1. N/A
2. N/A
3. The correlation function shifts in the opposite direction of the shift, by the same amount. This makes sense because the peak of the correlation function should show us the shift on the second function at which the signals look most like each other.
4. The output array is the length of the sum of the two arrays.
5. (a) Let $\alpha = e^{-2\pi ik/N}$, and let $S = \sum_{x=0}^{N-1} \alpha^x$, i.e.

$$S = 1 + \alpha + \alpha^2 + ... + \alpha^{N-2} + \alpha^{N-1}$$

Then,

$$\begin{aligned} S - \alpha S &= (1 + \alpha + \alpha^2 + ... + \alpha^{N-2} + \alpha^{N-1}) - (\alpha + \alpha^2 + \alpha^3 + ... + \alpha^{N-1} + \alpha^N) \\ &= 1 + (\alpha - \alpha) + (\alpha^2 - \alpha^2) + ... + (\alpha^{N-1} - \alpha^{N-1}) - \alpha^N \\ &= 1 - \alpha^N \end{aligned}$$

Now we can solve for S:

$$S = \frac{1 - \alpha^N}{1 - \alpha}$$

Plugging back our original values for S and α, we get the final result

$$\sum_{x=0}^{N-1} e^{-2\pi ikx/N} = \frac{1 - e^{-2\pi ik}}{1 - e^{-2\pi ik/N}} \tag{1}$$

(b) First we want to find the limit

$$\lim_{k\to\infty} \frac{1 - e^{-2\pi ik}}{1 - e^{-2\pi ik/N}}$$

As per l'Hôpital's rule, we can take the derivative of the top and bottom, which then gives us the desired result:

$$\lim_{k\to 0} \frac{1 - e^{-2\pi ik}}{1 - e^{-2\pi ik/N}} = \lim_{k\to 0} \frac{2\pi i e^{-2\pi ik}}{2\pi i e^{-2\pi ik/N}/N} = \frac{2\pi i N}{2\pi i} = N$$

For any integer k, we know $e^{-2\pi ik} = e^0 = 1$, so the numerator of (1) goes to 0. In the case k is a multiple of N, k/N is an integer, so the denominator will also go to zero because $e^{-2\pi ik/N} = 1$. There we must look at the limit as we found above. If k is not a multiple of N, then k/N is not an integer, so $e^{-2\pi ik/N} \neq 1$; hence the denominator is non-zero. But the numerator still is so the whole thing goes to 0.

(c) For a sine wave

$$f(x) = \sin(2\pi k_0 x/N) = \frac{e^{2\pi ik_0x/N} - e^{-2\pi ik_0x/N}}{2i}$$

we can write the DFT

$$\begin{aligned} F(k) &= \sum_{x=0}^{N-1} \frac{e^{2\pi ik_0x/N} - e^{-2\pi ik_0x/N}}{2i} e^{-2\pi ikx/N} \\ &= \frac{1}{2j}\Big[\sum_{x=0}^{N-1} e^{-2\pi i(k-k_0)x/N} - \sum_{x=0}^{N-1} e^{-2\pi i(k+k_0)x/N}\Big] \\ &= \frac{1}{2j}\Big[\frac{1 - e^{-2\pi i(k-k_0)}}{1 - e^{-2\pi i(k-k_0)/N}} - \frac{1 - e^{-2\pi i(k+k_0)}}{1 - e^{-2\pi i(k+k_0)/N}}\Big] \end{aligned}$$

The plotting is not working for me but I expect it to look like a spike but instead of a steep spike, we have a gradual slope down to 0. This is because the discontinuities on a discrete transform of a non-integer frequency look like a curve that goes asymptotically to 0 as k goes to infinity.

(d) Once again my plotting is not working but I expect the FT should actually go to zero with the window function because there is no longer any sharp discontinuity. The spike will still be smoothed out a bit because there are some other frequency components to the signal.

6. (a) Nothings working for me and I do not have time to finish oh well