2024 Specimen Paper

(pasted from Pearson Vue)

OXFORD UNIVERSITY

Physics Admissions Test

Welcome to the PAT

Time allowed: 2 hours

For candidates applying to Physics, Physics and Philosophy, Engineering, or Materials Science

Total 40 questions [100 Marks]

You should attempt as many questions as you can.

No tables, or formula sheets or physical calculators may be used.

An online scientific calculator is accessible by clicking on “Calculator” button near the top of the screen.

The numbers on the right-hand-side in square brackets indicate the marks expected to be assigned to each question. You are advised to divide your time according to the marks available.

1. What is the next number in the sequence $\frac{1}{5}, \frac{3}{25}, \frac{7}{125}, \frac{3}{125}, \frac{31}{3125}$

[2]

- $\frac{59}{15625}$
- $\frac{59}{3125}$
- $\frac{63}{15625}$
- $\frac{27}{3125}$
- $\frac{7}{125}$

the sequence is :

a bit more complicated, powers of 2 are 2, 4, 8, 16, 32 ... so linked to that

(n=1) (n=2) (n=3) (n=4) (n=5)

$$\frac{1}{5}, \frac{3}{25}, \frac{7}{125}, \frac{3}{125}, \frac{31}{3125}$$

notice these are ascending powers of 5

$$= \frac{2^1-1}{5^1}, \frac{2^2-1}{5^2}, \frac{2^3-1}{5^3}, \frac{2^4-1}{5^4}, \frac{2^5-1}{5^5}$$

$$\frac{16-1}{625} = \frac{15}{625} \overset{\div 5}{=} \frac{3}{125}$$

so the next term will be

$$\frac{2^6-1}{5^6} = \underline{\underline{\frac{63}{15625}}} \quad (c)$$

2. If $\frac{dy}{dx} = x^2 + \frac{1}{x^3}$ and $y = 0$ when $x = 1$, what is $\int_1^3 y\,dx$?

[3]

- $\frac{20}{3}$
- 8
- $\frac{8}{3}$
- $\frac{4}{3}$
- $\frac{22}{3}$

$\frac{dy}{dx} = x^2 + \frac{1}{x^3} \Rightarrow y = \int x^2 + x^{-3}\,dx$

$= \frac{1}{3}x^3 - \frac{1}{2}x^{-2} + c$

we know when $x=1, y=0$ – use to find c

$(0) = \frac{1}{3}(1)^3 - \frac{1}{2}(1)^{-2} + c \Rightarrow c = \frac{1}{6}$

$\therefore\ y = \frac{1}{3}x^3 - \frac{1}{2}x^{-2} + \frac{1}{6}$

now we know y, we can calculate

$\int_1^3 y\,dx = \int_1^3 \frac{1}{3}x^3 - \frac{1}{2}x^{-2} + \frac{1}{6}\,dx$

$= \left[\frac{1}{12}x^4 + \frac{1}{2}x^{-1} + \frac{1}{6}x\right]_1^3$

$= \left(\frac{3^4}{12} + \cancel{\frac{1}{2(3)}} + \cancel{\frac{1}{6}(3)}\right) - \left(\frac{1}{12} + \cancel{\frac{1}{2}} + \cancel{\frac{1}{6}}\right)$

$= \frac{81}{12} - \frac{1}{12} = \frac{80}{12} = \underline{\underline{\frac{20}{3}}}$ (A)

3. What is the (integer) m such that $\sum_{n=1}^{m} (3+2n) = 140$?

[3]

- ○ 14
- ○ 16
- ○ 8
- ○ 10
- ○ 12
- ○ 20
- ○ 6

important to note:

$$\sum_{r=1}^{k} r = \frac{1}{2}k(k+1)$$

$$\sum_{r=1}^{k} a = \underbrace{a+a+a+\cdots a}_{k \text{ times}} = ka$$

$$\sum_{n=1}^{m} (3+2n) = \sum_{n=1}^{m} 3 + 2\sum_{n=1}^{m} n = 140$$

$$3m + 2\left[\frac{1}{2}m(m+1)\right] = 140$$

$$3m + m^2 + m = 140$$

$$m^2 + 4m - 140 = 0$$

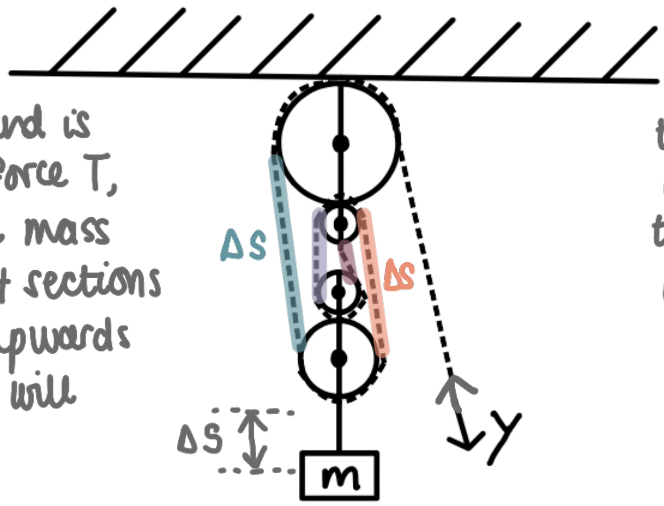
$$(m+14)(m-10) = 0$$

$$m = \cancel{-14}, \underline{10} \quad (c)$$

not an integer

4. Consider the pulley system in the diagram, containing 4 wheels. If you pull the free end a distance *y*, how far will *m* rise by?

If the free-end is pulled by a force T, then since the mass is held by 4 sections of rope, the upwards force on m will be 4T



the rope is 'folded' into 4 (coloured), but since it's all the same rope, the total distance moved by all sections must sum to y.

$$\Delta S + \Delta S + \Delta S + \Delta S = \Delta y$$

$$4\Delta S = \Delta y$$

$$\Delta S = \frac{1}{4}\Delta y \text{ (D)}$$

same tension throughout, so distance moved in each section the same.

[2]

- 2*y*
- *y*/2
- *y*/16
- *y*/4
- 16*y*
- 4*y*

5. The stable isotopes of carbon, nitrogen and oxygen are represented symbolically below:

$^{12}_{6}C$, $^{13}_{6}C$, $^{14}_{7}N$, $^{15}_{7}N$, $^{16}_{8}O$, $^{17}_{8}O$, $^{18}_{8}O$

Select **all** of the following statements that are true:

[3]

A) ☐ $^{18}_{8}O$ has a larger mass per unit charge than $^{12}_{6}C$

B) ☐ $^{14}_{7}N$ has a larger number of neutrons than $^{13}_{6}C$

C) ☐ $^{13}_{6}C$ has a larger number of protons than $^{12}_{6}C$

D) ☐ $^{16}_{8}O$ has a larger nuclear charge than $^{15}_{7}N$

E) ☐ $^{15}_{7}N$ has a larger mass than $^{14}_{7}N$

mass number, #(protons + neutrons)

$^{A}_{Z}X$

proton number, #protons

mass per unit charge $= \frac{\text{mass}}{\text{charge}}$

A) mass per unit charge, $m_p \approx m_n$

$$^{18}_{8}O = \frac{18m_p}{8e} = \frac{9}{4}\left(\frac{m_p}{e}\right) = 2.25\left(\frac{m_p}{e}\right)$$

$$^{12}_{6}C = \frac{12m_p}{6e} = 2\left(\frac{m_p}{e}\right) \quad \therefore \text{TRUE}$$

B) $^{14}_{7}N$ has $14-7=7$ neutrons, $^{13}_{6}C$ has $13-6=7$ neutrons $\therefore$ FALSE

C) $^{13}_{6}C$ has 6 protons, $^{12}_{6}C$ also has 6 $\therefore$ FALSE

D) $^{16}_{8}O$ has charge $8e$, $^{15}_{7}N$ has charge $7e$ $\therefore$ TRUE

E) $^{15}_{7}N$ has mass $\approx 15m_p$, $^{14}_{7}N$ has mass $\approx 14m_p$ $\therefore$ TRUE

6. A seed packet contains 100 seeds. When planted, 75 will successfully become plants, but of these only a third will have flowers, and of these only one fifth will produce fruit. How many seeds produce fruiting plants?

[2]

- ○ 10
- ○ 20
- ○ 5
- ○ 15
- ○ 50
- ○ 1
- ○ 25

100 seeds

$\downarrow$

75 plants

$\downarrow \div 3$

25 flowers

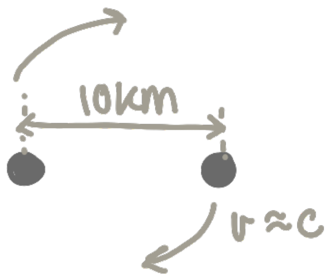
$\downarrow \div 5$

$\underline{\underline{5}}$ fruit

one orbit creates
one wave cycle

7. Two black holes orbit each other and emit gravitational waves arising from the periodic nature of the orbit. The orbital separation is around 10 km, the relative speeds of the black holes are close to the speed of light, and gravitational waves travel at the speed of light. Which of the following would best describe the frequency of the emitted radiation?

[3]



- ○ 10^{-2} Hz
- ○ 10^{7} Hz
- ○ 10^{10} Hz
- ○ 10^{4} Hz
- ○ 10 Hz

speed of orbit given by

$$v = \frac{s}{t} = \frac{2\pi r}{T} \leftarrow \frac{\text{circumfrence}}{\text{time period}}$$

$$= 2\pi r f$$

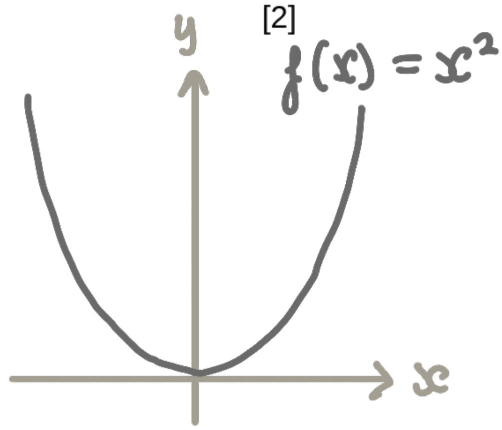
$$v \approx c = 2\pi (5 \times 10^3) f$$

$$f = \frac{3 \times 10^8}{2\pi \times 5 \times 10^3}$$

$$= 9.55 \times 10^3 \approx \underline{\underline{10^4 \text{ Hz}}} \text{ (D)}$$

8. Consider $f(x) = x^2$. You want to transform the function so you get a new function $g(x)$ stretched by a vertical scale factor of 2, with a line of symmetry about $x = 1$ and which is never positive. $g(x)$ would be equal to which of the following functions?

[2]



- ○ $-f(2x-2)$
- ○ $-2f(x+1)$
- ○ $-2f(x-1)$
- ○ $-f(x+1)$
- ○ $-f(x-1)$

- "stretched by a vertical scale factor of two"
  $Af(x)$ stretches a graph vertically, so we are looking for $A = 2$
- "line of symmetry about $x = 1$"
  the current line of symmetry is $x = 0$, so we want to translate it horizontally 1 to the right, using $f(x+B)$ where $B = -1$
- "never positive"
  i.e. flip it in the vertical direction so it is below the x-axis, using $-f(x)$

bringing this together, we get $g(x) = \underline{\underline{-2f(x-1)}}$ (c)

9. If $y = \left(2 + \frac{x}{2}\right)^4$, which of the following is $\frac{\mathrm{d}y}{\mathrm{d}x}$?

[2]

- $2 + x + \frac{3x^2}{8} + \frac{x^3}{8}$
- $32 + 24x + 6x^2 + \frac{x^3}{2}$
- $4 + 2x + \frac{3x^2}{4} + \frac{x^3}{4}$
- $8 + 6x + \frac{3x^2}{2} + \frac{x^3}{8}$
- $16 + 12x + 3x^2 + \frac{x^3}{4}$

chain rule, $\frac{d}{dx}\left[f g(x)\right] = g'(x) f' g(x)$

in this case, $f(x) = x^4$, $g(x) = 2 + \frac{1}{2}x$
$f'(x) = 4x^3$, $g'(x) = \frac{1}{2}$

$\therefore \frac{dy}{dx} = \left(\frac{1}{2}\right)4\left(2 + \frac{1}{2}x\right)^3 = 2\left(2 + \frac{1}{2}x\right)^3$

$= 2\left(2 + \frac{1}{2}x\right)\left(2 + \frac{1}{2}x\right)\left(2 + \frac{1}{2}x\right)$

we can expand fully, or, notice

constant term $= 2(2)(2)(2) = 16$

x^3 term $= 2\left(\frac{1}{2}x\right)\left(\frac{1}{2}x\right)\left(\frac{1}{2}x\right) = \frac{1}{4}x^3$

this matches up with (E)

10. What speed does a bull elephant (mass 4900 kg) have to move at to have the same kinetic energy as a cyclist (mass 100 kg) moving at $30\,\mathrm{km\,h^{-1}}$?

[2]

- ○ $0.6\ \mathrm{m\,s^{-1}}$
- ○ $8.3\ \mathrm{m\,s^{-1}}$
- ○ $16.6\ \mathrm{m\,s^{-1}}$
- ○ $4.2\ \mathrm{m\,s^{-1}}$
- ○ $1.2\ \mathrm{m\,s^{-1}}$

(E = elephant, C = cyclist)

if their kinetic energys are the same, then

$$\cancel{\tfrac{1}{2}} m_E v_E^2 = \cancel{\tfrac{1}{2}} m_C v_C^2$$

$$v_E = \sqrt{\frac{m_C}{m_E}}\, v_C$$

$$v_E = \sqrt{\frac{100}{4900}}\left(\frac{25}{3}\right)$$

$$= \frac{25}{21} \approx \underline{\underline{1.2\,\mathrm{ms^{-1}}}}\ (E)$$

$$30\,\mathrm{kmh^{-1}}$$

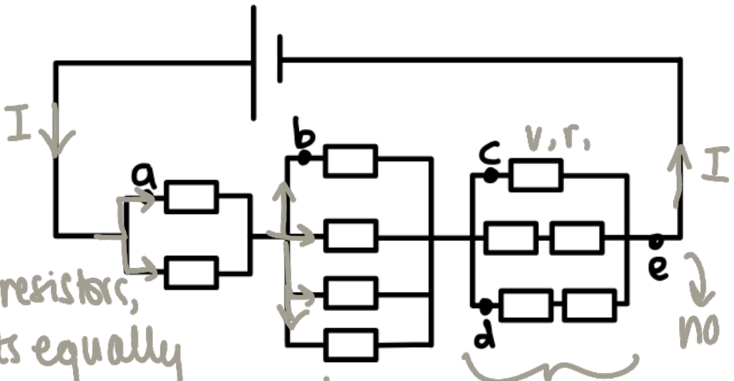
$$= 30 \times 10^3\,\mathrm{mh^{-1}}$$

$$= \frac{30 \times 10^3}{3600}\,\mathrm{ms^{-1}}$$

$$= \frac{25}{3}\,\mathrm{ms^{-1}}$$

11. All resistors in the circuit below have the same value. If an ammeter is placed in the circuit in turn at points (a) through to (e), which of the following sets of points will give the same reading?

KCL : current splits at junctions (i.e, $\Sigma I = 0$ at a node)



equal resistors, so splits equally in 2.

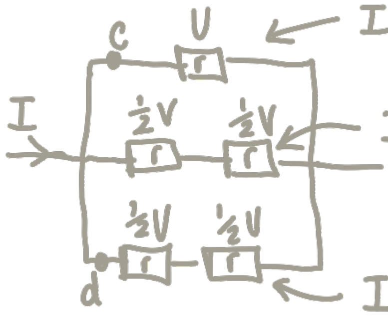
splits equally in 4

no split

KVL ⇒ same pd across parallel branches $[V = IR]$

[2]

- a,c — $I_a = \frac{1}{2}I$
- a,b — $I_b = \frac{1}{4}I$
- b,e — $I_c = \frac{1}{2}I$
- c,d — $I_d = \frac{1}{4}I$
- a,b,c — $I_e = I$



$I_1 = \frac{V}{r}$

$I_2 = \frac{\frac{1}{2}V}{r} = \frac{1}{2}\frac{V}{r}$

$I_3 = \frac{1}{2}\frac{V}{r}$

so current splits in ratio

2 : 1 : 1 (= 4)

& at c, $I_c = \frac{2}{4}I = \frac{1}{2}I$

d, $I_d = \frac{1}{4}I$

12. A particle of mass m , travelling freely at an initial speed v , can be stopped in a distance d by a constant retarding force F . What magnitude of force (applied in a direction perpendicular to the motion) would be needed to change the trajectory of the same particle (at the same speed v) into a circular arc of radius d ?

[2]

- $\sqrt{2}F$
- F
- $F/\sqrt{2}$
- $2F$
- $F/2$
- $4F$

applied force to change energy

$\Rightarrow$ work done $\rightarrow$ kinetic energy

$$Fd = \frac{1}{2}mv^2$$

$$2Fd = mv^2$$

centripetal force, $F_c = \frac{mv^2}{r}$

$$F_c = \frac{mv^2}{d} = \frac{2Fd}{d} = \underline{\underline{2F}} \text{ (D)}$$

13. A device uses 3kW of power at a voltage of 60V. It is connected to a power supply via an ideal transformer. The transformer has *N* turns on the winding connected to the device and 20*N* turns on the winding connected to the power supply. What current flows in the winding connected to the power supply?

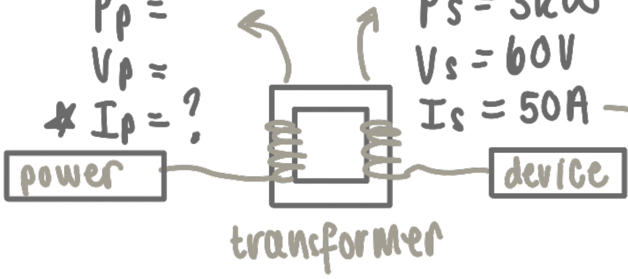
p = primary
s = secondary

[2]

- ○ 50A
- ○ 2.5A
- ○ 1kA
- ○ 0.4A
- ○ 1mA

$N_p = 20N$
$P_p =$
$V_p =$
$* \; I_p = ?$

$N_s = N$
$P_s = 3kW$
$V_s = 60V$
$I_s = 50A$ → calculate using $P = IV$



$$I_s = \frac{P_s}{V_s} = \frac{3 \times 10^3}{60} = 50A$$

(ideal) transformer equation,

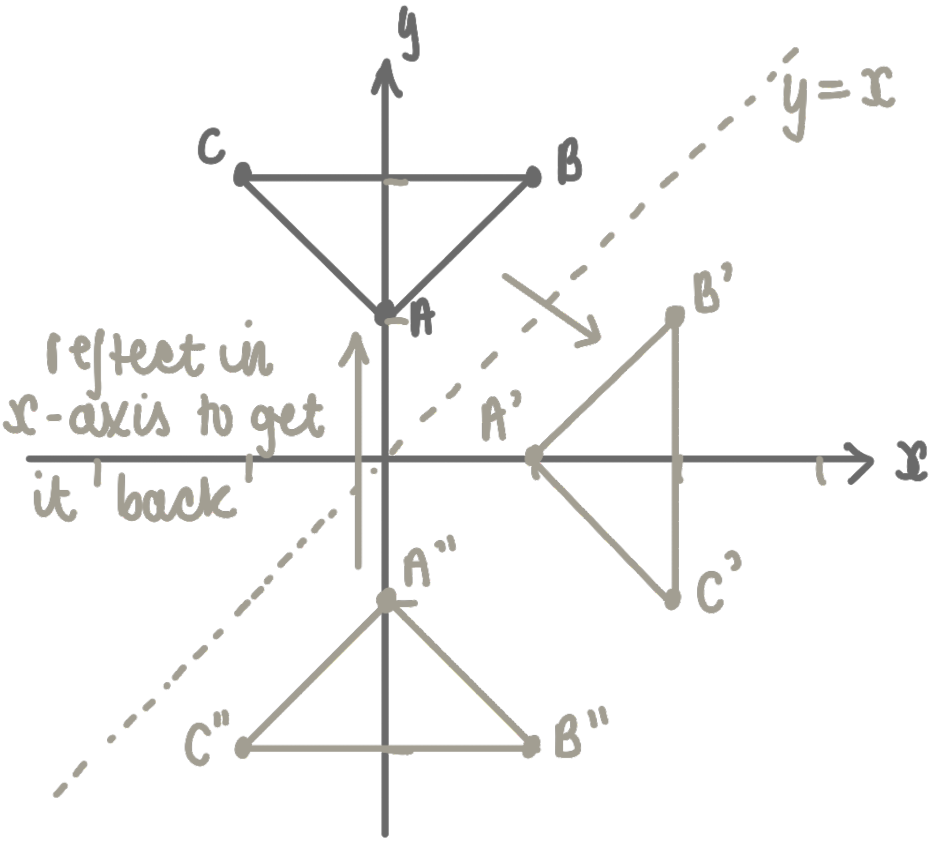
$$\frac{N_p}{N_s} = \frac{V_p}{V_s} = \frac{I_p}{I_s}$$

$$\Rightarrow I_p = \frac{I_s N_p}{N_s} = \frac{50 \times N}{20N} = \underline{\underline{2.5A}} \text{ (B)}$$

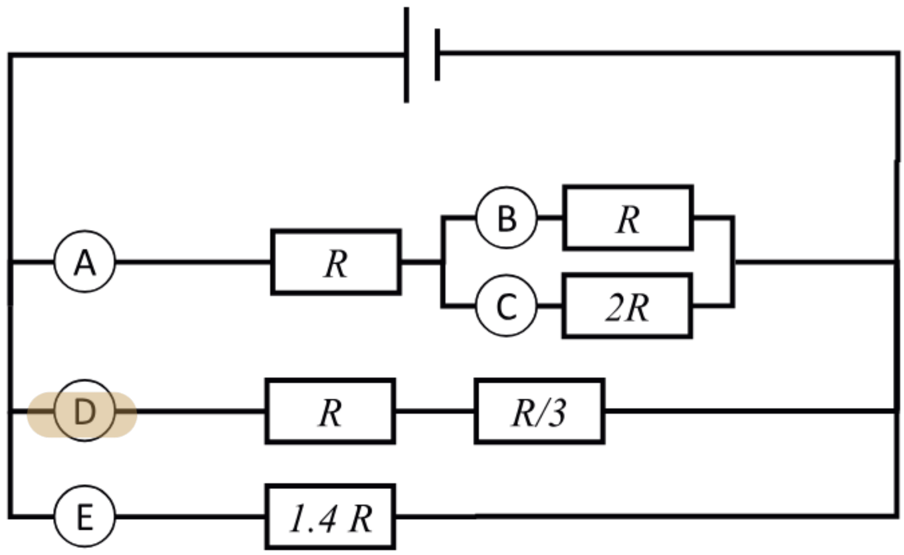
14. A triangle ABC has vertices at points in two-dimensional Cartesian co-ordinates: $A:(0,1)$, $B:(1,2)$, and $C:(-1,2)$. It is reflected in the line $y=x$ and then rotated around the origin by 90 degrees in a clockwise direction. Which single transformation maps the initial triangle to the final state of the above transformations?

[4]

- ○ rotation by $180°$ anti-clockwise around the origin
- ○ reflection in $x=0$
- ○ scale factor of -1
- ○ reflection in $y=0$
- ○ rotation by $90°$ anti-clockwise around $(2,0)$

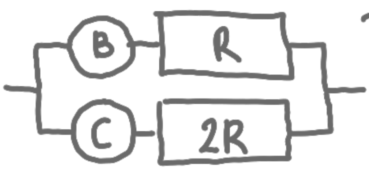


15. Which ammeter A, B, C, D, E gives the highest reading?



[3]

- KCL $\Rightarrow$ current splits at branches ($\Sigma I = 0$ at a node)
- Highest current reading comes from branch with lowest resistance



$$R_T = \frac{1}{\frac{1}{R} + \frac{1}{2R}} = \frac{2R}{3}$$

A : $R + \frac{2R}{3} = 1{\cdot}6R$

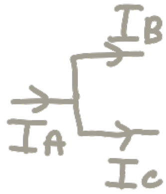
D : $R + \frac{1}{3}R = 1{\cdot}33R$

E : $1{\cdot}4R$

lowest resistance

$\Rightarrow$ highest current in <u>D</u>

B&C themselves will have a lower current than A since $I_A = I_B + I_C$



16. Solve $\log_2 x + \log_2(2x+3) = 1$ for x.

[3]

- $x = -2$
- $x = 0$
- $x = \frac{1}{2}$
- $x = -2$ and $\frac{1}{2}$
- $x = 1$

$$\left[\begin{array}{l} \log_a(x) + \log_a(y) = \log_a(xy) \\ \log_a(a) = 1 \\ \log_a(x) = \log_a(y) \Rightarrow x = y \end{array}\right]$$

$$\log_2(x) + \log_2(2x+3) = 1$$

$$\log_2[x(2x+3)] = \log_2(2)$$

$$x(2x+3) = 2$$

$$2x^2 + 3x - 2 = 0$$

$$(2x-1)(x+2) = 0$$

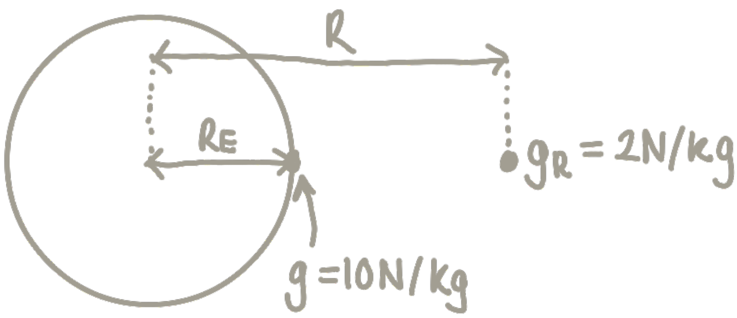
$$\underline{\underline{x = \frac{1}{2}}} \qquad \cancel{x = -2}$$

no solutions for $x < 0$ since $\log(x)$ only defined for $x > 0$.

17. If the gravitational field strength at the Earth's surface is $g_E = 10\,\mathrm{N/kg}$, and at a distance $R > R_E$ from its centre the field strength is $g_R = 2\,\mathrm{N/kg}$, what is the radius of the Earth R_E in terms of R ?

[2]

- ○ $R/\sqrt{2}$
- ○ $R/\sqrt{10}$
- ○ $R/5$
- ○ $R/\sqrt{5}$
- ○ R
- ○ $R/25$



gravitational field strength, $g = \frac{GM}{r^2}$

$$\Rightarrow g \propto \frac{1}{r^2} \Rightarrow gr^2 = \text{const.}$$

$$g_E R_E^2 = g_R R^2$$

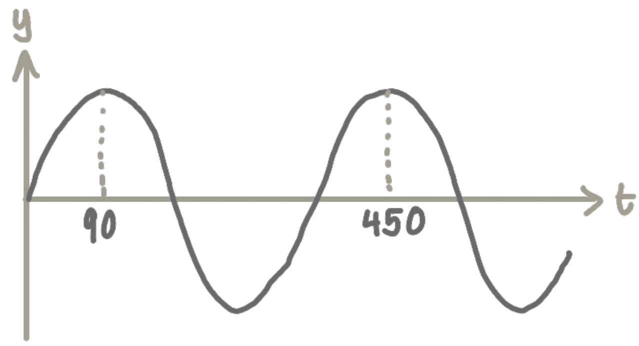
$$R_E = \sqrt{\frac{g_R}{g_E}}\, R = \sqrt{\frac{2}{10}}\, R = \underline{\underline{\frac{R}{\sqrt{5}}}}$$

18. Consider the function $y(x) = \sin(\frac{100}{x})$. The angle is in degrees, so that $\sin(180) = 0$. How many maxima of $y(x)$ occur for $x > 0.1$?

[3]

- 0
- 1
- 3
- 14
- ∞

$y = \sin\left(\frac{100}{x}\right)$ let $t = \frac{100}{x}$



maxima for $\sin(t)$ are for $t = 90 + 360n$

$\frac{100}{x} = 90 + 360n$

$x = \frac{100}{90 + 360n} > 0.1$

$100 > 0.1(90 + 360n)$

$100 > 9 + 36n$

$\frac{91}{36} > n \Rightarrow n < 2.53$

$\therefore n = 0, 1, 2$

i.e. $\underline{\underline{3}}$ solutions

19. What is the order, from shortest to longest, of the wavelengths of the peak electromagnetic emission from each of the following objects?

1. an electric torch VISABLE
2. a microwave oven MICROWAVE
3. a radioactive source GAMMA
4. a hot cooking stove INFRARED
5. a short-wave radio transmitter RADIO

[2]

- 3,1,2,4,5
- 5,4,2,1,3
- 5,2,4,1,3
- 3,4,1,5,2
- 3,1,4,2,5

electromagnetic spectrum is:

(5) ← (2) ← (4) ← (1) ← (3)

RADIO – MICROWAVE – INFRARED – VISABLE – ULTRAVIOLET – XRAY – GAMMA

+ ← wavelength, λ → −

20. A particle of type X decays with equal probability either to a pair of particles of type Y or a pair of particles of type Z. Both Y and Z particles are stable.

The decays of two X particles are observed. A pair of Y particles is found among the decay products. What is the probability that a pair of Z particles is among these decay products?

[2]

- ○ 1/2
- ○ 1/3
- ○ 2/3
- ○ 1/4
- ○ 1

$$X \xrightarrow{p=0.5} Y, Y$$
$$X \xrightarrow{p=0.5} Z, Z$$

all possible decay options (equally likely) are:

$$\left.\begin{array}{l} X_1 \rightarrow YY \;\&\; X_2 \rightarrow YY \\ X_1 \rightarrow ZZ \;\&\; X_2 \rightarrow YY \\ X_1 \rightarrow YY \;\&\; X_2 \rightarrow ZZ \end{array}\right\} \underline{\underline{\tfrac{2}{3}}} \text{ of these have a pair of Z particles.}$$

- ~~$X_1 \rightarrow ZZ \;\&\; X_2 \rightarrow ZZ$~~ ← "a pair of Y particles is found" so this one didnt happen

21. Ten students need to complete their compulsory practicals for their high school examinations as detailed in the table below:

No. of students	No. of different practicals to complete
2	1
4	2
4	3

The school only has one laboratory in which several different experiments can be set up simultaneously. A maximum of six students are allowed in the school's laboratory for a lesson. Each practical takes one lesson. What is the minimum number of lessons required to complete all the practicals?

[2]

- ○ 10
- ○ 4
- ○ 5
- ○ 3
- ○ 6

total number of practicals that need completing

$= 2(1) + 4(2) + 4(3) = 22$

number of lessons to complete these practicals

$= \frac{22}{6} = 3 \cdot 7 \Rightarrow$ 4 lessons

22. What is the next number in the sequence? $\underbrace{37, 41, 43, 47, 53, 59}$ [2]

- 64
- 65
- 62
- 61
- 67

these are all prime numbers

next prime number is 61

23. A stone of average diameter 10 cm is hit with a hammer and splits into pieces. Every time the stone or one of its pieces is hit, it splits into three further pieces of equal volume and similar shape. How many hits will it take before a piece reaches the size of a typical atom?

[3]

- 56
- 12
- 81
- 22
- 9

atom diameter $\approx 10^{-10}$ m

stone diameter $= 10^{-1}$ m

after one hit, the stone splits in three pieces so $V_f = \frac{1}{3} V_i$

and diameter is reduced to

$d_f = \left(\frac{1}{3}\right)^{1/3} d_i$ $\quad (V \propto d^3 \Rightarrow d \propto V^{1/3})$

so then after n hits, $d_n = \left[\left(\frac{1}{3}\right)^{1/3}\right]^n d_i$

$= 3^{-n/3} d_i$

to find how many hits until it reaches the diameter of an atom,

$$d_n = 3^{-n/3}(10^{-1}) = 10^{-10}$$

$$10^9 = 3^{n/3}$$

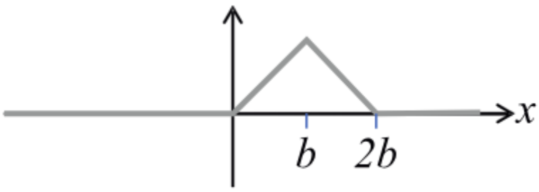
$$\log(10^9) = \log(3^{n/3})$$

$$\frac{\log(10^9)}{\log(3)} = \frac{1}{3}n \Rightarrow n = \frac{3\log(10^9)}{\log(3)}$$

$$= \underline{\underline{56.6}}$$

ORDER OF TRANSFORMATIONS

24. The graph below shows a function $f(x)$.



$g(x) = -f(a-x)$

translate $\begin{pmatrix} -a \\ 0 \end{pmatrix}$

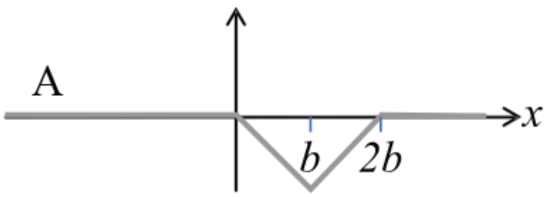
reflect in x-axis

reflect in y-axis

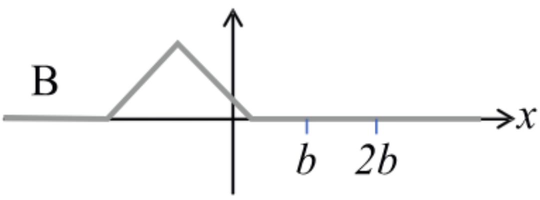
If a is a constant such that $0 < a < b$, identify the sketch of $g(x) = -f(a-x)$ from the sketches below.

[3]

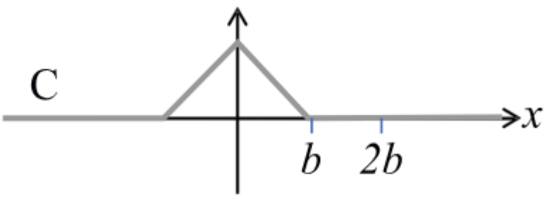
A



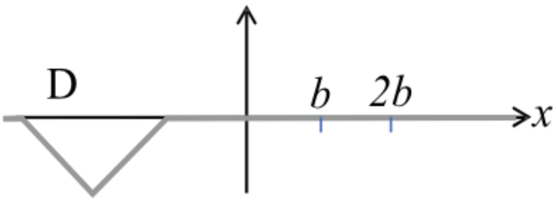
B



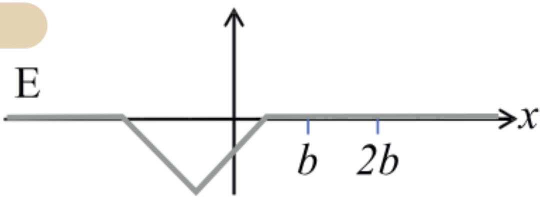
C



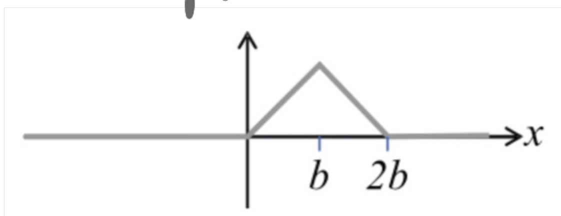
D



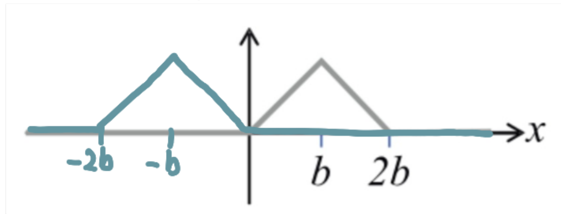
E



$f(x)$:

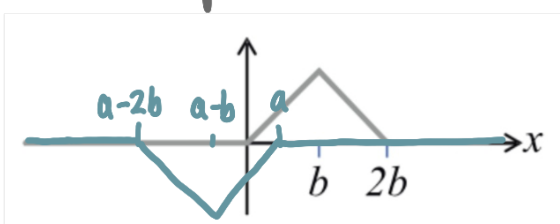


$f(-x)$



$f(a-x)$

$-f(a-x)$



25. What is the next number in the sequence? -972, 324, -108, 36, -12

[2]

- -4
- 9
- -3
- 4
- 3

$\frac{-972}{324} = 3$, $\frac{324}{-108} = -3$ ....

divide by 3 each time

next term is $\frac{-12}{3} = \underline{\underline{4}}$

26. Which values of x and y solve the following equations simultaneously:

$$\log x + 2\log y = \log 32$$
$$\log x - \log y = -\log 2$$

[3]

- $x = 2,\ y = 4$
- $x = -2,\ y = -4$
- $x = 2,\ y = -4$
- $x = -2,\ y = 4$
- No solution exists

$$\left[\begin{array}{l} \log(a) + \log(b) = \log(ab) \\ \log(a) - \log(b) = \log(a/b) \\ a\log(b) = \log(b^a) \\ \log(a) = \log(b) \Leftrightarrow a = b \end{array}\right]$$

$$\log(x) + 2\log(y) = \log(32)$$
$$\log(x) + \log(y^2) = \log(32)$$
$$\log(xy^2) = \log(32)$$
$$xy^2 = 32 \quad (1)$$

$$\log(x) - \log(y) = -\log(2)$$
$$\log\left(\frac{x}{y}\right) = \log\left(\frac{1}{2}\right)$$
$$\frac{x}{y} = \frac{1}{2}$$
$$y = 2x \quad (2)$$

Subsitute $(2) \to (1)$:

$$x(2x)^2 = 32$$
$$4x^3 = 32$$
$$x^3 = 8$$
$$\underline{\underline{x = 2}} \Rightarrow y = 2(2) \underline{\underline{= 4}}$$

27. Consider a system of many interacting particles. Let each particle have a potential energy $V(r)$ with respect to any other particle, where $V(r) \propto r^n$ where r is the distance to another particle and n is an integer. For such systems the Virial Theorem relates the time averaged total kinetic energy of all particles $\langle T_{\text{tot}} \rangle$ to the time averaged total potential energy $\langle V_{\text{tot}} \rangle$ as follows:

$$2\langle T_{\text{tot}} \rangle = n\langle V_{\text{tot}} \rangle$$

If the particles in our system interact only via gravity, what is the time averaged total energy $\langle E_{\text{tot}} \rangle$ of the system?

[3]

- ○ $\langle E_{\text{tot}} \rangle = 0$
- ○ $\langle E_{\text{tot}} \rangle = \langle V_{\text{tot}} \rangle / 2$
- ○ $\langle E_{\text{tot}} \rangle = 2\langle V_{\text{tot}} \rangle$
- ○ $\langle E_{\text{tot}} \rangle = -\langle V_{\text{tot}} \rangle$
- ○ $\langle E_{\text{tot}} \rangle = -2\langle V_{\text{tot}} \rangle$

potential energy, $V(r) \propto r^n$

particles only interact via gravity, and we know for gravity

$V(r) \propto r^{-1}$ $(n = -1)$

using the equation given, kinetic energy $\langle T_{tot} \rangle$:

$2\langle T_{tot} \rangle = -1\langle V_{tot} \rangle$

$\langle T_{tot} \rangle = -\frac{1}{2}\langle V_{tot} \rangle$

the total energy of the system will be the sum of the kinetic and potential energies

$\langle E_{tot} \rangle = \langle T_{tot} \rangle + \langle V_{tot} \rangle$

$= -\frac{1}{2}\langle V_{tot} \rangle + \langle V_{tot} \rangle = \underline{\underline{\frac{1}{2}\langle V_{tot} \rangle}}$

28. The acceleration g due to gravity on a spherical planet in any universe is given by:

$$g = \frac{GM}{R^2}$$

where M is the mass, R the radius of the planet and G is the gravitational constant in that planet's universe.

In a different universe the gravitational constant is G' and has twice the value of the gravitational constant in our Universe G .

Find the ratio $\frac{g_{\text{planet}}}{g_{\text{Earth}}}$ for a planet in the different universe which has half the radius and twice the density of the Earth.

[2]

$$\boxed{\begin{aligned} G' &= 2G \\ R' &= \tfrac{1}{2}R \\ \rho' &= 2\rho \end{aligned}}$$

- $\frac{g_{\text{planet}}}{g_{\text{Earth}}} = 2$
- $\frac{g_{\text{planet}}}{g_{\text{Earth}}} = \frac{1}{2}$
- $\frac{g_{\text{planet}}}{g_{\text{Earth}}} = \frac{1}{4}$
- $\frac{g_{\text{planet}}}{g_{\text{Earth}}} = 1$
- $\frac{g_{\text{planet}}}{g_{\text{Earth}}} = 4$

mass of planet will be (assuming it's spherical)

$$m' = \rho' V' = \rho' \tfrac{4}{3}\pi (R')^3 = \tfrac{4}{3}\pi (2\rho)(\tfrac{1}{2}R)^3$$

$$= \tfrac{1}{4}\left[\tfrac{4}{3}\pi\rho R^3\right] = \tfrac{1}{4}m$$

hence acceleration due to gravity

$$g' = \frac{G'm'}{(R')^2} = \frac{(2G)(\frac{1}{4}m)}{(\frac{1}{2}R)^2} = 2\frac{Gm}{R^2} = 2g$$

$$\Rightarrow \underline{\underline{\frac{g'}{g} = 2}}$$

29. In which range of α does the following equation have real solutions?

$\frac{1}{\cos^2\theta} + \alpha\tan\theta = 0$

[3]

- $\alpha \leq -2$ or $\alpha \geq 2$
- $\alpha \leq -2$
- $\alpha \geq 2$
- $\alpha \geq 0$
- $\alpha \leq 0$
- None

$$\frac{\sin^2(x)}{\cos^2(x)} + \frac{\cos^2(x)}{\cos^2(x)} = \frac{1}{\cos^2(x)}$$

$$\tan^2(x) + 1 = \frac{1}{\cos^2(x)}$$

$$\frac{1}{\cos^2(x)} + \alpha\tan(x) = 0$$

$$(\tan^2(x) + 1) + \alpha\tan(x) = 0$$

let $y = \tan(x)$

$$y^2 + \alpha y + 1 = 0$$

real solutions when discriminant $b^2 - 4ac \geq 0$

$$(\alpha)^2 - 4(1)(1) \geq 0$$

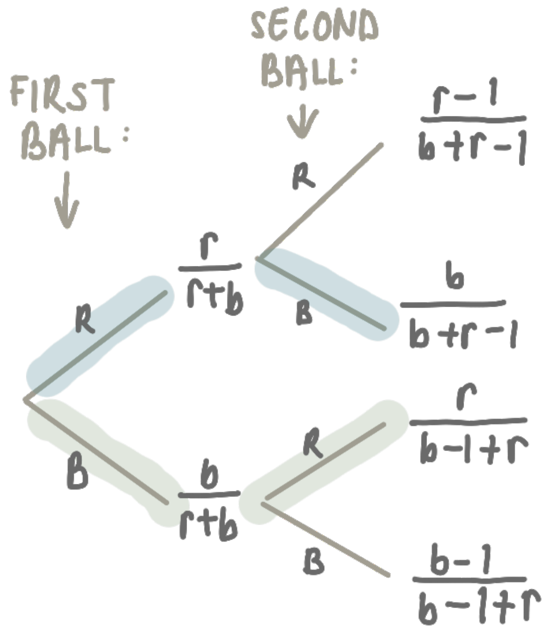
$$\alpha^2 \geq 4$$

$\underline{\underline{\alpha \leq -2}}$ or $\underline{\underline{a \geq 2}}$

30. A bag contains b blue balls and r red balls. If two balls are picked at random and removed from the bag, what is the probability P that they are different colours?

[3]

- $2br$
- $\frac{br}{(b+r)^2}$
- $\frac{2br}{(b+r)^2}$
- $\frac{br}{(b+r)(b+r-1)}$
- $\frac{2br}{(b+r)(b+r-1)}$



$$= \left(\frac{r}{r+b}\right)\left(\frac{b}{b+r-1}\right) = \frac{rb}{(r+b)(b+r-1)}$$

$$= \left(\frac{b}{r+b}\right)\left(\frac{r}{b-1+r}\right) = \frac{rb}{(r+b)(b+r-1)}$$

hence the probability they are different colours is

$$\frac{rb}{(r+b)(b+r-1)} + \frac{rb}{(r+b)(b+r-1)} = \underline{\underline{\frac{2rb}{(r+b)(b+r-1)}}}$$

31. We wish to represent integer numbers by using our ten fingers. A finger is assumed to be either stretched out or curled up. How many different integers can we represent with our fingers?

[3]

- ○ 32
- ○ 1024
- ○ 1000
- ○ 10
- ○ 512
- ○ 20

if each finger represents a binary digit (1 or 0)

then ten fingers can represent the digits of a 10 digit binary number

hence $N_{max} = 2^{10} = \underline{\underline{1024}}$

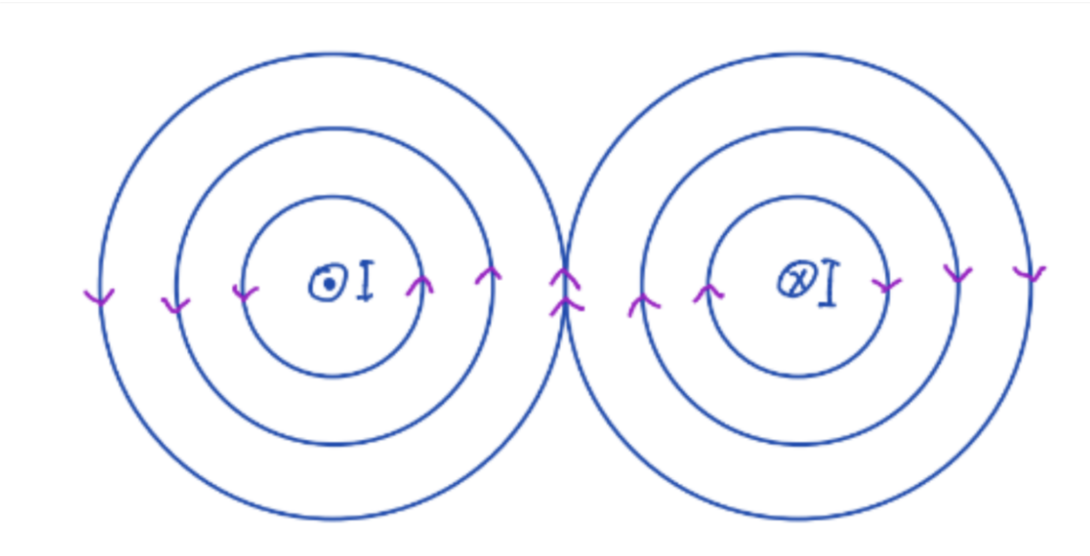
32. A long, thin, straight wire carrying an electric current I causes a magnetic field of flux density B at a perpendicular distance r from the wire. The magnitude of this flux density is given by the following relation:

$$B = \frac{\alpha I}{r}$$

where α is a constant. The magnetic field points circumferentially around the wire. A second, identical wire is placed parallel to the first one at a distance D. Find the current I_2 that has to flow in the second wire if the flux density at a line half way between and parallel to the wires is to double, compared to the flux density from only one wire at current I.

[3]

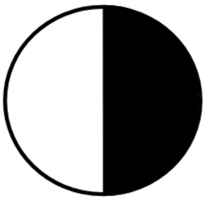
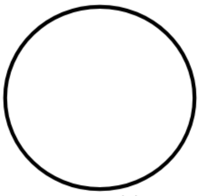
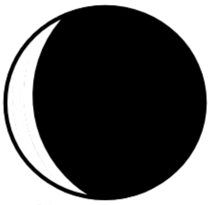
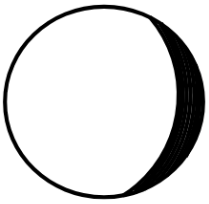
- $I_2 = I$
- $I_2 = -I/2$
- $I_2 = 2I$
- $I_2 = -I$
- $I_2 = -2I$

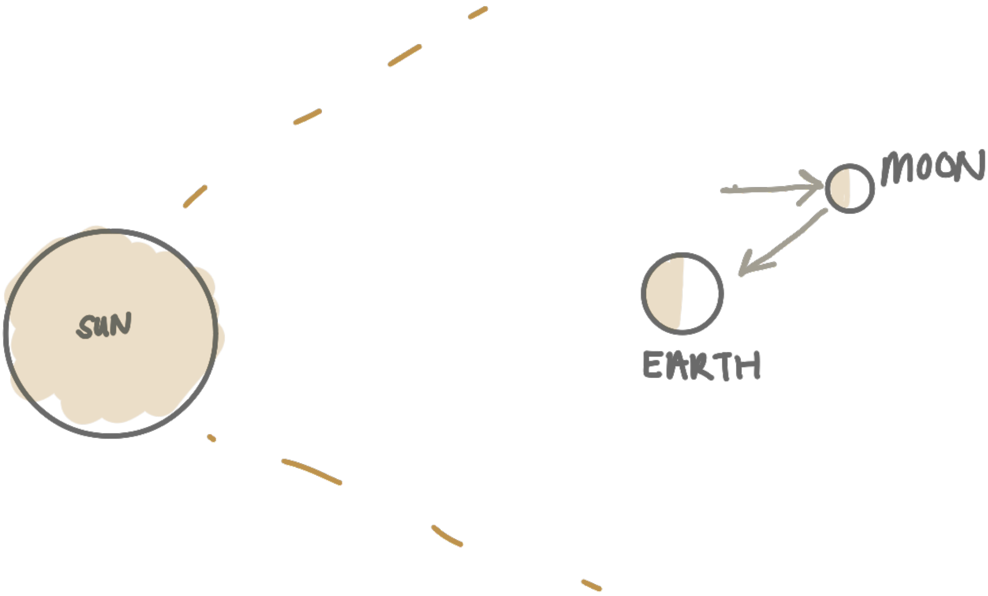


33. When the phase of the Moon as seen from the Earth is Full, what phase of the Earth is seen by an observer on the Moon?

(These symbols above show phases of the Earth as seen from the Moon.)

[2]

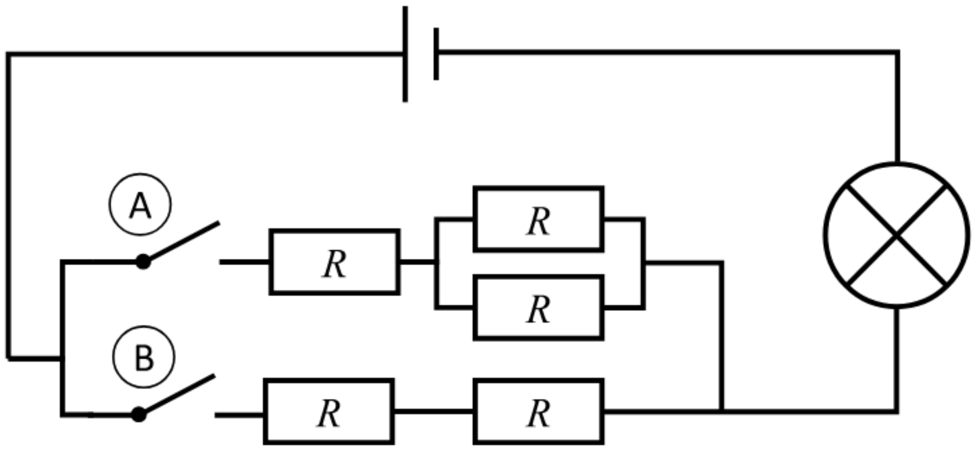
- ○ 
  Quarter (or ‘half’)
- ○ 
  Full
- ○ 
  Crescent
- ○ 
  Gibbous
- ○ 
  New



full moon ⇒ sun fully illuminates moon as seen from earth

from the moon, the sun will be behind the earth so will appear ‘new’.

34. In the circuit shown below all resistors have the same resistance R and the light bulb has a fixed resistance. You wish to change the state of the switches so that the brightness of the bulb increases from its minimum to its maximum. Which sequence of switch states will achieve this?



[3]

- both closed; then only B closed; then only A closed
- both closed; then only A closed; then only B closed
- only B closed; then only A closed; then both closed
- only A closed; then only B closed; then both closed
- all states have the same brightness

brightness $\propto$ power $= (V_{bulb})^2 / R_{bulb}$

only A : $R_T = R + \frac{1}{\frac{1}{R} + \frac{1}{R}} = \frac{3R}{2} = 1.5R$

only B : $R_T = R + R = 2R \Rightarrow$ lowest V_{bulb}

both A&B : $R_T = \frac{1}{\frac{1}{1.5R} + \frac{1}{2R}} = \frac{6R}{7} \approx 0.86R \Rightarrow$ highest V_{bulb}

35. An organ pipe is open at one end and closed at the other. The lowest note you can play on this pipe has frequency $f_{\min}$. If the speed of sound in the pipe is v , what is the length L of the pipe?

[2]

- $L = \frac{v}{4f_{\min}}$
- $L = \frac{4v}{f_{\min}}$
- $L = \frac{v}{f_{\min}}$
- $L = \frac{v}{2f_{\min}}$
- $L = \frac{2v}{f_{\min}}$

lowest node

$L = \frac{1}{4}\lambda$

$v = f\lambda \Rightarrow \lambda = v/f$

$L = \frac{1}{4}\lambda = \underline{\underline{v/4f}}$

36. Which combination of units is the odd one out?

[3]

- $\mathrm{kg\,m\,s^{-2}}$ ← only combination expressed in SI base units.
- $\mathrm{A\,T\,m}$
- $\mathrm{C\,m\,s^{-1}}$
- $\mathrm{J\,m^{-1}}$
- $\mathrm{A\,V\,m^{-1}}$

37. 90 people enter a maze. At each junction a third will go left and two thirds will go right. After three such junctions, what is the most likely combination of turns people will have taken?

[2]

- Gone right three times
- Gone left three times
- Gone right twice and once left
- Gone twice left and once right
- It is impossible to tell

$P(L) = \frac{1}{3}, \quad P(R) = \frac{2}{3}$

$P(RRR) = \left(\frac{2}{3}\right)^3 = \frac{8}{27} \approx 0.300$

$P(LLL) = \left(\frac{1}{3}\right)^3 = \frac{1}{27} \approx 0.037$

$P(RRL) + P(RLR) + P(LRR) = 3\left(\frac{2}{3}\right)^2\left(\frac{1}{3}\right) = \frac{4}{9} \approx \underline{\underline{0.444}}$

$P(LLR) + P(LRL) + P(RLL) = 3\left(\frac{2}{3}\right)\left(\frac{1}{3}\right)^2 = \frac{2}{9} \approx 0.222$

38. A person drinks many cups of tea. The first cup the person drinks is filled completely. They don't want to drink too much tea in total so the second cup is filled with only a fraction (α) of the tea in the first cup, the third cup contains the same fraction α of the second cup and so on. What is the maximum value of α so that the person drinks no more than 3 times the amount of tea in the first cup however many drinks they take?

[2]

- ○ $\alpha = \frac{2}{3}$
- ○ $\alpha = \frac{1}{2}$
- ○ $\alpha = \frac{1}{3}$
- ○ $\alpha = \frac{3}{4}$
- ○ $\alpha = \frac{1}{4}$

$$\left.\begin{aligned} 1^{st}\ \text{cup} &= c \\ 2^{nd}\ \text{cup} &= \alpha c \\ 3^{rd}\ \text{cup} &= \alpha^2 c \end{aligned}\right\}$$

total amount drank:

$$c + \alpha c + \alpha^2 c + \alpha^3 c + \cdots = 3c$$

$$1 + \alpha + \alpha^2 + \alpha^3 + \cdots = 3$$

geometric series with $a = 1$, $r = \alpha$

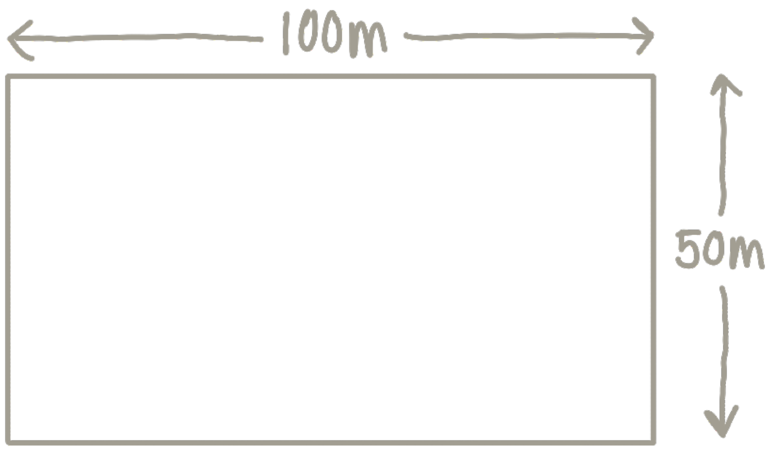
$$S_\infty = \frac{a}{1-r} \Rightarrow 3 = \frac{1}{1-\alpha}$$

$$3 - 3\alpha = 1 \Rightarrow \underline{\underline{\alpha = \frac{2}{3}}}$$

39. A rectangular building with sides 50 m and 100 m long has a flat roof on top of it.
The roof has a mass per unit area of $100\,\mathrm{kg\ m^{-2}}$. What is the total force on the vertical walls supporting the building?

[2]

- $4.9 \cdot 10^6\ \mathrm{N}$
- $5 \cdot 10^5\ \mathrm{N}$
- $1.2 \cdot 10^6\ \mathrm{N}$
- $2.5 \cdot 10^6\ \mathrm{N}$
- $9.8 \cdot 10^6\ \mathrm{N}$



area $= 100 \times 50 = 5000\,m^2$

mass of roof

$= 100 \times 5\,000$

$= 5 \times 10^5\,kg$

force on the roof

$= mg = 5 \times 10^5 \times 9.81$

$= \underline{\underline{4.905 \times 10^6\ N}}$

40. What is the equation of the line which intersects $y = 2x - 2$ at right angles and at position $x = 1$?

[3]

- ○ $y = -\frac{1}{2}x$
- ○ $y = x$
- ○ $y = -\frac{1}{2}x + \frac{1}{2}$
- ○ $y = \frac{1}{2}x - \frac{1}{2}$
- ○ $y = 2x$

$y = 2x - 2$ ← gradient $= 2$

when $x = 1$, $y = 2(1) - 2 = 0$

gradient of perpendicular line $= -\frac{1}{2}$

$y - y_1 = m(x - x_1)$

$y - 0 = -\frac{1}{2}(x - 1)$

$\underline{\underline{y = -\frac{1}{2}x + \frac{1}{2}}}$