Related, Constrained Probabilities in a System

A system consists of *n* independent sub-components, say C_1 through C_n, with the following individual probabilities of failure at the times indicated:

	2.5 hours	3 hours	3.5 hours	4 hours	…	q hours
C_1	p_{11}	p_{12}	p_{13}	p_{14}		p_{1q}
C_2	P_{21}	p_{12}	P_{23}	P_{24}		P_{2q}
…						
C_n	P_{n1}	P_{n2}	P_{n3}	P_{n4}		P_{nq}

Given that the failure probabilities (1 through q) for each component sum to 1 and failure of a component is independent of failures of others, what is the probability of system failure at any of the listed times? Notice that this scenario could be a mechanical system, people keying transactions, or drug reaction times in different people. As with all probability problems, accurate definition of the complete event space is critical. I considered two approaches: P(failure of C_1 or C_2 or … or C_n at time *t*, given failure of no component prior to *t*) and P(not failure of C_1 and not C_2 and … not C_n at time *t*, given no failure prior to *t*). The simpler of these is the second, giving an expression for cumulative probability of system failure to time *t* of

$$C(F_t) = 1\text{-}P(\text{not } F_t) = 1\text{-}[1\text{-}P(F_{C1t})][1\text{-}P(F_{C2t})]...[1\text{-}P(F_{Cnt})]$$

and for probability of failure at time *t* of

$$P(F_t)=C(F_t)\text{-}C(F_{t\text{-}1})$$

A simple check of the system failure event space is that their cumulative probabilities sum to 1, that is $C(F_{tq})=1$. I programmed the final calculations on the TI-89 app, Graph-89 by Dritan Hashorva, installed on my phone, being handy. If you haven't already used this app, you should consider it. It's a complete TI-89 in your pocket. By the way, the TI-89 will produce analytical results (it returns e^x for $\int e^x dx$ for instance) and correctly produces 1/*p* for $\sum_{k=1}^{\infty} kp(1-p)^{k-1}$, the mean of the Geometric pdf, for $0<p<1$. But $1<p$ causes this series to diverge and is no longer a pdf. When the TI-89 cannot find an analytical solution it returns your original expression or, as in this case, something algebraically equivalent, sometimes considerably more complicated. You should check out what it returns for $1<p$. It will surprise or, at least, amuse you. Finally, the solution to this failure problem is useful in modeling overall system reliability given changes in individual component failure models.