A Appendix

A.1 Proof of Theorem 1

Thanks to Section III.B and III.C, one can compute $C1$, S_2 and C_3 systems.

$$\begin{cases} \psi_1 = \psi_0 + \alpha_1 \\ X_1 = X_p^0 + \delta_1 R \begin{pmatrix} -\sin(\psi_0) + \sin(\psi_1) \\ \cos(\psi_0) - \cos(\psi_1) \end{pmatrix} \\ t_1 = t_0 + \frac{R}{v_p}\delta_1\alpha_1 \end{cases}$$

$$\begin{cases} \psi_2 = \psi_1 \\ X_2 = X_1 + d_2 \begin{pmatrix} cos(\psi_1) \\ sin(\psi_1) \end{pmatrix} \\ t_2 = t_1 + \frac{d_2}{v_p} \end{cases}$$

$$\begin{cases} \psi_3 = \psi_2 + \alpha_3 \\ X_3 = X_2 + \delta_3 R \begin{pmatrix} -\sin(\psi_2) + \sin(\psi_3) \\ \cos(\psi_2) - \cos(\psi_3) \end{pmatrix} \\ t_3 = t_2 + \frac{R}{v_p}\delta_3\alpha_3 \end{cases}$$

Note that:
— t_3 is the interception time noted t_f;
— during straight line, heading does not change so $\psi_2 = \psi_1$;
— pursuer and target final heading are equal so $\psi_3 = \psi_t$;
— pursuer and target final position are equal so $X_3 = X(t_f) = X_f$
Thus, one gets:

$$\psi_t = \psi_2 + \alpha_3 = \psi_1 + \alpha_3 = \psi_0 + \alpha_1 + \alpha_3$$

$$X_f = X_p^0 + \delta_1 R \begin{pmatrix} -\sin(\psi_0) + \sin(\psi_1) \\ \cos(\psi_0) - \cos(\psi_1) \end{pmatrix} + d_2 \begin{pmatrix} cos(\psi_1) \\ sin(\psi_1) \end{pmatrix} + \delta_3 R \begin{pmatrix} -\sin(\psi_1) + \sin(\psi_t) \\ \cos(\psi_1) - \cos(\psi_t) \end{pmatrix}$$

$$t_f = t_2 + \frac{R}{v_p}\delta_3\alpha_3 = t_1 + \frac{d_2}{v_p} + \frac{R}{v_p}\delta_3\alpha_3 = t_0 + \frac{d_2}{v_p} + \frac{R}{v_p}(\delta_1\alpha_1 + \delta_3\alpha_3)$$

Thus, one gets the system:

$$\begin{cases} \psi_t - \psi_0 = \alpha_1 + \alpha_3 \\ X_f = X_p^0 + R \begin{pmatrix} -\delta_1 \sin(\psi_0) + (\delta_1 - \delta_3)\sin(\psi_1) + \delta_3 \sin(\psi_t) \\ \delta_1 \cos(\psi_0) - (\delta_1 - \delta_3)\cos(\psi_1) - \delta_3 \cos(\psi_t) \end{pmatrix} + d_2 \begin{pmatrix} cos(\psi_1) \\ sin(\psi_1) \end{pmatrix} \\ t_f - t_0 = \frac{d_2}{v_p} + \frac{R}{v_p}(\delta_1\alpha_1 + \delta_3\alpha_3) \end{cases} \quad (1)$$

However, according to the target dynamics:

$$X_f = X_t^0 + v_t \begin{pmatrix} \cos(\psi_t) \\ \sin(\psi_t) \end{pmatrix} . (t_f - t_0) \tag{2}$$

Last line of (1) gives:

$$X_f = X_t^0 + \frac{v_t}{v_p} \begin{pmatrix} \cos(\psi_t) \\ \sin(\psi_t) \end{pmatrix} (d_2 + R(\delta_1 \alpha_1 + \delta_3 \alpha_3)) \tag{3}$$

But $\alpha_3 = \psi_t - \psi_0 - \alpha_1$, so:

$$X_f = X_t^0 + \frac{v_t}{v_p} \begin{pmatrix} \cos(\psi_t) \\ \sin(\psi_t) \end{pmatrix} (d_2 + R\alpha_1(\delta_1 - \delta_3) + R\delta_3(\psi_t - \psi_0)) \tag{4}$$

Finally, (4) and (1) give:

$$X_t^0 + \frac{v_t}{v_p} \begin{pmatrix} \cos(\psi_t) \\ \sin(\psi_t) \end{pmatrix} (d_2 + (\delta_1 - \delta_3) R\alpha_1 + R\delta_3(\psi_t - \psi_0)) = X_p^0 + R \begin{pmatrix} -\delta_1 \sin(\psi_0) + (\delta_1 - \delta_3)\sin(\psi_0 + \alpha_1) + \\ \delta_1 \cos(\psi_0) - (\delta_1 - \delta_3)\cos(\psi_0 + \alpha_1) - \end{pmatrix} \tag{5}$$

By putting the left part of the equality independent of α_1 and d_2, one gets:

$$X_t^0 - X_p^0 - R \begin{pmatrix} -\delta_1 \sin(\psi_0) + \delta_3 \sin(\psi_t) \\ \delta_1 \cos(\psi_0) - \delta_3 \cos(\psi_t) \end{pmatrix} + \frac{v_t}{v_p} R\delta_3 (\psi_t - \psi_0) \begin{pmatrix} \cos(\psi_t) \\ \sin(\psi_t) \end{pmatrix} = d_2 \left(\begin{pmatrix} \cos(\psi_0 + \alpha_1) \\ \sin(\psi_0 + \alpha_1) \end{pmatrix} - \frac{v_t}{v_p} \begin{pmatrix} \cos(\psi_t \\ \sin(\psi_t \end{pmatrix} \right. \tag{6}$$

A.2 Proof of Theorem 2

If $\delta = \delta_1 = \delta_3$, according to Theorem 1, one gets:

$$A = d_2 \left(\begin{pmatrix} cos(\psi_0 + \alpha_1) \\ sin(\psi_0 + \alpha_1) \end{pmatrix} - \frac{v_t}{v_p} \begin{pmatrix} \cos(\psi_t) \\ \sin(\psi_t) \end{pmatrix} \right) \tag{7}$$

Let us define $A = [A_x, A_y].\top = X_t^0 - X_p^0 - \delta R \begin{pmatrix} -\sin(\psi_0) + \sin(\psi_t) \\ \cos(\psi_0) - \cos(\psi_t) \end{pmatrix} - \frac{v_t}{v_p} R\delta(\psi_t - \psi_0) \begin{pmatrix} \cos(\psi_t) \\ \sin(\psi_t) \end{pmatrix}$. Note that A is a 2D vector independent from ψ_1 and d_2. Note $l = d_2^{-1}$ whose existence will be discussed after, one gets:

$$\begin{pmatrix} cos(\psi_1) \\ sin(\psi_1) \end{pmatrix} = Al + \frac{v_t}{v_p} \begin{pmatrix} \cos(\psi_t) \\ \sin(\psi_t) \end{pmatrix} \tag{8}$$

By taking the norm in (8), one gets:

$$1 = (A_x l + \frac{v_t}{v_p} cos\psi_t)^2 + (A_y l + \frac{v_t}{v_p} sin\psi_t)^2 \tag{9}$$

$$1 = (A_x^2 + A_y^2) l^2 + 2\frac{v_t}{v_p}(A_x cos\psi_t + A_y sin\psi_t) l + \left(\frac{v_t}{v_p}\right)^2 \tag{10}$$

$$(A_x^2 + A_y^2).l^2 + 2\frac{v_t}{v_p}(A_x cos\psi_t + A_y sin\psi_t) l + \left(\frac{v_t}{v_p}\right)^2 - 1 = 0 \tag{11}$$

One gets a second degree polynom in l to solve:.

$$\Delta = 4\left(\frac{v_t}{v_p}\right)^2 (A_x cos\psi_t + A_y sin\psi_t)^2 - 4(A_x^2 + A_y^2)\left(\frac{v_t^2}{v_p^2} - 1\right) \tag{12}$$

Note that $(A_x^2 + A_y^2) = \|A\|^2$and $(A_x cos\psi_t + A_y sin\psi_t)^2 = \left\langle A, \begin{pmatrix}\cos\psi_t\\ \sin\psi_t\end{pmatrix}\right\rangle^2$. So,

$$\Delta = 4\left(\frac{v_t}{v_p}\right)^2 \left\langle A, \begin{pmatrix}\cos\psi_t\\ \sin\psi_t\end{pmatrix}\right\rangle^2 + 4\left\|A\right\|^2\left(1 - \frac{v_t^2}{v_p^2}\right) \tag{13}$$

$$\Delta = 4\left(\frac{v_t}{v_p}\right)^2 \left\langle A, \begin{pmatrix}\cos\psi_t\\ \sin\psi_t\end{pmatrix}\right\rangle^2 + \frac{4}{v_p^2}\left\|A\right\|^2\left(v_p^2 - v_t^2\right) \tag{14}$$

Note that $\Delta \geq 0$ as $v_p \geq v_t \geq 0$, so:

$$l = \frac{-2\frac{v_t}{v_p}\left\langle A, \begin{pmatrix}\cos\psi_t\\ \sin\psi_t\end{pmatrix}\right\rangle + 2\sqrt{\left(\frac{v_t}{v_p}\left\langle A, \begin{pmatrix}\cos\psi_t\\ \sin\psi_t\end{pmatrix}\right\rangle\right)^2 + \left(\frac{\|A\|}{v_p}\right)^2\left(v_p^2 - v_t^2\right)}}{2\left\|A\right\|^2} \tag{15}$$

Or,

$$l = -\frac{2\frac{v_t}{v_p}\left\langle A, \begin{pmatrix}\cos\psi_t\\ \sin\psi_t\end{pmatrix}\right\rangle + 2\sqrt{\left(\frac{v_t}{v_p}\left\langle A, \begin{pmatrix}\cos\psi_t\\ \sin\psi_t\end{pmatrix}\right\rangle\right)^2 + \left(\frac{\|A\|}{v_p}\right)^2\left(v_p^2 - v_t^2\right)}}{2\left\|A\right\|^2} \tag{16}$$

This last case is impossible since the distance $d_2 = l^{-1}$ would be negative. Finally,

$$d_2 = \frac{\left\|A\right\|^2}{-\frac{v_t}{v_p}\left\langle A, \begin{pmatrix}\cos\psi_t\\ \sin\psi_t\end{pmatrix}\right\rangle + \sqrt{\left(\frac{v_t}{v_p}\left\langle A, \begin{pmatrix}\cos\psi_t\\ \sin\psi_t\end{pmatrix}\right\rangle\right)^2 + \left(\frac{\|A\|}{v_p}\right)^2\left(v_p^2 - v_t^2\right)}} \tag{17}$$

Note that $d_2$exists only iff :

$$\left(\frac{v_t}{v_p}\left\langle A, \begin{pmatrix}\cos\psi_t\\ \sin\psi_t\end{pmatrix}\right\rangle\right)^2 + \left(\frac{\|A\|}{v_p}\right)^2\left(v_p^2 - v_t^2\right) \geq 0$$

which is true since $v_p \geqslant v_t$, and

$$\sqrt{\left(\frac{v_t}{v_p}\left\langle A, \begin{pmatrix}\cos\psi_t\\ \sin\psi_t\end{pmatrix}\right\rangle\right)^2 + \left(\frac{\|A\|}{v_p}\right)^2\left(v_p^2 - v_t^2\right)} \geqq \frac{v_t}{v_p}\left\langle A, \begin{pmatrix}\cos\psi_t\\ \sin\psi_t\end{pmatrix}\right\rangle$$

i.e.

$$\left(\frac{v_t}{v_p}\right)^2 \left\langle A, \begin{pmatrix}\cos\psi_t\\ \sin\psi_t\end{pmatrix}\right\rangle^2 + \left(\frac{\|A\|}{v_p}\right)^2 \left(v_p^2 - v_t^2\right) \geq \left(\frac{v_t}{v_p}\right)^2 \left\langle A, \begin{pmatrix}\cos\psi_t\\ \sin\psi_t\end{pmatrix}\right\rangle^2$$

i.e.

$$\left(\frac{\|A\|}{v_p}\right)^2 \left(v_p^2 - v_t^2\right) \geq 0$$

which is true since $v_p \geqslant v_t$. Finally, d_2 exists.
Also, (8) can be split in two lines:

$$\begin{cases} cos\left(\psi_1\right) = \frac{A_x}{d_2} + \frac{v_t}{v_p} cos\psi_t \\ sin\left(\psi_1\right) = \frac{A_y}{d_2} + \frac{v_t}{v_p} sin\psi_t \end{cases} \tag{18}$$

Which gives:

$$\psi_1 = \arctan 2\left(\frac{A_y}{d_2} + \frac{v_t}{v_p} sin\psi_t, \frac{A_x}{d_2} + \frac{v_t}{v_p} cos\psi_t\right) \tag{19}$$

And by definition:

$$\alpha_1 = \psi_1 - \psi_0 \mod (2\pi)$$

$$\alpha_3 = \psi_f - \psi_1 \mod (2\pi)$$

And:

$$t_f - t_0 = \frac{d_2}{v_p} + \delta\frac{R}{v_p}\left(\alpha_1 + \alpha_3\right)$$

A.3 Proof of Theorem 3

If $\delta = \delta_1 = -\delta_3$, according to Theorem 1, one gets:

$$A = d_2\left(\begin{pmatrix} cos(\psi_0+\alpha_1)\\ sin(\psi_0+\alpha_1)\end{pmatrix} - \frac{v_t}{v_p}\begin{pmatrix}\cos\left(\psi_t\right)\\ \sin\left(\psi_t\right)\end{pmatrix}\right) + 2R\delta\left(\begin{pmatrix}\sin\left(\psi_0+\alpha_1\right)\\ -\cos\left(\psi_0+\alpha_1\right)\end{pmatrix} - \alpha_1\frac{v_t}{v_p}\begin{pmatrix}\cos\left(\psi_t\right)\\ \sin\left(\psi_t\right)\end{pmatrix}\right) \tag{20}$$

$$A = X_t^0 - X_p^0 - R\delta\begin{pmatrix}-\sin\left(\psi_0\right) - \sin\left(\psi_t\right)\\ \cos\left(\psi_0\right) + \cos\left(\psi_t\right)\end{pmatrix} - \frac{v_t}{v_p} R\delta\left(\psi_t - \psi_0\right)\begin{pmatrix}\cos\left(\psi_t\right)\\ \sin\left(\psi_t\right)\end{pmatrix} \tag{21}$$

And by definition:

$$\alpha_1 = \psi_1 - \psi_0 \mod (2\pi)$$

$$\alpha_3 = \psi_f - \psi_1 \mod (2\pi)$$

And, $\mathrm{t}_f - t_0 = \frac{d_2}{v_p} + \delta\frac{R}{v_p}\left(\alpha_1 - \alpha_3\right)$