AS COMPETITION PAPER 2007

Total Mark/50

SOLUTIONS

Section A: Multiple Choice

1. **C**
2. **D**
3. **B**
4. **B**

5. **B**
6. **A**
7. **A**
8. **C**

Section B: Written Answer

Question 9.

A mass *M* is attached to the end of a horizontal spring. The mass is pulled to the right, 8 cm from its rest position. It is then released so that the mass oscillates to the left and right, with the system gradually losing energy over many cycles.

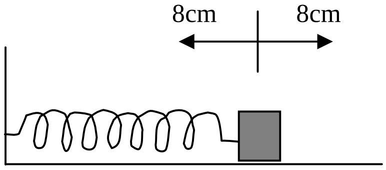

fig. 7

a) State the energy changes that take place over one complete cycle as the mass moves to the left and then back to the right.

Elastic pe → ke → elastic pe → ke → elastic pe

for correct energy changes ✓

for 1 cycle of changes only – no more and no less ✓ [2]

b) The energy stored in a stretched spring is proportional to the square of the extension of the spring. If after some time, the amplitude of the oscillation is reduced to 1 cm, what fraction of the initial energy has been lost? Show your working.

Amplitude reduces from 8 cm to 1 cm

Energy reduces from 64 to 1 unit ✓

So 1/64 left or 63/64 lost ✓ for either answer [2]

c) We will need to use a concept that you have met in radioactivity. State what is meant by the half-life of a radioactive substance.

The time taken for half of the (radioactive) nuclei (allow material) to decay ✓

[1]

d) Now we shall apply this concept to the loss of energy from the oscillating system. The amplitude decays away in the same manner as radioactive decay (exponentially). How many half-lives have passed for the amplitude to reduce to 1 cm?

8 cm $\rightarrow$ 4 cm $\rightarrow$ 2 cm $\rightarrow$ 1 cm ✓

3 half-lives ✓ [2]

e) The period of oscillation does not depend upon the amplitude of the oscillation, being the same for both large and small amplitudes. The period of oscillation is 0.5 seconds. The half-life for the amplitude loss is 5 seconds. How many oscillations have occurred by the time the amplitude has dropped down to 1cm?

10 oscillations per half-life ✓

30 oscillations in 3 half lives ✓ allow ecf [2]

f) The energy is also dissipated away exponentially with time. Using your answer to part (b) for the energy lost, how many energy loss half-lives have passed when the amplitude has reduced to 1 cm?

64 $\rightarrow$ 32 $\rightarrow$ 16 $\rightarrow$ 8 $\rightarrow$ 4 $\rightarrow$ 2 $\rightarrow$ 1 ✓ No ecf allowed

6 half-lives ✓

[2]

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Question 10.

a) At the earth's surface, the radiant power received from the Sun normally is 1.3×10^3 W per square metre. The power radiated by the Sun is the same everywhere over the Sun's surface. If the Earth orbits at a distance of 1.5×10^{11} m from the Sun, calculate the total energy radiated away by the Sun each second. (It may be useful to know that the surface area of a sphere is $4\pi r^2$).

Total power $= \mathbf{1.3 \times 10^3 \times area}$ **W** ✓

$= \mathbf{1.3 \times 10^3 \times 4 \times \pi \times r^2}$ **W**

$= \mathbf{3.7 \times 10^{26}}$ **W** $=$ $\mathbf{4 \times 10^{26}}$ **W** ✓ [2]

b) Although you may not have studied it yet, Einstein produced a famous equation relating mass and energy which we shall use, $E = mc^2$, where E is energy in joules, m is mass in kg, c is the velocity of light in a vacuum ($c = 3\text{x}10^8$ m/s). Using your answer to part (a), calculate the mass loss of the Sun due to the energy being radiated away each second.

$\mathbf{3.7 \times 10^{26} \div 9 \times 10^{16} = 4.1 \times 10^9}$ **kg (/s)**

$= \mathbf{4 \times 10^9}$ **kg (/s)** ✓ allow e.c.f.

= 4 million tonnes /s [1]

c) If the mass of the Sun is $2\text{x}10^{30}$ kg, what is the percentage of the Sun's mass that is lost by radiation each year?

$\mathbf{\dfrac{4 \times 10^9}{2 \times 10^{30}} \times 365 \times 24 \times 3600 \times 100\%}$ allow e.c.f.

✓ for 365 x 24 x 3600 seconds in a year

$= \mathbf{6.3 \times 10^{-12}\ \%}$ ✓

[2]

d) Assuming that this rate remains constant, what is the percentage loss of mass of the sun since it was formed, five thousand million years ago?

$\mathbf{6 \times 10^{-12}\ \% \times 5 \times 10^9}$ ✓ allow e.c.f.

= 0.03 % ✓

[2]

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Question 11.

The transformer, shown in fig. 9, outputs power at 415 V, along a copper cable 50 m in length, to an electrical machine. The total resistance of the copper conductor (100 m there and back) is 0.0493 Ω at an operating temperature of 60°C. The machine takes a current of 200 A.

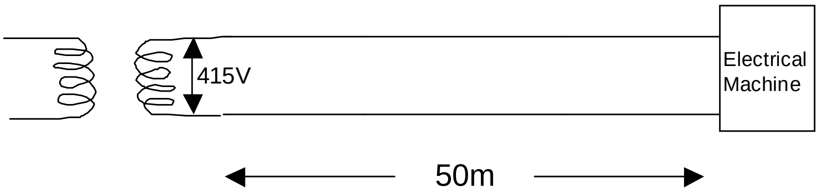

fig. 7

a) What is the total power that the transformer is supplying to the machine and cable?

Power = V x I ✓

= 415 x 200

= 83,000 W ✓ [2]

b) What is (i) the power loss in the cable, and (ii) what percentage is this of the total power supplied?

Power in cable $= I^2 R$ ✓

$= 200^2 \times 0.0493$

$= 1972\ W = 1.9\ kW$ ✓

Percentage is $\frac{1.9}{83{,}000} \times 100\% = 2.3 \times 10^{-3}\ \%$ ✓

[3]

c) Often the conductor size is chosen not on the basis of the steady current required, but on the short circuit current that might occur. If in our wiring, a short circuit occurred at the machine end of the cable, a current of 6000 A could be expected. Explain why this current is significantly less than that calculated from the 415 V supply and the 0.0493 Ω resistance of the cable.

The secondary of the transformer also has some resistance ✓

Resistance at the short circuit connection ✓

Do not accept cable heats up [2]

d) If the circuit breaker produces a delay of 0.4 s before it breaks the circuit, calculate the heat energy generated in this short time interval. Assume that the resistance of the wire does not change significantly as it heats up.

Energy converted = V I t ✓

= 415 x 6000 x 0.4

= 996 kJ ✓

[2]

e) The heat energy required to raise the temperature of 1kg of copper by $1^{o}C$ is called the specific heat capacity of copper. It has the value is 385 J $kg^{-1}\ {}^{o}C^{-1}$. We can use a simple formula,

heat energy supplied = mass **x** specific heat capacity **x** temperature rise

in order to determine the temperature rise of the copper cable, assuming no heat loss to the surroundings.

Calculate

(i) The mass of copper in the 100 m of cable, given that its

cross sectional area = 50 mm^2

density of copper = 8960 kg/m^3

mass = density x volume ✓ accept if used correctly numerically

= 8960 x 100 x 50 x 10^{-6}

= 44.8 kg ✓ [2]

(ii) Calculate the final temperature of the cable after 0.4s, if its initial temperature is $60^{o}C$.

996,000 = 44.8 x 385 x temperature rise ✓ allow e.c.f.

Hence temperature rise = $58^{o}C$

And so final temperature = 118 ^{o}C ✓

Mark acceptable just for the correct temperature rise [2]

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Question 12.

A single uniform underground cable linking A to B, 50 km long, has a fault in it at distance d km from end A. This is caused by a break in the insulation at X so that there is a flow of current through a fixed resistance R into the ground. The ground can be taken to be a very low resistance conductor. Potential differences are all measured with respect to the ground, which is taken to be at 0 V.

fig.8

In order to locate the fault, the following procedure is used. A potential difference of 200 V is applied to end A of the cable. End B is insulated from the ground, and it is measured to be at a potential of 40 V.

a) What is the potential at X? Explain your reasoning.

40 V ✓

B is at the same potential as X because **no current flows along BX** ✓

[2]

b) What is

(i) the potential difference between A and X?

200 – 40 = 160 V ✓

[1]

(ii) the potential gradient along the cable from A to X (i.e. the volts/km)?

$$\frac{(200-40)}{d} = \frac{160}{d}$$ ✓ allow reciprocal

[1]

c) The potential applied to end A is now removed and A is insulated from the ground instead. The potential at end B is raised to 300 V, at which point the potential at A is measured to be 40 V.

(i) What is the potential at X now?

40 V [1]

(ii) Having measured 40 V at end B initially, why is it that 40 V has also been required at end A for the second measurement?

So that X is at the same potential ✓

and then the same current flows into the ground through R ✓

(and the same currents will therefore flow along AX and BX - see part (e) (✓)) [2]

d) What is the potential gradient along the cable from B to X?

$$\frac{(300-40)}{(50-d)} = \frac{260}{(50-d)}$$ ✓

[1]

e) The potential gradient from A to X is equal to the potential gradient from B to X.

(i) Explain why this is true

Because the same currents flowed along AX and BX ✓

So $I = \frac{V_{AX}}{R_{AX}} = \frac{V_{BX}}{R_{BX}}$ ✓

and R is proportional to length (✓) [2]

(ii) From the two potential gradients that you obtained earlier, deduce the value of *d*.

$$\frac{160}{d} = \frac{260}{(50-d)}$$ **260** d **= 50 x 160 – 160** d

Hence d **= 19 km** ✓

[1]

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