

Mark Scheme September/October 2006

1. energy change = mgh (1)
power generated = $m/t \times g \times h$
 $826 \times 10^6 = m/t \times 9.81 \times 25$
 $m/t = 3.37 \times 10^6 \text{ kg}^3/\text{s}$ (1)
50% efficiency $\rightarrow 6.74 \times 10^6 \text{ kg}^3/\text{s}$ (1)
volume flow = mass flow/density
 $= 6740 \text{ m}^3/\text{s}$ (1)
per pipe $= 6740 \div 7 = 963 \text{ m}^3/\text{s}$ (1)
speed of flow = volume rate of flow/cross sectional area of pipe (1)
 $= 963/(\pi \times 3.7^2)$
 $= 22.4 \text{ m/s}$ (1)
2. There will be no change in the load on the supports. (1)
Using Archimedes Principle, the barge floats in the water by having an upthrust on it equal to the weight of water displaced. So as the barge floats by, it is indistinguishable weight-wise from the water that it has displaced. (1)
3. Energy of a photon $= 6.6 \times 10^{-34} \times 3 \times 10^{10}$
 $= 19.8 \times 10^{-24} \text{ J}$ (1)
$$\text{number of photons, } N = \frac{11}{19.8 \times 10^{-24}} = 5.5 \times 10^{23} \text{ s}^{-1}$$

$$\text{area at earth radius which receives these photons} = 10^{-4} \times 4\pi \times (10^{13})^2$$

$$= 4\pi \times 10^{22} \text{ m}^2$$

$$\text{dish area} = \pi \times 40^2 \text{ m}^2$$

$$\text{No of photons received by the dish per second} = \frac{\pi 40^2}{4\pi 10^{22}} \times 5.5 \times 10^{23} = 22,000$$
 (3)
4. Extra length is given by $2\pi(r + \Delta r) - 2\pi r = 2\pi\Delta r$
For $\Delta r = 1 \text{ m}$, extra length $= 6.3 \text{ m}$, independent of r . (3)
The earth's circumference would increase by 0.63 m (1)
Surface area of earth is given by $A = 4\pi r^2$
(i) algebraically

$$A + \Delta A = 4\pi(r + \Delta r)^2$$

$$A + \Delta A = 4\pi(r^2 + 2r\Delta r + (\Delta r)^2)$$

$$\Delta A \approx 8\pi r \Delta r$$


$$\frac{\Delta A}{A} = 2 \frac{\Delta r}{r}$$

or (ii) differentiate

$$\frac{dA}{dr} = 4\pi r$$

$$\frac{dA}{A} = 2 \frac{dr}{r}$$

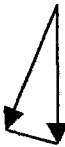
result $\frac{dA}{A} = 4 \times 10^{-5} \%$ (3)

5. (a) tangential arrows of equal length 

magnitude of change of velocity is $2v$

$$\text{acceleration} = \frac{2v}{T/2} = \frac{2v}{\pi r v} = \frac{2}{\pi} \frac{v^2}{r} \quad (3)$$

(b)  $\Delta v = \sqrt{2}v$ (2)

(c)  $(\Delta v)^2 = 2v^2 - 2v^2 \cos \theta$ (3)
 $(\Delta v)^2 = 2v^2(1 - \cos \theta)$

(d) $\frac{\Delta t}{T} = \frac{\theta}{2\pi}$ and $T = \frac{2\pi r}{v}$ (2)

(e)

$$a = \frac{\Delta v}{\Delta t} = \sqrt{2}v \frac{(1 - \cos \theta)^{\frac{1}{2}}}{r\theta/v}$$

$$= \sqrt{2} \frac{v^2}{r} \frac{(1 - \cos \theta)^{\frac{1}{2}}}{\theta}$$

$$= \sqrt{2} \frac{v^2}{r} \frac{(1 - 1 + \theta^2/2)^{\frac{1}{2}}}{\theta}$$

$$= \sqrt{2} \frac{v^2}{r} \frac{\theta}{\sqrt{2} \theta} = \frac{v^2}{r} \quad (4)$$

6. a) $R \propto V^{1/3}$ so that the ratio $\frac{R_{final}}{R_{initial}} = \sqrt[3]{\frac{1}{10^{15}}} = \frac{1}{10^5}$ (3)

b) $\Omega_{initial} R_{initial}^2 = \Omega_{final} R_{final}^2$
 and $\Omega_{final} = 3.6 \times 10^4$ radians/second
 and $T = 0.17$ ms (3)

c) R^2 decreases by a factor 10^{10}
 and so B increases by 10^{10} to a final field strength of 10^8 T (2)

d) Assume that the star remains spherical so that R describes its radius.
 Equate gravitational field strength with the centripetal acceleration at the equator.

$$\frac{v_{max}^2}{R} = \frac{GM}{R^2} \text{ with } v_{max} = \frac{2\pi r}{T_{min}} \Rightarrow T_{min} = \frac{2\pi}{\sqrt{G}} \frac{R^{\frac{3}{2}}}{M^{\frac{1}{2}}}$$

which evaluates to give $T_{min} = 0.46$ seconds (much longer period than the answer to part (b)) (4)

e) The initial BE is that for the two stars summed together. The factor relating the initial and final stars is the mass; the masses add whereas the radii do not.

$$BE_{initial} = k_1 \frac{GM_{initial}^2}{R_{initial}} \times 2 \text{ with } R_{initial} M_{initial}^{\frac{1}{3}} = k_2$$

giving

$$BE_{initial} = k_1 \frac{GM_{initial}^{2+\frac{1}{3}}}{k_2} \times 2$$

after collision

$$BE_{final} = k_1 \frac{GM_{final}^{2+\frac{1}{3}}}{k_2}$$

with $M_{final} = 2 \times M_{initial}$

hence

$$\begin{aligned} \frac{BE_{final}}{BE_{initial}} &= \frac{M_{final}^{\frac{7}{3}}}{2M_{initial}^{\frac{7}{3}}} \\ &= \frac{1}{2} \times 2^{\frac{7}{3}} = 2.52 \end{aligned} \quad (4)$$