Department of Mathematics

SF1626/SF1686
Several Variable Calculus
Academic year 2017/2018

Seminar 1

See www.kth.se/social/course/SF1626/SF1686 for information about how the seminars work and what you are expected to do before and during the seminars.

This seminar will start with a quiz on a variant of one of the recommended exercises from the text book Calculus by Adams and Essex (9th edition) which are marked by boldface in the following list:

Section	Recommended problems
10.1:	11, 17, 25, 27, **29**, 31, 33, 35, **37**, 39
10.6:	3, 5, 9, **13**
11.1:	17, 21, **33**
11.2:	3
11.3:	**5**, 7, **11, 13**, 15
12.1:	5, 9, 13,15, 17, **19, 23**, 27, 33
12.2:	5, 7, 9, **11, 14**, 15

PROBLEMS

Problem 1. Consider the sets in the xy-plane given by

$$\begin{aligned} D_1 &= \{(x,y)\colon 0 < y - x^2,\ y = x\} \\ D_2 &= \{(x,y)\colon 0 \le y^2 - x,\ x^2 + y^2 < 1\} \\ D_3 &= \{(x,y)\colon |x| \le 1,\ |y| \le 2\} \end{aligned}$$

(a) Sketch the sets D_1, D_2 och D_3.
(b) Mark the *inner points* of the sets.
(c) Mark the *boundary points* of the sets.
(d) Determine which of the sets that are *open*, *closed* or neither *open* nor *closed*.

Problem 2. Consider the cylinder S given by the equation $x^2 + y^2 = 1$ and the curve C that is given by the intersection between the cylinder and the plane given by the equation $ax + by + cz = 0$.

Express the parametric equation of the curve C as $\mathbf{r}(t)$ when

(a) $a = b = 0, \quad c = 1,$
(b) $b = c = 1, \quad a = 0,$
(c) $a = b = 1, \quad c = 0,$
(d) $a = b = c = 1.$

Problem 3. A particle travels in an orbit that is described by

$$\mathbf{r}(t) = (1 - t, \cos 2t, -\sin 2t),$$

(a) Compute the velocity $\mathbf{r}'(t)$.
(b) Compute the acceleration $\mathbf{r}''(t)$.
(c) Show that the velocity and the acceleration are perpendicular.

Problem 4. Let $f(x, y) = 10x^2 + 6xy + 13y^2$ for all (x, y) in $\mathbb{R}^2$.

(a) Make the change of variables given by $u = 3x + 2y$ och $v = x - 3y$, and write the new function in the variables u and v.
(b) Sketch some of the level curves of the function f.
(c) Sketch the graph of the function f.
(d) Determine a parametrization of the curve that is given by the intersection of the graph of the function f and the plane given by the equation $z = x - y$.

Problem 5. Find the domain of definition for the following functions by drawing a picture

$$f(x, y) = \ln |xy|, \qquad g(x, y) = \frac{1}{x^2 - y}, \qquad h(x, y) = \sqrt{|x - y|}, \qquad k(x, y) = \sqrt{xy},$$

$$t(x, y) = \sqrt{1 - x^2 - y^2}\left(\ln\left(x^2 + y^2\right)\right), \qquad s(x, y) = \log(x/y), \qquad p(x) = \arctan(y/x).$$

Vilka av dessa definitionsmängder är öppna, slutna eller ingetdera.

Problem 6. Find the limit for the function

$$f(x, y) = \frac{x^3 - y \sin y^2}{y - \sin x} \qquad \text{for } (x, y) \neq (0, 0),$$

when $(x, y) \to (0, 0)$ along the curve $y = 2x$.

Problem 7. Find the limit for the function

$$f(x, y) = \frac{x^2 \sin y}{x^4 + y^2} \qquad \text{för } (x, y) \neq (0, 0)$$

when $(x, y) \to (0, 0)$ along the curves $y = x$, and $y = x^2$. Is f continuous at the origin?

Problem 8. The function $f(x, y)$ is continuous in the entire plane. Further we know that the limit for the function f along the sequence $(x_j, y_j) \to (0, 0)$ is zero. Is it true that $f(0, 0) = 0$? Motivate your answer.