British Physics Olympiad 2018-19

A2 Challenge - Mark Scheme

September/October 2018

Instructions

Give equivalent credit for alternative solutions which are correct physics.
Generally allow leeway of ± 1 significant figure.

This is not a tight marking scheme of a competitive exam paper. It is to allow students to engage in problem solving and develop their physics by working through problems requiring explanations, and developing ideas or models. Mark generously to encourage ideas, determination and the willingness to have a go.

1. (i) $E = I(R + r)$ ✓

(ii) Rearrange to $R = \frac{E}{I} - r$. Plot R (ordinate, y-axis) vs I^{-1} ✓
gradient is E and intercept $-r$ ✓

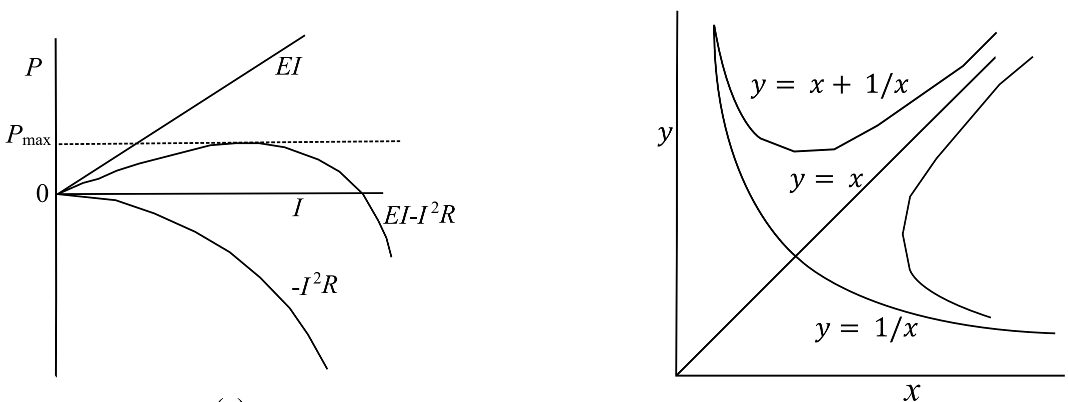
(iii) **Either** quote "maximum power theorem" ✓
Or any correct proof (several possibilities shown as (a) to (e) below) ✓✓✓

[6]

Suggestions for determining the value of R when maximum power is dissipated in the external resistor:

(a) By calculus (product/quotient rule): power dissipated in R is $P = I^2R = \frac{E^2}{(r+R)^2}R$
Differentiate w.r.t. R; $\frac{\mathrm{d}P}{\mathrm{d}R} = \frac{-2V^2R}{(r+R)^3} + \frac{V^2}{(r+R)^2}$ and with $\frac{\mathrm{d}P}{\mathrm{d}R} = 0$ at the maximum.
So $\frac{-2V^2R}{(r+R)^3} + \frac{V^2}{(r+R)^2} = 0$ which simplifies to $\frac{2R}{r+R} = 1$, so $R = r$.

(b) Calculus (simple differentiation); potential across the terminals (or across R) is $V = E - Ir$
Multiply through by I, so that $VI = P = EI - I^2r$, plotted in **Figure 1a**.



(a) **(b)**

Figure 1

But E and r are constants, so max power here is given by $\frac{\mathrm{d}P}{\mathrm{d}I} = E - 2Ir = 0$
Therefore $I = E/2r$ at maximum external power dissipated.

But we know that $I = E/(R + r)$
so equating the two expressions for I, we have $E/(r + R) = E/2r$, and hence $R = r$

(c) Non calculus; the resultant power graph shown in **Figure 1a**, from $V = E - Ir$ and $P = VI$, to give $P = -I^2R + EI$, can be written as $P = -I(Ir - E)$.
The graph is symmetric about a vertical line; the solutions for $P = 0$ are $I = 0$ and $I = E/r$. So at $P_{\max}$, $I = E/2r$. Hence we can use the result quoted above, that given also $I = E/(r + R)$, then $R = r$.

To show that the graph is symmetric, about a vertical line $I = E/2r$, substitute into the expression for P, $I = I' + E/2r$ to shift the graph along the x-axis by $-E/2r$. This gives $P = -(I' + E/2r)((I' + E/2r)r - E) = -(I' + E/2r)(I'r - E/2) = -r(I' + E/2r)(I' - E/2r)$, with solutions $I' = \pm E/2r$, indicating that the maximum of P is halfway between the two zeros of P.

(d) Non calculus; power dissipated in R is $P = I^2R = \frac{E^2}{(r+R)^2}R$
Max of P corresponds to minimum of reciprocal, i.e. min of $(r + R)^2)/R$, which is minimum of $r^2/R + 2rR/R + R^2/R = r^2/R + 2r + R$

So we want the minimum of $r^2/R + R$ (as the $2r$ term is a constant) or, by factoring out an r, min of $r/R + R/r$, which is the min of $1/x + x$. Sketching a graph (**Figure 1b**), the symmetry about the $y = x$ line shows that the minimum of $1/x + x$ is when $x = 1/x$, or in our case, $r/R = R/r$ so that $R = r$.

(e) Graphically; sketch V against I as shown in **Figure 2**. The value of a potential, V, and a current I are some point on the sloping line. Power is max area under the graph, and if the scales are chosen for equal height and width, that max area will be a square (the line is a $45°$ diagonal). The is $V = E/2$ and $I = E/2r$.
So if $V = E/2$, half the emf is across the external resistor, and half dropped across the internal resistor. So they must be the same value of resistance.

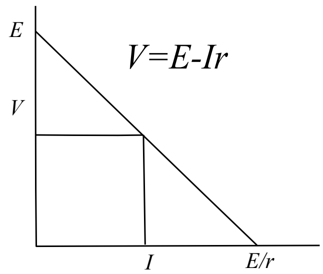


Figure 2

6 marks

2. a) (i) $40\,\mathrm{km\,h^{-1}}$ ✓

(ii) the higher speed is maintained for a shorter time **owtte** ✓

(The total length of the journey here does not make any difference provided that the outward and return journey are the same distance).

[2]

b) (i) bullet falls under gravity ✓

(ii) Time of flight $0.25\,\mathrm{s} \rightarrow s_{\mathrm{vert}} = \frac{1}{2}gt^2 = 0.5 \times 9.81 \times 0.25^2 = 0.31\,\mathrm{m}$ below target ✓

(iii) ✓

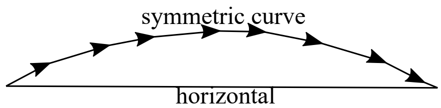


Figure 3

(iv) $v_v = at = g \times \frac{1}{2}\text{total travel time} = 9.81 \times 0.125 = 1.226 = 1.23\,\mathrm{m\,s^{-1}}$ ✓

(v) Aim up at angle θ. $v_v = v\sin\theta$ and $v = 400\,\mathrm{m\,s{-}1}$,
so $\sin\theta = 1.226/400$, and thus $\theta = 0.176° = 0.18°$ ✓

(the small vertical component here means that we have detracted from the $400\,\mathrm{m\,s^{-1}}$ horizontal velocity by an insignificant amount; $v_h \approx v$ because θ is a small angle.)

(vi) Exactly at the target as the target and bullet fall at the same rate ✓

(this is the well known monkey and hunter problem).

[6]

c) (i) $v_v = v\sin\theta$ and $v_h = v\cos\theta$

time of flight, $t =$ time to go up $+$ time to go down $= 2 \times \frac{v\sin\theta}{g}$ ✓

Range, $R = v_h \times$ flight time.

So $R = v\cos\theta \times t = v\cos\theta \times 2 \times \frac{v\sin\theta}{g} = \frac{v^2\sin 2\theta}{g}$ (using hint) ✓

(Students may have seen other approaches in maths;
vertically up, $s = v_v t - \frac{1}{2}gt^2$ with $t = \frac{R}{v_h\cos\theta}$. Going up and down, $s = 0$ and so $0 = v\sin\theta.\frac{R}{v\cos\theta} - \frac{1}{2}g\frac{R^2}{v^2\cos^2\theta}$

Thus $0 = R\tan\theta - \frac{gR^2}{2v^2}.\sec^2\theta$. Multiple through by $2\cos^2\theta$

Then $0 = R2\cos\theta\sin\theta - \frac{gR^2}{v^2}$. Using the hint, we get $R = \frac{v^2}{g}\sin 2\theta$.)

(ii) This maximises at $2\theta = 90°$ (i.e. $\sin 2\theta = 1$) ✓

So $R_{\mathrm{max}} = v^2/g = 30^2/9.81 = 91.7 = 92\,\mathrm{m}$ ✓

Hence shot always falls short of target

(iii) $\theta = \frac{1}{2}\sin^{-1}(\frac{Rg}{v^2}) = \frac{1}{2}\sin^{-1}(\frac{100\times 9.81}{400^2}) = 0.18°$ ✓

above the horizontal (agrees well with earlier approximate calculation)

(there is an alternative angle of $\theta = 89.82°$, which when plugged into the $sin2\theta$ gives the same range. However, this is almost vertically upwards, the bullet would therefore be in the air for about $80\,\mathrm{s}$ and is not the likely angle of fire. It does not correspond to a small angle away from the target either).

[5]

13 marks

3. a) (i) $\frac{1}{6}Nmc$ ✓

(ii) $\frac{1}{3}Nmc$ ✓

(iii) a/c ✓

(iv) $F = \Delta(mc)/\Delta t$ ✓

$= \frac{1}{3}Nmc/(a/c) = \frac{1}{3}Nmc^2/a$ ✓

(v) $p = F/A = \frac{1}{3}Nmc^2/a^3$ ✓

i.e. $pV = \frac{1}{3}Nmc^2$

(vi) $\frac{1}{2}mc^2 = 3RT/2N_\mathrm{A}$ which leads to $pV = NRT/N_\mathrm{A}$ ✓

(vii) as $n = N/N_\mathrm{A}$, ✓

$\rightarrow pV = nRT$

[8]

b) (i) volume of $18\,\mathrm{g}$ is $1.8 \times 10^{-5}\,\mathrm{m}^3$ ✓

so ratio is $1.8 \times 10^{-5}/0.024 = 1 : 1333$ ✓

(ii) linear ratio is cube root of volume ratio (approx), so $1 : 11.0$ ✓

(iii) Assuming Boyle's Law still holds ✓

V decreases by a factor 100, separation decreases by $\sqrt[3]{100}$ i.e. 4.64,

so answer becomes $1 : 2.37$ ✓

(iv) Again, 1333 atmospheres ✓

If Boyle assumed (but of course spherical or other shaped molecules won't tessellate, so this must be an approximation) **owtte** ✓

(v) $8 \times 4\pi r^3/3 = 32\pi r^3/3$ ✓

(vi) $16\pi r^3/3$ **owtte** ✓

(vii) Number of molecules is nN_A, so $b = nN_\mathrm{A} \times 16\pi r^3/3 = 16\pi N_\mathrm{A} r^3/3$ ✓

[10]

c) (i) It eventually condenses. ✓

(ii) $(p + \Delta p)(V - b) = nRT$ ✓

and for $+$ sign (the measured pressure p is less, so add the term a/V^2 to obtain the ideal pressure for the ideal gas equation/behaviour) ✓

(iii) both inversely proportional to V ✓

(iv) $\Delta p = a/V^2$ ✓

(v) $(p + a/V^2)(V - b) = nRT$ ✓

[6]

24 marks

4. (i) VI ✓

(ii) System above room temperature hence losses. ✓

Energy loss rate, L, depends on the temperature differences and the excess temperature, which are set to be the same. **owtte** ✓

(iii) $mc\Delta\theta = VI - L$ ✓

(iv) $m_1 c\Delta\theta = V_1 I_1 - L$ ✓

$m_2 c\Delta\theta = V_2 I_2 - L$ (no suffix on L, c, $\Delta\theta$ ✓

(v) $(m_1 - m_2)c\Delta\theta = (V_1 I_1 - V_2 I_2)$ ✓

the subtraction has eliminated the systematic error represented by L

[7]

7 marks

END OF SOLUTIONS