Physics Olympiad Competition 2012/2013
A2 Challenge Solutions
Mark Scheme Sept/Oct 2012

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Allow error carried forward where this gives sensible answers.

Answers should be to an appropriate number of significant figures. This should normally be the same number of significant figures given for the data in the question.

Question 1

Let the mass of each liquid be m

Density = mass / volume 1

Mass of mixture = $2m$

Volume of mixture $= \frac{m}{800} + \frac{m}{1200}$ 2

$= \frac{m}{480}$

$\Longrightarrow$ Density of mixture $= 2m \div \frac{m}{480}$

$= 960\ \mathrm{kg\ m^{-3}}$ unit needed in final answer 2

TOTAL 5

Question 2

a. 1 kW hr = 1,000 x 3,600 J = 3.6×10^6 J 2

b. Reactor electrical energy $= 15 \times 10^9$ kW hr in one year

$= 15 \times 10^9 \times 3.6 \times 10^6$ J in one year

$= 5.4 \times 10^{16}$ J in one year 1

Energy converted in the reactor $= 5.4 \times 10^{16} / 0.35$

$= 1.54 \times 10^{17}$ J 1

Rate of energy loss in one second $= \frac{1.54 \times 10^{17}}{365 \times 24 \times 3600}$

$= 4.9 \times 10^9\ \mathrm{J\ s^{-1}}$ 2

Mass loss per second = energy loss per second $\div c^2$ 1

$= \frac{4.9 \times 10^9}{9 \times 10^{16}}$ kg s^{-1}

= 5.4 x 10^{-8} kg s^{-1} 1

= 54 mg s^{-1}

TOTAL 8

Question 3

a. The speed of sound in sea water, v_{water} can be found by knowing the speed of sound in air, which is given as v_{air} = 330 m s^{-1}.

Speed = $\frac{\text{distance travelled}}{\text{time taken}}$ 1

Distance of charge behind vessel = v_{air} x 3.05

$\Longrightarrow \quad v_{water} = v_{air} \times \frac{3.05}{0.650}$ 2

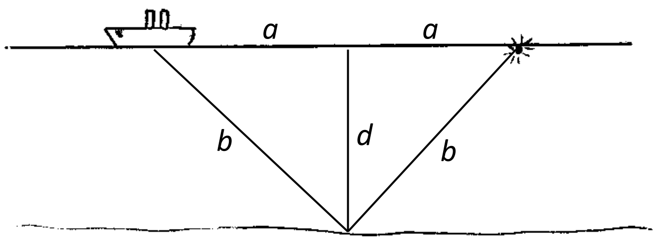
= 1550 m s^{-1} 1

b. Length of route (i) is 2a = v_{water} x 0.65

= 1007 m 2

Length of route (iii) is 2b = v_{water} x 3.26

= 5048 m 2



The depth of the ocean, d, may easily be found by Pythagoras: $d = \sqrt{b^2 - a^2}$ 1

= 2470 m 2

TOTAL 11

Question 4

a. The maximum expected discrepancy will occur when one of the resistors is 5% above its nominal value and the other is 5% below. 1

For example,

if R_1 = 10.5 kΩ and R_2 = 9.5 kΩ, the output voltage will be $V_{out} = \frac{R_1}{R_1 + R_2}$ x 12V 2

$= \frac{10.5}{20}$ x 12V

$= 6.30$ V

[if R_1 = 9.5 kΩ and R_2 = 10.5 kΩ, the output voltage will be V_{out} = 5.70 V]

⟹ the maximum discrepancy to be expected is 0.30 V 1

b. If V_{out} = 6.10 V then $\frac{R_1}{R_1 + R_2} = 0.5083$

⟹ R_2 = 9.67 kΩ, since R_1 is known to be 10 kΩ 2

To produce an output voltage of 6.00 V, the resistance of the lower part of the divider must also be 9.67 kΩ, which can be achieved by adding a resistance R in parallel with R_1. 1

⟹ $\frac{1}{9.67} = \frac{1}{10} + \frac{1}{R}$ 1

⟹ $\frac{1}{R} = 0.00339$ kΩ^{-1}

⟹ R = 295 kΩ (= 300 kΩ) 2

TOTAL 10

Question 5

$$f \propto \frac{\sqrt{T}}{d}$$ 2

$$\Longrightarrow \quad f_G = \frac{\sqrt{T_G/T_B}}{d_G/d_B} \times f_B$$ 2

$$= \frac{\sqrt{1.56}}{1.56} \times f_B$$ 2

$$= \frac{1}{\sqrt{1.56}} \times f_B$$

f_B = 245 Hz $\Longrightarrow$ f_G = 196 Hz 2

TOTAL 8

Question 6

$R = kE^4$ and when E becomes slightly greater by an amount e, then R will become slightly greater by an amount r 2

So we may write $R + r = k(E + e)^4$ 1
And this can be expanded when $e \ll E$ by writing it as

$R + r = kE^4(1 + e/E)^4$ and if $(1 + x)^n \approx 1 + nx$ 1

Then $R + r \approx kE^4(1 + 4e/E)$
Which becomes $R + r \approx kE^4 + 4kE^3e$

So that $r \approx 4kE^3e$ and using $R = kE^4$ 1

$$\frac{r}{R} = 4\,\frac{e}{E}$$ 1

So if $\frac{e}{E}$ is 1% then $\frac{r}{R}$ is 4% 2

TOTAL 8