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6. ~~a~~ For assertion 1:

~~If V and W share vector other than the $\vec{0}$ vector, then $\vec{u} = \vec{v} + \vec{w}$ can be written as $\vec{u} = \vec{w} + \vec{0}$ and $\vec{u} = \vec{v} + \vec{0}$ where $\vec{v} = \vec{w}$.~~

~~This means $\vec{u}$ could be written as $\vec{v} + \vec{w}$ in a non-unique way~~

Assume $\vec{V}_1 = \vec{\omega}_1 \neq 0 = \vec{x}$

$\vec{U} = \vec{V}_1 + \vec{\omega}_1 \quad \vec{U} = \vec{x} + \vec{x} = 2\vec{x}$

$\vec{U} = \frac{1}{2} V_1 + \frac{3}{2} \vec{\omega}_1 \quad \vec{U} = \frac{1}{2}\vec{x} + \frac{3}{2}\vec{x} = \frac{4}{2}\vec{x} = 2\vec{x}$

$\vec{U} = \vec{V} + \vec{\omega}$ is not unique if $\vec{V} = \vec{\omega}$ for any value.

Assertion 2:

Since $\vec{V} \neq \vec{\omega}$ for all vectors except $\vec{0}$

V and W may not share any coefficients/basis vectors.

Proof:

Assume $V = \text{span}(x_1, x_2, x_3, x_4 \ldots x_m)$

Assume $W = \text{span}(x_m, x_{m+1}, x_{m+3} \ldots x_n)$

If V and W share basis vector x_m, one could set all other basises to the $\vec{0}$ vector, and set x_m to $1x_m$. Then $\vec{V}$ would be equal to $\vec{\omega}$ which is a contradiction.

~~This means $\vec{v} + \vec{\omega}$ always ... where $\dim(V) = n$ and $\dim(W) = m$ ... $\dim(U)$ must be ...~~

If we therefore define V as $\text{span}(x_1, x_2, \ldots, x_n)$ and W as $\text{span}(y_1, y_2, \ldots, y_m)$ where $x \neq y$, and $\dim(V) = n$ and $\dim(W) = m$, any $\vec{V} + \vec{\omega}$ has dimension $n+m$, so any $\vec{U}$ has dimension $n+m$

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