Phys499A Report

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0.1 Introduction

The goal is to assess the viability of using wires to detect gravitational particles by measuring the force applied at the end of a wire due to a wave generated by an inciting particle.

If the particle is moving sufficiently at a sufficiently fast speed, v, the interaction can be approximated by the particle moving along a straight track in relation to the wire. I then define the coordinate system such that the wire lies on the $\hat{\mathbf{z}}$ axes and the distance of closest approach between the wire and the particle track, b, is centered at z = 0.

$$r_x = b\cos(\phi) + tv\sin(\theta)\sin(\phi) \tag{1}$$

$$r_y = b\sin(\phi) - tv\sin(\theta)\cos(\phi) \tag{2}$$

$$r_z = tv\cos(\theta) - z_{\text{wire}} \tag{3}$$

where \mathbf{r} is the vector from a point on the wire, z_{wire} to the particle, ϕ is the angle between the $\hat{\mathbf{x}}$ axes and the track at z=0, and θ is the angle between track and the $\hat{\mathbf{z}}$ axes.

The force the particle applies on a small piece of the wire is

$$\mathbf{F} = \frac{GM\partial m}{\mathbf{r}^2}\hat{\mathbf{r}} \tag{4}$$

where M is the mass of the inciting particle, ∂m is the mass of a small segment of the wire, and \mathbf{r} is the vector between the inciting particle and the segment of wire.

Since the particle is moving quick enough that it can be approximated as moving in a straight line in relation to the wire, it is safe to assume that it will apply and impulse to the wire. Taking \mathbf{F} from (4) and integrating from negative infinity to infinity with time gives

$$I_x = \frac{2GM\partial m(b\cos(\phi) + z_{\text{wire}}\sin(\theta)\cos(\theta)\sin(\phi))}{v(b^2 - z_{\text{wire}}^2\cos^2(\theta) + z_{\text{wire}}^2)}$$
(5)

$$I_y = \frac{2GM\partial m(b\sin(\phi) - z_{\text{wire}}\sin(\theta)\cos(\theta)\cos(\phi))}{v(b^2 - z_{\text{wire}}^2\cos^2(\theta) + z_{\text{wire}}^2)}$$
(6)

$$I_z = -\frac{2GM\partial m \ z_{\text{wire}} \cos^2(\theta) + z_{\text{wire}}^2}{v \left(b^2 - z_{\text{wire}}^2 \cos^2(\theta) + z_{\text{wire}}^2\right)}$$
(7)

0.2 Simple Example