# Coriolis and Centrifugal Forces

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# Derivation of Coriolis in a Rotating Framework

- Imagine a position vector **r (C in the figure),** rotating at a constant angular velocity ***Ω***
- From a stationary (Eulerian) view, we are viewing **r** rotate around the vertical axis at a constant speed ***Ω*** from some fixed point away from the system
- From a Lagrangian view, we are viewing the system from the moving position vector **r**
- Depending on frame (Eulerian or Lagrangian) of reference you view **r** from, ***we observe different motions: (WHY?)***
- The motion of the vector should be equivalent between the frames, so there should be an apparent force in one frame that equates them
- The change of a vector in the inertial (sub i) frame can be seen in the rotating frame (sub r) as
    - $(\delta \mathbf{B})_i = (\delta \mathbf{B})_r + \mathbf{\Omega} \times \mathbf{B}$
    - **B** is an arbitrary vector and $\mathbf{\Omega} \times \mathbf{B}$ is the velocity imparted by the rotating frame

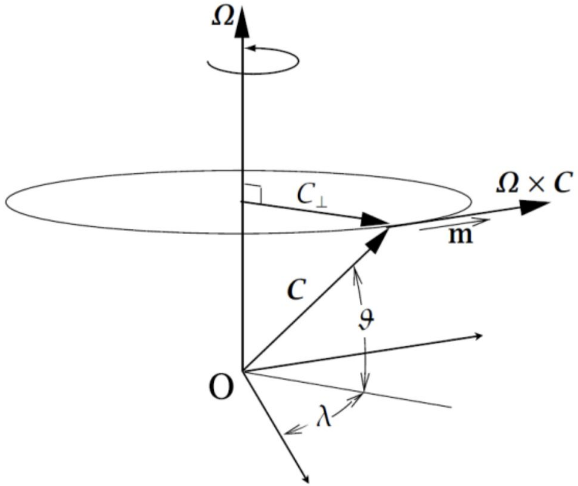



**Fig. 2.1** A vector $C$ rotating at an angular velocity $\Omega$. It appears to be a constant vector in the rotating frame, whereas in the inertial frame it evolves according to $(\mathrm{d}C/\mathrm{d}t)_I = \Omega \times C$.

- Now we can look at how a velocity vector $\mathbf{v}_i$ in the inertial frame changes in the rotating frame

$$\left(\frac{d\mathbf{v}_i}{dt}\right)_i = \boxed{\left(\frac{d\mathbf{v}_i}{dt}\right)_r} + \boldsymbol{\Omega} \times \boxed{\mathbf{v}_i}$$

$$= \boxed{\left(\frac{d}{dt}(\mathbf{v}_r + \boldsymbol{\Omega} \times \mathbf{r})\right)_r} + \boldsymbol{\Omega} \times \boxed{(\mathbf{v}_r + \boldsymbol{\Omega} \times \mathbf{r})}$$

$$= \left(\frac{d\mathbf{v}_r}{dt}\right)_r + \boldsymbol{\Omega} \times \left(\frac{d\mathbf{r}}{dt}\right)_r + \boldsymbol{\Omega} \times \mathbf{v}_r + \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r})$$

$$= \left(\frac{d\mathbf{v}_r}{dt}\right)_r + \boldsymbol{\Omega} \times \mathbf{v}_r + \boldsymbol{\Omega} \times \mathbf{v}_r + \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r})$$

$$= \left(\frac{d\mathbf{v}_r}{dt}\right)_r + 2\boldsymbol{\Omega} \times \mathbf{v}_r + \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r})$$

Recall: $(\delta\mathbf{B})_i = (\delta\mathbf{B})_r + \boldsymbol{\Omega}\times\mathbf{B}$

- Rearranging our in terms of $\mathbf{v}_i$ in the rotating reference frame, we see two new terms on the RHS which are the **Coriolis and Centrifugal Forces**
- There are **apparent** forces imparted by the rotating frame!

$$\boxed{\left(\frac{d\mathbf{v}_r}{dt}\right)_r = \left(\frac{d\mathbf{v}_i}{dt}\right)_i - 2\boldsymbol{\Omega} \times \mathbf{v}_r - \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r})}$$

# Magnitude of Coriolis and Centrifugal Forces

- Coriolis ~ 2$\boldsymbol{\Omega}$U = $2 * 7x10$^−5 $1/s * 10m/s$ = **1.4e-3 m/s**

$$\left(\frac{d\mathbf{v}_r}{dt}\right)_r = \left(\frac{d\mathbf{v}_i}{dt}\right)_i - 2\boldsymbol{\Omega} \times \mathbf{v}_r - \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r})$$

# What Does Coriolis do?

- In Cartesian coordinates, this is the vector components of Coriolis:

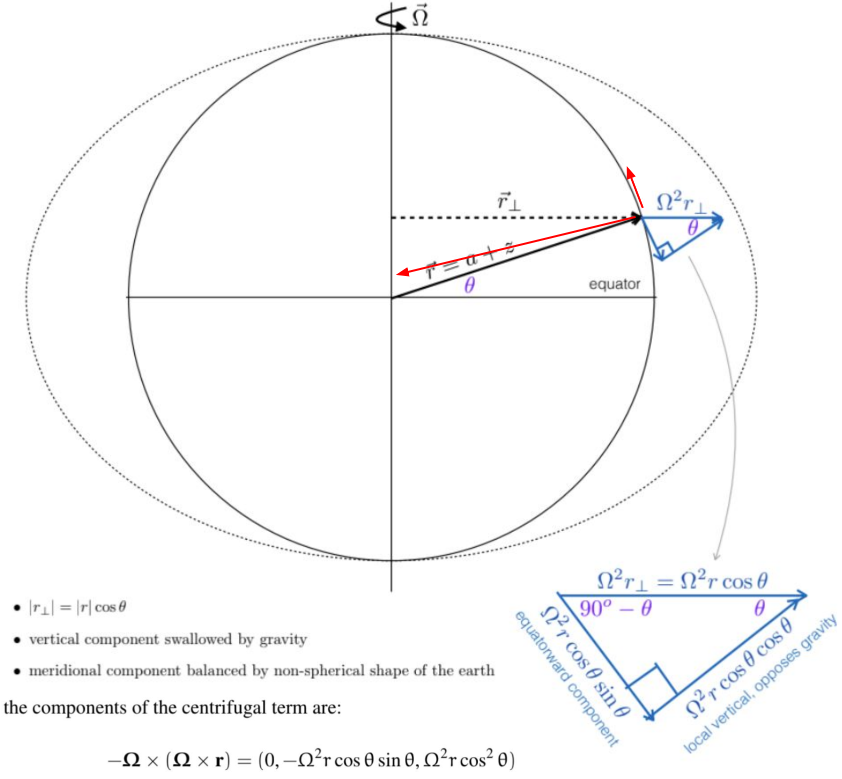
$$-2\boldsymbol{\Omega} \times \mathbf{v} = (-2\Omega w \cos\theta + 2\Omega v \sin\theta)\hat{\mathbf{i}} + -2\Omega u \sin\theta\hat{\mathbf{j}} + 2\Omega u \cos\theta\hat{\mathbf{k}}$$

- We can largely ignore the first term in ihat-direction and the khat term due to their small magnitudes compared to large-scale horizontal motions and gravity, respectively
- Therefore with:
    - +u (eastward motion) → effect of coriolis is southward motion
    - +v (northward motion) → effect of coriolis is eastward motion
        - This is why we see large systems being steering to the right in the Northern Hemisphere
    - And so on…
- Consider the acceleration the Coriolis force has on moving objects by
    - F=d^2x/dt^2 = dv/dt = 2$\boldsymbol{\Omega}$v_y*sin(Θ) → Integrate equation twice to get distance coriolis affects objects x = 2$\boldsymbol{\Omega}$v_y*sin(Θ); where v_y is velocity travelled in y-direction and a choose Θ = 45deg
    - A naval war gun is shot northward is swayed 40 whole meters to right east and even a rifle shot is affected by almost 4 meters

| Sporting event | $y$ | $v_y$ | $v_y$ | $t$ | $x$ |
|---|---|---|---|---|---|
| | (m) | (km h$^{-1}$) | (m s$^{-1}$) | (s) | (cm) |
| **Projectile type cases** | | | | | |
| Cricket: slow bowler's delivery[b] | 18 | 80 | 22.2 | 0.81 | 0.0752 |
| Archery | 65 | 300 | 83.3 | 0.78 | 0.261 |
| Cricket: throw from boundary[b] | 80 | 160 | 44.4 | 1.80 | 0.742 |
| Football: 60 m kick | 60 | 54 | 15 | 4.00 | 1.237 |
| Golf: drive from the tee[b] | 250 | 300 | 83.3 | 3.00 | 3.866 |
| Rifle shot | 500 | 1200 | 333 | 1.50 | 3.866 |
| World War 1 Naval gun | 24 000 | 2696 | 749 | 32.05 | 39.65 m |

# Derivation of Centrifugal in a Rotating Framework

- The centrifugal force is given by$Fce = -\Omega\times(\Omega\times r) = -\Omega\times(\Omega\times r\perp) = \Omega 2 r\perp$, where r⊥ is given by rcos(theta) where it is perpendicular to the axis of rotation
- This total vector is broken down into two components, one in the j-unit vector and one in the k-unit vector
- We notice a jhat vector pointing equatorward
    - So why are we sliding towards the Equator?
- This is because the Earth is an oblate spheroid and the mass has been distributed according to the long-term effect of the centrifugal force with there being more mass at the Equator
- The shape of the Earth now results in gravity that generates a term in the +jhat direction which opposes the -jhat centrifugal term
- For simplicity, the meridional terms (jhat) are considered to cancel out and are omitted
- By this we assume that the local vertical is now in the opposite direction of the effective gravity



# Centrifugal Force: Where is it?

- Centrifugal ~ ***Ω***^2a = (7*x*10^−5)^2 × 6 × 106m = **3e−2 m/s2**
- Coriolis ~ 2***Ω***U  = 2 ∗ 7*x*10^−5 1/*s* ∗ 10*m*/*s* = **1.4e-3 m/s**
- **As we can see Centrifugal > Coriolis by one order of magnitude**

So where is the Centrifugal term in the momentum equations?

- Centrifugal force is actually hidden in the potential on the RHS
- g is no longer the gravity constant 9.8m/s rather it is what is called effective gravity $\mathbf{g} \equiv \mathbf{g}_{eff} \equiv \mathbf{g}_{grav} + \Omega^2 \mathbf{r}_{\perp} = -\nabla\Phi$
- Since gravity (9.8m/s) is so much larger than the centrifugal force magnitude, it is conveniently placed in it

$$\frac{D\mathbf{v}}{Dt} + 2\boldsymbol{\Omega} \times \mathbf{v} = -\frac{1}{\rho}\nabla p - \boxed{\nabla\Phi}$$