

**PHYSICS ADMISSIONS TEST**  
**Thursday , 5<sup>th</sup> of November 2020**

**Time allowed: 2 hours**

*For candidates applying to Physics, Physics and Philosophy,  
Engineering, or Materials Science*

---

**Total 26 questions [100 Marks]**

Answers should be written on the question sheet in the spaces provided,  
and you are encouraged to show your working.  
You should attempt as many questions as you can.

**No tables, or formula sheets may be used.**

Answers should be given exactly and in simplest terms  
unless indicated otherwise.

Indicate multiple-choice answers by circling the best answer.  
Partial credit may be given for correct workings in multiple choice questions.

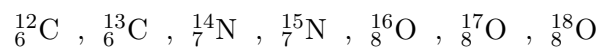
The numbers in the margin indicate the marks expected to be assigned  
to each question. You are advised to divide your time according to  
the marks available.

You may take the gravitational field strength  
on the surface of Earth to be  $g \approx 10 \text{ m s}^{-2}$

**Do NOT turn over until told that you may do so.**

This page is left intentionally blank

1. The stable isotopes of carbon, nitrogen and oxygen are represented symbolically below:



Which of the following statements are true?

[2]

1.  ${}^{13}_6\text{C}$  has a larger number of protons than  ${}^{12}_6\text{C}$ .
2.  ${}^{15}_7\text{N}$  has a larger mass than  ${}^{14}_7\text{N}$ .
3.  ${}^{16}_8\text{O}$  has a larger nuclear charge than  ${}^{15}_7\text{N}$ .
4.  ${}^{18}_8\text{O}$  has a larger mass per unit charge than  ${}^{12}_6\text{C}$ .
5.  ${}^{14}_7\text{N}$  has a larger number of neutrons than  ${}^{13}_6\text{C}$ .

A	B	C	D	E
1, 3, 4	3, 4, 5	2, 3, 4	1, 2, 3	2, 3, 5

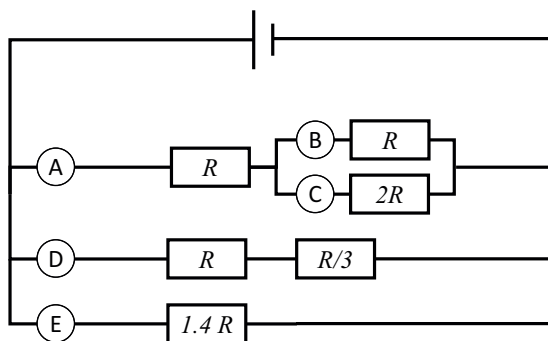
**2.** A triangle  $ABC$  has vertices at points in two-dimensional Cartesian co-ordinates  $A : (0, 1)$ ,  $B : (1, 2)$ , and  $C : (-1, 2)$ . It is reflected in the line  $y = x$  and then rotated around the origin by 90 degrees in a clockwise direction. Which single transformation maps the initial triangle to the final state of the above transformations?

[2]

A	B	C	D	E
reflection in $x = 0$	reflection in $y = 0$	rotation by $180^\circ$ anti-clockwise around the origin	rotation by $90^\circ$ anti-clockwise around $(2, 0)$	scale factor of $-1$

3. Which ammeter A, B, C, D, E gives the highest reading?

[2]



A	B	C	D	E
---	---	---	---	---

4. Solve  $\log_2 x + \log_2(2x + 3) = 1$  for  $x$ .
- [2]

A	B	C	D	E
$x = -2$	$x = \frac{1}{2}$	$x = 1$	$x = -2$ and $\frac{1}{2}$	$x = 0$

5. If the gravitational field strength at the Earth's surface is  $g_E = 10 \text{ N/kg}$ , and at a distance  $R > R_E$  from its centre the field strength is  $g_R = 2 \text{ N/kg}$ , what is the radius of the Earth  $R_E$  in terms of  $R$ ?

[2]

A	B	C	D	E
$R/25$	$R/5$	$R/\sqrt{10}$	$R/\sqrt{5}$	$R/\sqrt{2}$

- 6.** Consider the function  $y(x) = \sin(\frac{100}{x})$ . The angle is in degrees, so that  $\sin(180) = 0$ . How many maxima of  $y(x)$  occur for  $x > 0.1$ ?

[2]

A	B	C	D	E
0	1	3	14	$\infty$



7. What is the order, from shortest to longest, of the wavelengths of the peak electromagnetic emission from each of the following objects?

[2]

1. an electric torch
2. a microwave oven
3. a radioactive source
4. a hot cooking stove
5. a short-wave radio transmitter.

A	B	C	D	E
31425	52413	34152	31245	54213

8. A particle of type X decays with equal probability either to a pair of particles of type Y or a pair of particles of type Z. Both Y and Z particles are stable.

The decays of two X particles are observed. A pair of Y particles is found among the decay products. What is the probability that a pair of Z particles is among these decay products?

[2]

A	B	C	D	E
1/4	1/3	1/2	2/3	1

**9.** Ten students need to complete their compulsory practicals for their high school examinations as detailed in the table below:

No. of students	No. of different practicals to complete
2	1
4	2
4	3

The school only has one laboratory in which several different experiments can be set up simultaneously. A maximum of six students are allowed in the school's laboratory for a lesson. Each practical takes one lesson. What is the minimum number of lessons required to complete all the practicals?

[2]

<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>	<b>E</b>
3	4	5	6	10

10. What is the next number in the sequence? 37, 41, 43, 47, 53, 59 [2]

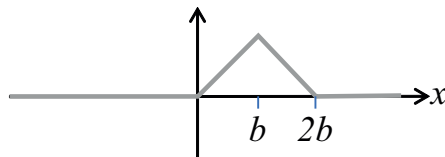
A	B	C	D	E
61	62	64	65	67

**11.** A stone of average diameter 10 cm is hit with a hammer and splits into pieces. Every time the stone or one of its pieces is hit, it splits into three further pieces of equal volume and similar shape. How many hits will it take before a piece reaches the size of a typical atom?

[2]

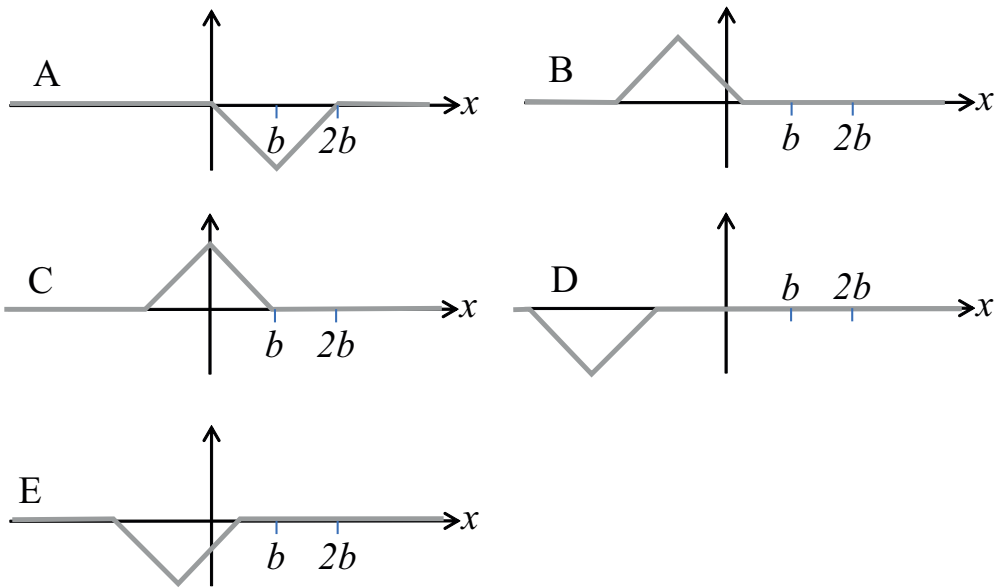
A	B	C	D	E
9	12	22	56	81

12. The graph below shows a function  $f(x)$ .



If  $a$  is a constant such that  $0 < a < b$ , identify the sketch of  $g(x) = -f(a - x)$  from the sketches below.

[2]



A	B	C	D	E
---	---	---	---	---

**13.** Consider a set of masses of three different values  $m_a$ ,  $m_b$  and  $m_c$ . Each of the following three combinations have the same total mass.

1. two masses of  $m_a$  plus three masses of  $m_b$
2. five masses of  $m_a$  plus one mass of  $m_c$
3. two masses of  $m_a$  plus one mass of  $m_b$  plus one mass of  $m_c$

Find  $m_b$  and  $m_c$  in terms of  $m_a$ .

[4]

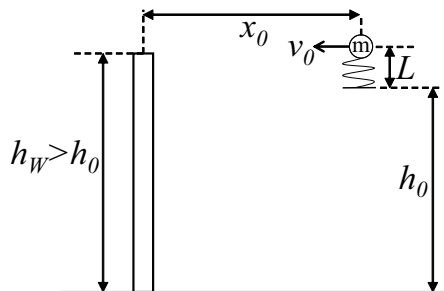
14. Find all solutions of the following equation in the range  $0 \leq \theta \leq 360^\circ$ .

$$4 \cos^2(\theta) + 2(\sqrt{3} - 1) \sin(\theta) = 4 - \sqrt{3}$$

[3]



15.



The above figure depicts a ball of mass  $m$  which is equipped on its lower side with a massless spring of uncompressed length  $L_0$  and spring constant  $k$ . The spring is initially compressed to a length  $L < L_0$ , as shown in the figure.

The ball-spring system is either dropped vertically or launched with a horizontal speed  $v_0$  from a height of  $h_0$  at a distance  $x_0$  from a wall of height  $h_W$ .

The spring remains compressed until it touches the ground. When it touches the ground the spring is released (by a mechanism not shown on the figure) and expands very quickly back to its uncompressed length  $L_0$  and is then held fixed at length  $L_0$ .

The bounce of the ball-spring system on the ground is assumed to be totally elastic.

- (a) When the ball is dropped vertically, find the maximum height  $h_{max}$  the ball-spring system could reach after its bounce in terms of the spring's compression  $\Delta L = L_0 - L$ , and the parameters  $m$ ,  $g$ ,  $k$  and  $h_0$ . In analogy to  $h_0$  in the figure,  $h_{max}$  should be the vertical distance between the floor and the lower end of the spring after the bounce. You may take  $g = 10 \text{ m s}^{-1}$ .

[3]

- (b) Find the optimal value for the distance  $x_0$  from which the ball would bounce over a wall with maximum value of  $h_W$ .

[6]

This page intentionally left blank for working.

This page intentionally left blank for working.

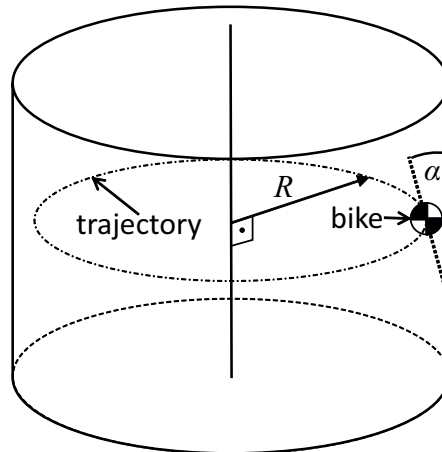
**16.** Given that

$$\begin{aligned}\frac{df(x)}{dx} &= -2x - x^{\frac{1}{2}} + \frac{1}{3} \\ \frac{dg(x)}{dx} &= f(x)\end{aligned}$$

determine  $g(x)$  such that  $f(1) = -1$  and  $g(1) = 0$ .

[4]

**17.** As indicated in the figure, a motor bike of total mass  $m$  ( $m$  is the mass of bike plus rider) is ridden along a horizontal trajectory of radius  $R$  on the inside of a cylindrical cage .



The bike and the rider are inclined at an angle  $\alpha$  to the wall. The angles  $\alpha$  that the bike can make with respect to the walls of the cage are limited by its handle bars to a certain minimum  $\alpha_{min} > 0$ . The tyres of the bike have a very high co-efficient of friction with the cage so that the tyres can only roll but not slip along the cage.

- (a) Show in a diagram the forces acting on the bike if it is to maintain a horizontal trajectory as shown in the figure. [2]
- (b) At what minimum speed  $v_{min}$  must the bike travel if it is not to fall down? [3]
- (c) Given that  $\alpha_{min} = 30^\circ$ ,  $R = 4 \text{ m}$ ,  $g = 10 \text{ m s}^{-2}$ ,  $m = 250 \text{ kg}$  find a numerical value for  $v_{min}$ . [1]

This page intentionally left blank for working.

**18.** For which  $x$  is the following inequality satisfied?

$$\frac{2x^2 + 3x - 2}{2x^2 - 3x - 2} > 0$$

[6]

**19.** The Euler number  $E_u$  has no units. It is used in fluid flow calculations. It depends on the pressure  $P$ , density  $\rho$ , and fluid velocity  $v$  such that:

$$E_u = P^a \rho^b v^c$$

where  $a$ ,  $b$  and  $c$  are constants.

Find the ratio of  $a : b : c$  in its simplest form, where  $a$ ,  $b$  and  $c$  are positive or negative integers.

[4]



**20.** Find the coordinates of the point(s) at which the line  $y = m(3x - 2)$  is tangent to the curve  $y = 9x^2 + 6x - 7$ . In the above  $m$  is a real constant.

[5]

This page intentionally left blank for working.

**21.** Mars' moons Phobos and Deimos are in equatorial, near-circular orbits around Mars. They both orbit in the direction of the planet's rotation. Phobos has an orbital period of  $1/3$  of a Martian day, and Deimos has an orbital period of  $5/4$  Martian days. An astronomer on Mars, near the equator, observes both moons during the course of a Martian night.

- (a) Would the two moons appear to move in the same direction in the sky? Explain your answer [2]
- (b) Describe qualitatively how the phases of the two moons might vary during the night [2]
- (c) Would it be possible for the astronomer to see Phobos both rise and set within a single night? [1]

This page intentionally left blank for working.

**22.** A number can be represented using base  $N$  as follows:

$$(x...cba)_N = a \cdot N^0 + b \cdot N^1 + c \cdot N^2 + .... + x \cdot N^w$$

In which base less than 10 is the following equation true?

$$(1101)_N - (313)_N = (344)_N$$

[5]

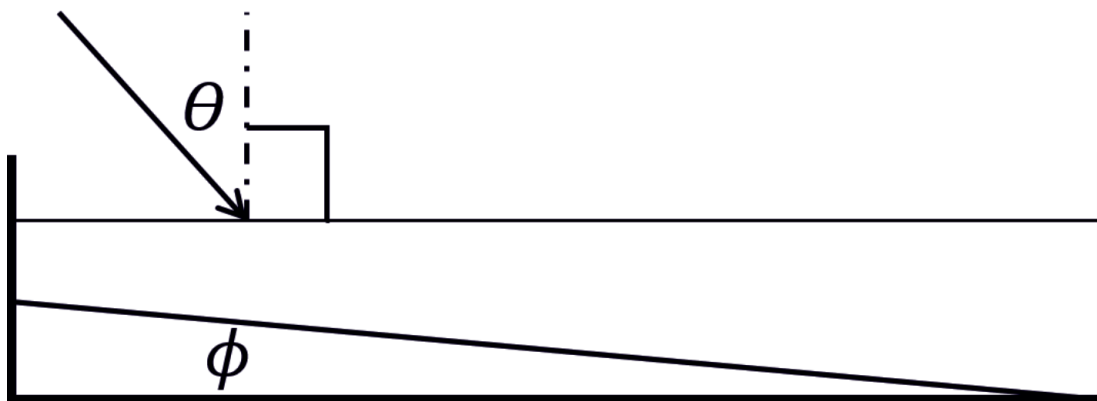
**23.** A monochromatic beam of light travels through air of refractive index  $n_a$  and strikes a liquid of refractive index  $n_l$  at an angle of incidence  $\theta$  as shown in the figure below. At the bottom of the tank which contains the liquid is a plane mirror at angle  $\phi$  to the horizontal. The tank can be considered infinitely long.



Beyond a certain value of  $\theta$  the light no longer leaves the tank after the first reflection in the mirror. Find the value of  $\theta$  for this case given  $\phi = 10^\circ$ ,  $n_a = 1$  and  $n_l = \frac{4}{3}$ .

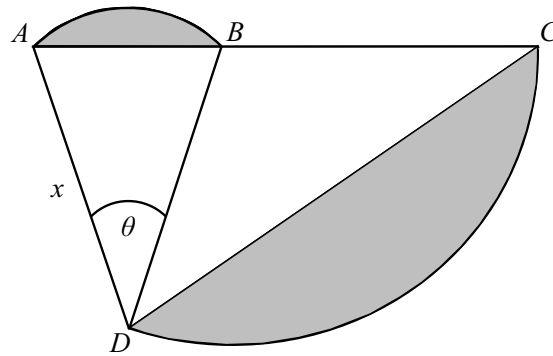
[5]

You may wish to use the larger diagram below to draw a ray diagram.



This page intentionally left blank for working.

24. The points  $A$ ,  $B$  and  $C$  lie on a straight line. The length of  $AD$  is  $x$ .



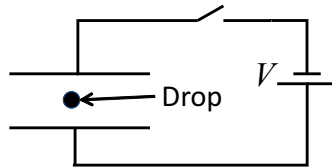
$D$  is the centre of the arc  $AB$ , and  $B$  is the centre of the arc  $CD$ . Find the total shaded area in terms of  $x$  and  $\theta$ .

[5]



This page intentionally left blank for working.

**25.** In Millikan's oil drop experiment, shown below, a spherical oil drop with charge  $+Q$ , radius  $r$  and density  $\rho_{oil}$  falls between two parallel conducting plates.



- (a) Initially the switch is open and the drop is falling with terminal velocity  $v_t$  in air of density  $\rho_{air}$  and viscosity  $\eta$ . We now measure its terminal velocity.

Write down an equation that relates all the forces on the drop while it falls at terminal velocity.

[1]

**Hint:** You may assume that the drag force  $f_D$  on the drop is given by  $f_D = 6\pi\eta r v_t$  and that the drop also experiences a buoyant force or upthrust  $f_B$  equal to the weight of the air displaced by the drop.

- (b) The switch is now closed. A uniform electric field  $E$  is applied to the drop and it becomes stationary. Write down a new equation relating the forces on the drop.

[1]

- (c) Derive an equation for  $Q$  which does not depend on the radius  $r$ .

[3]

This page intentionally left blank for working.

**26.**

- (a) On the same axes, sketch the functions  $y = x^2 + 1$ ,  $y = 2/x$ , and  $y = 3x + 1$ . [3]
- (b) Determine the exact  $x$  coordinates at which any of the graphs intersect and mark these on the  $x$ -axis. [3]
- (c) Find the exact area enclosed between  $y = 3x + 1$  and  $y = x^2 + 1$  that is also below the curve  $y = 2/x$ . [4]

This page intentionally left blank for working.

This page intentionally left blank for working.