



Department of Mathematics

SF1626/SF1686  
Several Variable Calculus

## Seminar 6

See [www.kth.se/social/course/SF1626/SF1686](http://www.kth.se/social/course/SF1626/SF1686) for information about how the seminars work and what you are expected to do before and during the seminars.

This seminar will start with a quiz on a variant of one of the recommended exercises from the text book Calculus by Adams and Essex (9th edition) which are marked by boldface in the following list:

Section	Recommended exercises
16.1	3, 7, 11
16.2	9, 15, 17
16.3	<b>3, 5</b> , 9
16.4	<b>5, 11, 15</b>
16.5	1, <b>3</b> , 5

In the seminar the following problems will be discussed.

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### PROBLEMS

**Problem 1.** Let  $\mathbf{F}$  be the vector field given by

$$\mathbf{F}(x, y, z) = (x^2 - yz, y^2 - xz, z^2 - xy).$$

for all  $(x, y, z)$  in  $\mathbb{R}^3$ .

- (a) Determine  $\text{rot } \mathbf{F} = \nabla \times \mathbf{F}$  and  $\text{div } \mathbf{F} = \nabla \cdot \mathbf{F}$ .
- (b) Determine whether there is a potential, i.e., a function  $g$  such that  $\mathbf{F} = \text{grad } g$ .
- (c) Compute the flux of  $\mathbf{F}$  out from the sphere given by

$$(x - a)^2 + (y - b)^2 + (z - c)^2 = r^2.$$

**Problem 2.** Let  $\mathbf{F} = \text{rot } \mathbf{G} = \nabla \times \mathbf{G}$  where

$$\mathbf{G}(x, y, z) = (z^2 - y^2, x^2 - z^2, y^2 - x^2)$$

- (a) Compute the flux of  $\mathbf{F}$  through the triangle with vertices in  $(1, 0, 0)$ ,  $(0, 1, 0)$  and  $(0, 0, 1)$ . (The normal direction is chosen so that it points away from the origin.)
- (b) Use Stoke's Theorem in order to relate the flux from part (a) to a line integral and compute this line integral by means of a parametrization.