

4a. $\begin{bmatrix} 1 & 2 & 4 & | & 1 \\ -1 & 2 & 1 & | & 1 \\ 0 & 4 & 5 & | & 1 \end{bmatrix} R_2 = R_2 + R_1 \rightarrow \begin{bmatrix} 1 & 2 & 4 & | & 1 \\ 0 & 4 & 5 & | & 2 \\ 0 & 4 & 5 & | & 1 \end{bmatrix}$

$$\begin{aligned} 0x_1 + 4x_2 + 5x_3 &= 2 \leftarrow \text{Contradiction!} \\ 0x_1 + 4x_2 + 5x_3 &= 1 \end{aligned}$$

\vec{v} does NOT lie in V

b. Since $R_3 = R_2 + R_1$, R_1 and R_2 are bases for V .

V is the span of $\left\{ \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} \right\}$ so V is in \mathbb{R}^3

$$\dim(V) = 2$$

c. To make R_1 and R_2 orthonormal, we must make each length 1. $\cos \theta = \frac{R_1 \cdot R_2}{|R_1||R_2|} = \frac{-1+8+4}{\sqrt{21}\sqrt{8}} = \frac{11}{\sqrt{168}}$

To make R_1 and R_2 orthonormal, we must make each length 1.

$$R_1 = \frac{R_1}{|R_1|} = \begin{bmatrix} 1/\sqrt{21} \\ 2/\sqrt{21} \\ 4/\sqrt{21} \end{bmatrix} \quad R_2 = \frac{R_2}{|R_2|} = \begin{bmatrix} -1/\sqrt{8} \\ 2/\sqrt{8} \\ 1/\sqrt{8} \end{bmatrix}$$