

1. Using the leapfrog scheme,

$$\begin{aligned}
f(x, t + dt) - f(x, t - dt) &= -v \frac{dt}{dx} [f(x + dx, t) - f(x - dx, t)] \\
\Rightarrow \xi^{t+dt} e^{ikx} - \xi^{t-dt} e^{ikx} &= -v \frac{dt}{dx} (\xi^t e^{ik(x+dx)} - \xi^t e^{ik(x-dx)}) \\
\Rightarrow \xi^{dt} - \xi^{-dt} &= -v \frac{dt}{dx} (e^{ikdx} - e^{-ikdx}) = -2iv \frac{dt}{dx} \sin(kdx) \\
\Rightarrow \xi^{2dt} - 1 &= -2i\xi^{dt} v \frac{dt}{dx} \sin(kdx)
\end{aligned}$$

Applying quadratic formula, we get

$$\xi^{dt} = -iv \frac{dt}{dx} \sin(kdx) \pm \sqrt{1 - (v \frac{dt}{dx} \sin(kdx))^2}$$

For  $\frac{vdt}{dx} > 1$ , the second term is complex so  $|\xi^{dt}|$ , and thus  $|\xi|$ , blows up. For  $\frac{vdt}{dx} < 1$ , the second term is real and we get exactly

$$\Rightarrow |\xi^{dt}|^2 = (v \frac{dt}{dx} \sin(kdx))^2 + 1 - (v \frac{dt}{dx} \sin(kdx))^2 = 1$$

Therefore  $|\xi| = 1$  as well, and since it is exactly 1, there is no energy change. That is to say, energy is preserved.

2. (a) I think my Green's function is a little off, but it worked for the other problems so I did not change it. I got  $V[1,0]=0.779$  and  $V[2,0]=0.669$ .  
(b) A 2D view of the charge can be seen in `square_box_charge.png`. The charge along one edge is shown in `oneside_charge.png`.  
(c) The potential everywhere in space can be seen in `square_box_potential.png`. The potential inside is very close to constant, with mean potential 1.0026 and standard deviation 0.00019. A detailed view of the interior potential can be seen in `interior_potential.png`, showing slight increases towards the corners. The field I calculated can be seen in `square_box_field.png`. I made it lower resolution so that the field lines would be clearer. It follows what we expect: field lines generally perpendicular to equipotentials except for edge effects, and stronger field strength near the corners.