Phys499A Report

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Introduction

In this thesis paper by Gosh *Insert ref to Gosh Thesis*, they use a matrix of sensor nodes to detect massive particles by measuring and mapping the displacement of the nodes. Inspired by this, the goal of this report is to assess the viability of detecting massive particles with a wire. A massive particle passing by a wire with sufficent speed will provide an impulse which will incite wave pulses along the wire that may be detectable by force sensors.

This could definitely be written better xd

Coordinate System

A massive particle moving past a wire will apply a force, $\mathbf{F}$, to the wire

$$\mathbf{F} = \frac{GM\partial m}{\mathbf{r}^2}\hat{\mathbf{r}} \tag{1}$$

where M is the mass of the inciting particle, ∂m is the mass of a small segment of the wire and $\mathbf{r}$ is the vector between the particle and that segment of the wire.

If the particle is moving at a sufficiently fast speed, v, the interaction can be approximated as an impulse and the particle trajectory as a straight track. We can define the coordinate system such that the wire lies on the $\hat{\mathbf{z}}$ axes and the distance of closest approach between the wire and the particle track, b, is centered at $z = 0$.

The vector between a point on the wire, z and the track, $\mathbf{r}$, is then defined as

$$\begin{aligned}\mathbf{r} =& (b\cos(\phi) + tv\sin(\theta)\sin(\phi))\hat{\mathbf{x}} \\ &+ (b\sin(\phi) - tv\sin(\theta)\cos(\phi))\hat{\mathbf{y}} \\ &+ (tw\cos(\theta) - z)\hat{\mathbf{x}}\end{aligned} \tag{2}$$

where ϕ is the angle between the $\hat{\mathbf{x}}$ axes and the particle track, and θ is the angle between the particle track and the $\hat{\mathbf{z}}$ axes.

Insert figure of wire and track with coord system labeled

Impulse

Taking $\mathbf{F}$ from (1) and integrating over all time gives the impulse, $\mathbf{I}$,

$$\begin{aligned}\mathbf{I} =& \frac{2GM\partial m}{v}\frac{(b\cos(\phi)+z\sin(\theta)\cos(\theta)\sin(\phi))}{(b^2-z^2\cos^2(\theta)+z^2)}\hat{\mathbf{x}}\\ &+\frac{2GM\partial m}{v}\frac{(b\sin(\phi)-z\sin(\theta)\cos(\theta)\cos(\phi))}{(b^2-z^2\cos^2(\theta)+z^2)}\hat{\mathbf{y}}\\ &-\frac{2GM\partial m}{v}\frac{z\sin^2(\theta)}{(b^2-z^2\cos^2(\theta)+z^2)}\hat{\mathbf{z}}\end{aligned} \tag{3}$$

Dividing these impulses by ∂m gives an initial velocity to each point on the wire which can be used to solve the wave equation as an initial value problem.

Wire Kinematics

Dividing $\mathbf{I}$ (3) by ∂m will give us an equation for initial velocity at every point along the wire, $\dot{\psi}(z,0)$. There will be two travelling waves of equal magnitude moving in opposite direction we can divide seperate $\dot{\psi}(z,0)$ into left and right travelling waves moving along the wire with speed w resulting in

$$\dot{\psi}(z,t) = \dot{\psi}(z,t)_L + \dot{\psi}(z,t)_R \tag{4}$$

$$\begin{aligned}\dot{\psi}_x(z,t)_L &= \frac{GM}{v}\frac{b\cos(\phi)+(z-wt)\sin(\theta)\cos(\theta)\sin(\phi)}{b^2-(z-wt)^2\sin^2(\theta)}\\ \dot{\psi}_x(z,t)_R &= \frac{GM}{v}\frac{b\cos(\phi)+(z+wt)\sin(\theta)\cos(\theta)\sin(\phi)}{b^2-(z+wt)^2\sin^2(\theta)}\end{aligned}$$

$$\begin{aligned}\dot{\psi}_y(z,t)_L &= \frac{GM}{v}\frac{(b\sin(\phi)-(z+wt)\sin(\theta)\cos(\theta)\cos(\phi))}{\left(b^2-(z+wt)^2\sin^2(\theta)\right)}\\ \dot{\psi}_y(z,t)_R &= \frac{GM}{v}\frac{(b\sin(\phi)-(z-wt)\sin(\theta)\cos(\theta)\cos(\phi))}{\left(b^2-(z-wt)^2\sin^2(\theta)\right)}\\ \dot{\psi}_z(z,t)_L &= \frac{GM}{v}\frac{(z+wt)\sin^2(\theta)}{\left(b^2-(z+wt)^2\sin^2(\theta)\right)}\\ \dot{\psi}_z(z,t)_R &= \frac{GM}{v}\frac{(z-wt)\sin^2(\theta)}{\left(b^2-(z-wt)^2\sin^2(\theta)\right)}\end{aligned}$$

$\dot{\psi}_x$ and $\dot{\psi}_y$ are transverse/shear waves and have a different wavespeed, w, than the longitudinal/pressure waves, $\dot{\psi}_x$.

Taking the time derivatives of $\dot{\psi}$ gives us the accelerations of the wavepulses, $\ddot{\psi}$

$$\ddot{\psi}(z,t) = \ddot{\psi}(z,t)_L + \ddot{\psi}(z,t)_R \tag{5}$$

$$\begin{aligned}\ddot{\psi}_x(z,t)_R = &- \frac{GMw\sin(\theta)\cos(\theta)\sin(\phi)}{v\left(b^2+\sin^2(\theta)(z-wt)^2\right)} \\ &+ \frac{2GMw\sin^2(\theta)(z-wt)(b\cos(\phi)+\sin(\theta)\cos(\theta)\sin(\phi)(z-wt))}{v\left(b^2+\sin^2(\theta)(z-wt)^2\right)^2}\end{aligned} \tag{6}$$

$$\begin{aligned}\ddot{\psi}_y(z,t)_R = &\frac{GMw\sin(\theta)\cos(\theta)\cos(\phi)}{v\left(b^2+\sin^2(\theta)(z-wt)^2\right)} \\ &+ \frac{2GMw\sin^2(\theta)(z-wt)(b\sin(\phi)-\sin(\theta)\cos(\theta)\cos(\phi)(z-wt))}{v\left(b^2+\sin^2(\theta)(z-wt)^2\right)^2}\end{aligned} \tag{7}$$

$$\ddot{\psi}_y(z,t)_R = \frac{GMw\sin^2(\theta)}{v\left(b^2+\sin^2(\theta)(z+wt)^2\right)} - \frac{2GMw\sin^4(\theta)(z-wt)^2}{v\left(b^2+\sin^2(\theta)(z+wt)^2\right)^2} \tag{8}$$

The left wave pulse accelerations are the same as the right with the signs of each fraction and w inverted. It should also be noted that when plugging values into a computer I factored out the $\frac{GM}{v}$ term to prevent IEEE precision errors when using a numerical solver such as sicpys solve ivp.

Force Detection

Acceleration on a point

The example that best shows each individual waveform is with $\phi = 90°$ and $\theta = 45°$. We can then analyze the the acceleration at single point to

*Need work here xd ***

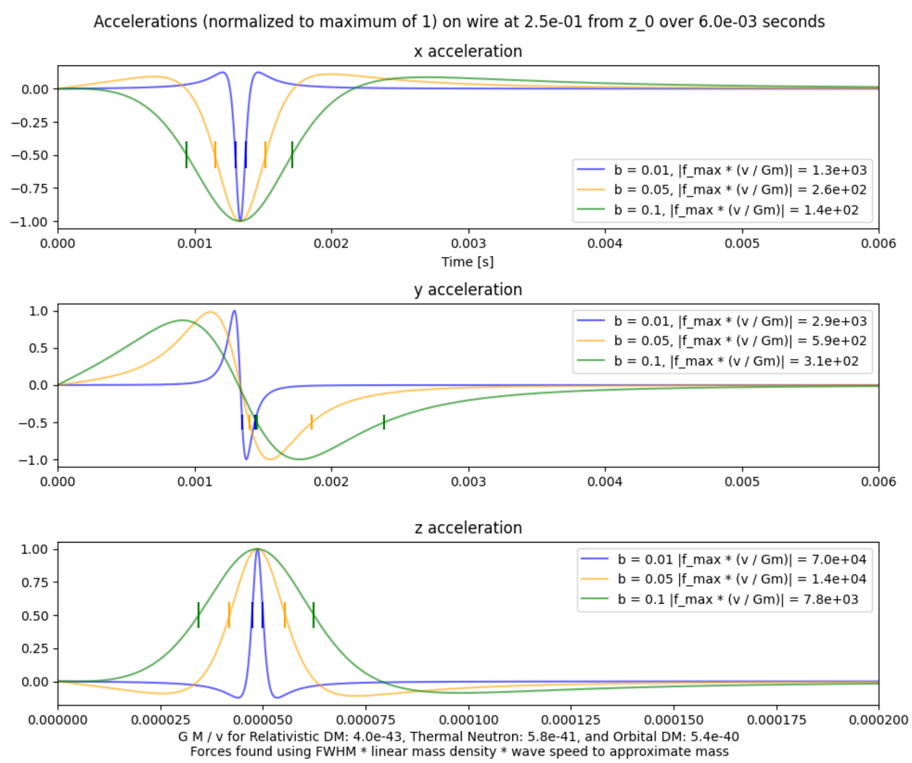

Figure 1: Acceleration normalized to a maximum magnitude of 1.

This figure shows that you can fit b, ϕ, andθ to the shape of the wave and then find $\frac{GM}{v}$ from the amplitude of force.

Recovering Force

By dimensional analysis, we can estimate mass by taking the Full Width Half Max of the acceleration pulse $\times$ linear mass density $\times$ wavespeed. The maximum force (with $\frac{GM}{v}$ factored out) can then be approximated by multiplying this mass with the maximum acceleration in 1. This combined with the magnitude of the $\frac{GM}{v}$ component can give us an estimation for how sensitive our force sensors would have to be.

Based on this *year* report by ** *AUTHOR* ** [2] most force sensors have a maximum sensitivity of milli-Newtons, with a few being able to detect micro-Newtons. This article from *year* by *AUTHOR* [1] states that a sensitivity of tens of zepto-Newtons can be achieved using carbon nanotubes and capacitive sensing. Based off the figure, the maximum detected force is around the order of 10^2 to 10^4 $\times \frac{GM}{v}$ Newtons. This gives somethings around 10^{-9} to 10^{-7} $\times \frac{M}{v}$. The inciting particle is likely to be travelling somewhere between 220 $\frac{km}{s}$ and $3e5$ $\frac{km}{s}$ leaving us with a force of approximately 10^{-17} to 10^{-12} $\times M$ Newtons. In order for this to be detectable with one of the detectors mentioned in [2] we would need a particle mass somewhere on the order of 10^{14} to 10^{6} kilograms. With another method such as the one from [1] we could perhaps allow particle mass to be as low as 10^{-6} to 10^{-3} kilograms.

Optimizable Parameters

By choosing a wire that maximizes cross-sectional area (increasing Tension and linear mass density) as well as Tensile strength and Elastic modulus (to maximize wave speed) we may be able to increase the range of detectable particle masses, as this will correspondingly increase the magnitude of the force.

Other Methods

Conclusion

Bibliography

[1] J. Moser et al. "Ultrasensitive force detection with a nanotube mechanical resonator". In: *Nature Nanotechnology* 8.7 (June 2013), pp. 493–496. DOI: `10.1038/nnano.2013.97`. URL: `https://doi.org/10.1038/nnano.2013.97`.

[2] Yuzhang Wei and Qingsong Xu. "An overview of micro-force sensing techniques". In: *Sensors and Actuators A: Physical* 234 (2015), pp. 359–374. ISSN: 0924-4247. DOI: `https://doi.org/10.1016/j.sna.2015.09.028`. URL: `https://www.sciencedirect.com/science/article/pii/S092442471530145X`.