

5a.  $A = PDP^{-1}$

$$P = \begin{bmatrix} 1 & a & a \\ 0 & b & b \\ -1 & c & c \end{bmatrix}$$

$$D = \begin{bmatrix} 3 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$A = \begin{bmatrix} d & g & h \\ g & e & i \\ h & i & f \end{bmatrix}$$

~~$(d-3)(e-3)(f-3) - i^2 = 0$~~   
 ~~$g(g(f-3) - i) + h(gi - h(e-3)) = 0$~~

Standard polynomial of A:

$$(\lambda - 3)(\lambda + 1)^2 = 0$$

when  $\lambda = 3$ ,  $\vec{x} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$

~~$\begin{bmatrix} d-3 & h & 0 \\ g & e-3 & i \\ 0 & 0 & f-3 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$~~

$A\vec{x} = \lambda\vec{x}$   
 $\begin{bmatrix} d & g & h \\ g & e & i \\ h & i & f \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$

$$d - h = 3 \quad h = d - 3$$

$$g - i = 0 \quad g = i$$

$$h - f = -3 \quad f = h + 3 = d - 3 + 3 = d$$

$$A = \begin{bmatrix} d & i & d-3 \\ i & e & i \\ d-3 & i & d \end{bmatrix}$$

$$\begin{aligned} & (d-\lambda)(e-\lambda)(d-\lambda) - i(i(f-\lambda) - i(d-\lambda)) + (d-\lambda)(i^2 - e(d-\lambda)) \\ & (d-\lambda)^2(e-\lambda) + (d-\lambda)(i^2 - e(d-\lambda)) \end{aligned}$$

b. Assume  $A = \begin{bmatrix} a & a & a \\ 0 & b & b \\ -1 & c & c \end{bmatrix}$  ~~if A was known (2 could figure it out)~~



$$5b. A^{99} = P^{99} D^{99} P^{-99} = D^{99} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}^{99}$$

$$\begin{bmatrix} 3 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 9 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}^2 = \begin{bmatrix} 3^2 & 0 & 0 \\ 0 & (-1)^2 & 0 \\ 0 & 0 & 1^2 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}^{99} = \begin{bmatrix} 3^{99} & 0 & 0 \\ 0 & (-1)^{99} & 0 \\ 0 & 0 & 1^{99} \end{bmatrix} = \begin{bmatrix} 3^{99} & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 3^{99} & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 3^{99} \\ -3 \\ 1 \end{bmatrix}$$

$$A^{99} \vec{w} = \begin{bmatrix} 3^{99} \\ -3 \\ 1 \end{bmatrix}$$