Mathematics Department, KTH

Final exam for SF1610 Discrete Mathematics for TCOMK1
2 juni 2021, kl 08.00–13.00.

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Aids: No aids are allowed during the final examination.
Writing time: 8:00 -13:00 (+30 min for uploading. The uploading time starts at 11:00 and closes at 13:30)
Extra writing time (Funka): 8:00 -15:30 (+30 min for uploading. The uploading time starts at 11:00 and closes at 16:00)

Toilet breaks are allowed throughout the exam writing time (but, first notify the exam guard, via chat).

If you have finished earlier (but after 11 am), you announce through chat in Zoom, to the exam guard, that you will take photos of your solutions. After uploading, notify the examiner that you are leaving the Zoom exam. After that, you must not return to the Zoom room or make changes to your solutions.

A student who wants to submit a ***blank*** exam may leave the zoom room at 9:00 the earliest.

Use paper and pen to solve the problems below. Scan or take pictures of your solutions (preferably PDF, do NOT compress files). **Please create separate file for each problem.**
Upload the files to the Canvas site for this exam:
https://kth.instructure.com/courses/30510/assignments

The final exam consists of 10 questions in three parts.
The total amount of points for this exam is 37p
NOTE: A complete solution with complete justifications is required for all problems.
Grading intervals:

13	points in total or more gives at least the lowest grade	Fx
15	points in total or more gives at least the grade	E
18	points in total or more gives at least the grade	D
22	points in total or more gives at least the grade	C
27	points in total or more gives at least the grade	B
32	points in total or more gives at least the grade	A

Write your name and personnummer (personal identity number) **on every page**. Declare that you have done the exam yourself: Write on the first submitted page *"I guarantee that I have done the exam myself"* and sign.
The parameters p and q in the information below are the last two digits of your personnummer. (Ex: If your personnummer is 751332 2248, then $p = 4$ and $q = 8$). Start by

replacing p and q with numbers from your id number and then solve the exercises.
Complete solutions are required for all exercises.

PART I
Each of the following five problems corresponds to a partial exam. Approved result on partial exam **nr. i** during the spring term 2021 automatically gives full points on problem **nr. i**
Solving a problem below that you already have from the partial exams gives no extra points.

1. (3p) Find all the integer solutions (x,y) for the Diophantine equation
$(p+2)x+(11-p)y=-p^2+9p+22$.
NOTE. A complete solution with complete justifications must be provided.

2. (3p) In a box there are *(100 + q)* notes; from which *(45 + q)* are marked A, *35* are marked B, and *20* are marked C. We choose *(8 + q)* notes randomly.
Compute the probability that we get exactly four A, two B and *(2 + q)* C among the chosen notes if we take the notes

a) Without replacement.
b) With replacement.

3. (3p) Let H be the set of all integers divisible by (13-p), i.e. $H=\{(13-p)\cdot k,\ k\in Z\}$
Prove that *(H, +)* is a group. (where + denotes addition of integers)

4. (3p) Let $A=$ *(p mod 2)*

a) Determine all code words defined by the check matrix
$$H=\begin{bmatrix}1&0&0&1&A&A\\1&1&0&1&0&0\\0&0&1&0&1&1\end{bmatrix}.$$
b) Determine the minimum distance between two code words in the above code.

5. (3p) In a planar connected graph (or multigraph) G, all nodes have degree 4. The number of edges is *(8p + 116)*. Determine the number of facets that a flat drawing of the graph has.

PART II

6. (4p)
a) Prove by mathematical induction that for all $n \geq 1$, $n^3 + 3p^2 - n + 3p$ is divisible by 6.

b)

Let $A = \begin{bmatrix} 1 & c & 0 \\ 0 & 1 & 0 \\ 0 & 0 & (q+2) \end{bmatrix}$ be a 3×3 matrix , where c is a real number.

Prove by mathematical induction that for every $n \geq 1$ the following equality holds

$$A^n = \begin{bmatrix} 1 & nc & 0 \\ 0 & 1 & 0 \\ 0 & 0 & (q+2)^n \end{bmatrix}$$

NOTE. You get 0 points if you do not use mathematical induction in a) and b).

7. (4p) Let *K=(p mod 4).* Determine the amount of integers *n*, in the interval $30 \leq n \leq (150 + 10K)$, that are divisible by at least one of the numbers 10, 12 or 18.

8. (4p) Write the following Boolean function
$f(x, y, z) = \overline{x}(xy + \overline{z}) + \overline{y}(y + \overline{z}) + xz$

a) in the disjunctive normal form.
b) in the conjunctive normal form.

PART III

9. (5p) Let (G, *) be a finite group with n elements, and assume that $n > 3$. Assume further that n **is not** a prime number.
Prove that G has at least one non-trivial subgroup (H, *) (i.e. a subgroup H that is different from G and different from $\{e\}$, where e denotes the unit element in G).

10. (5p) Let $c_1, c_2, ..., c_n$ be integers numbers, and let $d_1, d_2, ..., d_n$ be relatively prime positive integers. We are looking for integers x that satisfy the following system:

$$\begin{cases} x \equiv c_1 (\operatorname{mod} d_1) \\ x \equiv c_2 (\operatorname{mod} d_2) \\ \cdots \\ x \equiv c_n (\operatorname{mod} d_n). \end{cases} \qquad (sys\ 1)$$

Let $D = d_1 d_2 \cdots d_n$. For every $i \in \{1, ..., n\}$, we define $D_i = \dfrac{D}{d_i}$ and find a number s_i such that $s_i D_i + t_i d_i = 1$.

a) Show that
$$x = kD + \sum_{i=1}^{n} c_i s_i D_i \quad (*)$$
is a solution for the system ($sys\ 1$) for every integer k.
b) Prove that the system ($sys\ 1$) has no other solution different than those in (*).

Good luck!