Phys499A Report

Thomas Belvin

October 2023

0.1 Introduction

The goal is to assess the viability of using wires to detect gravitational particles by measuring the force applied at the end of a wire due to a wave generated by an inciting particle.

If the particle is moving sufficiently at a sufficiently fast speed, v, the interaction can be approximated by the particle moving along a straight track in relation to the wire. I then define the coordinate system such that the wire lies on the $\hat{\mathbf{z}}$ axes and the distance of closest approach between the wire and the particle track, b, is centered at $z = 0$.

$$r_x = b\cos(\phi) + tv\sin(\theta)\sin(\phi) \tag{1}$$

$$r_y = b\sin(\phi) - tv\sin(\theta)\cos(\phi) \tag{2}$$

$$r_z = tv\cos(\theta) - z_{\text{wire}} \tag{3}$$

where $\mathbf{r}$ is the vector from a point on the wire, z_{wire} to the particle, ϕ is the angle between the $\hat{\mathbf{x}}$ axes and the track at $z = 0$, and θ is the angle between track and the $\hat{\mathbf{z}}$ axes.

The force the particle applies on a small piece of the wire is

$$\mathbf{F} = \frac{GM\partial m}{\mathbf{r}^2}\hat{\mathbf{r}} \tag{4}$$

where M is the mass of the inciting particle, ∂m is the mass of a small segment of the wire, and $\mathbf{r}$ is the vector between the inciting particle and the segment of wire.

Since the particle is moving quick enough that it can be approximated as moving in a straight line in relation to the wire, it is safe to assume that it will apply and impulse to the wire. Taking $\mathbf{F}$ from (4) and integrating from negative infinity to infinity with time gives

$$I_x = \frac{2GM\partial m(b\cos(\phi) + z_{\text{wire}}\sin(\theta)\cos(\theta)\sin(\phi))}{v\,(b^2 - z_{\text{wire}}^2\cos^2(\theta) + z_{\text{wire}}^2)} \tag{5}$$

$$I_y = \frac{2GM\partial m(b\sin(\phi) - z_{\text{wire}}\sin(\theta)\cos(\theta)\cos(\phi))}{v\,(b^2 - z_{\text{wire}}^2\cos^2(\theta) + z_{\text{wire}}^2)} \tag{6}$$

$$I_z = -\frac{2GM\partial m\ z_{\text{wire}}\sin^2(\theta)}{v\,(b^2 - z_{\text{wire}}^2\cos^2(\theta) + z_{\text{wire}}^2)} \tag{7}$$

Dividing these impulses by ∂m gives an initial velocity to each point on the wire which can be used to solve the wave equation and show the time evolution of the wire after the impulse provided by the inciting particle. I also used pen and paper analysis to create a travelling wave pulse $\dot{\psi}$ which matches the solutions provided by scipy's solve_ivp method. It is easy to see that $\frac{I_i}{\partial m} = \dot{\psi}_i(z, 0)$

$$\dot{\psi}(z,t) = \dot{\psi}(z,t)_L + \dot{\psi}(z,t)_R \tag{8}$$

$$\dot{\psi}_x(z,t)_L = \frac{GM}{v}\frac{b\cos(\phi) + (z - wt)\sin(\theta)\cos(\theta)\sin(\phi)}{b^2 - (z - wt)^2\sin^2(\theta)}$$
$$\dot{\psi}_x(z,t)_R = \frac{GM}{v}\frac{b\cos(\phi) + (z + wt)\sin(\theta)\cos(\theta)\sin(\phi)}{b^2 - (z + wt)^2\sin^2(\theta)}$$

$$\dot{\psi}_y(z,t)_L = \frac{GM}{v}\frac{(b\sin(\phi) - (z + wt)\sin(\theta)\cos(\theta)\cos(\phi))}{\left(b^2 - (z + wt)^2\sin^2(\theta)\right)}$$
$$\dot{\psi}_y(z,t)_R = \frac{GM}{v}\frac{(b\sin(\phi) - (z - wt)\sin(\theta)\cos(\theta)\cos(\phi))}{\left(b^2 - (z - wt)^2\sin^2(\theta)\right)}$$
$$\dot{\psi}_z(z,t)_L = \frac{GM}{v}\frac{(z + wt)\sin^2(\theta)}{\left(b^2 - (z + wt)^2\sin^2(\theta)\right)}$$
$$\dot{\psi}_z(z,t)_R = \frac{GM}{v}\frac{(z - wt)\sin^2(\theta)}{\left(b^2 - (z - wt)^2\sin^2(\theta)\right)}$$

where w is the wave speed of the wire. The $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$ waves are transverse/shear waves and the $\hat{\mathbf{z}}$ is a longitudinal/pressure wave. Taking the time derivatives of

$\dot{\psi}$ gives the accelerations of the wavepulses;

$$\ddot{\psi}(z,t) = \ddot{\psi}(z,t)_L + \ddot{\psi}(z,t)_R \tag{9}$$

$$\ddot{\psi}_x(z,t)_R = -\frac{GMw\sin(\theta)\cos(\theta)\sin(\phi)}{v\left(b^2+\sin^2(\theta)(z-wt)^2\right)} + \frac{2GMw\sin^2(\theta)(z-wt)(b\cos(\phi)+\sin(\theta)\cos(\theta)\sin(\phi)(z-wt))}{v\left(b^2+\sin^2(\theta)(z-wt)^2\right)^2} \tag{10}$$

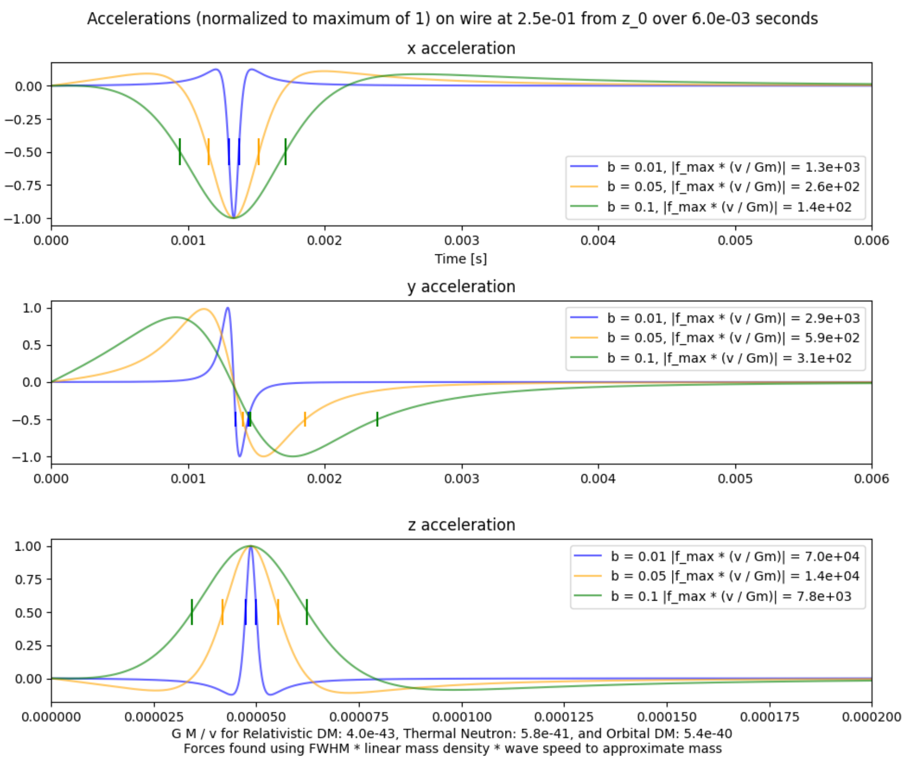
$$\ddot{\psi}_y(z,t)_R = \frac{GMw\sin(\theta)\cos(\theta)\cos(\phi)}{v\left(b^2+\sin^2(\theta)(z-wt)^2\right)} + \frac{2GMw\sin^2(\theta)(z-wt)(b\sin(\phi)-\sin(\theta)\cos(\theta)\cos(\phi)(z-wt))}{v\left(b^2+\sin^2(\theta)(z-wt)^2\right)^2} \tag{11}$$

$$\ddot{\psi}_y(z,t)_R = \frac{GMw\sin^2(\theta)}{v\left(b^2+\sin^2(\theta)(z+wt)^2\right)} - \frac{2GMw\sin^4(\theta)(z-wt)^2}{v\left(b^2+\sin^2(\theta)(z+wt)^2\right)^2} \tag{12}$$

The left wave pulse accelerations have the same structure as the right except with the signs of each fraction and wavespeed flipped. It should also be noted that when plugging values into a computer I factored out the $\frac{GM}{v}$ term to preven IEEE precision errors.

0.2 Example

The example that best shows each individual waveform is with $\phi = 90°$ and $\theta = 45°$.



This figure shows that you could identify b, ϕ, andθ by normalizing the data to a maximum of 1 and fitting. Additionally, the maximum force (without the GM/v component) caused by each wave has been approximated by taking the FWHM * linear mass density * wave speed. This combined with the magnitude of the GM/v component can give us an estimation for how sensitive our force sensors would have to be.