Phys499A Report

Thomas Belvin

October 2023

Introduction

In this thesis paper by Gosh *Insert ref to Gosh Thesis*, they use a matrix of sensor nodes to detect massive particles by measuring and mapping the displacement of the nodes. Inspired by this, the goal of this report is to assess the viability of detecting massive particles with a wire. A massive particle passing by a wire with sufficent speed will provide an impulse which will incite wave pulses along the wire that may be detectable by force sensors. By analyzing the wave pulses we may be able to determine properites of the inciting particle.

This could definitely be written better xd

Coordinate System

A massive particle moving past a wire will apply a force, $\mathbf{F}$, to the wire

$$\mathbf{F} = \frac{GM\partial m}{\mathbf{r}^2}\hat{\mathbf{r}} \tag{1}$$

where M is the mass of the inciting particle, ∂m is the mass of a small segment of the wire, and $\mathbf{r}$ is the vector between the particle and wire segment.

If the particle is moving at a sufficiently fast speed, v, the interaction will apply an impulse to the wire, with the particle track able to be approximated as a straight line. We can define the coordinate system such that the wire lies on the $\hat{\mathbf{z}}$ axes and the distance of closest approach between the wire and the particle track, b, is centered at $z = 0$.

The vector, $\mathbf{r}$, is the distance between the particle at a time, t, and a point on the wire, z.

$$\begin{aligned}\mathbf{r} =&(b\cos(\phi) + tv\sin(\theta)\sin(\phi))\hat{\mathbf{x}} \\ &+ (b\sin(\phi) - tv\sin(\theta)\cos(\phi))\hat{\mathbf{y}} \\ &+ (tv\cos(\theta) - z)\hat{\mathbf{z}}\end{aligned} \tag{2}$$

where b is the distance of closest approach between the wire and the particle, ϕ is the angle between particle trakc and the $\hat{\mathbf{x}}$ axes, and θ is the angle between the particle track and the $\hat{\mathbf{z}}$ axes.

Insert figure of wire and track with coord system labeled?

Impulse and Kinematics

Taking the force, $\mathbf{F}$ (1), applied to a segment of wire by the particle, and integrating over all time gives the impulse, $\mathbf{I}$,

$$\begin{aligned}\mathbf{I} =& \frac{2GM\partial m}{v}\frac{(b\cos(\phi)+z\sin(\theta)\cos(\theta)\sin(\phi))}{(b^2-z^2\cos^2(\theta)+z^2)}\hat{\mathbf{x}}\\ &+\frac{2GM\partial m}{v}\frac{(b\sin(\phi)-z\sin(\theta)\cos(\theta)\cos(\phi))}{(b^2-z^2\cos^2(\theta)+z^2)}\hat{\mathbf{y}}\\ &-\frac{2GM\partial m}{v}\frac{z\sin^2(\theta)}{(b^2-z^2\cos^2(\theta)+z^2)}\hat{\mathbf{z}}\end{aligned} \tag{3}$$

Dividing this impulse by the mass of the wire segment, ∂m, we get an equation for initial velocity at every point along the wire, $\dot{\psi}(z,0)$. We can use this as the initial velocity to solve the wave equation, and doing so yields two travelling wave pulses of equal magnitude moving in opposite directions.

Insert image from animation?

Because of this, we can seperate $\dot{\psi}(z,0)$ into left and right travelling waves moving along the wire with speed w resulting in

$$\dot{\psi}(z,t)=\dot{\psi}(z,t)_L+\dot{\psi}(z,t)_R \tag{4}$$

$$\begin{aligned}\dot{\psi}_x(z,t)_L &= \frac{GM}{v}\frac{b\cos(\phi)+(z-wt)\sin(\theta)\cos(\theta)\sin(\phi)}{b^2-(z-wt)^2\sin^2(\theta)}\\ \dot{\psi}_x(z,t)_R &= \frac{GM}{v}\frac{b\cos(\phi)+(z+wt)\sin(\theta)\cos(\theta)\sin(\phi)}{b^2-(z+wt)^2\sin^2(\theta)}\end{aligned}$$

$$\begin{aligned}\dot{\psi}_y(z,t)_L &= \frac{GM}{v}\frac{(b\sin(\phi)-(z+wt)\sin(\theta)\cos(\theta)\cos(\phi))}{\left(b^2-(z+wt)^2\sin^2(\theta)\right)}\\ \dot{\psi}_y(z,t)_R &= \frac{GM}{v}\frac{(b\sin(\phi)-(z-wt)\sin(\theta)\cos(\theta)\cos(\phi))}{\left(b^2-(z-wt)^2\sin^2(\theta)\right)}\\ \dot{\psi}_z(z,t)_L &= \frac{GM}{v}\frac{(z+wt)\sin^2(\theta)}{\left(b^2-(z+wt)^2\sin^2(\theta)\right)}\\ \dot{\psi}_z(z,t)_R &= \frac{GM}{v}\frac{(z-wt)\sin^2(\theta)}{\left(b^2-(z-wt)^2\sin^2(\theta)\right)}\end{aligned}$$

It should be kept in mind that $\dot{\psi}_x$ and $\dot{\psi}_y$ are transverse/shear waves and may have a different wavespeed, w, than the longitudinal/pressure waves, $\dot{\psi}_x$.

Taking the time derivatives of $\dot{\psi}$ gives us the accelerations of the wavepulses,

$\ddot{\psi}$

$$\begin{aligned}
\ddot{\psi}(z,t) =& \ddot{\psi}(z,t)_L + \ddot{\psi}(z,t)_R && (5)\\
\ddot{\psi}_x(z,t)_R =& -\frac{GMw\sin(\theta)\cos(\theta)\sin(\phi)}{v\left(b^2+\sin^2(\theta)(z-wt)^2\right)} \\
&+ \frac{2GMw\sin^2(\theta)(z-wt)(b\cos(\phi)+\sin(\theta)\cos(\theta)\sin(\phi)(z-wt))}{v\left(b^2+\sin^2(\theta)(z-wt)^2\right)^2} && (6)\\
\ddot{\psi}_y(z,t)_R =& \frac{GMw\sin(\theta)\cos(\theta)\cos(\phi)}{v\left(b^2+\sin^2(\theta)(z-wt)^2\right)} \\
&+ \frac{2GMw\sin^2(\theta)(z-wt)(b\sin(\phi)-\sin(\theta)\cos(\theta)\cos(\phi)(z-wt))}{v\left(b^2+\sin^2(\theta)(z-wt)^2\right)^2} && (7)\\
\ddot{\psi}_y(z,t)_R =& \frac{GMw\sin^2(\theta)}{v\left(b^2+\sin^2(\theta)(z+wt)^2\right)} - \frac{2GMw\sin^4(\theta)(z-wt)^2}{v\left(b^2+\sin^2(\theta)(z+wt)^2\right)^2} && (8)
\end{aligned}$$

The left wave pulse accelerations are the same as the right with the signs of each fraction and w inverted.

Force Detection

Acceleration on a point

Since the force is directly proportional to the acceleration on a point of the wire, analyzing the acceleration profiles should give insight on what properties of the particle we might be able to determine from measuring the magnitude and direction of the force on a point of the wire. It should be noted that $\frac{GM}{v}$ has been factored out to prevent IEEE imprecision due to the incredibly small numbers.

With $\phi = 90°$ and $\theta = 45°$ we get the following acceleration profiles.

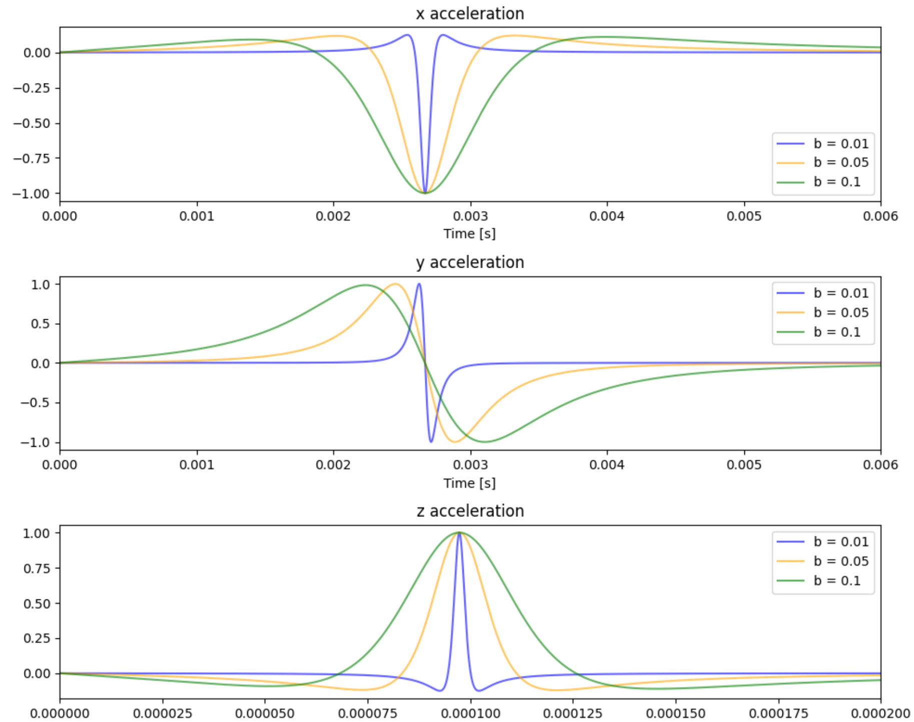

Figure 1: Acceleration normalized to a maximum magnitude of 1.

The unique shape of the accelerations at different values of b means we could fit b, ϕ, and θ from the shape of the curve and find the $\frac{M}{v}$ ratio of the particle from the magnitude of the force measured at a point on the wire.

Recovering Force

By dimensional analysis, we can estimate mass by taking the Full Width Half Max of the acceleration pulse $\times$ the linear mass density of the wire $\times$ the wavespeed. The maximum force (with $\frac{GM}{v}$ factored out) can then be approximated by multiplying this mass with the maximum acceleration in 1. This combined with the magnitude of the $\frac{GM}{v}$ component can give us an estimation for how sensitive our force sensors would have to be to detect the waves incited by a massive particle.

Based on this *year* report by *** AUTHOR *** [2] most force sensors have a maximum sensitivity of milli-Newtons, with a few being able to detect micro-Newtons. This article from *year* by *AUTHOR* [1] states that a sensitivity of tens of zepto-Newtons can be achieved using carbon nanotubes and capacitive sensing.

Applying this to figure 1 gives the maximum detected force as around the order of 10^2 to 10^4 $\times \frac{GM}{v}$ Newtons or around 10^{-9} to 10^{-7} $\times \frac{M}{v}$ once we factor in G. The inciting particle is likely to be travelling somewhere between 220 $\frac{km}{s}$ and $3e5$ $\frac{km}{s}$ leaving us with a force of approximately 10^{-17} to 10^{-12} $\times M$ Newtons.

Is there something I can reference for these particle speeds?

In order for this to be detectable with one of the detectors mentioned in [2] we would need a particle mass somewhere on the order of 10^{14} to 10^6 kilograms. With a more detector such as the one from [1] we could perhaps allow particle mass to be as low as 10^{-6} to 10^{-3} kilograms.

Optimizable Parameters

By choosing a wire that maximizes cross-sectional area (increasing Tension and linear mass density) as well as Tensile strength and Elastic modulus (to maximize wave speed) we may be able to increase the range of detectable particle masses, as this will correspondingly increase the magnitude of the force.

TRIPLE CHECK THAT THESE ARE ACTUALLY WHAT WOULD OPTIMIZE IT

Other Methods

Conclusion

Bibliography

[1] J. Moser et al. "Ultrasensitive force detection with a nanotube mechanical resonator". In: *Nature Nanotechnology* 8.7 (June 2013), pp. 493–496. DOI: 10.1038/nnano.2013.97. URL: https://doi.org/10.1038/nnano.2013.97.

[2] Yuzhang Wei and Qingsong Xu. "An overview of micro-force sensing techniques". In: *Sensors and Actuators A: Physical* 234 (2015), pp. 359–374. ISSN: 0924-4247. DOI: https://doi.org/10.1016/j.sna.2015.09.028. URL: https://www.sciencedirect.com/science/article/pii/S092442471530145X.