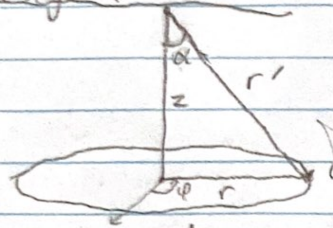
Ring $\vec{E}$ field:



$dq = \lambda dl = \lambda r d\varphi$

Horizontal components cancel by symmetry. We are left with vertical electric field. From one dq, we get

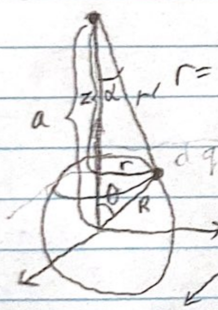
$$dE_z = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r'^2}\cos\alpha = \frac{1}{4\pi\varepsilon_0}\frac{\lambda r d\varphi}{r^2+z^2}\frac{z}{\sqrt{r^2+z^2}}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{\lambda r z\, d\varphi}{(r^2+z^2)^{3/2}}$$

Taking the integral about the ring, we get

$$E_z = \frac{\lambda r z}{4\pi\varepsilon_0(r^2+z^2)^{3/2}}\int_0^{2\pi} d\varphi = \frac{\lambda r z}{2\varepsilon_0(r^2+z^2)^{3/2}}$$

Spherical Shell $\vec{E}$ field:



$r = R\sin\theta$, $z = a - R\cos\theta$

$\lambda = \sigma R d\theta$

$$\Rightarrow dE_z = \frac{\lambda r z}{2\varepsilon_0(r^2+z^2)^{3/2}} = \frac{\sigma R^2 \sin\theta\,(a - R\cos\theta)\,d\theta}{2\varepsilon_0\left(R^2\sin^2\theta + a^2 - 2aR\cos\theta + R^2\cos^2\theta\right)^{3/2}}$$

$$= \frac{\sigma R^2 \sin\theta\,(a - R\cos\theta)\,d\theta}{2\varepsilon_0\,(R^2 + a^2 - 2aR\cos\theta)^{3/2}}$$

$$\Rightarrow E_z = \frac{\sigma R^2}{2\varepsilon_0}\int_0^{\pi}\frac{\sin\theta\,(a - R\cos\theta)\,d\theta}{(R^2+a^2-2aR\cos\theta)^{3/2}} = \frac{\sigma R^2}{2\varepsilon_0}\underbrace{\int_{-1}^{1}\frac{(a - Ru)\,du}{(R^2+a^2-2aRu)^{3/2}}}_{}$$

This is the integral that will be solved in the question