Department of Mathematics

SF1626/SF1686
Several Variable Calculus

Seminar 5

See www.kth.se/social/course/SF1626/SF1686 for information about how the seminars work and what you are expected to do before and during the seminars.

This seminar will start with a quiz on a variant of one of the recommended exercises from the text book Calculus by Adams and Essex (9th edition) which are marked by boldface in the following list:

Section	**Recommended problems**
15.1	3, **5** , **15** , 17,
15.2	**3, 5**, 7, **21**
15.3	5, **7**, 9, **11**
15.4	1, **5**, 7, **15**, 17, 22
15.5	1, **7** , **13**
15.6	**5, 9**, 13, 15

PROBLEMS

Problem 1. Let $\mathbf{F}$ and $\mathbf{G}$ be the vector fields given by

$$\mathbf{F}(x,y) = \nabla f(x,y) \quad \text{and} \quad \mathbf{G}(x,y) = \left(y^2, -x^2\right)$$

where $f(x,y) = x^4y^2 + xy$ for (x,y) in $\mathbb{R}^2$. Let C_1 be the curve given by the staight line segment from $(2,4)$ to $(-1,3)$ and let C_2 be the closed curve given by the unit circle centered at the origin oriented counter-clockwise.

(a) Compute the line integral $\displaystyle\int_{C_1} \mathbf{F} \cdot d\mathbf{r}$.

(b) Compute the line integral $\displaystyle\int_{C_1} \mathbf{G} \cdot d\mathbf{r}$.

(c) Compute the line integral $\displaystyle\int_{C_2} \mathbf{F} \cdot d\mathbf{r}$.

(d) Compute the line integral $\displaystyle\int_{C_2} \mathbf{G} \cdot d\mathbf{r}$.

Problem 2. The function f is given by

$$f(x,y) = 2x + x^2 + 2y^2, \qquad \text{for all } (x,y) \text{ in } \mathbb{R}^2.$$

(a) Determine a vector field such that its field lines are level curves of f.
(b) Are there several such vector fields?

Problem 3. Let $\mathbf{F}$ be the vector field which is given by

$$\mathbf{F}(x,y) = \left(\frac{ax+by}{x^2+y^2}, \frac{cx+dy}{x^2+y^2}\right)$$

away from the origin. Let C be the curve given by

$$\mathbf{r}(t) = (1,1) + e^{-t}(\cos t, \sin t),$$

for $t \geq 0$. Let $D_1 = \{(x,y):\ y > 0\}$ adn $D_2 = \{(x,y):\ y > -1\}$.

(a) For which values of a, b, c, d is $\mathbf{F}$ conservative in the region D_1?
(b) For which values of a, b, c, d is $\mathbf{F}$ conservative in the region D_2?
(c) Use the potential of $\mathbf{F}$ in order to compute the line integral

$$\int_C \mathbf{F} \cdot d\mathbf{r}.$$

when $a = d = 1$, $b = c = 0$.
In order to compute the line integral for an infinite curve you can first restrict to a finite part $0 \leq t \leq T$ and then compute the limit as $T \to \infty$.

Problem 4. Let S be the surface which in spherical coordinates [1] is given by

$$r = 3, \qquad -\pi/4 \leq \theta \leq \pi/4.$$

The orientation of the surface is such that its normal vector is pointing away from the origin.

(a) Sketch the surface S and compute its area.
(b) Compute the flux of the vector field $\mathbf{F}(x,y,z) = (x^2, y^2, z^2)$ through S.

[1] In the text book they use R instead of r for the distance to the origin. Here the spherical coordinates are written as r, θ and ϕ where ϕ is the angle to the positive z-axis.