1. Using the leapfrog scheme,

$$\begin{aligned}
&f(x,t+dt) - f(x,t-dt) = -v\frac{dt}{dx}[f(x+dx,t) - f(x-dx,t)] \\
\implies &\xi^{t+dt}e^{ikx} - \xi^{t-dt}e^{ikx} = -v\frac{dt}{dx}(\xi^t e^{ik(x+dx)} - \xi^t e^{ik(x-dx)}) \\
\implies &\xi^{dt} - \xi^{-dt} = -v\frac{dt}{dx}(e^{ikdx} - e^{-ikdx}) = -2iv\frac{dt}{dx}\sin(kdx) \\
\implies &\xi^{2dt} - 1 = -2i\xi^{dt}v\frac{dt}{dx}\sin(kdx)
\end{aligned}$$

Applying quadratic formula, we get

$$\xi^{dt} = -iv\frac{dt}{dx}\sin(kdx) \pm \sqrt{1 - (v\frac{dt}{dx}\sin(kdx))^2}$$

For $\frac{vdt}{dx} > 1$, the second term is complex so $|\xi^{dt}| > 1$, and thus $|\xi| > 1$, so our solution is unstable. For $\frac{vdt}{dx} < 1$, the second term is real and we get exactly

$$\implies |\xi^{dt}|^2 = (v\frac{dt}{dx}\sin(kdx))^2 + 1 - (v\frac{dt}{dx}\sin(kdx))^2 = 1$$

Therefore $|\xi| = 1$ as well, and since it is exactly 1, there is no energy change. That is to say, energy is preserved.

2. (a) I got V[1,0]=0 and V[2,0]=-0.5.

   (b) A 2D view of the charge can be seen in square_box_charge.png. The charge along one edge is shown in oneside_charge.png.

   (c) The potential everywhere in space can be seen in square_box_potential.png. The potential inside is very close to constant, with mean potential 1.0026 and standard deviation 0.00019. A detailed view of the interior potential can be seen in interior_potential.png, showing slight increases towards the corners. The field I calculated can be seen in square_box_field.png. I made it lower resolution so that the field lines would be clearer. It follows what we expect: field lines generally perpendicular to equipotentials except for edge effects, and stronger field strength near the corners.