2}{h}$$

$$\frac{d}{2} = \left(\frac{D}{2}\right)\left(\frac{h}{H}\right)$$

$$v = \frac{V}{2}$$

$$\cancel{\frac{1}{3}}\cancel{\pi}\left(d/2\right)^2 h = \frac{1}{2}\cdot\cancel{\frac{1}{3}}\cancel{\pi}\left(\frac{D}{2}\right)^2 H$$

$$\cancel{\left(\frac{D}{2}\right)^2}\left(\frac{h}{H}\right)^2 \cdot h = \frac{1}{2}\cancel{\left(\frac{D}{2}\right)^2} H$$

$$h = \frac{H}{\sqrt[3]{2}}$$

23. In an imaginary water filtration process a fraction of $1/n$ of an impurity is removed in the first pass of the water through the system. In each succeeding pass, the amount of impurity removed is $1/n$ of the amount removed in the preceding pass. Show that if $n = 2$ the water can be made arbitrarily pure but if $n = 3$, at least half of the impurity will remain. [5]

Let S_r be the fraction of impurity removed in pass r.

$S_1 = \frac{1}{n}$, $S_2 = \left(\frac{1}{n}\right)^2$, $S_3 = \left(\frac{1}{n}\right)^3$, ...

Total fraction removed:

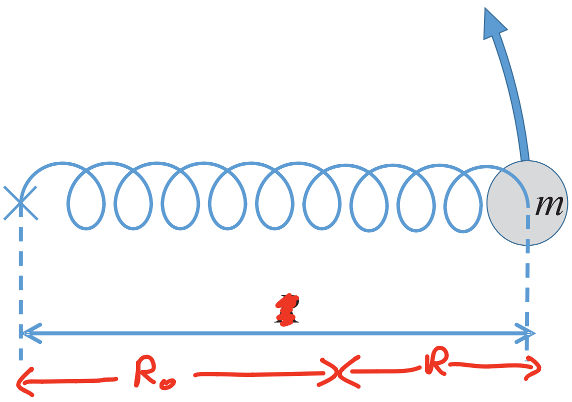
$$S = S_1 + S_2 + \dots + S_\infty$$
$$= \frac{1}{n} + \left(\frac{1}{n}\right)^2 + \left(\frac{1}{n}\right)^3 + \dots$$
$$= \frac{1/n}{1 - 1/n}$$
$$= \frac{1}{n-1}$$

For $n = 2$, $S = 1$

$n = 3$, $S = \frac{1}{2}$

$\therefore$ All the impurities can be removed if $n = 2$, but half remains if $n = 3$

24. The sketch below shows a ball of mass m on a spring of unextended length R_0 and spring constant k. The spring is pivoted on the left on a central axis marked with a cross. The axis is perpendicular to the plane of the sketch. The ball and spring rotate around the central axis on a smooth horizontal table as indicated by the arrow in the sketch. The spring will break if it is stretched with a force larger than F_{max}.



(a) Find the equilibrium extension R of the system when it rotates with angular frequency ω. [3]

(b) Find the equilibrium angular frequency ω_c at which the spring will break. [1]

(c) Sketch ω_c against F_{max} in the range from zero to one Newton for the following parameters $m = 1\,\mathrm{kg}$, $R_0 = 1\,\mathrm{m}$, $k = 1\,\mathrm{N\,m^{-1}}$. Label your axes. [2]

(d) Under some conditions the system can only achieve a maximum angular frequency $\omega_i < \omega_c$. Find a relationship between k, m and ω_i and explain what happens to the system as the angular frequency increases to ω_i. [4]

a) $F = kR = m\omega^2(R_0 + R)$

$kR - m\omega^2 R = m\omega^2 R_0$

$R = \dfrac{m\omega^2 R_0}{k - m\omega^2}$

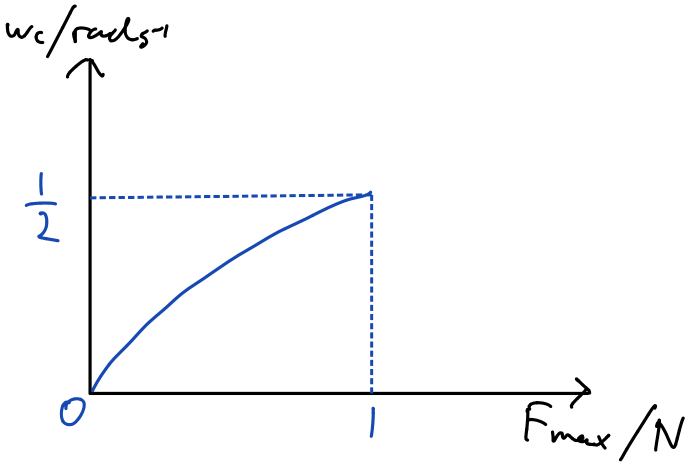
b) $F_{max} = k R_{max}$

$= \dfrac{k m \omega_c^2 R_0}{k - m\omega_c^2}$

$k F_{max} = k m \omega_c^2 R_0 + F_{max} m \omega_c^2$

$\omega_c = \sqrt{\dfrac{k F_{max}}{m(kR_0 + F_{max})}}$

c) $\omega_c = \sqrt{\dfrac{F_{max}}{1 + F_{max}}}$



d) $k - m\omega_i^2 = 0$

$\omega_i = \sqrt{\dfrac{k}{m}}$

This is the natural freq. of the spring. System rotates in circular motion with increasing R until ω_i. After this, the spring cannot provide enough centripetal force to keep the mass in circular motion