I have done the exam myself.

TZC

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1a.

	0	1	2	3	4
0	0	0	0	0	0
1	0	1	2	3	4
2	0	2	4	1	3
3	0	3	1	4	2
4	0	4	3	2	1

$p = 3$

$q = 8$

b. $a = 31 + 3 = 34$

$b = 40$

$gcf(40, 34)$

$40 = 34 \cdot 1 + 6$

$34 = 6 \cdot 5 + 4$

$6 = 4 \cdot 1 + 2$

$4 = 2 \cdot 2 + 0$

$gcf(40, 34) = 2$

c. $(3+4)^4 \bmod (3+2) = 7^4 \bmod 5$

$7^4 \bmod 5 = 2^4 \bmod 5$

$2^4 \bmod 5 = (2^2)^2 \bmod 5$

$= 4^2 \bmod 5$

$= 16 \bmod 5$

$= 1$

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2. $(8+2)x + (8+4)y = 16+4$

$10x + 12y = 20$

$5x + 6y = 10$

$\gcd(6,5) = 1$ ← solution exists!

$6 = 5\cdot1+1$ $\qquad 1 = 6\cdot1 - 5\cdot1$

$10 = 6\cdot10 - 5\cdot10 \;\to\; 20 = 12\cdot10 - 10\cdot10$

$(-10, 10)$ ← one solution

$\mathrm{lcm}(10,12) = \frac{10\cdot12}{2} = \frac{120}{2} \cancel{= 12\cdot10 = 4\cdot3\cdot10 = 4\cdot30}$

$= 60 = 6\cdot10$

$20 = 12\cdot10 - 10\cdot10 + 60k - 60k$

$20 = 12\cdot10 + 60k - 10\cdot10 - 60k$

$20 = 12\underbrace{(10+5k)}_{y} - 10\underbrace{(10+6k)}_{-x}$

full solution: $(-10-6k,\ 10+5k)$

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3. ~~a~~

$a = qj$

$b = qk$ where $gcd(a,b) = q$ and $(k, j) \in \mathbb{Z}$

$b - a = qk - qj = q(k-j)$

$gcd(a, b-a) = gcd(qj, q(k-j))$

$gcd(k, j) = 1$ since q is gcd of a and b

so $j \mid (k-j)$ is a false statement.

Therefore, q is the gcd of a and $b-a$

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