British Physics Olympiad

Paper 1, September/October 2006

Answer all the Questions

Allow 1 hour Total 50 marks

($g = 9.81\ ms^{-2}$ or $N\ kg^{-1}$)

1. The Oahe hydroelectric power plant in South Dakota has a total peak capacity of 826 MW from its seven generators. The water intakes are 25m below the surface level of the lake. There are seven 7.4m diameter tunnels, which are the intakes for the turbines; that is one tunnel for each turbine. Calculate the speed of the water flowing along the tunnels, assuming the generators are 50% efficient.

density of water = 10^3 kg m^{-3}

(7 marks)

2. A barge loaded with iron ore floats along a canal and crosses a valley by passing along an aqueduct. What happens to the load on the supports of the aqueduct as the barge approaches and then passes across the aqueduct? Explain your reasoning.

(2 marks)

3. The Voyager 2 satellite has an 11W transmitter and beams back a signal to earth via a parabolic dish that focuses the radio waves in a diverging beam that is about 2° wide. This angular width of beam, when reaching the surface area of a sphere centred upon the satellite, corresponds to covering one ten thousandth of the surface area of the sphere. The satellite is at a distance of 10^{10} km from the earth. If it transmits at a frequency of 3×10^{10} Hz, and the energy of a photon is given by $E = hf$, then determine the maximum number of photons per second that arrive at an 80m diameter radio telescope dish on earth.

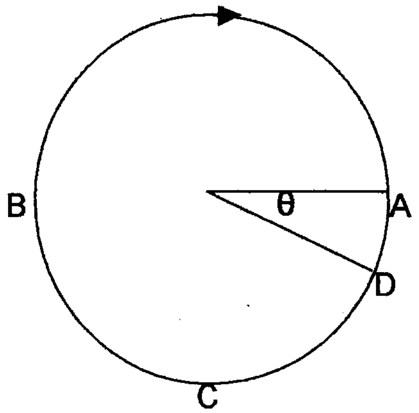
$h = 6.6 \times 10^{-34}$ Js

(4 marks)

4. A piece of elastic string is laid around the circumference of a sphere of radius, r. It is then stretched so that it is raised 1m above the surface. How much longer must the string need to be? How does this extra length relate to the radius, r, of the sphere?
How much greater would the circumference of the earth become if it was to expend slightly so that its radius increased by 0.1m ?
If in expanding, the radius of the earth increased by 2×10^{-5} %, then what is the percentage increase in the surface area?

(7 marks)

5. A particle moving at constant speed, v, follows a clockwise circular path of radius, r, with period of orbit, T.



a) Sketch the circle shown on the left and draw the velocity vectors at both A and B. Write down the magnitude of the velocity change, and hence the magnitude of the average acceleration between A and B in terms of v and T, and hence show that the average acceleration in terms of v and r is given by,

$$a_{average} = \frac{2}{\pi}\frac{v^2}{r}$$

b) Draw a vector diagram of the velocities at A and C, and determine the magnitude of the change of velocity between A and C.

c) Draw a vector diagram of the velocities at A and D, and write down the magnitude of the change of velocity, Δv, between A and D in terms of v and θ.

d) Show that the time taken, Δt, for the particle to move from A to D through angle θ (in radians) is given by $\Delta t = \frac{r\theta}{v}$.

e) Write down the magnitude of the acceleration between A and D, and find the limiting value as θ tends to zero.

(14 marks)

Hint: the cosine rule is $a^2 = b^2 + c^2 - 2bc\cos A$

and as θ tends towards zero, $\cos\theta \approx 1 + \frac{\theta^2}{2}$

6. This question contains several sections that are independent of each other.
A star more massive than the sun can collapse under its own gravity to form a neutron star. Here the electrons and protons combine to form neutrons. A star which is initially rotating and collapses will rotate with a shorter period, as its rotational momentum (called angular momentum) is **conserved**. The angular momentum of the star is given by, $J=\Omega R^2$ (ignoring some constant), where R is the radius of the star and Ω is the angular rate of rotation in radians per second (2π radians is one full rotation).

a) If the volume of the star decreases by 15 orders of magnitude, and the shape of the star remains the same, then what is the ratio of the final to initial radii, $\frac{R_{final}}{R_{initial}}$?

b) If J is conserved, and $\Omega_{initial} = \dfrac{2\pi}{20days}$, then calculate Ω_{final} in radians/second and also the new period of rotation in seconds.

c) As the core of the star collapses to form the neutron star, the electrical conductivity becomes very high. In this case the star's magnetic field lines become frozen into the material of the star and collapse down with the star, increasing the flux density. The neutron star will thus have a very strong magnetic field. If we take the flux $\Phi = BR^2$, with B being the magnetic field strength whose initial value is 10^{-2} T, then determine the final magnetic field strength after the collapse.

d) If a neutron star spins too fast, it will start losing material from its equatorial region. Show that this implies a minimum period, $T_{\min}$ given by

$$T_{\min} = const \cdot M^{-\frac{1}{2}} R^{\frac{2}{3}}$$

$$\text{where } const = \frac{2\pi}{\sqrt{G}}$$

Taking M=1.4 solar masses, R=10 km, then calculate $T_{\min}$.

e) The binding energy of a neutron star is the gravitational potential energy lost when it is formed from a cloud of atoms all separated a great distance apart. The binding energy of a star of mass, M, and radius, R, is given by $BE = k_1 \dfrac{GM^2}{R}$, where k_1 is a numerical constant. Neutron stars do not behave in the same way as ordinary matter and they have a mass-radius relationship given by $RM^{\frac{1}{3}} = k_2$ where k_2 is a constant. Two neutron stars of identical mass collide and form a more massive star. Assuming the mass-radius constraint holds, what is the ratio of the final binding energy of the star to the total initial binding energy?

Mass of the sun = 2.0×10^{30} kg
$G = 6.67 \times 10^{-11}$ Nm^2kg^{-2}

(16 marks)