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6a. Base case $n=1$

$$1^3+3\cdot 9-1+3\cdot 3=1+27-1+9=36 \leftarrow \text{divisible by } 6$$

Assume for $n=r$, $r^3+27-r+9=6a$ where $a\in\mathbb{Z}$

$$(r+1)^3-(r+1)+27+9$$

$$=r^3+3r^2+3r+\cancel{1}-r-\cancel{1}+27+9$$

$$=r^3+3r^2+2r+27+9$$

$$=r^3+27-r+9+3r^2+3r$$

$$=6a+3r(r+1)$$

either r or $r+1$ is a factor of two (even number) which means that $3r(r+1)$ can be rewritten as $3\cdot 2b$ where $b\in\mathbb{Z}$

$$6a+3r(r+1)=6a+3\cdot 2b=6a+6b=\boxed{6(a+b)}$$

where $a,b\in\mathbb{Z}$

6. $A=\begin{bmatrix}1 & c & 0\\ 0 & 1 & 0\\ 0 & 0 & 10\end{bmatrix}$

Base case $n=1$

$A^1=\begin{bmatrix}1 & 1c & 0\\ 0 & 1 & 0\\ 0 & 0 & 10^1\end{bmatrix}$ ✓

Inductive step... $n=r+1$

$$A^{r+1}=A^r\begin{bmatrix}1 & c & 0\\ 0 & 1 & 0\\ 0 & 0 & 10\end{bmatrix}=\begin{bmatrix}1 & rc & 0\\ 0 & 1 & 0\\ 0 & 0 & 10^r\end{bmatrix}\begin{bmatrix}1 & c & 0\\ 0 & 1 & 0\\ 0 & 0 & 10\end{bmatrix}$$

$$=\begin{bmatrix}1 & c+rc & 0\\ 0 & 1 & 0\\ 0 & 0 & 10\cdot 10^r\end{bmatrix}=\begin{bmatrix}1 & c(r+1) & 0\\ 0 & 1 & 0\\ 0 & 0 & 10^{r+1}\end{bmatrix}$$

$A^{r+1}=\begin{bmatrix}1 & (r+1)c & 0\\ 0 & 1 & 0\\ 0 & 0 & 10^{r+1}\end{bmatrix}$ when $n=r+1$ if $A^r=\begin{bmatrix}1 & rc & 0\\ 0 & 1 & 0\\ 0 & 0 & 10^r\end{bmatrix}$