BPhO
British Physics Olympiad

BRITISH PHYSICS OLYMPIAD 2016-17

A2 Challenge

September/October 2016

Instructions

Time: 1 hour.

Questions: Answer ALL questions.

Marks: Total of 50 marks.

Solutions: These questions are about problem solving. Draw diagrams to get to understand the questions. You must write down the questions in terms of symbols and equations; then try calculating quantities in order to work towards a solution.

In these questions you will need to explain your reasoning by showing clear working. Even if you cannot complete the question, show how you have started your thinking, with ideas and, generally, by drawing a diagram.

Formula sheet: You are allowed any standard exam board data/formula sheet.

Note: This exam paper is not an A level paper or equivalent and you may struggle to get anywhere near the end. You will probably not achieve a high mark unless you have practised lots of problem solving already. The first rule of problem solving is to have a go. The second rule is to have another go. You will improve with practice, not by seeing someone else's solution.

Important Constants

Speed of light	c	3.00×10^{8}	$\mathrm{m\ s^{-1}}$
Planck constant	h	6.63×10^{-34}	$\mathrm{J\ s}$
Electronic charge	e	1.60×10^{-19}	C
Mass of electron	m_e	9.11×10^{-31}	kg
Gravitational constant	G	6.67×10^{-11}	$\mathrm{N\ m^{2}\ kg^{-2}}$
Acceleration of free fall	g	9.81	$\mathrm{m\ s^{-2}}$
Permittivity of the vacuum	ε_0	8.85×10^{-12}	$\mathrm{F\ m^{-1}}$
Avogadro constant	N_A	6.02×10^{23}	$\mathrm{mol^{-1}}$

Qu. 1 This question relates to Newton's Laws of Motion.

(a) Newton's 1st Law suggests the results of the action of a force by describing the consequences of the absence of a force. We now look at the consequences which may arise from the action of a force.

(i) **Figure 1** shows a body (shaded) with an instantaneous velocity v moving past a fixed point C. Copy the diagram in **figure 1** and add an arrow to represent a force that could cause the speed of the body to increase with no other change. Mark this arrow F_A.

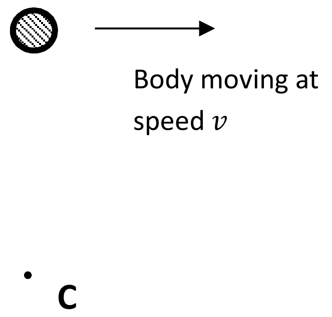

Figure 1. Body moving at instantaneous speed v passing a fixed point C.

(ii) On the same diagram draw an arrow to represent a force causing the body to follow a circular path around the point **C** at a constant speed. Mark this arrow F_C.

(iii) Circular motion is often described as an accelerated motion. In what respect is the motion caused by F_C an accelerated motion if the speed is unaltered?

(iv) Why does F_C cause no change to the speed of the body?

2 marks

(b) Newton's 2nd Law may be stated as *force equals rate of change of momentum,* i.e.

$$F = \frac{dp}{dt}$$

(i) By differentiating the product mv find an expression for F containing two terms.

(ii) State which term corresponds to the simplified version of Newton's 2nd Law, $F = ma$.

(iii) Using the *second* term from part (i) or otherwise, solve these two problems:

A. A paint-spraying machine projects 5.0 cm^3 of paint, of density 1200 kg m^{-3}, horizontally at a speed of 30 m s^{-1} onto a vertical surface each second. All the paint sticks to the surface. What force does the impact of the paint stream exert on the surface?

B. A metal-hardening machine projects 5.0 cm^3 of small, hard steel balls, of density 8000 kg m^{-3}, horizontally at a speed of 30 m s^{-1} onto a vertical surface each second. All the balls rebound elastically (i.e. no loss of kinetic energy) from the surface. What force does the stream of balls exert on the surface?

5 marks

(c) Newton's 3rd Law is often stated in its traditional form: *to every action there is an equal and opposite reaction.*

(i) The diagram in **figure 2** represents a person sitting at rest on a table on the surface of the Earth (not to scale). Copy the diagram and add arrows to represent the weight, $\boldsymbol{W}$, of the person and $\boldsymbol{R}$, the Newton's 3rd Law 'equal and opposite reaction' to $\boldsymbol{W}$.

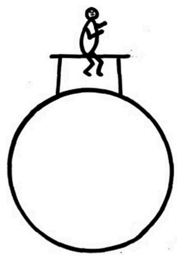

Figure 2. Person sitting on a table at rest on the Earth.

(ii) Now draw a second diagram of the person alone, featuring $\boldsymbol{W}$ and any other force, $\boldsymbol{F}$, needed to maintain the equilibrium of the person.

(iii) State the origin and nature of $\boldsymbol{F}$ and $\boldsymbol{W}$, by making **two** statements like 'electrostatic repulsion between two like charges'.

(iv) Explain briefly why $\boldsymbol{F}$ and $\boldsymbol{R}$ are not the same force.

4 marks

11

Qu. 2 This question looks at some implications of thermal expansion.

Thermal expansion is measured by a quantity, α, called the *coefficient of linear expansion,* which may be defined as *fractional increase of length (i.e. strain) per unit temperature rise.*

(a) Write an equation for α in terms of l, l_0 and ΔT, where l is the length at temperature T, l_0 is the length at temperature T_0 and ΔT is the temperature rise $(T - T_0)$. Show that

$$l = l_0(1 + \alpha\Delta T)$$

(b) Expansion of railway lines is a serious problem in hot weather.

(i) Calculate how much longer the 632 km of track from London to Edinburgh will be on a hot summer's day (25 ^{o}C), compared with a winter's day (0 ^{o}C).
The linear coefficient of expansion of steel is 1.2 x 10^{-5} $^{o}C^{-1}$ for this temperature range.

(ii) The difficulty is overcome by laying the track in a stretched condition so that it just goes slack in the summer. Calculate the tensile force at 0 ^{o}C in a single rail of cross-sectional area 100 cm^2.
Young Modulus of steel is 200 GPa

4 marks

(c) Clearly, flat sheet material will also expand when heated, thus increasing its area. A flat sheet of steel has a circular disc cut out, leaving a circular hole. The sheet is then heated. Comment on whether:

(i) The unheated disc will fit into the hole in the heated sheet.

(ii) The heated disc will fit into the hole when the sheet is cooler.

2 marks

The last part of this question explores the expansion of a liquid.

(d) Imagine a hypothetical, isolated cube of liquid (i.e. ignore the need for a container) of side, l_0.

(i) What is the volume of the cube in terms of l_0?

(ii) Using the equation from part (a) above, find the volume of the cube when it is heated through a temperature rise of ΔT, in terms of l_0 and ΔT.

(iii) Hence find the increase of volume of the liquid.

Thermal expansion of a liquid is measured by a quantity, γ**,** called the *coefficient of volumetric expansion,* which may be defined as the *fractional increase of volume per unit temperature rise*.

(iv) Write an equation for γ in terms of V, V_0 and ΔT, where V is the volume at temperature T, V_0 is the volume at temperature T_0 and ΔT is the temperature rise $(T - T_0)$. Show that

$$\gamma = 3\alpha$$

Hint: $\alpha \Delta T$ is very small, so you might consider ignoring some terms in your algebra.

7 marks

13

Qu. 3 This question explores some consequences of the principle of superposition.

(a) Two sources of sound emitting frequencies of 200 Hz and 202 Hz, but of equal amplitude, are placed close to each other. They are heard by a nearby observer.

(i) How many waves from each source have passed the observer 1 s, ½ s, and ¼ s after the start of his observations?

(ii) At the start of these observations (time $t = 0$ s) the waves were in phase at the position of the observer. Comment on the phase difference of the two waves at that position after 1 s, ½ s, and ¼ s.

(iii) Hence describe what the observer hears during the first second.

(iv) What is the frequency of the **changes** the observer hears?

(v) Write a rule relating this frequency to the frequencies of the two original sound waves.

6 marks

(b) Two sources of sound, of equal amplitude and of frequency 10 kHz, are set up at opposite ends of a tube containing a liquid in which the speed of sound is 1 km s^{-1}. A standing (or stationary) wave along the line joining the two sources arises from the superposition of the two sound waves. (You can ignore reflections from the ends of the tube).

(i) Calculate the wavelength of the waves and hence state the separation of the nodes in the standing wave.

(ii) A small microphone is moved along the axis of the tube at 5 m s^{-1}. The output of the microphone rises and falls as it passes through the standing wave system. What is the frequency of this variation of output?

(iii) As the microphone moves through the sound waves, the two individual sounds will have their frequencies modified by the *Doppler effect*. The change in frequency, Δf, caused by moving at a speed v through waves travelling at speed c, for this purpose is given by:

$$\frac{\Delta f}{f} = \frac{v}{c}$$

Calculate the frequencies of each of the two sounds as received by the moving microphone.

(iv) Use your rule from (a)(v) above to determine the resultant output of the microphone.

(v) Compare the results of (ii) and (iv) above and comment.

5 marks

(c) When incident light grazes across the surface of a compact disc (CD) or a vinyl record, coloured fringes are seen.

In **figure 3** this effect is modelled by a series of equally spaced reflecting points, with separation d, scattering the incident beam in a direction defined by the angle θ, in a manner similar to the light emerging from a diffraction grating.

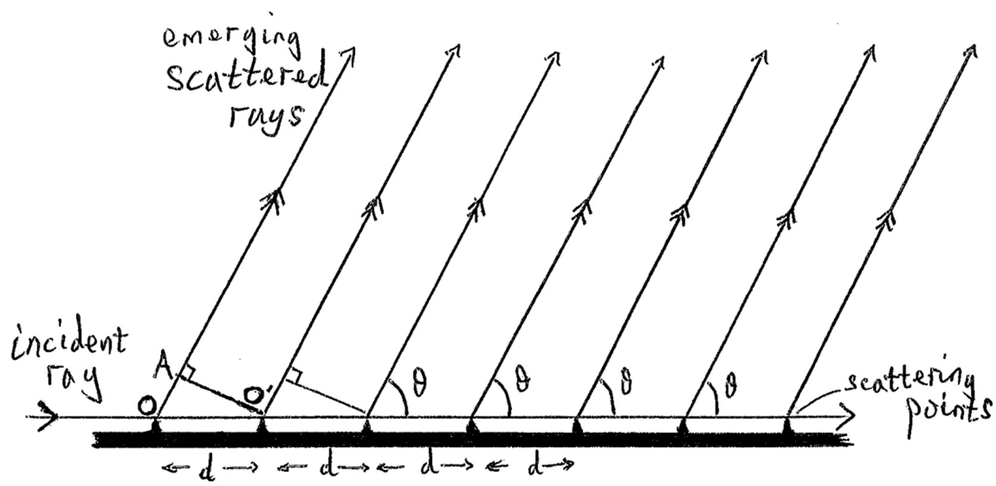

Figure 3. Scattering from a regular set of reflecting points.

(i) Obtain an equation for the length of the path OA.

(ii) Hence obtain an equation for the path difference between any two successive scattered emergent rays e.g. the rays scattered at O and O'.

(iii) Now write down a formula for the position of diffraction maxima in this situation (like the familiar $d \sin\theta = n\lambda$ for a diffraction grating).

(iv) The smallest angle, θ, at which maximum of intensity is seen (i.e. the first order, $n = 1$), using light of wavelength 500 nm, is 12°. Calculate the separation, d, of the scattering points.

4 marks

15

Qu. 4 This question explores the properties of some resistor networks.

(a) (i) What is the resistance of two 12 Ω resistors connected in parallel?

(ii) What is the resistance of three 12 Ω resistors connected in parallel?

(iii) Hence devise a rule for the resistance of n resistors of resistance R**,** connected in parallel.

(iv) Two "series pairs" of 12 Ω resistors are connected in parallel, as in **figure 4**. What is the resistance of this '2 x 2 array' of resistors?

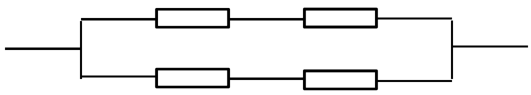

Figure 4.

(v) What can you deduce about the resistance of any square (i.e. $n \times n$) array of equal resistors, each of value R?

5 marks

(b) Here are some electrical puzzles which may be solved most easily by the use of symmetry. i.e. find points which are at the same potential.

The diagram in **figure 5** shows a cube formed of twelve wires each of resistance 12 Ω.

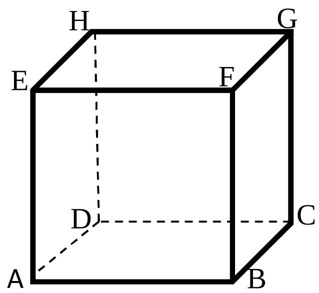

Figure 5.

(i) What is the resistance between the corners, C, H of the cube?

(ii) What is the resistance between the corners, F, D of the cube?

(iii) The six wires joined to D and F are now replaced by wires of resistance 6 Ω, while the other six remain as 12 Ω. What is the new value of resistance between D and F?

6 marks

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