

Final Exam – Part C – SF1610 Discrete Mathematics - TCOMK

Examiner: Armin Halilovic

Course responsible: Ivan Martino

Date: 2020-05-27

You may join for the second part of the exam only if you have attended the first one.

Part C – Time: 11:30 – 13:30 (Extra-time students: 13:00 – 16:00)

Right after, you have 20 minutes to take pictures and upload your solution on Canvas.

Short summary of the rules of the exam:

1. Use your computer only to read the questions of the exam.
 2. If you may, use your phone for the Zoom-meeting call and place it so that your desk is visible; if you are not using your phone for the Zoom-meeting call, then you cannot use your phone for the whole exam.
 3. No calculator, books, notes, lecture notes are allowed.
 4. You may use your phone during the 20 minute break to take picture of your solution. You still need to be visible while doing so, hence you need to be in the Zoom-call from another device.
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The exam consists of 10 questions in three parts: A, B and C. **Problems are not ordered by difficulty.** This exam provides a total of 39 points, plus any extra bonus points from the partial exams. The bonus point collected during the semester will be extra to the final exam score.

IMP. The full points will be given only to complete and fully explained solutions.

Evaluation table:

13 points total or more – give at least the grade of Fx
15 points total or more – give at least the grade of E
18 points total or more – give at least the grade of D
22 points total or more – give at least the grade of C
27 points total or more – give at least the grade of B
32 points total or more – give at least the grade of A

You should use paper and pen to solve the following exercises. You scan/take pictures of your solutions (jpg, jpeg, png, pdf format). Then, you have to upload your solutions gathered in a folder and compressed (as a zip or rar file) to Canvas / tasks / Final Exam – May 27, 2020 – Part 2: C.

Extra-time students should use a different folder Canvas / tasks / Final Exam – May 27, 2020 – Part 2: C – Extra time students.

IMPORTANT: The folder name should contain your last name and name; in other words use the NAME_FIRSTNAME as name of the folder with your solutions.

Write names and social security numbers on each sheet. On the first sheet write "I have done this Final exam by myself" and sign it. So you declare that you have made final exam by yourself.

The parameters p and q in the information below are the last two digits of your social security number. For example: If your social security number is 751332 2248 then $p = 4$ and $q = 8$.

PART C

9. (2p) Let G be a graph with $n = 7 + (p \bmod 3)$ vertices.

a) Determine the minimum number of edges that G can have to be a non-planar (connected or not-connected) graph with n vertices.

b) Determine the minimum number of edges that G can have to be non-planar connected graph with n vertices.

Note. A not-connected graph could be a planar graph.

IMP. Only a full explained complete solution will get points.

10. (2p) Let $K = 20 + p$. Compute the following sums

a)
$$\sum_{0 \leq r \leq n \leq K} \binom{K}{n} \cdot \binom{n}{r}$$

b)
$$\sum_{0 \leq r \leq n \leq m \leq K} \binom{m}{n} \cdot \binom{n}{r}$$

IMP. Only a full explained complete solution will get points.

11. (3p) Let $K = 3 + (q \bmod 2)$. Find all integers solutions x to the following system of modular equations:

$$\begin{cases} x \equiv 2 \pmod{K} \\ x \equiv 3 \pmod{7} \\ x \equiv 5 \pmod{11}. \end{cases}$$

Note. In other words, x must be a solution of all the above modular equations.

IMP. Only a full explained complete solution will get points.

12. (5p) Two groups $(G_1, *)$ and (G_2, \circ) are isomorphic if there is a bijection $f: G_1 \rightarrow G_2$ such that $f(x * y) = f(x) \circ f(y)$ for every x and y in G_1 .

Let us consider the groups $(G_1, *)$ and (G_2, \circ) , defined by the following operation tables:

G_1					
*	x	y	z	w	
x	x	y	z	w	
y	y	x	w	z	
z	z	w	x	y	
w	w	z	y	x	

G_2					
\circ	a	b	c	d	
a	a	b	c	d	
b	b	d	a	c	
c	c	a	d	b	
d	d	c	b	a	

a) (3p) Show that $(G_1, *)$ and (G_2, \circ) are **not** isomorphic.

b) (2p) Let $M = \{-i, i, -1, 1\}$ where i is the imaginary complex number. This is a complex number such that $i^2 = -1$. Show that (M, \cdot) is a group that is isomorphic to (G_2, \circ) ; here the operation \cdot is the complex multiplication.

IMP. Only a full explained complete solution will get points.

Have fun and good luck!