Final Exam – Part A and B – SF1610 Discrete Mathematics – TCOMK

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Course responsible: Ivan Martino
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Part A and B – Time: 8:00 – 11:00 (Extra-time students: 8:00 – 12:30)
Right after, you have 20 minutes to take pictures and upload your solution on Canvas.
Part C will be sent to you later. Time: 11:30 – 13:30 (Extra-time students: 13:00 – 16:00)

Short summary of the rules of the exam:

1. Use your computer only to read the questions of the exam.
2. If you may, use your phone for the Zoom-meeting call and place it so that your desk is visible; if you are not using your phone for the Zoom-meeting call, then you cannot use your phone for the whole exam.
3. No calculator, books, notes, lecture notes are allowed.
4. You may use your phone during the 20 minute break to take picture of your solution. You still need to be visible while doing so, hence you need to be in the Zoom-call from another device.

The exam consists of 10 questions in three parts: A, B and C. **Problems are not ordered by difficulty.** This exam provides a total of 39 points, plus any extra bonus points from the partial exams. The bonus point collected during the semester will be extra to the final exam score.

IMP. The full points will be given only to complete and fully explained solutions.

Evaluation table:
13 points total or more – give at least the grade of Fx
15 points total or more – give at least the grade of E
18 points total or more – give at least the grade of D
22 points total or more – give at least the grade of C
27 points total or more – give at least the grade of B
32 points total or more – give at least the grade of A

You should use paper and pen to solve the following exercises. You scan/take pictures of your solutions (jpg, jpeg, png, pdf format). Then, you have to upload your solutions gathered in a folder and compressed (as a zip or rar file) to Canvas / tasks / Final Exam – May 27, 2020 – Part 1: A and B.

Extra-time students should use a different folder Canvas / tasks / Final Exam – May 27, 2020 – Part 1: A and B – Extra time students.

IMPORTANT: The folder name should contain your last name and name; in other words use the NAME_FIRSTNAME as name of the folder with your solutions.

Write names and social security numbers on each sheet. On the first sheet write "I have done this Final exam by myself" and sign it. So you declare that you have made final exam by yourself.

The parameters p and q in the information below are the last two digits of your social security number. For example: If your social security number is 751332 2248 then p = 4 and q = 8.

PART A – If you have passed all partial exams, move to PART B.

If you passed the n^{th} partial exam then you get automatically 3 points for question number n in Part A. (So, If you passed the n^{th} partial exam, then you don't need to solve question number n of Part A.)

If you did not pass the n^{th} partial exam number, then you can try and solve the specific question number n on the final exam and get some points.

Part A will give you a total of 15 points that is equivalent to passing the course with E.

If you have passed all partial exams, then you should move to Part B.

1. (3p) Determine all integer solutions (x, y) of the Diophantine equation
$(q+3)x + (10-q)y = -q^2 + 7q + 30$.
IMP. Only a full explained complete solution will get points.

2. (3p) Find all integers n such that $100 + p \leq n \leq 200 + p$ which are divisible by 3 or 4.
Note. For instance, n=123 is one of such numbers because it is divisible by 3.
IMP. Only a full explained complete solution will get points.

3. (3p) Let $n = 5p + 15$ and consider the group $G = (Z/n, +)$
3.a) Determine a subgroup H to G such that $|H| = p + 3$.
3.b) Determine all cosets of H.
3.c) Determine an element in the group (Z/n, +) that has order 5.
Note: in item 3.c) you need to find only one element, you don't need to list all of them.

4. (3p) Let $A = p \bmod 2$.
4.a) Find all the words of the linear binary code defined by the matrix

$$H = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & A & 1 \\ 0 & 1 & 1 & 1 & 1 \end{bmatrix}.$$

4.b) Find the minimal distance between two words of the above defined code.
IMP. Only a full explained complete solution will get points.

5. (3p) Draw a connected graph with at least (p + 5) edges...
5.a) which is Eulerian but not Hamiltonian graph.
5.b) which is Hamiltonian graph but not Eulerian graph.
5.c) which is neither Eulerian nor Hamiltonian graph.
Note. The number of edges of the graph is larger or equal to (p+5).
Note. You have to draw, if possible, three graphs; one for each item of the exercise.
IMP. Only a full explained complete solution will get points.

PART B – If you have passed all partial exams, you start here.

6. (4p)
Show using the induction method that

$$q \cdot 3^{2n+3} + 27 \cdot 9^n + (q+1) \cdot (40n - 27)$$

is divisible by 64 for all integers n with $n \geq 0$.
IMP. You get no point if you do not use the induction method.

7. (4p)

7.a) (2p) Let K = 7+ (p mod 2). Determine the number of words of length K that can be formed using the letters a, b, c, d and e, such that each of the letters a, b, c and d occur at least once in the word, while the letter e occurs exactly twice in the word.

7.b) (2p) In one class there are (20 + q) students from Stockholm, 12 students from Uppsala and 14 students from Sundsval. You want to select a team that contains 7 students, which includes at least one student from each of the three cities. Determine the number of such teams (with 7 students and at least 1 from each city).
Also, determine the number of such teams (with 7 students and at least 1 from each city) where the number of students from Stockholm is greater than the number of students from Uppsala and Sundsval together.

Note. In assignment 7.a and 7.b, you may use the binomial coefficient and Stirling number notations in the answer.
IMP. Only a full explained complete solution will get points.

8. (4p)

An RSA-method has the following parameter, n = 143. Select an appropriate encryption key e where $12 \leq e \leq 20 + p$. For such encryption key e, find the decryption key d. Finally, encrypt the message "2".
IMP. Only a full explained complete solution will get points.

Good luck and have fun!