Department of Mathematics

SF1626/SF1686
Several Variable Calculus

Seminar 3

See www.kth.se/social/course/SF1626/SF1686 for information about how the seminars work and what you are expected to do before and during the seminars.

This seminar will start with a quiz on a variant of one of the recommended exercises from the text book Calculus by Adams and Essex (9th edition) which are marked by boldface in the following list:

Section	Recommended problems
12.8	**13**, 17
12.9	**1**, 3, **5**, 7, 11
13.1	5, **7, 9**, 19, 23, 25
13.2	3, **5, 9**, 15
13.3	3, **9, 11**, 15
13.4	1, 3

PROBLEMS

Problem 1. Find $\frac{\partial u}{\partial x}$, $\frac{\partial v}{\partial y}$ at $(u, v) = (1, 1)$ where $x = v^2 - uv$, and $y = v - u^2$.

Problem 2. Argue without computation why the function $f(x, y) = (x^2 + y^2)e^{-x^2-y^2}$ takes its maximum and minimum in $\mathbb{R}^2$.

Problem 3. Let f be given by $f(x, y) = ax^3y^2 + y^2 - 4x^3$ for all (x, y) in $\mathbb{R}^2$, where $a \in \mathbb{R}$.

(a) Find all stationary points for f, when $a = 1$.
(b) Show that the only stationary point for $a < 0$ is the origin.
(c) Find the Taylor polynomial of second order for f close to all stationary points when $a = -1$.

Contemplate: For $a \geq 0$ decide what type of stationary points there are.

Problem 4. The function f is given by
$$f(x, y) = (\sin 2x - \sin 2y)^2$$
for all (x, y) in $\mathbb{R}^2$.

(a) Find all stationary points to f.
(b) Single out by sketching the stationary points to f along with the level curves for $f(x, y) = 0$. Mark which of them are maximum, minimum or neither of these.

Problem 5. Consider the problem of finding the largest and the smalles value for the function $f(x, y, z) = 2x - y + z$ subject to the constraints $x^2 + y^2 + z^2 \leq 1$ and $2y \leq 1$.

(a) Sketch the domain D that is given by the constraints.
(b) Search and find all stationary points to f in the interior of D.
(c) Find possible extremal points on the boundary of D by parametrizing the boundary. (Think of the boundary as surface that consists of two parts that intersect each other along a circle. Parametrize both sides separately. The circle is like the boundary of both parts.)
(d) Search possible extremal points on the boundary of D using Lagranges method. (Exactly as in (c) both parts and their intersection are needed to be handled.)
(e) Make conclusion about largest and smallest value for f. How can one be sure that the method leads to a correct answer?

Problem 6. Let
$$f(x, y) = \frac{2 - xy}{2x^2 + y^2 + 2}$$
for all (x, y) in the closed disc $C : \{2x^2 + y^2 \leq 4\}$.

(a) Find all stationary points to f.
(b) Use a parametrization of the boundary of C to find extremal points to f on the boundary.
(c) Use Lagrange's method to find candidates for the extremal points to the boundary of C. (Observe that the denominator is constant on the boundary.)
(d) Use the results from (a), (b) and (c) to decide the maximum and the minimum for f on C.