How-To: **Game Theory**

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The following slides cover…

Game Theory: Prisoner's Dilemma (PD)

Game Theory: Game of Chicken & Battle of the Sexes

Normal Form to Extensive Form Games

Today, we will practice:

- Solving games using best response analysis
- Identifying different types of games based on equilibria (PD, Chicken, Battle of Sexes)
- Solving extensive form games using backwards induction

The Prisoner's Dilemma (PD) Game

- **Players**: 2 (P1 and P2)
- **Strategies for each player**: Cooperate (C) or Defect (D)
- **Payoffs**: Depicted in each square of a 2x2 game
  - P1/row player→ first number in payoff box
  - P2/column player → second number in payoff box
  - In this example…payoff ranking is: T>R>P>S

TABLE 4.1 **The Prisoner's Dilemma**

P2 = column player = 2nd number

P1 =row player = 1st number

	C Pat cooperates	D Pat defects
Chris cooperates C	R, R	S, T
Chris defects D	T, S	P, P

Note: $T > R > P > S$. T = temptation; R = reward; P = punishment; S = sucker's payoff.

Prisoner's Dilemma has 4 Possible Outcomes

If both prisoners cooperate (**C,C**)	• Both get 1 year in prison
If both prisoners defect (**D,D**)	• Both get 10 years in prison
If Prisoner 1 cooperates, while Prisoner 2 defects (**C,D**)	• P1 gets 25 years in prison, while P2 goes free
If Prisoner 1 defects, while Prisoner 2 cooperates (**D,C**)	• P1 goes free, while P2 gets 25 years in prison

How does each player rank the outcomes?

- Key interest for each player is **to get the shortest possible prison sentence**
- Going free>1 yr in prison>10 yrs in prison>25 yrs in prison
- Let's rank their outcomes:

Jail sentence + Payoffs (ranked from most (4) to least desirable (1))	Which outcome produces that sentence for Prisoner 1?	Which outcome produces that sentence for Prisoner 2?
4 = Go free = TEMPTATION	**D,C**	**C,D**
3 = 1 year jail = REWARD	**C,C**	**C,C**
2 = 10 years jail = PUNISHMENT	**D,D**	**D,D**
1 = 25 years jail = SUCKER	**C,D**	**D,C**

- **P1: Temptation (D,C) > Reward (C,C) > Punishment (D,D) > Sucker (C,D)**
- **P2: Temptation (C,D) > Reward (C,C) > Punishment (D,D) > Sucker (D,C)**

Setting up the "game"…

Jail sentence + Payoffs (ranked from most (4) to least desirable (1))	Which outcome produces that sentence for Prisoner 1?	Which outcome produces that sentence for Prisoner 2?
4 = Go Free	D,C	C,D
3 = 1 year jail time	C,C	C,C
2 = 10 years jail time	D,D	D,D
1 = 25 years jail time	C,D	D,C

Player 2

Player 1

	C (Cooperate)	D (Defect)
C (Cooperate)		
D (Defect)		

Setting up the "game"…

Jail sentence + Payoffs (ranked from most (4) to least desirable (1))	Which outcome produces that sentence for Prisoner 1?	Which outcome produces that sentence for Prisoner 2?
4 = Go Free	D,C	C,D
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2 = 10 years jail time	D,D	D,D
1 = 25 years jail time	C,D	D,C

Player 2

Player 1

	C (Cooperate)	D (Defect)
C (Cooperate)	3,	
D (Defect)		

Setting up the “game”…

Jail sentence + Payoffs (ranked from most (4) to least desirable (1))	Which outcome produces that sentence for Prisoner 1?	Which outcome produces that sentence for Prisoner 2?
4 = Go Free	D,C	C,D
3 = 1 year jail time	C,C	C,C
2 = 10 years jail time	D,D	D,D
1 = 25 years jail time	C,D	D,C

Player 2

Player 1

	C (Cooperate)	D (Defect)
C (Cooperate)	3, 3	
D (Defect)		

Setting up the "game"…

Jail sentence + Payoffs (ranked from most (4) to least desirable (1))	Which outcome produces that sentence for Prisoner 1?	Which outcome produces that sentence for Prisoner 2?
4 = Go Free	D,C	C,D
3 = 1 year jail time	C,C	C,C
2 = 10 years jail time	D,D	D,D
1 = 25 years jail time	C,D	D,C

		Player 2	
		C (Cooperate)	D (Defect)
Player 1	C (Cooperate)	3, 3	
	D (Defect)		

Setting up the "game"…

Jail sentence + Payoffs (ranked from most (4) to least desirable (1))	Which outcome produces that sentence for Prisoner 1?	Which outcome produces that sentence for Prisoner 2?
4 = Go Free	D,C	C,D
3 = 1 year jail time	C,C	C,C
2 = 10 years jail time	D,D	D,D
1 = 25 years jail time	C,D	D,C

Player 2

Player 1

	C (Cooperate)	D (Defect)
C (Cooperate)	3, 3	1,
D (Defect)		

Setting up the "game"…

Jail sentence + Payoffs (ranked from most (4) to least desirable (1))	Which outcome produces that sentence for Prisoner 1?	Which outcome produces that sentence for Prisoner 2?
4 = Go Free	D,C	C,D
3 = 1 year jail time	C,C	C,C
2 = 10 years jail time	D,D	D,D
1 = 25 years jail time	C,D	D,C

		Player 2	
		C (Cooperate)	D (Defect)
Player 1	C (Cooperate)	3, 3	1, 4
	D (Defect)		

Setting up the “game”…

Jail sentence + Payoffs (ranked from most (4) to least desirable (1))	Which outcome produces that sentence for Prisoner 1?	Which outcome produces that sentence for Prisoner 2?
4 = Go Free	D,C	C,D
3 = 1 year jail time	C,C	C,C
2 = 10 years jail time	D,D	D,D
1 = 25 years jail time	C,D	D,C

Player 2

Player 1

	C (Cooperate)	D (Defect)
C (Cooperate)	3, 3	1, 4
D (Defect)		

Setting up the "game"…

Jail sentence + Payoffs (ranked from most (4) to least desirable (1))	Which outcome produces that sentence for Prisoner 1?	Which outcome produces that sentence for Prisoner 2?
4 = Go Free	D,C	C,D
3 = 1 year jail time	C,C	C,C
2 = 10 years jail time	D,D	D,D
1 = 25 years jail time	C,D	D,C

Player 2

Player 1

	C (Cooperate)	D (Defect)
C (Cooperate)	3, 3	1, 4
D (Defect)	4, 1	

Setting up the "game"…

Jail sentence + Payoffs (ranked from most (4) to least desirable (1))	Which outcome produces that sentence for Prisoner 1?	Which outcome produces that sentence for Prisoner 2?
4 = Go Free	D,C	C,D
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2 = 10 years jail time	D,D	D,D
1 = 25 years jail time	C,D	D,C

Player 2

Player 1

	C (Cooperate)	D (Defect)
C (Cooperate)	3, 3	1, 4
D (Defect)	4, 1	

Setting up the "game"…

Jail sentence + Payoffs (ranked from most (4) to least desirable (1))	Which outcome produces that sentence for Prisoner 1?	Which outcome produces that sentence for Prisoner 2?
4 = Go Free	D,C	C,D
3 = 1 year jail time	C,C	C,C
2 = 10 years jail time	D,D	D,D
1 = 25 years jail time	C,D	D,C

Player 2

Player 1

	C (Cooperate)	D (Defect)
C (Cooperate)	3, 3	1, 4
D (Defect)	4, 1	2, 2

The final game set up

Jail sentence + Payoffs (ranked from most (4) to least desirable (1))	Which outcome produces that sentence for Prisoner 1?	Which outcome produces that sentence for Prisoner 2?
4 = Go Free	D,C	C,D
3 = 1 year jail time	C,C	C,C
2 = 10 years jail time	D,D	D,D
1 = 25 years jail time	C,D	D,C

		Player 2	
		C (Cooperate)	D (Defect)
Player 1	C (Cooperate)	3, 3 Reward, Reward	1, 4 Sucker, Temptation
	D (Defect)	4, 1 Temptation, Sucker	2, 2 Punishment, Punishment

1 = Sucker
2 = Punishment
3 = Reward
4 = Temptation

Remember:
4>3>2>1 so…

Temptation > Reward > Punishment > Sucker

How do we “solve” a normal-form game?

- “Solving” a game means finding its Nash Equilibrium(s)
- **Nash Equilibrium** → A combination of strategies of all players (e.g. P1 picks C or D and P2 picks C or D) in which no player has incentive to *unilaterally* change its behavior, *given what they expect the other player to do.*
- To find the Nash Equilibrium (NE), we use **Best Response Analysis**
  - **If Player 1 plays __ (C or D), what should Player 2 do (play C or D)?**
  - **If Player 2 plays __ (C or D), what should Player 1 do (play C or D)?**

Let's "solve" the PD game we set up earlier using best response analysis

Player 2

		C (Cooperate)	**D (Defect)**
Player 1	**C (Cooperate)**	**3, 3**	**1, 4**
	D (Defect)	**4, 1**	**2, 2**

Let's "solve" the PD game we set up earlier using "best response analysis"

		Player 2	
		C (Cooperate)	**D (Defect)**
Player 1	**C (Cooperate)**	**3, 3**	**1, 4**
	D (Defect)	**4, 1**	**2, 2**

Ask yourself…if P2 cooperates, what would P1 want to do?

Let's "solve" the PD game...("best response analysis")

		Player 2	
		C (Cooperate)	**D (Defect)**
Player 1	**C (Cooperate)**	**3, 3**	**1, 4**
	D (Defect)	**4* 1**	**2, 2**

If P2 cooperates, P1 will want to defect (4>3)

Let's "solve" the PD game...("best response analysis")

Player 2

		C (Cooperate)	D (Defect)
Player 1	**C (Cooperate)**	**3, 3**	**1, 4**
	D (Defect)	**4*, 1**	**2, 2**

If P2 defects, what would P1 want to do?

Let's "solve" the PD game...("best response analysis")

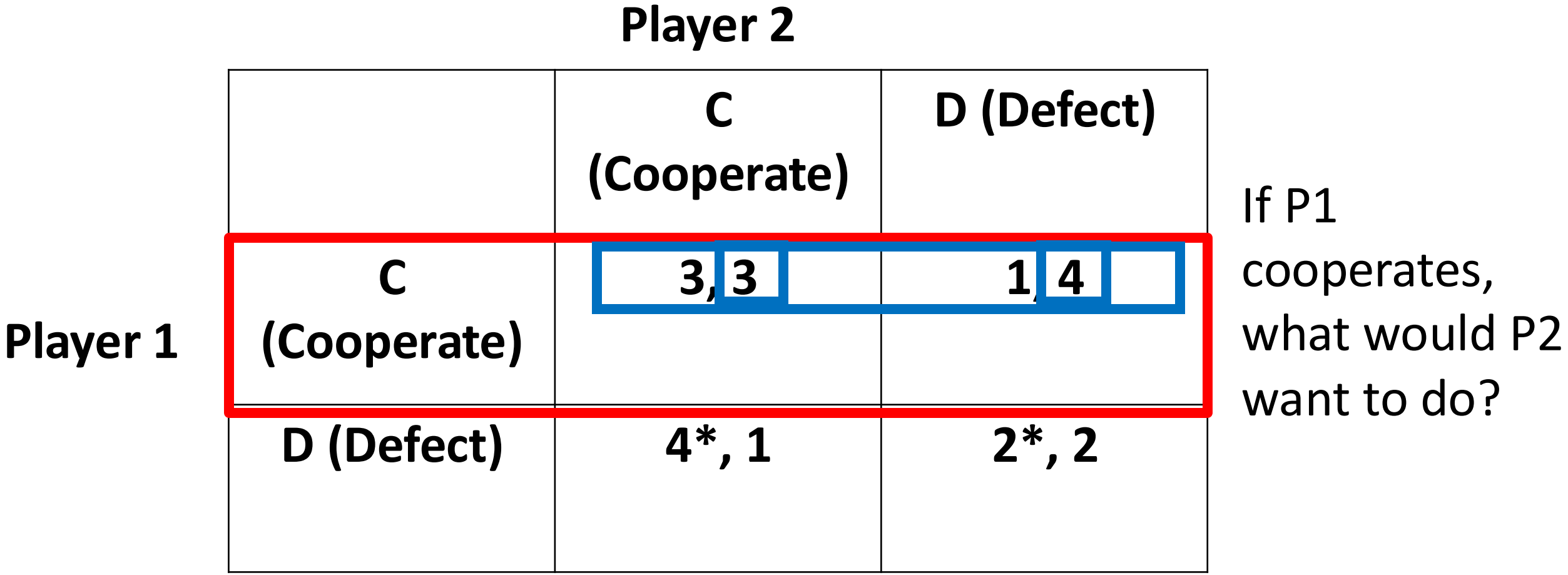
Player 2

	C (Cooperate)	**D (Defect)**
C (Cooperate)	**3, 3**	**1, 4**
D (Defect)	**4*, 1**	**2*, 2**

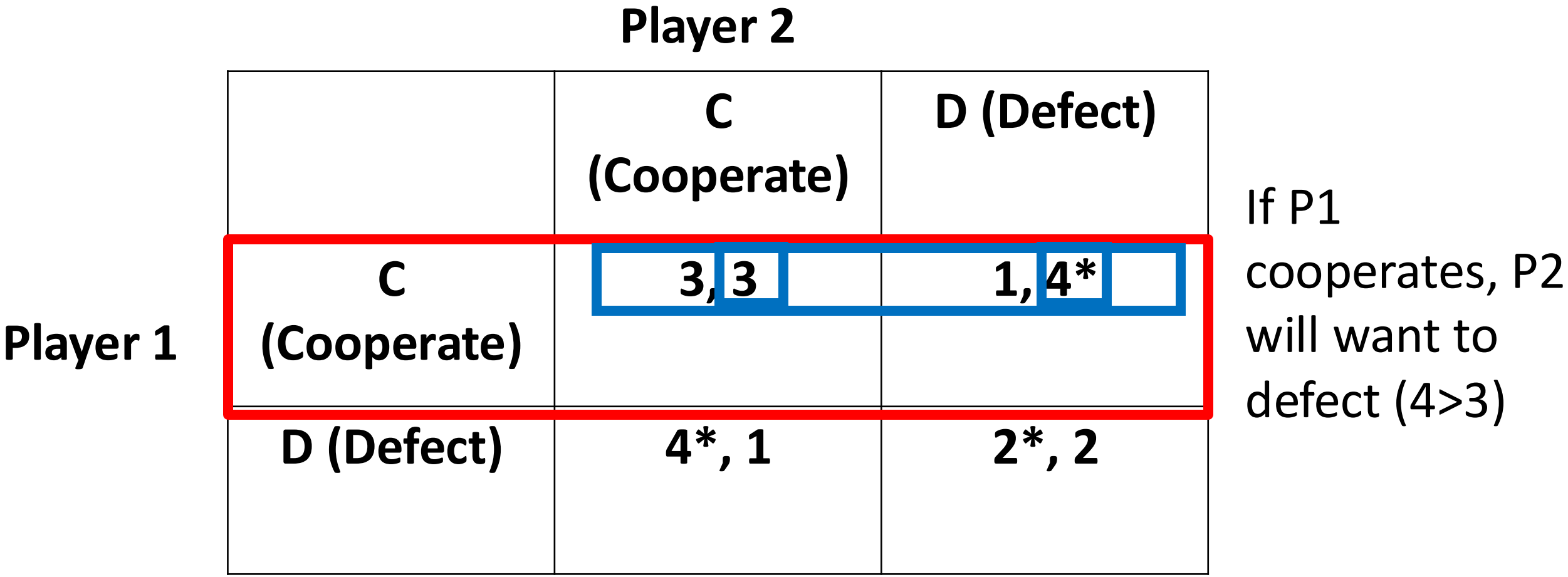
Player 1

If P2 defects, P1 will want to defect (2>1)

Let's "solve" the PD game...("best response analysis")



Let's "solve" the PD game...("best response analysis")



Let's "solve" the PD game...("best response analysis")

Player 2

		C (Cooperate)	D (Defect)
Player 1	**C (Cooperate)**	**3, 3**	**1, 4***
	D (Defect)	**4*, 1**	**2*, 2**

If P1 defects, what would P2 want to do?

Let's "solve" the PD game...("best response analysis")

		Player 2	
		C (Cooperate)	**D (Defect)**
Player 1	**C (Cooperate)**	**3, 3**	**1, 4***
	D (Defect)	**4*, 1**	**2*, 2***

If P1 defects, P2 will want to defect (2>1)

The “Solved” PD Game = The Equilibrium = Defect, Defect → If it has 2 asterisks, then it is an equilibria!

		Player 2	
		C (Cooperate)	**D (Defect)**
Player 1	**C (Cooperate)**	**3, 3**	**1, 4***
	D (Defect)	**4*, 1**	**2*, 2***

The “Solved” PD Game = The Equilibrium = Defect, Defect → If it has 2 asterisks, then it is an equilibria!

Player 2

Player 1		**C (Cooperate)**	**D (Defect)**
	C (Cooperate)	**3, 3**	**1, 4***
	D (Defect)	**4*, 1**	**2*, 2***

Can you explain why this is considered a “Nash equilibrium”?

Hint: Does either player have incentive to *unilaterally* change their behavior?

What about other types of games?

- The Prisoner's Dilemma (PD) is just one possible 2x2, normal-form game, which so happens to have ONE equilibrium (D,D/defect, defect)
- But there are other games, some with multiple equilibria
- Solving other games is just like solving the PD → use **best response analysis** to find the equilibrium!

Let's practice solving a different game….

Steps to Solve a Normal-Form Game

1

Step 1 - Solve the game for its equilibrium(s) using **best response analysis**

2

Step 2 - Determine what “type” of game it is

- Prisoner’s Dilemma (PD)
- Game of Chicken
- Battle of Sexes

Let's solve this game using "best response analysis"

If P2 cooperates (C), what does P1 want to do?

		Player 2	
		Cooperate	Defect
Player 1	Cooperate	2, 2	1, 3
	Defect	3, 1	0, 0

Let's solve this game using "best response analysis"

If P2 cooperates (C), what does P1 want to do?

		Player 2	
		Cooperate	Defect
Player 1	Cooperate	2, 2	1, 3
	Defect	3*, 1	0, 0

Let's solve this game using "best response analysis"

If P2 defects (D), what does P1 want to do?

		Player 2	
		Cooperate	Defect
Player 1	Cooperate	2, 2	1, 3
	Defect	3*, 1	0, 0

Let's solve this game using "best response analysis"

If P2 defects (D), what does P1 want to do?

		Player 2	
		Cooperate	Defect
Player 1	Cooperate	2, 2	1*, 3
	Defect	3*, 1	0, 0

Let's solve this game using "best response analysis"

If P1 cooperates (C), what does P2 want to do?

		Player 2	
		Cooperate	Defect
Player 1	Cooperate	2, 2	1, 3*
	Defect	3*, 1	0, 0

Let's solve this game using "best response analysis"

If P1 cooperates (C), what does P2 want to do?

		Player 2	
		Cooperate	Defect
Player 1	Cooperate	2, 2	1, 3*
	Defect	3*, 1	0, 0

Let's solve this game using "best response analysis"

If P1 defects (D), what does P2 want to do?

		Player 2	
		Cooperate	Defect
Player 1	Cooperate	2, 2	1*, 3*
	Defect	3*, 1	0, 0

Let's solve this game using "best response analysis"

If P1 defects (D), what does P2 want to do?

		Player 2	
		Cooperate	Defect
Player 1	Cooperate	2, 2	1*, 3*
	Defect	3*, 1*	0, 0

Putting it all together, we have two equilibria: (C,D) and (D,C)

- What type of game is this? Why?
  - Prisoner's Dilemma?
  - Battle of the Sexes?
  - Chicken?

		Player 2	
		Cooperate	Defect
Player 1	Cooperate	2, 2	1*, 3*
	Defect	3*, 1*	0, 0

It’s a game of chicken! Chicken games have two equilibria (unlike PD which only has 1) where the two players play opposite strategies (C,D or D,C)

		Player 2	
		Cooperate	Defect
Player 1	Cooperate	2, 2	1*, 3*
	Defect	3*, 1*	0, 0

In the classic example, we have: **Swerve vs. Drive Straight** instead of **Cooperate vs. Defect**

What about the “Battle of the Sexes” game?

- Unlike Chicken, BOS is a 2x2 *coordination* game where both players get a positive payout if they *coordinate* and do the *same* thing
- Example: You and your significant other are deciding where to go out on a date. You would like to go axe throwing, while your partner would like to go see the new Antman movie. Both of you would prefer to go to the same place rather than different places (e.g. both go axe throwing or both go see Antman). Assuming you cannot communicate, what should you do?
- How many equilibria? → Two ”pure strategy” equilibria

For more info: https://www.youtube.com/watch?v=2BdBWmL8tJc (Good walk through)

The Battle of the Sexes game looks like this and has two equilibria (where players do the same thing):

		Your Partner	
		Axe Throwing	**Antman movie**
You	Axe Throwing	2,1	0,0
	Antman movie	0,0	1,2

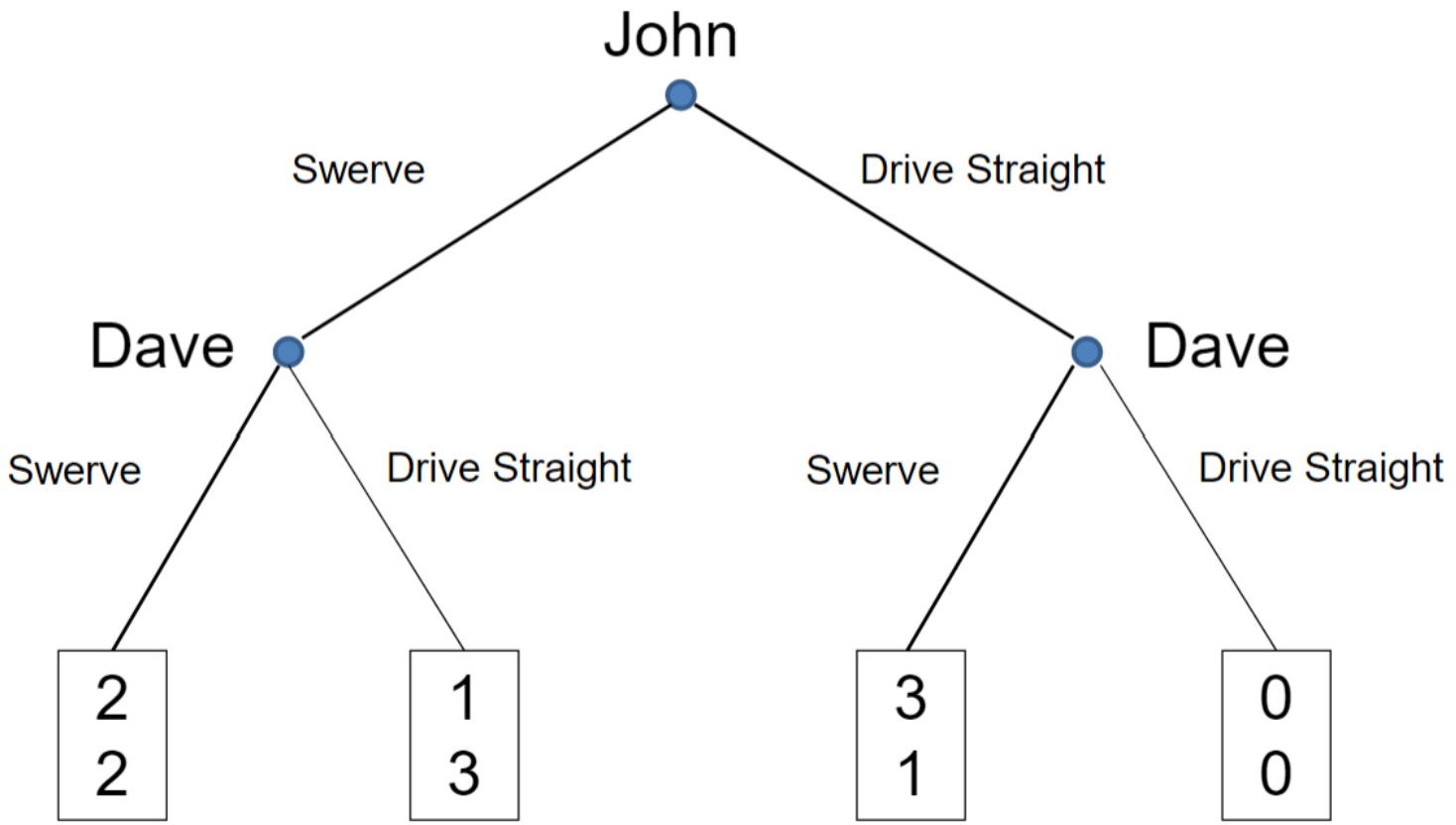
Normal Form → Extensive Form Games

Normal-form game → Extensive-form game

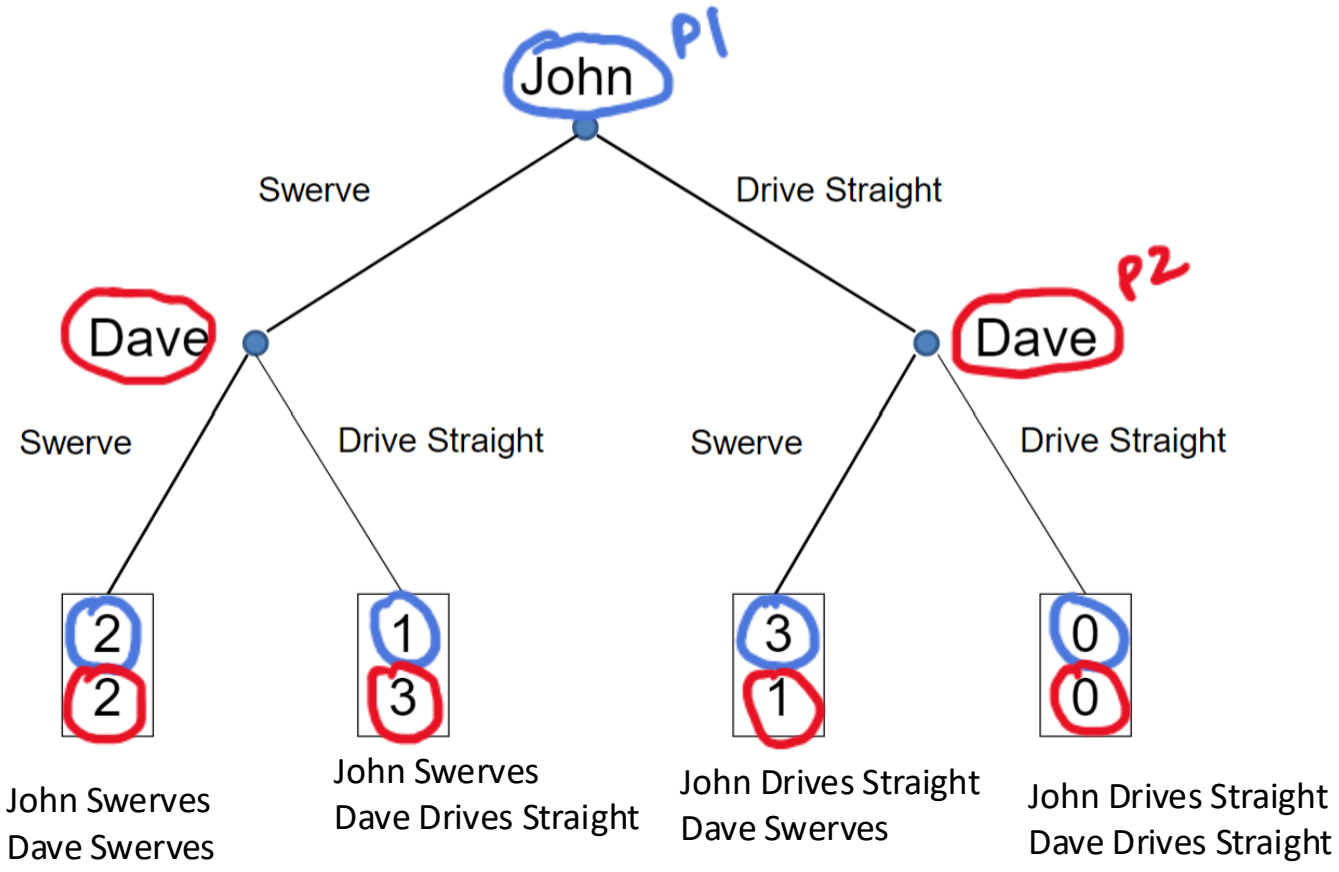
- In a normal-form game we assume both players move *simultaneously*
- In extensive-form games we assume that *one player moves first*
- Extensive form games thus give us more precise solutions
- Recall John and Dave in the Game of Chicken to the right
  - What happens if we assume that John moves first, then followed by Dave?

		Dave	
		Swerve	Drive Straight
John	Swerve	2, 2	1, 3
	Drive Straight	3, 1	0, 0

Here is the extensive form game all set up…



The Payoffs



*Remember, the top number is Player 1/first mover (AKA John) and the bottom number is Player 2/second mover (AKA Dave)

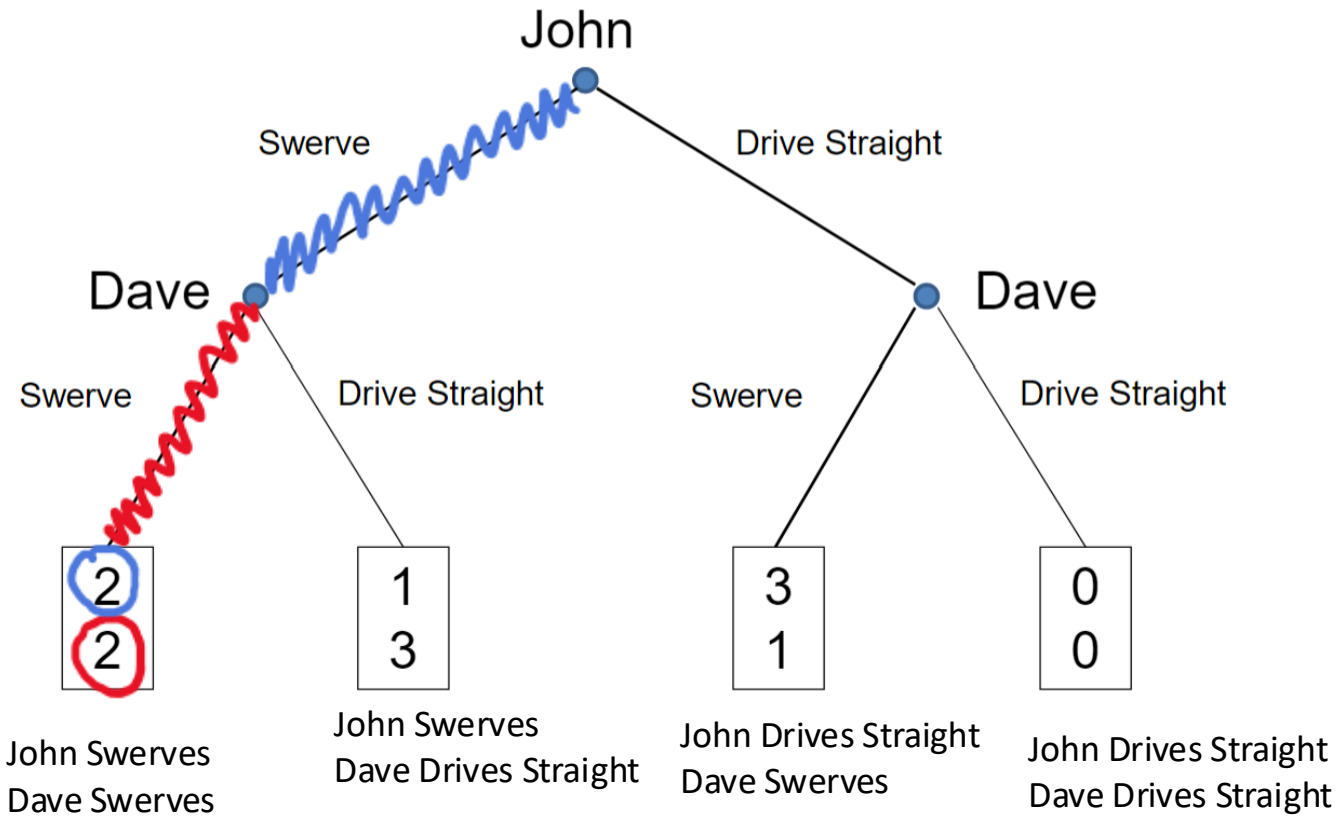
2,2: John Swerves and gets 2, Dave Swerves and gets 2

1,3: John Swerves and gets 1, Dave Drives Straight and gets 3

3,1: John Drives Straight and gets 3, Dave Swerves and gets 1

0,0: John Drives Straight and gets 0, Dave Drives Straight and gets 0

John Swerves, Dave Swerves



The payoffs

*Remember, the top number is Player 1/first mover, AKA John, and the bottom number is Player 2/second mover AKA Dave

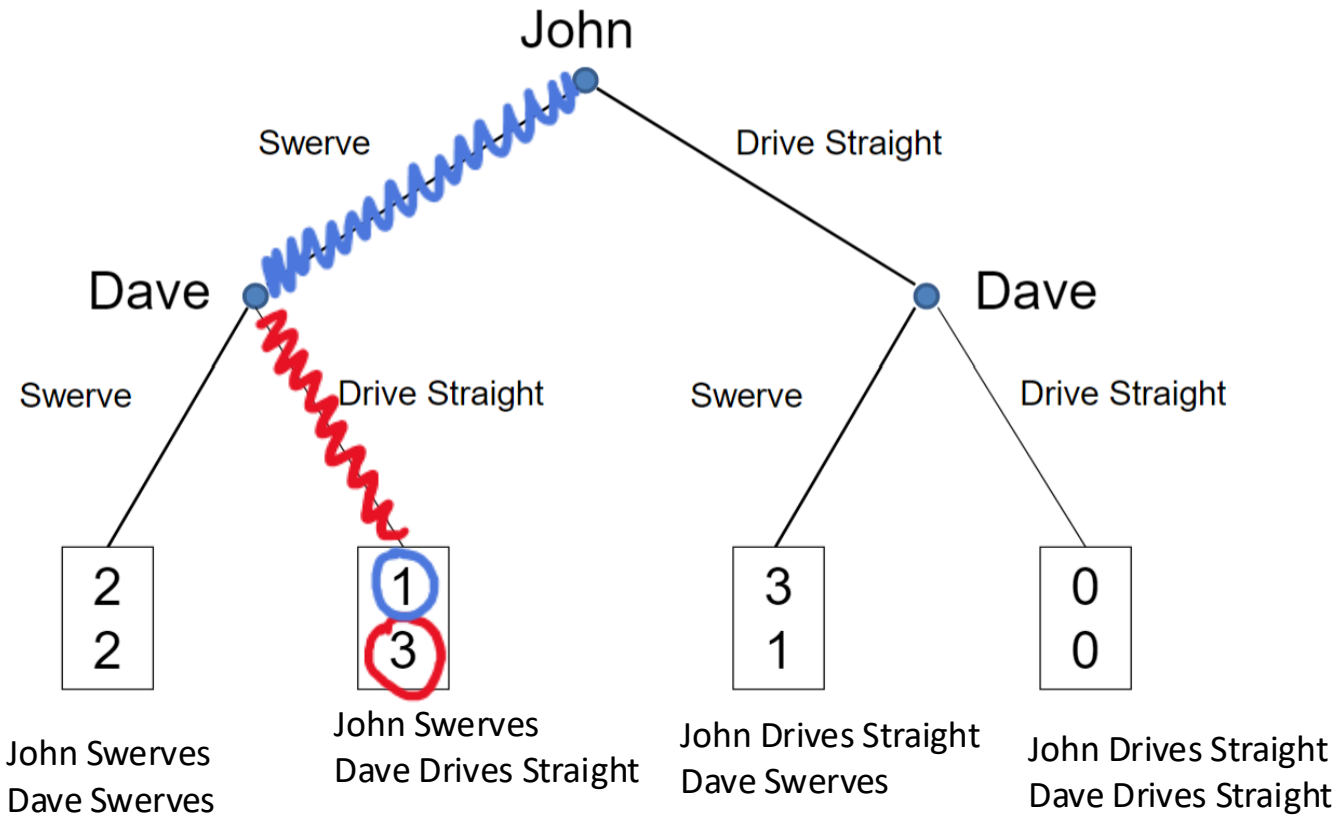
2,2: John Swerves and gets 2, Dave Swerves and gets 2

1,3: John Swerves and gets 1, Dave Drives Straight and gets 3

3,1: John Drives Straight and gets 3, Dave Swerves and gets 1

0,0: John Drives Straight and gets 0, Dave Drives Straight and gets 0

John Swerves, Dave Drives Straight



The payoffs

*Remember, the first number is Player 1/first mover, AKA John, and the second number is Player 2/second mover AKA Dave

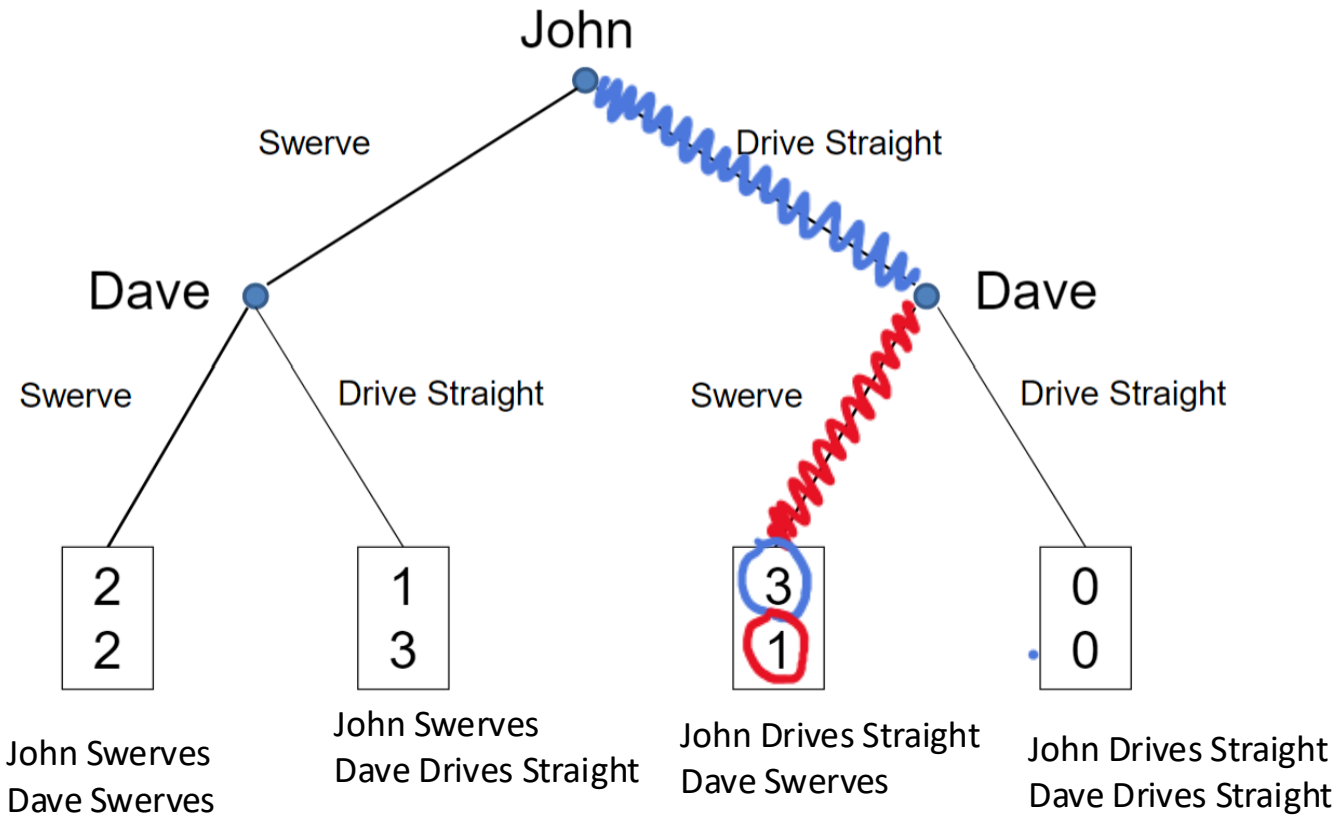
2,2: John Swerves and gets 2, Dave Swerves and gets 2

1,3: John Swerves and gets 1, Dave Drives Straight and gets 3

3,1: John Drives Straight and gets 3, Dave Swerves and gets 1

0,0: John Drives Straight and gets 0, Dave Drives Straight and gets 0

John Drives Straight, Dave Swerves



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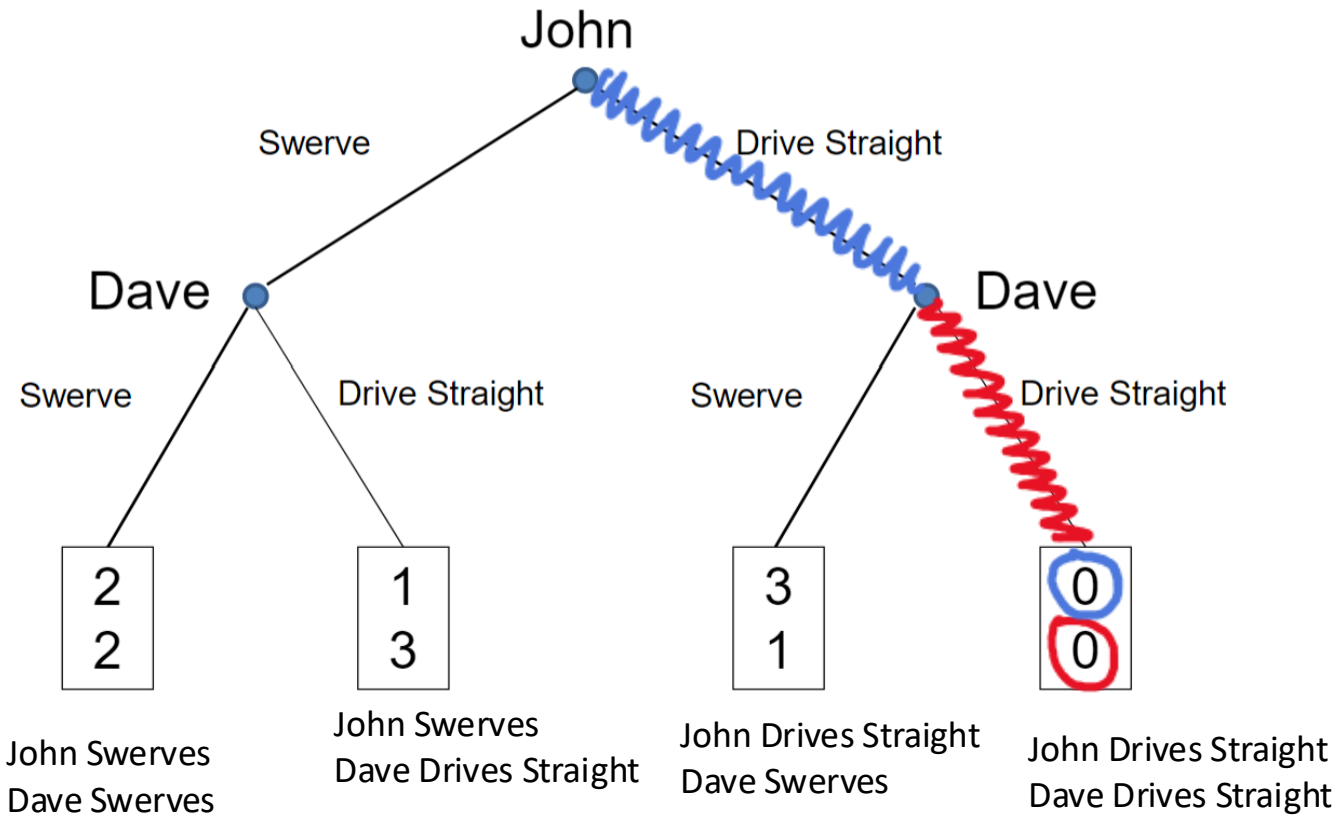
0,0: John Drives Straight and gets 0, Dave Drives Straight and gets 0

John Drives Straight, Dave Drives Straight



The payoffs

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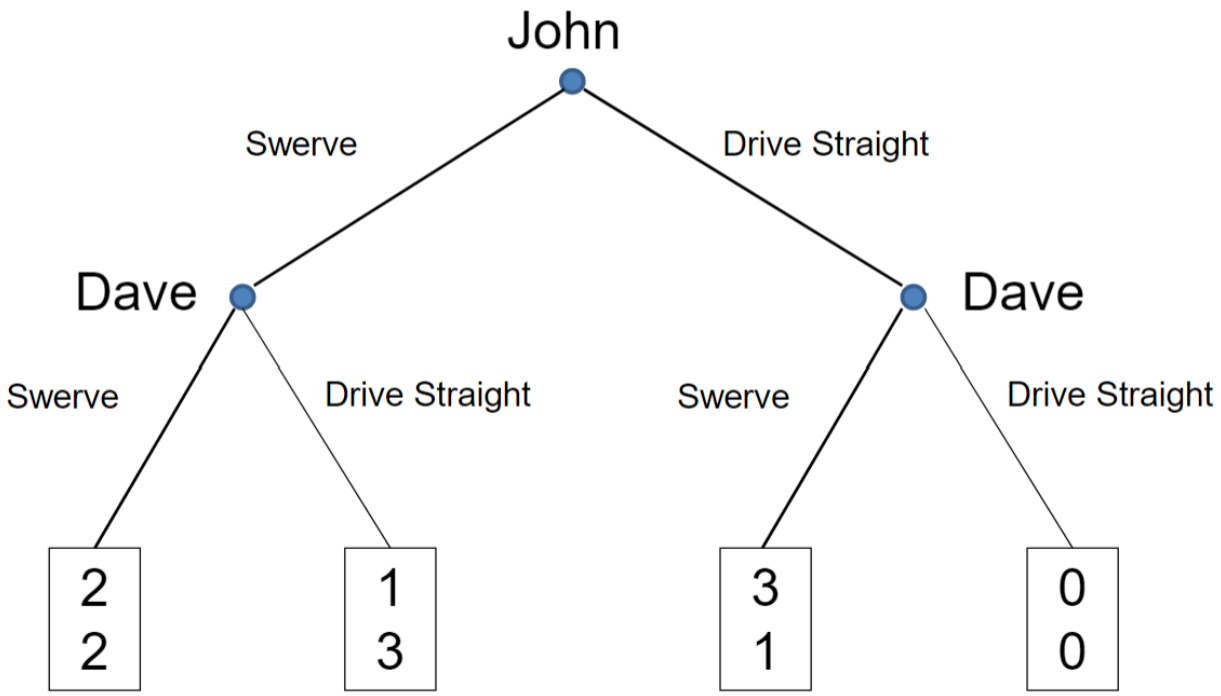
2,2: John Swerves and gets 2, Dave Swerves and gets 2

1,3: John Swerves and gets 1, Dave Drives Straight and gets 3

3,1: John Drives Straight and gets 3, Dave Swerves and gets 1

0,0: John Drives Straight and gets 0, Dave Drives Straight and gets 0

What is the equilibrium? How do we find it?

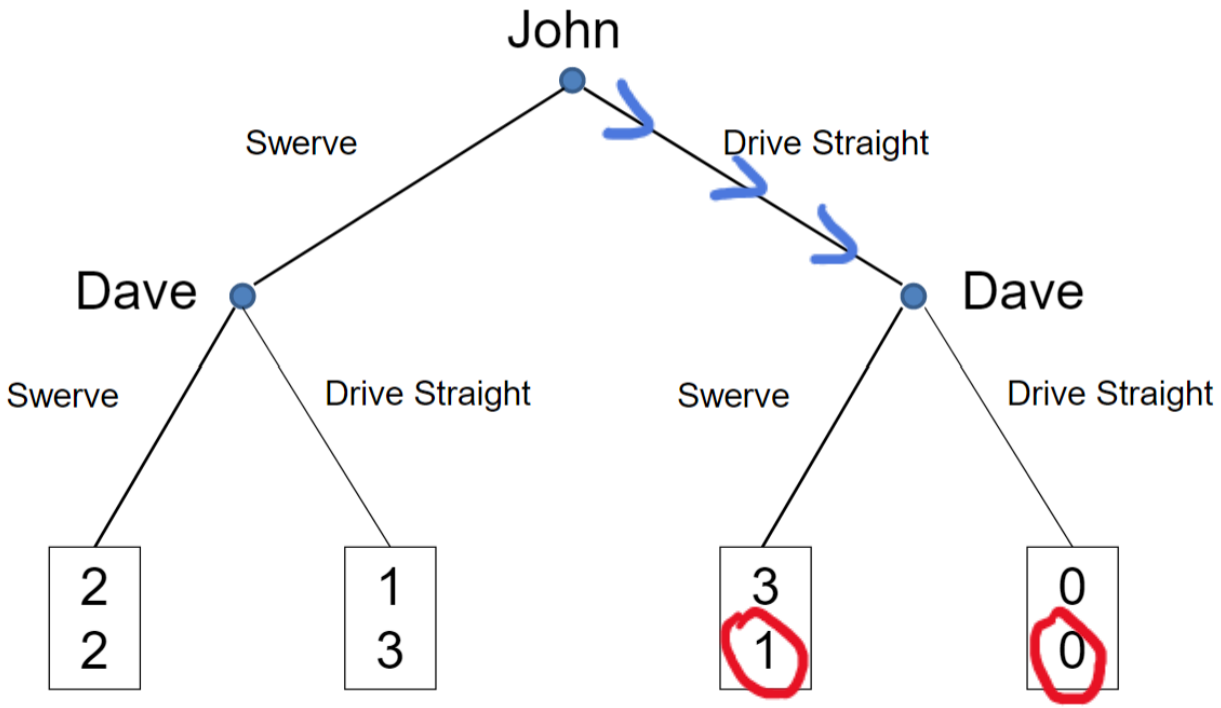


Using **backward induction**!!

Must ask ourselves:

1. If John goes first and drives **straight**, what will Dave do?
2. If John goes first and **swerves**, what will Dave do?
3. Then, given what we know Dave will do in each case, **what should John do as their "best" first move?**

If John goes first and drives straight, what should Dave do?



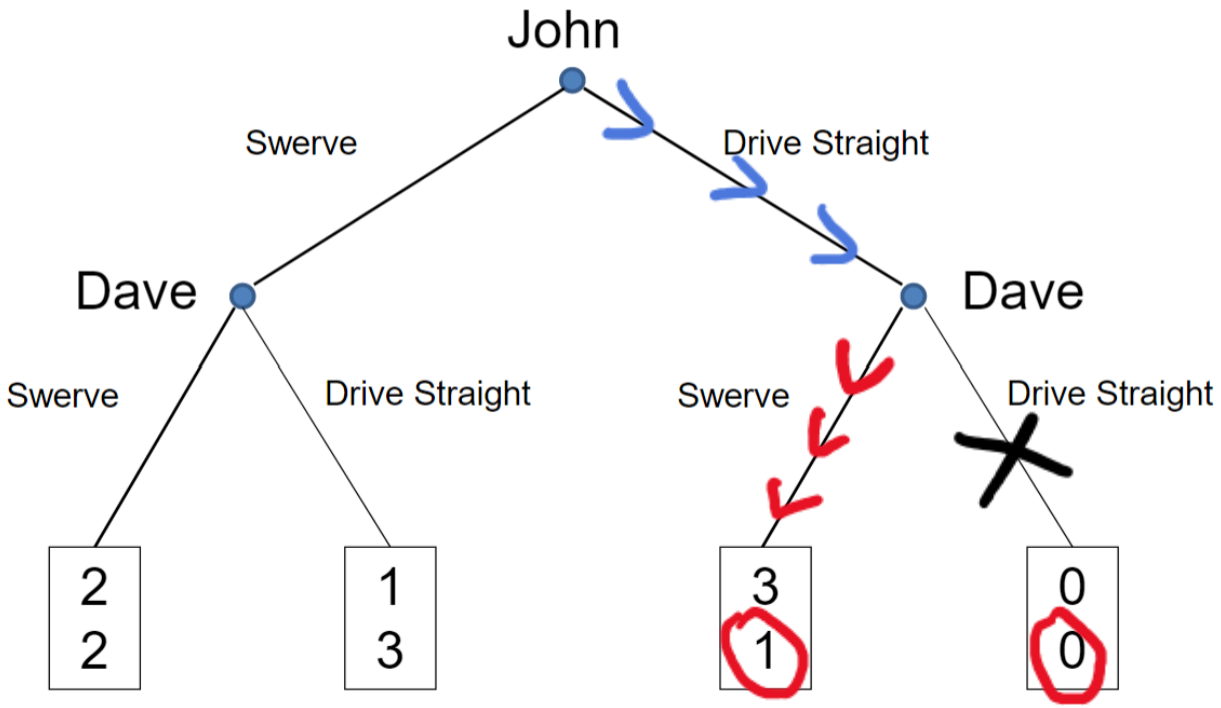
Given that John Drives Straight…

If Dave Swerves, Dave gets **1**.

If Dave Drives Straight, Dave gets **0**.

What should Dave do?

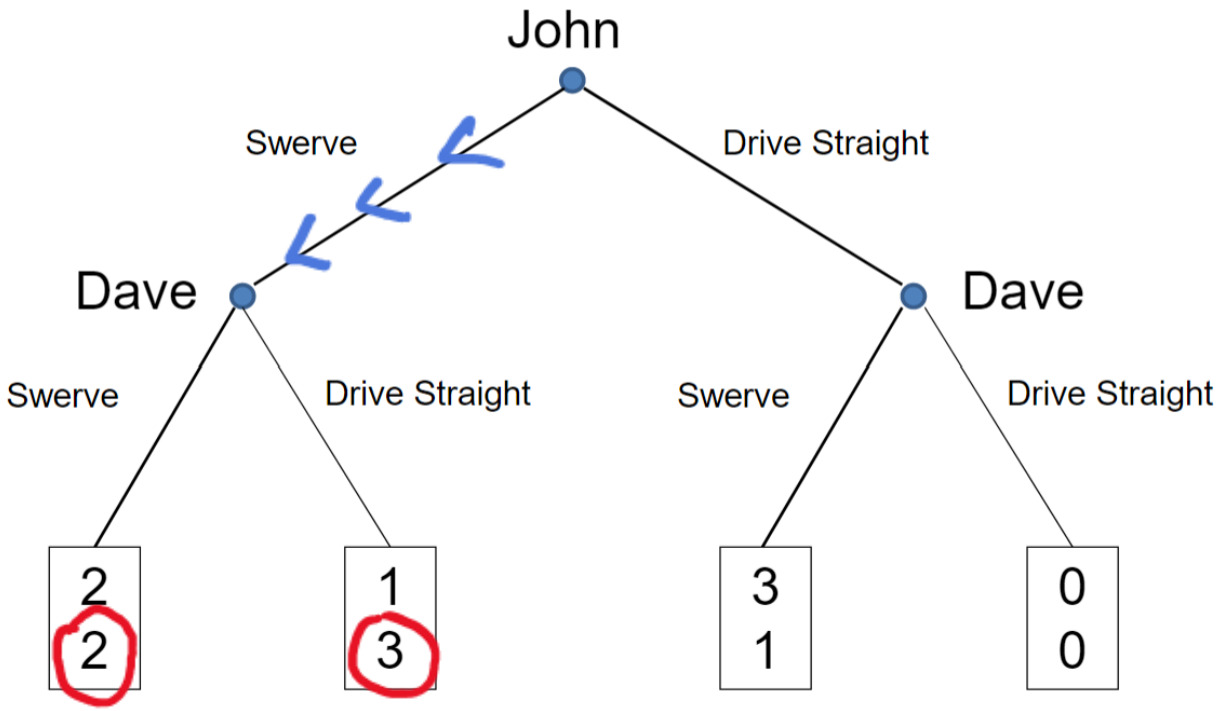
If John goes first and drives straight, what should Dave do?



Dave should Swerve b/c 1>0

Thus, Driving Straight is the **dominated strategy** (it is *dominated by* Swerving)

If John goes first and Swerves, what should Dave do?



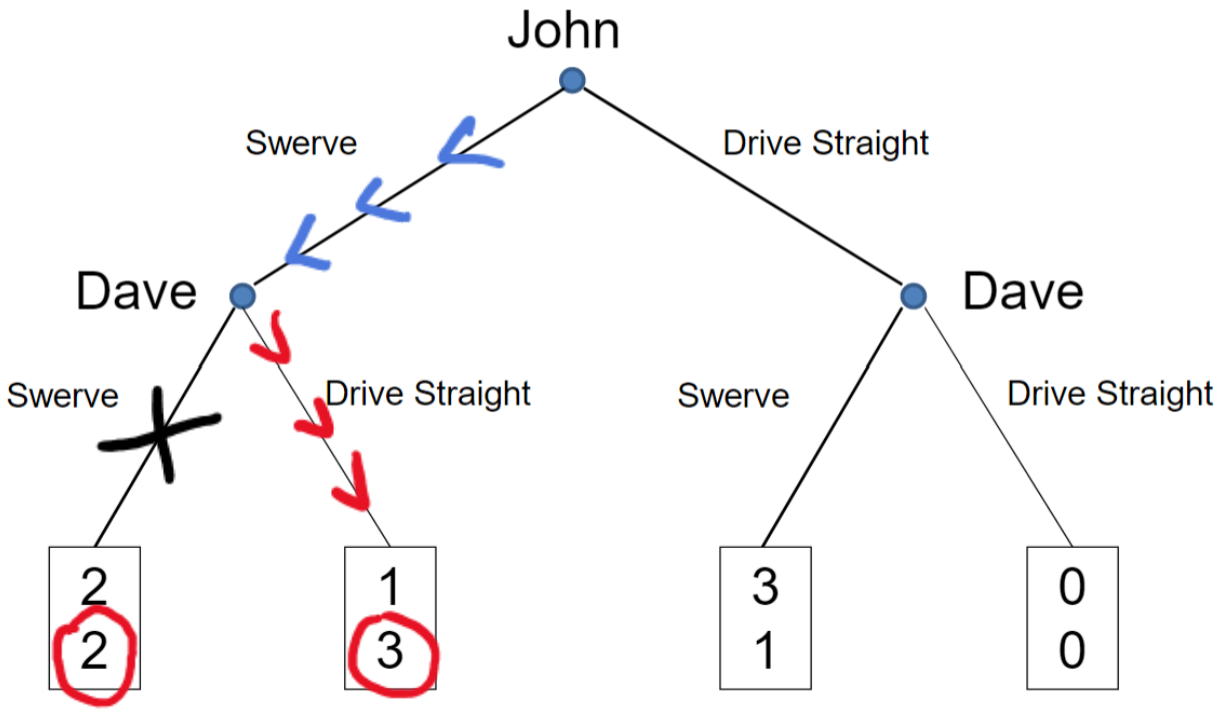
Given that John Swerves…

If Dave Swerves, Dave gets 2.

If Dave Drives Straight, Dave gets 3.

What should Dave do?

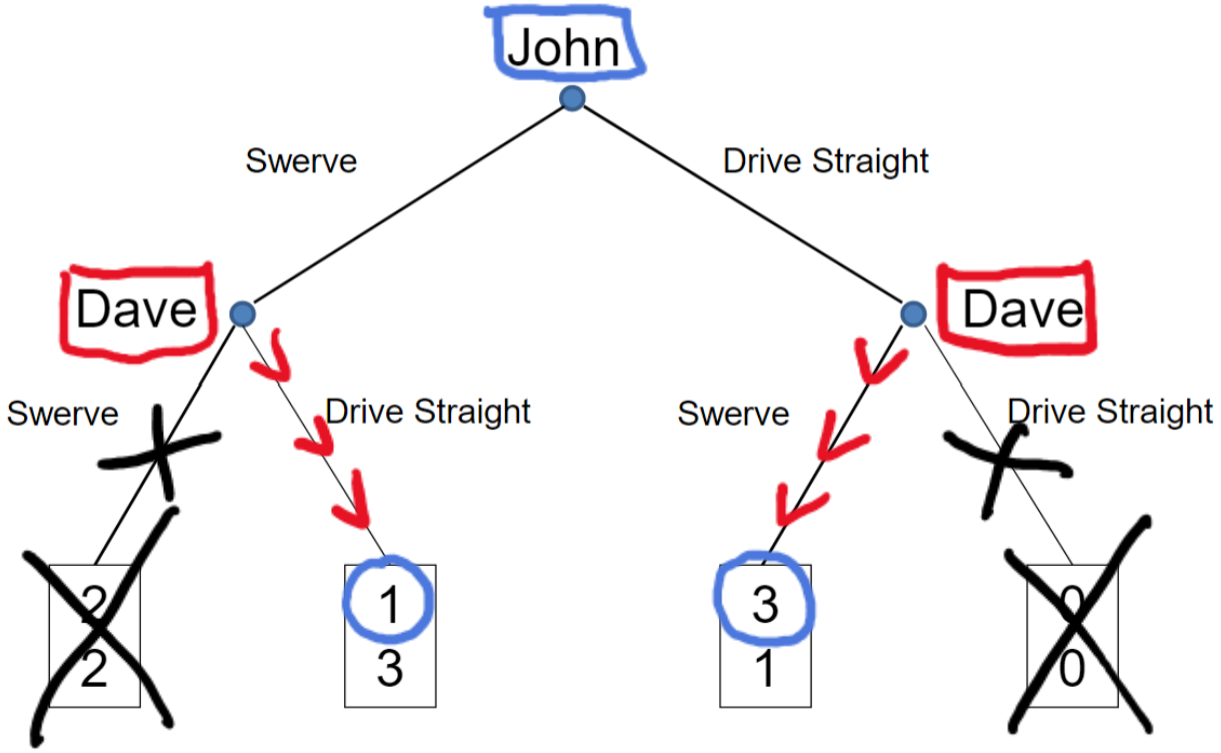
If John goes first and Swerves, what should Dave do?



Dave should Drive Straight b/c 3>2

Thus, Swerving is the **dominated strategy** (it is *dominated by* Driving Straight)

Given what we know Dave will do if John Swerves or Drives Straight, what should John do?

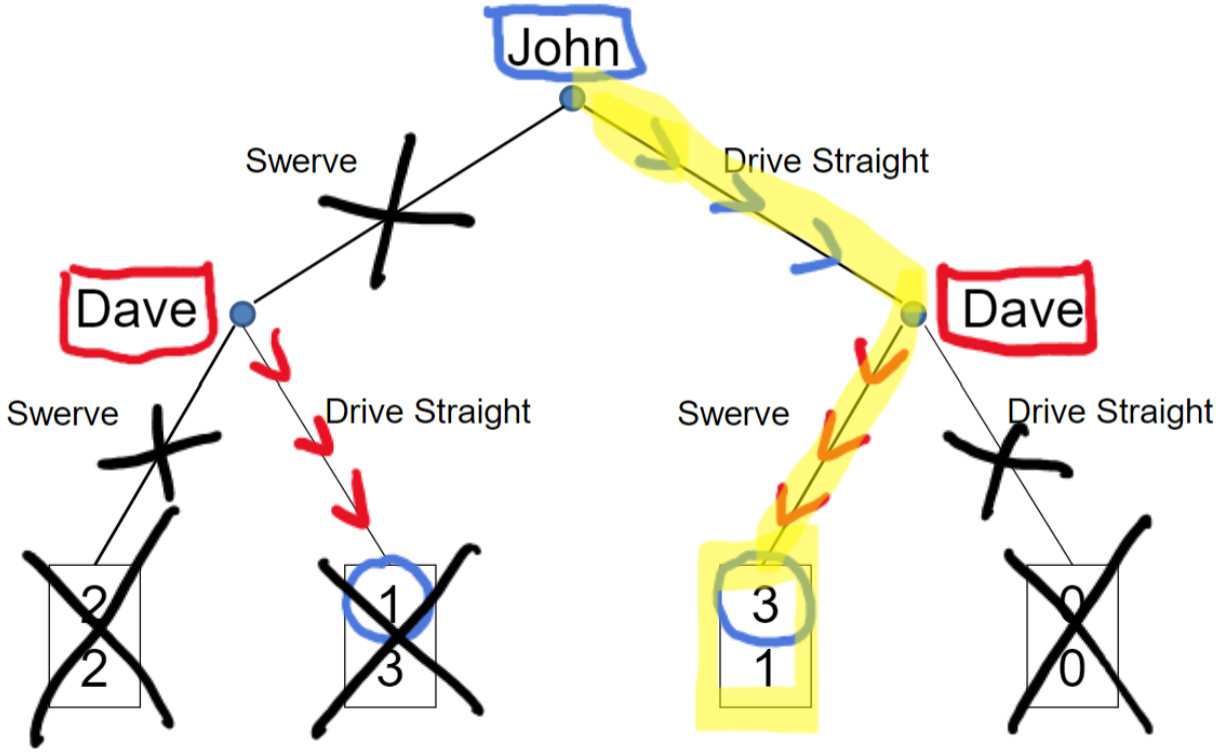


If John Swerves, we know Dave will Drive Straight, so John gets **1**.

If John Drives Straight, we know Dave will Swerve, so John gets **3.**

Should John Swerve or Drive Straight?

Given what we know Dave will do if John Swerves or Drives Straight, what should John do?



John will Drive Straight b/c 3>1.

Thus, for John, Swerving is a **dominated Strategy** (*dominated by* Driving Straight)

All we have time for now! Any questions?