Phys499A Report

Thomas Belvin

October 2023

Introduction

The goal is to assess the viability of using wires to detect gravitational particles by measuring the force applied at the end of a wire due to a wave generated by an inciting particle.

If the particle is moving sufficiently at a sufficiently fast speed, v, the interaction can be approximated by the particle moving along a straight track in relation to the wire. I then define the coordinate system such that the wire lies on the $\hat{\mathbf{z}}$ axes and the distance of closest approach between the wire and the particle track, b, is centered at $z = 0$.

$$r_x = b\cos(\phi) + tv\sin(\theta)\sin(\phi) \tag{1}$$

$$r_y = b\sin(\phi) - tv\sin(\theta)\cos(\phi) \tag{2}$$

$$r_z = tv\cos(\theta) - z_{\text{wire}} \tag{3}$$

where $\mathbf{r}$ is the vector from a point on the wire, z_{wire} to the particle, ϕ is the angle between the $\hat{\mathbf{x}}$ axes and the track at $z = 0$, and θ is the angle between track and the $\hat{\mathbf{z}}$ axes.

The force the particle applies on a small piece of the wire is

$$\mathbf{F} = \frac{GM\partial m}{\mathbf{r}^2}\hat{\mathbf{r}} \tag{4}$$

where M is the mass of the inciting particle, ∂m is the mass of a small segment of the wire, and $\mathbf{r}$ is the vector between the inciting particle and the segment of wire.

Since the particle is moving quick enough that it can be approximated as moving in a straight line in relation to the wire, it is safe to assume that it will apply and impulse to the wire. Taking $\mathbf{F}$ from (4) and integrating from negative infinity to infinity with time gives

$$I_x = \frac{2GM\partial m}{v}\frac{(b\cos(\phi) + z_{\text{wire}}\sin(\theta)\cos(\theta)\sin(\phi))}{(b^2 - z_{\text{wire}}^2\cos^2(\theta) + z_{\text{wire}}^2)} \tag{5}$$

$$I_y = \frac{2GM\partial m}{v}\frac{(b\sin(\phi) - z_{\text{wire}}\sin(\theta)\cos(\theta)\cos(\phi))}{(b^2 - z_{\text{wire}}^2\cos^2(\theta) + z_{\text{wire}}^2)} \tag{6}$$

$$I_z = -\frac{2GM\partial m}{v}\frac{z_{\text{wire}}\sin^2(\theta)}{(b^2 - z_{\text{wire}}^2\cos^2(\theta) + z_{\text{wire}}^2)} \tag{7}$$

Dividing these impulses by ∂m gives an initial velocity to each point on the wire which can be used to solve the wave equation and show the time evolution of the wire after the impulse provided by the inciting particle. I also used pen and paper analysis to create a travelling wave pulse $\dot{\psi}$ which matches the solutions provided by scipy's solve_ivp method. It is easy to see that $\frac{I_i}{\partial m} = \dot{\psi}_i(z,0)$

$$\dot{\psi}(z,t) = \dot{\psi}(z,t)_L + \dot{\psi}(z,t)_R \tag{8}$$

$$\dot{\psi}_x(z,t)_L = \frac{GM}{v}\frac{b\cos(\phi) + (z-wt)\sin(\theta)\cos(\theta)\sin(\phi)}{b^2 - (z-wt)^2\sin^2(\theta)}$$
$$\dot{\psi}_x(z,t)_R = \frac{GM}{v}\frac{b\cos(\phi) + (z+wt)\sin(\theta)\cos(\theta)\sin(\phi)}{b^2 - (z+wt)^2\sin^2(\theta)}$$

$$\dot{\psi}_y(z,t)_L = \frac{GM}{v}\frac{(b\sin(\phi) - (z+wt)\sin(\theta)\cos(\theta)\cos(\phi))}{\left(b^2 - (z+wt)^2\sin^2(\theta)\right)}$$
$$\dot{\psi}_y(z,t)_R = \frac{GM}{v}\frac{(b\sin(\phi) - (z-wt)\sin(\theta)\cos(\theta)\cos(\phi))}{\left(b^2 - (z-wt)^2\sin^2(\theta)\right)}$$
$$\dot{\psi}_z(z,t)_L = \frac{GM}{v}\frac{(z+wt)\sin^2(\theta)}{\left(b^2 - (z+wt)^2\sin^2(\theta)\right)}$$
$$\dot{\psi}_z(z,t)_R = \frac{GM}{v}\frac{(z-wt)\sin^2(\theta)}{\left(b^2 - (z-wt)^2\sin^2(\theta)\right)}$$

where w is the wave speed of the wire. The $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$ waves are transverse/shear waves and the $\hat{\mathbf{z}}$ is a longitudinal/pressure wave. Taking the time derivatives of

$\dot{\psi}$ gives the accelerations of the wavepulses;

$$\ddot{\psi}(z,t) = \ddot{\psi}(z,t)_L + \ddot{\psi}(z,t)_R \tag{9}$$

$$\ddot{\psi}_x(z,t)_R = -\frac{GMw\sin(\theta)\cos(\theta)\sin(\phi)}{v\left(b^2+\sin^2(\theta)(z-wt)^2\right)} + \frac{2GMw\sin^2(\theta)(z-wt)(b\cos(\phi)+\sin(\theta)\cos(\theta)\sin(\phi)(z-wt))}{v\left(b^2+\sin^2(\theta)(z-wt)^2\right)^2} \tag{10}$$

$$\ddot{\psi}_y(z,t)_R = \frac{GMw\sin(\theta)\cos(\theta)\cos(\phi)}{v\left(b^2+\sin^2(\theta)(z-wt)^2\right)} + \frac{2GMw\sin^2(\theta)(z-wt)(b\sin(\phi)-\sin(\theta)\cos(\theta)\cos(\phi)(z-wt))}{v\left(b^2+\sin^2(\theta)(z-wt)^2\right)^2} \tag{11}$$

$$\ddot{\psi}_y(z,t)_R = \frac{GMw\sin^2(\theta)}{v\left(b^2+\sin^2(\theta)(z+wt)^2\right)} - \frac{2GMw\sin^4(\theta)(z-wt)^2}{v\left(b^2+\sin^2(\theta)(z+wt)^2\right)^2} \tag{12}$$

The left wave pulse accelerations have the same structure as the right except with the signs of each fraction and wavespeed flipped. It should also be noted that when plugging values into a computer I factored out the $\frac{GM}{v}$ term to preven IEEE precision errors.

0.1 Finding a force estimate

The example that best shows each individual waveform is with $\phi = 90°$ and $\theta = 45°$. I am analyzing the acceleration at a single point on the wire to estimate the force applied. If a force sensor was placed on a fixed end of the wire the magnitude of the forces would???

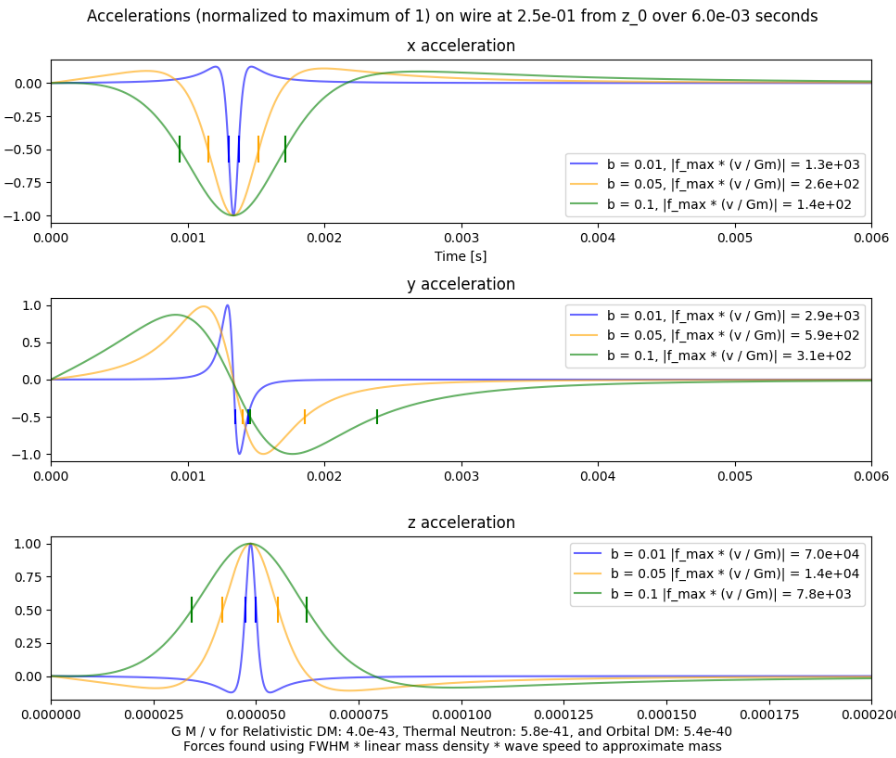

Figure 1: Acceleration normalized to a maximum magnitude of 1.

This figure shows that you can fit b, ϕ, andθ to the shape of the wave and then find $\frac{GM}{v}$ from the amplitude of force.

Recovering Force

The maximum force (with $\frac{GM}{v}$ factored out) caused by each wave has been approximated by taking the FWHM * linear mass density * wave speed. This combined with the magnitude of the $\frac{GM}{v}$ component can give us an estimation for how sensitive our force sensors would have to be.

Based on [2] most force sensors work best within a range of milli-Newtons, with more sensitive ones being able to detect micro-Newtons. The most sensitive force sensor I found was able to detect about 10 zepto-Newtons [1].

Based off the figure, the maximum detected force is around the order of 10^2 to 10^4 $\times\frac{GM}{v}$ Newtons. This gives somethings around 10^{-9} to 10^{-7} $\times\frac{M}{v}$. The inciting particle is likely to be travelling somewhere between 220 $\frac{km}{s}$ and $3e5$ $\frac{km}{s}$ leaving us with a force of approximately 10^{-17} to 10^{-12} $\times M$ Newtons. In order for this to be detectable with one of the detectors mentioned in ** Reference article going over force sensors ** we would need a particle mass somewhere on the order of 10^{14} to 10^{6} kilograms. With a specialized sensor such as the one from [1] we could allow particle mass to be as low as 10^{-6} to 10^{-3} kilograms.

By choosing a wire that maximizes cross-sectional area (increasing Tension and linear mass density) as well as Tensile strength and Elastic modulus (to maximize wave speed) we may be able to increase the range of detectable particle masses, as this will correspondingly increase the magnitude of the force.

Bibliography

[1] J. Moser et al. "Ultrasensitive force detection with a nanotube mechanical resonator". In: *Nature Nanotechnology* 8.7 (June 2013), pp. 493–496. DOI: 10.1038/nnano.2013.97. URL: https://doi.org/10.1038/nnano.2013.97.

[2] Yuzhang Wei and Qingsong Xu. "An overview of micro-force sensing techniques". In: *Sensors and Actuators A: Physical* 234 (2015), pp. 359–374. ISSN: 0924-4247. DOI: https://doi.org/10.1016/j.sna.2015.09.028. URL: https://www.sciencedirect.com/science/article/pii/S092442471530145X.