ENGAA 2019
Section 1

Model Solutions

PART A Mathematics and Physics

1 Evaluate

$$\left(\sqrt{7}+\sqrt{3}\right)^2 - \left(\sqrt{7}-\sqrt{3}\right)^2$$

A 0

B $2\sqrt{7}$

C $4\sqrt{7}$

D $2\sqrt{21}$

E 10

F $4\sqrt{21}$

G 20

$(a+b)^2 = a^2 + 2ab + b^2$

$\Rightarrow \left(\sqrt{7}\right)^2 + 2\sqrt{7}\sqrt{3} + \left(\sqrt{3}\right)^2 - \left(\left(\sqrt{7}\right)^2 - 2\sqrt{7}\sqrt{3} + \left(\sqrt{3}\right)^2\right)$

$\Rightarrow 7 + 2\sqrt{21} + 3 - 7 + 2\sqrt{21} - 3$

$\Rightarrow 4\sqrt{21}$

2 The current–voltage graph for a diode is shown.
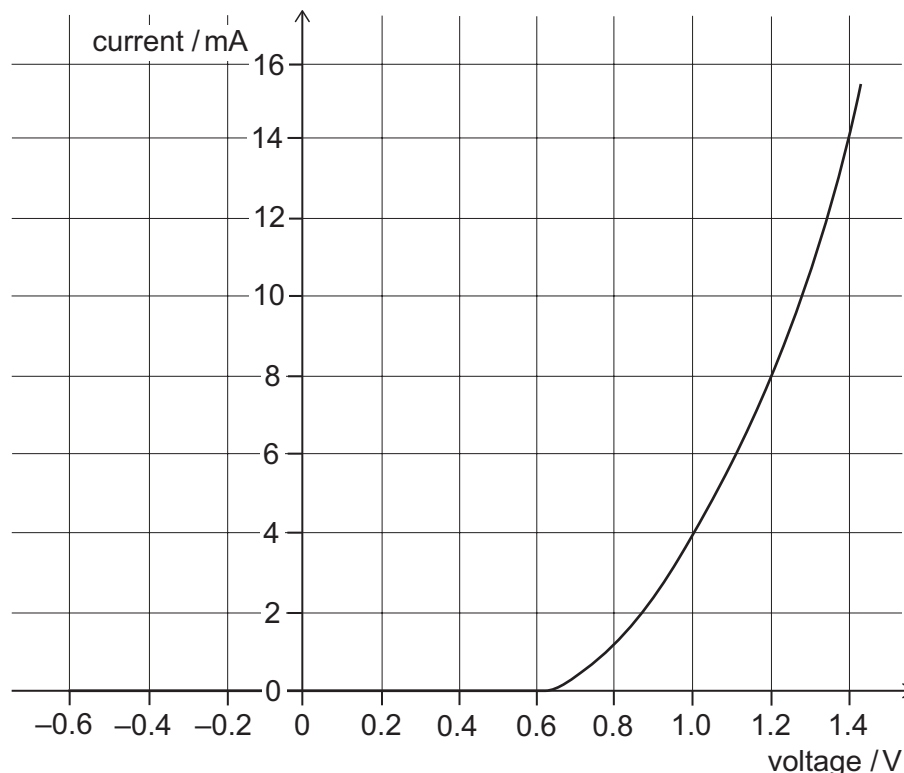
current /mA

The diode is connected in series with a resistor and a 6.0 V battery. The current in the circuit is 8.0 mA.

What is the resistance of the resistor?

(Assume that the battery has negligible resistance.)

A 0.15 Ω

B 0.60 Ω

C 0.75 Ω

D 4.8 Ω

E 150 Ω

F 600 Ω

G 750 Ω

$$V = IR$$
$$\Rightarrow R = \frac{V}{I}$$

At 8.0 mA, p.d. across diode = 1.2V
(from graph).

∴ Pd across Res. = 6 - 1.2 = 4.8V

∴ $R = \frac{V}{I} = \frac{4.8}{0.008} = \frac{4800}{8} = 600\,\Omega$

3 The equation gives y in terms of x:

$$y = 3 - 4\left(1 - \frac{x}{2}\right)^2$$

Which one of the following is a rearrangement for x in terms of y?

A $x = -2 \pm 2\sqrt{\dfrac{3-y}{4}}$

B $x = -2 \pm 2\sqrt{\dfrac{4-y}{3}}$

C $x = 1 \pm \sqrt{\dfrac{3-y}{4}}$

D $x = 1 \pm 2\sqrt{\dfrac{3-y}{4}}$

E $x = 2 \pm 2\sqrt{\dfrac{3-y}{4}}$

F $x = 2 \pm 2\sqrt{\dfrac{4-y}{3}}$

G $x = 2 \pm 2\sqrt{\dfrac{3+y}{4}}$

$$y = 3 - 4\left(1 - \frac{x}{2}\right)^2$$

$$y - 3 = -4\left(1 - \frac{x}{2}\right)^2$$

$$\frac{3-y}{4} = \left(1 - \frac{x}{2}\right)^2$$

$$\pm\sqrt{\frac{3-y}{4}} = 1 - \frac{x}{2}$$

$$\frac{x}{2} = 1 \pm \sqrt{\frac{3-y}{4}}$$

$$x = 2 \pm 2\sqrt{\frac{3-y}{4}}$$

4 Two electromagnetic waves P and Q travel in a vacuum and the ratio of their wavelengths is:

$$\frac{\text{wavelength of P}}{\text{wavelength of Q}} = 1.0 \times 10^8$$

Which row in the table shows the ratio of their speeds, the ratio of their frequencies, and identifies the possible natures of P and Q?

	$\dfrac{\text{speed of P}}{\text{speed of Q}}$	$\dfrac{\text{frequency of P}}{\text{frequency of Q}}$	nature of P	nature of Q
A	1.0	1.0×10^{-8}	microwave	X-ray
B	1.0	1.0×10^{-8}	microwave	radio wave
C	1.0	1.0×10^{8}	infrared	ultraviolet
D	1.0	1.0×10^{8}	visible light	infrared
E	1.0×10^{8}	1.0	gamma	X-ray
F	1.0×10^{8}	1.0	gamma	infrared
G	1.0×10^{8}	1.0×10^{16}	infrared	radio wave
H	1.0×10^{8}	1.0×10^{16}	visible light	ultraviolet

Ratio of Speed is 1 because they are E.M. waves.
$v = \lambda f$, *∴ ratio of f. must be 1×10⁻⁸. ∴ Must be A as ratio of λ is so big.*

5 The resistance to the motion of a car is directly proportional to the square of the speed of the car.

$$R \propto v^2 \implies R = Kv^2$$

The car increases its speed by 20%.

What is the percentage increase in the resistance to the motion of the car?

A 20%

B 24%

C 44%

D 120%

E 224%

F 240%

G 400%

$$R' = K(1.2v)^2$$
$$R' = K(1.44v^2)$$

∴ R must increase by 44% to keep K constant.

6   A water-tight cylinder with a thin, freely moving piston contains $2.0 \times 10^{-3}\,m^3$ of trapped air at atmospheric pressure of $1.0 \times 10^5\,Pa$.

When the cylinder is submerged in water of constant density $1000\,kg\,m^{-3}$, the volume of air in the cylinder decreases to $4.0 \times 10^{-4}\,m^3$.

The piston is at a depth h below the surface of the water and the water surface is open to the atmosphere.

What is the depth h?

(gravitational field strength = $10\,N\,kg^{-1}$; assume that the temperature of the air remains constant and that air is an ideal gas)

A   40 m

B   50 m

C   60 m

D   400 m

E   500 m

F   600 m

$$P_1 V_1 = P_2 V_2$$

$$\Rightarrow \quad 1 \times 10^5 \times 2 \times 10^{-3} = P_2 \times 4 \times 10^{-4}$$

$$\Rightarrow \quad \overset{2}{\cancel{2}} \times 10^2 = P_2 \times \overset{2}{\cancel{4}} \times 10^{-4}$$

$$\Rightarrow \quad 0.5 \times 10^6 = 5 \times 10^5 = P_2$$

$$P_2 = P_1 + h\rho g \Rightarrow 1 \times 10^5 + h(1000)(10) = 5 \times 10^5$$

$$\Rightarrow 4 \times 10^5 = h(10000) \Rightarrow h = \underline{\underline{40\,m}}$$

7   The equation of a curve is $y = px^2 + qx$ where p and q are constants.

The curve passes through the points $(2, 6)$ and $(4, -4)$.

What is the value of $q - p$?

A   1

B   2

C   5

D   6

E   9

F   16

$$6 = 4p + 2q \Rightarrow 3 = 2p + q \quad — (i)$$

$$-4 = 16p + 4q \Rightarrow -1 = 4p + q \quad — (ii)$$

$$3 = 2p + q$$
$$-1 = 4p + q$$
$$\frac{(+) \quad (-) \quad (-)}{4 = -2p}$$

$$p = -2 \Rightarrow 3 = 2(-2) + q$$

$$\Rightarrow q = 7$$

$$\Rightarrow q - p = \underline{\underline{9}}$$

8   The secondary coil of an ideal, 100% efficient transformer is connected to a resistor by cables of total resistance $1500\,\Omega$. The current in the primary coil is $4.0\,A$. There are 240 turns in the primary coil and 4800 turns in the secondary coil.

What is the power produced as heat in the cables?

A 60 W

B 300 W

C 6000 W

D 24 000 W

E 120 000 W

F 9 600 000 W

Handwritten working:

$Pd \uparrow \quad \therefore I \downarrow$

$\therefore I_s = \dfrac{240}{4800} \times 4 = \dfrac{1}{5} = 0.2\,A$

$\left(\dfrac{240}{4800} = \dfrac{20}{5}\right)$

$P_{dissipated} = I^2 R = (0.2)^2 \times 1500$

$= \dfrac{4}{100} \times 1500 = 60\,W$

9   Which of the following is a simplification of

$$4 - \frac{x(3x+1)}{x^2(3x^2 - 2x - 1)}$$

A $\dfrac{12x^3 - 8x^2 - 7x - 1}{x(3x-1)(x-1)}$

B $\dfrac{4x^2 + 4x - 1}{x(x+1)}$

C $\dfrac{4x^2 + 4x + 1}{x(x+1)}$

D $\dfrac{4x^2 - 4x - 1}{x(x-1)}$

E $\dfrac{4x^2 - 4x + 1}{x(x-1)}$

F $\dfrac{12x^3 - 8x^2 - x + 1}{x(3x-1)(x-1)}$

Handwritten working:

$3x^2 - 2x - 1$

$\Rightarrow 3x^2 - 3x + x - 1$

$\Rightarrow 3x(x-1) + 1(x-1)$

$\Rightarrow (3x+1)(x-1)$

$\Rightarrow 4 - \dfrac{x(3x+1)}{x^2(3x+1)(x-1)}$

$\Rightarrow 4 - \dfrac{x}{x^2(x-1)}$

$\Rightarrow \dfrac{4(x^2)(x+1) - x}{x^2(x-1)}$

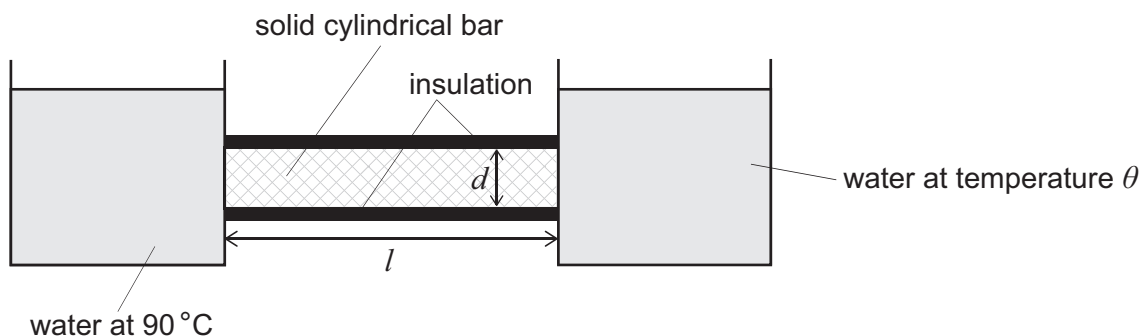$\Rightarrow \dfrac{4x^3 + 4x^2 - x}{x^2(x-1)}$

$\Rightarrow \dfrac{4x^2 + 4x - 1}{x(x-1)}$

10 Two tanks of water are connected by a solid cylindrical copper bar of length l and diameter d.

The bar is insulated.

One tank contains water at 90 °C and the other tank contains water at temperature θ.

For which of the following conditions is thermal energy conducted along the bar at the lowest rate?

	l / m	d / cm	θ / °C
A	0.40	4.0	20
B	0.40	4.0	40
C	0.40	8.0	20
D	0.40	8.0	40
E	0.80	4.0	20
F	0.80	4.0	40
G	0.80	8.0	20
H	0.80	8.0	40

Temp difference must be lowest, i.e. θ should be largest – 40°C.

l should be the greatest – 0.8m.

d should be the smallest – 4 cm.

11 The ball for a garden game is a solid sphere of volume $192\,cm^3$.

For the children's version of the game the ball is a solid sphere made of the same material, but the radius is reduced by 25%.

What is the volume, in cm^3, of the children's ball?

A   48

B   81

C   96

D   108

E   144

$$V = \frac{4}{3}\pi r^3$$

$$V_2 = \frac{4}{3}\pi \left(\frac{3}{4}r\right)^3$$

$$\Rightarrow V \times \frac{3^3}{4^3} = V_2 = 192 \times \frac{27}{64} = \underline{81\,cm^3}$$

12 The radioactive isotope X becomes the stable isotope Y after a succession of decays involving only the emission of alpha and beta (β⁻) particles.

During the decay of one nucleus from X to Y, a total of seven particles are emitted. It is known that more of these particles are alpha particles than beta particles.

The atomic number of X is Z and the mass number of X is A.

Which row in the table could give the atomic number and the mass number of Y?

	atomic number of Y	mass number of Y
A	$Z - 2$	$A - 12$
B	$Z - 5$	$A - 8$
C	$Z - 8$	$A - 20$
D	$Z - 10$	$A - 24$
E	$Z - 11$	$A - 16$

$$_{Z}^{A}X \rightarrow n\,_{2}^{4}\alpha + (7-n)\,_{-1}^{0}\beta + Y$$
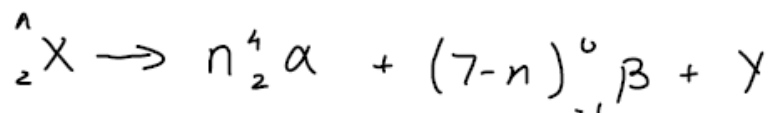
$n \geq 4$ (given) $\therefore$ $A \rightarrow A - 4n$ $\therefore$ must be one of last 3 options.

Checking C, D, E: C: A-20 means n is 5

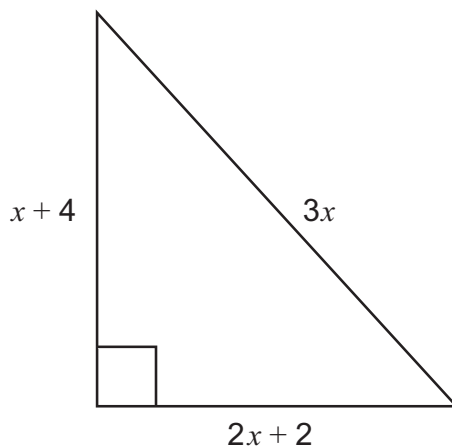so $Z - (5\times2) - 2(-1) = \underline{Z - 8}$

↳ As in C

$\therefore$ C is correct.

13 The diagram shows a right-angled triangle, with sides of length $x + 4$, $2x + 2$ and $3x$, all in cm.

[diagram not to scale]

What is the area, in cm^2, of the triangle?

A 10

B 12

C 28

D 36

E 40

(F) 64

G 70

By pythagoras:

$(x+4)^2 + (2x+2)^2 = (3x)^2$

$x^2 + 16 + 8x + 4x^2 + 4 + 8x = 9x^2$

$\Rightarrow 4x^2 - 16x - 20 = 0$

$\Rightarrow x^2 - 4x - 5 = 0$

$\Rightarrow (x-5)(x+1) = 0$

$\Rightarrow x = 5$ (Sides are '+'ve)

Base = 12
Height = 9

Area $= \frac{1}{2} \times 9 \times 12$

$= 54$ cm^2

14 The kinetic energy of an object of mass 4.0 kg, travelling in a straight line, increases from 32 J to 200 J in 3.0 seconds due to a constant resultant force.

What is the value of this resultant force?

A 2.0 N

B 4.0 N

(C) 8.0 N

D 24 N

E 28 N

F 56 N

$\frac{1}{2}mu^2 = 32 \Rightarrow 4u^2 = 64 \Rightarrow u = 4$

Final KE $= \frac{1}{2}mv^2 = 200 \Rightarrow 4v^2 = 400 \Rightarrow v = 10$

$m(v - u) = F \Delta t$

$4(10 - 4) = F \times 3$

$24 = 3F$

$F = 8N$

15 *PR* and *QS* are the diagonals of a rhombus *PQRS*.

PR = (3*x* + 2) cm

QS = (8 − 2*x*) cm

The area of *PQRS* is 11 cm².

What is the difference, in cm, between the two possible lengths of *PR*?

A $2\frac{2}{3}$

B $4\frac{1}{2}$

C $5\frac{1}{3}$

D 8

E 14

Area = $\frac{1}{2}$ × PR × QS = 11

⟹ $\frac{1}{2}$ × (3x+2)(8−2x) = 11

⟹ 24x − 6x² + 16 − 4x = 22

⟹ −6x² + 20x − 6 = 0

⟹ 6x² − 20x + 6 = 0

⟹ 3x² − 10x + 3 = 0

⟹ 3x² − 9x − x + 3 = 0

⟹ 3x(x−3) − 1(x−3) = 0

⟹ (3x − 1)(x − 3) = 0

⟹ x = $\frac{1}{3}$ or x = 3

PR = 3x + 2

= 3×$\frac{1}{3}$ + 2   OR   3×3 + 2
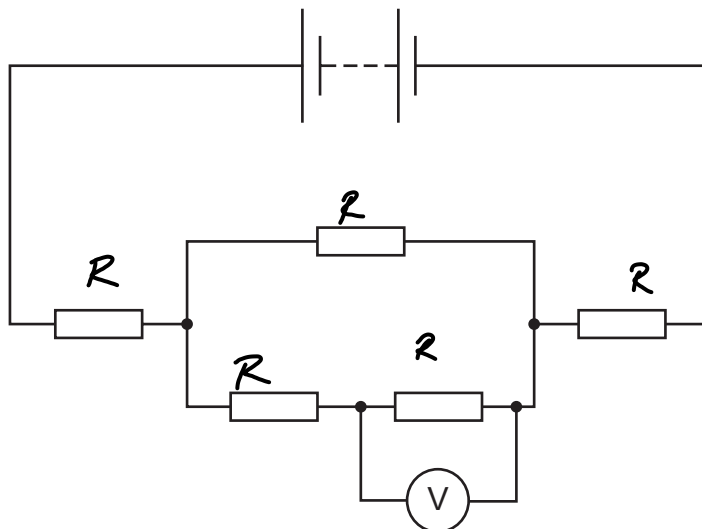
= 3   = 11

Diff. = 11 − 3 = $\underline{\underline{8}}$

16   In the following circuit, all five resistors have the same resistance.

The reading on the voltmeter is 1.0 V.

What is the voltage across the battery?

A   4.0 V

B   5.0 V

C   6.0 V

D   7.0 V

Ⓔ   8.0 V

F   9.0 V

G   10 V

Res. of parallel part:

$$\left(\frac{1}{R} + \frac{1}{2R}\right)^{-1}$$

$$= \left(\frac{2+1}{2R}\right)^{-1} = \frac{2R}{3} \ \Omega$$

∴ By potential divider formula,
pd across parallel part =

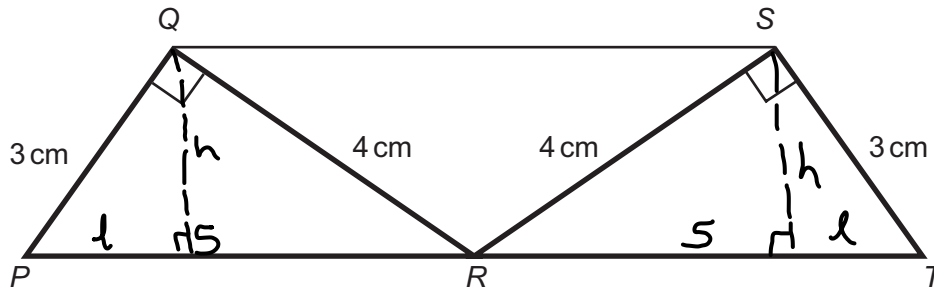$$\frac{2}{3}R \div \left(2R + \frac{2}{3}R\right) \times V$$

$$\frac{2R}{3} \times \frac{3}{48R} \times V = \frac{1}{4} \times V$$

∴ Reading of voltmeter = ½ × ¼ × V

⟹ V = 8V

17

[diagram not to scale]

The diagram shows two congruent right-angled triangles *PQR* and *TSR* with right angles at *Q* and *S*, respectively.

PQ = *TS* = 3 cm

QR = *SR* = 4 cm

PRT is a straight line.

What is the length, in cm, of *QS*?

A 4

B $3\sqrt{2}$

C 5.2

D $4\sqrt{2}$

E 6.4

F 8.2

G 10

$RP = RT = 5cm \quad (3-4-5 \ \triangle s)$

By inverse pythagoras:

$$\frac{1}{h^2} = \frac{1}{3^2} + \frac{1}{4^2}$$

$$\frac{1}{h^2} = \frac{16+9}{9 \times 16}$$
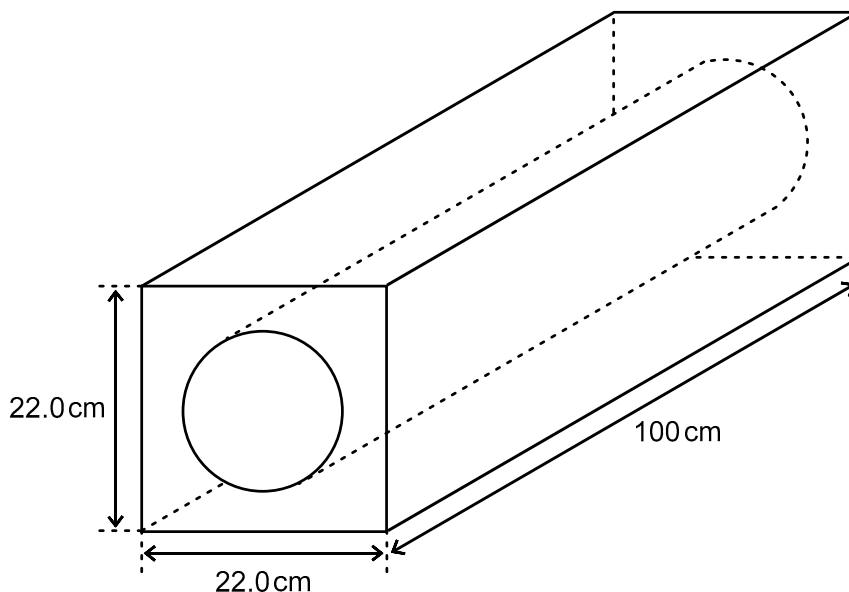
$$h^2 = \frac{9 \times 16}{25}$$

By pythagoras: $l^2 = 3^2 - h^2$

$$= 9 - \frac{9 \times 16}{25}$$

$$= \frac{(25-16) \ 9}{25}$$

$$l = \frac{9}{5}$$

$\therefore \quad QS = 10 - 2l = 10 - \frac{18}{5} = \frac{32}{5}$

18   A block is designed with a cylindrical channel to accommodate a hot-water pipe. The block is 100 cm long and it has a square cross-section of side 22.0 cm with a cylindrical hole in the middle, as shown in the diagram:

22.0 cm

22.0 cm

100 cm

[diagram not to scale]

The diameter of the cylindrical hole is 14.0 cm and the density of the material from which the block is made is 0.100 g cm^{-3}.
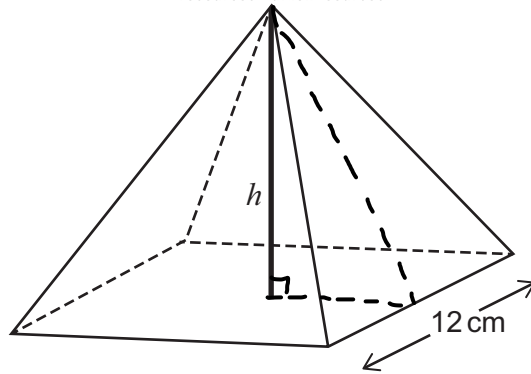
What is the mass of the block?

(take π to be $\frac{22}{7}$)

A   1.32 kg

B   3.30 kg

C   13.2 kg

D   33.0 kg

E   132 kg

F   330 kg

G   1320 kg

H   3300 kg

Vol of block = Vol of material − Vol of cylinder

$= (22 \times 22 \times 100) - \frac{22}{7} \times 7 \times 7 \times 100$

$= 22 \times (2200 - 700)$

$= 22 \times (1500) \, cm^3$

$\rho = \frac{m}{v} \implies \rho v = m$

$\implies 0.1 \times 22 \times 1500 = m$

$\implies 3300 \, g = 3.3 \, kg$

$\begin{array}{r} 22 \\ \times \, 15 \\ \hline 110 \\ 22 \times \\ \hline 330 \end{array}$

19

[diagram not to scale]

A solid pyramid has a square base of side length 12 cm and a vertical height of h cm.

The volume of the pyramid, in cm³, is equal to the total surface area of the pyramid, in cm².

What is the value of h?

(volume of pyramid = $\frac{1}{3}$ × area of base × vertical height)

A $\dfrac{72}{35}$

B $2\sqrt{3}$

C 6

D $\dfrac{144}{23}$

E 8

F $2\sqrt{21}$

$\dfrac{\cancel{144}\;48}{\cancel{3}}$

$\dfrac{\cancel{144}\;6}{24}$

$SA = 12^2 + 4 \times \text{Area of triangle}$

$= 12^2 + 4 \times \frac{1}{2} \times 12 \times \left(\sqrt{6^2 + h^2}\right)$

$= 144 + 24\sqrt{6^2 + h^2}$

$SA = Vol.$

$\Rightarrow 144 + 24\sqrt{6^2 + h^2} = \frac{1}{3} \times 12^2 \times h$

$\Rightarrow 144 + 24\sqrt{6^2 + h^2} = 48h$

$\Rightarrow 6 + \sqrt{6^2 + h^2} = 2h$

$\Rightarrow 6^2 + h^2 = (2h - 6)^2$

$\Rightarrow 6^2 + h^2 = 4h^2 + 36 - 24h$

$\Rightarrow 3h^2 - 24h = 0$

$3h(h - 8) = 0$

$h = 0 \quad \text{or} \quad \underline{\underline{h = 8}}$

20 A sample initially contains equal numbers of atoms of a radioactive isotope X and a stable isotope Y.

Isotope X has a half-life of 3 years and decays in a single stage to the stable isotope Y.

What is the ratio

$$number\ of\ atoms\ of\ X : number\ of\ atoms\ of\ Y$$

in the sample 6 years later?

A The sample contains only isotope Y.

B 1 : 7

C 1 : 4

D 1 : 3

E 7 : 4

6 yrs ⟹ 2 half lives

∴ X ⟹ ¹/₄ X

⟹ Y ⟹ ³/₄ Y + Y = $\frac{7}{4}$ Y

∴ $\frac{x}{y}$ ⟹ $\frac{1}{4} \div \frac{7}{4}$ = 1 : 7
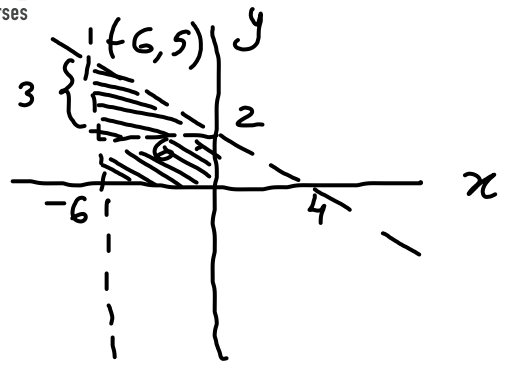
PART B Advanced Mathematics and Advanced Physics

21 Find the area of the shape bounded by the four lines:

$$2y + x = 4$$

$$x = -6$$

$$x = 0$$

$$y = 0$$

A 4

B 12

C 21

D 25

E 27

F 30

Handwritten working:

(6,5)

$2y + x = 4$
$2y = -x + 4$
$y = -\frac{1}{2}x + 2$
↳ $(0,2)$
$(4,0)$

$x = -6$ $y = -\frac{1}{2}(-6) + 2 = 5$

⟹ Area $= \frac{1}{2} \times 6 \times 3 + 2 \times 6$

$= 9 + 12$

$= 21$

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22 A hydroelectric power station uses the water in a reservoir to power the generators. The water falls through a vertical height of 150 m to the turbines which power the generators.

The efficiency of the power station is 90% and the output power of the power station is 1800 MW.

The gravitational field strength is $10\,\text{N}\,\text{kg}^{-1}$ and the density of water is $1000\,\text{kg}\,\text{m}^{-3}$.

What volume of water passes through the turbines in one minute?

A $6.48 \times 10^4\,\text{m}^3$

B $7.20 \times 10^4\,\text{m}^3$

C $8.00 \times 10^4\,\text{m}^3$

D $6.48 \times 10^7\,\text{m}^3$

E $7.20 \times 10^7\,\text{m}^3$

F $8.00 \times 10^7\,\text{m}^3$

Handwritten working:

Total output power in a minute
$= 1800 \times 60 = 108000\,\text{MW}$
$= 108 \times 10^9\,\text{W}$

Total Input $= \frac{108 \times 10^9}{0.9} = 120 \times 10^9\,\text{W}$

All power is from gravitational potential energy.

⟹ $mgh = 120 \times 10^9$
⟹ $m \times 10 \times 150 = 120 \times 10^9$
⟹ $m = \frac{12 \times 10^8}{15} = 0.8 \times 10^8\,\text{kg}$

Vol $= \frac{m}{\rho} = \frac{0.8 \times 10^8}{1000} = 8 \times 10^4\,\text{m}^3$

23 The curve

$$y = x^3 + px^2 + qx + r$$

has a local maximum when $x = -1$ and a local minimum when $x = 3$

What is the value of p?

A -9

B -3

C -1

D 1

E 3

F 9

$$\frac{dy}{dx} = 3x^2 + 2px + q = 0 \text{ at st. pts.}$$

$$\Rightarrow 3x^2 + 2px + q = 0$$

$x = -1$

$$\Rightarrow 3 - 2p + q = 0 \Rightarrow q - 2p = -3$$

$x = 3$

$$\Rightarrow 27 + 6p + q = 0 \Rightarrow q + 6p = -27$$
$$\underline{(-) \quad (-) \qquad (+)}$$
$$-8p = 24 \Rightarrow \underline{p = -3}$$

24 A car P of mass 1000 kg is travelling north at 30 m s⁻¹ along a straight, horizontal road when it hits another car Q which is directly ahead of P and travelling in the same direction. Car Q has a mass of 500 kg and is travelling at 20 m s⁻¹.

The collision lasts for 0.20 s and immediately after the collision car Q is moving north at 30 m s⁻¹.

What is the speed of P immediately after the collision and what is the size of the average resultant force that acts on Q during the collision?

(Assume that no external forces act on the cars during the collision.)

North ↑ +

Impulse on Q = $F\Delta t = mv - mu$

$$= 500(30 - 20)$$

$$= 5000 \text{ Ns}$$

$F\Delta t = 5000$

$$F = \frac{5000}{0.2} \quad 25000N$$

∴ F on Q = $\underline{25000 N}$

PCLM:

$$1000(30) + 500(20) = 500(30) + 1000(v)$$

$$\Rightarrow 40000 = 15000 + 1000v$$

$$\Rightarrow \underline{v = 25 \text{ ms}^{-1}}$$

	speed of P / m s⁻¹	average force on Q / N
A	20	25 000
B	20	50 000
C	20	100 000
D	20	125 000
E	25	25 000
F	25	50 000
G	25	100 000
H	25	125 000

25 When simplified, $\dfrac{1}{\left(1-\sqrt{2}\right)^3}$ is written in the form $a+b\sqrt{2}$ where a and b are integers.

What is the value of b?

A   −7

B   −5

C   −1

D   1

E   5

F   7

Rationalising:

$$\dfrac{1}{\left(1-\sqrt{2}\right)^3} \times \dfrac{\left(1+\sqrt{2}\right)^3}{\left(1+\sqrt{2}\right)^3}$$

$$= \dfrac{\left(1+\sqrt{2}\right)^3}{\left(\left(1-\sqrt{2}\right)\left(1+\sqrt{2}\right)\right)^3} = \dfrac{\left(1+\sqrt{2}\right)^3}{(1-2)^3}$$

$$= -\left(1+\sqrt{2}\right)^3 = -\left(1+3\sqrt{2}+6+2\sqrt{2}\right)$$

$$= -\left(7+5\sqrt{2}\right) \Rightarrow \underline{\underline{b=-5}}$$

26 A metal wire of length 0.50 m has a uniform cross-sectional area of $4.0 \times 10^{-7}\,\text{m}^2$.

There is a current of 4.0 A in the wire.

What is the potential difference across the ends of the wire?

(resistivity of the metal = $1.6 \times 10^{-7}\,\Omega\,\text{m}$)

A   0.05 V

B   0.20 V

C   0.80 V

D   3.2 V

E   5.0 V

F   20 V

$$\rho = \dfrac{RA}{\ell}$$

$$\dfrac{\rho\ell}{A} = R$$

$$V = IR \Rightarrow V = \dfrac{\rho\ell I}{A}$$

$$= \dfrac{1.6\times10^{-7} \times 4 \times 0.5}{4 \times 10^{-7}}$$

$$= \underline{\underline{0.8\,\text{V}}}$$

27   It is given that

$$7\cos x + \tan x \sin x = 5$$

where $0° < x < 90°$

What are the possible values of $\tan x$?

A $\dfrac{1}{2}$ or $\dfrac{1}{3}$

B $\dfrac{1}{\sqrt{3}}$ or $\dfrac{1}{2\sqrt{2}}$

C $\dfrac{\sqrt{3}}{2}$ or $\dfrac{2\sqrt{2}}{3}$

D $\sqrt{3}$ or $2\sqrt{2}$

E   3 or 2

$\tan x = \dfrac{\sin x}{\cos x}$

$\Rightarrow 7\cos x + \dfrac{\sin^2 x}{\cos x} = 5$

$\Rightarrow 7\cos^2 x + \sin^2 x = 5\cos x$

$\Rightarrow 7\cos^2 x + 1 - \cos^2 x = 5\cos x$

$\Rightarrow 6\cos^2 x - 5\cos x + 1 = 0$

Let $\cos x = a$

$6a^2 - 5a + 1 = 0$

$6a^2 - 3a - 2a + 1 = 0$

$3a(2a - 1) - 1(2a - 1) = 0$

$a = \tfrac{1}{2}$ $a = \tfrac{1}{3}$
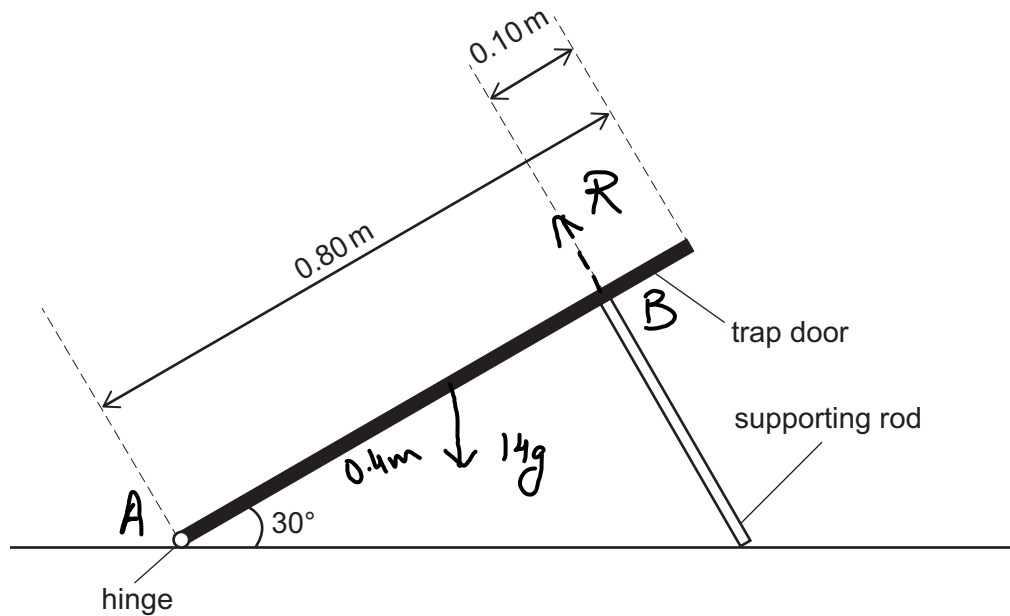
$\cos x = \tfrac{1}{2}$ $\cos x = \tfrac{1}{3}$

$\tan^2 x = \sec^2 x - 1$

$ = \dfrac{1}{\cos^2 x} - 1$

$ = 3 \quad \text{or} \quad 8$

$\Rightarrow \tan x = \sqrt{3} \text{ or } 2\sqrt{2}$

28 A uniform square trap door of side 0.80 m and mass 14 kg has a smooth hinge at one edge and is held open at an angle of 30° to the horizontal. It is supported by a single rigid rod placed so that it meets the surface of the trap door at 90° at a distance 0.10 m from the top edge of the trap door, as shown.

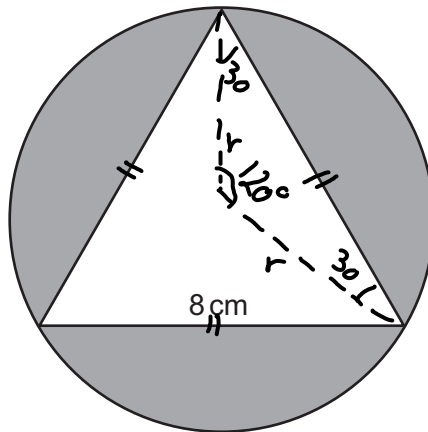What is the normal contact force exerted on the trap door by the rod?

(gravitational field strength = 10 N kg^{-1})

A   40 N

B $35\sqrt{3}$ N

C $40\sqrt{3}$ N

D   80 N

E $80\sqrt{3}$ N

F $280\dfrac{\sqrt{3}}{3}$ N

Handwritten working:

$\circlearrowleft A : (14g \cos 30)0.4 = R(0.7)$

$140 \cos 30 \times 4 = R(7)$

$\dfrac{80\sqrt{3}}{2} = R$

$40\sqrt{3} = R$

29 An equilateral triangle of side 8 cm is drawn so that its vertices lie on the circumference of a circle, as shown in the diagram.

8 cm

What is the total of the three areas shaded in the diagram, in cm^2?

A $8(2\pi - 3)$

B $24(\pi - \sqrt{3})$

C $48(4\pi - \sqrt{3})$

D $\frac{16}{3}(4\pi - 6 - 3\sqrt{3})$

E $\frac{16}{3}(4\pi - 3\sqrt{3})$

Area of equilateral $\triangle = \frac{1}{2} \times 8^2 \sin 60$

$= 32 \sin 60$

$= 16\sqrt{3}$

By Sine rule:

$\frac{r}{\sin 30} = \frac{8}{\sin 120}$

$\sin 120 = \cos 30$

$2r = \frac{16}{\sqrt{3}}$

$\Rightarrow r = \frac{8}{\sqrt{3}}$

Area of circle $= \pi r^2 = \frac{64}{3}\pi$

$\therefore$ Shaded area $= \frac{64}{3}\pi - 16\sqrt{3}$
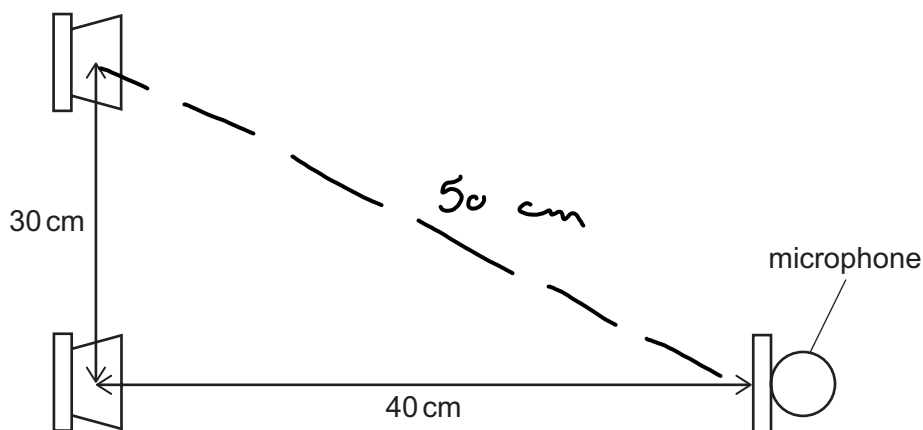
$= \frac{16}{3}(4\pi - 3\sqrt{3})$

30 Two small loudspeakers are placed side by side 30 cm apart.

They are connected to the same signal generator so that they emit sound of frequency 400 Hz in phase with one another.

The sounds both reach a microphone placed 40 cm directly in front of one of the two loudspeakers as shown.

What is the phase difference between waves from the loudspeakers as they arrive at the microphone?

(speed of sound = 320 m s^{-1})

A 30°

B 36°

C 45°

D 60°

E 72°

F 90°

G 120°

Path difference = 10 cm

$\lambda = \dfrac{320 \times 10^2}{400} = 80\,cm$

∴ phase difference = $\dfrac{10}{80} \times 2\pi = \pi/4^{\,c}$

31 Which one of the following is the real solution of the equation

$$3 \times 5^{2x+1} - 5^x - 2 = 0$$

A $\quad x = \log_5\left(\dfrac{1}{3}\right)$

B $\quad x = \log_5\left(\dfrac{2}{5}\right)$

C $\quad x = \log_5\left(\dfrac{3}{5}\right)$

D $\quad x = \log_5\left(\dfrac{2}{3}\right)$

E $\quad x = \log_5\left(\dfrac{5}{3}\right)$

F $\quad x = \log_5\left(\dfrac{5}{2}\right)$

$\Rightarrow 3 \times 5^{2x} \times 5 - 5^x - 2 = 0$

$\Rightarrow 15 \times 5^{2x} - 5^x - 2 = 0$

Let $5^x = a$

$\Rightarrow 15a^2 - a - 2 = 0$

$\Rightarrow 15a^2 + 5a - 6a - 2 = 0$

$\Rightarrow 5a(3a+1) - 2(3a+1) = 0$

$\Rightarrow (5a-2)(3a+1) = 0$

Invalid

$\Rightarrow a = \dfrac{2}{5} \quad \Rightarrow 5^x = \dfrac{2}{5} \Rightarrow x = \log_5\left(\dfrac{2}{5}\right)$

32 An astronaut on the Moon throws a ball vertically upwards. The ball has a mass of 2.0 g and is thrown upwards at $80\,\mathrm{m\,s^{-1}}$.

What is the maximum height gained by the ball?

(gravitational field strength close to the Moon's surface = $1.6\,\mathrm{N\,kg^{-1}}$)

A $\quad 25\,\mathrm{m}$

B $\quad 50\,\mathrm{m}$

C $\quad 320\,\mathrm{m}$

D $\quad 2000\,\mathrm{m}$

E $\quad 3200\,\mathrm{m}$

F $\quad 4000\,\mathrm{m}$

$v = 0$

$u = 80$

$a = -1.6$

$S = ?$

$v^2 = u^2 + 2aS$

$0 = 80^2 - 3.2S$

$3.2S = 6400$

$S = \dfrac{6400}{3.2} = 2000\,\mathrm{m}$

33 For a particular function $f(x)$, it is given that:

$$\int_{-2}^{2} 2f(x)\,dx + \int_{2}^{4} f(x)\,dx = 4$$

and also:

$$\int_{-2}^{2} 5f(x)\,dx - \int_{-2}^{4} f(x)\,dx = 7$$

Find the value of $\int_{2}^{4} f(x)\,dx$

A $\dfrac{1}{3}$

B $\dfrac{11}{7}$

C $\dfrac{11}{6}$

D $\dfrac{13}{6}$

E $\dfrac{13}{3}$

$$\int_{-2}^{4} f(x)\,dx = \int_{2}^{4} f(x)\,dx + \int_{-2}^{2} f(x)\,dx$$

$(i) \longrightarrow 2\int_{-2}^{2} f(x)\,dx + \int_{2}^{4} f(x)\,dx = 4$

$$5\int_{-2}^{2} f(x)\,dx - \left(\int_{2}^{4} f(x)\,dx + \int_{-2}^{2} f(x)\,dx \right) = 7$$

$(ii) \longrightarrow \Rightarrow 4\int_{-2}^{2} f(x)\,dx - \int_{2}^{4} f(x)\,dx = 7$

$(i) + (ii) \Rightarrow 6\int_{-2}^{2} f(x)\,dx = 11$
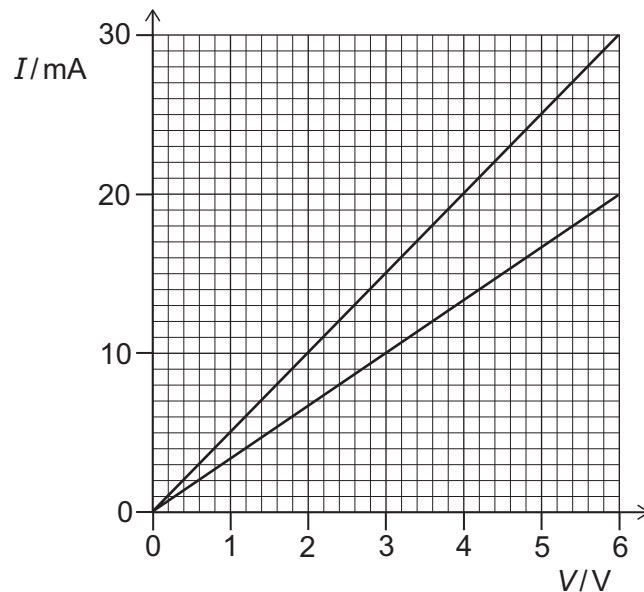
$$\int_{-2}^{2} f(x)\,dx = \frac{11}{6}$$

$$\Rightarrow \int_{2}^{4} f(x)\,dx = 4 - 2 \times \frac{11}{6} = \frac{2}{6} = \frac{1}{3}$$

34 A student has one $300\,\Omega$ resistor and another resistor of resistance R.

The student plots a graph of current I against potential difference V for the $300\,\Omega$ resistor and then for both resistors connected in parallel.

What is the resistance R?

- **A** $3.3\,\Omega$
- **B** $5.0\,\Omega$
- **C** $10\,\Omega$
- **D** $100\,\Omega$
- **E** $200\,\Omega$
- **F** $600\,\Omega$
- **G** $1000\,\Omega$

Steeper line
as $R = \dfrac{1}{grad}$.

Resistance is lower in parallel.

$$R_T = \frac{6}{30 \times 10^{-3}} = 200\,\Omega$$

$$\Rightarrow \left(\frac{1}{300} + \frac{1}{R}\right)^{-1} = 200$$

$$\Rightarrow \frac{R + 300}{300R} = \frac{1}{200}$$

$$\Rightarrow 2R + 600 = 3R$$

$$R = \underline{\underline{600\,\Omega}}$$

35 Given that

$$f(x) = \int_0^x (3+2t)^7 \, dt$$

what is the coefficient of x^4 in the expansion of $f(x)$ in powers of x?

A   70

B   162

C $\dfrac{2835}{4}$

D   3024

(E) 5670

F   15 120

G   22 680

Handwritten working:

Must be the t^3 term in $(3+2t)^7$ because power increases after $\int$.

$\Rightarrow t^3: \ ^7C_3 \times 3^4 \times (2t)^3$

$= \dfrac{7!}{3!4!} \times 81 \times 8t^3 = 35 \times 81 \times 8t^3$

$\int 35 \times 81 \times 8t^3 \, dt = \dfrac{35 \times 81 \times 8}{4} t^4 = 70 \times 81 \, t^4$

$= 5670 \, t^4$

$\times \dfrac{81}{7}$
$\overline{567}$

36 A light, vertical, copper wire of length 2.4 m and uniform cross-sectional area 2.0×10^{-6} m^2 supports a load of mass 4.0 kg.

The Young modulus of copper is 1.2×10^{11} Pa.

What is the strain energy in the wire?

(gravitational field strength = 10 N kg^{-1}; assume that the wire obeys Hooke's law and that the cross-sectional area remains constant)

A 8.0×10^{-5} J

B 1.7×10^{-4} J

C 4.0×10^{-4} J

(D) 8.0×10^{-3} J

E 4.0×10^{-2} J

F 1.6×10^{-2} J

Handwritten working:

$Y = \dfrac{Fl}{Ax} = \dfrac{4 \times 10 \times 2.4}{2 \times 10^{-6} \times x}$

$\Rightarrow \dfrac{4 \times 10^7 \times 2.4}{2x} = 1.2 \times 10^{11} \text{ Pa}$

$\Rightarrow \dfrac{4 \times 10^7}{x} = 10^{11}$

$\Rightarrow 4 \times 10^{-4} = x$

Strain energy $= \frac{1}{2} \times F \times x = \frac{1}{2} \times 4 \times 10 \times 4 \times 10^{-4}$

$= 8 \times 10^{-3}$ J

37   The three internal angles in a triangle are α, β and θ, and

$$\times 3 \begin{cases} 3\tan\alpha - 2\sin\beta = 2 \\ 5\tan\alpha + 6\sin\beta = 8 \end{cases} \times 1$$

What is the value of θ in degrees?

A   15

B   45

C   75

D   105

E   135

$$9\tan\alpha - 6\sin\beta = 6$$
$$5\tan\alpha + 6\sin\beta = 8$$
$$\overline{14\tan\alpha \qquad = 14}$$
$$\underline{\tan\alpha = 1}$$

$$3 - 2\sin\beta = 2$$
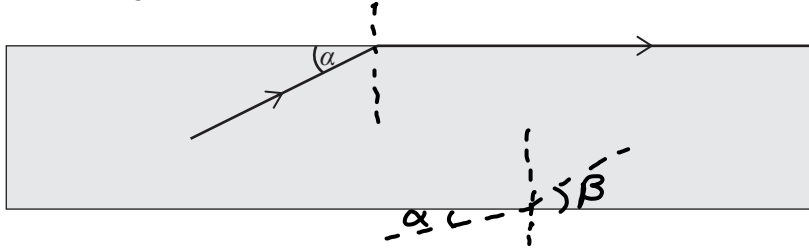$$1 = 2\sin\beta$$
$$\underline{\sin\beta = \tfrac{1}{2}}$$

$$\rightarrow \beta = 30° \qquad \alpha = 45°$$
$$\rightarrow \theta = 180 - (75) = \underline{\underline{105°}}$$

38 Light travelling in a transparent liquid strikes the surface from below. The angle between the surface of the liquid and the direction of travel of the light is α.

The light then travels along the surface between the liquid and the air as shown in the diagram.

Now, light travelling in air strikes the surface from above so that the angle between the surface and the direction of travel of this light is also α.

After the light strikes the surface from above, the angle between the surface and the direction of travel of the refracted light is β.

Which expression gives β?

(all angles are in degrees)

A $\cos\beta = \cos^2\alpha$

B $\cos\beta = \cos\alpha \sin^{-1}\left(\dfrac{1}{\alpha}\right)$

C $\sin\beta = \sin^2\alpha$

D $\beta = 90 - (90 - \alpha)^2$

E $\beta = 0$

F $\beta = \alpha$

Handwritten: Angles given and required one between light and boundary, not light and normal as in Snell's Law.
$\Rightarrow$ replace sin with cos:
$\cos\alpha = n\cos\beta \Rightarrow n = \dfrac{\cos\alpha}{\cos\beta}$
and $n\cos\alpha = 1 \Rightarrow n = \dfrac{1}{\cos\alpha}$
$\dfrac{\cos\alpha}{\cos\beta} = \dfrac{1}{\cos\alpha} \Rightarrow \cos^2\alpha = \cos\beta$

39 Find the complete set of values of x for which

$$x^3 - 2x^2 - 7x - 4 > 0$$

A $x < -1$

B $x > -1$

C $-1 < x < 4$
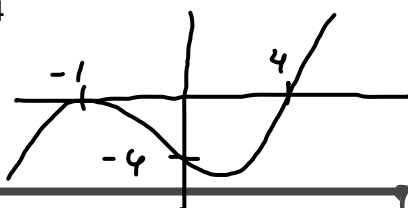
D $x < -1$ or $x > 4$

E $x < 4$

F $x > 4$

Handwritten: $(x-4)$ is a root
$(x-4)\sqrt{x^3-2x^2-7x-4}\;\; x^2+2x+1$
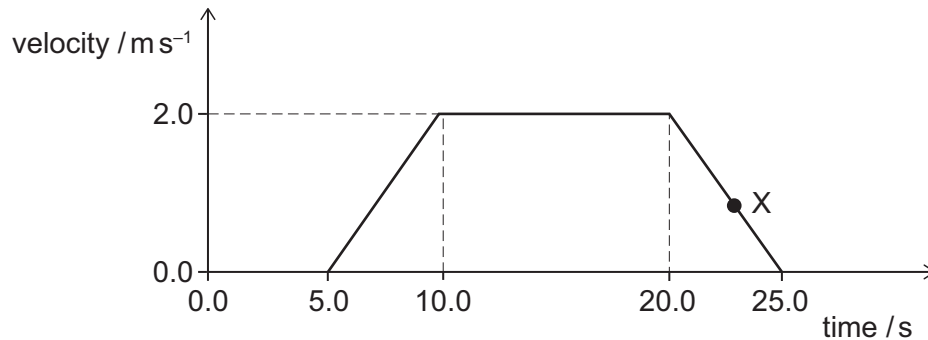$x^3 - 4x^2$
$2x^2 - 7x$
$2x^2 - 8x$
$x - 4$
$x^2 + 2x + 1 = (x+1)^2$
$x > 4$ (from graph)

40 The velocity–time graph is for an 80 kg person in a lift that is moving vertically upwards.

velocity / m s^{-1}

2.0

0.0

0.0   5.0   10.0   20.0   25.0

X

time / s

What is the magnitude of the contact force between the person and the lift floor at the time corresponding to X?

(gravitational field strength = 10 N kg^{-1})

A   640 N

B   768 N

C   800 N

D   832 N

E   960 N

Net force is down as lift & person are decelerating.

Deceleration = $\frac{2}{5}$ ms^{-2}

$F = ma = \cancel{80}^{16} \times \frac{2}{\cancel{5}} = 32$ N

$\Rightarrow$ Reaction = $80g - 32 = 800 - 32 = 768$ N

END OF TEST