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1a. $f'(x) = \frac{d}{dx}\left(1 + \frac{x}{1+\sin^2 x}\right)^{\frac{1}{2}}$

$= \frac{1}{2}\left(1 + \frac{x}{1+\sin^2 x}\right)^{-1/2} \cdot \frac{d}{dx}\left(1 + \frac{x}{1+\sin^2 x}\right)$

$= \frac{\frac{d}{dx}\left(\frac{x}{1+\sin^2 x}\right)}{2\sqrt{1+\frac{x}{1+\sin^2 x}}}$

$\frac{d}{dx}\frac{x}{1+\sin^2 x} = \frac{\frac{d}{dx}x \cdot (\sin^2 x+1) - x\frac{d}{dx}(\sin^2 x+1)}{(\sin^2 x+1)^2}$

$= \frac{\sin^2 x + 1 - x\sin 2x}{(\sin^2 x+1)^2}$

$= \boxed{\frac{\sin^2 x \cancel{+} x\sin 2x + 1}{2(\sin^2 x+1)^2\sqrt{1+\frac{x}{1+\sin^2 x}}}}$

b. $f'(0) = \frac{\sin^2(0) + 0 + 1}{2(\sin^2(0)+1)^2\sqrt{1+\frac{0}{1}}} = \frac{0+0+1}{2(1)\sqrt{1}} = \frac{1}{2}$ ← slope of tangent line

$f(0) = \sqrt{1+\frac{0}{1+0}} = \sqrt{1} = 1$ ← point (0,1) line goes through

$y = m(x-x_1) + y_1$

$y = \frac{1}{2}(x-0) + 1$

$\boxed{y = \frac{1}{2}x + 1}$

2. $y = -2x$ has slope of -2

$\frac{d}{dx}\, x^3 - 6x^2 + 7x = 3x^2 - 12x + 7$

$-2 = 3x^2 - 12x + 7$

$3x^2 - 12x + 9 = 0$

$x^2 - 4x + 3 = 0$

$(x-3)(x-1) = 0$

equation has tangent line with slope -2 at points $x=3$ and $x=1$

$(1)^3 - 6(1)^2 + 7(1) = 1 - 6 + 7 = 2$

$-2(1) = -2$ $\qquad$ $-2 \neq 2$ X not tangent at point (1,2) or (1,-2)

$(3)^3 - 6(3)^2 + 7(3) = 27 - 54 + 21 = -6$

$-2(3) = -6$ $\qquad$ $\boxed{-6 = -6 \checkmark \text{ tangent at point } (3,-6)}$

Tomas Belmar 020513-5338

3 a. Domain defined for all x where $x \neq 1$

b. $f'(x) = 4 + \frac{d}{dx}\frac{1}{x-1} = 4 + \frac{d}{dx}(x-1)^{-1} = 4 - (x-1)^{-2} \cdot \frac{d}{dx}(x-1)$

$= 4 - \frac{1}{(x-1)^2}$ defined for all x where $x \neq 1$

c. $4 - \frac{1}{(x-1)^2} = 0$

$4 = \frac{1}{(x-1)^2}$ $\quad 4(x-1)^2 = 1$ $\quad 4(x^2 - 2x + 1) = 1$

$4x^2 - 8x + 4 = 1$

$4x^2 - 8x + 3 = 0$

$(2x-1)(2x-3) = 0$

$x = \{\frac{1}{2}, \frac{3}{2}\} \Leftarrow$ sign changes at these points

when $x < \frac{1}{2} \Rightarrow 4 - \frac{1}{(\frac{1}{4}-1)^2} = 4 - \frac{1}{(-\frac{3}{4})^2} = 4 - \frac{1}{9/16} = 4 - \frac{16}{9} = \frac{20}{9} > 0$

when $\frac{1}{2} < x < 1 \Rightarrow 4 - \frac{1}{(\frac{3}{4}-1)^2} = 4 - \frac{1}{(-\frac{1}{4})^2} = 4 - \frac{1}{1/16} = 4 - 16 = -8 < 0$

when $1 < x < \frac{3}{2} \Rightarrow 4 - \frac{1}{(\frac{4}{3}-1)^2} = 4 - \frac{1}{(\frac{1}{3})^2} = 4 - 9 = -5 < 0$

when $\frac{3}{2} < x \Rightarrow 4 - \frac{1}{(5-1)^2} = 4 - \frac{1}{4^2} = 4 - \frac{1}{16} = \frac{63}{16} > 0$

$x < \frac{1}{2}$	$\frac{1}{2} < x < 1$	$1 < x < \frac{3}{2}$	$\frac{3}{2} < x$
+	−	−	+
increasing	decreasing	decreasing	increasing

4. ~~$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h} = \lim_{h \to 0} \frac{x^2 - (x+h)^2}{h}$~~

~~$= \lim_{h \to 0} \frac{x^2 - (x^2 + 2xh + h^2)}{h} = \lim_{h \to 0} \frac{\cancel{x^2} - \cancel{x^2} - 2xh - h^2}{h} = \lim_{h \to 0} \frac{-2xh - h^2}{h}$~~

~~$= \lim_{h \to 0} -2x - h = -2x$~~

Tomas Belmar 020513-5338

4. $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h} = \lim_{h\to 0} \frac{\frac{x^2-(x+h)^2}{(x+h)^2x^2}}{h}$

$= \lim_{h\to 0} \frac{x^2-(x+h)^2}{(x+h)^2x^2h} = \lim_{h\to 0} \frac{\cancel{x^2}-\cancel{x^2}-2xh-h^2}{hx^4+2x^3h^2+x^2h^3} = \lim_{h\to 0} \frac{-h(2x+h)}{h(x^4+2x^3h+x^2h^2)}$

$= \lim_{h\to 0} \frac{-2x-h}{x^4+2x^3h+x^2h^2} = \frac{-2x}{x^4} = \boxed{\frac{-2}{x^3}}$

5. $(y^2+x^2)^3 = 8x^2y^2$

$3(y^2+x^2)^2(2y\frac{dy}{dx}+2x) = 16x^2y\frac{dy}{dx} + 16xy^2$

$3(y^2+x^2)^2 \cdot 2(\frac{dy}{dx}y+x) = \cancel{16xy(x\frac{dy}{dx}+y)}\ 16x^2y\frac{dy}{dx} + 16xy^2$

$6(y^2+x^2)^2(\frac{dy}{dx}y+x) = \cancel{16xy(x\frac{dy}{dx}+y)}\ 16x^2y\frac{dy}{dx} + 16xy^2$

$\cancel{6y^3\frac{dy}{dx}+6y^2x+6x^2y\frac{dy}{dx}+6x^3 = 16x^2y\frac{dy}{dx}+16xy^2}$

$\cancel{\frac{dy}{dx}(6y^3+6x^2y-16x^2y) = 16xy^2-6y^2x-6x^3}$

$\cancel{\frac{dy}{dx} = \frac{16xy^2-6y^2x-6x^3}{6y^3+6x^2y-16x^2y}}$

$(6y^4+12y^2x^2+6x^4)(\frac{dy}{dx}y+x) = 16x^2y\frac{dy}{dx}+16xy^2$

$6y^5\frac{dy}{dx}+12y^3x^2\frac{dy}{dx}+6x^4y\frac{dy}{dx}+6y^4x+12y^2x^3+6x^5 = 16x^2y\frac{dy}{dx}+16xy^2$

$\frac{dy}{dx}(6y^5+12y^3x^2+6x^4y-16x^2y) = 16xy^2-6y^4x-12y^2x^3-6x^5$

$\frac{dy}{dx} = \frac{16xy^2-6y^4x-12y^2x^3-6x^5}{6y^5+12y^3x^2+6x^4y-16x^2y}$

point $(-1,1)$: $\frac{16(-1)(1)-6(1)(-1)-12(1)(-1)-6(-1)}{6(1)+12(1)(1)+6(1)(1)-16(1)(1)}$

$= \frac{-16+6+12+6}{6+12+6-16} = \frac{8}{8} = 1$ ← slope!

3

$y = 1(x - x_1) + y_1$

$y = 1(x - (-1)) + 1$

$y = x + 1 + 1$

$\boxed{y = x + 2}$