ENGAA 2017
Section 1

Model Solutions

1

$$\frac{(\sqrt{12}+\sqrt{3})^2}{(\sqrt{12}-\sqrt{3})^2}$$
$$= \frac{(2\sqrt{3}+\sqrt{3})^2}{(2\sqrt{3}-\sqrt{3})^2}$$
$$= \frac{(3\sqrt{3})^2}{(\sqrt{3})^2}$$
$$= \frac{27}{3}$$
$$= 9$$

2

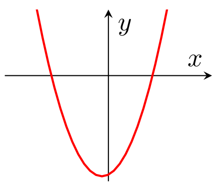
The area underneath the graph represents displacement, and the gradient represents acceleration. The car is therefore decelerating where the gradient is negative; for $110 \leq t \leq 130$. Therefore:

$$\text{Distance} = \frac{1}{2} \cdot (30+20) \cdot (20)$$
$$= 500m$$

3

$$2x^2 \geq 15 - x$$
$$2x^2 + x - 15 \geq 0$$
$$(2x-5)(x+3) \geq 0$$

$$\text{Critical values: } x = \frac{5}{2}, x = -3$$



$$\therefore x \leq -3, x \geq \frac{5}{2}$$

4

$$\rho = \frac{m}{v} \text{ where } \rho \text{ is density, } m \text{ is mass, } v \text{ is volume.}$$

Heating the liquid increases the thermal vibration of water molecules, thereby increasing the spacing between adjacent molecules. This leads to an *increase* in volume, and therefore a *decrease* in density, for a fixed mass of water. Therefore statements 2 and 3 are true. The less dense, hot water rises because it is lighter, allowing cooler water to replace it. This process creates a convection current.

For fixed v, as ρ decreases, m must also decrease, so statement 1 is false.

5

Making x the subject:

$$
\begin{aligned}
y &= 3\left(\frac{x}{2}-1\right)^2 - 5 \\
\frac{y+5}{3} &= \left(\frac{x}{2}-1\right)^2 \\
1 \pm \sqrt{\frac{y+5}{3}} &= \frac{x}{2} \\
x &= 2 \pm 2\sqrt{\frac{y+5}{3}}
\end{aligned}
$$

6

$$
\begin{aligned}
\text{GPE gained} &= mgh \\
&= 1200 \cdot 10 \cdot 1 \\
&= 12000 \\
\text{Work-energy principle:} & \\
12000 &= 28000 - \text{Energy lost} \\
\therefore \text{Energy lost} &= 16000J
\end{aligned}
$$

7

$$
\begin{aligned}
2x + 5y &= P && (1) \\
3x + 2y &= Q && (2)
\end{aligned}
$$

To find y, the cost of one pear, eliminate x from both equations:

$$
\begin{aligned}
3 \times (1) : 3P &= 6x + 15y && (3) \\
2 \times (2) : 2Q &= 6x + 4y && (4)
\end{aligned}
$$

subtract (4) from (3)

$$
\begin{aligned}
11y &= 3P - 2Q \\
y &= \frac{3P - 2Q}{11}
\end{aligned}
$$

8

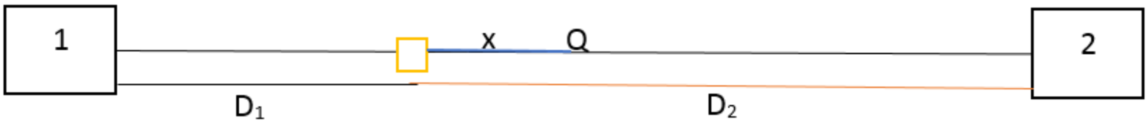


Figure 1: x is the distance between Q and the source

$$t_1 = \text{the time taken for a gamma ray to travel } D_1$$
$$t_2 = \text{the time taken for a gamma ray to travel } D_2$$
$$\text{Time difference} = t_2 - t_1 = 4.0 \times 10^{-10} s$$

$$\begin{aligned}
\text{distance} &= \text{speed} \times \text{time} \\
\therefore D_2 - D_1 &= c \times (t_2 - t_1) \\
&= (3 \times 10^8) \times (4 \times 10^{-10}) \\
&= 12 \times 10^{-2} m \\
&= 12cm \\
D_1 &= 1.5 - x \\
D_2 &= 1.5 + x \\
D_2 - D_1 &= (1.5 + x) - (1.5 - x) \\
12cm &= 2x \\
\therefore x &= 6cm
\end{aligned}$$

9

$$P \propto Q^2 \qquad\qquad Q \propto \frac{1}{R}$$
$$P = kQ^2 \qquad\qquad Q = \frac{k}{R}$$
$$2 = 16k \qquad\qquad 2 = \frac{k}{5}$$
$$k = \frac{1}{8} \qquad\qquad k = 10$$
$$P = \frac{1}{8}Q^2 \qquad\qquad Q = \frac{10}{R}$$

$$\begin{aligned}
\therefore P &= \frac{1}{8} \cdot (\frac{10}{R})^2 \\
&= \frac{100}{8R^2} \\
&= \frac{25}{2R^2}
\end{aligned}$$

10

a) False as $w + y + z = 240$ due to the conservation of nucleon number

b) **True** as it is a rearrangement of the above equation

c) False as neutrons do not contribute to proton number, therefore $x = 40$ due to the conservation of proton number

d) False as $94 = 54 + x + 0$

e) False as it combines both mass and proton number

f) False as it combines both mass and proton number

11

$$\begin{aligned}
&2 - \frac{x^2(9x^2-4)}{x^3(2-3x)} \\
&= 2 - \frac{x^2(3x+2)(3x-2)}{-x^3(3x-2)} \\
&= 2 + \frac{(3x+2)}{x} \\
&= 2 + \frac{2}{x} + 3 \\
&= 5 + \frac{2}{x}
\end{aligned}$$

12

$$\begin{aligned}
\text{Gain in GPE} &= 20 \cdot g \cdot 6 = 1200J \\
0.8 &= \frac{1200J}{\text{total input from motor}} \\
\text{Total energy input by motor} &= 1500J \\
\text{Power of motor} &= IV = \frac{E}{t} \\
I \cdot 12 \cdot 5 &= 1500J \\
I &= 25A
\end{aligned}$$

13

$$\begin{aligned}
2^{3+2x} \cdot 4^x \cdot 8^{-x} &= 4\sqrt{2} \\
2^{3+2x} \cdot 2^{2x} \cdot 2^{-3x} &= 2^2 \cdot 2^{0.5} \\
3 + 2x + 2x - 3x &= 2 + 0.5 \\
x &= -0.5
\end{aligned}$$

14

A is not possible because alpha decay results in P-4, Q-2. To get an atomic number of Q-1 you would need one beta decay. But as the atomic mass number does not change in beta decay, its not possible for atomic mass to be P.

15

	Boys	Girls	Total
German	2Y	*2X+Y-35*	?
French		X	
Spanish	Y	*35 -Y*	35
Total		3X	100

$$\begin{aligned} \text{Number of girls studying Spanish} &= 35 - Y \\ \text{Number of girls studying German} &= 3X - X - (35 - Y) \\ &= 2X + Y - 35 \\ \text{Total number of German students} &= (2X + Y - 35) + 2Y \\ &= 2X + 3Y - 35 \end{aligned}$$

16

Atomic Radius (Ra) $= 3 \times 10^4$ Nuclear Radius (Rn)

$$\begin{aligned} \frac{Ra}{Rn} &= 3 \times 10^4 \\ \rho &= \frac{m}{v} \\ \rho &= \frac{m}{\frac{4}{3}\pi r^3} \\ \therefore \text{For a constant mass, } \rho &\propto \frac{1}{r^3} \end{aligned}$$

$$\begin{aligned} \frac{\rho_a}{\rho_n} &= \left(\frac{Rn}{Ra}\right)^3 \\ &= (3 \times 10^4)^{-3} \end{aligned}$$

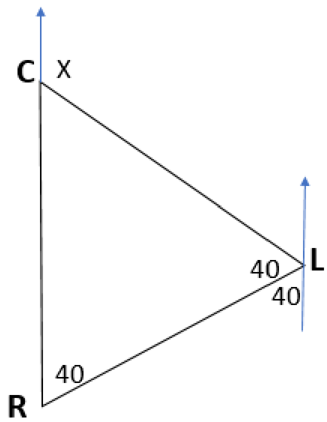
17

$$\begin{aligned} \frac{360}{n} - 4 &= \frac{360}{n+3} \\ (360 - 4n)(n+3) &= 360n \\ -4n^2 - 12n + 1080 + 360n &= 360n \\ 4n^2 + 12n - 1080 &= 0 \\ n^2 + 3n - 270 &= 0 \\ (n+18)(n-15) &= 0 \\ n &= 15 \end{aligned}$$

18

$$v = f\lambda$$
$$\text{Time period} = \frac{2(t_2 - t_1)}{3}$$
$$\text{Frequency} = \frac{3}{2(t_2 - t_1)}$$
$$\lambda = 2(x_2 - x_1)$$
$$v = 2(x_2 - x_1) \cdot \frac{3}{2(t_2 - t_1)}$$
$$= \frac{3(x_2 - x_1)}{t_2 - t_1}$$

19



From diagram, $\angle CRL = 40^o$ due to alternate angles rules
Triangle CRL is isosceles, therefore $\angle CLR = 40^o$
$\therefore x = 080^o$

20

$$P = IV$$
$$I = \frac{150}{12}$$

$$Q = It$$
$$= \frac{150}{12} \cdot 20 \cdot 60$$
$$= 15000C$$

21

Take 4 as a reference,
Angle between 4 and 8 $= 30 \times 4 = 120$
Angle between 4:00 and 4:40 $= 30 \times \frac{2}{3} = 20$
Angle between the hour and minute hand $= 100^o$

22

Mass of freight train $(M_f) = 7 \times 30 + 3 \times 130 = 600$
Mass of passenger train (M_p)= $2 \times 70 + x \times 10 = 140 + 10x$

By the Principle of Conservation of Linear Momentum, we know that the two trains have momentum equal in magnitude:

$$2M_f - 5M_p = 0$$
$$\frac{2}{5} \cdot 600 = 140 + 10x$$
$$x = 10$$

23

$$\frac{x}{4+x} \cdot \frac{x-1}{3+x} = \frac{1}{3}$$
$$3x(x-1) = (4+x)(3+x)$$
$$3x^2 - 3x = 12 + 7x + x^2$$
$$2x^2 - 10x - 12 = 0$$
$$x^2 - 5x - 6 = 0$$
$$(x-6)(x+1) = 0$$
$$x = 6$$

24

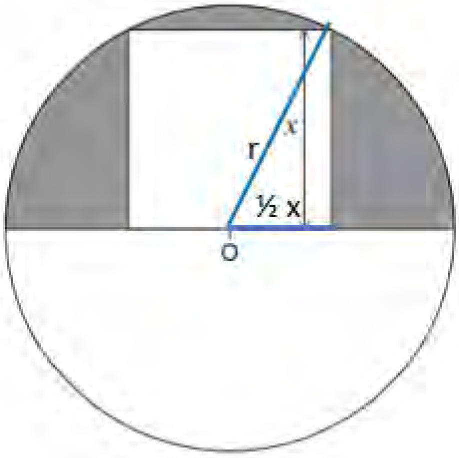
$$KE = 0.5 \times 72 \times 5^2$$
$$= 900J$$
$$\therefore 1 \text{ is } \textbf{false}$$

Each second, loss of height is 5m:

$$\text{GPE lost per second} = 72 \times 10 \times 5$$
$$= 3600Js^-1$$
$$\therefore 2 \text{ is } \textbf{true}$$

The two forces are not of the same type and act on the same body, therefore 3 is **false**

25



$$\begin{aligned} radius &= \sqrt{x^2 + (\frac{1}{2}x)^2} \\ &= \sqrt{\frac{5}{4}x^2} \\ Area &= \frac{1}{2}\pi r^2 - x^2 \\ &= \frac{1}{2}\pi\left(\frac{5}{4}x^2\right) - x^2 \\ &= x^2\left(\frac{5\pi - 8}{8}\right) \end{aligned}$$

26

If there are initially N molecules of x and N molecules of Y:
6 hours later X undergoes 2 half lives, so there are $\frac{N}{4}$ molecules
6 hours later Y undergoes 3 half lives, so there are $\frac{N}{8}$ molecules
Number of Z = 2N - $\frac{3}{8}N = (\frac{13}{8})N$

$$\begin{aligned} \text{Fraction of mixture made of Z} &= (\frac{13}{8})N \div 2N \\ &= \frac{13}{16} \end{aligned}$$

27

$$\begin{aligned} \text{Volume of metal} &= (\pi \cdot 5^2 - \pi \cdot 4^2) \cdot 16 \\ &= 144\pi cm^3 \end{aligned}$$

$$\begin{aligned} \text{Mass} &= 8g/cm^3 \cdot 144\pi cm^3 \\ &= 1152\pi g \end{aligned}$$

28

$$
\begin{aligned}
KE_y &= \frac{1}{2}mv^2 \\
KE_x &= \frac{1}{2} \cdot \frac{4m}{5} \cdot (\frac{3v}{2})^2 \\
&= KE_y \cdot (\frac{4}{5} \cdot \frac{9}{4}) \\
&= 1.8 \cdot KE_y
\end{aligned}
$$

29

$$
\begin{aligned}
1 - \left(\frac{3+\sqrt{3}}{6-2\sqrt{3}}\right)^2 &= 1 - \frac{6(2+\sqrt{3})}{24(2-\sqrt{3})} \\
&= \frac{4(2-\sqrt{3})}{4(2-\sqrt{3})} - \frac{(2+\sqrt{3})}{4(2-\sqrt{3})} \\
&= \frac{6-5\sqrt{3}}{4(2-\sqrt{3})} \\
&= \frac{(6-5\sqrt{3})(2+\sqrt{3})}{4(4-3)} \\
&= -\frac{3}{4} - \sqrt{3}
\end{aligned}
$$

30

$$
\begin{aligned}
\text{Additional anti-clockwise moment} &= 400 \cdot 10 \cdot 5 \\
&= 20000Nm \\
\therefore \text{Additional clockwise moment needed to balance} &= 20000Nm \\
20000 &= 2000 \cdot 10 \cdot x \\
x &= 1m
\end{aligned}
$$

Direction is to the right to provide a greater clockwise moment

31

$$
\begin{aligned}
2sinx + 1 &= 0 \\
sinx &= -\frac{1}{2} \\
x &= 210, 330
\end{aligned}
$$

$$
\begin{aligned}
2cos2x &= 1 \\
cos2x &= \frac{1}{2} \\
2x &= 60, 300, 420 \\
x &= 30, 150, 210
\end{aligned}
$$

$$\therefore k = 210$$

30, 150, and 210 are solutions for at least one of the equations in the range $0 \leq x \leq 210$ $\therefore$ **3 values of** x

32

- Acceleration is constant (g) so the graph should comprise of straight lines (not B, C, H)
- The ball is thrown up into the air so line must be above time axis initially (not F,G)
- When the ball falls back down the velocity is negative (not E, D)
- **A** is the correct graph

33

$$\begin{aligned} 3^{2x+1} &= 2 \cdot 3^{x+1} \\ \frac{3^{2x+1}}{3^{x+1}} &= 2 \\ 3^x &= 2 \\ x &= \log_3 2x \end{aligned}$$

34

Resultant force is zero because the aircraft is climbing at a "constant speed" so acceleration is zero.

35

$$\begin{aligned} \text{Length of arc} &= r\theta \\ r \cdot \frac{11\pi}{6} &= 22\pi \\ r &= 12 \end{aligned}$$

$$\begin{aligned} \text{Area QOR (Major Sector)} &= \frac{1}{2} r^2 \theta \\ &= \frac{1}{2} \cdot (12)^2 \cdot \frac{11\pi}{6} \\ &= 132\pi \end{aligned}$$

$$\begin{aligned} \text{Area POS (Triangle)} &= \frac{1}{2} \cdot PQ \cdot SQ \cdot sin(\angle PQS) \\ &= \frac{1}{2} \cdot (12+18) \cdot (12+18) \cdot sin\frac{\pi}{6} \\ &= 225 \end{aligned}$$

$$\therefore \text{Total Area} = 132\pi + 225$$

36

Taking moments about the pivot:

$$\begin{aligned} 60 \cdot 10 \cdot 2 &= F sin60 \cdot 4 \\ F &= \frac{300}{sin60} \end{aligned}$$

37

$$\begin{aligned}\frac{(8^p)^3}{((\frac{1}{2})^{2q})^2} &= \frac{8^{3p}}{(\frac{1}{2})^{4q}}\\ &= \frac{(2^3)^{3p}}{(2^{-1})^{4q}}\\ &= \frac{2^{9p}}{2^{-4q}}\\ &= 2^{9p+4q}\end{aligned}$$

$$\begin{aligned}\log_2(2^{9p+4q}) &= (9p+4q)(\log_2 2)\\ &= 9p+4q\end{aligned}$$

38

$$u = 40ms^{-1} \qquad a = -14.4ms^{-2}$$
$$s = 20m \qquad v = ?$$

$$\begin{aligned}v^2 &= u^2 + 2as\\ &= 40^2 + 2\cdot(-14.4)\cdot 20\\ &= 1024\\ \therefore v &= \sqrt{1024} = 32\end{aligned}$$

39

$$\text{Let } f(x) = x^3 + px^2 + qx + 6$$

$$\begin{aligned}f'(x) &= 3x^2 + 2px + q\\ f'(2) &= 12 + 4p + q = 0(1)\\ f'(4) &= 48 + 8p + q = 0(2)\end{aligned}$$

$$\begin{aligned}(2)-(1): 36 + 4p &= 0\\ 4p &= -36\\ p &= -9\\ q &= 24\end{aligned}$$

40

Resolving forces on the suspended load, we see that the tension in the string is $10N$ because the load is in equilibrium and therefore experiences no resultant force. The pulley is frictionless and the string inextensible, therefore the tension throughout the string is $10N$. The force meter reads $10N$.

41

$$\begin{aligned} Area &= \frac{1}{2} \cdot (8-3x) \cdot (4x) \cdot sin60 \\ &= \frac{\sqrt{3}}{4} \cdot 4x \cdot (8-3x) \\ &= \sqrt{3}(8x-3x^2) \end{aligned}$$

Completing the Square:

$$-3\sqrt{3}\left(x-\frac{4}{3}\right)^2 + \frac{16\sqrt{3}}{3}$$

$$\therefore \text{ maximum area } = \frac{16\sqrt{3}}{3}$$

42

Using Work-Energy Principle:

$$\begin{aligned} ME_i &= 4J \\ ME_f &= 3.2J \\ \text{Work done against resistive forces} &= 4-3.2 \\ &= 0.8J \end{aligned}$$

43

Using binomial expansions:

$$\begin{aligned} \text{Coefficient of } x^4 &= \binom{6}{4} \cdot 2^2 \cdot (3x)^4 \\ &= 4860 \\ \text{Once differentiated:} & \\ \text{Coefficient of } x^3 &= 4860 \times 4 \\ &= 19440 \end{aligned}$$

44

$$u = 13ms^{-1} \text{ due to the conservation of energy}$$

$$a = 10ms^{-2}$$

$$s = 6m$$

$$t = ?$$

$$\begin{aligned} s &= ut + \frac{1}{2}at^2 \\ 6 &= 13t + \frac{1}{2}(10)t^2 \\ 0 &= 5t^2 + 13t - 6 \\ &= (5t-2)(t+3) \\ \therefore t &= 0.4s \end{aligned}$$

45

$$\begin{aligned}
\frac{4}{3} &= \frac{1}{1-r} \\
4(1-r) &= 3 \\
4 - 2sin2x &= 3 \\
1 &= 2sin2x \\
sin2x &= \frac{1}{2} \\
2x &= \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6} \\
x &= \frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12} \\
\therefore \text{In the given range } x &= \frac{13\pi}{12}, \frac{17\pi}{12}
\end{aligned}$$

46

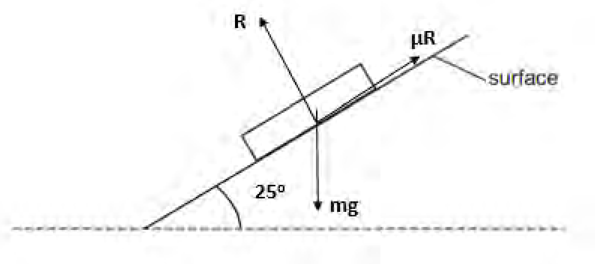
$$\begin{aligned}
\text{Work Done} &= \text{area under F-d graph} \\
&= \frac{1}{2} \cdot 0.4 \cdot 192 \\
&= 38.4J \\
\therefore \text{Kinetic Energy} &= 38.4J \\
\text{GPE gained} &= \text{ KE loss} \\
38.4 &= 0.024 \cdot 10 \cdot h \\
\therefore h &= 160m
\end{aligned}$$

47

$$\begin{aligned}
U_1 &= 2 \\
U_2 &= 2P + 3 \\
U_3 &= 2P^2 + 3P + 3 \\
U_4 &= 2P^3 + 3P^2 + 3P + 3 = -7
\end{aligned}$$

$$\begin{aligned}
\text{let f(p)} &= 2P^3 + 3P^2 + 3P + 10 \\
f(-2) &= 0 \therefore \text{(P+2) is a factor} \\
\therefore P &= -2 \\
U_2 &= -1 \\
U_3 &= 5 \\
\therefore Total &= -1
\end{aligned}$$

48



$$\textit{When angle of tilt is } 20^o:$$
$$\text{Resolving perpendicular to the slope: } R = mgcos20$$
$$\text{Resolving parallel to the slope: } mgsin20 - \mu(mgcos20) = 0$$
$$sin20 = \mu cos20$$
$$\mu = tan20$$

$$\textit{When angle of tilt is } 25^o:$$
$$\text{Resolving perpendicular to the slope: } R = mgcos25$$
$$\text{Resolving parallel to the slope: } mgsin25 - \mu(mgcos25) = ma$$
$$g(sin25 - \mu cos25) = a$$
$$a = g(sin25 - tan20 \cdot cos25)$$

49

From the answers we can surmise that the root to the equation on the numerator is x = 1 and x = 4. Then sketching $f(x) = (x-1)^2(x-4)$ and $g(x) = \frac{1}{x}$

- for x>4, f(x) and g(x) both greater than 0, so the function > 0
- for 0≤x≤4, f(x)<0 and g(x)>0, so the function is < 0
- for x<0, f(x) and g(x) both less than 0, so the function > 0

50

- Resolving the component of weight (mg) parallel to the slope gives $mgsin\theta$
- To travel at a constant speed the resultant force on the suitcase must be zero
- $\therefore$ Frictional force acting up the slope (opposite to the parallel component of weight) is also $mgsin\theta$

51

$$f(x) = sinx$$
$$\text{Stretch: } f(2x) = sin(2x)$$
$$\text{Translation: } f\left(2\left(x+\frac{\pi}{4}\right)\right) = sin\left(2x+\frac{\pi}{2}\right)$$

52

$$\begin{aligned}\text{Change in Momentum} &= \text{Area underneath } F-t \text{ Graph}\\ &= F(t_2-t_1)\\ \text{Momentum before} &= m(-u)\\ \text{Momentum after} &= mv\\ F(t_2-t_1) &= mv-(-mu)\\ \therefore mv &= F(t_2-t_1)-mu\end{aligned}$$

53

Perpendicular implies the product of the gradients is -1

$$(2p^2-p)(p-2) = -1$$
$$2p^3-p^2-4p^2+2p+1 = 0$$
$$\therefore f(p) = 2p^3-5p^2+2p+1$$

$$f(1) = 0 \therefore (p-1) \text{ is a factor by the Factor Theorem}$$
$$\therefore f(p) = (p-1)(2p^2-3p-1)$$

$$\begin{aligned}\text{Using the quadratic formula, the remaining two roots } &= \frac{3\pm\sqrt{9+8}}{4}\\ &= \frac{3}{4}\pm\frac{\sqrt{17}}{4}\end{aligned}$$
$$\text{We know that } \sqrt{16} = 4 \text{ so } \frac{\sqrt{17}}{4} \approx 1$$
$$\text{The greatest root is therefore roughly } 1.75$$

54

For the situation in which the ball bounces off the ground at t=0.5, and reaches maximum height.

$$v = 0 \qquad a = -10 \qquad t = \frac{1.3-0.5}{2} = 0.4$$

$$\begin{aligned}s &= vt-\frac{1}{2}at^2\\ &= 0-\left(\frac{1}{2}\right)(-10)(0.4)^2\\ &= 0.8\end{aligned}$$

$$\begin{aligned}v &= u+at\\ 0 &= u+(-10)(0.4)\\ u &= 4\end{aligned}$$