ENGAA 2016
Section 2

Model Solutions

1

1a

The stiffest sample has the least extension per tension force applied (steepest gradient) hence the correct answer is S1, as it has the steepest gradient of the lines shown on the graph.

1b

$$\begin{aligned} strain &= \frac{extension}{\text{original length}} \times 100 \\ 2 &= \frac{x}{100} \times 100 \\ \therefore x &= 2mm \end{aligned}$$

From the graph, S2 is not a straight line up to 2mm of extension, hence it does not obey Hooke's law up to 2% strain.

1c

$$\begin{aligned} E &= \frac{Fl}{Ax} \\ &= \frac{100}{5} \cdot \frac{250}{25 \times 10{-}6} \\ &= 200 \times 10^6 Pa \\ &= 200MPa \end{aligned}$$

1d

The work done is equal to the area underneath the force-extension curve, which is equal to the definite integral from 0 to $10^{-2}m$.

$$\begin{aligned} &\int_0^{10^{-2}} ax - bx^2 \\ &= \left[\frac{a}{2}x^2 - \frac{b}{3}x^3\right]_0^{10^{-2}} \\ &= \frac{a}{2}(10^{-2})^2 - \frac{b}{3}(10^{-2})^3 \\ &= \frac{a}{2}(10^{-4}) - \frac{b}{3}(10^{-6}) \end{aligned}$$

2

2a

The current flowing through each of the three resistors differs because the current is divided at the junctions according to the size of the resistance in the parallel branches. The voltage across R_2 and R_3 is the same as they are connected across the same potential in the circuit.

2b

$$\begin{aligned}\text{Effective Resistance} &= R_1 + \frac{R_2 \cdot R_3}{R_2 + R_3}\\ &= \frac{R_1 \cdot (R_2 + R_3) + R_2 \cdot R_3}{R_2 + R_3}\\ &= \frac{R_1R_2 + R_1R_3 + R_3R_2}{R_2 + R_3}\end{aligned}$$

$$\begin{aligned}V &= IR\\ I &= \frac{V}{R}\\ &= V \div \frac{R_1R_2 + R_1R_3 + R_3R_2}{R_2 + R_3}\\ &= \frac{V(R_2 + R_3)}{R_1R_2 + R_1R_3 + R_3R_2}\end{aligned}$$

2c

Using Potential Dividers Equation:

$$\begin{aligned}\text{V across } R_3 &= \frac{\frac{R_1R_3}{R_1+R_3}}{\frac{R_1R_3}{R_1+R_3} + R_1} \cdot V\\ V_3 &= \frac{R_1R_3}{R_1R_3 + R_1(R_1 + R_3)} \cdot V\\ &= \frac{R_3}{2R_3 + R_1} \cdot V\\ V_3^2 &= \frac{R_3^2}{(2R_3 + R_1)^2} \cdot V^2\end{aligned}$$

$$\begin{aligned}P &= \frac{V_3^2}{R_3}\\ &= \frac{R_3^2}{(2R_3 + R_1)^2} \cdot V^2 \div R_3\\ &= \frac{V^2R_3}{(2R_3 + R_1)^2}\end{aligned}$$

2d

$$
\begin{aligned}
P &= \frac{V^2 R_3}{(2R_3 + R_1)^2} \\
\frac{1}{P} &= \frac{4R_3^2 + 4R_3R_1 + R_1^2}{V^2 R_3} \\
&= (4R_3 + 4R_1 + \frac{R_1^2}{R_3}) \cdot \frac{1}{V^2} \\
\frac{\mathrm{d}\frac{1}{P}}{\mathrm{d}R_3} &= (4 - R_1^2 \cdot R_3^{-2})\frac{1}{V^2} \\
&= 0 \text{ (at a minimum point)} \\
4 - \frac{R_1^2}{R_3^2} &= 0 \\
4R_3^2 &= R_1^2 \\
R_3 &= \frac{R_1}{2}
\end{aligned}
$$

3

3a

$$
\begin{aligned}
v &= \frac{d}{t} \\
3 \times 10^8 ms^{-1} &= \frac{d}{1 \times 10^{-9} s} \\
d &= 3 \times 10^{-1} m \\
&= 0.30m
\end{aligned}
$$

3b

$$
\begin{aligned}
1.5 &= \frac{3 \times 10^8}{v} \\
v &= 2 \times 10^8 ms^{-1} \\
2 \times 10^8 &= \frac{9 \times 10^3}{t} \\
t &= 4.5 \times 10^{-5} s \\
&= 45 \mu s
\end{aligned}
$$

3c

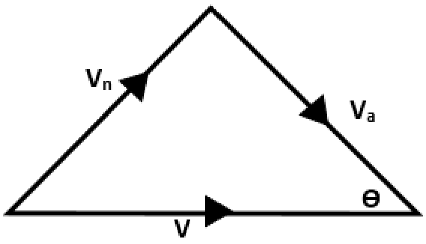
$$
\begin{aligned}
T &= \frac{d}{v} \text{ where d is a constant} \\
T &\propto \frac{1}{v} \\
n &= \frac{c}{v} \\
v &= \frac{c}{n} \text{ where c is a constant}
\end{aligned}
$$

$\therefore T \propto n :$

$$
\begin{aligned}
n_{red} < n_{blue} < n_{nom} \\
T_{red} < T_{blue} < T_{nom}
\end{aligned}
$$

4

4a



$$\text{Gain in KE} = \text{Loss of GPE} + \text{Energy input}$$

$$\frac{1}{2}Mv^2 = Mgh + E$$

$$v^2 = \frac{2(Mgh + E)}{M}$$

$$v = \sqrt{\frac{2(Mgh + E)}{M}}$$

$$v = \sqrt{2\left(gh + \frac{E}{M}\right)}$$

$$cos\theta = \frac{V_a}{V}$$

$$\therefore V_a = \sqrt{2\left(gh + \frac{E}{M}\right)} \cdot cos\theta$$

4b

Considering motion perpendicular to the slope:

$$u = V_n = Vsin\theta$$
$$a = -gcos\theta$$
$$s = 0$$
$$t = ?$$

$$s = ut + \frac{1}{2}at^2$$

$$0 = Vsin\theta t - \frac{gcos\theta}{2}t^2$$

$$t = \frac{2Vsin\theta}{gcos\theta}$$

$$= \frac{2Vtan\theta}{g}$$

4c

Considering motion parallel to the slope:

$$u = V_a = V cos\theta$$
$$a = gsin\theta$$
$$s = L$$
$$t = \frac{2V tan\theta}{g}$$

$$s = ut + \frac{1}{2}at^2$$
$$L = V cos\theta \cdot \frac{2V tan\theta}{g} + \frac{gsin\theta}{2}\left(\frac{2V tan\theta}{g}\right)^2$$
$$= \left(\frac{2V^2}{g}\right)\left[sin\theta + tan^2\theta \cdot sin\theta\right]$$
$$= \left(\frac{2V^2}{g}\right) \cdot sin\theta \cdot \left[1 + \frac{sin^2\theta}{cos^2\theta}\right]$$
$$= \left(\frac{2V^2}{g}\right) \cdot sin\theta \cdot \left[\frac{cos^2\theta + sin^2\theta}{cos^2\theta}\right]$$
$$= \left(\frac{2V^2}{g}\right)\left[\frac{sin\theta}{cos^2\theta}\right]$$

4d

Considering motion parallel to the slope:

$$u = V_a = V cos\theta$$
$$a = gsin\theta$$
$$\text{Acceleration is constant} \quad \therefore t = \frac{2V tan\theta}{g} \div 2$$
$$= \frac{V tan\theta}{g}$$

$$s = ut + \frac{1}{2}at^2$$
$$= V cos\theta \cdot \frac{V tan\theta}{g} + \frac{gsin\theta}{2} \cdot \left(\frac{V tan\theta}{g}\right)^2$$
$$= \left(\frac{V^2}{g}\right)\left[cos\theta tan\theta + \frac{1}{2}tan^2\theta sin\theta\right]$$
$$= \left(\frac{V^2}{g}\right) \cdot sin\theta \cdot \left[1 + \frac{1}{2}tan^2\theta\right]$$