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CS 517

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Machine Assignment I

Problem 4:

$$\gamma_n = \int_0^1 t^n e^{-t}\, dt, \quad n = 0, 1, 2, \ldots$$

- $\gamma_0 = \int_0^1 t^0 e^{-t}\, dt$

$$\gamma_0 = -e^{-t}\Big|_0^1$$

$$\gamma_0 = -e^{-1} - -e^{0}$$

$$\boxed{\gamma_0 = \frac{-1}{e} + 1} \leftarrow \text{Base Case}$$

- $\gamma_1 = \int_0^1 t\, e^{-t}\, dt$

$u = t \qquad v = -e^{-t}$
$du = dt \qquad dv = e^{-t}\, dt$

$$\gamma_1 = -t e^{-t}\Big|_0^1 + \int_0^1 +e^{-t}\, dt$$

$$\gamma_1 = -t e^{-t}\Big|_0^1 + \int_0^1 \gamma_0\, dt$$

$$\gamma_1 = \left(-(1)e^{-1} - 0\right) + \left(\frac{-1}{e} + 1\right)$$

Π will always be zero except when $n = 0$

$$\gamma_1 = \frac{-1}{e}\left[\frac{1}{e} + 1\right] \gamma_0$$

- $\gamma_2 = \int_0^1 t^2 e^{-t}\,dt$

$u = t^2 \qquad v = -e^{-t}$
$du = 2t \qquad dv = e^{-t}\,dt$

$$\gamma_2 = -t^2 e^{-t}\Big|_0^1 + 2\int_0^1 t e^{-t}\,dt$$

$$\gamma_2 = -t^2 e^{-t}\Big|_0^1 + \underbrace{2}_{n}\int_0^1 \underbrace{\gamma_1}_{\text{we already have this in terms of } \gamma_0}\,dt \ \ldots$$

- $\gamma_3 = \int_0^1 t^3 e^{-t}\,dt$

$u = t^3 \qquad v = -e^{-t}$
$du = 3t^2 dt \qquad dv = e^{-t}\,dt$

$$\gamma_3 = \underbrace{-t^3 e^{-t}\Big|_0^1}_{\text{always } (-1)} + \underbrace{3}_{n}\int t^2 e^{-t}\,dt$$

$\int t^2 e^{-t}\,dt = \gamma_2$

$$\therefore \boxed{\gamma_K = \frac{-1}{e} + K\left[\gamma_{K-1}\right]}$$