

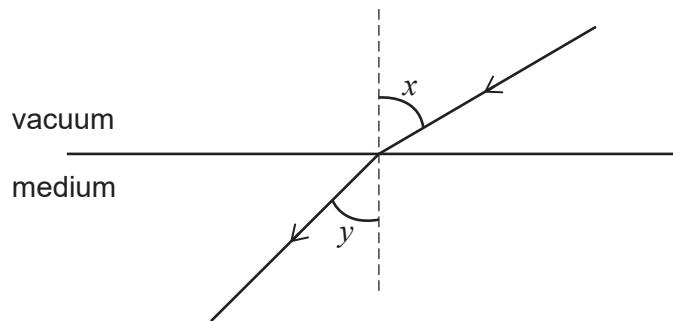
# ENGAA 2019

## Section 2

### Model Solutions



- 1 The ray diagram shows light passing from a vacuum into a medium.



Two angles,  $x$  and  $y$ , are shown on the diagram.

When  $x$  is  $60^\circ$ ,  $y$  is  $45^\circ$ .

When  $x$  is  $45^\circ$ , what is the value of  $\sin y$ ?

A  $\frac{1}{\sqrt{3}}$

B  $\frac{2}{\sqrt{3}}$

C 1

D  $\frac{\sqrt{3}}{2}$

E  $\sqrt{3}$

$$\text{Refractive index } x = \frac{\sin i}{\sin r}$$

$$= \frac{\sin x}{\sin y}$$

$$= \frac{\sin(60)}{\sin(45)}$$

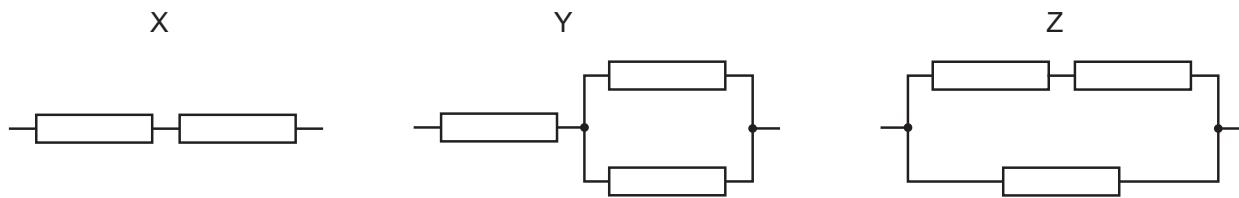
$$\Rightarrow \frac{\sin(60)}{\sin(45)} = \frac{\sin(45)}{\sin y}$$

$$\Rightarrow \sin y = \frac{\sin(45) \sin(45)}{\sin(60)}$$

$$= \frac{1}{2} \times \frac{2}{\sqrt{3}} = \frac{1}{\sqrt{3}}$$



- 2 Identical resistors are used to produce three different arrangements X, Y and Z.



Each arrangement is connected, in turn, across the same battery which has a negligible internal resistance.

The total power developed in each of the arrangements is determined.

What is the order of the arrangements when placed in order of **increasing** power?

- (A) X, Y, Z
- (B) X, Z, Y
- (C) Y, X, Z
- (D) Y, Z, X
- (E) Z, X, Y
- (F) Z, Y, X

$$X : 2R$$

$$Y : \left( \frac{1}{R} + \frac{1}{R} \right)^{-1} = \frac{R}{2} \Rightarrow \frac{3}{2}R$$

$$Z : \left( \frac{1}{2R} + \frac{1}{R} \right)^{-1} = \left( \frac{1+2}{2R} \right)^{-1} = \frac{2}{3}R$$

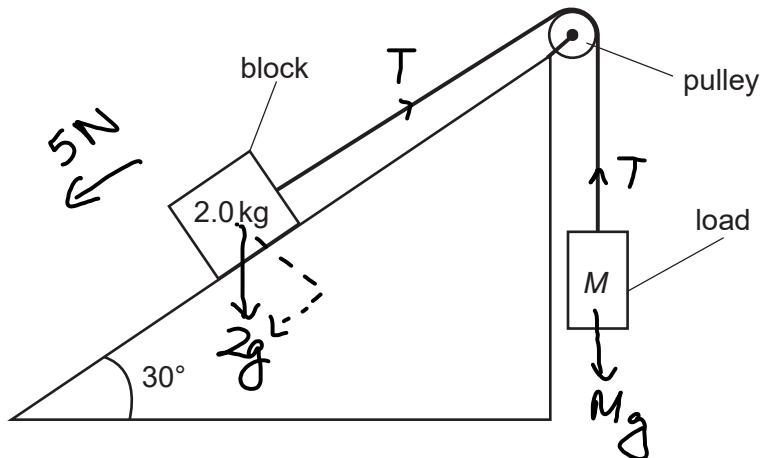
$P = \frac{V^2}{R} \Rightarrow$  Smaller the R, bigger the power lost, hence greater the developed power.

$$X < Y < Z$$



- 3 A block of mass 2.0kg is on a plane which is inclined to the horizontal at an angle of  $30^\circ$ .

The block is attached to a load of mass  $M$  by a light, inextensible string which passes over a smooth pulley.



The load moves downwards at a constant speed.  $\Rightarrow \alpha = 0 \Rightarrow Mg = T$

A constant friction force of  $5.0\text{N}$  acts on the block while it moves.

What is the value of  $M$ ?

(gravitational field strength =  $10\text{Nkg}^{-1}$ ; assume that air resistance is negligible)

A 0.50kg

B 1.0kg

C 1.5kg

D 2.5kg

E 4.0kg

F 6.0kg

$$\text{N2L on block: } T - (5 + 2g \sin 30) = 0$$

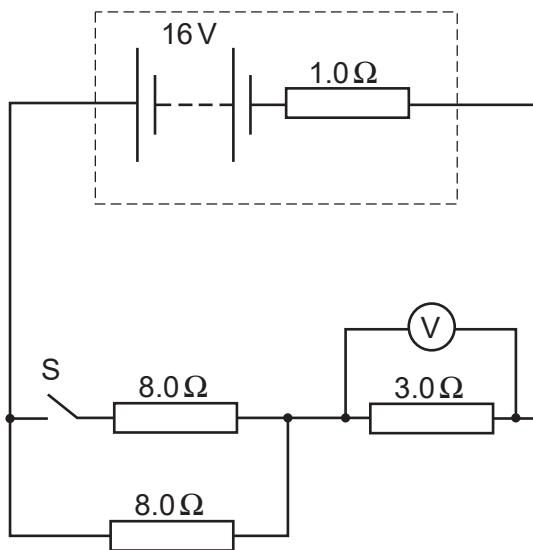
$$\Rightarrow Mg = 5 + 2g \sin 30$$

$$\Rightarrow Mg = 5 + 10$$

$$\Rightarrow M = \frac{15}{10} = \frac{3}{2} = \underline{\underline{1.5\text{kg}}}$$



- 4 The battery in the circuit shown has an emf of 16V and an internal resistance of  $1.0\Omega$ .



Which line in the table gives the voltmeter readings when switch S is in its open and closed states?

	voltmeter reading /V	
	when S is open	when S is closed
A	$\frac{4.0}{3.0}$	2.0
B	4.0	6.0
C	4.0	2.4
D	6.0	2.4
E	6.0	4.0
F	$\frac{48}{11}$	$\frac{48}{19}$
G	$\frac{48}{11}$	$\frac{48}{7.0}$
H	$\frac{128}{11}$	$\frac{64}{7.0}$

OPEN: No current through that branch.

By potential divider:

$$\frac{3}{3+8+1} \times 16 = \underline{\underline{4V}}$$

Closed: 8 and 8 make a pair of parallel resistors where their total resistance =  $4\Omega$

Potential divider:

$$\frac{3}{3+4+1} \times 16 = \underline{\underline{6V}}$$



- 5 A stationary wave is set up in a medium in which the speed of the wave is  $3.2 \text{ ms}^{-1}$ .

The stationary wave is formed by the superposition of two longitudinal waves, each of amplitude 1.5 cm, travelling in opposite directions.

The distance between adjacent nodes in the stationary wave is 4.0 cm.

What is the total distance travelled by a particle at an antinode during a time interval of 1.0 minute?

A 0 m

$$\lambda = 8 \text{ cm} = 0.08 \text{ m}$$

B 72 m

$$v = 3.2 \text{ ms}^{-1}$$

C 144 m

$$v = \lambda f \Rightarrow f = \frac{v}{\lambda} = \frac{3.2}{0.08}$$

D 192 m

$$= 40 \text{ Hz}$$

E 288 m

$$\therefore 40 \text{ waves in } 1 \text{ sec} \Rightarrow 40 \times 60 \\ = 2400 \text{ waves per min.}$$

F 576 m

Amp of 1 wave = 1.5 cm  
 ∵ stationary waves of 2, 1.5 cm  
 waves will have amp = 3 cm

$$\therefore \text{Total distance} = 3 \times 4 \times 2400 \text{ cm}$$

$$= 12 \times 2400$$

$$= 28800 \text{ cm}$$

$$= 288 \text{ m}$$

$$\begin{array}{r} 24 \\ 12 \\ \hline 48 \\ 24 \times \\ \hline 288 \end{array}$$



- 6 A ray of light of single frequency  $f$  is travelling in a block of transparent material.

The ray strikes the boundary between the block and air at an angle  $\theta$  to the boundary.

When  $\theta = 65^\circ$  the ray is just at the threshold of being totally internally reflected.

Which of the following is an expression for the wavelength of the light in the material?

(The speed of light in air is  $v_{\text{air}}$ .)

A  $\frac{v_{\text{air}} \cdot \cos 65^\circ}{f}$

B  $\frac{v_{\text{air}} \cdot \sin 65^\circ}{f}$

C  $\frac{f}{v_{\text{air}} \cdot \cos 65^\circ}$

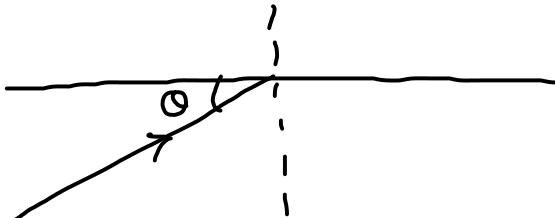
D  $\frac{f}{v_{\text{air}} \cdot \sin 65^\circ}$

E  $\frac{v_{\text{air}}}{f \cdot \cos 65^\circ}$

F  $\frac{v_{\text{air}}}{f \cdot \sin 65^\circ}$

G  $\frac{f \cdot \cos 65^\circ}{v_{\text{air}}}$

H  $\frac{f \cdot \sin 65^\circ}{v_{\text{air}}}$



If  $\theta$  is with boundary,  
 $90 - \theta = \theta_c = 25^\circ$

$$\lambda = \frac{v}{f}$$

$$\frac{n_{\text{material}}}{n_{\text{air}}} = \frac{v_{\text{air}}}{v_{\text{material}}} = \frac{1}{\sin 25}$$

$$\Rightarrow v_{\text{material}} = v_{\text{air}} \sin 25$$

$$\sin \theta = \cos 90 - \theta$$

$$\sin 25 = \cos 65$$

$$\lambda = \frac{v_{\text{material}}}{f} = \frac{v_{\text{air}} \cos 65}{f}$$



- 7 A solid pyramid of height 140 m has a square base.

The density of the stone from which the pyramid is made is  $2100 \text{ kg m}^{-3}$ .

Atmospheric pressure is 100 kPa.

What is the average pressure on the ground under the pyramid?

(gravitational field strength =  $10 \text{ N kg}^{-1}$ ; volume of a pyramid =  $\frac{1}{3} \times \text{base area} \times \text{vertical height}$ )

A 98 kPa

B 108 kPa

C 198 kPa

D 980 kPa

E 1080 kPa

F 2940 kPa

G 3040 kPa

$$\frac{\frac{1}{3} \times \cancel{\text{base area}} \times 140 \times \rho \times g}{\cancel{\text{base area}}} + \text{atm}$$

$$= \frac{140}{3} \times 2100 \times 10 + 100000$$

$$= 980000 + 100000$$

$$= 1080000 \text{ Pa}$$

$$= \underline{1080 \text{ kPa}}$$

$$\times \frac{14}{7} \\ 98$$



- 8 The pressure exerted by a gas at constant temperature is directly proportional to its density.

A spherical bubble of gas forms at the bottom of a glass containing a fizzy drink.

The radius of the bubble at the point of formation, at the bottom of the drink, is  $R$ .

The depth of the liquid in the glass is  $h$ , and the density of the liquid of the drink is  $\rho$ .

Atmospheric pressure is  $P$ . *let new radius = r*

As the bubble rises, its radius changes.

Which expression gives the radius of the bubble when it is at a distance  $x$  below the surface of the drink?

(gravitational field strength =  $g$ ; volume of sphere =  $\frac{4}{3}\pi r^3$  where  $r$  is the radius; the mass and the temperature of the gas in the bubble remain constant)

A  $R \left( \frac{h\rho g - P}{x\rho g - P} \right)^{\frac{1}{3}}$

$$\rho V = \text{constant} \quad (\text{for bubble})$$

B  $R \left( \frac{h}{x} \right)^{\frac{1}{3}}$

$$(i) (h\rho g + P) \times \frac{4}{3}\pi R^3 = C \rightarrow \text{constant}$$

C  $R \left( \frac{h\rho g + P}{x\rho g + P} \right)^{\frac{1}{3}}$

$$(ii) (x\rho g + P) \times \frac{4}{3}\pi r^3 = C$$

D  $R \left( \frac{x\rho g - P}{h\rho g - P} \right)^{\frac{1}{3}}$

$$(i) = (ii) \Rightarrow (h\rho g + P) \cancel{\frac{4}{3}\pi h R^3} = (x\rho g + P) \cancel{\frac{4}{3}\pi r^3}$$

E  $R \left( \frac{x}{h} \right)^{\frac{1}{3}}$

$$\Rightarrow \frac{(h\rho g + P) R^3}{(x\rho g + P)} = r^3$$

F  $R \left( \frac{x\rho g + P}{h\rho g + P} \right)^{\frac{1}{3}}$

$$\Rightarrow R \left( \frac{h\rho g + P}{x\rho g + P} \right)^{\frac{1}{3}} = r$$



- 9 A block of mass 2.0kg slides directly down a smooth slope.

The slope is at an angle of  $30^\circ$  to the horizontal.

The block reaches a speed of  $8.0\text{ m s}^{-1}$ , at which point the slope becomes rough and the block begins to decelerate.

After travelling a distance of 4.0m down the rough slope the block comes to rest.

What is the magnitude of the average friction force between the block and the rough slope?

(gravitational field strength =  $10\text{ N kg}^{-1}$ ; assume that air resistance is negligible)

A  $2.0\text{ N}$

B  $6.0\text{ N}$

C  $10\text{ N}$

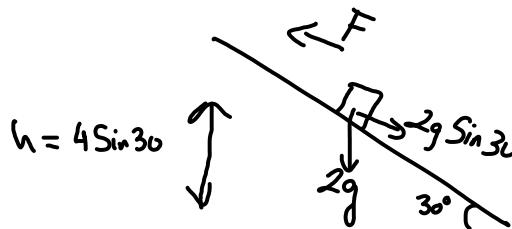
D  $12\text{ N}$

E  $16\text{ N}$

F  $10\sqrt{3}\text{ N}$

G  $26\text{ N}$

H  $(16 + 10\sqrt{3})\text{ N}$



$$S = 4$$

$$h = 4 \sin 30 = 2$$

Conservation of energy:  
 $\text{Initial KE} + \text{Initial PE} = \text{Final KE} + \text{Final PE} + \text{Work against F}$

$$\Rightarrow \frac{1}{2} \times 2 \times 8^2 + 2 \times g \times 2 = \frac{1}{2} \times 2 \times 0^2 + 0^2 + F(4)$$

$$\Rightarrow 64 + 40 = 4F$$

$$\Rightarrow 104 = 4F$$

$$\Rightarrow \underline{\underline{26 = F}}$$

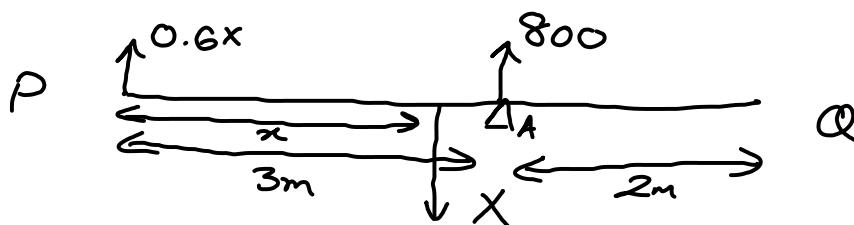


- 10 A non-uniform beam PQ of length 5.0m and weight  $X$  rests on a pivot placed 3.0m from end P. It is kept in equilibrium in a horizontal position by an upward force of magnitude  $0.60X$  acting at end P.

The normal contact force at the pivot is 800N.

What is the weight of the beam and how far is the centre of gravity of the beam from the pivot?

	weight of beam / N	distance from pivot / m
A	500	0.50
B	500	1.8
C	500	3.0
D	2000	0.50
E	2000	1.8
F	2000	3.0



$\Rightarrow$  dist of c.o.m from P

$$\text{At } A: X(3-x) = 0.6X \times 3$$

$$3 - x = 1.8$$

$$x = \underline{1.2 \text{ m}}$$

$$\Rightarrow 3 - x = \underline{1.8 \text{ m}} \quad (\text{Dist of C.o.M from pivot})$$

Resolving  $\uparrow$ :

$$0.6X + 800 = X$$

$$\Rightarrow 800 = 0.4X$$

$$\Rightarrow X = \underline{2000 \text{ N}}$$



- 11 A car is travelling along a straight road with constant acceleration. It passes a road sign.

It travels 12.2 m in the 3<sup>rd</sup> second and 14.4 m in the 4<sup>th</sup> second after passing the road sign.

What was the speed of the car as it passed the road sign?

A  $2.20 \text{ ms}^{-1}$

$$S = ut + \frac{1}{2}at^2$$

B  $4.50 \text{ ms}^{-1}$

$$12.2 = u(1) + \frac{1}{2}(a)(1)$$

C  $6.70 \text{ ms}^{-1}$

$$12.2 = u + \frac{a}{2} - (i)$$

D  $7.80 \text{ ms}^{-1}$

$$(12.2 + 14.4) = u(2) + \frac{1}{2}(a)(4)$$

$$26.6 = 2u + 2a$$

$$13.3 = u + a - (ii)$$

$$\begin{array}{rcl} u + 0.5a & = & 12.2 \\ \cancel{u} + \cancel{a} & = & 13.3 \\ \hline -0.5a & = & -1.1 \end{array}$$

$$a = \frac{11}{5} \text{ ms}^{-2}$$

$$u = 13.3 - \frac{11}{5} = 13.3 - 2.2 = \underline{\underline{11.1 \text{ ms}^{-1}}}$$

$u$  is velocity at start of the 3<sup>rd</sup> second.

$u_0$  = velocity when sign is crossed

$$u = u_0 + at$$

$$11.1 = u_0 + \frac{11}{5} \times 2$$

$$11.1 - 4.4 = u_0$$

$$u_0 = \underline{\underline{6.7 \text{ ms}^{-1}}}$$



- 12 A light spring has unstretched length 0.40m and spring constant  $50\text{ N m}^{-1}$ .

The spring is stretched by a varying tension force that starts at a value of zero and increases at a constant rate of  $0.20\text{ N s}^{-1}$  up to a maximum value.

When the force reaches its maximum value, the strain energy of the spring is 0.25J.

What is the average power used to stretch the spring?

(Assume that the spring obeys Hooke's law.)

A 0.010W

$$\frac{1}{2} \times F \times \Delta x = 0.25$$

B 0.020W

C 0.040W

$$\frac{1}{2} \times k \times \Delta x^2 = 0.25$$

D 0.080W

$$25 \times \Delta x^2 = 0.25$$

E 1.0W

$$\Delta x = 10\text{ m}$$

F 2.0W

$$\frac{1}{2} \times F \times 10 = 0.25$$

G 4.0W

$$F = \frac{0.5}{10} = 0.05\text{ N}$$

H 8.0W

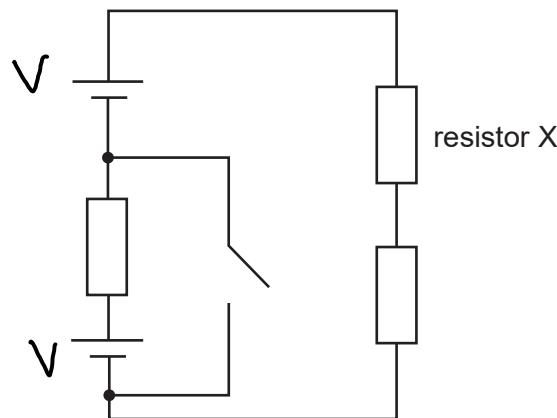
$$\text{Rate} = 0.2 \text{ N s}^{-1}$$

$$\Rightarrow t = \frac{0.05}{0.2} = 25$$

$$P = \frac{E}{t} = \frac{0.25}{25} = \underline{\underline{0.01\text{ W}}}$$



- 13 The circuit below contains three identical resistors, and two identical cells. When the switch is open, the power dissipated by resistor X is P.



What is the power dissipated by resistor X after the switch is closed?

- A  $\frac{P}{4}$
- B  $\frac{9P}{16}$
- C  $\frac{3P}{4}$
- D  $P$
- E  $\frac{16P}{9}$
- F  $\frac{9P}{4}$

When switch closes, no current goes through the resistor next to the switch.

When closed.  $R_T = 3X$

$$V = IR \Rightarrow \frac{2V}{3X} = I$$

$$\begin{aligned} P &= I^2 R = \left(\frac{2V}{3X}\right)^2 \times \\ &= \frac{4V^2}{9X} \end{aligned}$$

When open:

Pd across X =  $\frac{V}{2}$  (potential divider)

$$P = \frac{V^2}{R} = \frac{V^2}{4X} \div X = \frac{V^2}{4X}$$

$$\frac{V^2}{4X} \div \frac{4V^2}{9X} = \frac{V^2}{4X} \times \frac{9X}{4X} = \frac{9}{16}$$

$$\begin{aligned} \Rightarrow P_{\text{lost}} &\text{ when switch open} \\ &= \frac{9}{16} P \end{aligned}$$



- 14 A car of mass  $m$  is pulling a caravan of mass  $M$ .

The caravan is connected to the car by a metal bar of length  $l$  and cross-sectional area  $A$ .

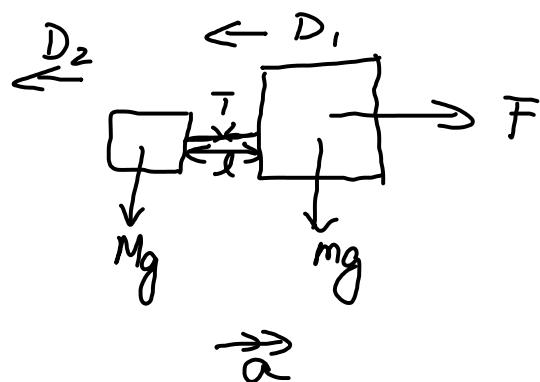
The Young modulus of the metal from which the bar is made is  $E$ .

The car and caravan have a constant forward acceleration  $a$  and there are total resistive forces  $D_1$  acting on the car and  $D_2$  acting on the caravan.

What is the extension of the bar?

(Assume that the bar obeys Hooke's law and that the cross-sectional area of the bar remains unchanged.)

- A  $\frac{Mal}{EA}$
- B  $\frac{Ma}{EA}$
- C  $\frac{(Ma + D_2)l}{EA}$
- D  $\frac{Ma + D_2}{EA}$
- E  $\frac{(Ma + ma + D_1 + D_2)l}{EA}$
- F  $\frac{Ma + ma + D_1 + D_2}{EA}$



NZL on caravan.

$$T - (D_2) = ma$$

$$E = \frac{T \times l}{A \times \Delta x} \Rightarrow \frac{EA \Delta x}{l} = T$$

$$\Rightarrow \frac{EA \Delta x}{l} - D_2 = ma$$

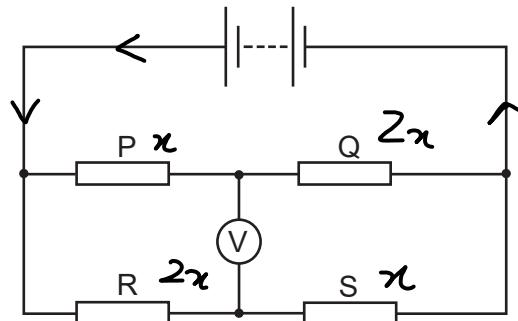
$$\Rightarrow \frac{EA \Delta x}{l} = ma + D_2$$

$$\Rightarrow \Delta x = \frac{(ma + D_2)l}{EA}$$



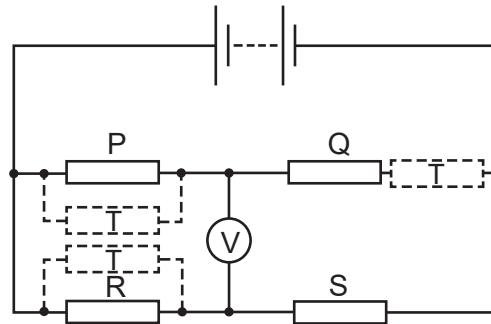
- 15 Four resistors, P, Q, R and S, are connected to a battery with negligible internal resistance, as shown in the diagram.

P and S each have resistance  $x$ . Q and R each have resistance  $2x$ .



A fifth resistor, T, which has resistance  $x$ , is to be added to the circuit in one of the following listed positions, as shown in the diagram:

- 1 in parallel with P  $\text{Res of II part} = 0.5x \Rightarrow \text{Diff b/w } R \text{ and } P \uparrow$
- 2 in series with Q  $Pd \text{ across } P \downarrow (\text{divider}) \Rightarrow \uparrow \text{ Diff b/w } R \text{ & } P$
- 3 in parallel with R  $\text{Res of II part } \downarrow \Rightarrow \text{Diff b/w } R \text{ & } P \downarrow$



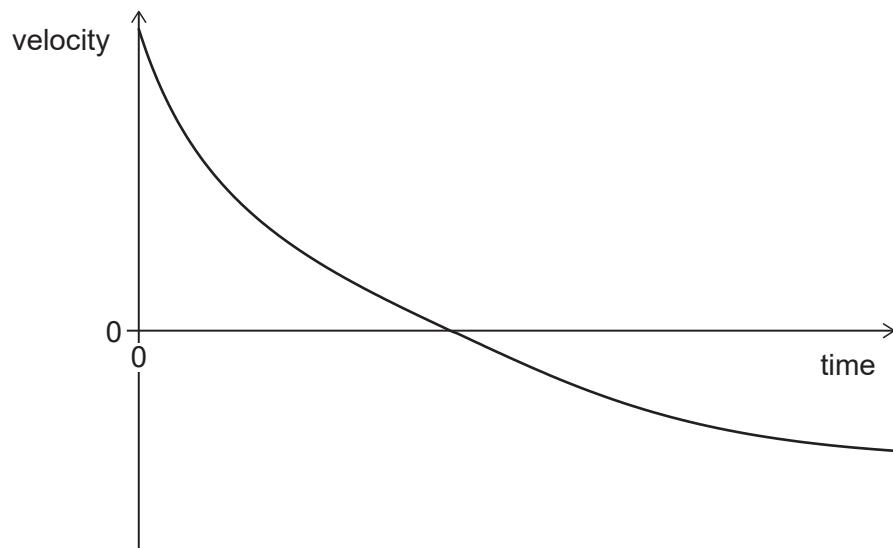
Which of the positions for resistor T causes an increase in the magnitude of the voltmeter reading?

$$Pd \text{ across } R - Pd \text{ across } P = V$$

- A none of them
- B 1 only
- C 2 only
- D 3 only
- E** 1 and 2 only
- F 1 and 3 only
- G 2 and 3 only
- H 1, 2 and 3



- 16 A ball is thrown vertically upwards in air. The ball travels upwards to reach its highest point and then falls back down to its initial starting position. The velocity–time graph for the ball is shown.



Which of the following statements is/are correct?

- 1 The magnitude of the acceleration of the ball is only equal to the magnitude of the acceleration of free fall when it is at its highest point. ✓ (*It's always g*)
- 2 The time taken for the upward journey of the ball is equal to the time taken for the journey back down to its starting position. ✗
- 3 The maximum increase in the gravitational potential energy of the ball is less than its initial kinetic energy and greater than its kinetic energy when it returns to its starting position. ✓ (*energy lost to air resistance during motion*)

A none of them

B 1 only

C 2 only

D 3 only

E 1 and 2 only

F 1 and 3 only

G 2 and 3 only

H 1, 2 and 3



- 17 A stone is projected from level ground at an angle of  $30^\circ$  to the horizontal.

After 1.0s the stone lands on a ledge at height  $h$  above the level ground.

During this journey the vertical component of velocity of the stone is upwards for the first 0.60s and downwards for the remaining 0.40s.

What is the value of  $h$ ?

(gravitational field strength =  $10 \text{ N kg}^{-1}$ ; assume that air resistance is negligible)

A 1.0m

B 1.6m

C 2.0m

D 3.0m

E 3.2m

F 6.0m

G 7.0m

H 11m



$$v_x = v \cos 30$$

$$v_y = v \sin 30$$

$$h = v_y t - \frac{1}{2} g t^2$$

$$h = v \sin 30 t - \frac{1}{2} g t^2 \quad (\text{i})$$

at top  $\Rightarrow$  at  $t = 0.6\text{s}$  (acc to question)

$$v_y = 0$$

$$\Rightarrow v_y = u_y - gt = v \sin 30 - 10(0.6) = 0$$

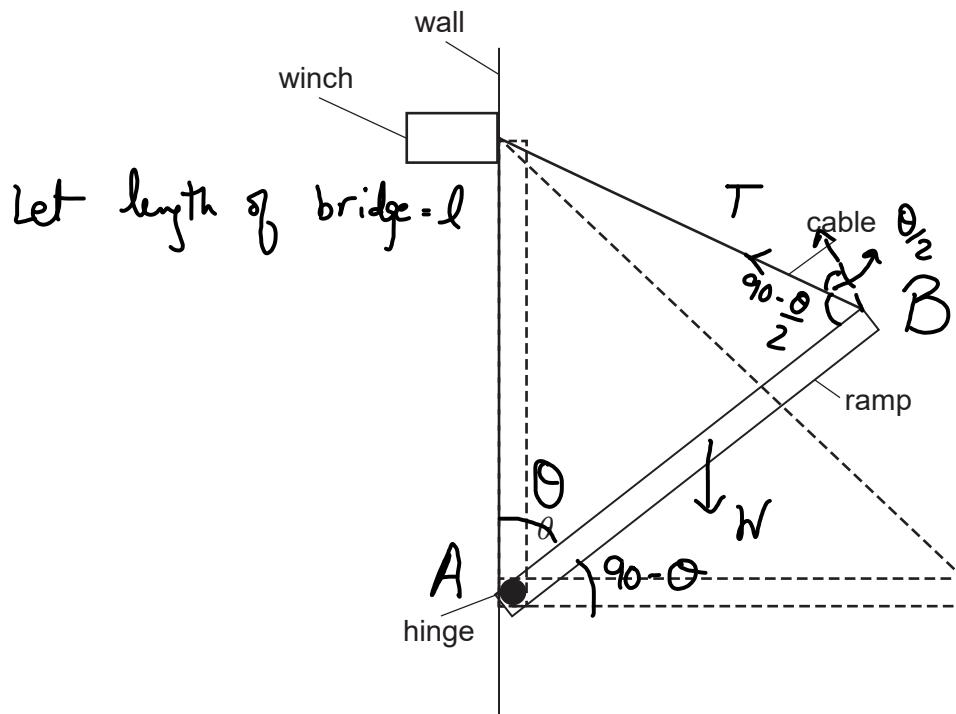
$$v \sin 30 = 6$$

$$v = \underline{\underline{12 \text{ ms}^{-1}}}$$

Solve in (i):  $h = \frac{1}{2} (1) - 5 = \underline{\underline{1\text{m}}}$



- 18 A drawbridge system consists of a uniform ramp, of weight  $W$ , that is smoothly hinged at its lower end. The upper end is connected by a light, inextensible cable to a winch that is fixed to the wall in the position shown in the diagram.



The ramp is lowered slowly, at constant speed, from its closed (vertical) position ( $\theta = 0^\circ$ ) to its open (horizontal) position ( $\theta = 90^\circ$ ).

What is the maximum tension in the cable during this process?

(double-angle identities:  $\sin 2\theta = 2\sin \theta \cos \theta$ ;  $\cos 2\theta = \cos^2 \theta - \sin^2 \theta$ )

A  $\frac{W}{2}$

~~A):  $W \times \cancel{\cos(90-\theta)} = T \cos \frac{\theta}{2} \times \cancel{\frac{1}{2}}$~~

B  $\frac{W}{\sqrt{2}}$

~~$W \sin \theta = T \cos \frac{\theta}{2}$~~

C  $\frac{\sqrt{3}W}{2}$

~~$W \sin \frac{\theta}{2} \cos \frac{\theta}{2} = T \cos \frac{\theta}{2}$~~

D  $W$

~~$W \sin \frac{\theta}{2} = T$~~

E  $\frac{2W}{\sqrt{3}}$

F  $\sqrt{2}W$

G  $2W$

Max value of  $\sin \frac{\theta}{2}$  is @  $\theta = 90^\circ$

$\sin \frac{90}{2} = \frac{1}{\sqrt{2}}$  ( $\theta$  of bridge can't exceed  $90^\circ$ )

$$\frac{W}{\cancel{F}} = T$$

- 19 A particle of mass  $m$  has kinetic energy  $E$  when it collides with a stationary particle of mass  $M$ . The two particles coalesce.

Which of the following expressions gives the total kinetic energy transferred to other forms of energy in the collision?

A  $0$

B  $\frac{ME}{(M+m)}$

C  $\frac{mE}{(M+m)}$

D  $\frac{(M+m)E}{m}$

E  $\frac{(M+m)E}{M}$

F  $\frac{mME}{(M+m)^2}$

G  $E$

$$\frac{1}{2} mu^2 = E$$

$$mu^2 = 2E$$

$$u = \sqrt{\frac{2E}{m}}$$

PCLM:

$$mu + 0 = (m+M)v$$

$$mu = (m+M)v$$

$$\frac{mu}{m+M} = v$$

$$\Rightarrow \text{Final KE} = \frac{1}{2} \times (m+m) \times m^2 \times \frac{2E}{m} \times \frac{1}{(m+M)^2}$$

$$= \frac{mE}{m+M}$$

$$\text{Energy transferred to other forms} = E - \frac{mE}{m+M}$$

$$= \frac{Em + EM - Em}{m+M} = \frac{EM}{m+M}$$



- 20 The critical angle for light incident on a boundary from medium X to air is  $45^\circ$ .

The critical angle for light of the same frequency incident on a boundary from medium Y to air is  $60^\circ$ .

There is a boundary between medium X and medium Y. Light of the same frequency travelling in one of these mediums is incident on this boundary.

In which direction of incidence is there a critical angle at this boundary, and within what range is this critical angle?

	<i>direction of incidence</i>	<i>critical angle</i>
A	X to Y	between $0^\circ$ and $30^\circ$
B	X to Y	between $30^\circ$ and $45^\circ$
C	X to Y	between $45^\circ$ and $60^\circ$
D	X to Y	between $60^\circ$ and $90^\circ$
E	Y to X	between $0^\circ$ and $30^\circ$
F	Y to X	between $30^\circ$ and $45^\circ$
G	Y to X	between $45^\circ$ and $60^\circ$
H	Y to X	between $60^\circ$ and $90^\circ$

$$n_x = \frac{1}{\sin 45}$$

$$n_y = \frac{1}{\sin 60}$$

$$\sin 60 > \sin 45$$

$$\Rightarrow \frac{1}{\sin 60} < \frac{1}{\sin 45}$$

$$\Rightarrow n_y < n_x$$

$\theta_c$  is from slower to faster medium. In

this case, X to Y,

$$\frac{n_x}{n_y} = \frac{1/\sin 45}{1/\sin 60} = \frac{\sin 60}{\sin 45} = \frac{\sqrt{2}}{\sqrt{3}} \approx \frac{1.4...}{1.7...} \approx 0.8$$

$$\frac{\sqrt{3}}{2} \times \frac{1.7...}{2} \approx 0.85... \therefore \theta_c \text{ slightly less than } 60^\circ$$

**END OF TEST**

