Department of Mathematics

SF1626
Several Variable Calculus

Seminar 4

See www.kth.se/social/course/SF1626 for information about how the seminars work and what you are expected to do before and during the seminars.

This seminar will start with a quiz on a variant of one of the recommended exercises from the text book Calculus by Adams and Essex (9th edition) which are marked by boldface in the following list:

Section	Recommended exercises
14.1	15, 19, 21
14.2	**3**, **5**, 15, **23**
14.3	1, **3**, 13, 27
14.4	5, **9**, 15, **19**, 21
14.5	5, **7**, 9
14.6	3, 7, **11**
14.7	5, 9, **13**, 21, 27

In the seminar the following problems will be discussed.

PROBLEMS

Problem 1. Let H be a regular hexagon in the xy-plane with all six vertices on the unit circle, one of which is in $(1, 0)$.

(a) Compute the integral $\iint_H xy\,dxdy$.

(b) Compute the integral $\iint_H (x^2 + y^2)\,dxdy$.

(c) Compute the integral $\iint_H (x - y)^2\,dxdy$.

Discuss further what happens for regular n-gons for other n than 6.

Problem 2. In order to compute an integral over a triangle in the plane it is possible to first carry out a change of variables that moves the triangle to the triangle Δ with

vertices $(0,0)$, $(1,0)$ and $(0,1)$. There are many changes of variables that make this possible, but the easiest way is to use an *affine* change of variables, i.e., one that is given by a linear transformation plus a constant. In other words, this can be expressed as

$$\begin{cases} x = a + bs + ct, \\ y = d + es + ft. \end{cases}$$

where a, b, c, d, e and f are constants and s and t are the new variables.

Consider the triangle T with vertices $(1,2)$, $(2,3)$ and $(-1,-1)$.

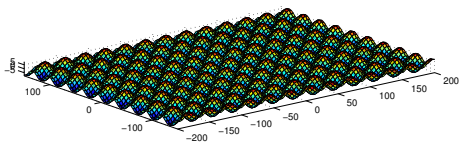
(a) Choose an affine change of variables that transforms the triangle T into the triangle Δ.

(b) Compute the Jacobian $\dfrac{\partial(x,y)}{\partial(s,t)}$ for the change of variables in (a).

(c) Use the change of variables in (a) in order to compute

$$\iint_T (xy - y^2)\, dxdy.$$

Problem 3. A rectangular sheet of metal is shaped as the graph of the function $f(x,y) = a(\cos kx + \cos ky)$ where $a = 4{,}5$ mm and $k = 0{,}2$ mm^{-1}.



The measures of the sheet are 400 mm in the x-direction and 300 mm in the y-direction. When the sheet is placed horisontally the holes can carry some water.

(a) How many holes are there in the sheet?

(b) Use an integral in order to estimate how much water the sheet can carry.