SF1686 Calculus in several variables
Exam
27 October 2021

Time: 08.00-11.00
Pen and paper exam. No calculators or formula sheets etc allowed
Examiner: Lars Filipsson

This exam consists of six problems, each worth six points, hence the maximal score is 36. Part A consists of the two first problems. To the score on part A your bonus points are added, up to a maximum of 12. The score on part A is at most 12, bonus points included. The bonus points are added automatically.

The following two problems constitute part B and the last two problems part C. You need a certain amount of points from part C to obtain the highest grades, as is seen in this table:

Grade	A	B	C	D	E	Fx
Total score	27	24	21	18	16	15
score on part C	6	3	–	–	–	–

To obtain a maximal 6 points for a solution to a problem on the exam, your solution must be well presented and easy to follow. Notation must be explained, the logical structure of the solution must be clearly described in words or in symbols and the reasoning leading up to the conclusion must be well motivated and clearly explained.

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PART A

1. Let $f(x, y, z) = x^2 + xy + 2y^2 - z^2$.
   A. Compute the maximal directional derivative of f at the point $(0, 1, 1)$.
   B. Find an equation for the tangent plane to the level surface $f(x, y, z) = 1$ at the point $(0, 1, 1)$.

2. We study the line integral
$$\int_\gamma \mathbf{F} \cdot d\mathbf{r}$$
where the vector field $\mathbf{F} = (yz, xz, xy + 1)$ and γ is the spiral curve parametrized by $(x, y, z) = (\cos t, \sin t, t)$ where t runs from 0 to $\pi/4$.
   A. Compute the line integral directly using the parametrization of the curve.
   B. Compute the line integral using a potential for $\mathbf{F}$.

PART B

3. Determine the maximum and minimum values (if they exist) of $f(x, y) = xy^2$ when (x, y) varies in the region given by the inequality $\dfrac{x^2}{3} + \dfrac{y^2}{2} \leq 1$.

4. Compute the volume of the region K given by the inequalities $x^2 \leq z \leq 4 - y^2$.

PART C

5. Compute the flux of the vector field $\mathbf{F} = (x^2, x^2 + y^2, x^2 + y^2 + z^2)$ out of the region K given by the inequalities $\sqrt{x^2 + y^2} \leq z \leq \sqrt{2 - x^2 - y^2}$.

6. Give a proof of the theorem that states that a real-valued function of two variables with continuous partial derivatives in a neighborhood of a point must be differentiable at that point.