PHYSICS ADMISSIONS TEST
November 2022

Time allowed: 2 hours

For candidates applying to Physics, Physics and Philosophy, Engineering, or Materials Science

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Total 23 questions [100 Marks]

Answers should be written on the question sheet in the spaces provided, and you are encouraged to show your working.
You should attempt as many questions as you can.

No tables, or formula sheets may be used.

Answers should be given exactly and in simplest terms unless indicated otherwise.

Indicate multiple-choice answers by circling the best answer.
Partial credit may be given for correct workings in multiple choice questions.

The numbers in the margin indicate the marks expected to be assigned to each question. You are advised to divide your time according to the marks available.

You may take the gravitational field strength on the surface of Earth to be $g \approx 10\,\mathrm{m\,s^{-2}}$

Do NOT turn over until told that you may do so.

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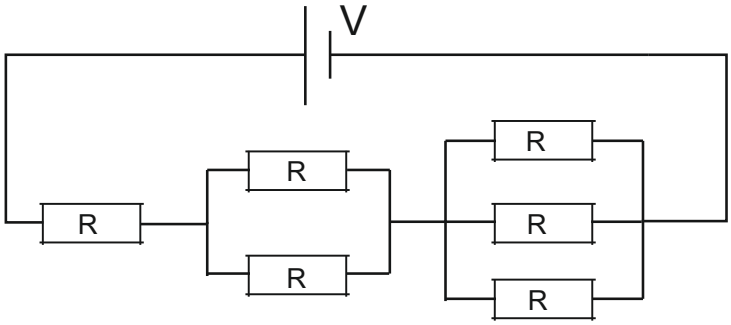
1. What is the total resistance of the circuit? [2]



A	**B**	**C**	**D**	**E**
$\frac{11R}{6}$	$6R$	$\frac{6R}{11}$	$3R$	$\frac{R}{3}$

2. For which values of x is $\left(24 - 14x - 3x^2\right)^{-1}$ positive? [2]

A	**B**	**C**	**D**	**E**
$x < -4/3$ and $x > 6$	$x < -6$ and $x > 4/3$	$-4/3 < x < 6$	$-6 < x < 4/3$	$-\infty < x < \infty$

3. Molecules of oxygen in the atmosphere absorb solar radiation in bands centred at about 80 nm, 650 nm and 1000 nm. In which parts of the electromagnetic spectrum are these absorption bands? [2]

A: Visible, Infrared and Microwave

B: Visible and Infrared

C: Ultraviolet and Infrared

D: Ultraviolet, Visible and Infrared

E: X-ray, Ultraviolet and Visible

4. Which of these polynomial functions has the largest second derivative at $x = 0$? [2]

A	**B**	**C**	**D**	**E**
$5x^5 - x^3 + 4x$	$3x^4 + x^2 + 16$	$4x^6 + x^2 - 1$	$x^3 + 2x^2 - 5x + 10$	$10x^5 + 3x^3 - 7x + 2$

5. An asteroid of mass $10^3\,\mathrm{kg}$ is moving towards a space station at $1\,\mathrm{m\,s^{-1}}$. It is proposed to stop it by firing a 1 MW laser at it. For how long must the laser be fired? You may assume that the surface of the asteroid is perfectly reflective, all photons are incident perpendicular to the surface of the asteroid, and a photon's momentum is related to its energy by $p = \frac{E}{c}$, where $c = 3 \times 10^8\,\mathrm{m\,s^{-1}}$ is the speed of light. [2]

A	**B**	**C**	**D**	**E**
$3 \times 10^{-3}\,\mathrm{s}$	$7.5 \times 10^4\,\mathrm{s}$	$1.5 \times 10^5\,\mathrm{s}$	$3 \times 10^5\,\mathrm{s}$	$3 \times 10^{11}\,\mathrm{s}$

6. Which expression correctly represents the sum $\sum_{k=0}^{n} ar^{2k}$? [2]

A	**B**	**C**	**D**	**E**
$\frac{a}{1-r^2}k$	$\frac{a(1-r^{2n})}{1-r}$	$\frac{a(1-r^{2n})}{1-r^2}$	$\frac{a(1-r^{2n+2})}{1-r}$	$\frac{a(1-r^{2n+2})}{1-r^2}$

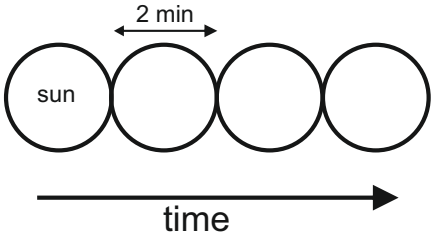
7. In a cathode ray tube, an electron (mass 9.1×10^{-31} kg, charge -1.6×10^{-19} C) is accelerated from rest by a uniform electric field of strength $20\,\mathrm{kV\,m^{-1}}$. How much time does it take to travel 50 cm? [2]

A	**B**	**C**	**D**	**E**
1.1×10^{-18} s	2.8×10^{-16} s	1.7×10^{-8} s	5.3×10^{-7} s	3.2×10^{-5} s

8. If a function $y = f(x)$ has a stationary point at (x_0, y_0), what are the co-ordinates of the corresponding stationary point of the function $y = af(bx + c)$? [2]

A	**B**	**C**	**D**	**E**
$\left(\frac{x_0}{b} - c, ay_0\right)$	$(bx_0 + c, ay_0)$	$\left(\frac{x_0 - c}{b}, ay_0\right)$	$\left(x_0 - \frac{c}{b}, ay_0\right)$	$\left(\frac{x_0 + c}{b}, ay_0\right)$

9. As it appears to move across the sky, the Sun moves through an angle equal to that subtended by its diameter in about two minutes, as in the diagram. In a solar eclipse, the Moon covers the Sun almost exactly in the sky. Using this, what is the approximate ratio of the Moon's radius to its orbital distance from Earth? [2]



A	B	C	D	E
0.0014	0.0022	0.0028	0.0044	0.0056

10. What is the next number in the sequence $0, \frac{3}{4}, \frac{3}{8}, \frac{9}{16}, \frac{15}{32}, \frac{33}{64}$? [2]

A	**B**	**C**	**D**	**E**
$\frac{51}{128}$	$\frac{53}{128}$	$\frac{63}{128}$	$\frac{65}{128}$	$\frac{71}{128}$

11. Two moons occupy circular orbits around a planet. The smaller moon has mass 1.5×10^{15} kg and orbital radius 2.3×10^{4} km. The larger moon has mass 1.1×10^{16} kg and orbital radius 9.4×10^{3} km. If the gravitational force exerted by the planet on the smaller moon is 10^{14} N, what force does the planet exert on the larger moon? [2]

A	**B**	**C**	**D**	**E**
2.4×10^{14} N	6.0×10^{14} N	7.3×10^{14} N	1.8×10^{15} N	4.4×10^{15} N

12. What is the derivative of $y = x^6 + 6x^5 + 12x^4 + 8x^3$? [2]

A: $(3x + 3)(x^2 + 2x)^2$

B: $(2x + 2)(x^2 + 2x)^2$

C: $(6x + 6)(x^2 + 2x)$

D: $(2x + 2)(x^2 + 2x)^3$

E: $(6x + 6)(x^2 + 2x)^2$

13.

(a) Draw the functions $y_1(x) = x^2 - 1$, $y_2(x) = 4x - 2$ and $y_3(x) = -\frac{x}{2} - 2$ on a common set of axes. Label where they cross the axes. [3]

(b) Work out the x-values of the intersection points of these three functions. [3]

(c) Write down a single integral which describes a finite area bounded by two of the three functions. You do *not* need to evaluate the integral. [2]

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14. The Trojan asteroids share Jupiter's orbit around the Sun: approximately circular with a mean radius 5.2 AU (1 AU $= 1.5 \times 10^{11}$ m is the mean radius of the Earth's orbit around the Sun). The Trojans are clustered around two points labelled L4 and L5, where the L4 point is 60° ahead of Jupiter in its orbit and the L5 point is 60° behind Jupiter in its orbit.

(a) Determine the mean distance between the asteroids *588 Achilles* (at the L4 point) and *617 Patroclus* (at the L5 point). [2]

(b) A spacecraft travels in a straight line between the two asteroids, accelerating at $10\,\mathrm{m\,s^{-2}}$ until the half-way point between the asteroids, and decelerating at $10\,\mathrm{m\,s^{-2}}$ from there to the end-point. Assuming that the asteroids are approximately stationary on the timescale of the journey, and neglecting any gravitational effects of Jupiter or the Sun, find the total travel time. [3]

(c) Explain why the assumption that the asteroids are approximately stationary during the journey is well-justified. [1]

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15. A projectile is launched at speed v and angle θ (as measured from the horizontal) outwards from the top of a high cliff.

(a) Sketch the trajectory of the projectile for launch angles $\theta = 5°$, $45°$ and $85°$. Use $x(t)$ for the horizontal displacement from the launch point and $y(t)$ for the vertical displacement from the launch point. [2]

(b) Using separate axes, now sketch the absolute distance, $r(t) = \sqrt{x(t)^2 + y(t)^2}$, from its launch point as a function of time for all of the three launch angles above. [2]

(c) Obtain an expression for $r(t)$. For which angles does $r(t)$ have a stationary point? [5]

(d) For angles below these, what happens to $r(t)$ as time increases? [1]

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16. Suppose $f(t) = 4t$ and $g(x) = \frac{3}{2}\left(3x - x^2\right)$. Consider the inequality

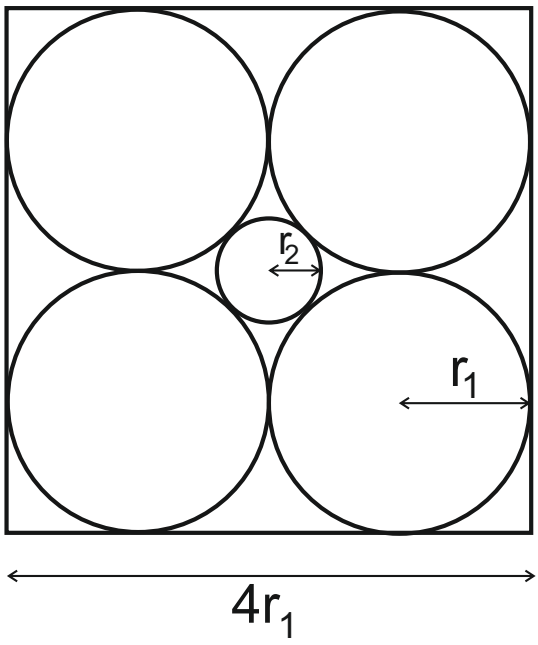
$$\frac{\mathrm{d}g(x)}{\mathrm{d}x} > \int_{3/2}^{x} f(t)\,\mathrm{d}t\,.$$

For which values of x is this inequality satisfied? [4]

17. Four circles of radius r_1 are inscribed inside a square of side $4r_1$ as shown in the diagram below.

(a) What is the radius r_2 of the largest circle that can fit in the space at the centre of the square, bounded by the outer circles? [3]

(b) If 8 spheres of radius r_1 are now similarly arranged inside a cube of edge length $4r_1$, what is the radius r_3 of the largest sphere that can fit in the space at the centre of the cube? [3]



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18. Consider the function

$$f(x) = -\frac{P}{x^3} + \frac{Q}{x^2} - \frac{R}{x}$$

in the region $x > 0$, where P, Q and R are all positive constants.

(a) Find an inequality satisfied by P, Q and R in order for $f(x)$ to have at least one real root. [2]

(b) Find a relationship between P, Q and R in order for $f(x)$ to have exactly one stationary point. [3]

(c) If the relationshiop of the previous part holds, so that exactly one stationary point exists, what is the nature of that stationary point and at what value of x (expressed in terms of P, Q and R) is it? *It is not necessary to work out a second derivative to answer this.* [3]

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19. Following Bohr, we assume that a hydrogen-like atom may be modelled as a single electron (mass m and charge $-e$) in a circular orbit around a much more massive nucleus (charge $+Ze$).

(a) By balancing forces, find the speed v of the electron in terms of its orbital radius. [3]

(b) Show that the total energy of the electron is equal to the negative of its kinetic energy. You may assume that its potential energy U is given by (where r is the radius of the orbit) [3]

$$U = -\frac{Ze^2}{4\pi\epsilon_0 r}$$

(c) Assuming that for the electron the product $mvr = n\hbar$, where n is an integer and $\hbar$ (pronounced h-bar) is a constant, find an expression for the electron energy in terms of n (and which does not depend on either v or r). [2]

(d) If $E(n = 1) = -13.6\,\text{eV}$ for hydrogen, what is $E(n = 3)$ for once-ionised helium (He^+)? [2]

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20. Two unbiased dice are rolled. The numbers obtained are multiplied.

(a) What is the probability that the product is even? [1]

(b) Which product has a probability of $\frac{1}{12}$ to occur? [1]

(c) What is the probability that the product is greater than 28? [1]

(d) Which product(s) has(ve) the highest probability to occur? [2]

(e) If the product is known to be even, what is the probability that it is also divisible by 4? [2]

21. A ball of mass $2m$ slides along a frictionless track with speed u. Starting from a long distance away, it collides elastically with a stationary ball of mass m.

(a) Calculate the final speeds of both balls (you may neglect any rotation of the balls). [5]

(b) If both balls were now positively electrically charged, describe qualitatively either how the results would change or why you would leave the results unaltered. [2]

22. Consider the following set of equations:

$$\begin{aligned} 2x + y &= z, \\ x^2 &= y, \\ z + 2y &= 2x^3. \end{aligned}$$

Find the possible values of x which satisfy these equations. [5]

23. The number of atoms N_x in a sample of a radioactive substance x decays with time according to the equation,

$$N_x(t) = N_x(0)e^{-\lambda_x t},$$

where $N_x(0)$ is the number of atoms at time $t = 0$ and λ_x is a constant for substance x.

The half-life of a substance is defined as the time taken for N_x to reach half of its initial value. Substance a has a half-life of 1 hour. 36% of its decays emit an alpha particle and 64% of its decays emit a beta particle. Substance b has a half-life of 15 minutes. 56% of its decays emit an alpha particle and 44% of its decays emit a beta particle.

If the total particle emission rate of substance x (where $x = a, b$) is $\lambda_x N_x(t)$ and $N_a(0) = N_b(0)$, what time in minutes passes before the beta particle emission rates from the two samples are equal? [5]

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