ENGAA 2016
Section 1

Model Solutions

1

$$-8 < 6 - \frac{x}{2}$$
$$-16 < 12 - x$$
$$x < 28$$

2

Change in nucleon number = 214-210 = 4
Change in proton number = 0

Alpha emission leads to a decrease of nucleon number by 4 and decrease of proton number by 2
Beta emission leads to an increase of proton number by 1 and no change in nucleon number

a) change in nucleon number $= 3 \times 4 = 12$, therefore not A

b) change in nucleon number $= 2 \times 4 = 8$, therefore not B

c) change in nucleon number $= 2 \times 4 = 8$, therefore not C

d) change in nucleon number $= 1 \times 4 = 8$, change in proton number $= 2 - 2 = 0$, therefore D is correct

3

$$\begin{aligned}(\sqrt{3} - \sqrt{2})^2 &= 3 - 2\sqrt{3}\sqrt{2} + 2 \\ &= 5 - 2\sqrt{3}\sqrt{2}\end{aligned}$$

4

The graph is a straight line through the origin so the relationship between x and y must be direct proportionality:

1) KE $= \frac{1}{2} \cdot 10v^2$, therefore this relationship is not directly proportional

2) GPE $= mgh = 20 \cdot 10 \cdot h$
When h=2, GPE= 400, not 10, therefore cannot be represented by graph

3) $F = ma$ and $a = \frac{v-u}{t} \therefore 100 = 20 \cdot \frac{v-0}{t}$, 5t = v (when accelerated from rest), therefore could be represented by graph

4) W = 5d, therefore can be represented by graph

Therefore, only 3 and 4 are correct

5

$$
\begin{aligned}
Q : R &= 5 : 2 \\
\frac{Q}{R} &= \frac{5}{2} \\
R &= \frac{2Q}{5} (1) \\
R : S &= 3 : 10 \\
\frac{R}{S} &= \frac{3}{10} \\
R &= \frac{3S}{10} (2) \\
\text{Sub (1) in (2)} & \\
\frac{3S}{10} &= \frac{2Q}{5} \\
\frac{3}{4} &= \frac{Q}{S} \\
Q : S &= 3 : 4
\end{aligned}
$$

6

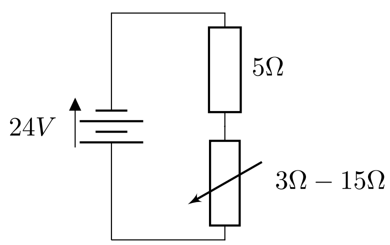
1) Conservation of nucleon number: Before fission = 236, After fission = 235; not possible
2) Conservation of nucleon number: Before fission = 236= After fission
   Conservation of charge: Before fission = 92 = After fission; possible
3) Conservation of nucleon number: Before fission = 236, After fission = 235; not possible

Therefore, 2 only

7

$$
\begin{aligned}
\text{Total of age} &= 28 \times 20 = 560 \\
\text{Total age with two extra} &= 30 \times 22 = 660 \\
\text{Sum of ages of two extra people} &= 660 - 560 = 100 \\
\text{Mean of two} &= \frac{100}{2} = 50yrs
\end{aligned}
$$

8



$P = \frac{V^2}{R}$ ∴ Maximum power dissipated when maximum PD across 5Ω resistor

$$V_5(max) = \frac{5}{5+3} \cdot 24$$
$$= 15\Omega$$
$$P = \frac{15^2}{5}$$
$$= 45W$$

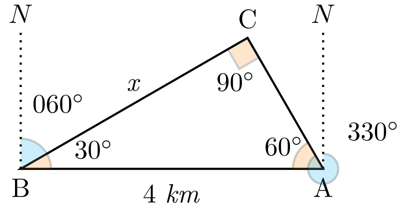
9

$$\text{Value after 2 years} = £15000 \times (0.8)^2 = £9600$$
$$£15000 - £9600 = £5400$$

10

$$\frac{P_1}{P_0} = \frac{kR_1^2T_1^4}{kR_0^2T_0^4}$$
$$\frac{P_1}{4.0 \times 10^{26}} = \frac{k \cdot (100R_0)^2 \cdot \left(\frac{T_0}{2}\right)^4}{k \cdot R_0^2 \cdot T_0^4}$$
$$P_1 = \frac{10^4 R_0^2 \cdot \left(\frac{1}{16}\right) T_0^4}{R_0^2 \cdot T_0^4} \cdot 4.0 \times 10^{26}$$
$$= 10^4 \cdot \left(\frac{1}{16}\right) \cdot 4.0 \times 10^{26}$$
$$= 2.5 \times 10^{29} W$$

11



$$cos30 = \frac{x}{4}$$
$$\frac{\sqrt{3}}{2} = \frac{x}{4}$$
$$2\sqrt{3} = x$$

12

$$\text{Number of complete cycles in 60 seconds} = 60 \times 5 = 300$$
$$\text{Distance travelled by a particle in one cycle} = 3.0cm \times 4 = 12cm$$
$$\text{Total Distance} = 12 \times 300 = 3600cm$$

13

$$
\begin{aligned}
x &\propto \frac{1}{\sqrt{y}} \\
x &= \frac{k}{\sqrt{y}} \\
8 &= \frac{k}{\sqrt{9}} \\
24 &= k
\end{aligned}
$$

$$
\begin{aligned}
6 &= \frac{24}{\sqrt{y}} \\
\sqrt{y} &= 4 \\
y &= 16
\end{aligned}
$$

14

Resolving upwards on the 5.0kg mass:

$$
\begin{aligned}
T - 50 &= 5 \cdot 0.8 \\
T &= 54N \\
&\therefore \text{54N force tension either side of the pulley} \\
\therefore \text{total force} &= 54N + 54N = 108N
\end{aligned}
$$

15

$$
\begin{aligned}
Area &= (x-1)x + \frac{1}{2} \cdot 6x = 120 \\
120 &= x^2 + 2x \\
0 &= x^2 + 2x - 120 \\
&= (x+12)(x-10) \\
x &= -12, 10 \\
\therefore RS &= 15cm
\end{aligned}
$$

16

$$
\begin{aligned}
V &= IR \\
I &= \frac{6}{15} = 0.4A \\
Q &= It \\
&= 0.4 \times 180 \\
&= 72C \\
V &= \frac{W}{Q} \\
&= \frac{180}{72} \\
&= 2.5V
\end{aligned}
$$

17

$$\begin{aligned} a &= \frac{b^2+2}{3b^2-1} \\ a(3b^2-1) &= b^2+2 \\ 3ab^2-a &= b^2+2 \\ (3a-1)b^2 &= 2+a \\ b &= \pm\sqrt{\frac{2+a}{3a-1}} \end{aligned}$$

18

$$\begin{aligned} mass &= \frac{30}{10} = 3\text{kg or }3000\text{g} \\ volume &= 10^3-(5^2)10 = 750 \\ density &= \frac{3000}{750} = 4gcm^{-3} \end{aligned}$$

19

$$\begin{aligned} circumference &= 5 = 2\pi r \\ r &= \frac{5}{2\pi} \\ Volume &= \pi\cdot\left(\frac{5}{2\pi}\right)^2\cdot 10 \\ &= \frac{125}{2\pi} \end{aligned}$$

20

P and Q lose thermal energy via convection because the temperature of the balls is greater than that of the surrounding air. The balls heat up the air in contact with them, causing air particles to rise and, through the process of convection, consequently transfer thermal energy away from the balls. S has the greatest rate of emission of thermal radiation; dull black surfaces are better emitters of infrared radiation than surfaces which are shiny and/or white, and hotter objects emit more thermal radiation than cooler objects (irrespective of the temperature of the surrounding air).

21

$$\begin{aligned} 4+\frac{4-x^2}{x^2-2x} &= 4+\frac{(2+x)(2-x)}{-x(2-x)} \\ &= 4+\frac{(2+x)}{-x} \\ &= 4+\frac{2}{-x}+\frac{x}{-x} \\ &= 3-\frac{2}{x} \end{aligned}$$

22

Displacement is equal to the area under a velocity-time graph:

$$\begin{aligned}
\text{Total Distance} &= \frac{1}{2}(20)(8) + \frac{1}{2}(10)(2) \\
&= 90m \\
\text{Distance from Starting Position} &= \frac{1}{2}(20)(8) - \frac{1}{2}(10)(2) \\
&= 70m \\
\text{Average Speed} &= \frac{90}{30} \\
&= 3ms^{-1}
\end{aligned}$$

23

	Swimming	**Archery**	**Tennis**	**Total**
Girl	25	9	12	46
Boy	32	18	24	74
Total	57	27	36	120

$$\text{Probability that a chosen boy does swimming} = \frac{32}{74} = \frac{16}{37}$$

24

$$\begin{aligned}
X &= \frac{M_{cop}}{0.9V} \\
Y &= \frac{M_{tin}}{0.1V} \\
\text{Proportion of mass of tin} &= \frac{M_{tin}}{\text{Total Mass}} \times 100 \\
&= \frac{0.1YV}{(0.9X + 0.1Y)V} \times 100 \\
&= \frac{Y}{9X + Y} \times 100
\end{aligned}$$

25

$$\begin{aligned}
\frac{9^{2n+1} \times 3^{4-3n}}{27^{2-n}} &= \frac{3^{2(2n+1)} \times 3^{4-3n}}{3^{3(2-n)}} \\
&= \frac{3^{4n+2+4-3n}}{3^{6-3n}} \\
&= 3^{4n+2+4-3n-6+3n} \\
&= 3^{4n}
\end{aligned}$$

26

Using the Principle of the Conservation of Momentum:

$$\begin{aligned}
0 &= 234mV_t - 4mV_\alpha \\
4V_\alpha &= 234V_t \\
\frac{V_\alpha}{V_t} &= \frac{234}{4} \\
\frac{KE_\alpha}{KE_t} &= \frac{0.5}{0.5} \times \frac{4m}{234m} \times \left(\frac{234}{4}\right)^2 \\
&= \frac{234}{4} \\
4KE_\alpha &= 234KE_t \\
234KE_\alpha + 4KE_\alpha &= 234KE_t + 234KE_\alpha \\
238KE_\alpha &= 234E \\
KE_\alpha &= \frac{234E}{238}
\end{aligned}$$

27

The sum of exterior angles is 360^o:

$$\begin{aligned}
\therefore \text{Angle } RQT &= \frac{360}{n} \\
x &= 180 - \frac{2 \times 360}{n} \\
n &= \frac{720}{180 - x}
\end{aligned}$$

28

Time taken for signal reflected off left building to reach student:

$$T_1 = \frac{48 \times 2}{320} = 0.3s$$

Time taken for signal reflected off right building to reach student:

$$T_2 = \frac{(128 - 48) \times 2}{320} = 0.5s$$

You would need to choose a frequency such that the reflected clicks which begin to pass the student after $0.3s$ and $0.5s$ will coincide with a new click. This requires finding the highest common factor of $0.3s$ and $0.5s$ which is $0.1s$. This means $f = 10Hz$.

29

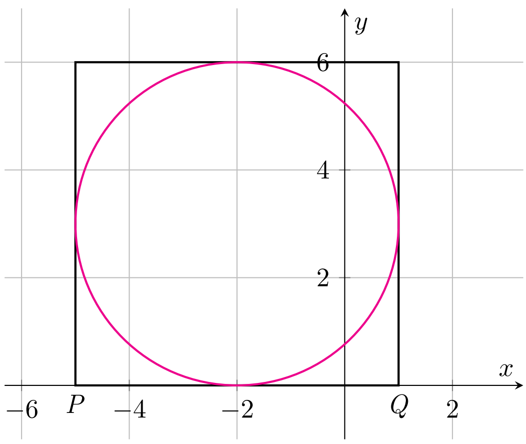
$$\begin{aligned}
\text{let f(x)} &= x^3 + px^2 + qx + p^2 \\
f(2) &= 8 + 4p + 2q + p^2 = 0 \ (1) \\
f(1) &= 1 + p + q + p^2 \qquad\qquad\qquad\qquad = -3.5 \\
p + q + p^2 &= -\frac{9}{2} \ (2) \\
2 \times (2) &: \\
2p + 2q + 2p^2 &= -9 \ (3) \\
(3) - (1) &: \\
-2p + p^2 &= -1 \\
p^2 - 2p + 1 &= 0 \\
(p-1)^2 &= 0 \\
\therefore p &= 1
\end{aligned}$$

30

1) No, because the forces are of different types on the same body
2) No, because the forces are of different types on the same body
3) No, because they act in the same direction and have different magnitudes
4) Yes, because the force is of the same type, on different bodies, equal in magnitude and opposite in direction
5) No, because forces are of different types

Therefore, only equation 4 is a direct application of Newton's third law.

31



Side length of the square is 6 $\therefore$ the centre of the circle is 3 units to the left and 3 units up from Q

$$\begin{aligned}
\text{Centre of circle} &: (-2, 3) \\
\text{Equation of circle} : (x - (-2))^2 + (y-3)^2 &= 3^2 \\
x^2 + 4x + 4 + y^2 - 6y + 9 &= 9 \\
x^2 + y^2 + 4x - 6y + 4 &= 0
\end{aligned}$$

32

Resolving upwards on the crate:

$$\begin{aligned} T - 800g &= 800 \times 2 \\ T &= 1600 + 8000 \\ &= 9600N \end{aligned}$$

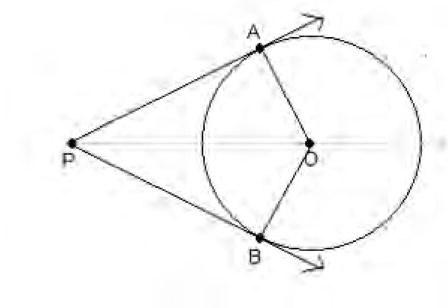
33

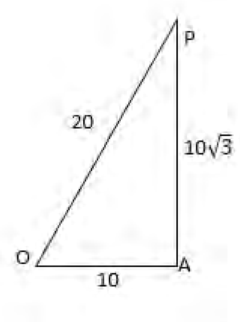
$$\begin{aligned} a &= 8 \\ U_5 &= 2 = 8r^4 \\ r &= \pm\frac{1}{\sqrt{2}} \\ U_6 &= 8r^5 > 0 \\ \therefore r &= \frac{1}{\sqrt{2}} \\ S_\infty &= \frac{8}{1-\frac{1}{\sqrt{2}}} \\ &= \frac{8 \times 2}{2-\sqrt{2}} \\ &= \frac{8 \times 2(2+\sqrt{2})}{(2-\sqrt{2})(2+\sqrt{2})} \\ &= \frac{8 \times 2(2+\sqrt{2})}{4-2} \\ &= 8(2+\sqrt{2}) \end{aligned}$$

34

$$\begin{aligned} \text{Work Done} &= 50cos37 \cdot 15 \\ &= 600J \\ \text{Vertical Force} &= 50sin37 + 350 \\ &= 380N \end{aligned}$$

35





$$\text{Area of kite AOBP} = \frac{1}{2} \cdot 10 \cdot 10\sqrt{3} \times 2$$
$$= 100\sqrt{3}$$
$$tan\theta = \frac{10}{10\sqrt{3}} = \frac{\sqrt{3}}{3}$$
$$\theta = \frac{\pi}{6}$$
$$\therefore \text{Angle AOB} = \left(\frac{\pi}{2} - \frac{\pi}{6}\right) \times 2 = \frac{2\pi}{3}$$
$$\text{Area of sector AOB} = \frac{1}{2}(10)^2 \left(\frac{2\pi}{3}\right)$$
$$= \frac{100\pi}{3}$$
$$\text{Shaded Area} = 100\sqrt{3} - \frac{100\pi}{3}$$
$$= \frac{100}{3}(3\sqrt{3} - \pi)$$

36

Let x be the distance of the centre of mass from the pivot, to the right of the pivot.
Taking moments about the pivot:

$$600 \cdot 0.8 = 150x + 350 \cdot 1.2$$
$$60 = 150x$$
$$x = 0.4m \text{ to the right of the pivot (as the answer is non-negative)}$$

Resolving vertically:

$$\text{Force on pivot } = 600 + 150 + 350 = 1100N$$

37

$$7cos\theta - 3tan\theta sin\theta = 1$$
$$7cos\theta - 3\frac{sin\theta}{cos\theta} \cdot sin\theta = 1$$
$$7cos^2\theta - 3sin^2\theta = cos\theta$$
$$7cos^2\theta - 3(1 - cos^2\theta) = cos\theta$$
$$10cos^2\theta - cos\theta - 3 = 0$$
$$(5cos\theta - 3)(2cos\theta + 1) = 0$$
$$cos\theta = \frac{3}{5}$$
$$cos\theta = -\frac{1}{2}$$

38

$$ME_i = \frac{1}{2} \cdot 200 \cdot 5^2 + 200g \cdot 8$$
$$= 18500$$
$$ME_f = \frac{1}{2} \cdot 200 \cdot 9^2 + 200g \cdot 2$$
$$= 12100$$

Using Work-Energy Principle:

$$\text{Work Done against Resistive Forces} = 18500 - 12100$$
$$= 6400J$$

39

$$3x^2 = (a + 2)x - 3$$
$$3x^2 - (a + 2)x + 3 = 0$$

For two distinct real roots:

$$b^2 - 4ac = (a + 2)^2 - 4 \cdot 3 \cdot 3 > 0$$
$$a^2 + 4a + 4 - 36 > 0$$
$$a^2 + 4a - 32 > 0$$
$$(a - 4)(a + 8) > 0$$
$$\therefore a < -8, a > 4$$

40

1) False: without frictional forces the cube would slide - not tilt
2) True: Friction acts opposite to the direction of motion
3) False: Friction cannot act in the same direction as motion
4) False: The perpendicular distance from the pivot is not d

41

$$\text{The two lines are perpendicular so: } mp = -1$$
$$p = -\frac{1}{m}$$

$$\text{at (M,0): } Mp + 2 = 0$$
$$M = -\frac{2}{p}$$
$$\text{at (L,0): } Lm + 3 = 0$$
$$L = -\frac{3}{m}$$

$$\begin{aligned} 5 &= -\frac{2}{p} + \frac{3}{m} \\ 5 &= \frac{-2m + 3p}{mp} \\ &= \frac{-2m + 3p}{-1} \\ -5 &= -2m - \frac{3}{m} \\ 2m^2 - 5m + 3 &= 0 \\ (2m-3)(m-1) &= 0 \\ m &= \frac{3}{2} \text{ (or 1)} \\ p &= -\frac{2}{3} \\ m + p &= \frac{5}{6} \end{aligned}$$

42

$$\begin{aligned} \text{arrangement 2:} &= 2mga \\ \text{arrangement 1:} &= 2mgb \\ \text{difference} &= 2mg(a-b) \end{aligned}$$

43

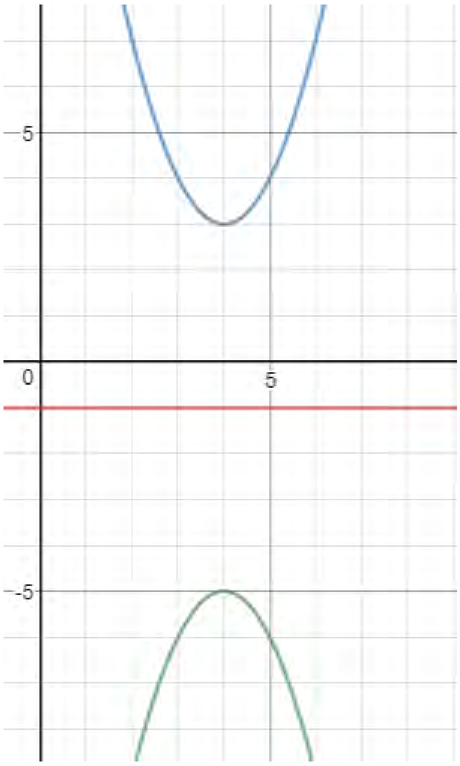
$$f'(x) = 3x^2 - a^2 \geq 0 \text{for an increasing function}$$
$$x \geq \pm\sqrt{\frac{a^2}{3}}$$
$$x \leq -\frac{a}{\sqrt{3}}$$
$$x \geq \frac{a}{\sqrt{3}}$$

44

$$u = 8$$
$$v = 2$$
$$a = -g$$
$$s = ?$$

$$v^2 = u^2 + 2as$$
$$4 = 64 - 20s$$
$$\therefore s = 3$$
$$v = u + at$$
$$-2 = 8 - 10t$$
$$t = 1$$

45



$$\text{Let f(x)} = x^2$$
$$\text{Translation: } f((x-4)+3) = (x-4)^2 + 3$$
$$\text{Reflection: } = -(x-4)^2 - 5$$
$$= -5 - (x-4)^2$$

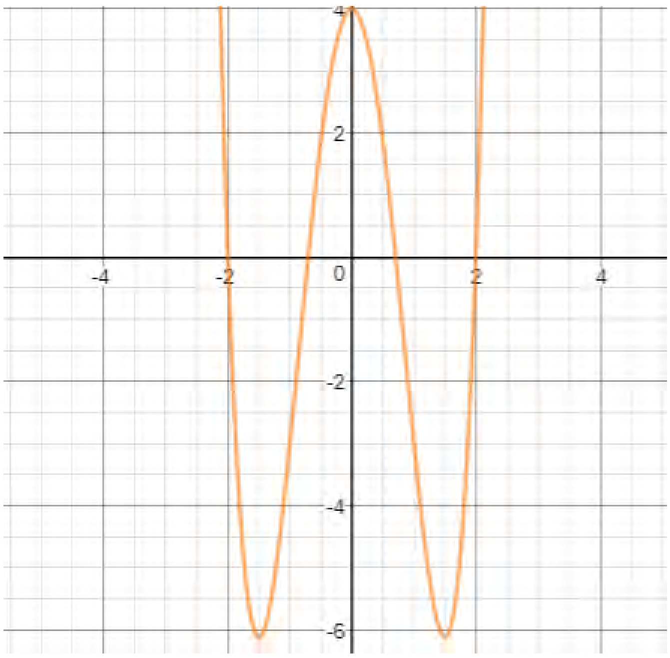
46

Using the Principle of the Conservation of Momentum and resolving with the right direction as positive:

$$\begin{aligned} 10 \times 4 &= 4V_p + 10 \times 2 \\ V_p &= 5 \text{ to the right} \\ KE_{loss} &= \frac{1}{2} \times 4 \times 10^2 - \frac{1}{2} \times 2 \times 10^2 - \frac{1}{2} \times 4 \times 5^2 \\ &= 50J \end{aligned}$$

47

$$\begin{aligned} 2x^4 - 9x^2 + 4 &> 0 \\ (x^2 - 4)(2x^2 - 1) &> 0 \\ \therefore \text{x } &= \pm 2 \text{ or x } = \pm\frac{1}{\sqrt{2}} \end{aligned}$$



The graph is greater than zero for:

$$\begin{aligned} x &< -2, \\ -\frac{1}{\sqrt{2}} < x &< \frac{1}{\sqrt{2}}, \\ x &> 2 \end{aligned}$$

This result can be deduced using the general shape of a quartic curve

48

$$\begin{aligned}
\text{Resolve vertically: } R &= 20\\
\text{Friction} &= \mu R = 5N\\
\text{Resultant Horizontal Force: } &= \sqrt{9^2+12^2} = 15N\\
\text{Total Resultant Force: } &= 15-5 = 10N\\
F &= ma\\
a &= \frac{10}{2}\\
&= 5ms^{-2}
\end{aligned}$$

49

$$\begin{aligned}
\text{Let f(x) } &= 4x^3-12x^2-36x-15\\
f'(x) &= 12x^2-24x-36 = 0\\
x^2-2x-3 &= 0\\
x &= 3,-1\\
f''(x) &= 24x-24\\
f''(-1) &= -48 < 0 \therefore \text{ turning point at (-1,5) is a maximum}
\end{aligned}$$

The curve has a maximum at $x=-1$ and therefore a minimum at $x=3$. For $x<3$, the greatest value of y is therefore at $x=-1$.

$$\begin{aligned}
f(-1) &= 4(-1)^3-12(-1)^2-36(-1)-15\\
&= 5
\end{aligned}$$

The curve is closest to $y=10$ at the point $(-1,5)$

$$\begin{aligned}
\therefore \text{Minimum distance} &= 10-5\\
&= 5
\end{aligned}$$

50

$$\begin{aligned}
\text{Resolving the 30kg object vertically: } 300-T &= 30\cdot 2.5\\
T &= 225N\\
\text{Resultant Force on 20kg object} &= 20\times 2.5\\
&= 50N\\
\text{Resolving parallel to the slope on 20kg object: } 225-20gsin30-F &= 50\\
F &= 75N
\end{aligned}$$

51

$$\begin{aligned}
3(y-1)^2+4 &= 3[y^2-2y+1]+4\\
&= 3y^2-6y+7 = 7\\
3y(y-2) &= 0\\
y &= 0,2\\
\int_0^2 3y^2-6y+7dy &= [y^3-3y^2+7y]\\
&= 10\\
\therefore Area &= 14-10 = 4
\end{aligned}$$

52

Let m be the mass of fuel. Resolving momentum relative to the spaceship's speed.

$$75(4000 - m) = 1425m$$
$$75(40) = 15m$$
$$200 = m$$

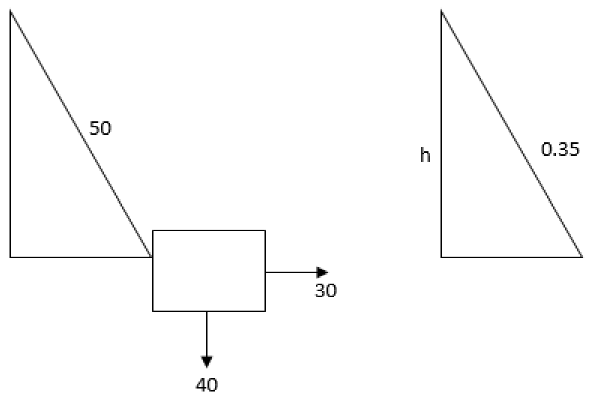
53

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 12x^3 - 12x^2 - 24x = 0 \text{ (at turning points)}$$
$$x(x-2)(x+1) = 0$$

$$\text{when x=0: } y = 20 - k$$
$$\text{when x=2: } y = 52 - k$$
$$\text{when x=-1: } y = 15 - k$$

For a positive quartic graph to have four distinct real roots, two of the turning points are below the x-axis while the 'middle' turning point is above. The turning point at x=0 will be positive for $20 - k > 0$ (so $k < 20$). The other two turning points are both negative as long as $15 - k < 0$ is satisfied. Therefore, $k > 15$ and the complete inequality is $15 < k < 20$.

54



$$\text{Resultant force on object: } = \sqrt{40^2 + 30^2} = 50N$$
Comparing ratios of similar triangles:
$$\frac{h}{40} = \frac{0.35}{50}$$
$$h = 0.28$$
$$\text{Change in height} = 0.35 - 0.28 = 0.07$$
$$\text{Change in GPE} = 2.8J$$