SF1686 Calculus in several variables
Exam
20 December 2021

Time: 14.00-17.00
Pen and paper exam. No calculators or formula sheets etc allowed
Examiner: Lars Filipsson

This exam consists of six problems, each worth six points, hence the maximal score is 36. Part A consists of the two first problems. To the score on part A your bonus points are added, up to a maximum of 12. The score on part A is at most 12, bonus points included. The bonus points are added automatically.

The following two problems constitute part B and the last two problems part C. You need a certain amount of points from part C to obtain the highest grades, as is seen in this table:

Grade	A	B	C	D	E	Fx
Total score	27	24	21	18	16	15
score on part C	6	3	–	–	–	–

To obtain a maximal 6 points for a solution to a problem on the exam, your solution must be well presented and easy to follow. Notation must be explained, the logical structure of the solution must be clearly described in words or in symbols and the reasoning leading up to the conclusion must be well motivated and clearly explained.

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PART A

1. Find an equation for the tangent plane at the point $(1, -1, 0)$ to the surface
$$z = \ln(1 + x^2 + y^3).$$

2. Compute the line integral $\int_\gamma \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F} = (e^{x+y} + y^2 + x + 1, e^{x+y} + x^2 + x + 2)$ and γ is the ellips $2x^2 + 3y^2 = 6$.

PART B

3. Compute the integral
$$\iiint_K \frac{1}{1 + x^2 + y^2 + z^2}\, dV,$$
where K is the region given by the inequalities $x^2 + y^2 + z^2 \leq 1$ and $z \leq 0$.

4. Find the maximum and minimum values of $f(x, y, z) = x^2 + y + z$ on the unit sphere $x^2 + y^2 + z^2 = 1$.

PART C

5. Use Stokes' theorem to compute $\int_\gamma \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F} = (-x^3, -z^3, y^3)$ and γ is the intersection of the cylinder $y^2 + z^2 = 1$ and the plane $x + 2y + 2z = 3$, positively oriented when viewed from the top of the positive x-axis.

6. Give a precise formulation and a proof of the theorem that states that the gradient of a two-variable function at a point is normal to the level curve of the function passing through that point.