

$$3a. \begin{matrix} & A & & \vec{u} \\ \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} & \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} & = & \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} \end{matrix}$$

$j, k,$ and l are any row in the original matrix.
coefficients of

$$A\vec{u} = \vec{u} \rightarrow \begin{aligned} a - 2b + c &= 1 \\ d - 2e + f &= -2 \\ g - 2h + i &= 1 \end{aligned}$$

$$\begin{bmatrix} 1 & -2 & 1 \\ 2 & 0 & -1 \\ -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} \begin{matrix} j \\ k \\ l \end{matrix}$$

$$A\vec{u} = 2\vec{u} \rightarrow \begin{aligned} 2a - c &= 4 \\ 2d - f &= 0 \\ 2g - i &= -2 \end{aligned}$$

$$\begin{bmatrix} 1 & -2 & 1 & | & 1 \\ -1 & 1 & 0 & | & 1 \\ 2 & 0 & -1 & | & 4 \end{bmatrix} \begin{matrix} j \\ k \\ l \end{matrix} \rightarrow \begin{bmatrix} 1 & -2 & 1 & | & 1 \\ 0 & -1 & 1 & | & 2 \\ 0 & 4 & -3 & | & 2 \end{bmatrix} \begin{matrix} j \\ k+2j \\ k-2j \end{matrix}$$

$$A\vec{u} = -\vec{u} \rightarrow \begin{aligned} -a + b &= 1 \\ -d + e &= -1 \\ -g + h &= 0 \end{aligned}$$

$$\rightarrow \begin{bmatrix} 1 & -2 & 1 & | & 1 \\ 0 & 1 & -1 & | & -2 \\ 0 & 0 & 1 & | & 10 \end{bmatrix} \begin{matrix} j \\ -2-j \\ k-2j+4(2+j) \\ k+2j+4(2) \end{matrix}$$

~~$$\begin{bmatrix} 1 & -2 & 1 & | & -2 \\ 0 & 1 & -1 & | & 0 \\ 0 & 0 & 1 & | & 1 \end{bmatrix} \begin{matrix} j \\ k \\ l \end{matrix}$$

$$\begin{aligned} f &= -1 \\ e + 1 &= 0 \rightarrow e = -1 \\ d + 2 - 1 &= -2 \rightarrow d = -3 \end{aligned}$$~~

~~$$\begin{bmatrix} 1 & -2 & 1 & | & -2 \\ 2 & 0 & -1 & | & 4 \\ -1 & 1 & 0 & | & 1 \end{bmatrix} \begin{matrix} j \\ k \\ l \end{matrix}$$

$$\begin{bmatrix} 1 & -2 & 1 & | & j \\ 0 & 1 & -1 & | & -k-j \\ 0 & 0 & 1 & | & k+2j+4(2) \end{bmatrix}$$~~

~~$$\begin{bmatrix} 1 & -2 & 1 & | & 1 \\ 0 & 1 & -1 & | & -2 \\ 0 & 0 & 1 & | & 0 \end{bmatrix} \begin{matrix} j \\ k \\ l \end{matrix}$$~~

$$\begin{bmatrix} 1 & -2 & 1 & | & -2 \\ 0 & 1 & -1 & | & 3 \\ 0 & 0 & 1 & | & -8 \end{bmatrix} \rightarrow \begin{aligned} d &= -2 - 5 + 8 = 1 \\ e &= 3 - 8 = -5 \\ f &= -8 \end{aligned}$$

$$\begin{aligned} j &= -2 \\ k &= 0 \\ l &= -1 \end{aligned}$$

$$\begin{bmatrix} 1 & -2 & 1 & | & 1 \\ 0 & 1 & -1 & | & -1 \\ 0 & 0 & 1 & | & 0 \end{bmatrix} \begin{aligned} g &= 1 - 2 + 0 = -1 \\ h &= -1 \\ i &= 0 \end{aligned}$$

$$\begin{aligned} j &= 1 \\ k &= -2 \\ l &= 0 \end{aligned}$$

$$A = \begin{bmatrix} 25 & 8 & 10 \\ 1 & -5 & -8 \\ -1 & -1 & 0 \end{bmatrix}$$

b. $A = PDP^{-1}$

$$\det \left(\begin{bmatrix} 25-\lambda & 8 & 10 \\ 1 & -5-\lambda & -8 \\ -1 & -1 & -\lambda \end{bmatrix} \right) = 0$$

$$= (25-\lambda)(-5-\lambda)(-\lambda) - 8(-\lambda-8) + 10(-1-(5+\lambda)) = 0$$

$$= (25-\lambda)(5\lambda + \lambda^2 - 8) + 8\lambda + 64 - 10 - 10\lambda = 0$$

$$= (25-\lambda)(5\lambda + \lambda^2 - 8) - 2\lambda + 4 = 0$$

Assume eigenvalues are: $\lambda_1 = 25$, $\lambda_2 = 8$, $\lambda_3 = 1$

(I believe I got the wrong matrix therefore I will use random eigenvalues)

$$\vec{e}_1 = \begin{bmatrix} 0 & 8 & 10 \\ 1 & -30 & -8 \\ -1 & -1 & -25 \end{bmatrix} = \begin{bmatrix} 1 & -30 & -8 \\ 0 & 8 & 10 \\ -1 & -1 & -25 \end{bmatrix} = \begin{bmatrix} 1 & -30 & -8 \\ 0 & 8 & 10 \\ 0 & -29 & -33 \end{bmatrix}$$

$$A = \begin{bmatrix} \vec{e}_{11} & \vec{e}_{21} & \vec{e}_{31} \\ \vec{e}_{12} & \vec{e}_{22} & \vec{e}_{32} \\ \vec{e}_{13} & \vec{e}_{23} & \vec{e}_{33} \end{bmatrix} \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix} \begin{bmatrix} \vec{e}_1 & \vec{e}_2 & \vec{e}_3 \\ \vec{e}_{12} & \vec{e}_{22} & \vec{e}_{32} \\ \vec{e}_{13} & \vec{e}_{23} & \vec{e}_{33} \end{bmatrix}^{-1}$$

Wrong eigenvalues means I can't diagonalize the matrix or find its eigenvectors. This is how I would do it if I had the correct values.