BPhO Physics Challenge

(formerly A2 Challenge)

September/October 2020

Instructions

Time: 1 hour.

Questions: Answer ALL questions.

Marks: Total of **50 marks**.

Instructions: You are allowed any standard exam board data/formula sheet.

Equipment: Any standard non-graphical calculator may be used.
Ruler and pencil may be needed.

Solutions: These questions are about problem solving. Draw diagrams in order to understand the questions. You must write down the questions in terms of symbols and equations; then try calculating quantities in order to work quickly towards a solution. In these questions you will need to explain your reasoning by showing clear working. Even if you cannot complete the question, show how you have started your thinking, with ideas and, generally, by drawing a diagram.

Clarity: Solutions must be written legibly and set out properly with a "narrative" which links one step to the next (and, so, therefore, hence, but, also, using equ 5, etc.).

Important Constants

Constant	Symbol	Value
Speed of light in free space	c	$3.00 \times 10^{8}\,\mathrm{m\,s^{-1}}$
Elementary charge	e	$1.60 \times 10^{-19}\,\mathrm{C}$
Mass of electron	m_e	$9.11 \times 10^{-31}\,\mathrm{kg}$
Atomic mass unit	u	$1.66 \times 10^{-27}\mathrm{kg}$
Earth's gravitational field strength	g	$9.81\,\mathrm{N\,kg^{-1}}$
Planck constant	h	$6.63 \times 10^{-34}\,\mathrm{J\,s}$
Avogadro constant	N_{A}	$6.02 \times 10^{23}\,\mathrm{mol^{-1}}$
Molar Gas constant	R	$8.31\,\mathrm{J\,mol^{-1}\,K^{-1}}$

Qu 1. This question is about estimations.

Especially at the start of research or when solving a complex problem, it is useful to have an estimate of the sort of outcome to expect; making approximate calculations is a useful skill.
Use the suggestions given and any other estimated values of your own to answer the following:

a) "Hanging on by a hair's breadth" is a well known phrase that is not meant to be taken literally.
However, what would be the stress in a single hair supporting a person?

Begin by giving your own estimated values of the mass of a person, and the diameter of a human hair (for approximation purposes, you may assume a square cross-section).

[4]

b) An inventor claims to have a microprocessor that will operate at $100\,\mathrm{THz}$. Comment on whether this is sensible, based on the likely time for information to travel from one part of the chip to another.

Begin by giving your estimated values of the size of a microprocessor chip, and considering how fast information might travel across it.

[4]

c) What is the density of a neutron?

Begin by finding the mass of a hydrogen atom by using the common chemical information [H=1].

Also, make an estimate of the diameter of a nucleus.

[4]

[12 marks]

Qu 2. This question is about elastic collisions between equal masses.

a) What is meant by a (perfectly) elastic collision? [1]

In the novelty toy, known as Newton's Cradle, illustrated in **Figure 1**, equal masses collide head-on, in a manner which may be assumed to be perfectly elastic.

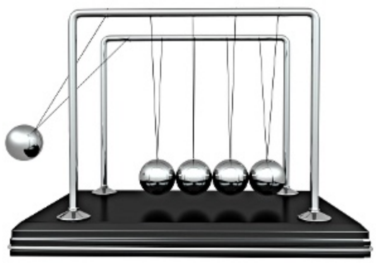

Figure 1: Newton's cradle
Credit: https://www.real-world-physics-problems.com/newtons-cradle.html

b) Show that a moving ball meeting a stationary ball head-on will come to a standstill, while the stationary ball will exactly acquire the velocity of the incoming ball. [2]

c) Hence explain the behaviour of the toy when one ball is released from the position shown. [1]

Neutrons, being uncharged, are difficult to detect and it was not until 1932 that James Chadwick announced their identification. In his experiment, neutrons were allowed to impact paraffin wax and protons were ejected, which, being charged, are much easier to detect.

d) Use your ideas about Newton's cradle to explain what is happening in Chadwick's experiment. [2]

e) Of course, all collisions are not head on. Draw a diagram to show a perfectly elastic, *oblique* collision between a moving ball and a stationary one of equal mass. Label the initial velocity $\vec{u}$ and the final velocities $\vec{v}_1$ and $\vec{v}_2$.

[1]

f) By drawing a vector triangle to represent the conservation of momentum in this process, show that the two balls must part on perpendicular paths. [3]

[10 marks]

Qu 3. This question relates to ways in which light energy may be concentrated by the refracting behaviour of a 'crystal ball'.

The diagram below, **Figure 2**, shows part of a transparent glass sphere of radius R with refractive index $n = 2.0$.

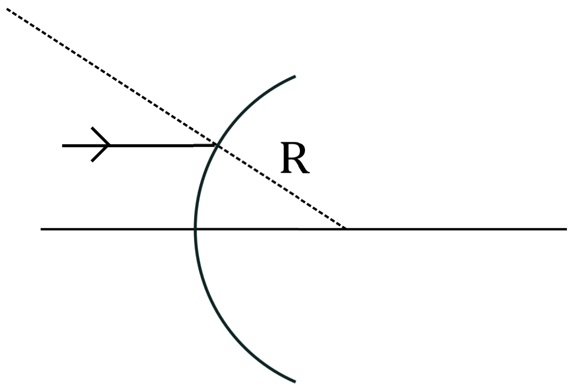

Figure 2: A ray is shown entering the spherical surface, parallel and close to a diameter. A radius is provided as a reference line.

a) Copy **Figure 2** and complete the path of the ray beyond where it intersects the given diameter, marking the angle of incidence i and the angle of refraction r. [1]

b) Calculate the critical angle for the glass of this sphere in air. [1]

c) Bearing in mind that the angles involved are small, calculate the angle of refraction. (You should use the small angle approximation $\sin\theta \approx \theta$). [1]

d) Use this information obtained about to r to copy and complete this ray diagram for **Figure 3**, indicating the point of intersection of the ray and the diameter. [3]

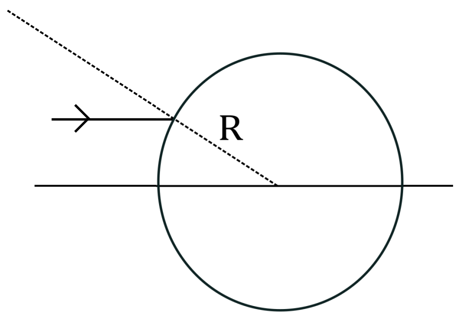

Figure 3: A ray is shown entering the sphere, parallel and close to a diameter (a paraxial ray) and a radius is provided as a reference line.

The hours of sunshine each day can be recorded by such an instrument, know as the Campbell-Stokes sunshine recorder. An example is shown in the illustration of **Figure 4**.

e) Explain how this works, with specific reference to the strip of paper around the circumference. [2]

Figure 4: A Campbell-Stokes sunshine recorder
Credit: Bureau of Meteorology Campbell-Stokes recorder (Darwin Airport Met Office)

f) Another version of this sunshine recorder is shown in **Figure 5**. Explain where this might be located on the Earth, giving reasons for your suggestion. [2]

Figure 5: A pair of Campbell-Stokes sunshine recorders
Credit: CambridgeBayWeather

[10 marks]

Qu 4. This question investigates a simple model of metallic conduction.

The diagram of **Figure 6** shows a lattice of metallic ions, which are essentially fixed, surrounded by a 'gas' of free electrons which have been contributed by the original metal atoms, whose outer electrons are only loosely bound (delocalised). These 'free' electrons move around randomly at a high speed in a manner similar to gas molecules.

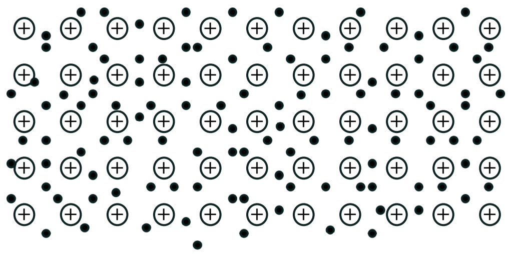

Figure 6: A model of an atomic lattice of ions ($\oplus$) and free electrons ($\cdot$)

When a potential difference is applied across two opposite faces of the metal, the electrons have a drift velocity superimposed on the random motion already described. This organised additional velocity is responsible for the transfer of charge which we commonly call an electric current.

a) Why does the rapid motion of electrons, present whether or not a p.d. is applied, **not** contribute to any current? [1]

b) Given that an air molecule has a mass of $\approx 30\,\mathrm{u}$ and a speed of $500\,\mathrm{m\,s^{-1}}$ at room temperature, using our " free electron model" estimate the random velocity (often referred to as the thermal velocity) of the electrons in a conductor at $\approx 300\,\mathrm{K}$. [3]

The diagram of **Figure 7** shows a section of conductor of cross-sectional area A, containing n free electrons per unit volume (the number density) all travelling to the left with an average drift velocity, v_d. This drift constitutes a conventional current, I.

c) Use the formula $I = nv_\mathrm{d}Ae$ to estimate the drift velocity of the electrons in a lighting cable that has a cross-sectional area of $1.0\,\mathrm{mm}^2$, carrying a current of $1.0\,\mathrm{A}$.
(Copper contains approximately 10^{29} free electrons per m^3.) [1]

d) Comment on your answer to part (c) in relation to your answer for part (b). [1]

e) Explain why room lights come on instantly even if they are several metres from the switch. [1]

This model can also give some insight into resistivity and Joule heating. Suppose a p.d. of V is applied across a length, ℓ, of conductor

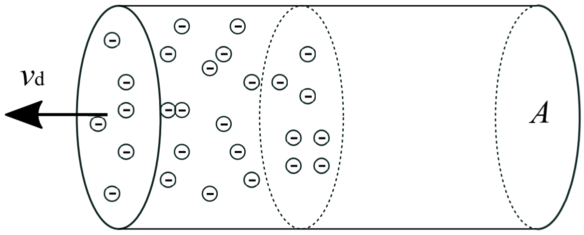

Figure 7: Model of a wire with free electrons flowing.

f) State the electric field strength in the conductor and thus show that the acceleration a of the electrons, of mass, m, which is superimposed on their thermal motion, is given by the expression

$$a = \frac{Ve}{m\ell}$$

[3]

g) Sketch a graph of the drift velocity of such an electron starting from rest. [1]

Now, after a while an interaction with the lattice will randomise the drift velocity, effectively returning it to zero: this stop-start acceleration process then repeats at **irregular intervals** for as long as the external p.d. is applied.

h) By considering the kinetic energy of the electrons, comment on the effect of resistive heating of a current-carrying conductor. [1]

i) Extend the graph of part (g) to show several cycles of the stop-start drift motion. [1]

This random interaction frequency is plainly difficult for calculation purposes, so we define τ, the mean free time, which is the average time between the randomising interactions.

j) From part (f) find an expression for the average speed of the electrons over a time interval, τ. This is the drift velocity v_{d}. [2]

k) Using the formula in part (c) find an expression for the current I. [1]

l) Hence, determine the resistivity of the metal in terms of m, e, n and τ. [2]

[18 marks]

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