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9. If $|G| = n$ and n is not prime, then some integer $k > 1$ divides n. $g_i \in G$

If we let $H = k \cdot G$ ~~[illegible]~~ (so that the set H is defined as every element in G that is a multiple of k), then $|H| = \frac{n}{k}$ and $|H|$ divides $|G|$ (since $n = (\frac{n}{k}) \cdot k$)
We can define $h_i \in H = kg_i \in G$
Now we must check that H fits all the requirements of a group.

1. Closed:
   - $\rightarrow h_i \cdot h_j = kg_i \cdot kg_j = k^2(g_i \cdot g_j)$
   - $\rightarrow g_i \cdot g_j \in G$ (since G is a group under multiplication)
   - $\rightarrow k(g_i \cdot g_j) \in G$ (since kg_i or kg_j is a part of the group)
   - $\rightarrow k(k(g_i \cdot g_j)) \in H$ (since $H = kG$)

2. Associative:
   - $\rightarrow h_i (h_j \cdot h_z) = h_i \cdot h_j \cdot h_z = (h_i \cdot h_j) h_z$
   - $\rightarrow$ follows from multiplication properties

3. Identity:
   - $\rightarrow$ If e is the unit element in G then
     $g_i \cdot e = g_i = e \cdot g_i$
   - $\rightarrow$ It follows from group rules that $e \cdot h_i = e \cdot kg_i = kg_i = h_i$.
     All we have to do is include e in H

4. Inverse:
   - $\rightarrow h_i = kg_i$
     $g_i \cdot g_i^{-1} = e$
     $h_i \cdot h_i^{-1} = (kg_1)(kg_1)^{-1} = e$
   - $\rightarrow h_i$ has inverse in the form of $(kg_1)^{-1}$