a) We can expand the equation:

$$\begin{aligned} z &= a(x^2 - 2xx_0 + x_0^2 + y^2 - 2yy_0 + y_0^2) + z_0 \\ &= a(x^2 + y^2) - 2ax_0x - 2ay_0y + (ay_0^2 + ax_0^2 + z_0) \\ &= a(x^2 + y^2) - bx - cy + d \end{aligned}$$

Where a, b, c, and d are our new parameters, and the original parameters can be found:

$$\begin{aligned} a &= a \\ x_0 &= \frac{b}{2a} \\ y_0 &= \frac{c}{2a} \\ z_0 &= d - \frac{1}{4a}(b^2 + c^2) \end{aligned}$$

This problem is now linear in these new parameters!

b) The best-fit parameters are shown in p3_out.txt. First are the modified parameters a, b, c, and d, and then the original parameters a, x_0, y_0, and z_0.

c) Using the noise matrix I obtained from my fit, I estimated the error on a to be about 6.45×10^{-8}. For our paraboloid, we can rewrite

$$z - z_0 = a((x - x_0)^2 + (y - y_0)^2) = \frac{1}{4f}((x - x_0)^2 + (y - y_0)^2)$$

so $f = \frac{1}{4a}$. Then $\frac{df}{da} = -\frac{1}{4a^2}$ and we approximate $df = |\frac{df}{da}|da = \frac{1}{4a^2}da \approx 0.58\,m$. See p3_out.txt for output of my program. The focal length found was very close to 1.5 metres.