PHYSICS ADMISSIONS TEST
Thursday, 2 November 2017

Time allowed: 2 hours

For candidates applying to Physics, Physics and Philosophy, Engineering, or Materials

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Total 23 questions [100 Marks]

Answers should be written on the question sheet in the spaces provided, and you are encouraged to show your working.
You should attempt as many questions as you can.

No calculators, tables, or formula sheets may be used.

Answers should be given exactly and in simplest terms unless indicated otherwise.

Indicate multiple-choice answers by circling the best answer.

The numbers in the margin indicate the marks expected to be assigned to each question. You are advised to divide your time according to the marks available.

Do NOT turn over until told that you may do so.

1. Differentiate $y = 2x\cos x$ with respect to x: [2]

A	B	C	D	E
$-2\sin x$	$2\cos x$	$2\cos x + 2x\sin x$	$2\cos x - 2x\sin x$	$-2x\sin x$

$\frac{dy}{dx} = 2\cos x - 2x\sin x$

use the product rule

2. Which equation has the same solutions as $2x^2 - 2x - 12 = 0$? [2]

A	B	C
$(x+2)(x-3) = 0$	$2(x-2)(x+3) = 0$	$(x-6)(x+1) = 0$

D	E
$(x - 1 - 2\sqrt{10})(x - 1 + 2\sqrt{10}) = 0$	$(x - 2 - \sqrt{10})(x - 2 + \sqrt{10}) = 0$

$x^2 - x - 6 = 0$

$(x-3)(x+2) = 0$

3. Evaluate the following sum:

$$\sum_{n=0}^{10} (-e^{-1})^n$$

[2]

A	B	C	D	E
$\dfrac{1}{1+e^{-1}}$	$\dfrac{1-e^{-10}}{1-e^{-1}}$	$\dfrac{1+e^{-10}}{1+e^{-1}}$	$\dfrac{1+e^{-9}}{1-e^{-1}}$	$\dfrac{1+e^{-11}}{1+e^{-1}}$

G.P. with $a = 1$, $r = -e^{-1}$

$S_n = \frac{a(1-r^n)}{1-r}$

$S_{10} = \frac{1-(-e^{-1})^{11}}{1-(-e^{-1})} = \frac{1+e^{-11}}{1+e^{-1}}$

4. If

$$a^{3-x}b^{5x} = a^{x+5}b^{3x}$$

with a and b both real and positive, and $a \neq b$, what is x? [2]

A	B	C	D	E
$\frac{2\log(a-b)}{\log b}$	$2\log a - \log b$	$\frac{2\log b}{\log a - \log b}$	$\frac{\log a}{\log b - \log a}$	$\frac{\log a + \log b}{\log a}$

$a^3 \cdot a^{-x} \cdot b^{5x} = a^x \cdot a^5 \cdot b^{3x}$ $\quad \div a^3 b^{3x} a^{-x}$

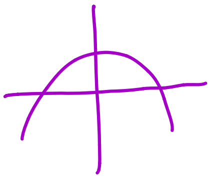
$b^{2x} = a^{2x} \cdot a^2$

$\log(b^{2x}) = \log(a^{2x}a^2)$

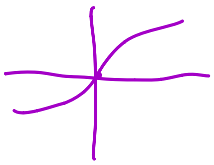
$2x \log b = 2x \log a + 2 \log a$

$x(\log b - \log a) = \log a$

5. Which of the following integrals are equal to zero?



even



odd

$$I_1 = \int_{-1}^{1} x^3 dx \quad \text{odd}$$

$$I_2 = \int_{-\infty}^{\infty} e^{-x^2} dx \quad \text{even}$$

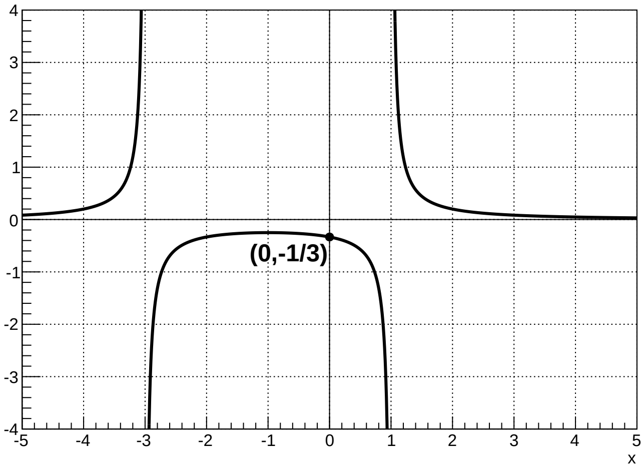
$$I_3 = \int_{-\pi}^{\pi} x \sin x dx \quad \text{even}$$

$$I_4 = \int_{-\pi/2}^{\pi/2} x \cos x dx \quad \text{odd}$$

[2]

A	B	C	D	E
None of these	I_1 and I_4	I_1, I_2, and I_4	I_2, I_3, and I_4	All of these

6. The graph below could represent which of the following functions? [2]



A	~~**B**~~	**C**	~~**D**~~	~~**E**~~
$\frac{1}{x-1}+\frac{2}{x+3}$	$\frac{-1}{x^2-2x+3}$	$\frac{1}{x^2+2x-3}$	$\frac{1}{x^2-1}+\frac{2}{x+3}$	$\frac{3}{x^2-9}$
	$(x-3)(x+1)$	$(x+3)(x-1)$		

vertical asymptotes at $x=-3$ and $x=1$

$\Rightarrow$ denominator roots are -3 and 1

$\Rightarrow$ not B, D or E

When $x=-2$, $\frac{1}{-2-1}+\frac{2}{-2+3} = \frac{-1}{3}+\frac{2}{1} = \frac{5}{3}$

$\Rightarrow$ not A

7. An astronaut on the surface of the Moon lightly tosses a ball of mass m upward. What happens to the ball? [2]

A	**B**	**C**
The ball enters an orbit around the Earth.	The ball eventually falls toward the Earth, burning up in the atmosphere.	The ball falls to the surface of the Moon.

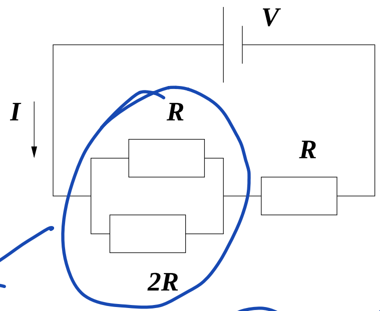
D	**E**
The ball rises slowly until it hovers above the astronaut.	The ball enters an orbit around the Moon.

8. In which of the following lists are the parts of the electromagnetic spectrum ordered correctly from shortest wavelength (at the top) to longest wavelength (at the bottom)? [2]

A	**B**	**C**	**D**	**E**
ultraviolet, X-ray, visible, radio, infrared	X-ray, ultraviolet, visible, radio, infrared	ultraviolet, X-ray, visible, infrared, radio	infrared, radio, visible, ultraviolet, X-ray	X-ray, ultraviolet, visible, infrared, radio

[**Turn over**]

9. What is the value of the current I in the circuit below? [2]



A	B	C	D	E
$\frac{V}{4R}$	$\frac{3V}{5R}$	$\frac{4V}{3R}$	$\frac{5V}{3R}$	$\frac{3V}{4R}$

$$\frac{2R \cdot R}{2R+R}$$

$$R_T = \frac{2R^2}{3R} + R$$

$$= \frac{5R}{3}$$

$$I = \frac{V}{R_T}$$

$$= V \cdot \frac{3}{5R}$$

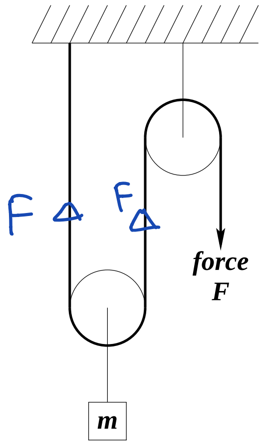
10. A capacitor is constructed with two conducting plates of equal area A separated by an insulator. The capacitance is measured to be C.

The conducting plates are then shrunk to half the original area. What is the capacitance now? [2]

A	B	C	D	E
$\frac{C}{2}$	C	$2C$	C^2	$\frac{1}{C^2}$

$C \propto A$

11. Consider the pulley system below supporting an object with mass m. Assume gravitational acceleration to be g, that the pulleys are massless and frictionless, and the string is massless and inextensible. With how much force F must the string be pulled to keep the mass at the same height? [2]



A	B	C	D	E
$\frac{mg}{3}$	$\frac{mg}{2}$	mg	$2mg$	$3mg$

$mg = 2F$

$F = \frac{mg}{2}$

12. A particle with charge q and initial speed v is stopped by a potential difference V in a distance d and time t. What was its initial momentum? [2]

A	B	C	D	E
$\frac{qVt}{d}$	$\frac{qV}{v}$	$\frac{qVd}{t}$	$2qVv$	$\frac{qV}{2v}$

$F = \frac{\Delta p}{\Delta t}$ $\quad F = Eq$ $\quad E = \frac{V}{d}$

$\frac{p}{t} = Eq$

$= \frac{Vq}{d}$

$p = \frac{Vqt}{d}$

13. Expand $(3+2x)^5$ as a sum of powers of x. [3]

$$(3+2x)^5 = 3^5 + 5(3)^4(2x)^1 + 10(3)^3(2x)^2 + 10(3)^2(2x)^3 + 5(3)^1(2x)^4 + (2x)^5$$

$$= 243 + 810x + 1080x^2 + 720x^3 + 240x^4 + 32x^5$$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1

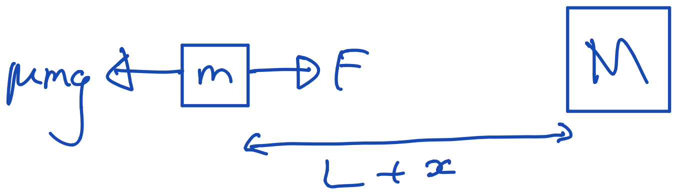
14. Person A is busy for 50% of the week, Person B 75%, and Person C 20%. If a time for a meeting is picked at random, what is the probability that (a) all three people are busy, and (b) all three people can attend the meeting? [4]

a) $0.5 \times 0.75 \times 0.2 = 0.075$

b) $0.5 \times 0.25 \times 0.8 = 0.1$

15. A spring with spring constant k and natural length L joins two blocks of mass m and M. The two blocks lie on a horizontal table, initially L apart. The maximum force for static friction between a block and the table is given by the coefficient of static friction μ_s multiplied by the block's weight.

How far must the mass M be displaced to cause mass m to move? [5]



$F = kx$

$x = \dfrac{F}{k}$

$= \dfrac{\mu m g}{k}$

[Turn over]

16. A cone has a height equal to the diameter of a sphere. If the volumes of the two objects are equal, and the radius of the sphere is r, what is the radius of the base of the cone? [4]

$$V_{cone} = \frac{1}{3}\pi R^2 h = \frac{1}{3}\pi R^2 (2r)$$

$$V_{sphere} = \frac{4}{3}\pi r^3$$

$$\Rightarrow \frac{\cancel{4}^{2}}{\cancel{3}} \cancel{\pi} r^{\cancel{3}^{2}} = \frac{\cancel{2}}{\cancel{3}} \cancel{\pi} R^2$$

$$R^2 = 2r^2$$

$$R = \sqrt{2}\, r$$

17. A parachutist jumps out of a plane at height h. She is subject to air resistance with a force of $-\alpha v^2$. The equation of her motion is given by

$$m\frac{dv}{dt} = mg - \alpha v^2.$$

a) What are the units of α?

b) Calculate the terminal velocity of the parachutist.

c) Estimate how much work is done by the air resistance as she falls, assuming that she is falling at near terminal velocity by the time she reaches the ground. [7]

a) αv^2 is a force $\Rightarrow$ units $N \equiv kg\,m\,s^{-2}$

$$\alpha (m s^{-1})^2 = kg\,m\,s^{-2}$$

$$\alpha = \frac{kg\,m\,s^{-2}}{m^2 s^{-2}} = kg\,m^{-1}$$

b) $\alpha v^2 = mg$

$$v = \sqrt{\frac{mg}{\alpha}}$$

c) At plane: $GPE = mgh$ $\quad KE = 0$

On ground level: $GPE = 0$ $\quad KE = \frac{1}{2}mv^2$

$$= \frac{1}{2}m\frac{mg}{\alpha}$$

$\Rightarrow$ Work done $= mgh - \frac{1}{2}\frac{m^2 g}{\alpha}$

18. Consider two sound waves travelling with the same speed and amplitude but having similar but slightly different wavelengths, λ_1 and λ_2, and angular frequencies, ω_1 and ω_2. The two waves are described with the functions

a) $\omega = 2\pi f$

$f = \frac{\omega}{2\pi}$

$$y_1(x,t) = A\cos\left(\frac{2\pi x}{\lambda_1} - \omega_1 t\right)$$
$$y_2(x,t) = A\cos\left(\frac{2\pi x}{\lambda_2} - \omega_2 t\right).$$

$v = f\lambda = \frac{\omega_1 \lambda_1}{2\pi}$

$= \frac{\omega_2 \lambda_2}{2\pi}$

a) What is the speed v in terms of the angular frequencies and wavelengths?

b) Sketch $y_1 + y_2$ as a function of x, at some time t.

c) If you stood in the path of these sound waves, what frequency would you hear (assuming you can hear it)? What is the distance between points where the sound disappears?

[Hint: you can use the formula $\cos x + \cos y = 2\cos\frac{x+y}{2}\cos\frac{x-y}{2}$.] [8]

b) Let $C = \frac{2\pi x}{\lambda_1} - \omega_1 t$, $D = \frac{2\pi x}{\lambda_2} - \omega_2 t$

$$\frac{C+D}{2} = \pi x\frac{(\lambda_2+\lambda_1)}{\lambda_1\lambda_2} - \frac{t(\omega_1+\omega_2)}{2}$$

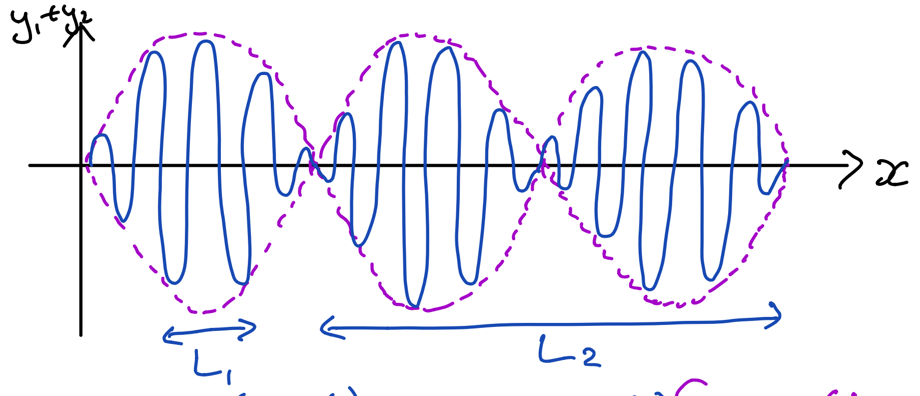
$$\frac{C-D}{2} = \pi x\frac{(\lambda_2-\lambda_1)}{\lambda_1\lambda_2} - t(\omega_1-\omega_2)$$

$$\Rightarrow y_1+y_2 = 2A\underbrace{\cos\left[x\frac{\pi(\lambda_2+\lambda_1)}{\lambda_1\lambda_2} - t\frac{(\omega_1+\omega_2)}{2}\right]}_{\text{let wavelength } = L_1}\underbrace{\cos\left[x\frac{\pi(\lambda_2-\lambda_1)}{\lambda_1\lambda_2} - t\frac{(\omega_1-\omega_2)}{2}\right]}_{\text{let wavelength } = L_2}$$

$\frac{2\pi}{L_1} = \frac{\pi(\lambda_2+\lambda_1)}{\lambda_1\lambda_2}$

$L_1 = \frac{2(\lambda_1\lambda_2)}{\lambda_2+\lambda_1}$

$L_2 = \frac{2(\lambda_1\lambda_2)}{\lambda_2-\lambda_1}$



c) freq heard $= \frac{v}{L_1} = \frac{\omega_1\lambda_1}{2\pi} \times \frac{(\lambda_2+\lambda_1)}{2\lambda_1\lambda_2} = \frac{\omega_1(\lambda_2+\lambda_1)}{4\pi\lambda_2}$ $\left[= \frac{\omega_2(\lambda_2+\lambda_1)}{4\pi\lambda_1}\right]$

distance = node to node $= \frac{L_2}{2} = \frac{\lambda_1\lambda_2}{\lambda_2-\lambda_1}$

19. A curve is defined parametrically:

$$x = a(\omega t - \sin \omega t)$$
$$y = a(\sqrt{3} - 2\cos \omega t)$$

with non-zero constants a and ω. At what values of x is y equal to zero? [9]

$0 = a(\sqrt{3} - 2\cos wt)$

$\frac{\sqrt{3}}{2} = \cos wt$

$wt = \frac{\pi}{6} + 2\pi n, \ \frac{11\pi}{6} + 2\pi n$

When $wt = \frac{\pi}{6} + 2\pi n$,

$x = a\left[\frac{\pi}{6} + 2\pi n - \sin\left(\frac{\pi}{6} + 2\pi n\right)\right] = a\left(\frac{\pi}{6} + 2\pi n - \frac{1}{2}\right)$

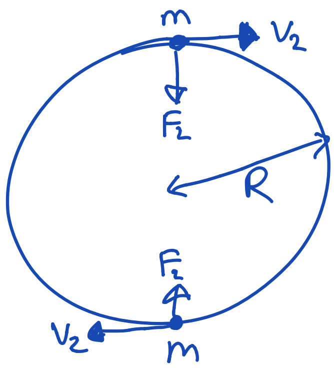
When $wt = \frac{11\pi}{6} + 2\pi n$,

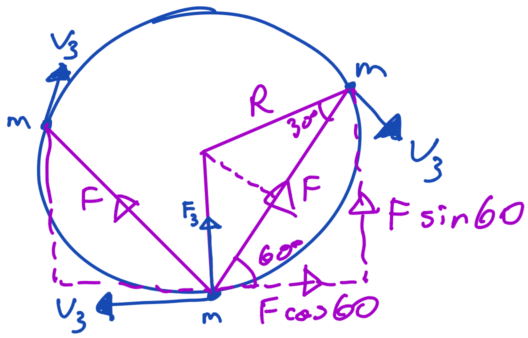
$x = a\left[\frac{11\pi}{6} + 2\pi n - \sin\left(\frac{11\pi}{6} + 2\pi n\right)\right] = a\left(\frac{11\pi}{6} + 2\pi n + \frac{1}{2}\right)$

20. In a certain binary star system, two stars with identical mass m have equal and opposite velocities v_2 on the opposite sides of the same circular orbit with radius R.

In another system with three identical stars of the same mass m as before, it is observed that all three stars are equally spaced around a circular orbit with the same radius R as before. What is the speed v_3 of these stars in terms of v_2?

[Hint: consider the direction and magnitude of the force exerted on one star by the other two.] [9]





$$F_2 = \frac{mv_2^2}{R} = \frac{Gm.m}{(2R)^2}$$

$$\frac{\cancel{m}v_2^2}{\cancel{R}} = \frac{Gm^{\cancel{2}}}{4R^{\cancel{2}}}$$

$$v_2^2 = \frac{Gm}{4R}$$

$$F_3 = \frac{mv_3^2}{R} = 2 \times F\sin 60$$

$$= 2\frac{\sqrt{3}}{2}\frac{Gmm}{(2R\cos 30)^2}$$

$$\frac{\cancel{m}v_3^2}{\cancel{R}} = \sqrt{3}\frac{Gm^{\cancel{2}}}{3R^{\cancel{2}}}$$

$$v_3^2 = \frac{\sqrt{3}\,Gm}{3R}$$

$$\Rightarrow \frac{v_3^2}{v_2^2} = \frac{\sqrt{3}\,Gm}{3R} \times \frac{4R}{Gm} = \frac{4\sqrt{3}}{3}$$

$$v_3 = \sqrt{\frac{4\sqrt{3}}{3}}\,v_2$$

$$= \frac{2}{3^{1/4}}\,v_2$$

21. Evaluate the following expression:

$$\frac{d}{dt}\int_0^{2t^2}(xt)^4 dx.$$

[9]

$$\frac{d}{dt}\int_0^{2t^2} t^4x^4\,dx = \frac{d}{dt}\left[t^4\,\frac{x^5}{5}\right]_0^{2t^2}$$

$$= \frac{d}{dt}\left[\frac{t^4}{5}(2t^2)^5\right]$$

$$= \frac{d}{dt}\left(\frac{32}{5}t^{14}\right)$$

$$= \frac{448}{5}t^{13}$$

22. The equation of circle C_1 is

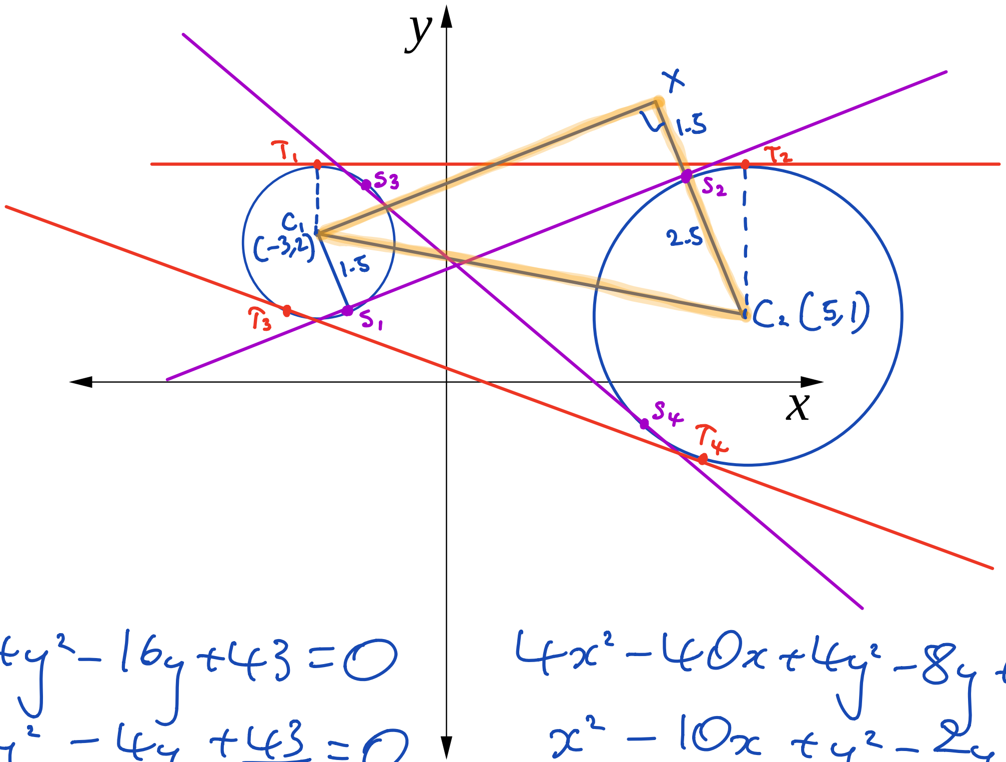
$$4x^2 + 24x + 4y^2 - 16y + 43 = 0$$

while the equation of circle C_2 is

$$4x^2 - 40x + 4y^2 - 8y + 79 = 0.$$

Sketch a diagram of these circles on the axes below, along with all lines which are tangent to both circles.

For each line, calculate the length of the line segment joining the tangent points. [9]



$$4x^2+24x+4y^2-16y+43=0$$
$$x^2+6x+y^2-4y+\frac{43}{4}=0$$
$$(x+3)^2-9+(y-2)^2-4+\frac{43}{4}=0$$
$$(x+3)^2+(y-2)^2=\left(\frac{3}{2}\right)^2$$

$$4x^2-40x+4y^2-8y+79=0$$
$$x^2-10x+y^2-2y+\frac{79}{4}=0$$
$$(x-5)^2-25+(y-1)^2-1+\frac{79}{4}=0$$
$$(x-5)^2+(y-1)^2=\left(\frac{5}{2}\right)^2$$

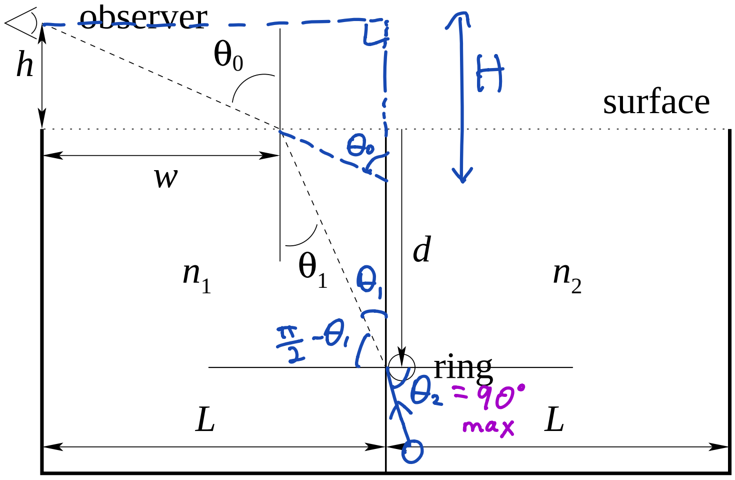
$$(C_1C_2)^2=(5+3)^2+(2-1)^2=65$$

$$S_1S_2=S_3S_4=C_1X=\sqrt{65-(2.5+1.5)^2}=7$$

$$T_1T_2=T_3T_4=5-(-3)=8$$

23. An experimental setup consists of two deep tanks, each of width L, separated by a thin, transparent membrane, as shown in the figure below. The left tank is filled with a transparent liquid with refractive index n_1, and the right tank with a transparent liquid with refractive index n_2. The membrane has refractive index n_1. Assume that the refractive index of air is 1, and $1 < n_2 < n_1$.

A gold ring is dropped in the right pool (with refractive index n_2), near the membrane, and drops straight down. An observer, at height h above the left edge of the experimental setup, watches the ring drop. The dashed line in the figure indicates the path of a light ray from the ring to the observer, with lengths and angles indicated.



At a certain apparent depth, the ring will appear to the observer to stop descending. At what apparent depth does this happen? [9]

$$n_2 \sin\left(\frac{\pi}{2}\right) = n_1 \sin\left(\frac{\pi}{2}-\theta_1\right)$$

$$n_2 = n_1 \cos\theta_1$$

$$\sin\theta_0 = n_1 \sin\theta_1$$

$$(n_2)^2 + (\sin\theta_0)^2 = n_1^2(\cos^2\theta_1 + \sin^2\theta_1)$$

$$\sin^2\theta_0 = n_1^2 - n_2^2$$

$$\sin^2\theta_0 = \frac{L^2}{H^2+L^2}$$

$$H^2 + L^2 = \frac{L^2}{n_1^2 - n_2^2}$$

$$H^2 = \frac{L^2(1 + n_1^2 - n_2^2)}{n_1^2 - n_2^2}$$

Apparent depth $= H - h = L\sqrt{\dfrac{1+n_1^2-n_2^2}{n_1^2-n_2^2}} - h$