BRITISH PHYSICS OLYMPIAD 2013-14

A2 Challenge

September/October 2013

Instructions

Time: 1 hour.

Questions: Answer ALL questions.

Marks: Total of 50 marks.

Solutions: These questions are about problem solving. You must write down the questions in terms of symbols and equations, and try calculating quantities in order to work towards a solution.

In these questions you will need to explain your reasoning by showing your working. Even if you cannot complete the question, show how you have started your thinking; with ideas and, generally, by drawing a diagram.

Setting the paper: You are allowed any standard exam board data/formula sheet.

Important Constants

Speed of light	c	3.00×10^8	m s^{-1}
Planck constant	h	6.63×10^{-34}	J s
Electronic charge	e	1.60×10^{-19}	C
Mass of electron	m_e	9.11×10^{-31}	Kg
Gravitational constant	G	6.67×10^{-11}	N m^2 kg^{-2}
Acceleration of free fall	g	9.81	m s^{-2}
Permittivity of a vacuum	ε_0	8.85×10^{-12}	F m^{-1}
Avogadro constant	N_A	6.02×10^{23}	mol^{-1}

Q1.

This question explores some consequences of the equivalence of mass and energy, contained in Einstein's famous equation $E = mc^2$.

a) What is the mass equivalent to an energy of one *joule*?

b) What minimum amount of energy is needed in a system to make possible the formation of an electron-positron pair? Express your answer both in *joules* and *electron-volts.*

c)
   i) A power station with an output of 2 GW may be expected to have a lifetime of 30 years. What is the mass equivalent of the energy the power station generates in its lifetime?

   ii) Burning 1 kg of coal liberates approximately 4×10^7 J of energy, which is used with an efficiency of 25% in producing the output from the power station. What mass of coal will the power station consume in its lifetime?

   iii) Comment on the comparison between your two previous answers.

8 marks

Q2.

This question refers to a novel form of environmentally friendly home heating.

It is proposed that houses having an outdoor swimming pool could be heated in winter by extracting the thermal energy of the pool, which has been accumulated by natural solar heating in the summer months.

For the purposes of this problem we will assume that such a pool has dimensions 8 m x 20 m and a uniform depth of 2 m, and that a typical, well insulated house requires an average of 5 kW of heating power for the 200 days of colder weather each year.

a) What is the mass of the water in the pool?

b) How much energy must be removed from the pool for its temperature to decrease by 1 K?

c) How much energy is required by the house during the period of colder weather?

d) Hence, calculate the fall in temperature of the pool over the winter period, if it were the only source of heating for the house, and state one assumption you have made.

e) Suggest a reason why the temperature of the pool cannot be reduced indefinitely. Therefore, what is the lowest reasonable temperature of the pool at the end of the period of colder weather?

f) Hence, what does the maximum temperature of the pool need to be at the end of the period of warmer weather, if this system is to be viable?

g) Comment of the feasibility of this approach to domestic heating in the British climate.

   Useful data:

   Density of water $1\,000$ kg m^{-3}

   Specific heat capacity of water $4\,200$ J kg^{-1}K^{-1}

10 marks

Q3.

This question investigates the effect of scaling in living organisms and engineering models.

a)

    (i)   A human femur (thigh bone) has a diameter of approximately 30 mm. Assuming that it carries half the weight of a person of mass 70 kg, calculate the stress in the femur when the person is standing normally.

    (ii) Without calculation, suggest what will happen to this stress when the person stands on one leg, then begins to run.

b) In the story *Gulliver's Travels*, the land of *Brobdingnag* is inhabited by giants. Gulliver estimates that the giants are approximately 100 times as big as humans in all dimensions.

    (i)   By what factor does the mass of an average Brobdingnagian exceed the mass of a human?

    (ii) Therefore, by what factor does the stress in a Brodingnagian's femur exceed that of a human femur?

    (iii) The breaking stress of bone may be taken as 100 MPa. What does this tell us about the way a Brobdingnagian might walk and run?

c) Engineers sometimes make use of scale models to examine aspects of new designs. Use the outcomes of the illustration above to comment on the usefulness of scale models in predicting the strength of a lightweight bridge structure.

12 marks

Q4.

This question looks at the behaviour of light passing through a diffraction grating which has openings of finite size.

A grating consists of a screen with many transparent openings 2 μm wide, separated by opaque strips 4 μm wide. The screen is illuminated by monochromatic light of wavelength 6×10^{-7} m. The simple pattern of maxima and minima of transmitted intensity is modified by diffraction at each individual opening.

a) Theory predicts that each *individual* opening will have diffraction *minima* at angular positions, ϑ, given by:
$$b \sin \vartheta = m\lambda$$

where b is the width of the opening and m is a non-zero integer. Calculate the angle between the central (zero-order) maximum and the first order ($m = 1$) minimum of intensity when illuminated as described above.

b)
   (i) For light passing through the many openings of the grating, the angle at which the intensity is a *maximum*, θ is given by:

   $$d \sin \theta = n\lambda$$

   where n is any integer, including zero, and d is the slit separation (i.e. separation of the centres of the openings). State the value of the slit separation.

   (ii) Maxima of intensity, as predicted by the equation in (b.i) above, are expected in positions where the emergent wave output from the individual openings is in phase, leading to constructive interference. Calculate the angle between the central (zero-order) maximum and the third order ($n = 3$) position for constructive interference, when illuminated with the same monochromatic light as above.

   (iii) The result you have just calculated is the position for constructive interference of the outputs from the combination of the slits, each of which is described in (a) above. Considering the answers to (a) and (b.ii) together, what is the light intensity observed at this position?

c) What general relationship between the dimensions b and d will lead to the effect on certain 'maxima' predicted in (b.iii)?

5 marks

Q5.

This question explores the behaviour of some simple electrical circuits.

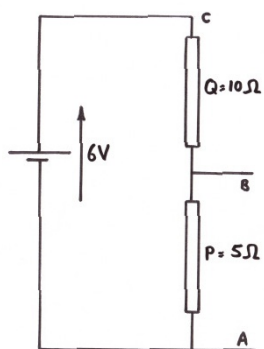a) Fig 5.1 is a commonly used system known as a potential divider.

Fig 5.1

(i) What would you expect a voltmeter to read if connected across AB?

(ii) What condition for the current passing between A and B is necessary if your calculation above is to be valid? How well does the use of a voltmeter fulfil the condition you have stated?

(iii) Without making any calculation, what would you expect to happen to the potential difference across AB if a light bulb were connected in parallel with the voltmeter? Explain your answer in qualitative terms.

b) Fig 5.2 is a circuit comprising two potential dividers: ABC, as before, with the addition of DEF
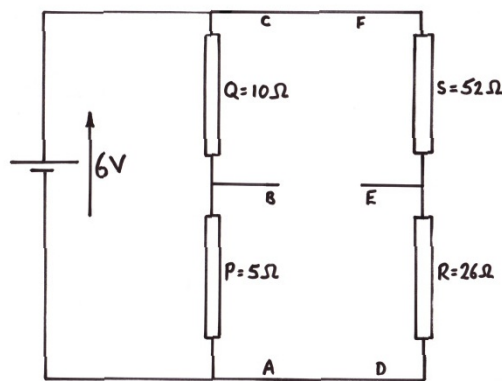

Fig 5.2

(i) What is the potential difference between D and E?

(ii) Hence state the potential difference between B and E.

(iii) What current would you expect to be flowing in any conductor connected between B and E?

(iv) What do you notice about the values of resistors P,Q and R,S which leads to the outcome you describe in (iii) above?

c) The system we have examined in (b) above was historically known as a *Wheatstone Bridge* and for the particular value of current described in (b.iii) above, the bridge is said to be 'balanced'. The Wheatstone Bridge was classically drawn as in Fig 5.3 and the galvanometer, G, was used to detect any current flowing across the diagonal of the bridge. We will now develop this context to solve a practical problem.
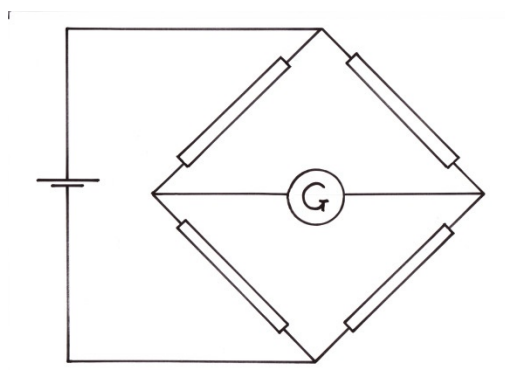

Fig 5.3

An open regular tetrahedron, WXYZ, is made from six identical pieces of wire each having a resistance of 10 Ω.

(i) Sketch WXYZ as a triangular pyramid with base WXY and vertex Z.

(ii) Now sketch the tetrahedron viewed along the line joining the mid-points of WX and YZ.

(iii) A source of electric current is now connected across WX. What current flows in YZ? (You may wish to use this result to simplify your next calculation on the electrical behaviour of the system).

(iv) Calculate the resistance between W and X.

15 marks

End of Questions