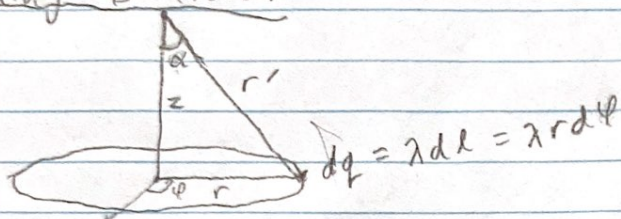


Ring \vec{E} field:



Horizontal components cancel by symmetry. We are left with vertical electric field. From one dq , we get

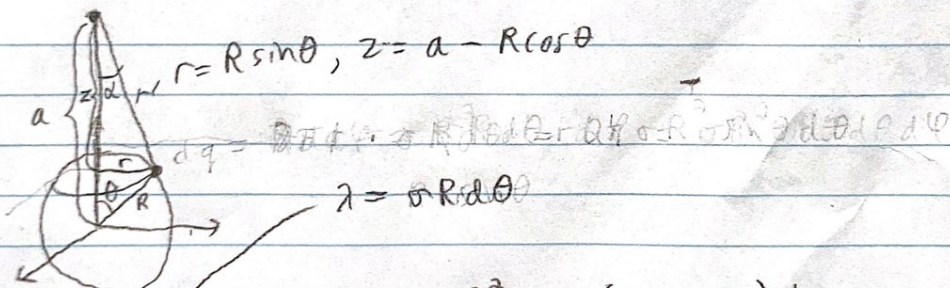
$$dE_z = \frac{1}{4\pi\epsilon_0} \frac{dq}{r'^2} \cos\alpha = \frac{1}{4\pi\epsilon_0} \frac{\lambda r d\phi}{r^2 + z^2} \frac{z}{\sqrt{r^2 + z^2}}$$

$$= \frac{\lambda r z d\phi}{4\pi\epsilon_0 (r^2 + z^2)^{3/2}}$$

Taking the integral about the ring, we get

$$E_z = \frac{\lambda r z}{4\pi\epsilon_0 (r^2 + z^2)^{3/2}} \int_0^{2\pi} d\phi = \frac{\lambda r z}{2\epsilon_0 (r^2 + z^2)^{3/2}}$$

Spherical shell \vec{E} field:



$$\Rightarrow dE_z = \frac{\lambda r z}{2\epsilon_0 (r^2 + z^2)^{3/2}} = \frac{\sigma R^2 \sin\theta (a - R \cos\theta) d\theta}{2\epsilon_0 (R^2 \sin^2\theta + a^2 - 2aR \cos\theta + R^2 \cos^2\theta)^{3/2}}$$

$$= \frac{\sigma R^2 \sin\theta (a - R \cos\theta) d\theta}{2\epsilon_0 (R^2 + a^2 - 2aR \cos\theta)^{3/2}}$$

$$\Rightarrow E_z = \frac{\sigma R^2}{2\epsilon_0} \int_{-\pi/2}^{\pi/2} \frac{\sin\theta (a - R \cos\theta) d\theta}{(R^2 + a^2 - 2aR \cos\theta)^{3/2}}$$

This is the integral that will be solved in the question