Predicting Plays in Regular Season NFL Games with Bayesian Logistic Regression

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1. Introduction

In NFL games, defending teams have are at an advantage if they can correctly predict whether the team with possession will pass or run the ball. Accessible data and literature on statistical learning methods enables spectators off the field to make informed predictions by modeling the play system as a classification problem.

Previous literature has explored this problem using frequentist statistical learning methods. Fernandes et al. (2020) developed models achieving 75.3% accuracy with neural networks, while Goyal (2019) reached 80% accuracy. The consensus is that neural networks deliver the highest OOS accuracy, but at the cost of interpreting features.

2. The Gap

In frequentist statistics, parameters are fixed unknown constants and probability refers to observation frequencies in the long-run. Put simply, “if I repeated this experiment infinitely, how often would I see a certain result?” In Bayesian statistics, parameters are seen as random variables with probability distributions. A prior belief about a variable is made, data is observed, and then updated to a posterior distribution using Bayes’ Theorem. In this way, statements like “there is a 90% chance the coefficient is between 0.3 and 0.7” can be directly made.

While black-box models like neural networks achieve high accuracy, one cannot draw linear relationships between predictor values and outcome probabilities. For fans watching games with access to a computer, an interpretable model that explains the why behind what may be very valuable.

3. Rationale and Approach

This analysis examines a Bayesian logistic regression as an interpretable alternative. The Bayesian framework offers several advantages: uncertainty quantification and transparent coefficient interpretation. We use weakly informative Cauchy(0, 2.5) priors following the Gelman et al. (2008) recommendation for routine logistic regression. The goal is to demonstrate that Bayesian logistic regression can balance interpretability with competitive predictive performance.

4. Methods

4.1 Data

We use pre-processed NFL play-by-play data from nflfastR merged with EA Madden player ratings. The dataset includes 44 predictors describing possession team pass/run ability, opponent blocking ability, in-game statistics, and field position. The model is trained on seasons 2013–2022 and tested on 2023.

```
set.seed(632)
# Pre-processed data sourced from nflfastR and maddenratings.weebly.com
nfl_data <- read_csv("nfl_data.csv", show_col_types = F)
# train data 2013-2022
train <- nfl_data %>%
  filter(season < 2023)
# test data 2023
test <- nfl_data %>%
  filter(season == 2023)
# train predictors (columns that start with "_char")
train_X <- train %>%
  dplyr::select(starts_with("char_"))
# test predictors
test_X <- test %>%
  dplyr::select(starts_with("char_"))
# train dependent variable
train_y <- train$target
# test dependent variable
test_y <- test$target
```

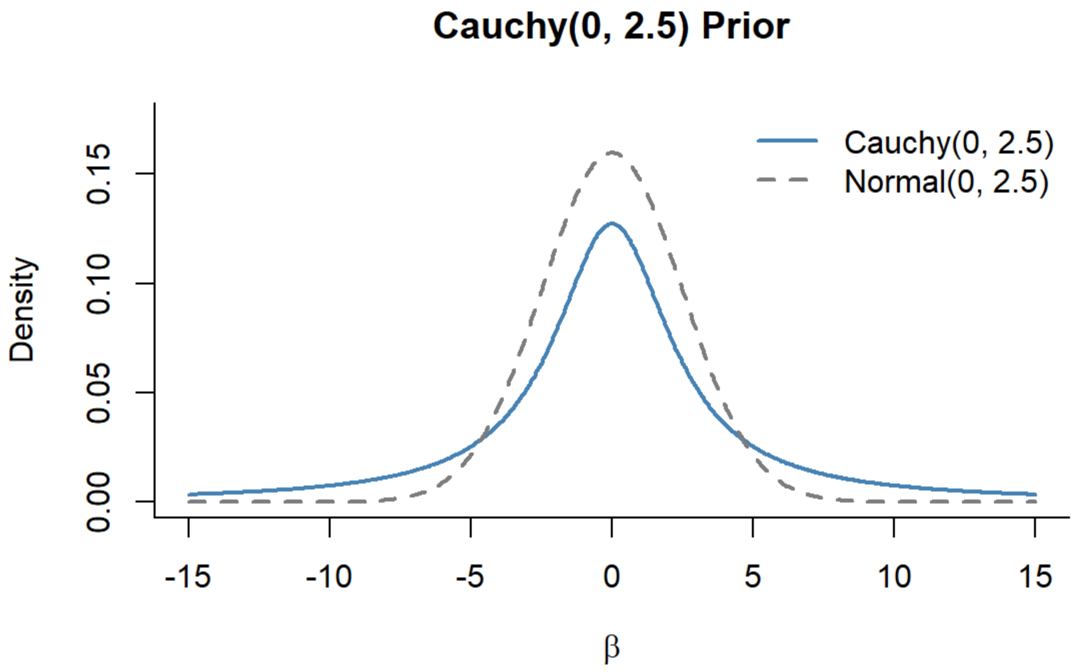
4.2 Model Specification

The logistic regression model assumes $y_i \in \{0, 1\}$ follows a Bernoulli distribution with probability $\pi_i = \frac{1}{1+e^{-X_i^\top \beta}}$.

We specify independent Cauchy(0, 2.5) priors on all coefficients. This is a weakly informative choice that regularizes estimates while allowing for potentially large coefficient effects.

```
# visualize the Cauchy(0, 2.5) prior
x <- seq(-15, 15, length.out = 500)
cauchy_density <- dcauchy(x, location = 0, scale = 2.5)
normal_density <- dnorm(x, mean = 0, sd = 2.5)

plot(x, cauchy_density, type = "l", lwd = 2, col = "steelblue",
     xlab = expression(beta), ylab = "Density",
     main = "Cauchy(0, 2.5) Prior",
     bty = "l", ylim = c(0, max(normal_density) * 1.1))
lines(x, normal_density, lwd = 2, col = "gray50", lty = 2)
legend("topright", legend = c("Cauchy(0, 2.5)", "Normal(0, 2.5)"),
       col = c("steelblue", "gray50"), lwd = 2, lty = c(1, 2), bty = "n")
```



The plot above compares our prior to a Normal with the same scale. Both are centered at zero, expressing a prior belief that coefficients are probably small, potentially uninformative. The Cauchy distribution has heavier tails and it doesn't aggressively penalize large coefficients the way a Normal would. This matters in logistic regression when the following is plausible: a coefficient of $\beta = 5$ on a standardized predictor implies that a one standard deviation increase changes the log-odds by 5, which is huge but not impossible. The Cauchy distribution says "probably small, but I won't be shocked by a big effect." A Normal prior would pull such estimates back toward zero more forcefully. Gelman et al. (2008) recommend this as a sensible default when you don't have strong prior information.

4.3 Sampling Algorithm

We implement a Metropolis-Hastings sampler with Gaussian proposals. Multiple independent chains are run from different initializations to assess convergence. The first half of each chain is discarded as burn-in.

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### Define Functions ###
# scale continuous columns
binary_cols <- sapply(train_X, function(col) all(col %in% c(0, 1))) # identify binary cols
train_X_scaled <- train_X
train_X_scaled[, !binary_cols] <- scale(train_X[, !binary_cols]) # scaled value = (x - x_bar) /
s
test_X_scaled <- test_X
test_X_scaled[, !binary_cols] <- scale(test_X[, !binary_cols])
# include a column of ones to X for the intercept
train_X_scaled <- cbind(Intercept = 1, train_X_scaled)
test_X_scaled <- cbind(Intercept = 1, test_X_scaled)
# define log likelihood function
log_likelihood <- function(y, X, beta){
  linear_comb <- X %*% beta
  probs <- 1 / (1 + exp(-linear_comb))
  epsilon <- 1e-10
  probs <- pmax(pmin(probs, 1 - epsilon), epsilon) # to prevent rare case of log(0)
  sum(y * log(probs) + (1 - y) * log(1 - probs))
}
# define log prior function
log_prior <- function(beta){
  sum(dcauchy(beta, location = 0, scale = 2.5, log = TRUE))
}
# define log posterior
log_posterior <- function(y, X, beta){
  log_likelihood(y, X, beta) + log_prior(beta)
}
# define Metropolis algorithm
metro_sampler <- function(y, X, n_iter, init_beta = NULL, proposal_sd = 0.1){
  K <- ncol(X) # number of betas
  beta_samples <- matrix(0, nrow = n_iter, ncol = K) # init beta sample storage
  beta <- if (is.null(init_beta)) rep(0, K) else init_beta # init beta
  pb <- txtProgressBar(min = 0, max = n_iter, style = 3)
  # run Metropolis steps for n_iter iterations
  for (i in 1:n_iter){
    setTxtProgressBar(pb, i)
    beta_proposed <- beta + rnorm(K, mean = 0, sd = proposal_sd)
    accept_beta <- log_posterior(y, X, beta_proposed) - log_posterior(y, X, beta)
    if (log(runif(1)) < accept_beta){
      beta <- beta_proposed
    }
    beta_samples[i, ] <- beta
  }
  close(pb)
  list(
    # drop the first 1/2 of chain samples as burn-in
    beta_samples = beta_samples[(n_iter / 2 + 1):n_iter, , drop = FALSE]
  )
}
# define multiple chain function
run_multiple_chains <- function(y, X, n_iter, n_chains, proposal_sd = 0.1){
  results <- list()
  for (chain in 1:n_chains){
    cat("Running chain", chain, "...\n")
    results[[chain]] <- metro_sampler(
      y, X, n_iter,
      init_beta = rnorm(ncol(X), 0, 1),
      proposal_sd = proposal_sd
    )
  }
  results # list of n_chains independent MCMC chains made with the sampler
}
### Run the Model
# change from df to matrix
X <- as.matrix(train_X_scaled)
y <- train_y
# run 5 chains for 10,000 iterations each (do 1,000 for a shorter run time)
chains <- run_multiple_chains(y, X, n_iter = 1000, n_chains = 5)
```

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```

```
# extract the samples
beta_samples_all <- lapply(chains, function(chain) chain$beta_samples)
```

5. Results

5.1 Posterior Summaries

We compute posterior means, standard deviations, 90% credible intervals, effective sample size (ESS), and the Gelman-Rubin statistic ($\hat{R}$) for convergence assessment.

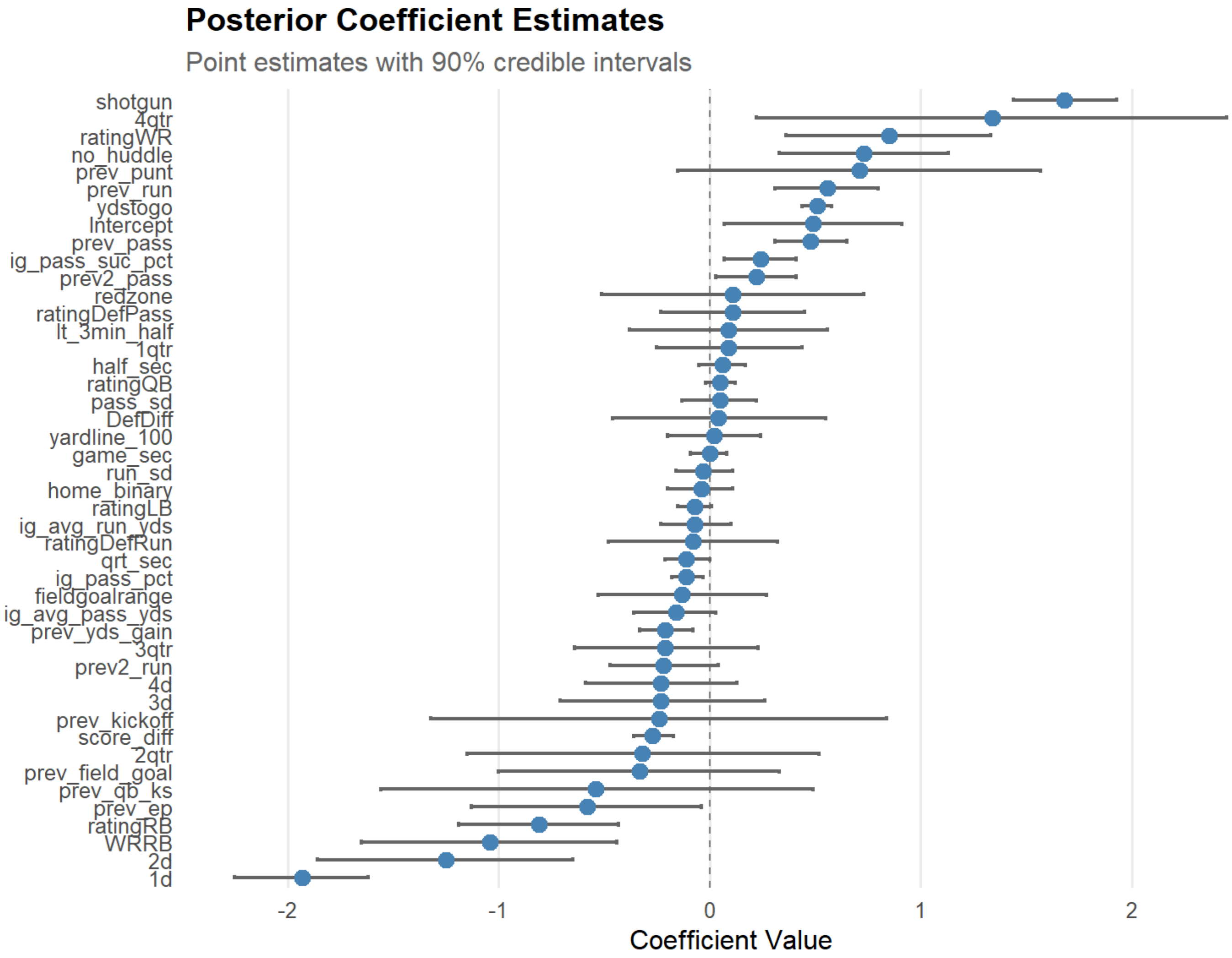
```r
# calculate the mean for each beta
beta_hat <- colMeans(do.call(rbind, beta_samples_all))
# calculate the standard deviation for each beta
beta_sd <- apply(do.call(rbind, beta_samples_all), 2, sd)
# calculate the 90% CI boundaries for each beta
beta_lower <- beta_hat - qnorm(0.95) * beta_sd / sqrt(length(beta_samples_all))
beta_upper <- beta_hat + qnorm(0.95) * beta_sd / sqrt(length(beta_samples_all))
# convert the beta_samples_all list of matrices into an mcmc.list using coda
mcmc_chains <- lapply(beta_samples_all, function(mat) {
  mcmc(mat)
})
combined_mcmc <- mcmc.list(mcmc_chains)
# R hat
rhat <- gelman.diag(combined_mcmc, multivariate = FALSE)$psrf[, "Point est."]
# ESS
ess <- effectiveSize(combined_mcmc)
# make matrix, use to extract column names
X_test <- as.matrix(test_X_scaled) # Includes intercept column of 1s
# Table 2 in the paper
beta_summary <- data.frame(
  Beta = colnames(as.data.frame(X_test)),
  Mean = round(beta_hat, 2),
  SD = round(beta_sd,2),
  CI_Lower = round(beta_lower,2),
  CI_Upper = round(beta_upper,2),
  ESS = round(ess, 2),
  R_hat = round(rhat, 2)
)
# drop "char_" prefix from the beta names
beta_summary$Beta <- gsub("char_", "", beta_summary$Beta)
# sort the data by mean
beta_summary <- beta_summary[order(beta_summary$Mean), ]
rownames(beta_summary) <- NULL
beta_summary
```

```
##                Beta  Mean   SD CI_Lower CI_Upper   ESS R_hat
## 1               1d -1.93 0.42    -2.25    -1.62 15.20 13.50
## 2               2d -1.25 0.82    -1.86    -0.65 27.11 28.87
## 3             WRRB -1.04 0.82    -1.65    -0.44 19.76 23.00
## 4         ratingRB -0.81 0.52    -1.19    -0.43 31.01 13.09
## 5          prev_ep -0.58 0.74    -1.13    -0.04 46.44 16.20
## 6       prev_qb_ks -0.54 1.39    -1.56     0.49 21.66 34.65
## 7  prev_field_goal -0.33 0.91    -1.00     0.33 16.02 15.03
## 8             2qtr -0.32 1.14    -1.15     0.52 21.56 21.04
## 9       score_diff -0.27 0.13    -0.36    -0.17 20.26  2.14
## 10    prev_kickoff -0.24 1.47    -1.32     0.84 16.66 23.88
## 11              3d -0.23 0.66    -0.71     0.26 20.52 22.73
## 12              4d -0.23 0.49    -0.59     0.13 14.64 10.41
## 13       prev2_run -0.22 0.35    -0.47     0.04 28.68  6.36
## 14            3qtr -0.21 0.59    -0.64     0.23 12.50 12.33
## 15   prev_yds_gain -0.21 0.17    -0.33    -0.08 33.54  3.97
## 16 ig_avg_pass_yds -0.16 0.27    -0.36     0.03 18.13  3.67
## 17  fieldgoalrange -0.13 0.54    -0.53     0.27 14.32  7.07
## 18         qrt_sec -0.11 0.14    -0.21     0.00 40.44  3.23
## 19     ig_pass_pct -0.11 0.10    -0.18    -0.03 15.89  1.67
## 20    ratingDefRun -0.08 0.54    -0.48     0.32 17.51 16.77
## 21  ig_avg_run_yds -0.07 0.22    -0.23     0.10 25.22  4.94
## 22        ratingLB -0.07 0.11    -0.15     0.01 30.03  1.50
## 23     home_binary -0.04 0.21    -0.20     0.11 20.38  6.28
## 24          run_sd -0.03 0.18    -0.16     0.11 27.57  2.78
## 25        game_sec  0.00 0.12    -0.09     0.08 15.96  2.00
## 26    yardline_100  0.02 0.30    -0.20     0.24 31.17 10.39
## 27         DefDiff  0.04 0.69    -0.46     0.55 99.33 20.86
## 28         pass_sd  0.05 0.24    -0.13     0.22 16.18  1.57
## 29        ratingQB  0.05 0.09    -0.02     0.12 20.68  3.05
## 30        half_sec  0.06 0.15    -0.05     0.17 32.30  3.95
## 31            1qtr  0.09 0.47    -0.25     0.44 33.94  6.30
## 32    lt_3min_half  0.09 0.64    -0.38     0.56 14.83  9.75
## 33         redzone  0.11 0.84    -0.51     0.73 16.90 17.52
## 34   ratingDefPass  0.11 0.46    -0.23     0.45 16.07  9.67
## 35      prev2_pass  0.22 0.26     0.03     0.41 19.86  4.53
## 36 ig_pass_suc_pct  0.24 0.24     0.07     0.41 12.85  4.36
## 37       prev_pass  0.48 0.23     0.31     0.65 22.04  7.55
## 38       Intercept  0.49 0.58     0.07     0.91 17.92 10.26
## 39         ydstogo  0.51 0.09     0.44     0.58 16.24  4.15
## 40        prev_run  0.56 0.33     0.31     0.80 28.88  7.22
## 41       prev_punt  0.71 1.17    -0.15     1.57 22.21 19.02
## 42       no_huddle  0.73 0.54     0.33     1.13 25.47 15.16
## 43        ratingWR  0.85 0.66     0.36     1.33 35.25 21.31
## 44            4qtr  1.34 1.52     0.22     2.45 22.95 34.43
## 45         shotgun  1.68 0.33     1.44     1.93 20.24  5.93
```

5.2 Coefficient Estimates

The plot below displays posterior mean estimates with 90% credible intervals. Coefficients whose intervals exclude zero provide stronger evidence of an effect on play-calling.

```r
ggplot(beta_summary, aes(x = reorder(Beta, Mean), y = Mean)) +
  geom_hline(yintercept = 0, linetype = "dashed", color = "gray50", linewidth = 0.5) +
  geom_errorbar(aes(ymin = CI_Lower, ymax = CI_Upper),
                width = 0.25, color = "gray40", linewidth = 0.8) +
  geom_point(color = "steelblue", size = 3) +
  coord_flip() +
  theme_minimal(base_size = 12) +
  labs(
    title = "Posterior Coefficient Estimates",
    subtitle = "Point estimates with 90% credible intervals",
    x = NULL,
    y = "Coefficient Value"
  ) +
  theme(
    plot.title = element_text(face = "bold"),
    plot.subtitle = element_text(color = "gray40"),
    panel.grid.major.y = element_blank(),
    panel.grid.minor = element_blank()
  )
```



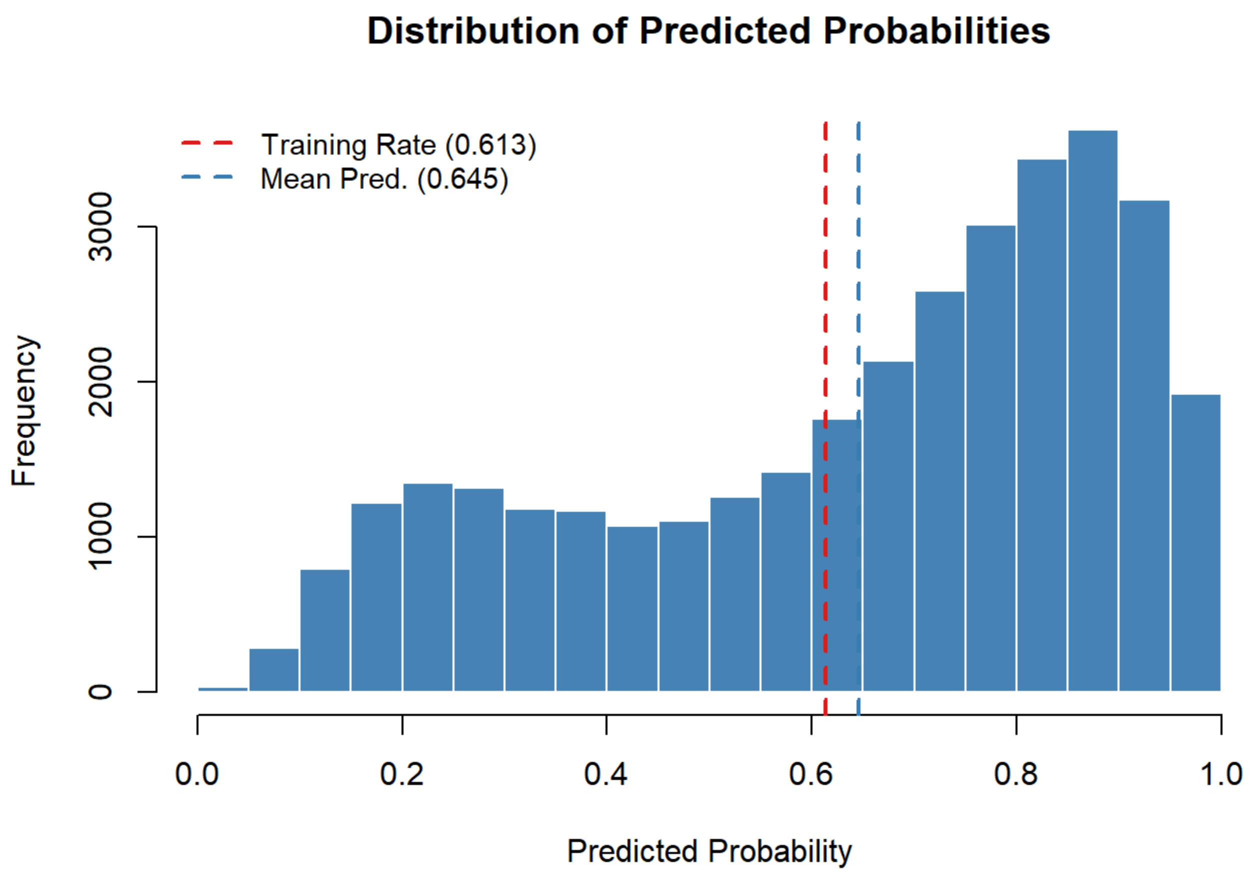
5.3 Predictive Performance

We evaluate predictions on the 2023 test set using a threshold equal to the training set pass rate (61.2%).

```
# compute probabilities using logistic regression
logit_probs <- 1 / (1 + exp(-(X_test %*% beta_hat)))
# convert probabilities to predicted classes
predicted_classes <- ifelse(logit_probs > mean(train_y), 1, 0)
# confusion matrix
confusion_matrix <- table(Predicted = predicted_classes, Actual = test_y)
print(confusion_matrix)
```

```
##          Actual
## Predicted     0     1
##         0  7918  4673
##         1  5223 16022
```

```
# histogram of predicted probabilities
hist(logit_probs,
     main = "Distribution of Predicted Probabilities",
     xlab = "Predicted Probability",
     ylab = "Frequency",
     breaks = 20,
     col = "steelblue",
     border = "white")
abline(v = mean(train_y), col = "#E41A1C", lwd = 2, lty = 2)
abline(v = mean(logit_probs), col = "#377EB8", lwd = 2, lty = 2)
legend("topleft",
       legend = c(paste0("Training Rate (", round(mean(train_y), 3), ")"),
                  paste0("Mean Pred. (", round(mean(logit_probs), 3), ")")),
       col = c("#E41A1C", "#377EB8"),
       lwd = 2, lty = 2,
       bty = "n", cex = 0.9)
```



```
# pull true pos, false neg, false pos, true neg values from the confusion matrix
TP <- confusion_matrix[1, 1]
FN <- confusion_matrix[1, 2]
FP <- confusion_matrix[2, 1]
TN <- confusion_matrix[2, 2]
# performance metrics
accuracy <- (TP + TN) / (TP + TN + FP + FN)
precision <- TP / (TP + FP)
sensitivity <- TP / (TP + FN)
specificity <- TN / (TN + FP)
cat("Accuracy:", round(accuracy, 3), "\n")
```

```
## Accuracy: 0.708
```

```
cat("Precision:", round(precision, 3), "\n")
```

```
## Precision: 0.603
```

```
cat("Sensitivity:", round(sensitivity, 3), "\n")
```

```
## Sensitivity: 0.629
```

```
cat("Specificity:", round(specificity, 3), "\n")
```

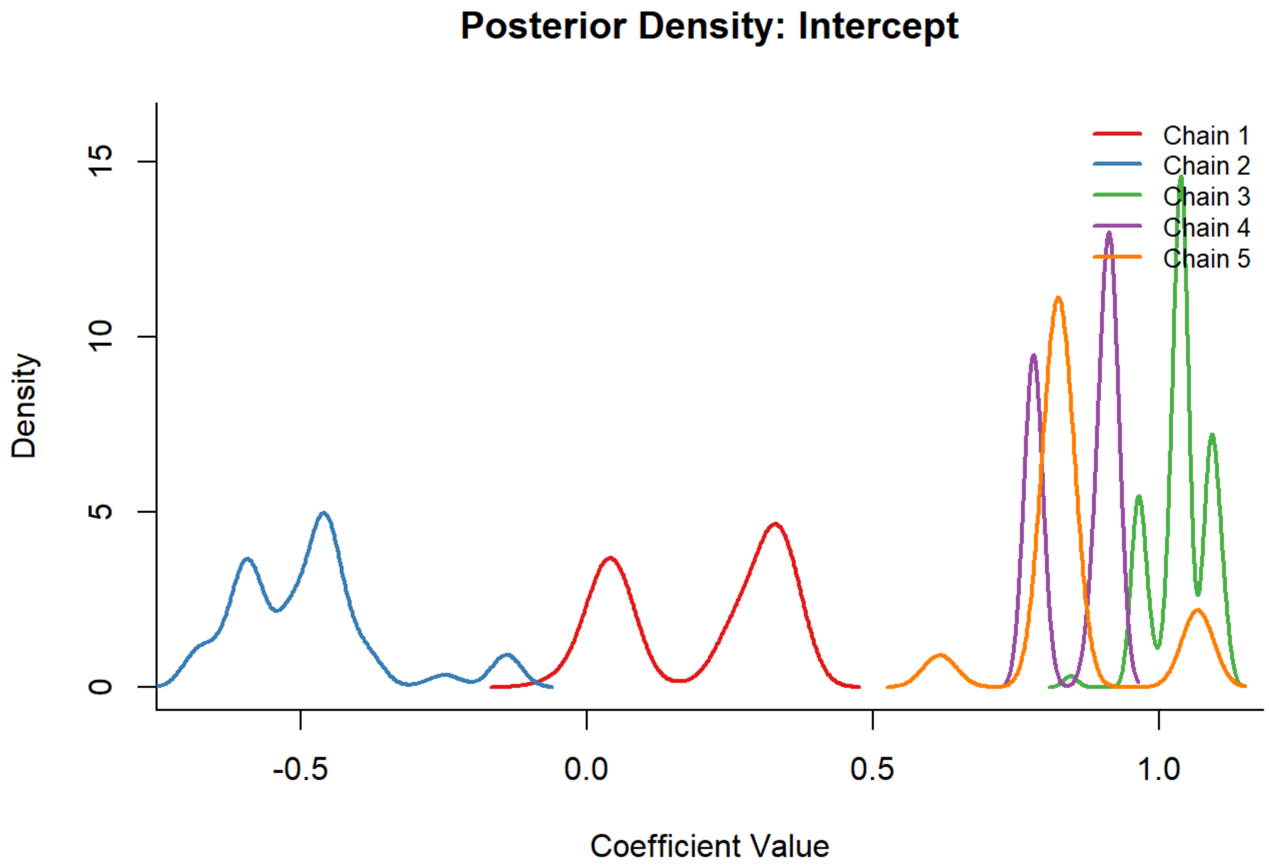
```
## Specificity: 0.754
```

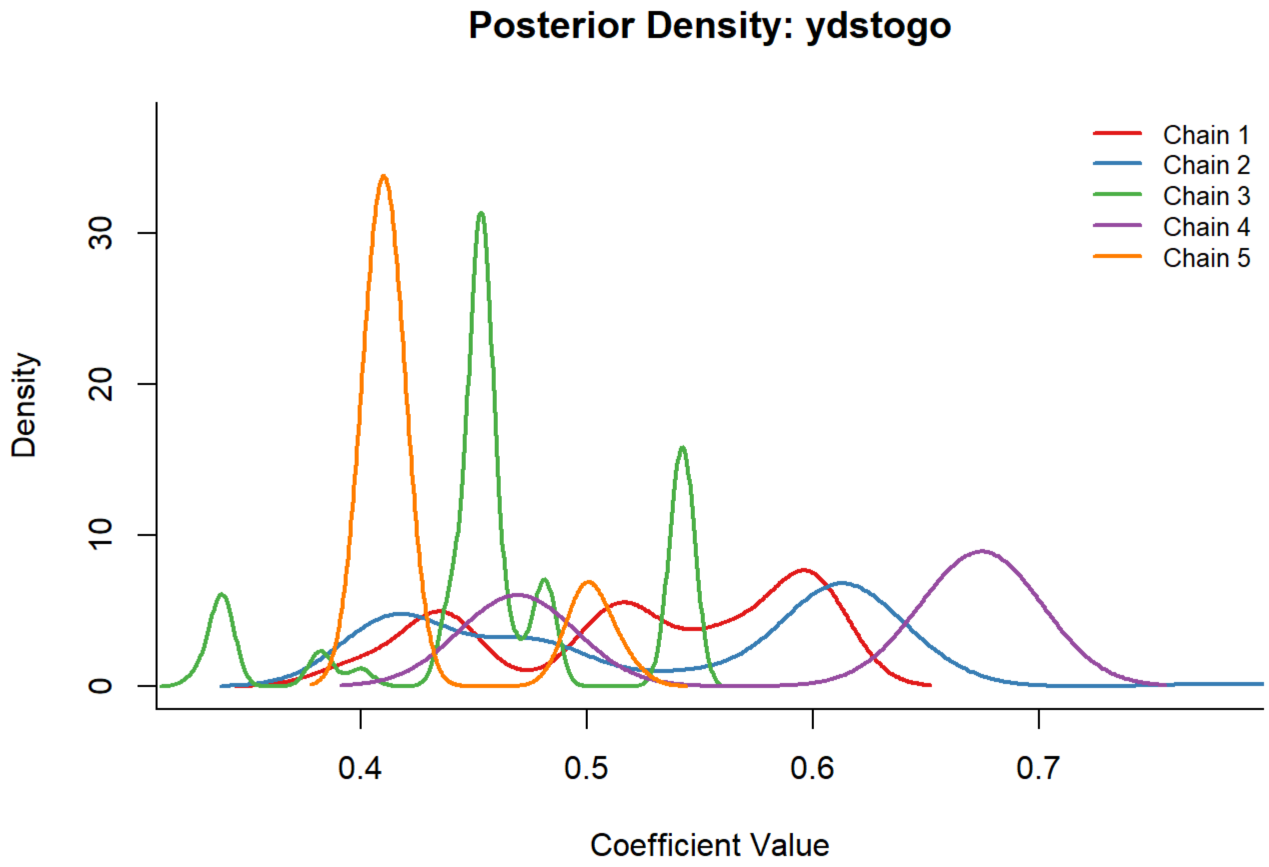
5.4 MCMC Diagnostics

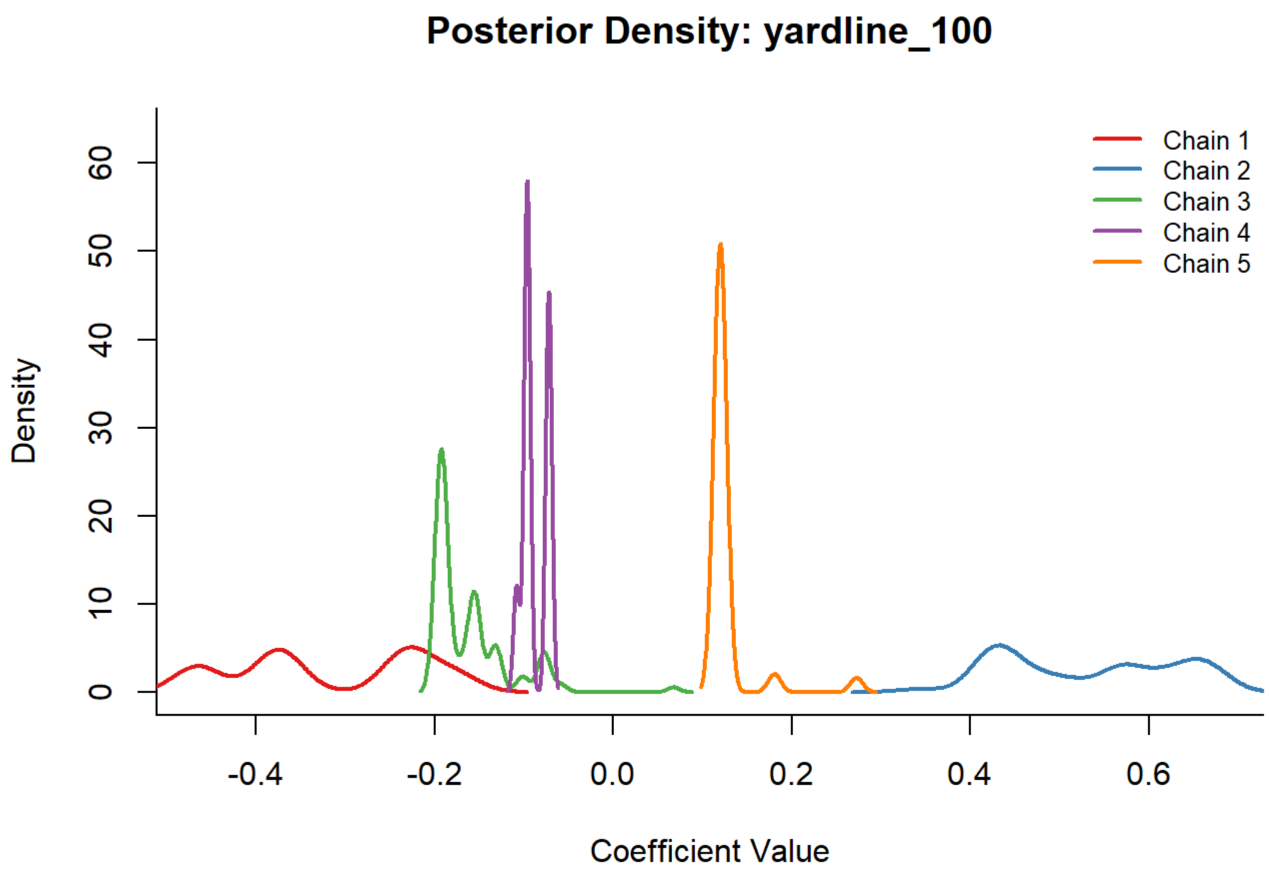
The density and trace plots below visualize chain behavior for each coefficient. Good mixing is indicated by overlapping densities across chains and trace plots resembling “hairy caterpillars” without trends.

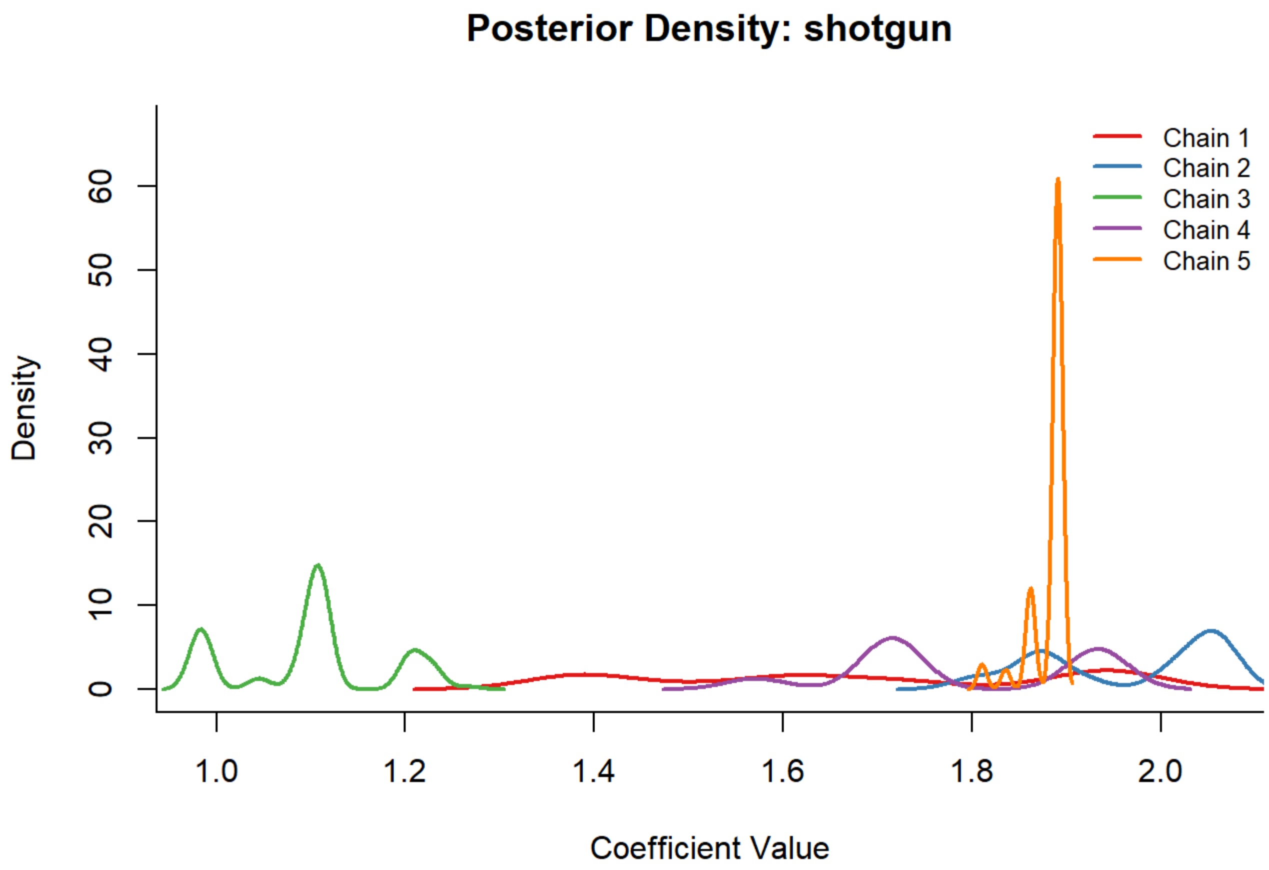
```
# color palette for chains
chain_colors <- c("#E41A1C", "#377EB8", "#4DAF4A", "#984EA3", "#FF7F00")

plot_smoothed_histograms <- function(beta_samples_all, X_test){
  variable_names <- colnames(as.data.frame(X_test))
  variable_names <- gsub("char_", "", variable_names)
  num_vars <- ncol(as.matrix(beta_samples_all[[1]]))
  for (var_idx in 1:num_vars) {
    plot(NULL,
         xlim = range(unlist(lapply(beta_samples_all, function(chain) chain[, var_idx]))),
         ylim = c(0, max(sapply(beta_samples_all, function(chain) {
           max(density(chain[, var_idx])$y)
         })) * 1.1),
         main = paste("Posterior Density:", variable_names[var_idx]),
         xlab = "Coefficient Value",
         ylab = "Density",
         bty = "l")
    for (chain_idx in seq_along(beta_samples_all)) {
      lines(density(beta_samples_all[[chain_idx]][, var_idx]),
            col = chain_colors[chain_idx], lwd = 2)
    }
    legend("topright", legend = paste("Chain", seq_along(beta_samples_all)),
           col = chain_colors[seq_along(beta_samples_all)], lwd = 2, bty = "n", cex = 0.8)
  }
}

plot_trace_plots <- function(beta_samples_all, X_test){
  variable_names <- colnames(as.data.frame(X_test))
  variable_names <- gsub("char_", "", variable_names)
  num_vars <- ncol(as.matrix(beta_samples_all[[1]]))
  num_chains <- length(beta_samples_all)
  for (var_idx in 1:num_vars) {
    plot(NULL, type = "n",
         xlim = c(1, nrow(beta_samples_all[[1]])),
         ylim = range(sapply(beta_samples_all, function(chain) chain[, var_idx])),
         xlab = "Iteration (post burn-in)",
         ylab = "Coefficient Value",
         main = paste("Trace Plot:", variable_names[var_idx]),
         bty = "l")
    for (chain_idx in 1:num_chains) {
      lines(beta_samples_all[[chain_idx]][, var_idx], col = chain_colors[chain_idx], lwd = 1)
    }
  }
}
```

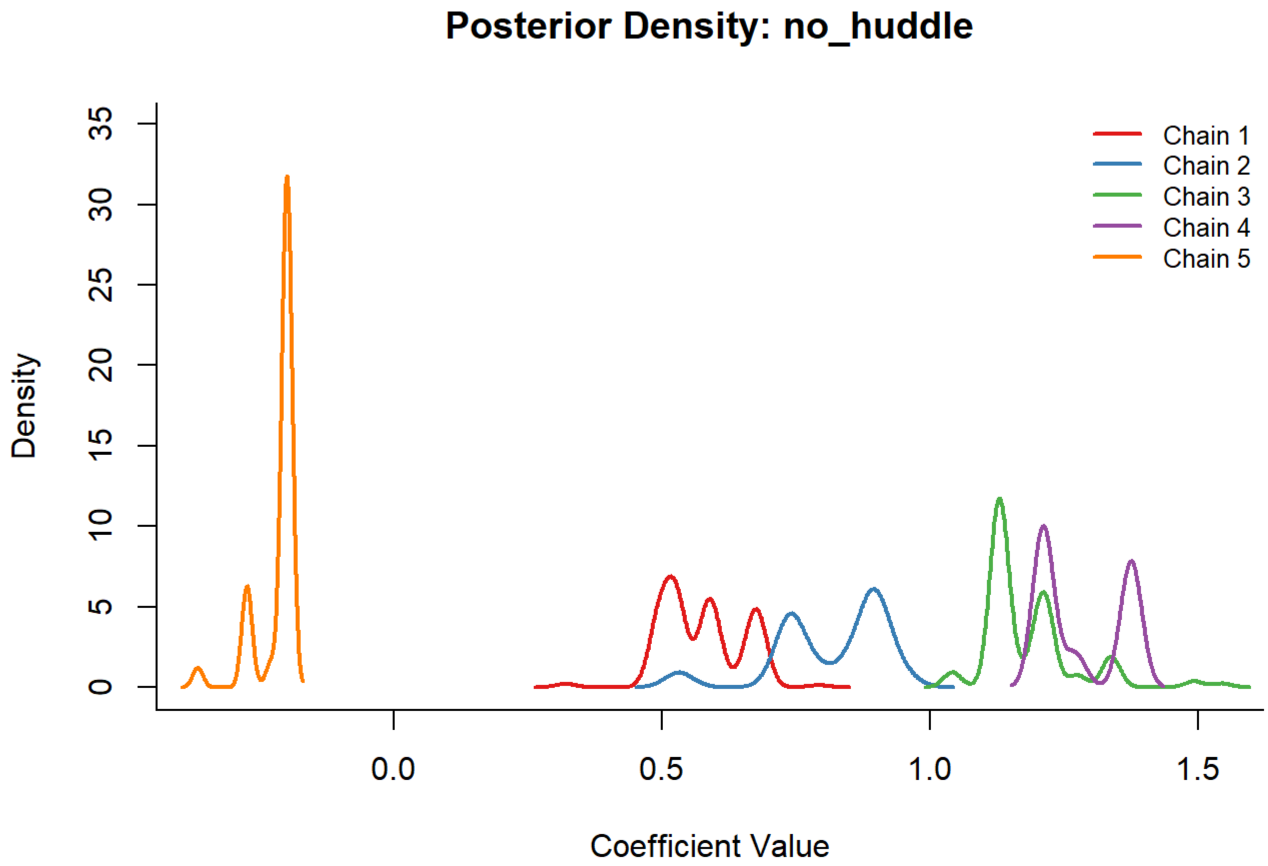
```
plot_smoothed_histograms(beta_samples_all, X_test)
```

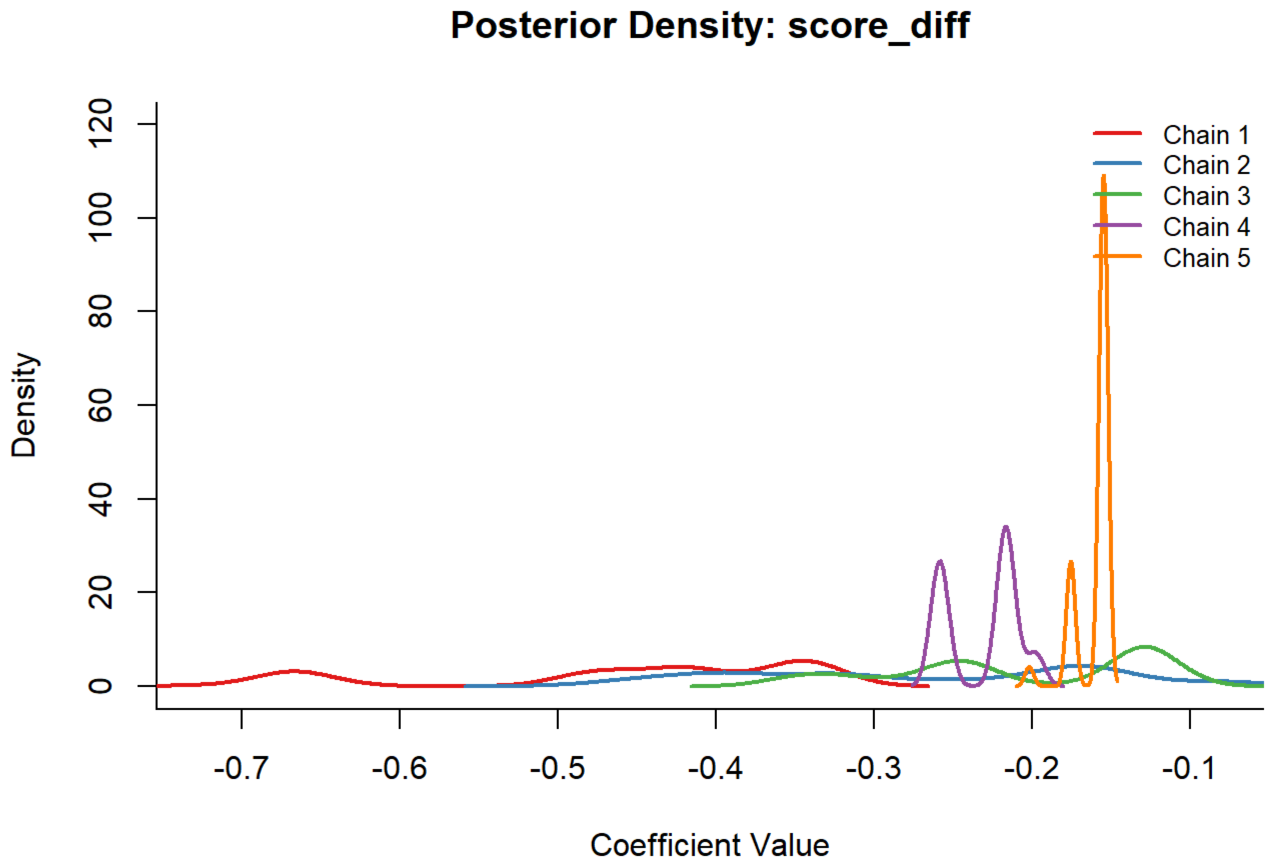

Posterior Density: Intercept
Density
0
5
10
15
-0.5
0.0
0.5
1.0
Coefficient Value
Chain 1
Chain 2
Chain 3
Chain 4
Chain 5

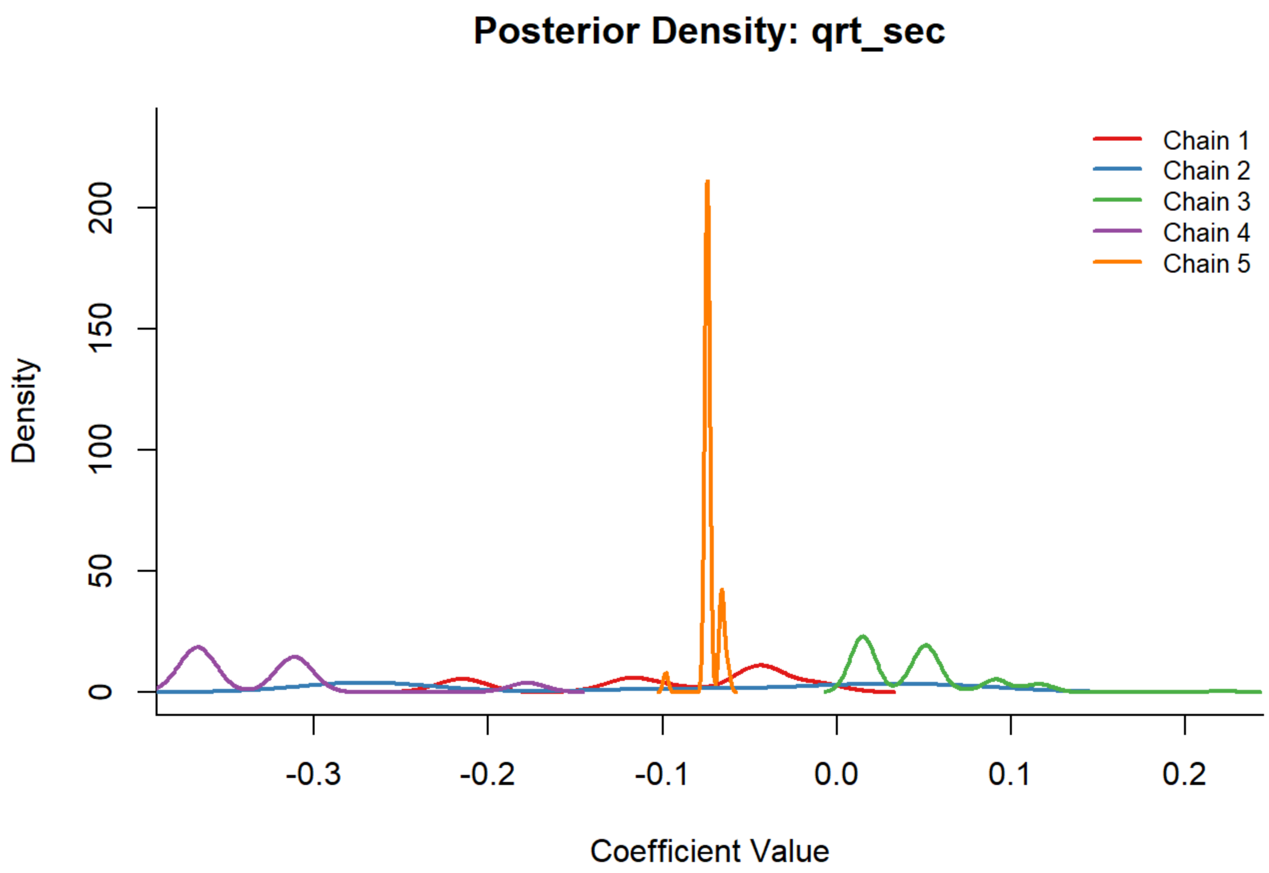

Posterior Density: ydstogo
Density
0
10
20
30
0.4
0.5
0.6
0.7
Coefficient Value
Chain 1
Chain 2
Chain 3
Chain 4
Chain 5

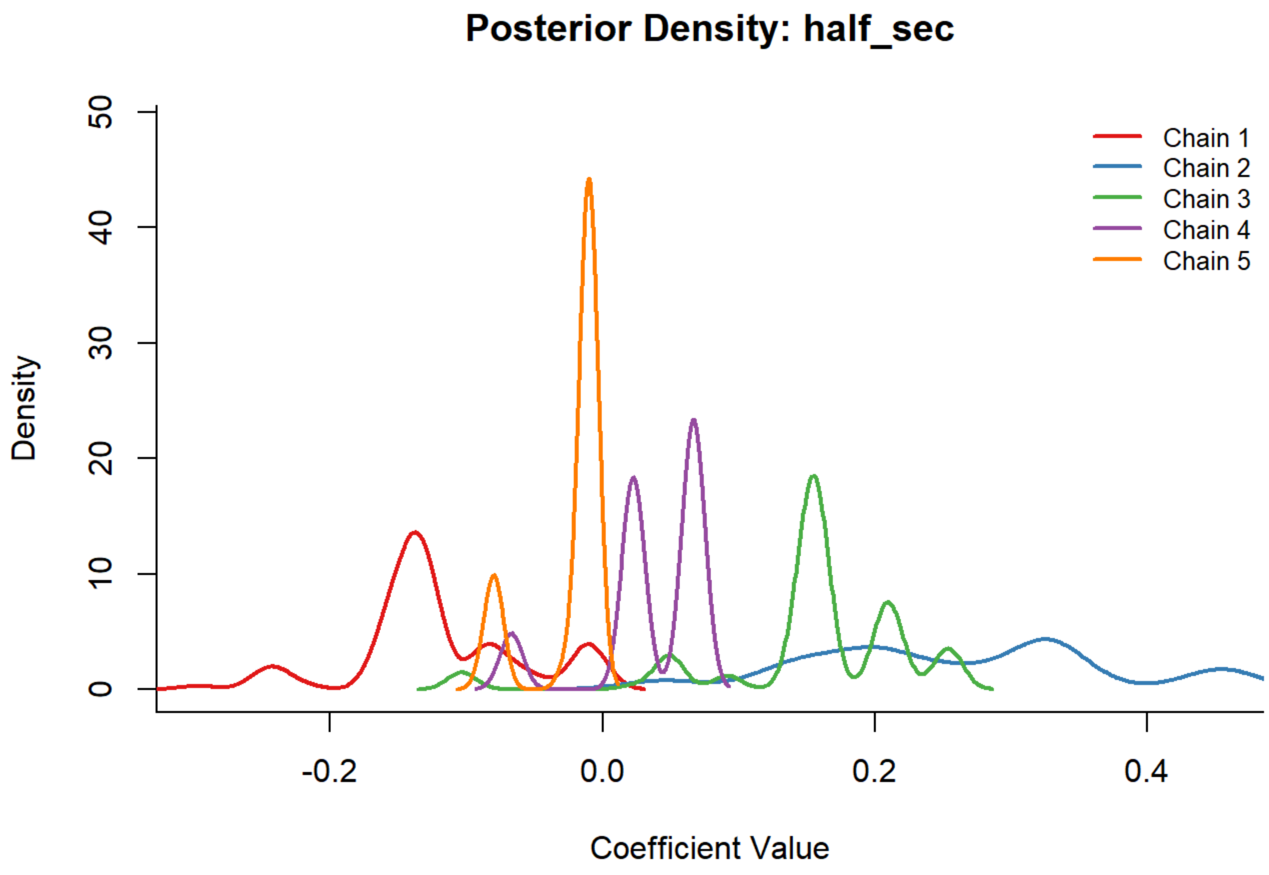

Posterior Density: yardline_100
Chain 1
Chain 2
Chain 3
Chain 4
Chain 5
Density
0
10
20
30
40
50
60
-0.4
-0.2
0.0
0.2
0.4
0.6
Coefficient Value

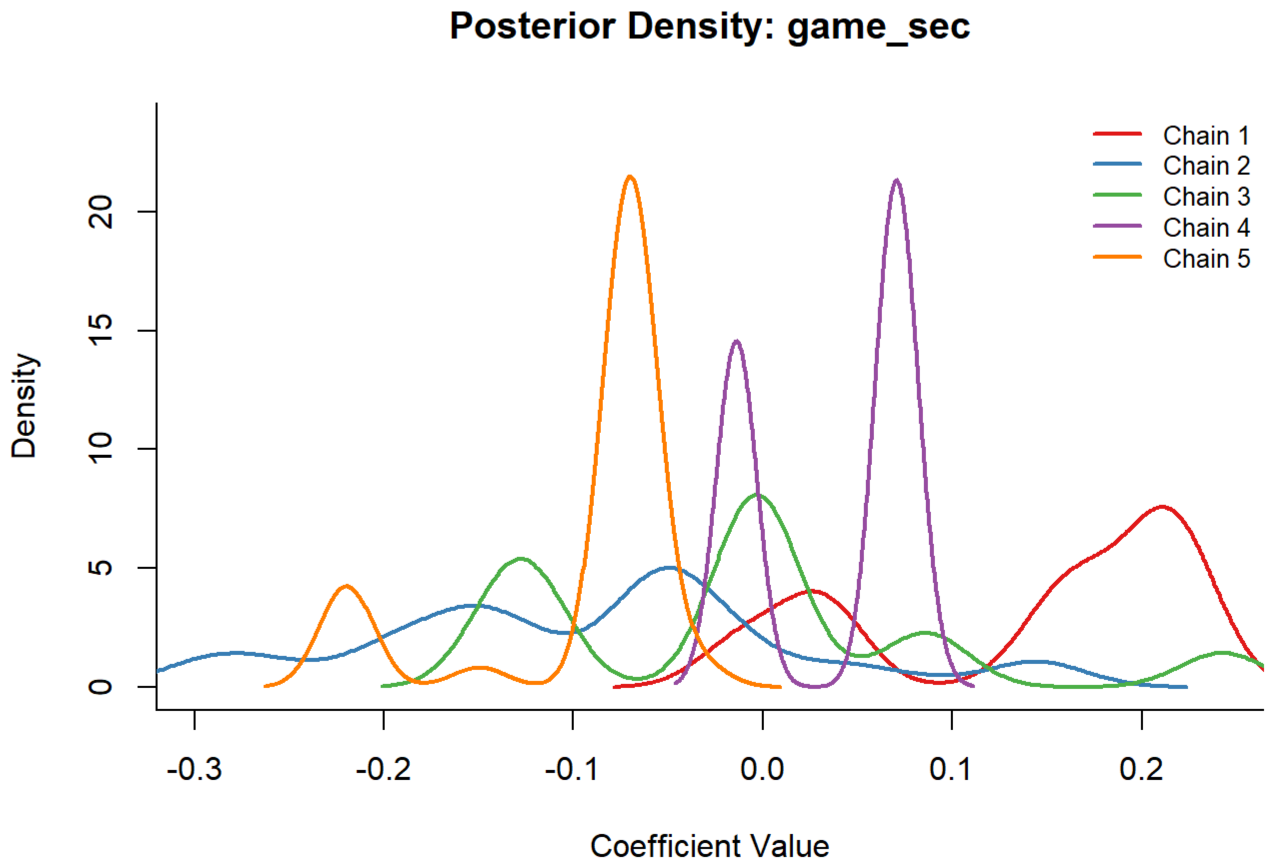

Posterior Density: shotgun
Chain 1
Chain 2
Chain 3
Chain 4
Chain 5
Density
0
10
20
30
40
50
60
1.0
1.2
1.4
1.6
1.8
2.0
Coefficient Value


Posterior Density: no_huddle
Density
0
5
10
15
20
25
30
35
0.0
0.5
1.0
1.5
Coefficient Value
Chain 1
Chain 2
Chain 3
Chain 4
Chain 5

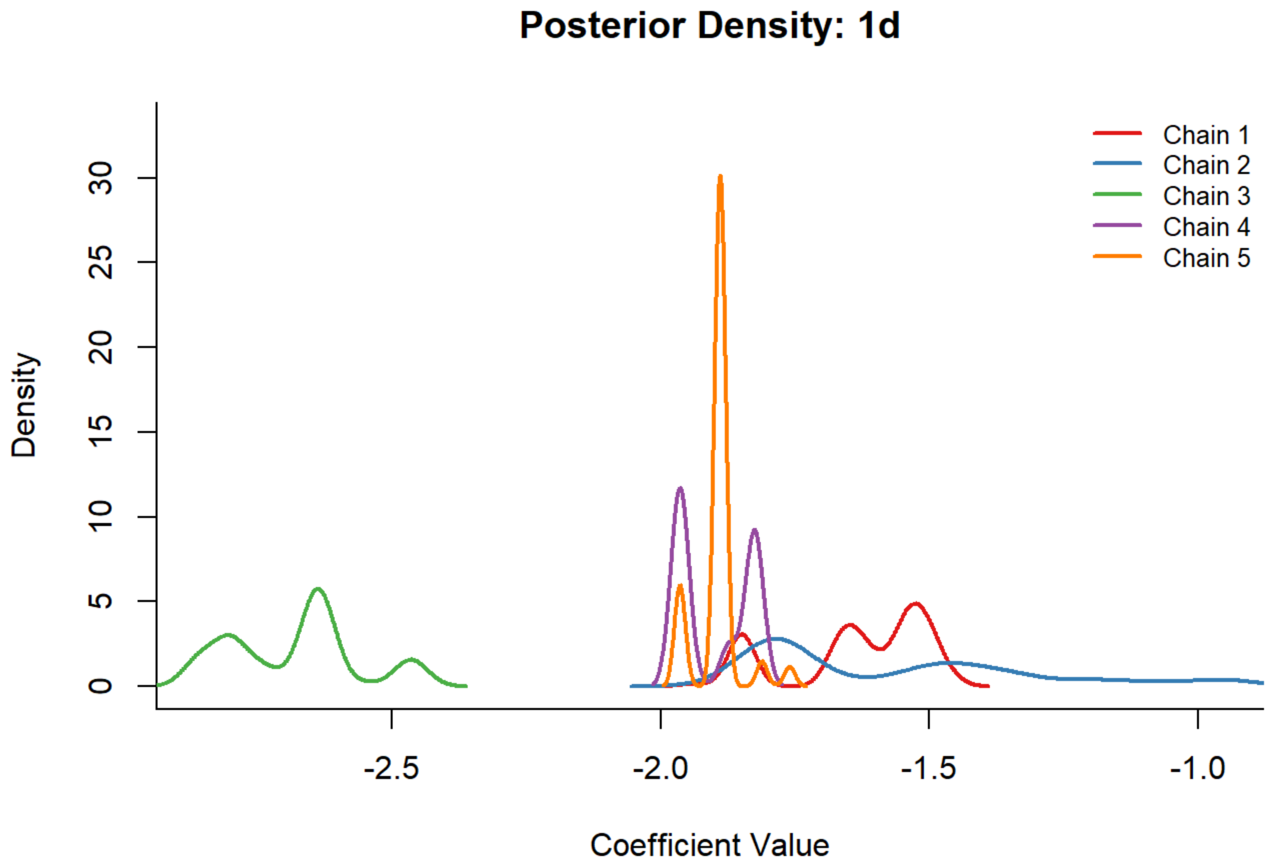

Posterior Density: score_diff
Density
0
20
40
60
80
100
120
-0.7
-0.6
-0.5
-0.4
-0.3
-0.2
-0.1
Coefficient Value
Chain 1
Chain 2
Chain 3
Chain 4
Chain 5


Posterior Density: qrt_sec
Density
0
50
100
150
200
-0.3
-0.2
-0.1
0.0
0.1
0.2
Coefficient Value
Chain 1
Chain 2
Chain 3
Chain 4
Chain 5

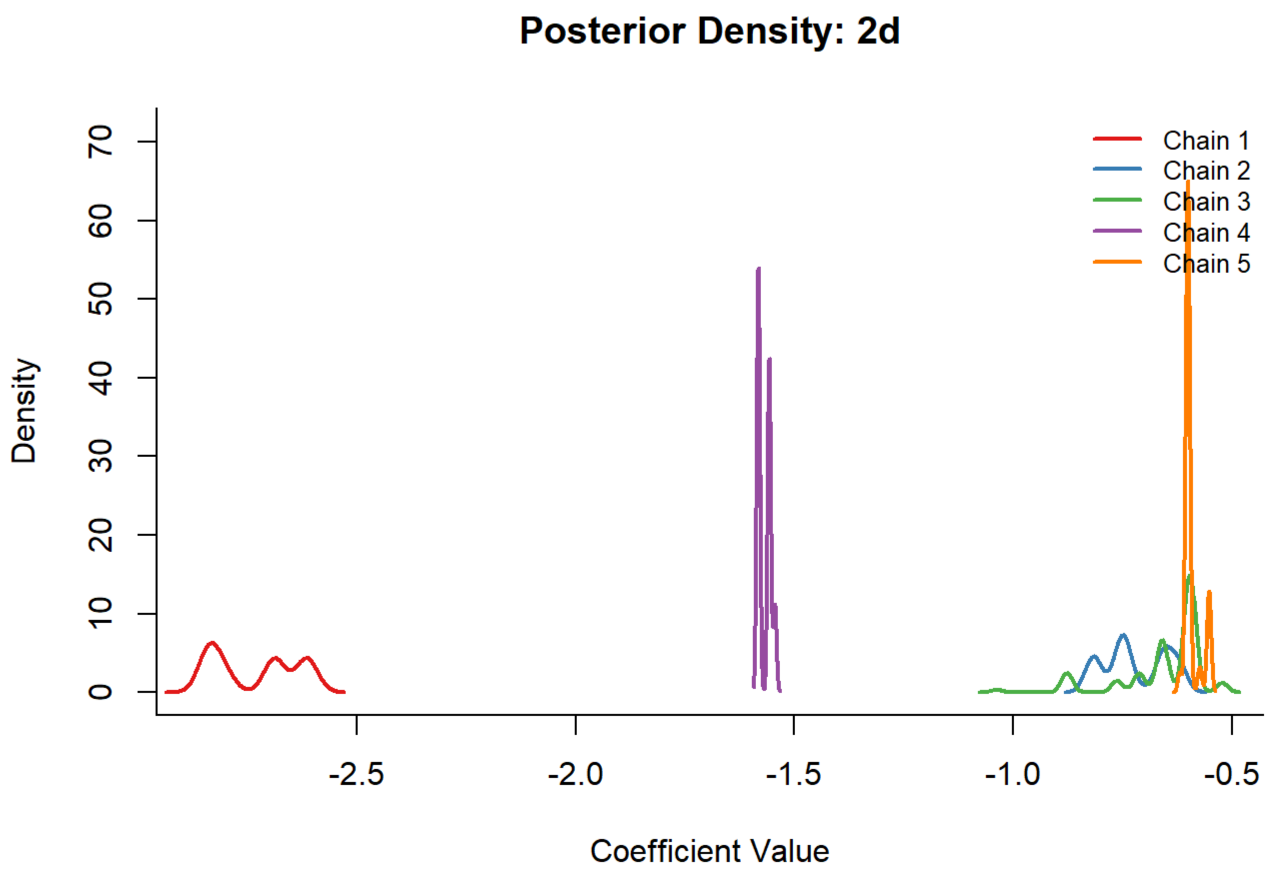

Posterior Density: half_sec
Density
0
10
20
30
40
50
-0.2
0.0
0.2
0.4
Coefficient Value
Chain 1
Chain 2
Chain 3
Chain 4
Chain 5



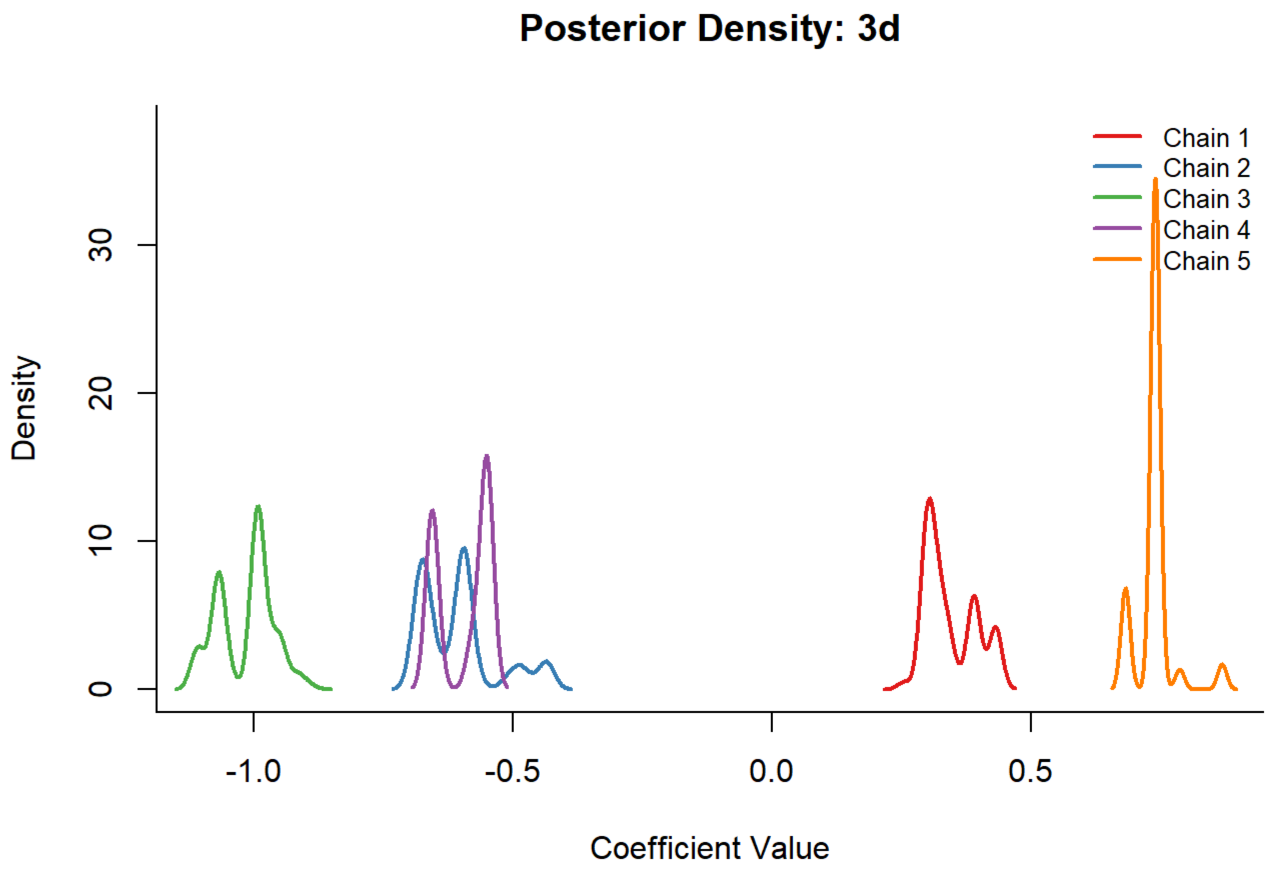
Posterior Density: game_sec



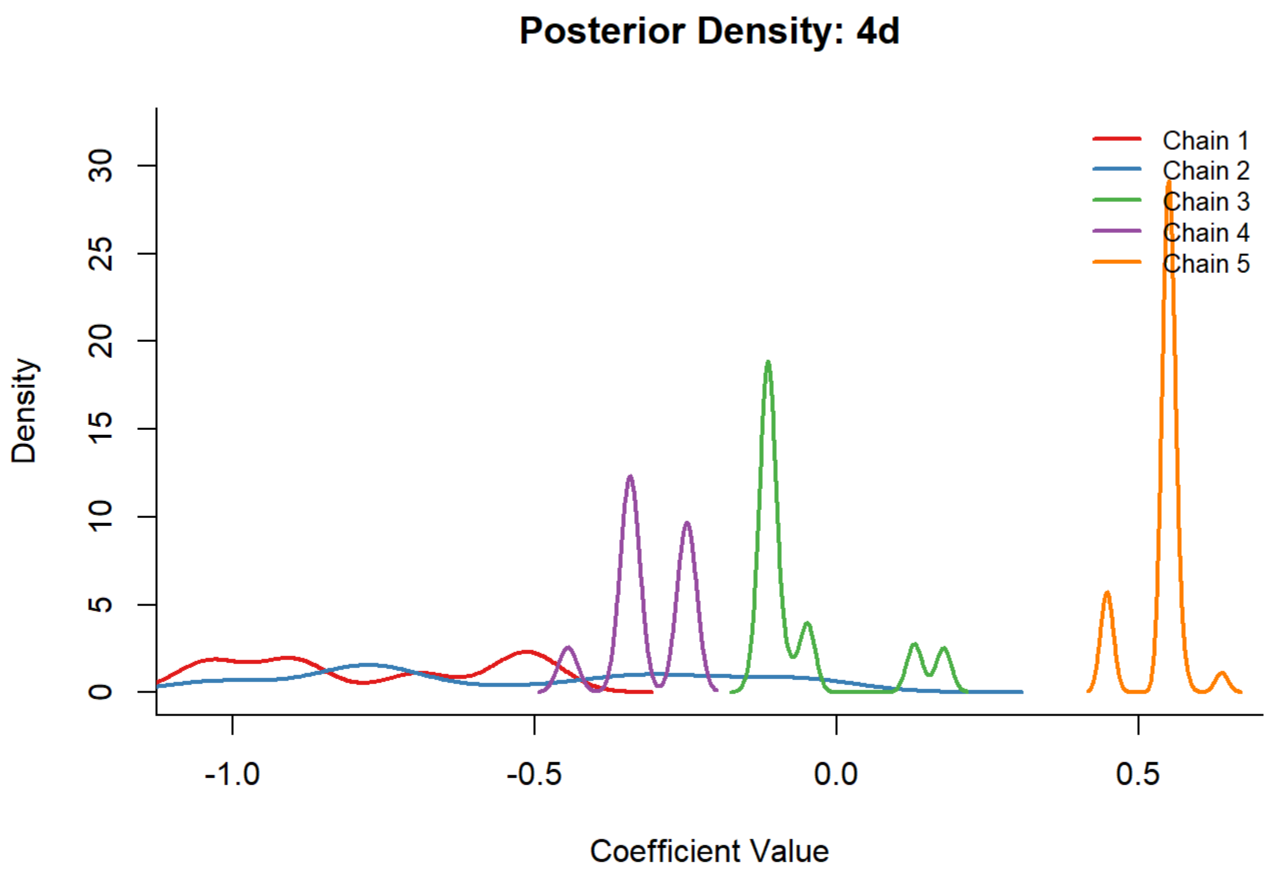
Posterior Density: 1d

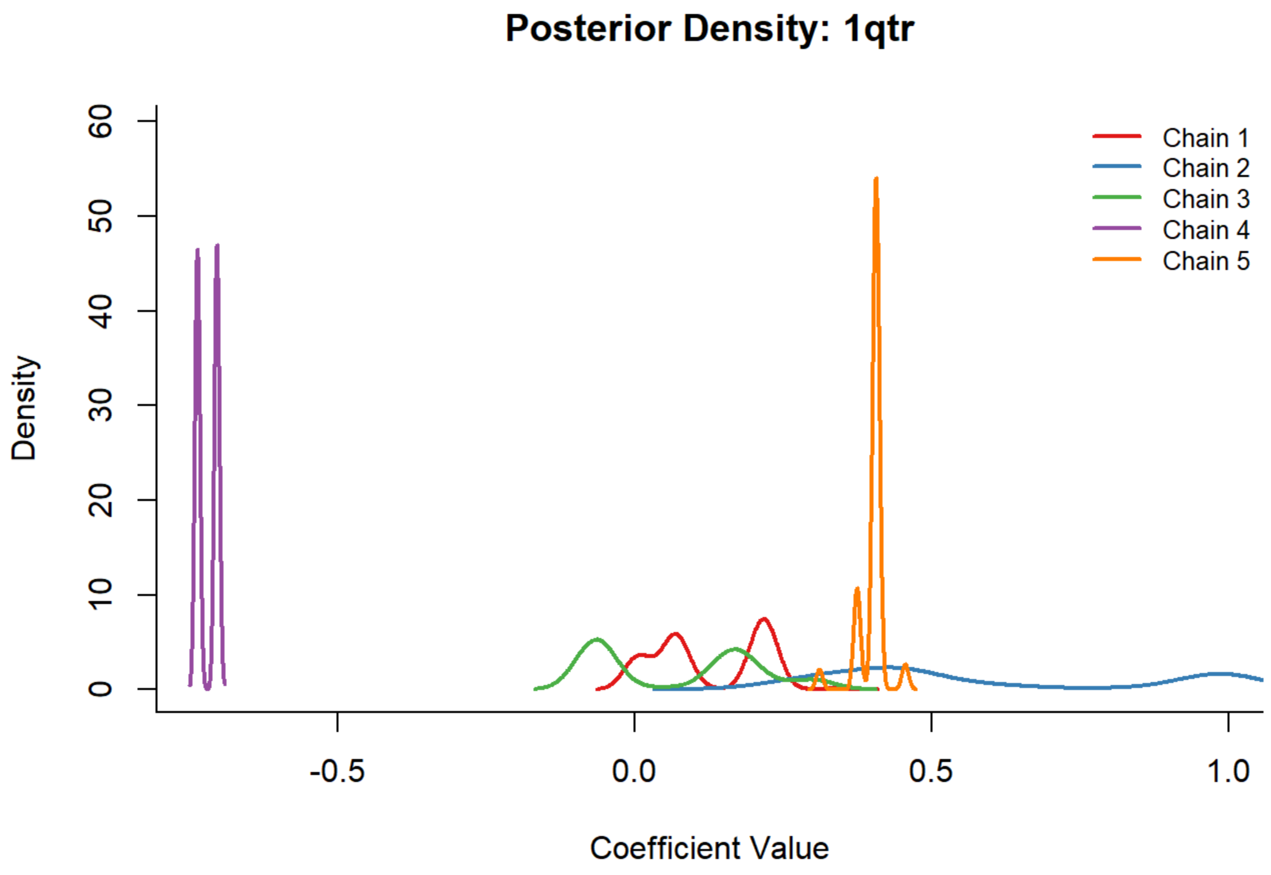


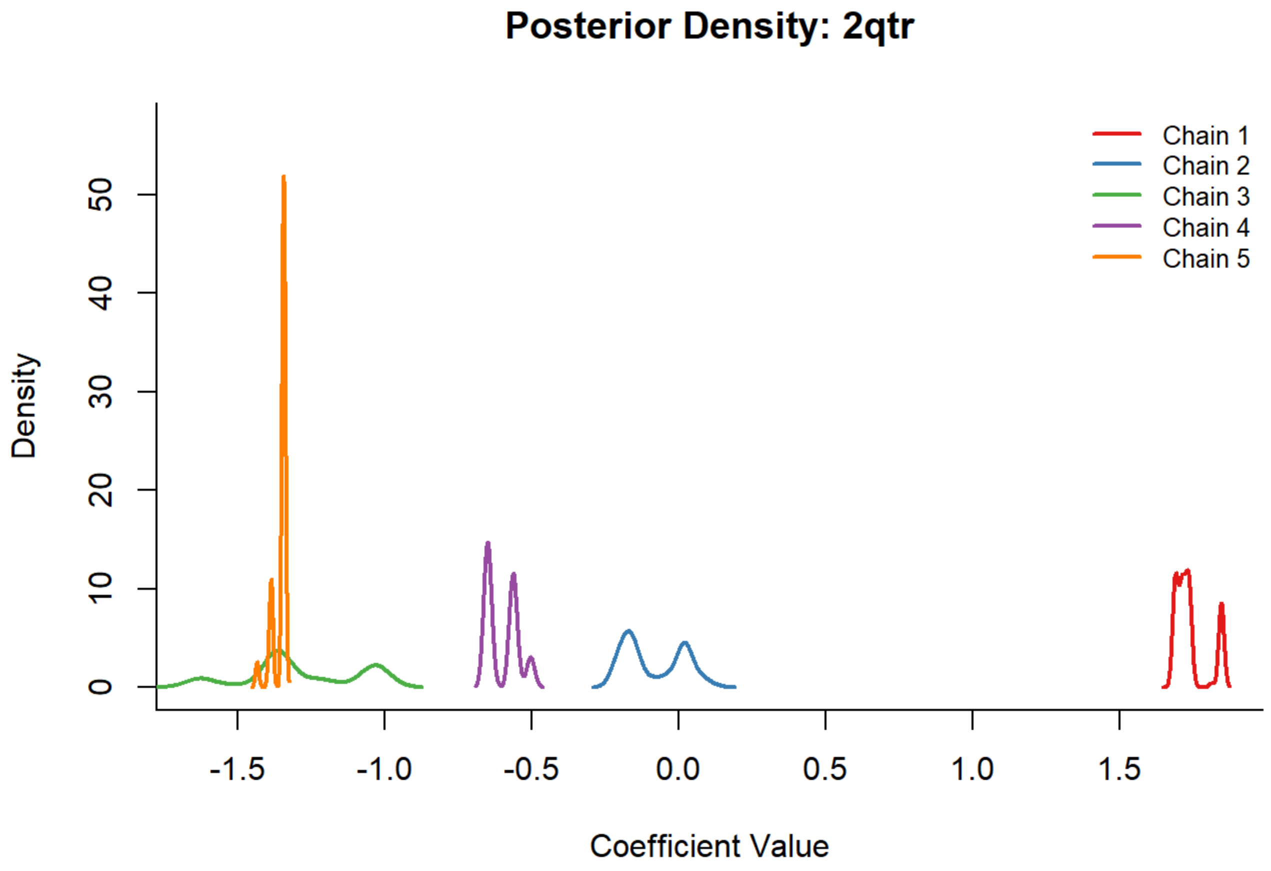
Posterior Density: 2d

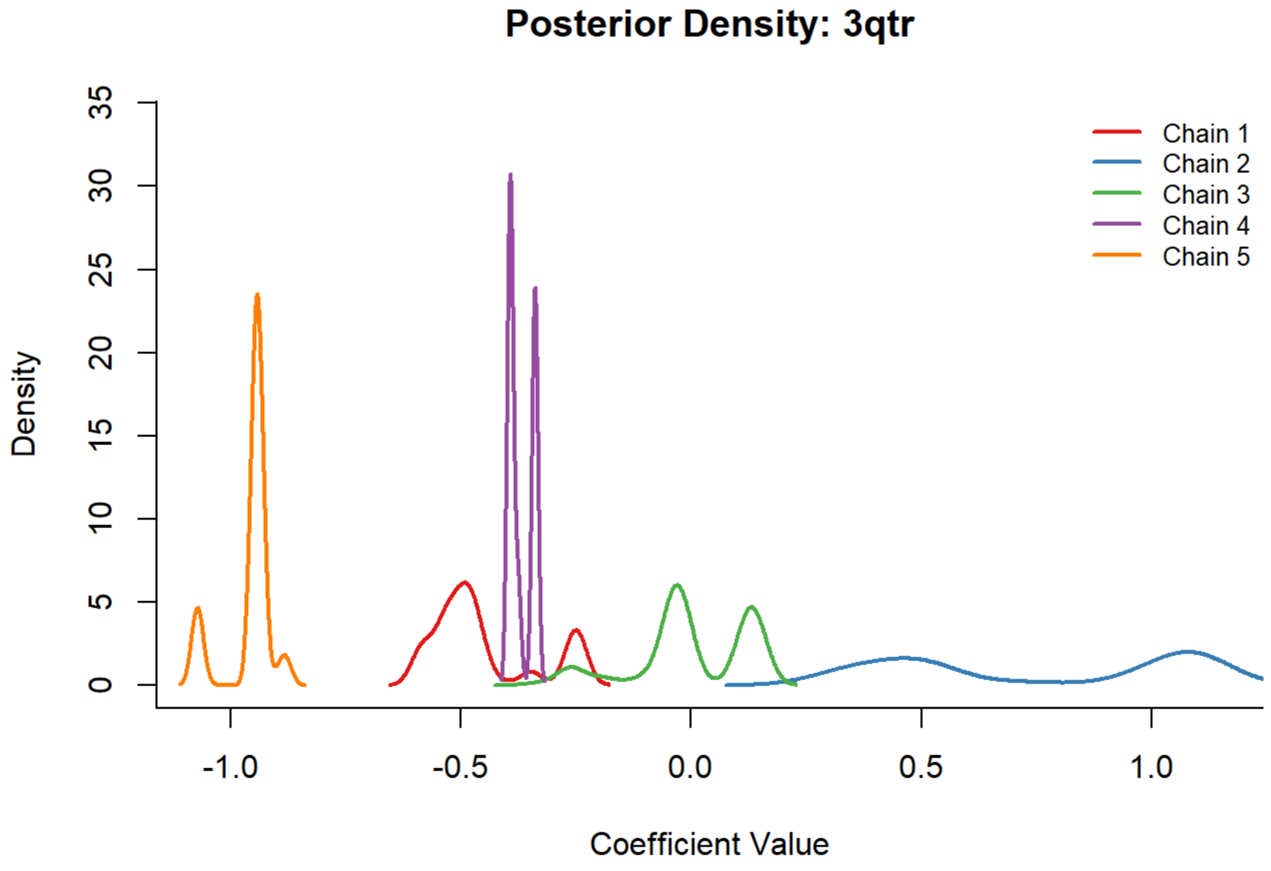


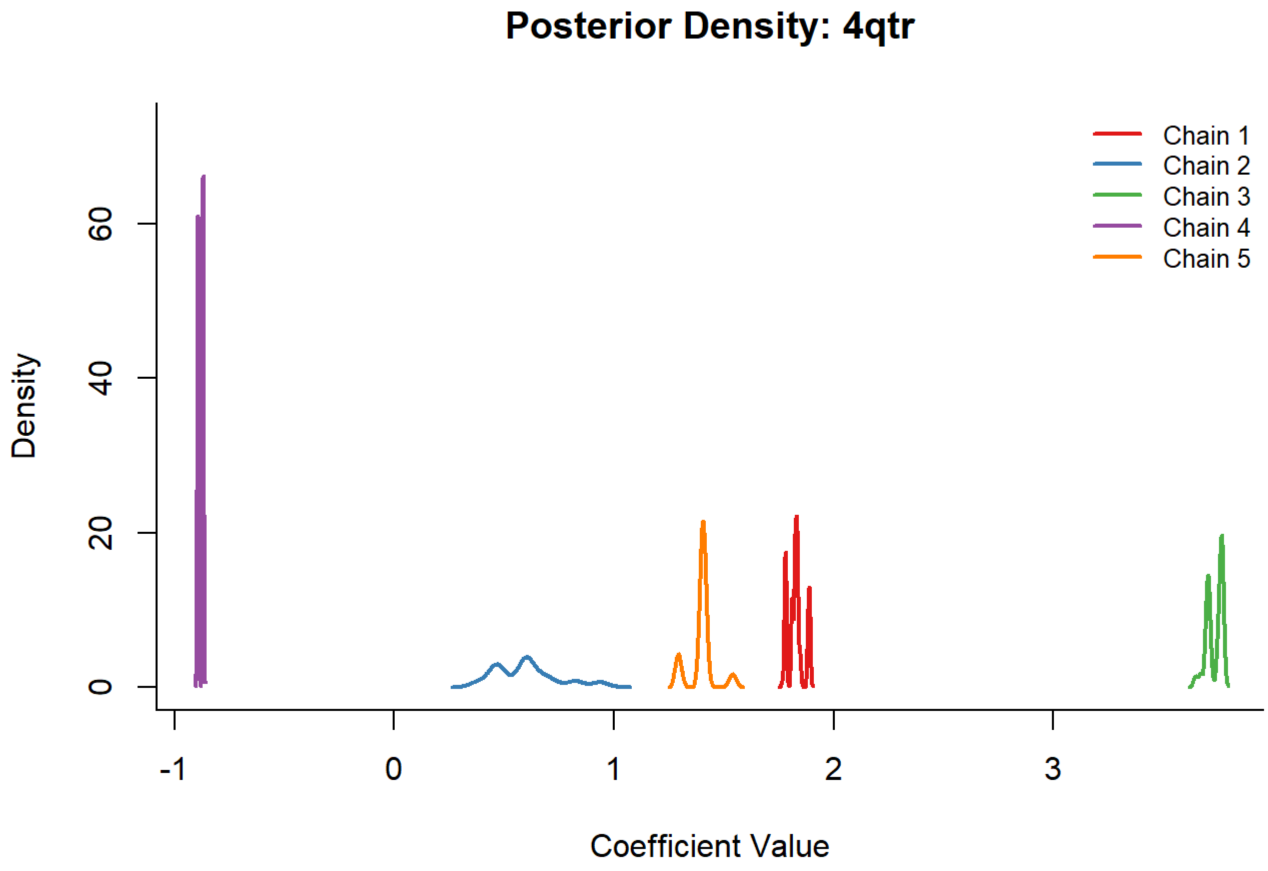
Posterior Density: 3d

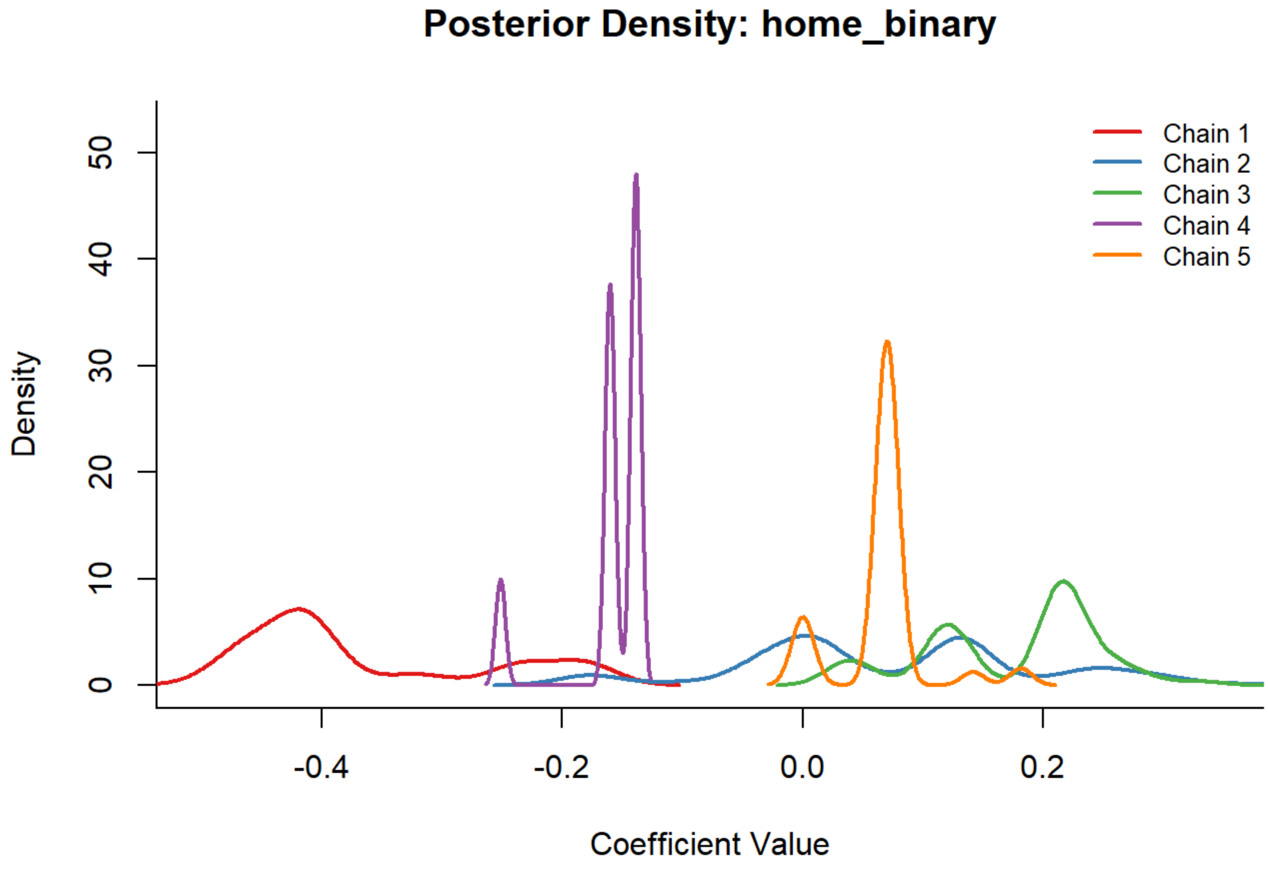

Posterior Density: 4d
Density
0
5
10
15
20
25
30
-1.0
-0.5
0.0
0.5
Coefficient Value
Chain 1
Chain 2
Chain 3
Chain 4
Chain 5


Posterior Density: 1qtr
Density
0
10
20
30
40
50
60
-0.5
0.0
0.5
1.0
Coefficient Value
Chain 1
Chain 2
Chain 3
Chain 4
Chain 5

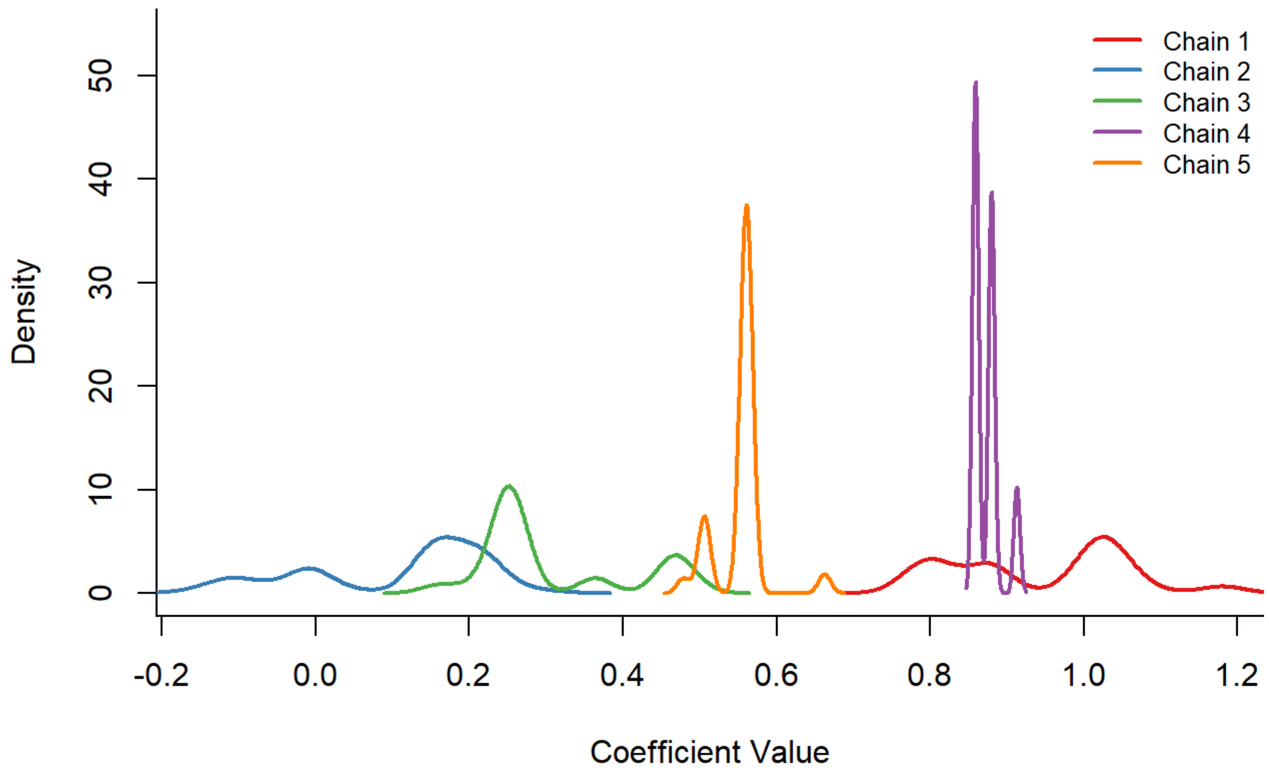

Posterior Density: 2qtr
Density
0
10
20
30
40
50
-1.5
-1.0
-0.5
0.0
0.5
1.0
1.5
Coefficient Value
Chain 1
Chain 2
Chain 3
Chain 4
Chain 5


Posterior Density: 3qtr
Density
0
5
10
15
20
25
30
35
-1.0
-0.5
0.0
0.5
1.0
Coefficient Value
Chain 1
Chain 2
Chain 3
Chain 4
Chain 5

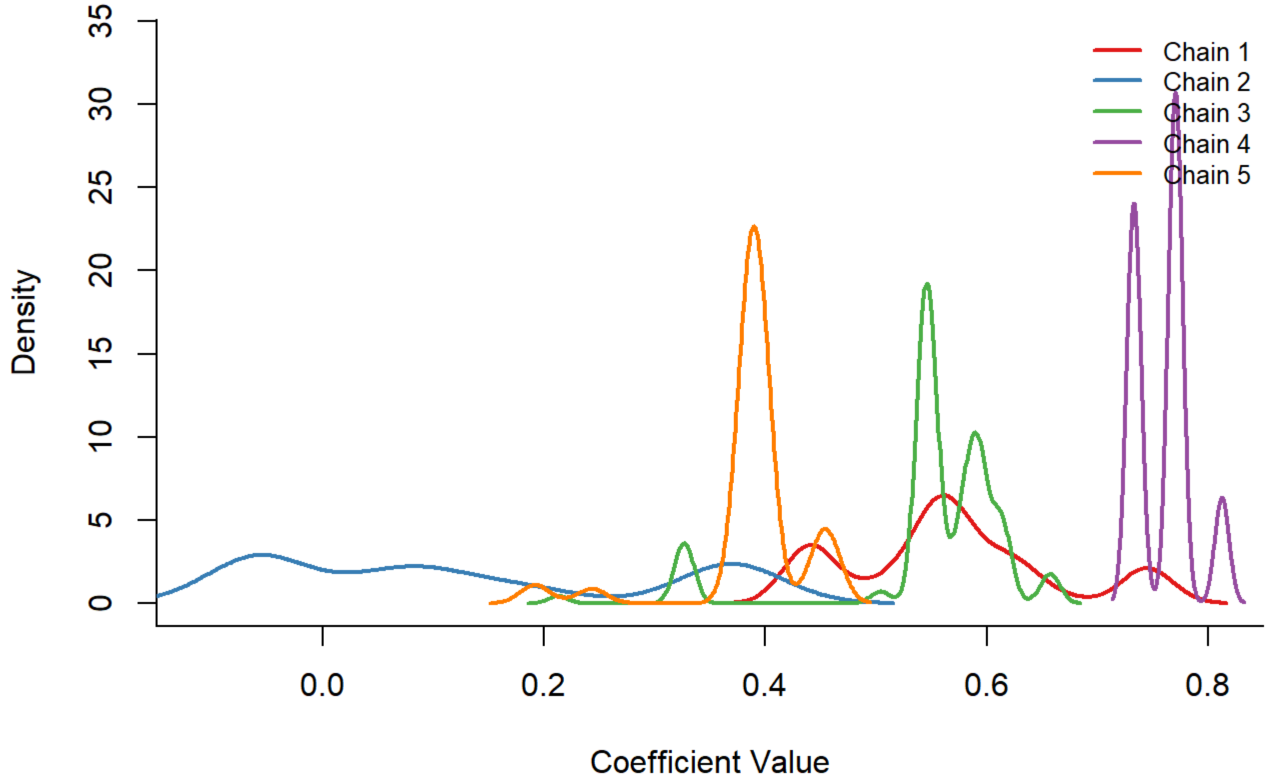

Posterior Density: 4qtr
Chain 1
Chain 2
Chain 3
Chain 4
Chain 5
Density
0
20
40
60
-1
0
1
2
3
Coefficient Value

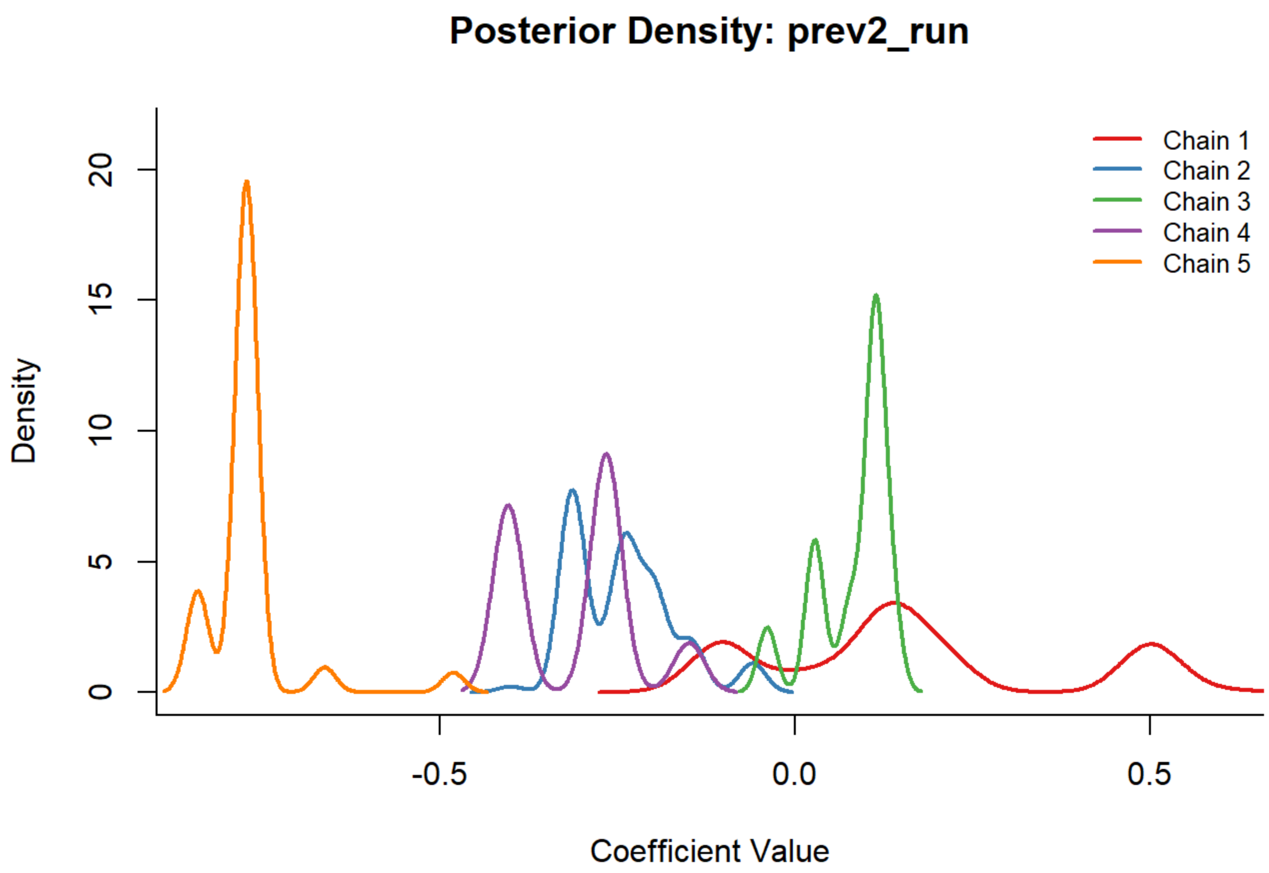

Posterior Density: home_binary
Chain 1
Chain 2
Chain 3
Chain 4
Chain 5
Density
0
10
20
30
40
50
-0.4
-0.2
0.0
0.2
Coefficient Value

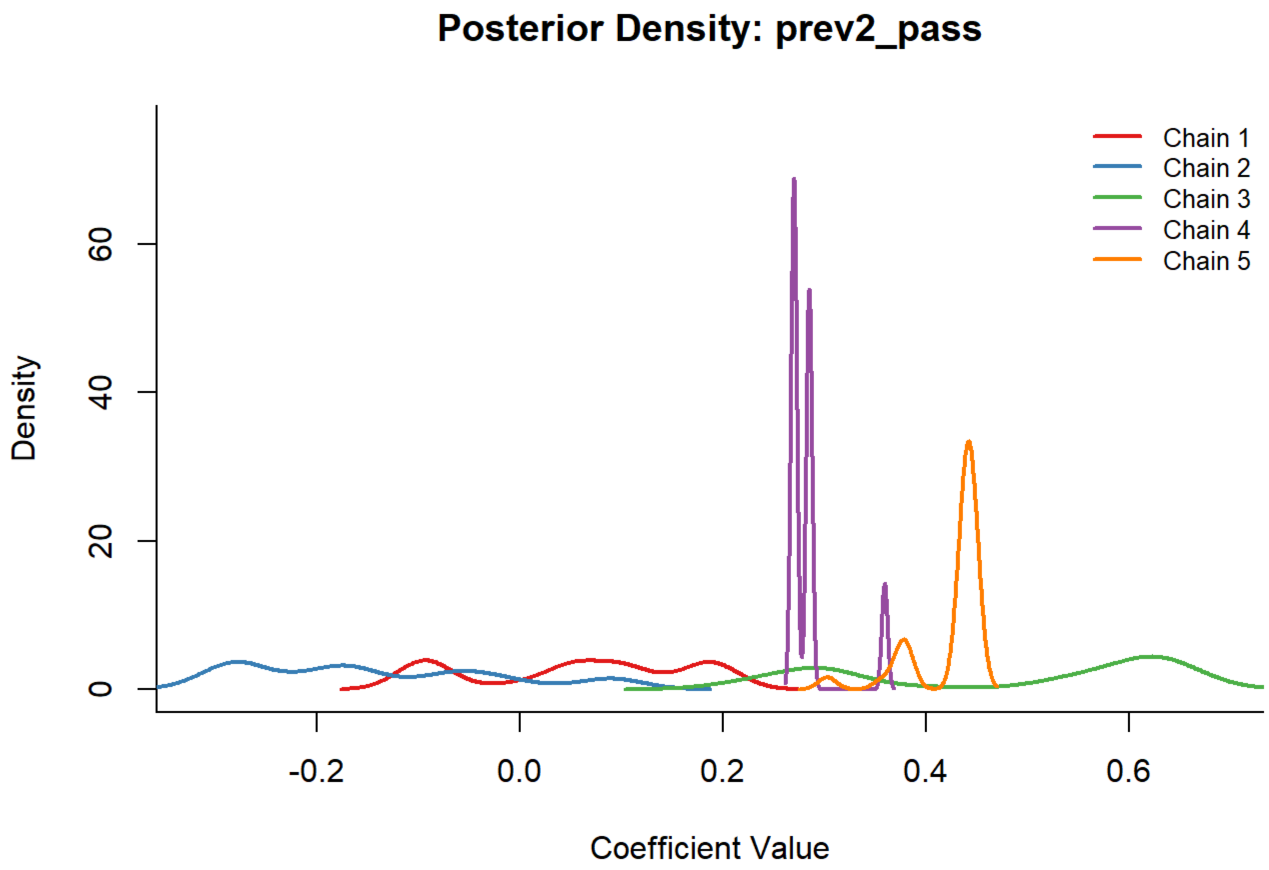
Posterior Density: prev_run

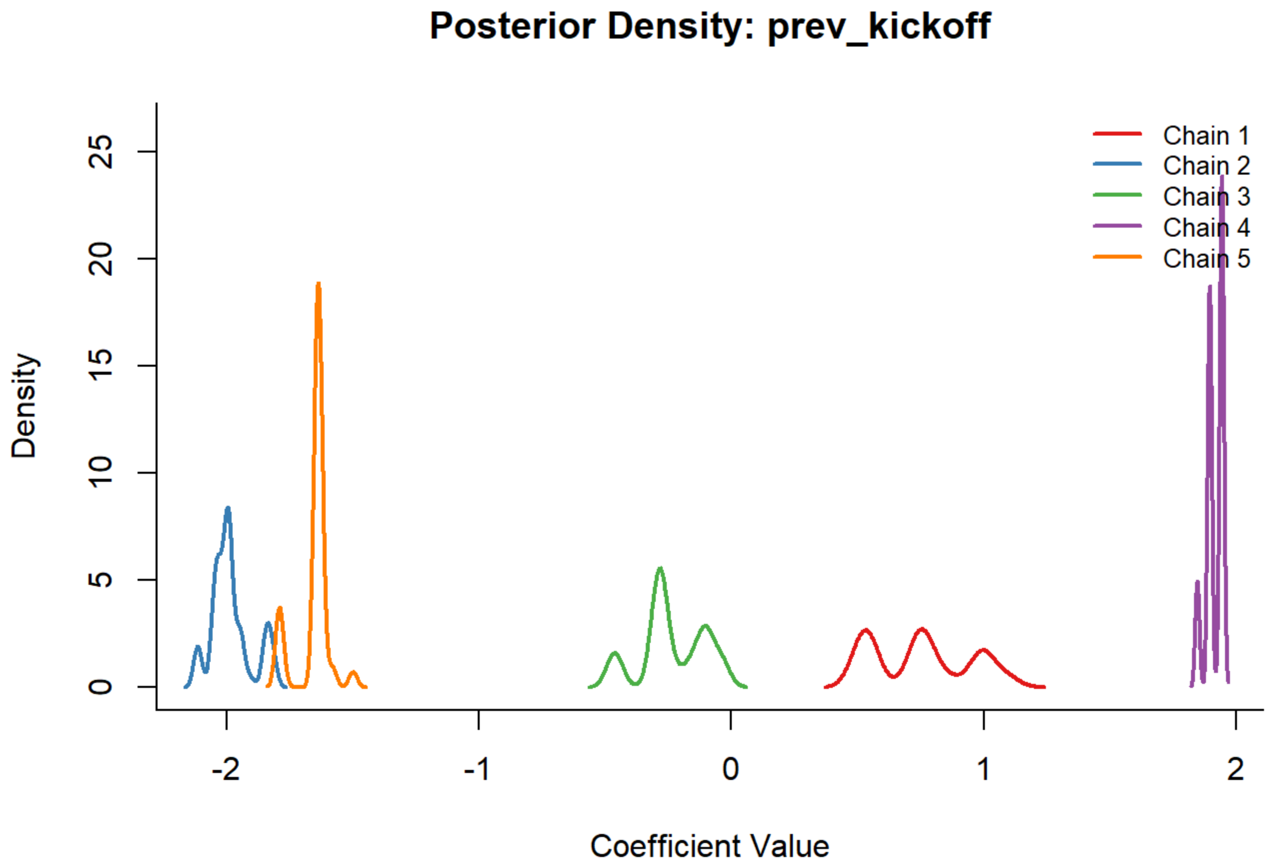


Posterior Density: prev_pass

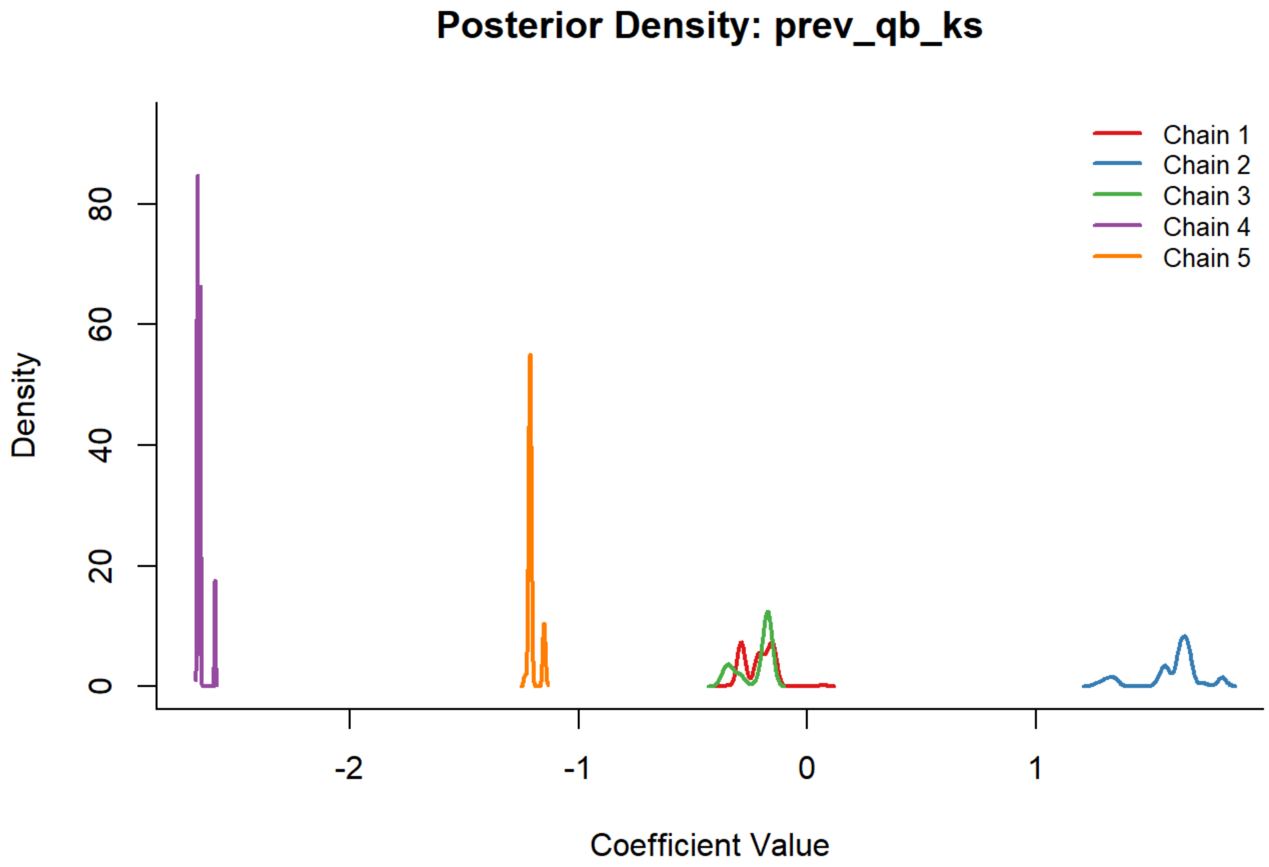



Posterior Density: prev2_run
Density
0
5
10
15
20
-0.5
0.0
0.5
Coefficient Value
Chain 1
Chain 2
Chain 3
Chain 4
Chain 5

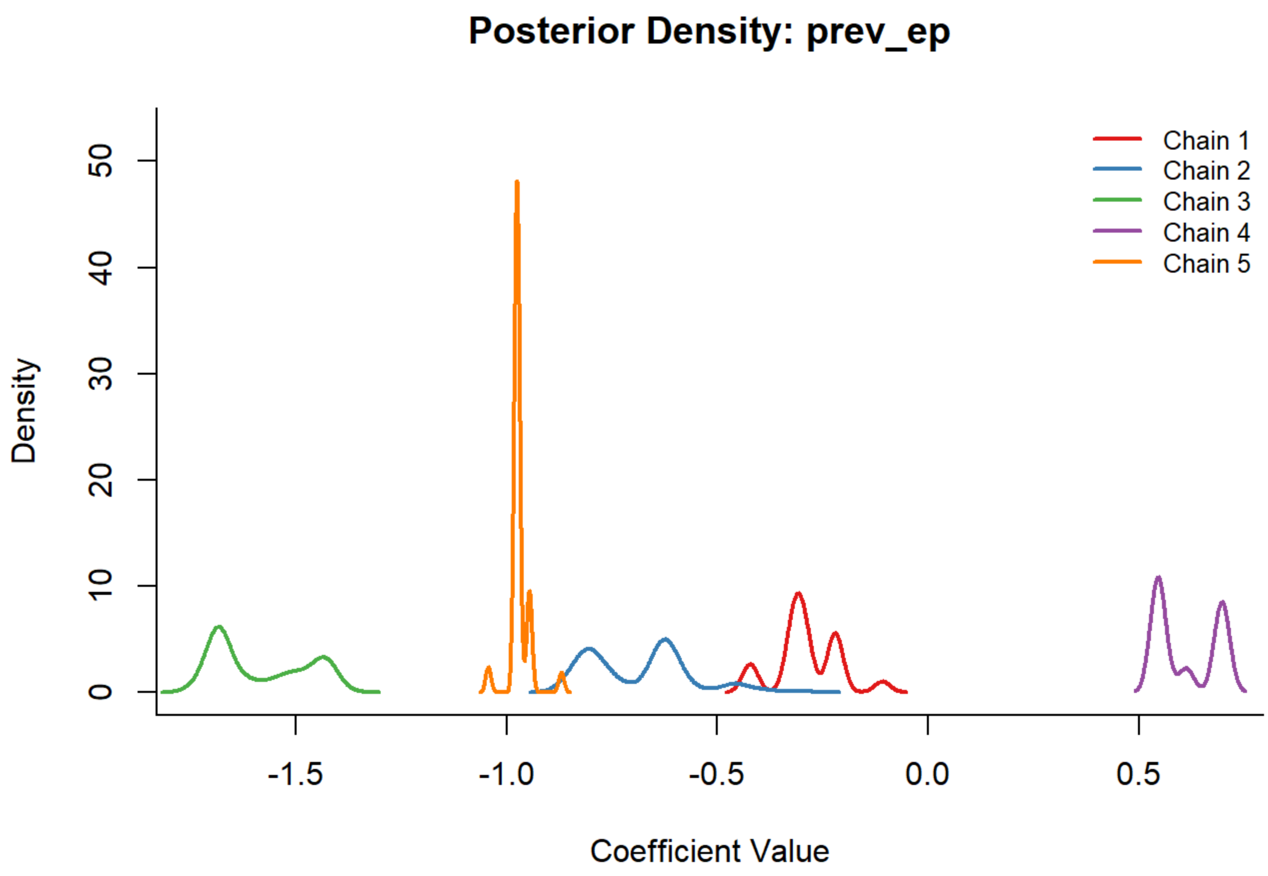

Posterior Density: prev2_pass
Density
0
20
40
60
-0.2
0.0
0.2
0.4
0.6
Coefficient Value
Chain 1
Chain 2
Chain 3
Chain 4
Chain 5

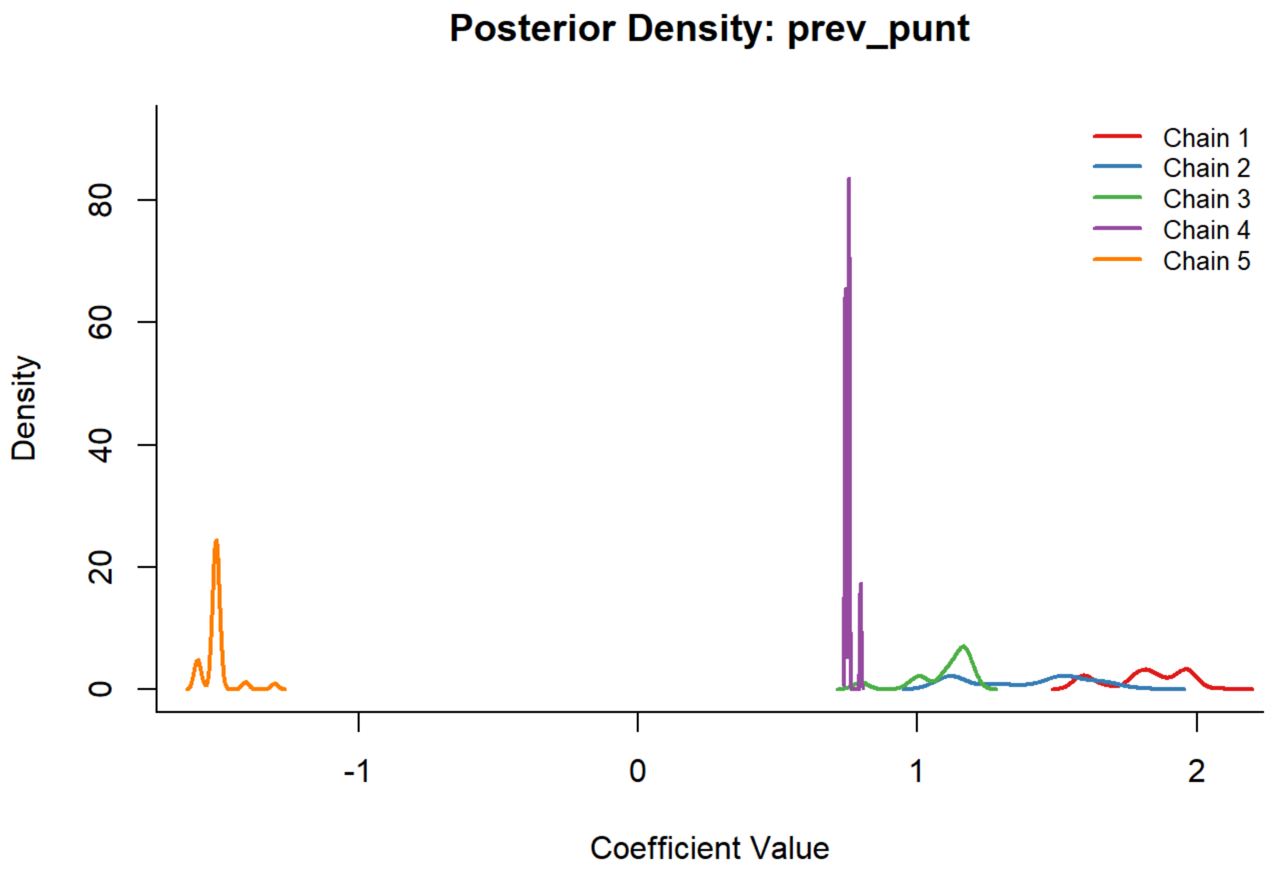


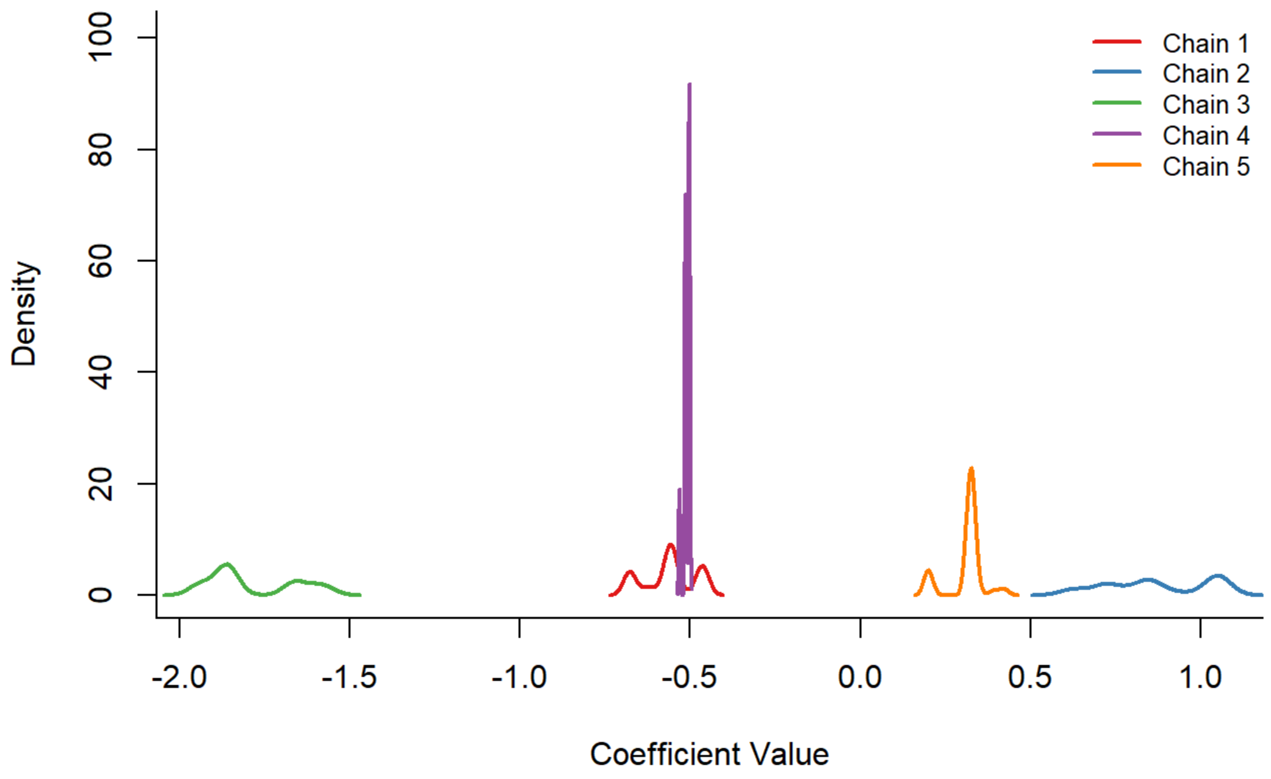
Posterior Density: prev_kickoff



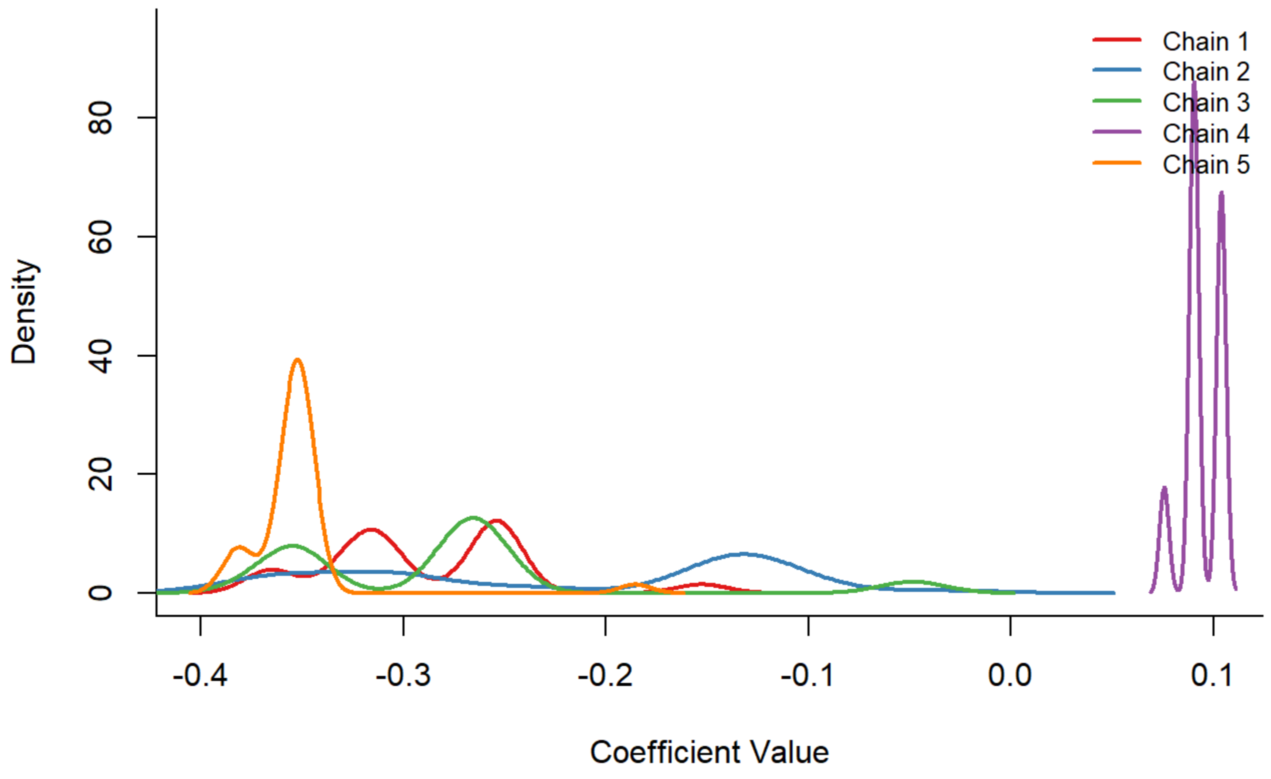
Posterior Density: prev_qb_ks


Posterior Density: prev_ep
Density
Coefficient Value
0
10
20
30
40
50
-1.5
-1.0
-0.5
0.0
0.5
Chain 1
Chain 2
Chain 3
Chain 4
Chain 5

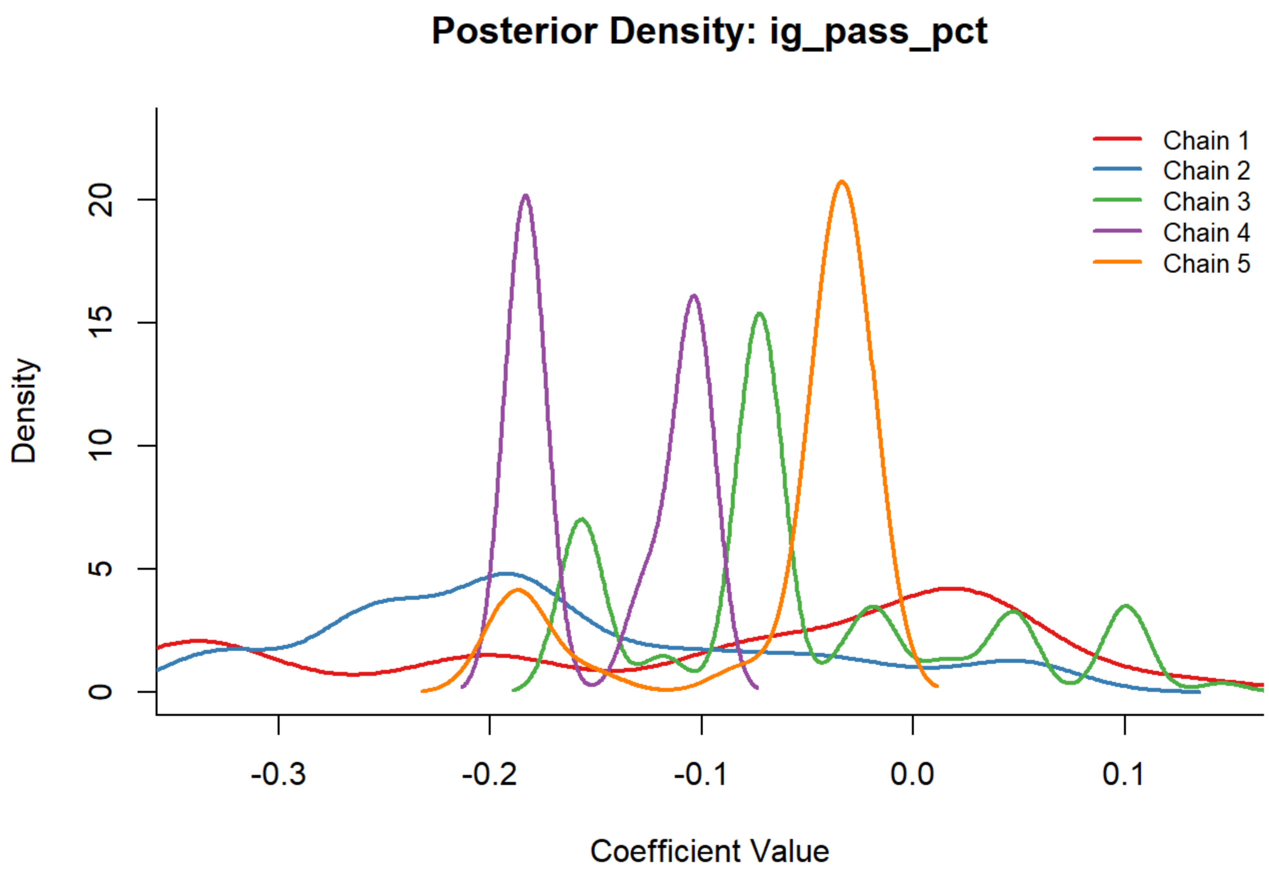

Posterior Density: prev_punt
Density
Coefficient Value
0
20
40
60
80
-1
0
1
2
Chain 1
Chain 2
Chain 3
Chain 4
Chain 5



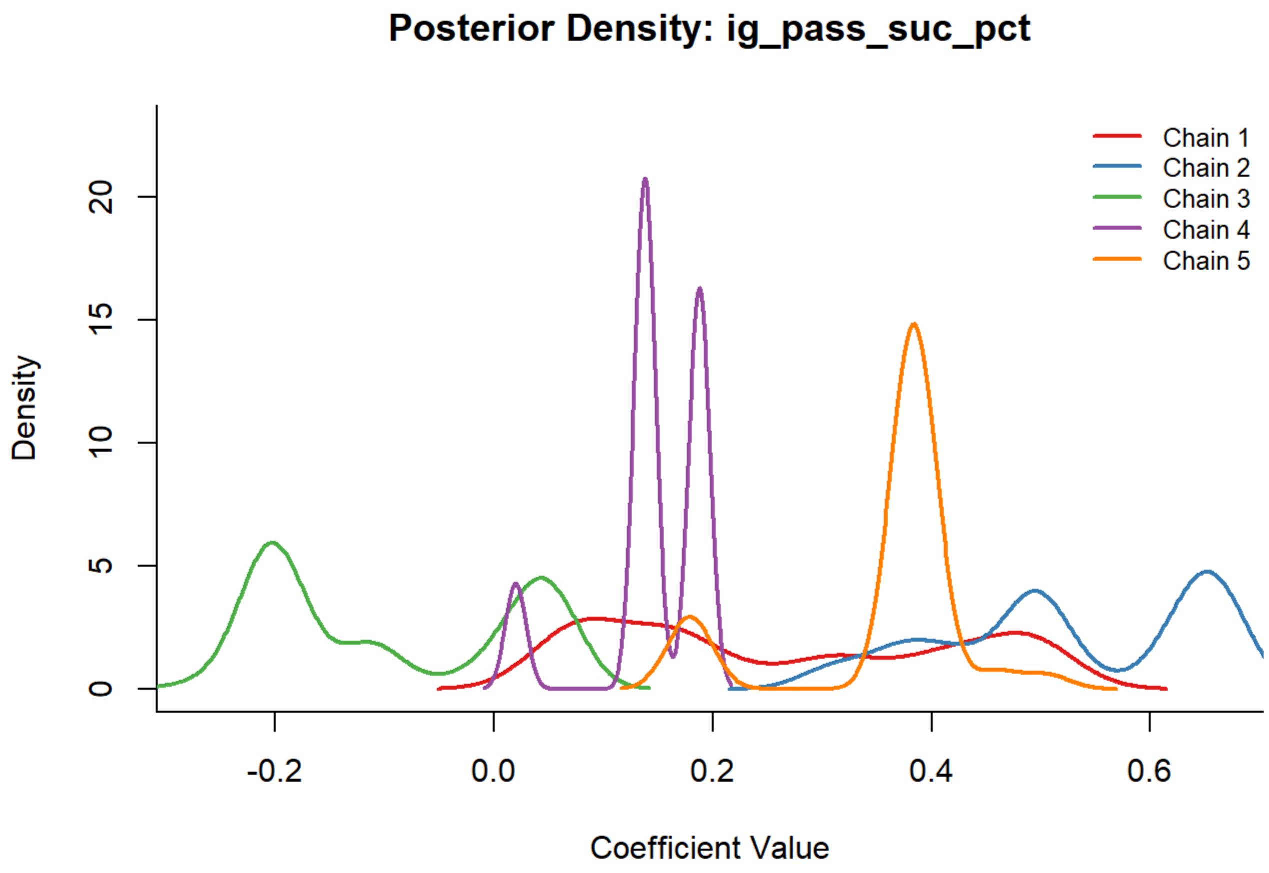
Posterior Density: prev_field_goal



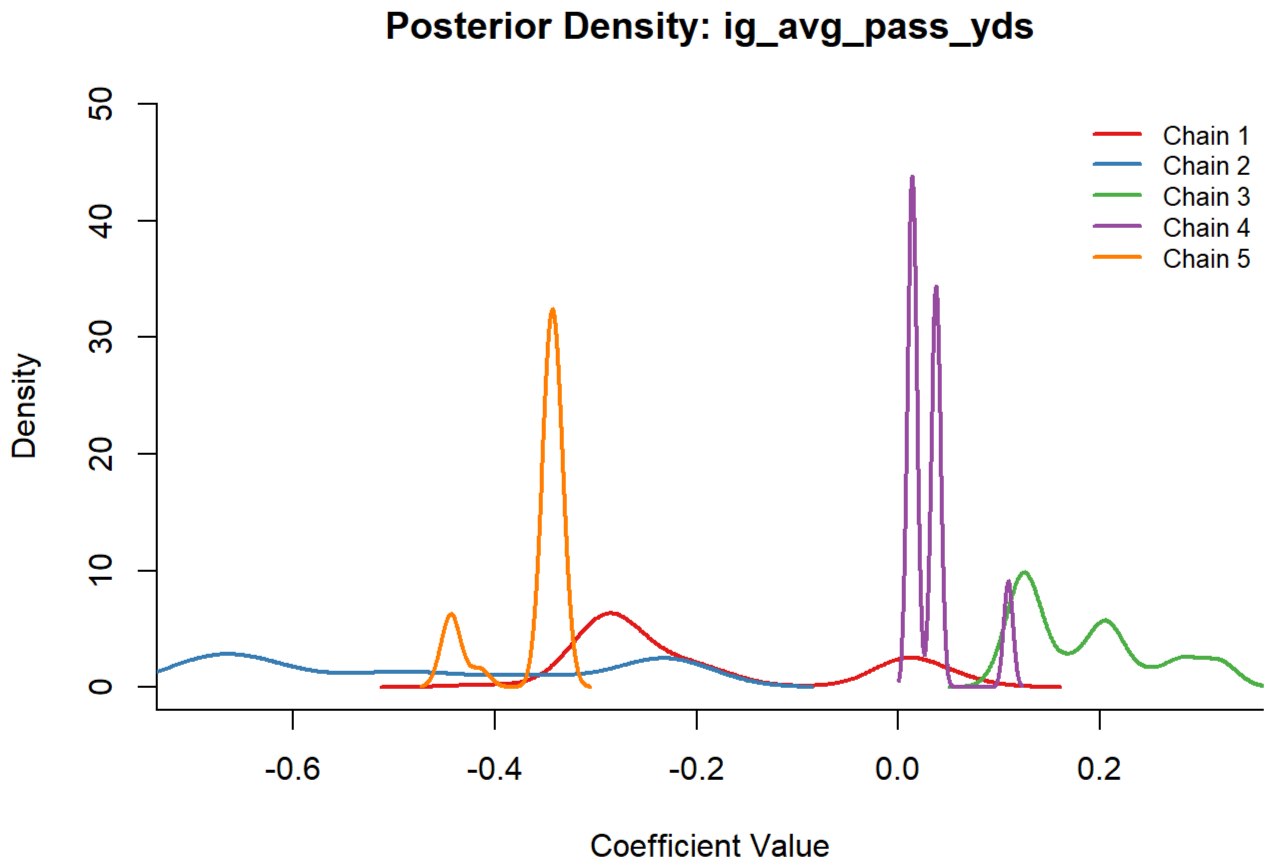
Posterior Density: prev_yds_gain

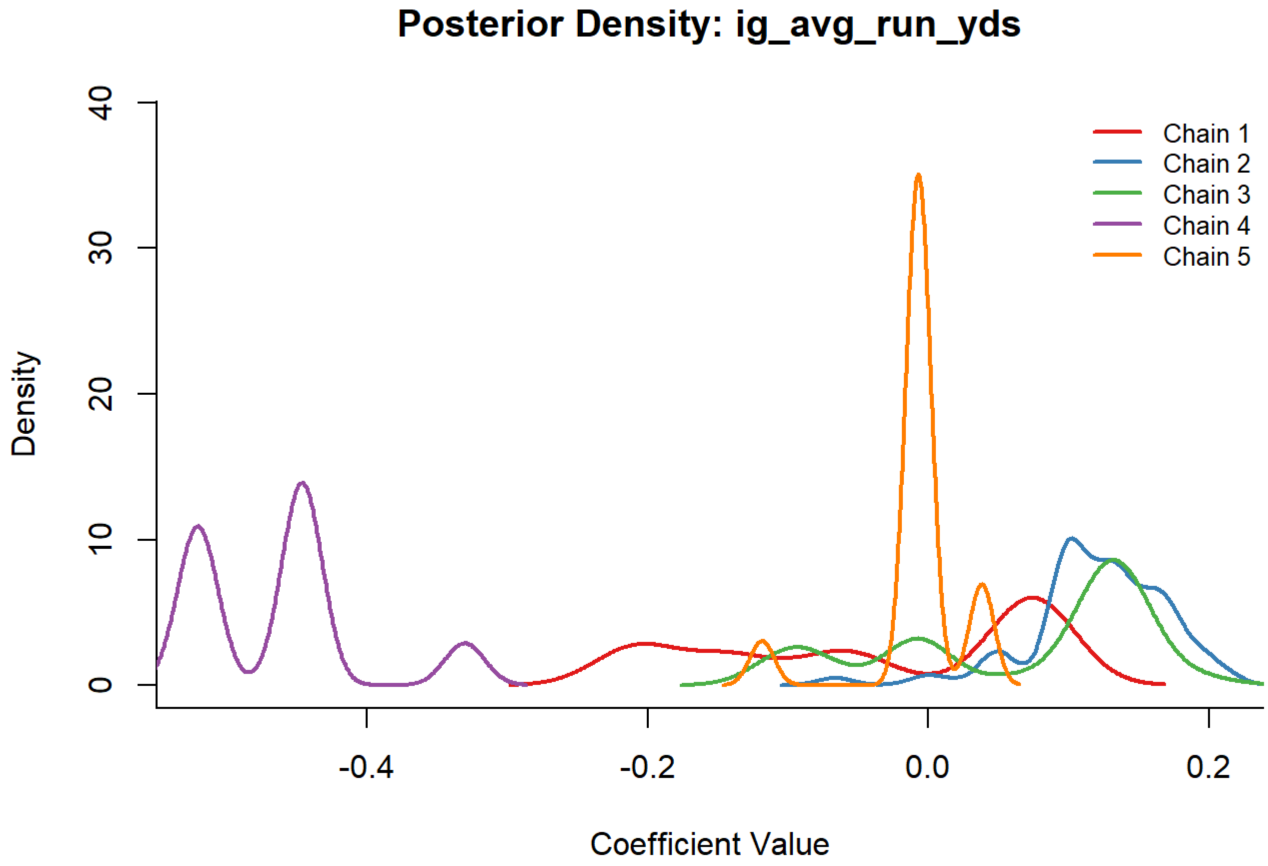


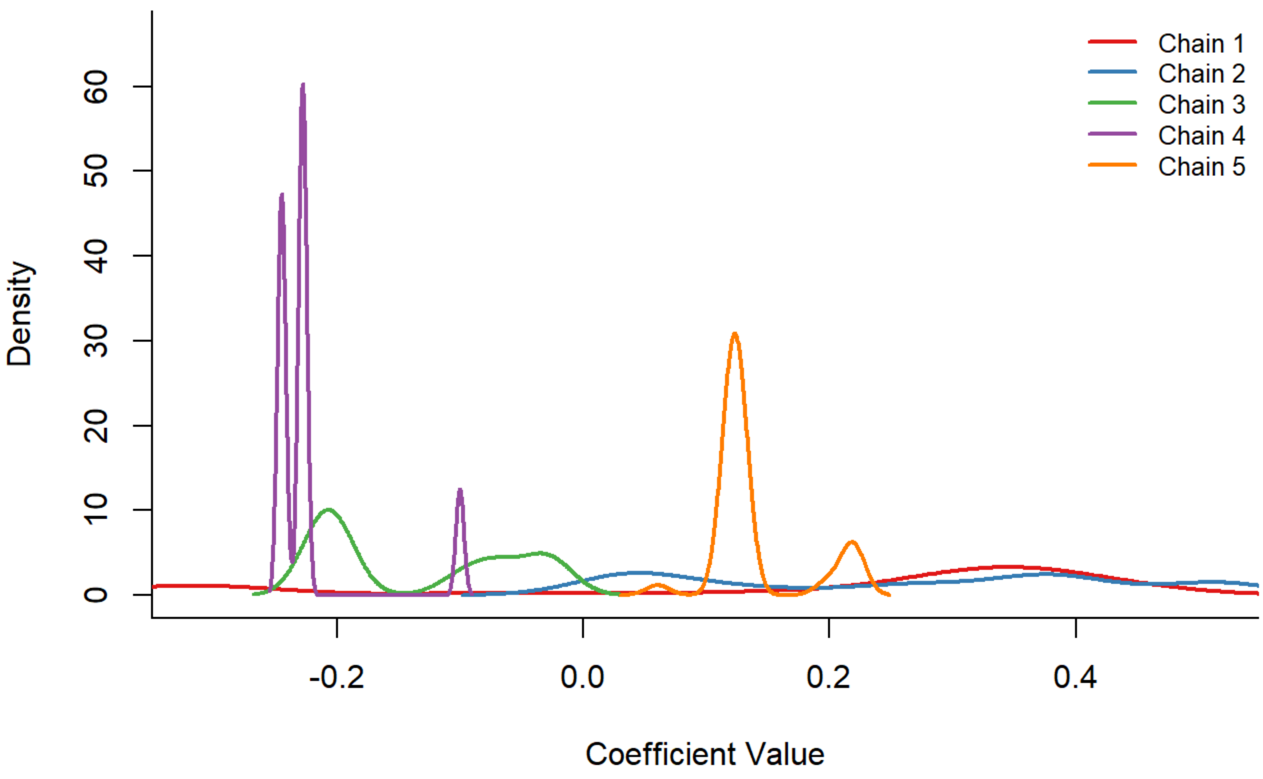
Posterior Density: ig_pass_pct



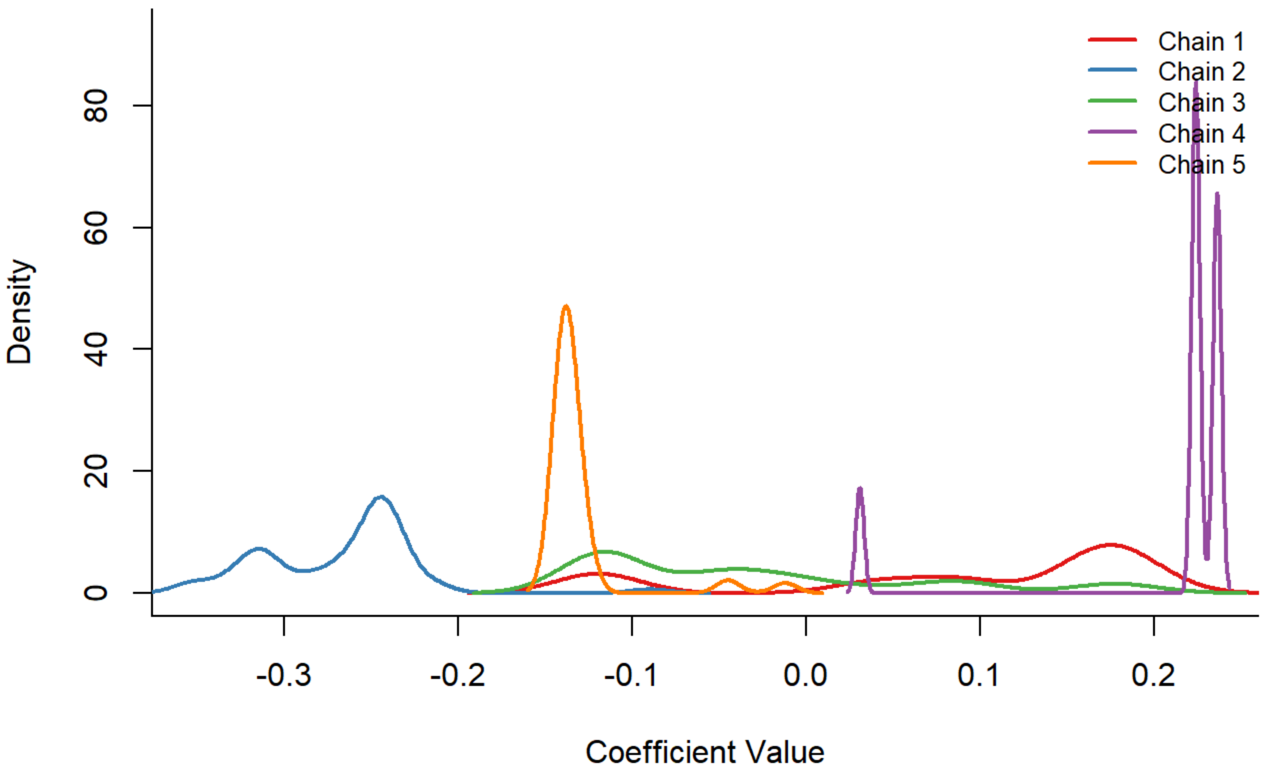
Posterior Density: ig_pass_suc_pct


Posterior Density: ig_avg_pass_yds
Density
0
10
20
30
40
50
-0.6
-0.4
-0.2
0.0
0.2
Coefficient Value
Chain 1
Chain 2
Chain 3
Chain 4
Chain 5

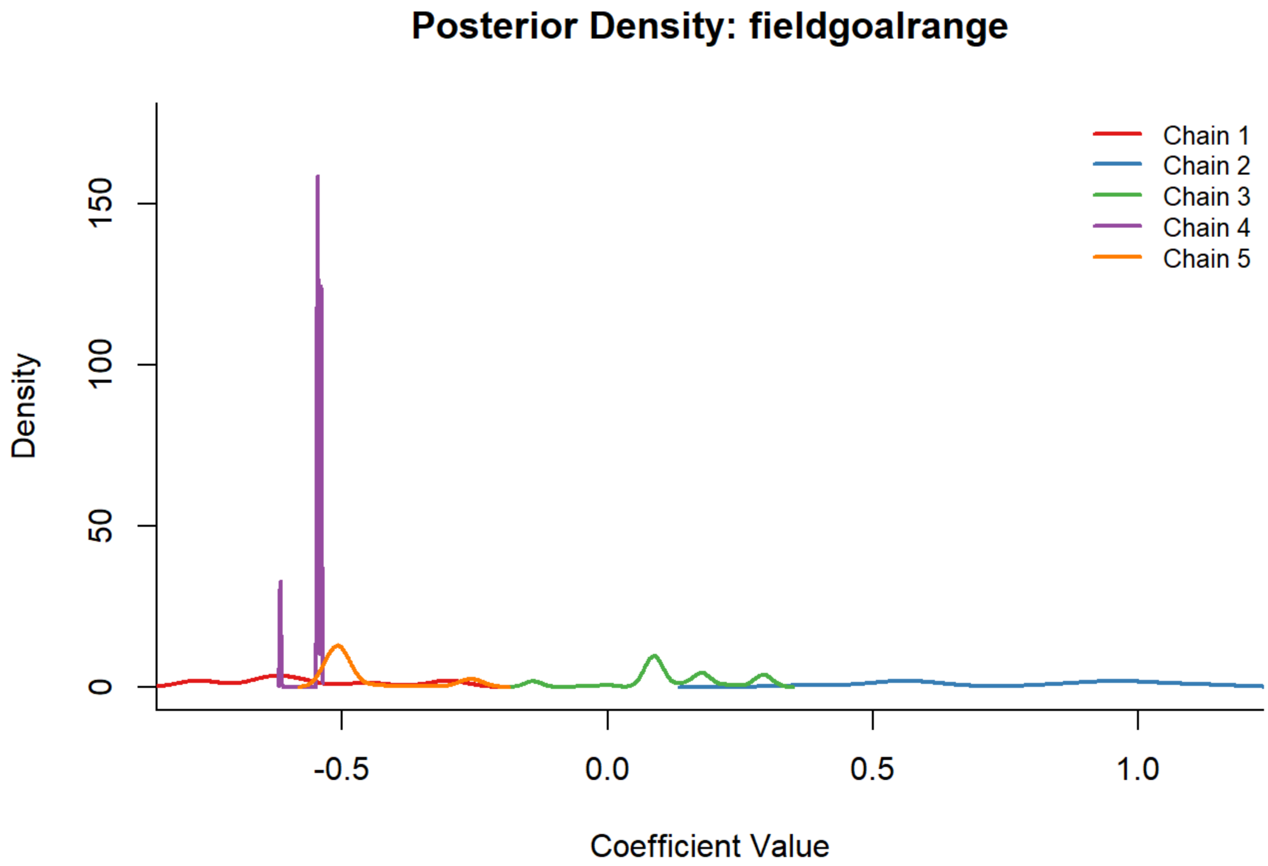

Posterior Density: ig_avg_run_yds
Density
0
10
20
30
40
-0.4
-0.2
0.0
0.2
Coefficient Value
Chain 1
Chain 2
Chain 3
Chain 4
Chain 5

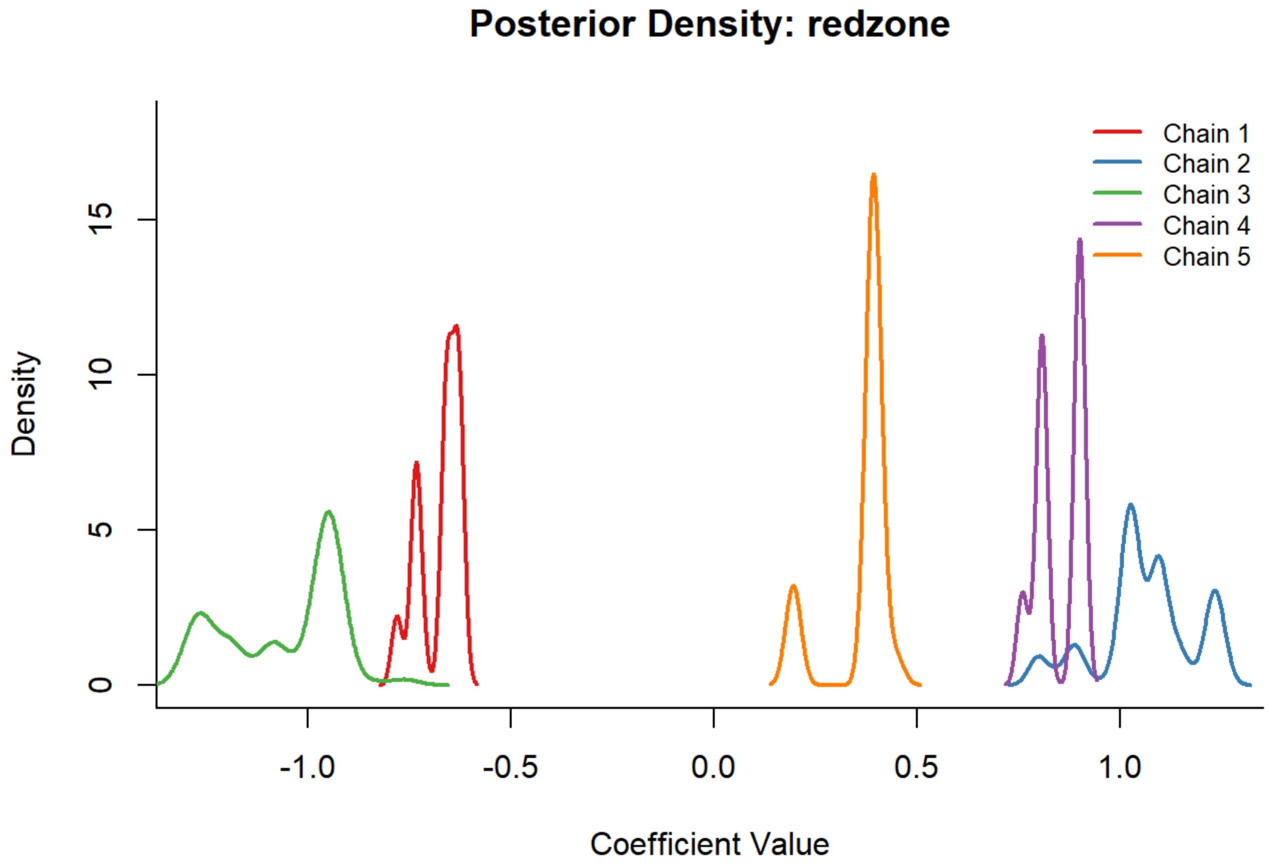


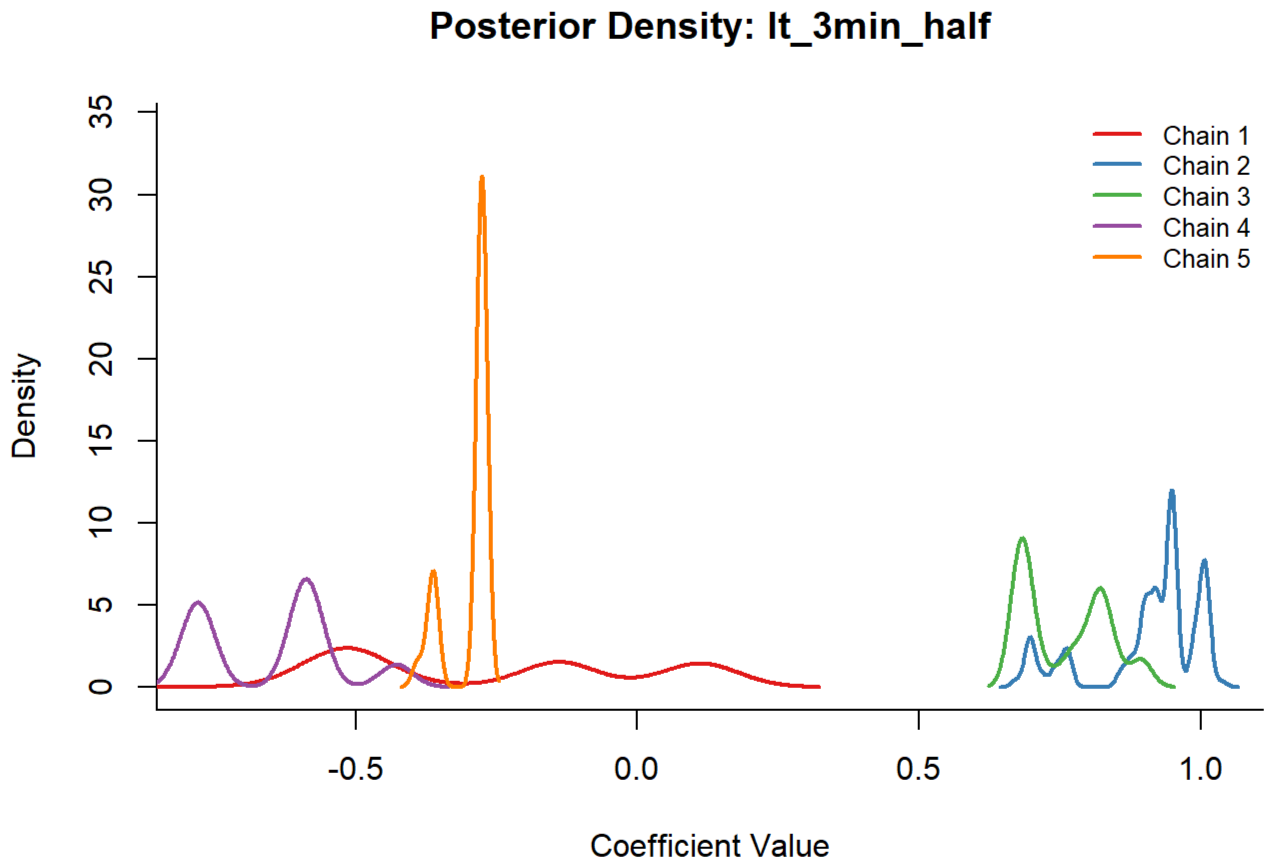
Posterior Density: pass_sd



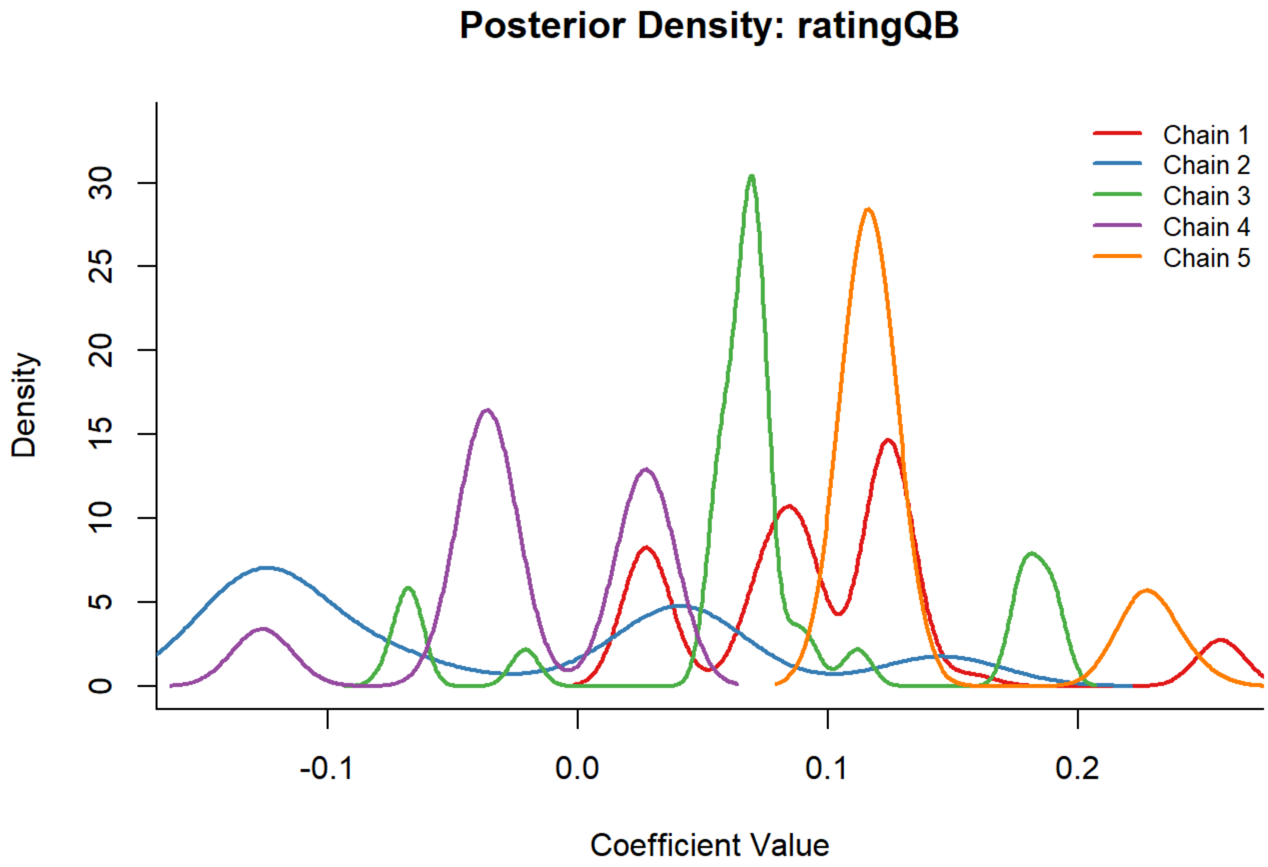
Posterior Density: run_sd


Posterior Density: fieldgoalrange
Density
0
50
100
150
-0.5
0.0
0.5
1.0
Coefficient Value
Chain 1
Chain 2
Chain 3
Chain 4
Chain 5

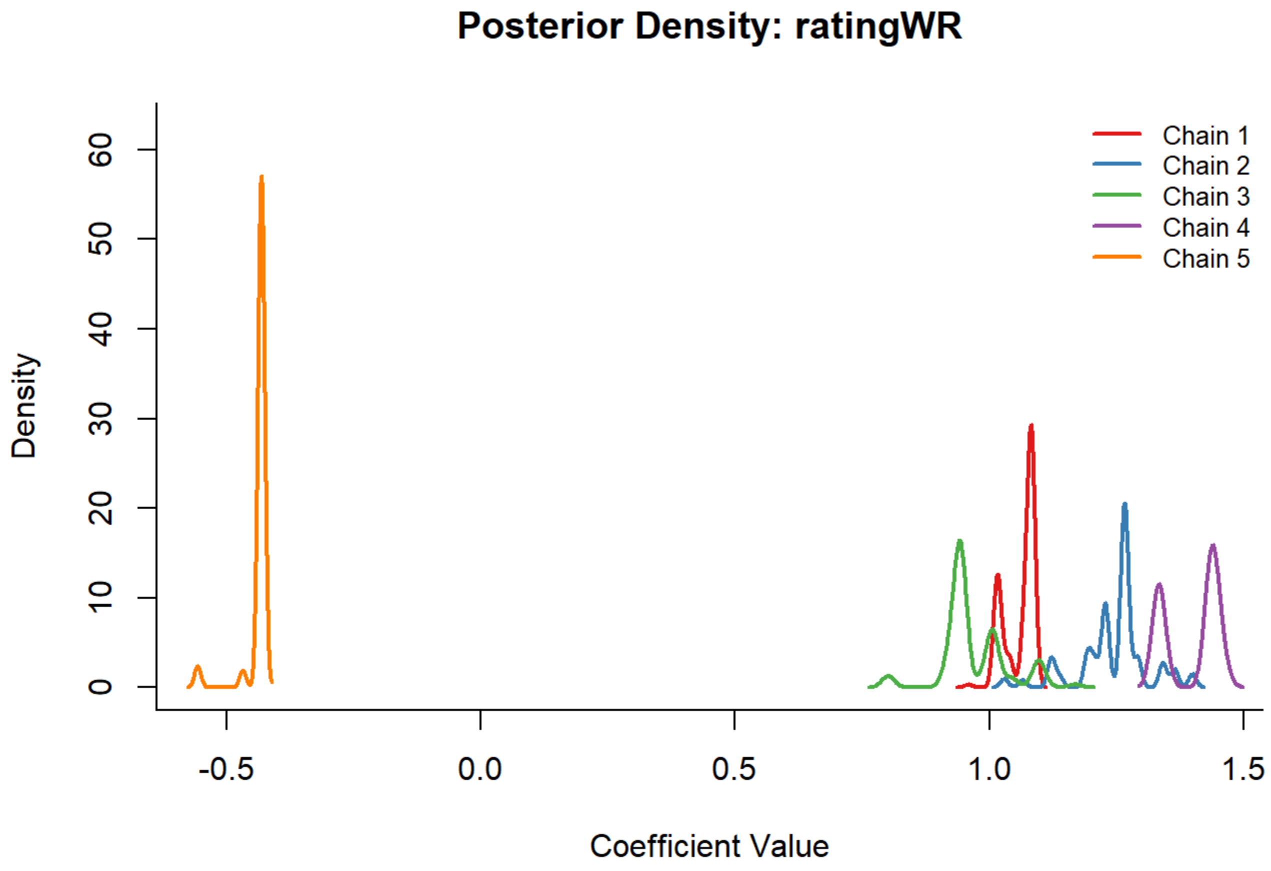

Posterior Density: redzone
Density
0
5
10
15
-1.0
-0.5
0.0
0.5
1.0
Coefficient Value
Chain 1
Chain 2
Chain 3
Chain 4
Chain 5

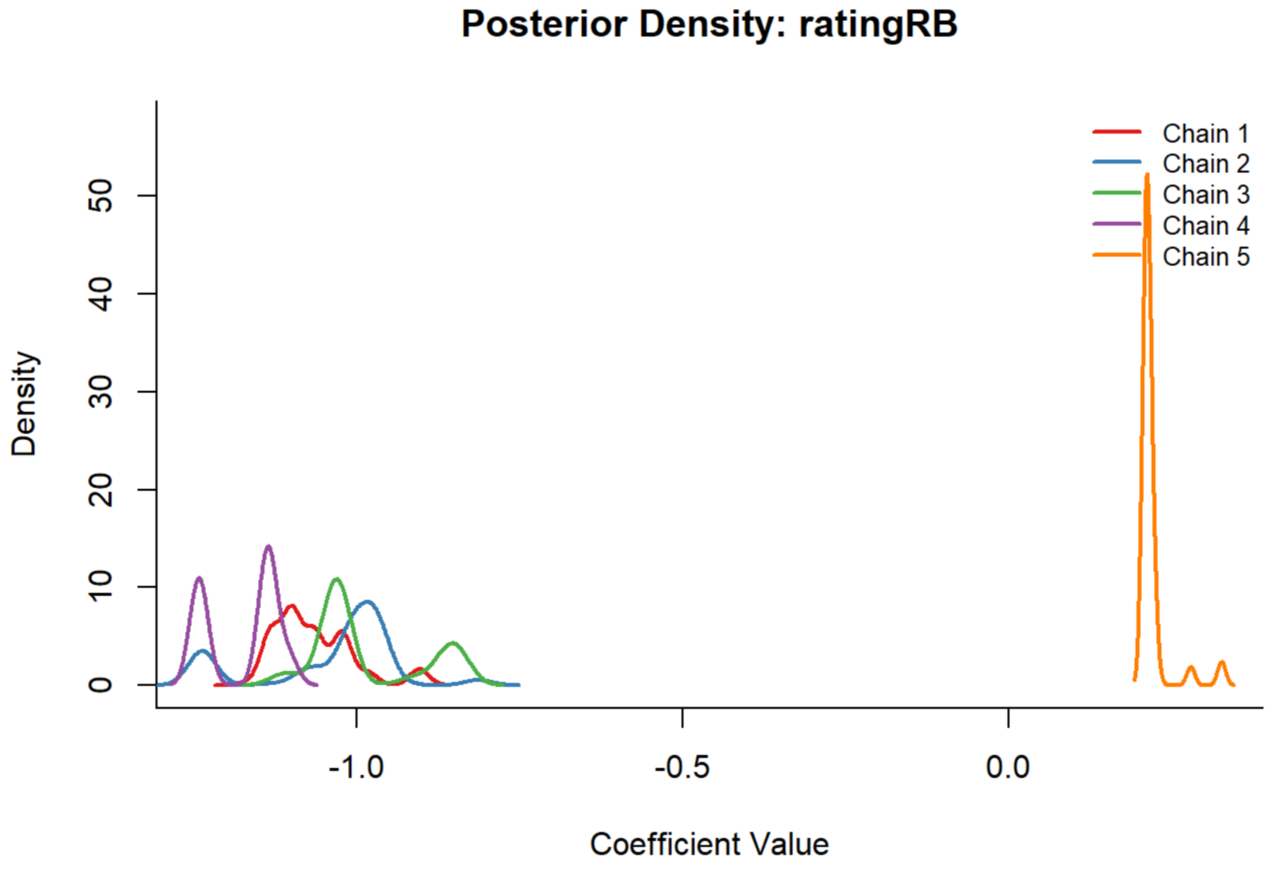


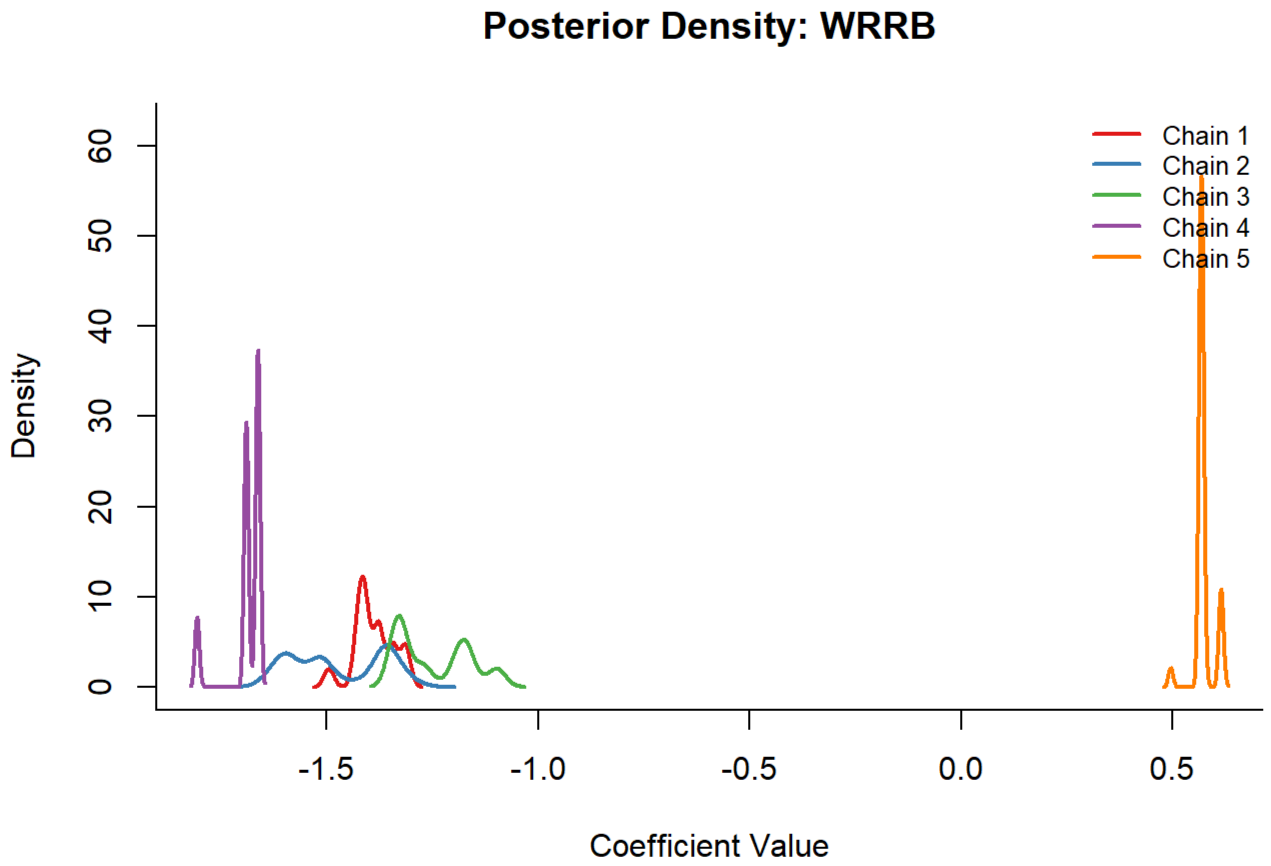
Posterior Density: lt_3min_half

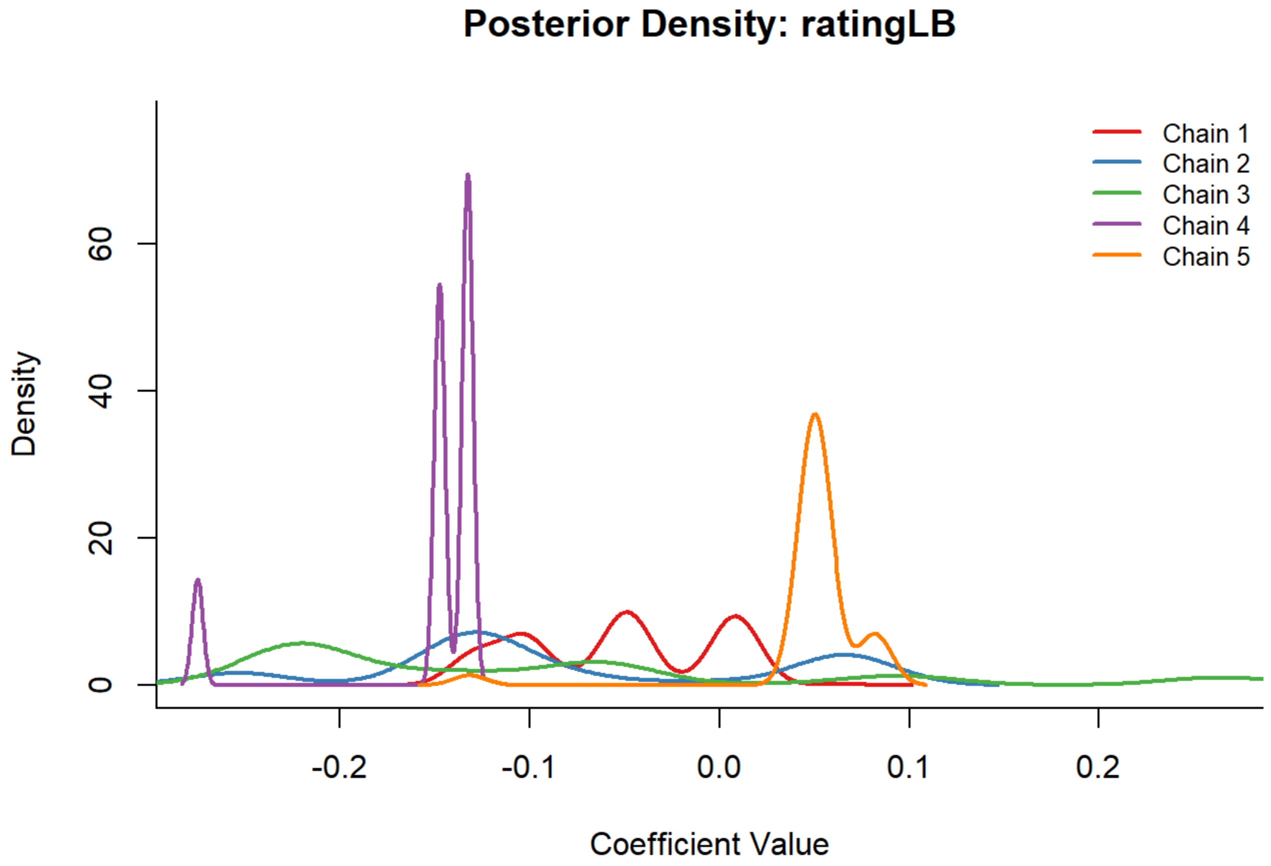


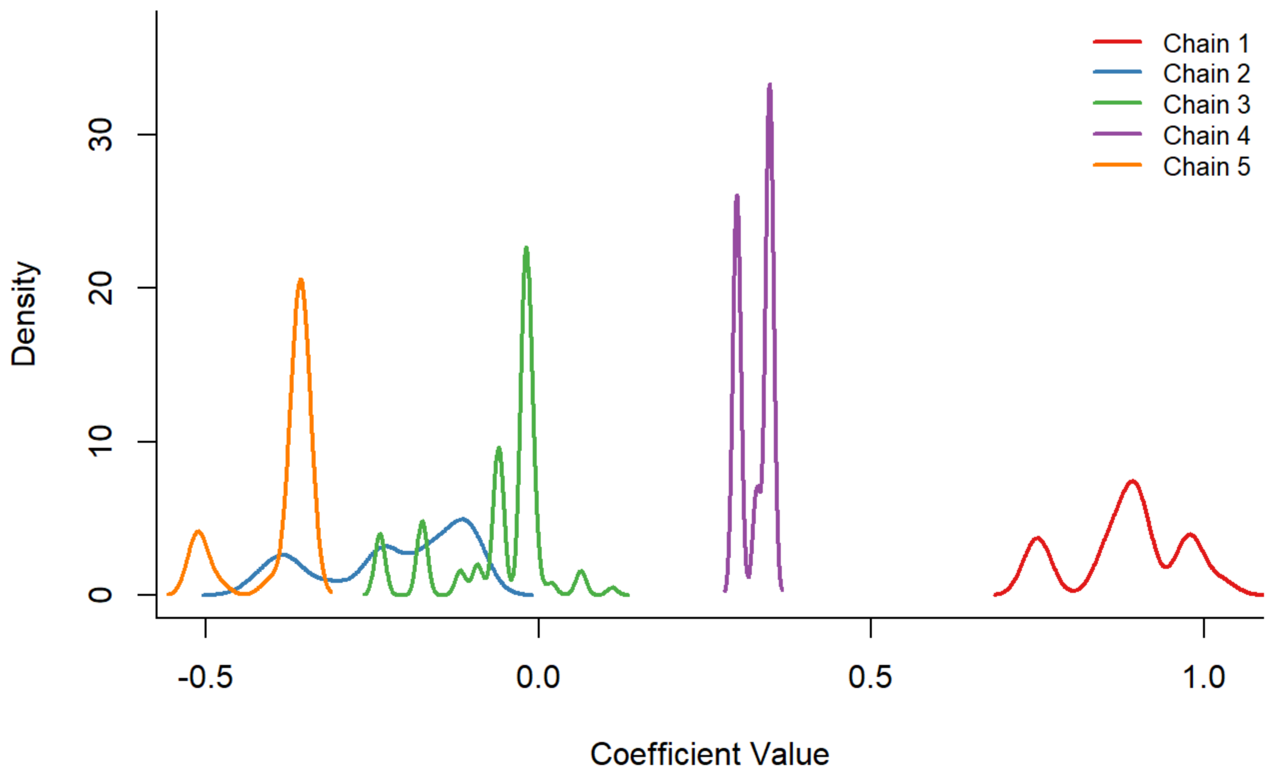
Posterior Density: ratingQB


Posterior Density: ratingWR

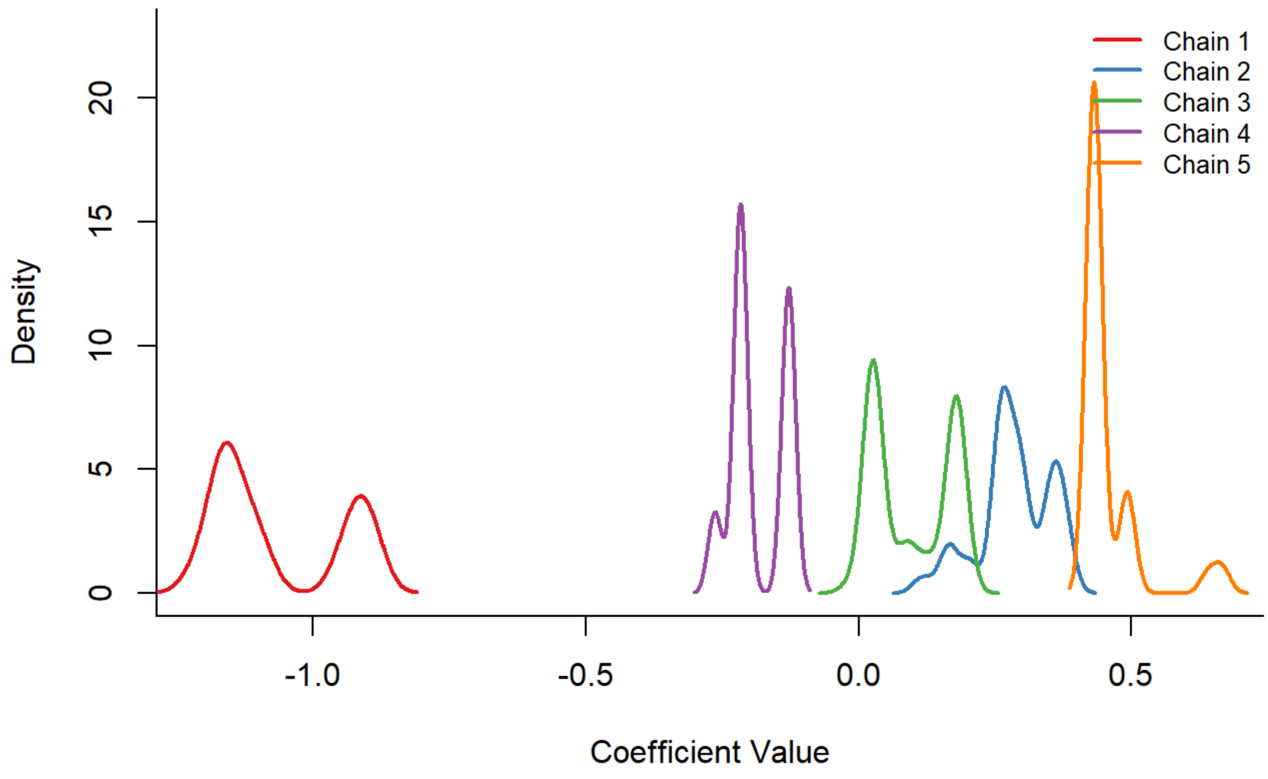

Posterior Density: ratingRB


Posterior Density: WRRB
Density
0
10
20
30
40
50
60
Chain 1
Chain 2
Chain 3
Chain 4
Chain 5
-1.5
-1.0
-0.5
0.0
0.5
Coefficient Value

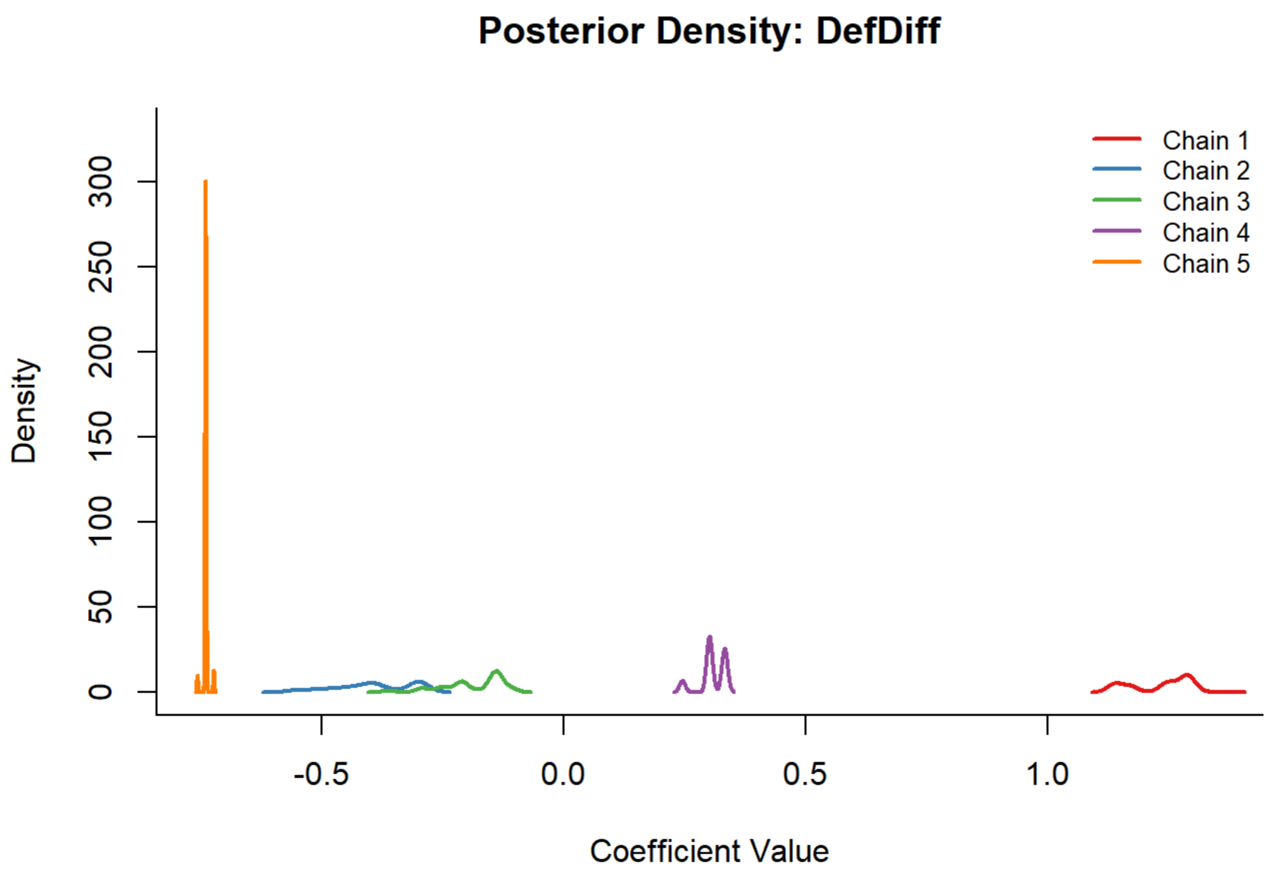

Posterior Density: ratingLB
Density
0
20
40
60
Chain 1
Chain 2
Chain 3
Chain 4
Chain 5
-0.2
-0.1
0.0
0.1
0.2
Coefficient Value



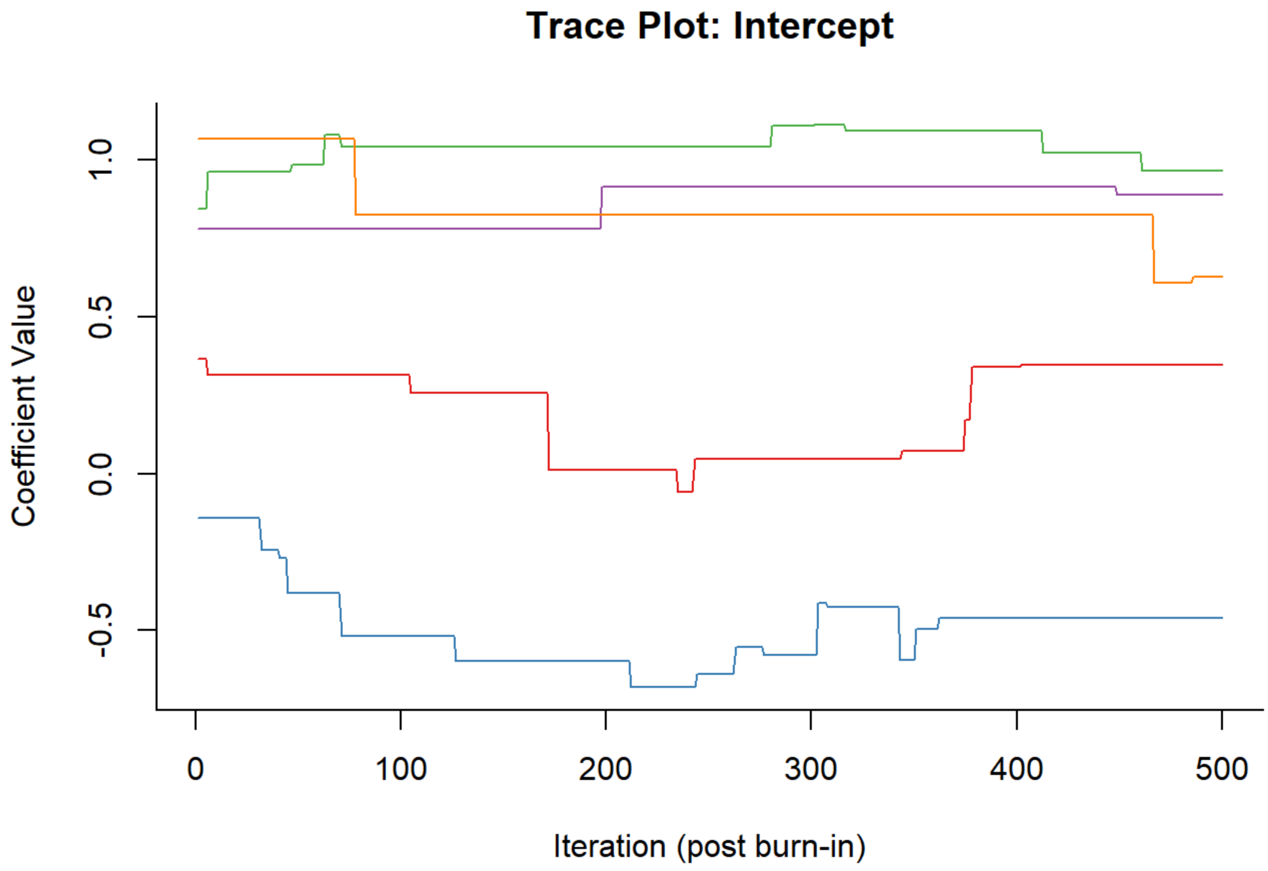
Posterior Density: ratingDefPass

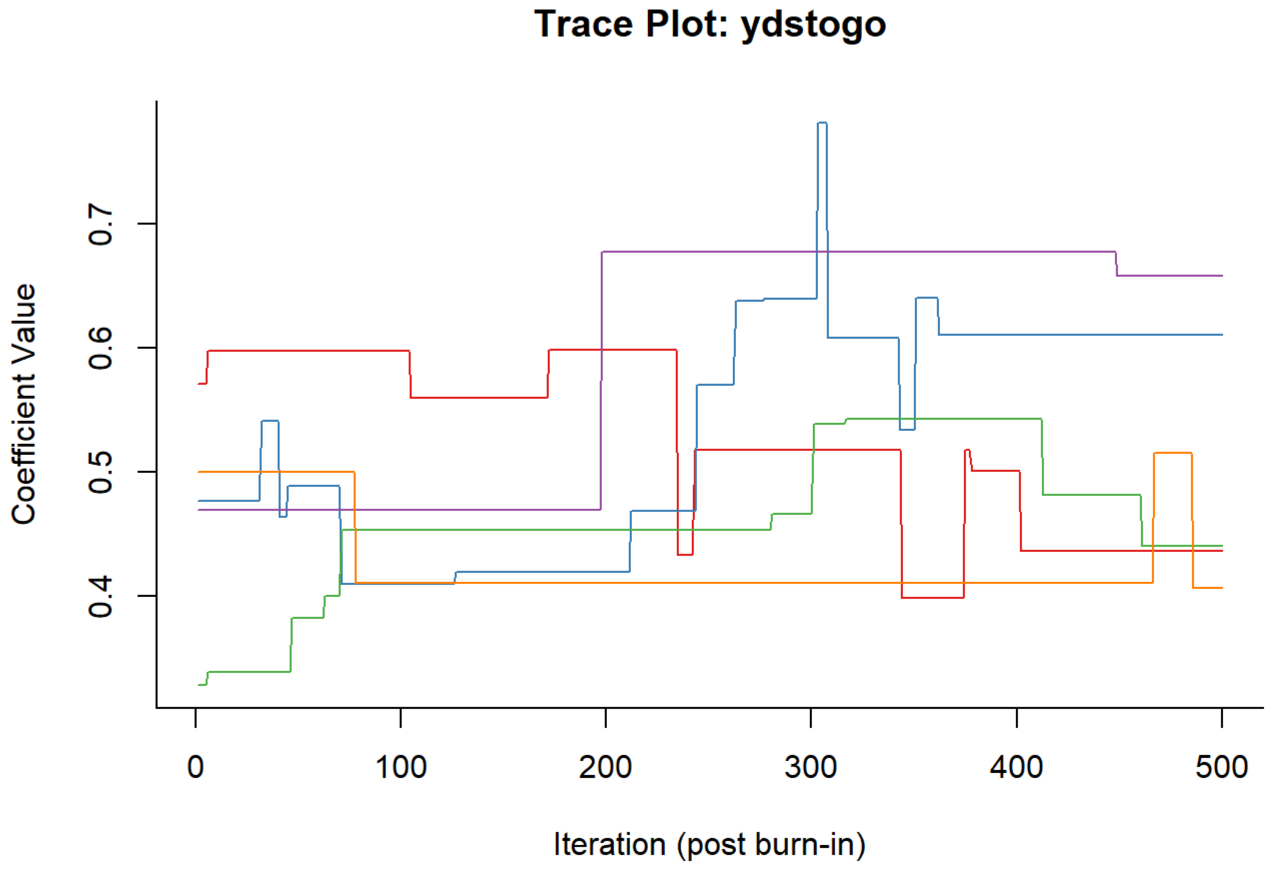


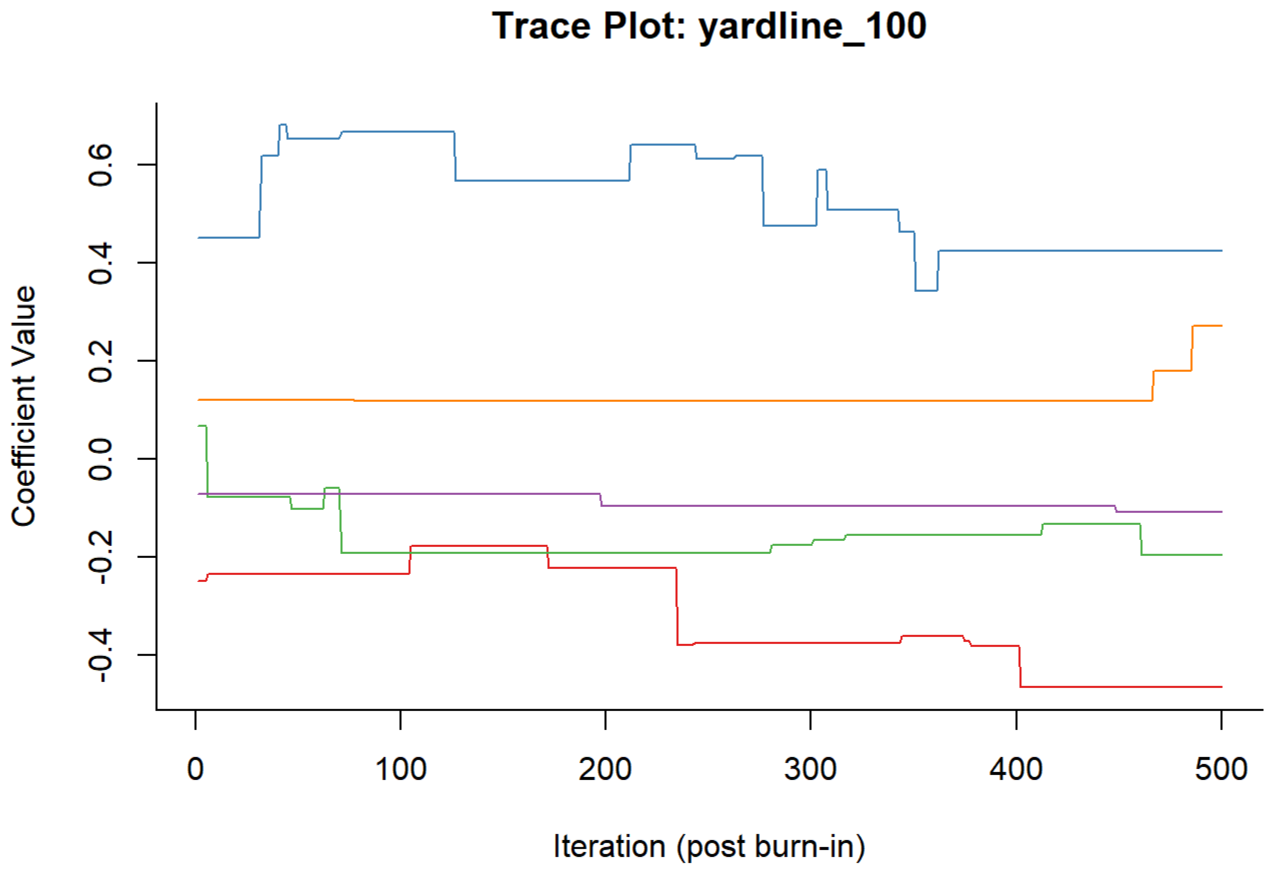
Posterior Density: ratingDefRun

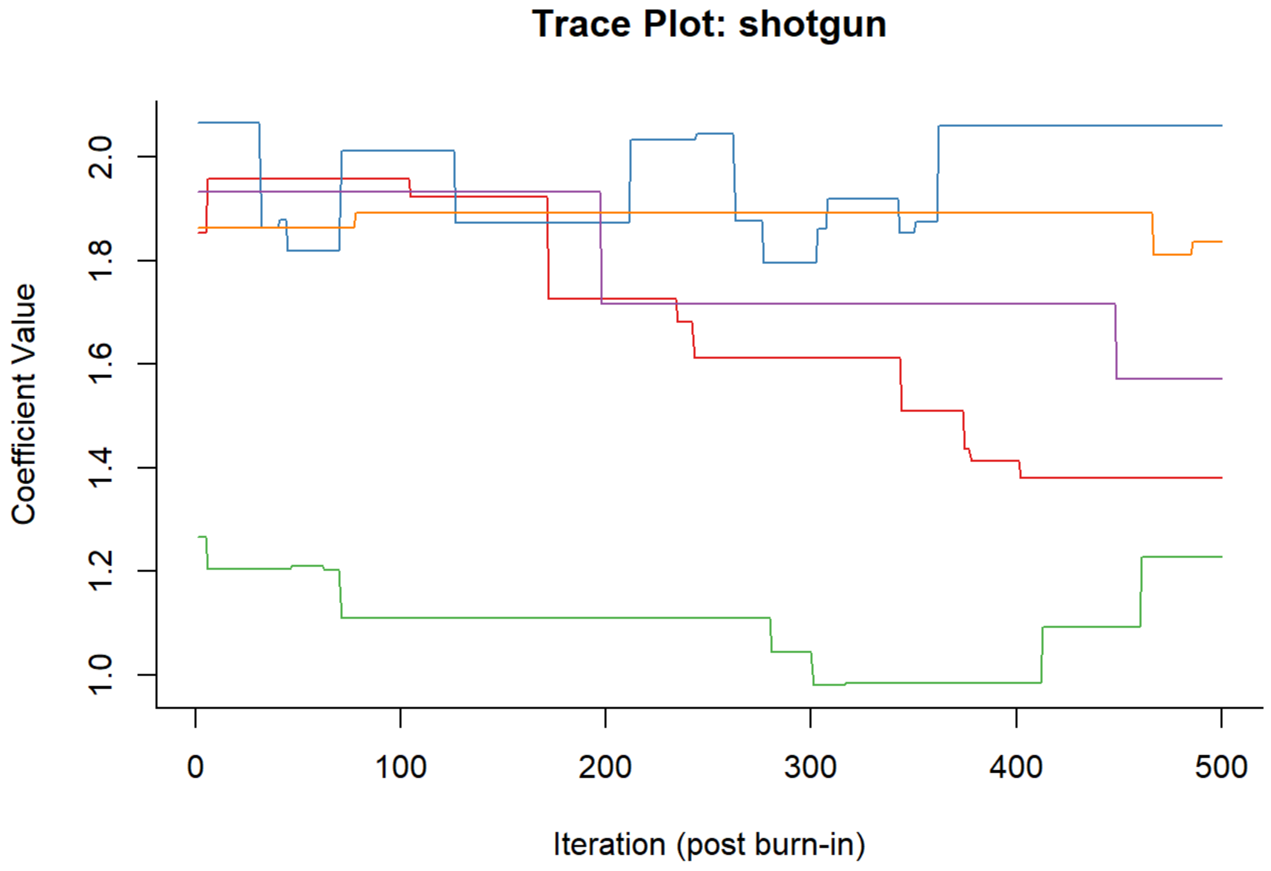


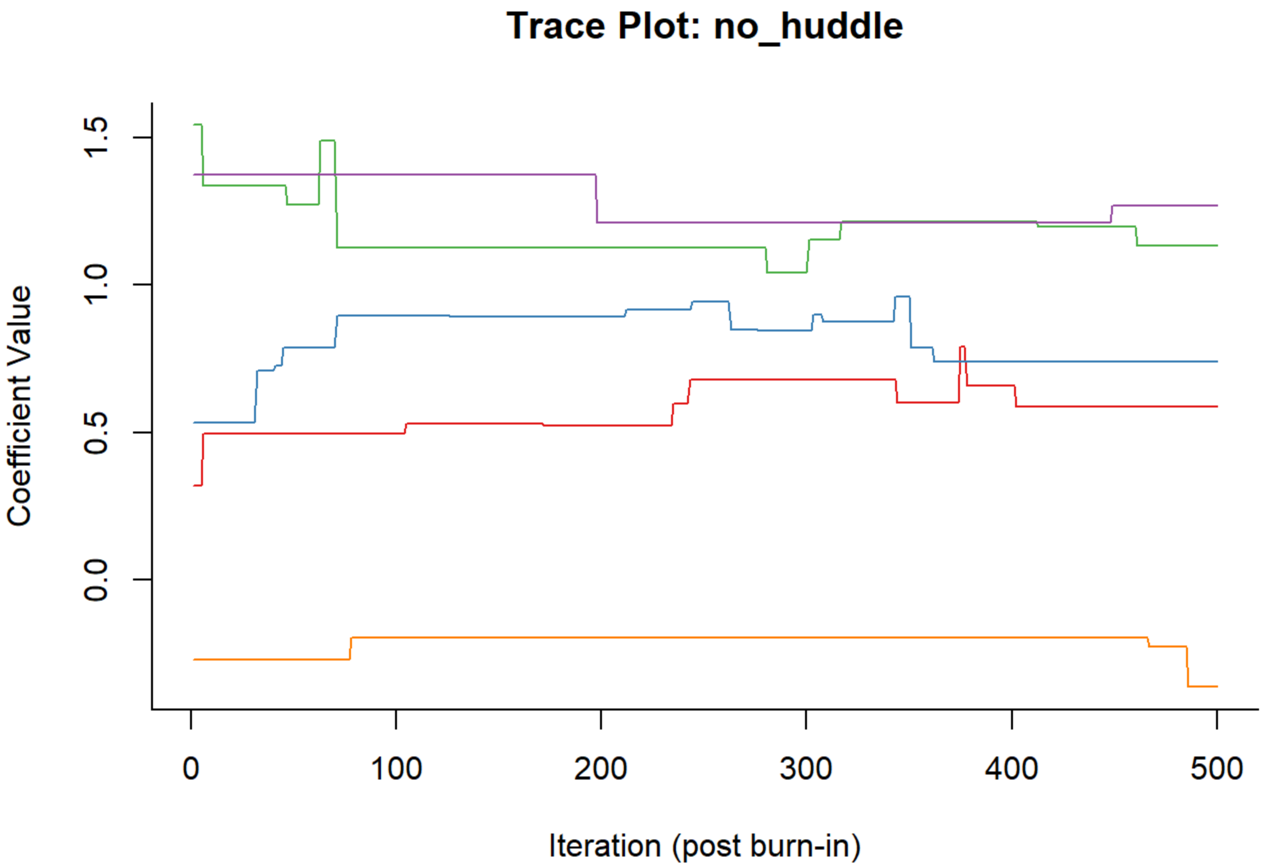
```
plot_trace_plots(beta_samples_all, X_test)
```

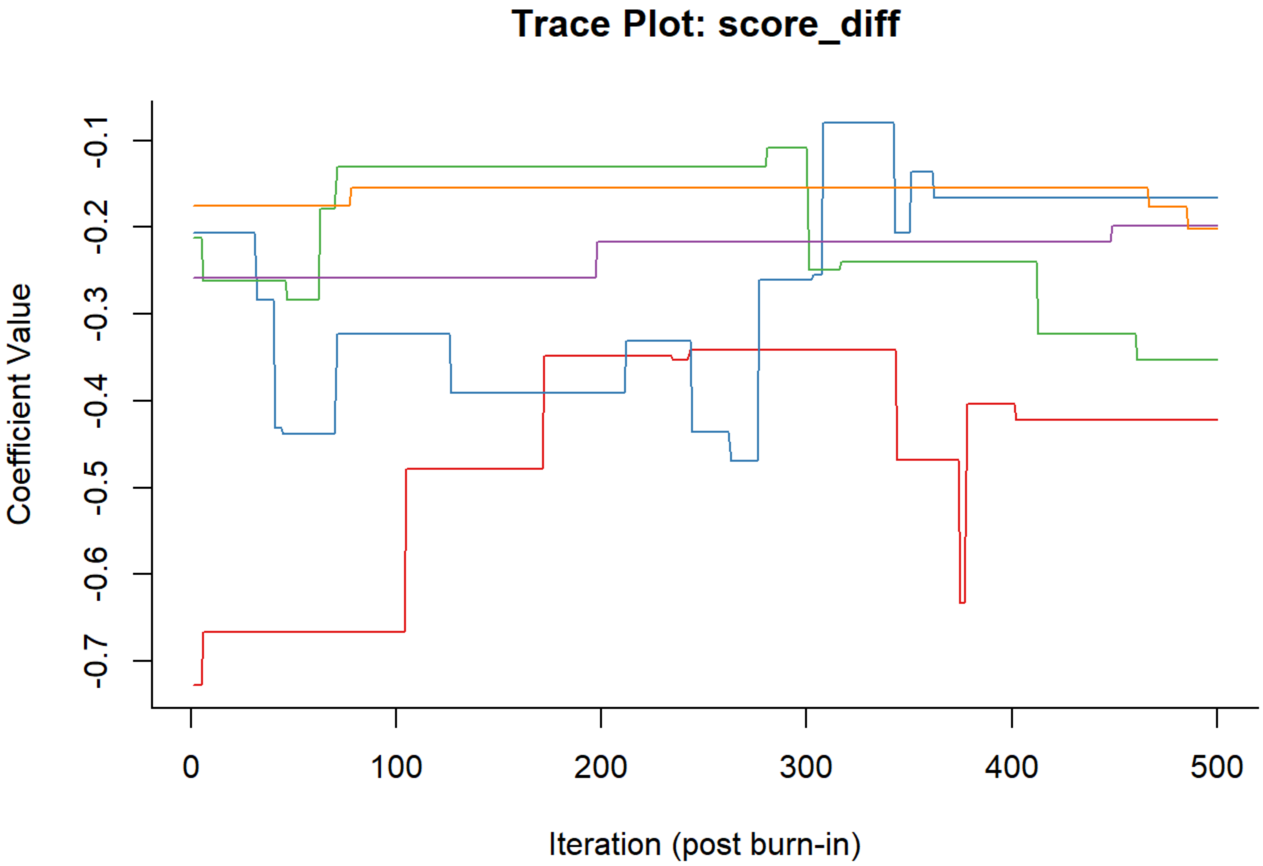

Trace Plot: Intercept
Coefficient Value
1.0
0.5
0.0
-0.5
0
100
200
300
400
500
Iteration (post burn-in)


Trace Plot: ydstogo
Coefficient Value
0.7
0.6
0.5
0.4
0
100
200
300
400
500
Iteration (post burn-in)

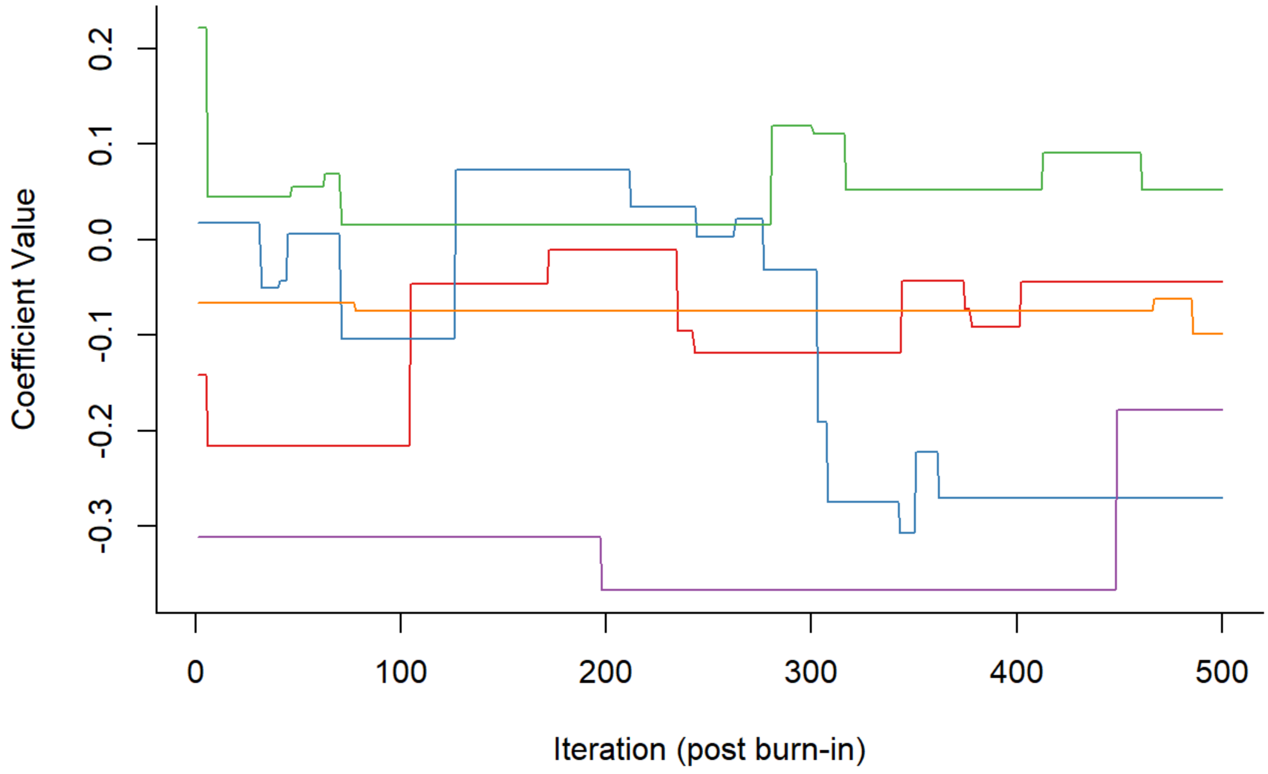

Trace Plot: yardline_100


Trace Plot: shotgun

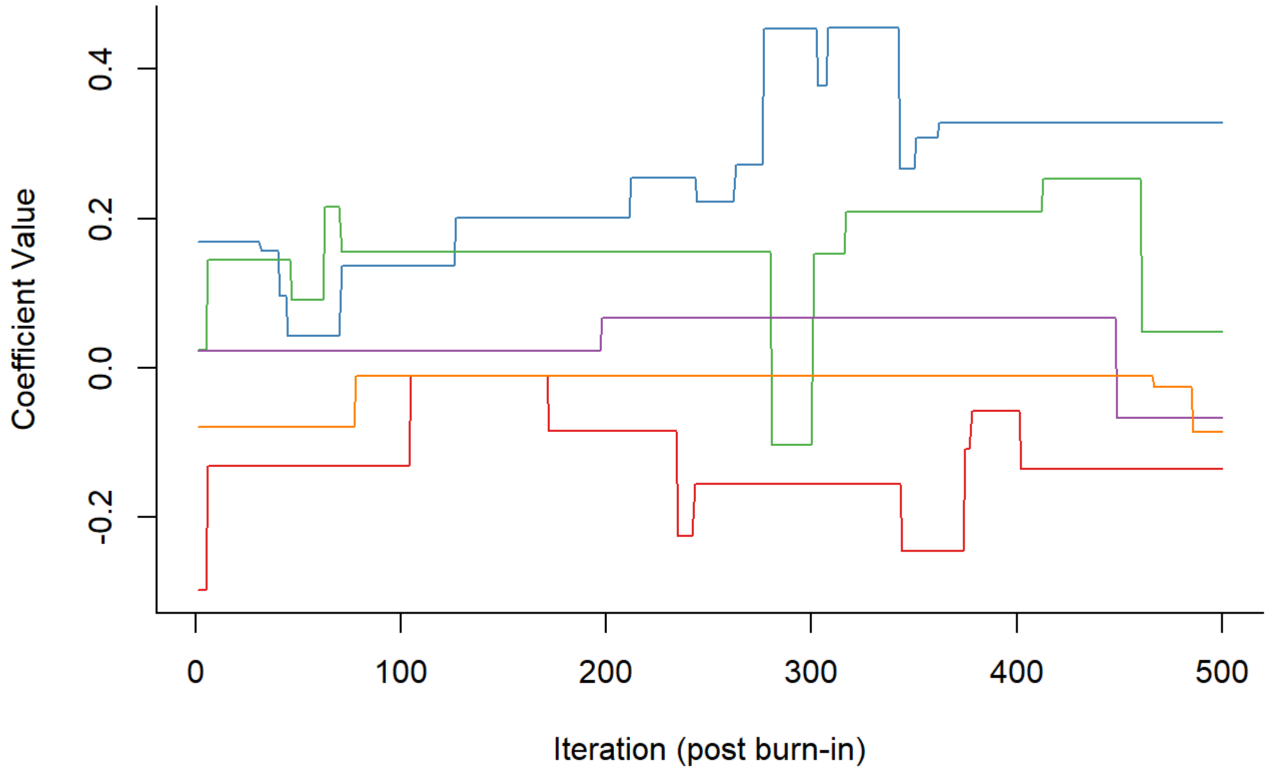

Trace Plot: no_huddle
Coefficient Value
1.5
1.0
0.5
0.0
0
100
200
300
400
500
Iteration (post burn-in)

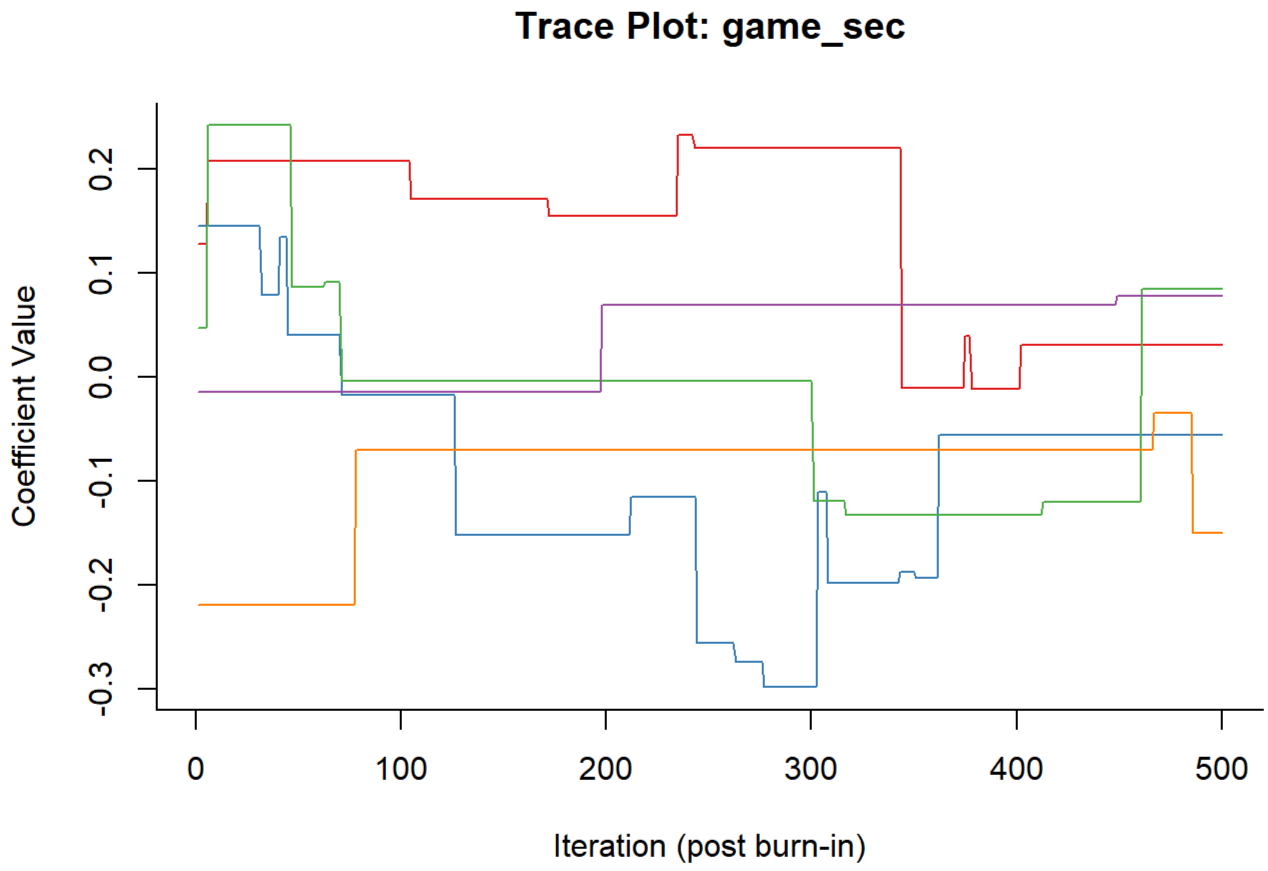

Trace Plot: score_diff
Coefficient Value
-0.1
-0.2
-0.3
-0.4
-0.5
-0.6
-0.7
0
100
200
300
400
500
Iteration (post burn-in)

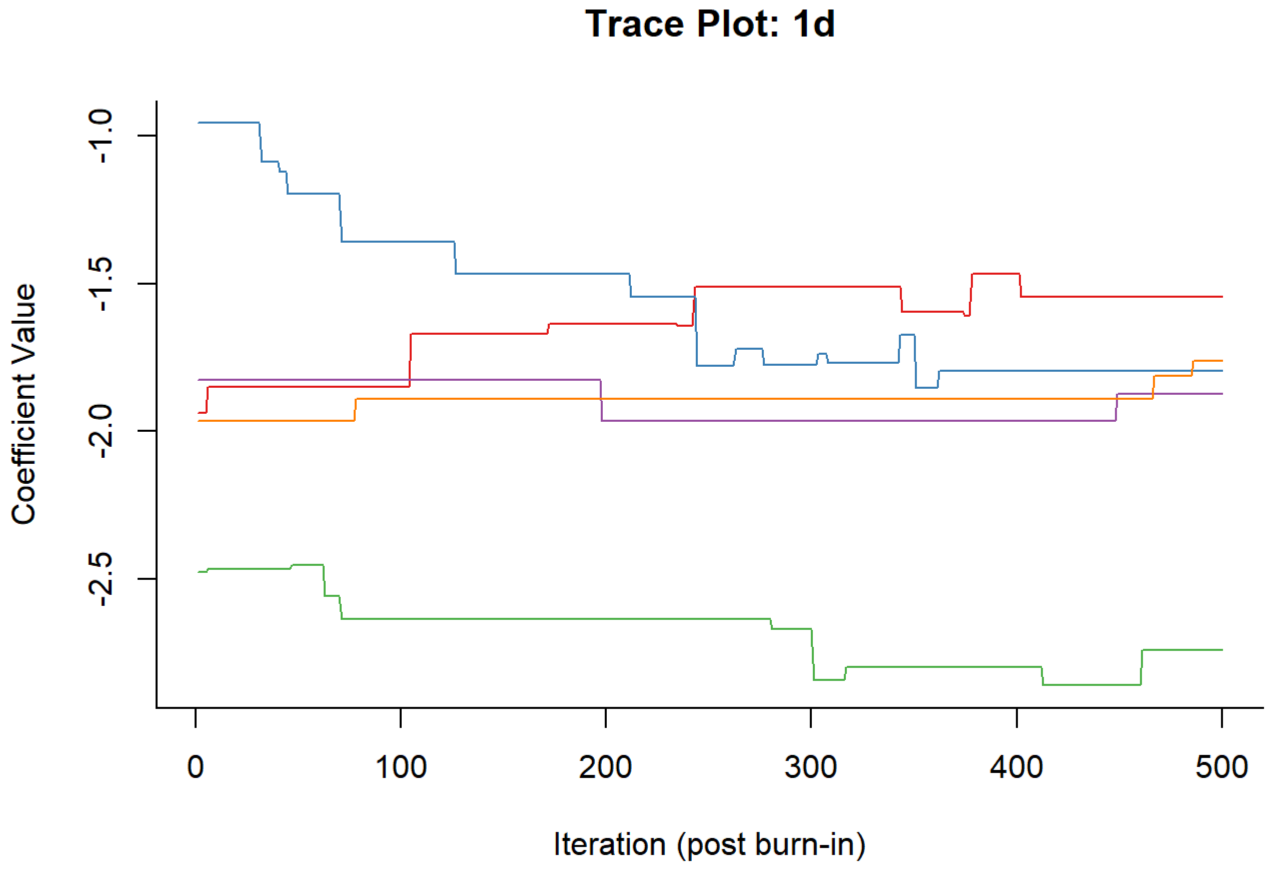
Trace Plot: qrt_sec

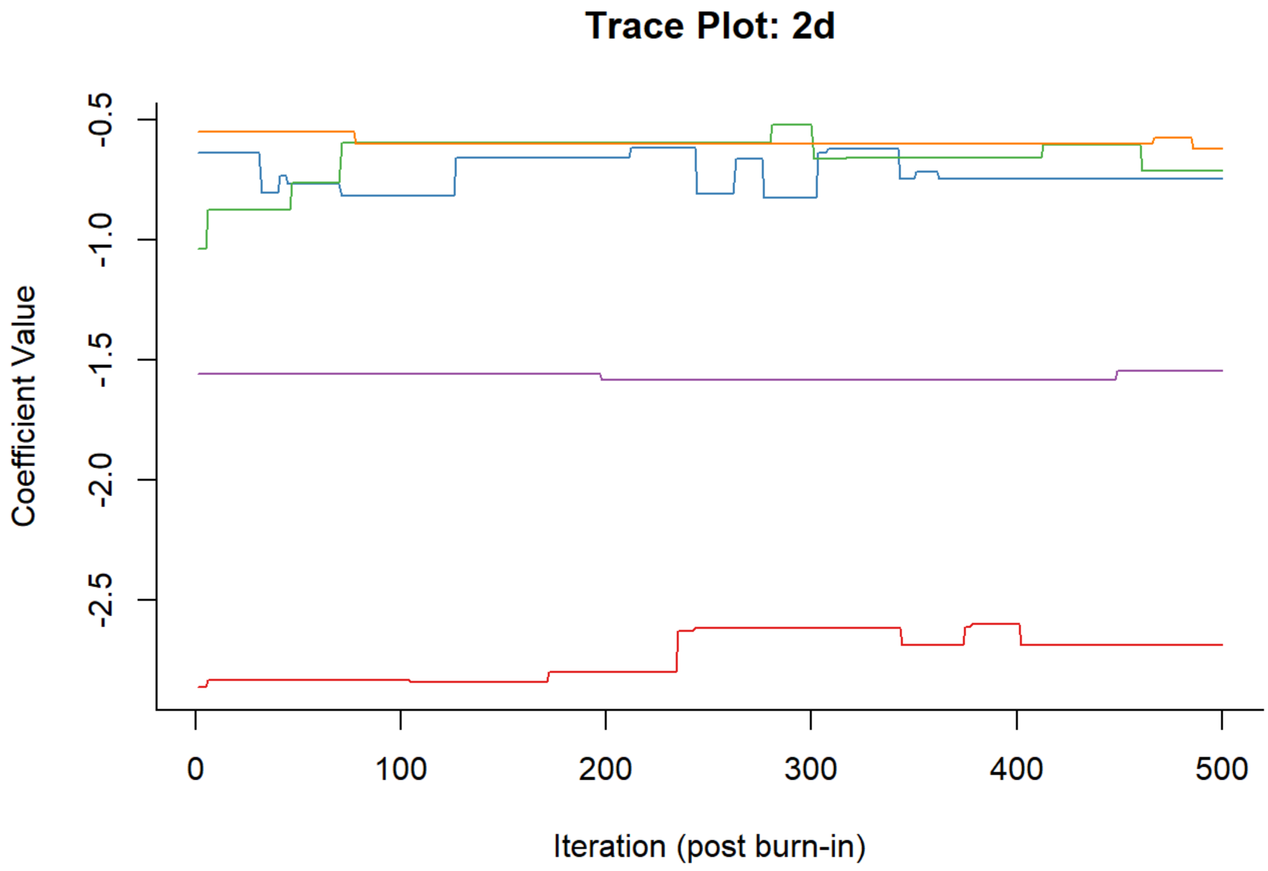


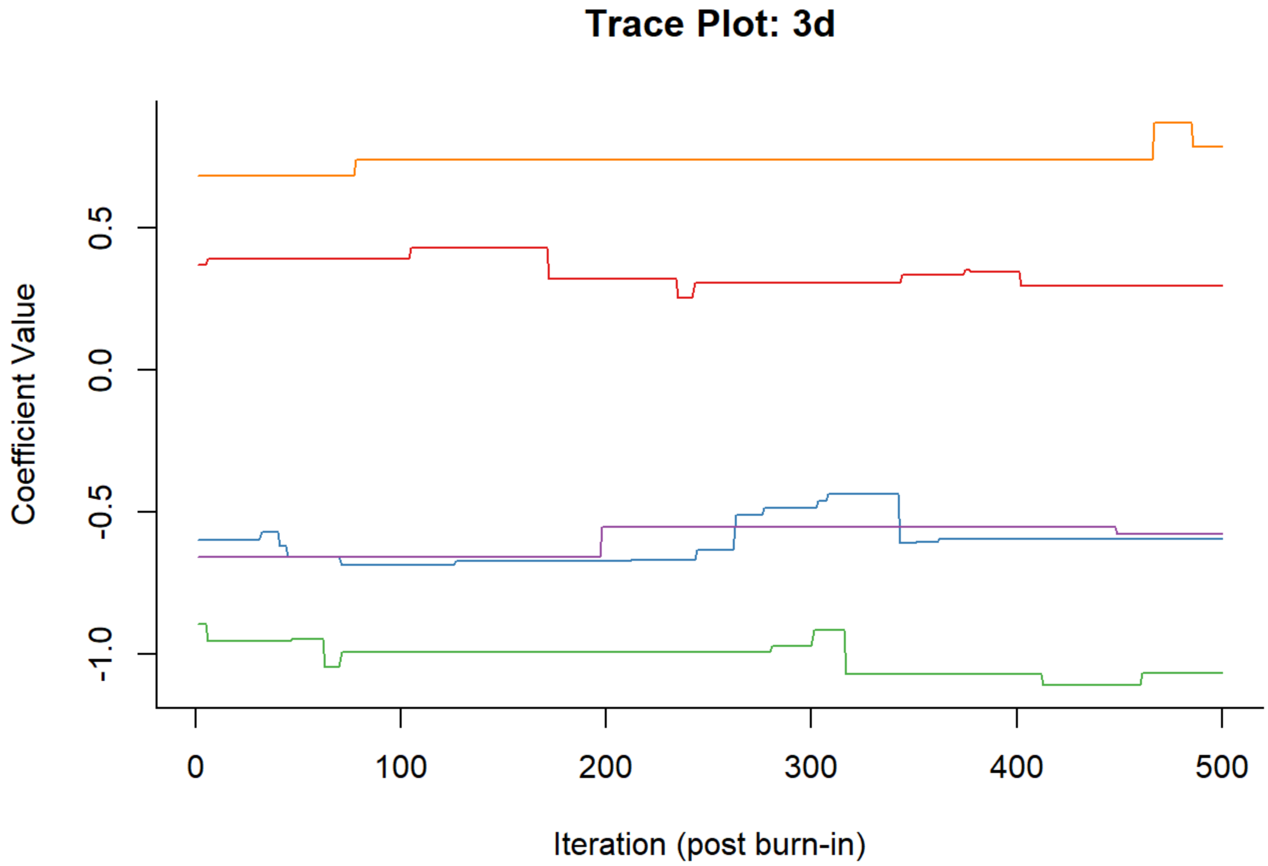
Trace Plot: half_sec

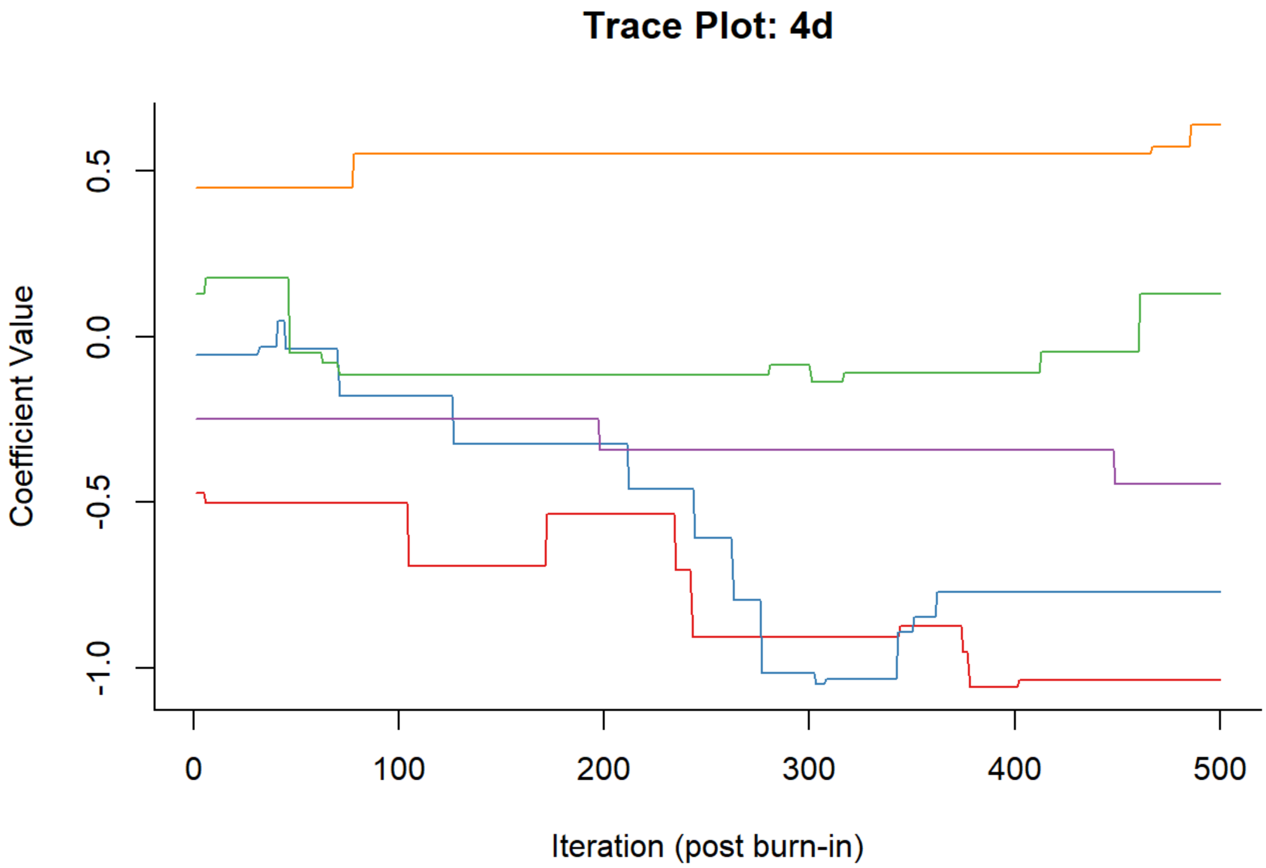


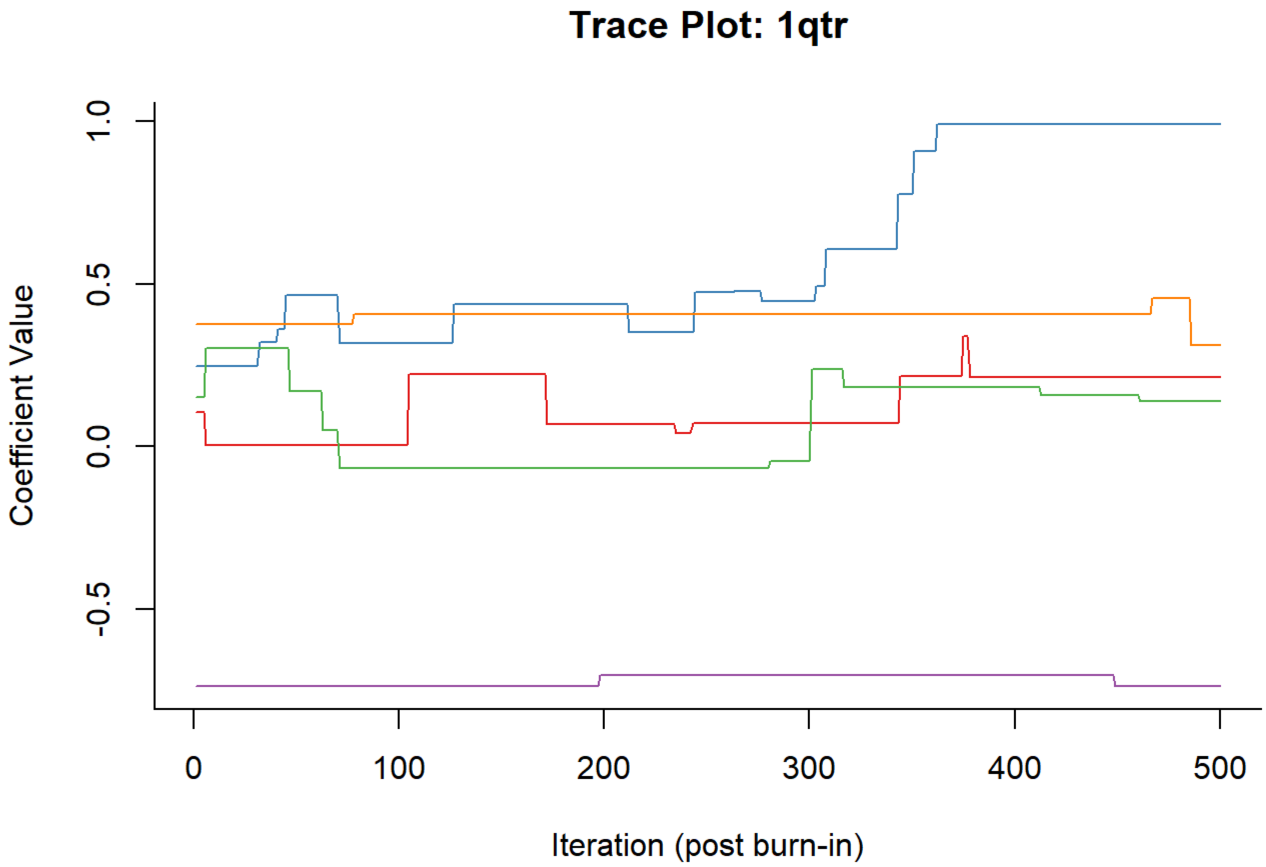

Trace Plot: game_sec
Coefficient Value
0.2
0.1
0.0
-0.1
-0.2
-0.3
0
100
200
300
400
500
Iteration (post burn-in)

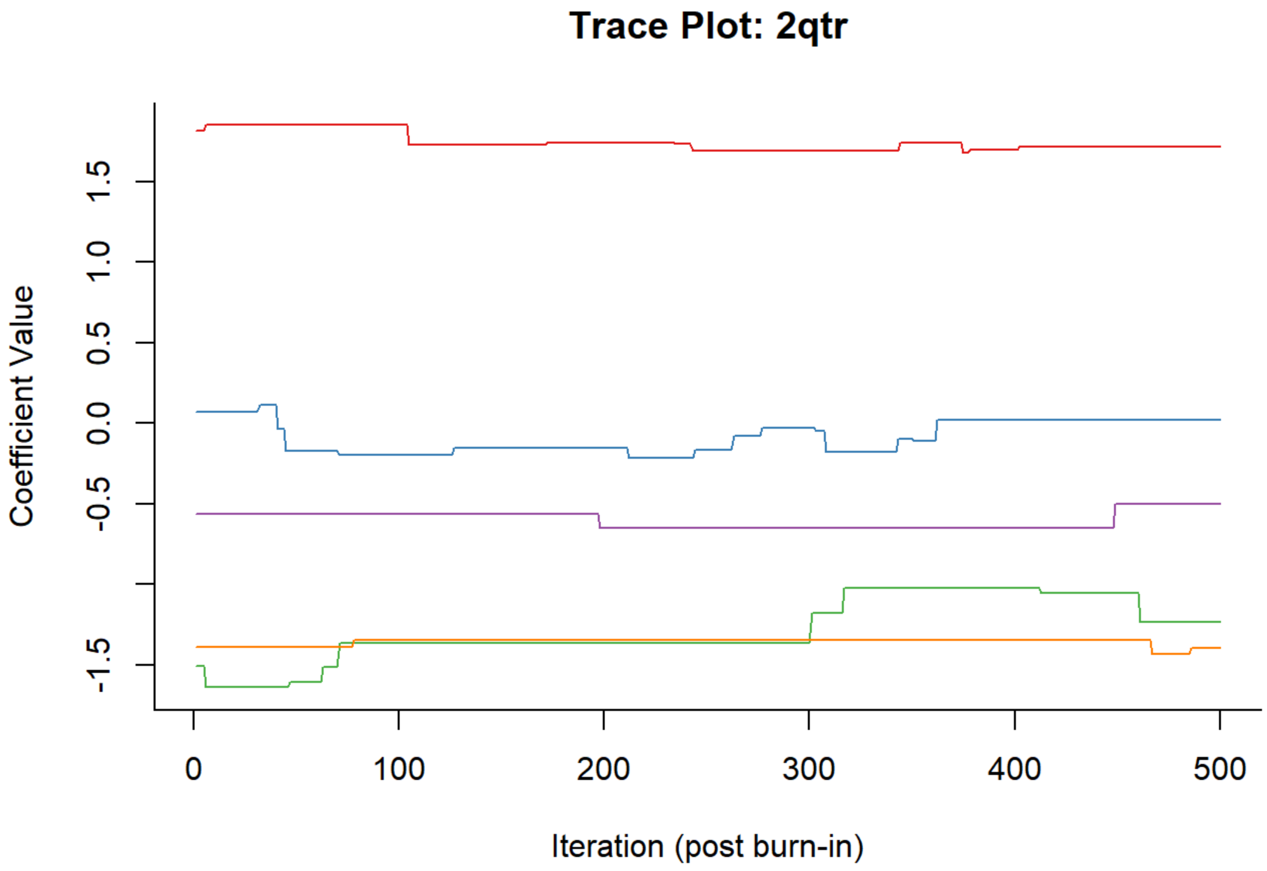

Trace Plot: 1d
Coefficient Value
-1.0
-1.5
-2.0
-2.5
0
100
200
300
400
500
Iteration (post burn-in)

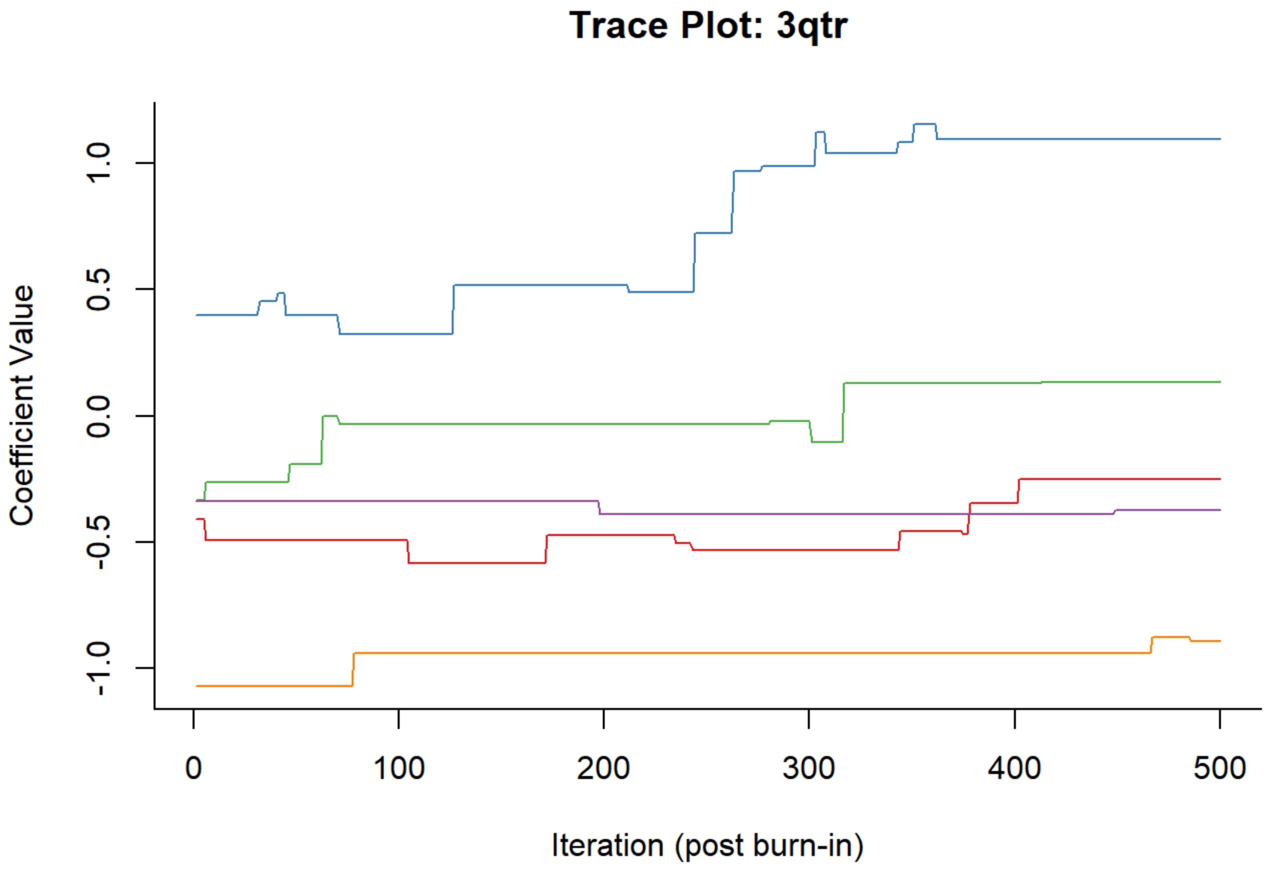

Trace Plot: 2d
Coefficient Value
-0.5
-1.0
-1.5
-2.0
-2.5
0
100
200
300
400
500
Iteration (post burn-in)

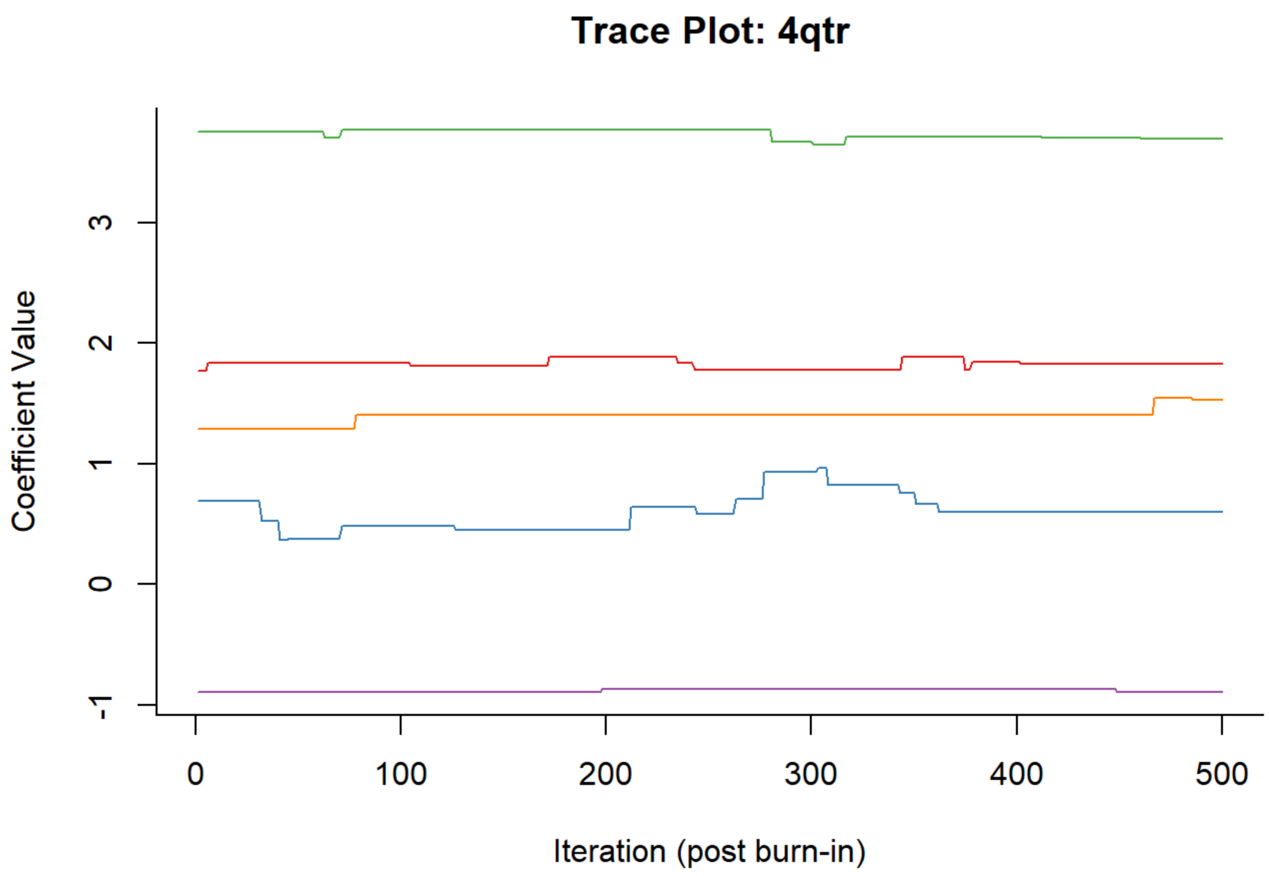

Trace Plot: 3d
Coefficient Value
0.5
0.0
-0.5
-1.0
0
100
200
300
400
500
Iteration (post burn-in)

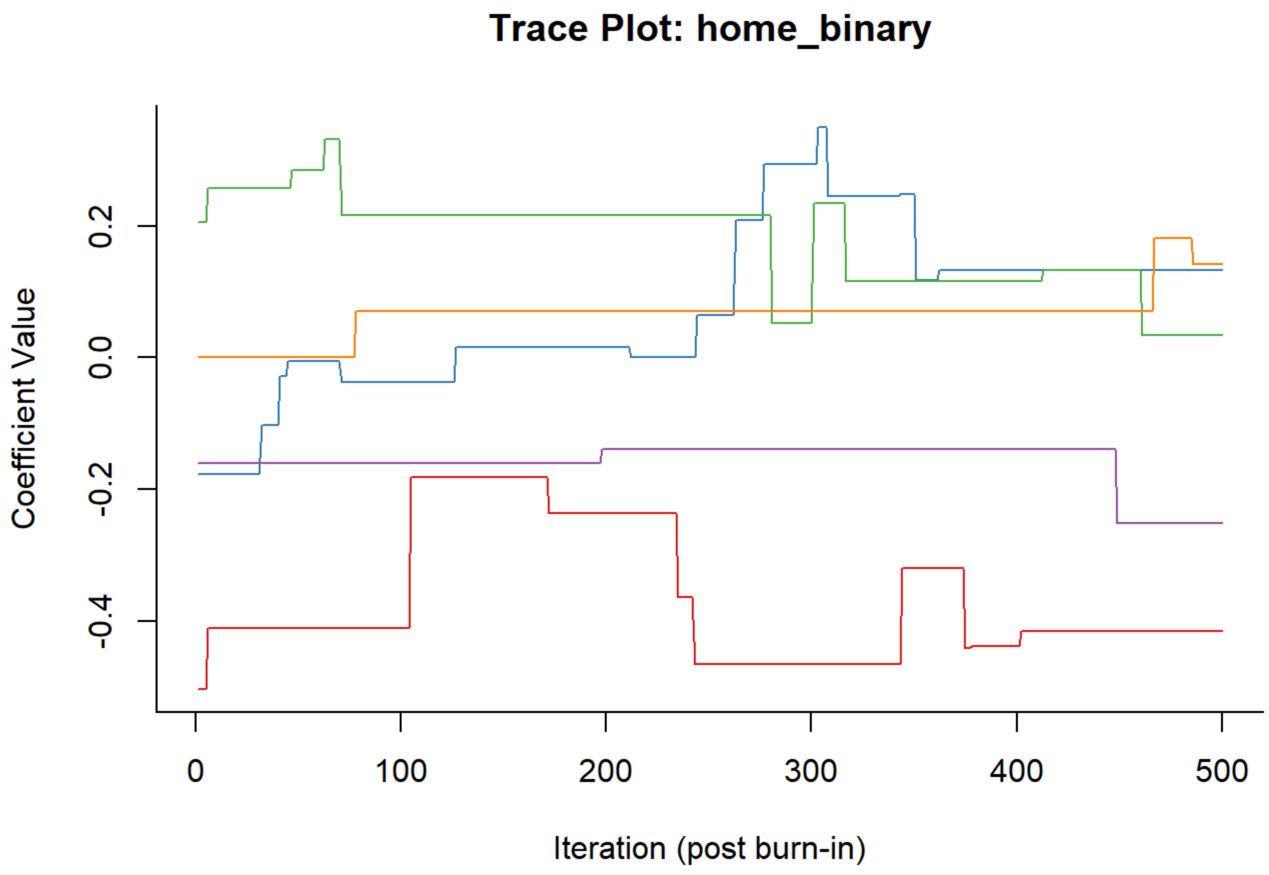

Trace Plot: 4d
Coefficient Value
0.5
0.0
-0.5
-1.0
0
100
200
300
400
500
Iteration (post burn-in)


Trace Plot: 1qtr
Coefficient Value
1.0
0.5
0.0
-0.5
0
100
200
300
400
500
Iteration (post burn-in)

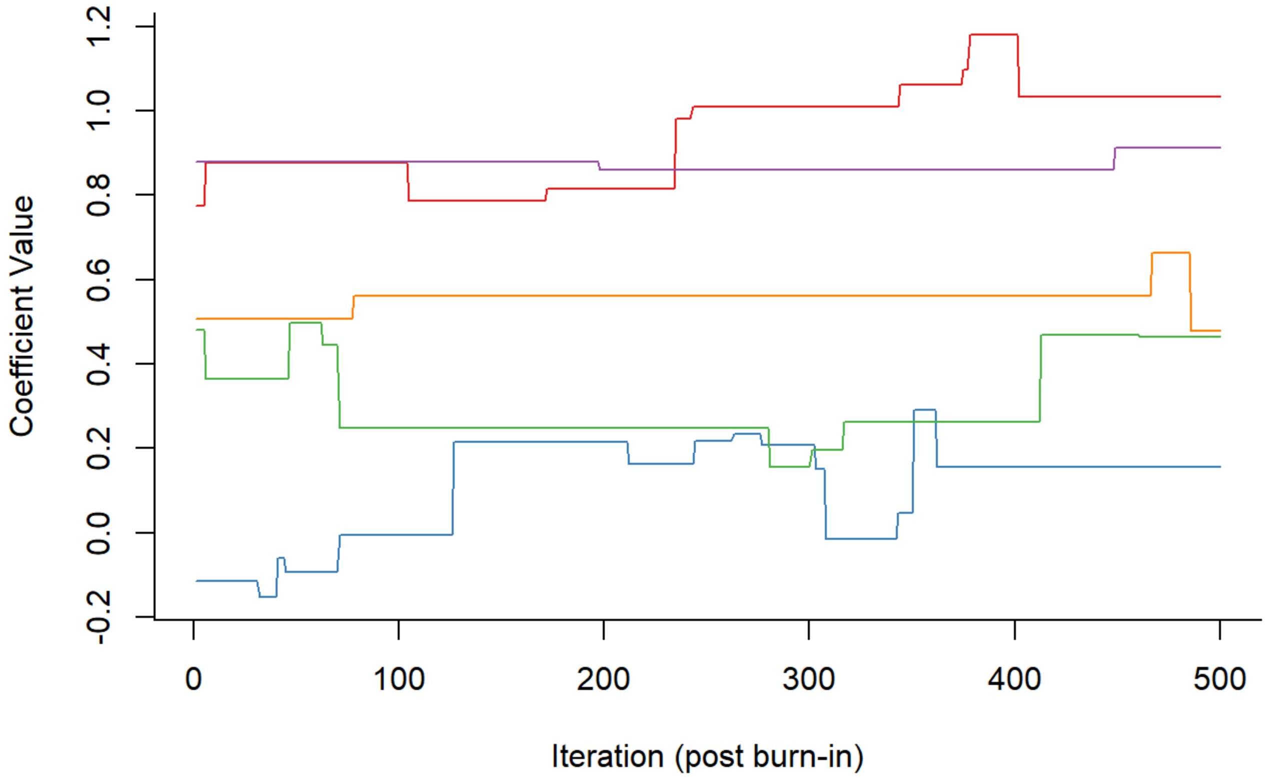

Trace Plot: 2qtr


Trace Plot: 3qtr

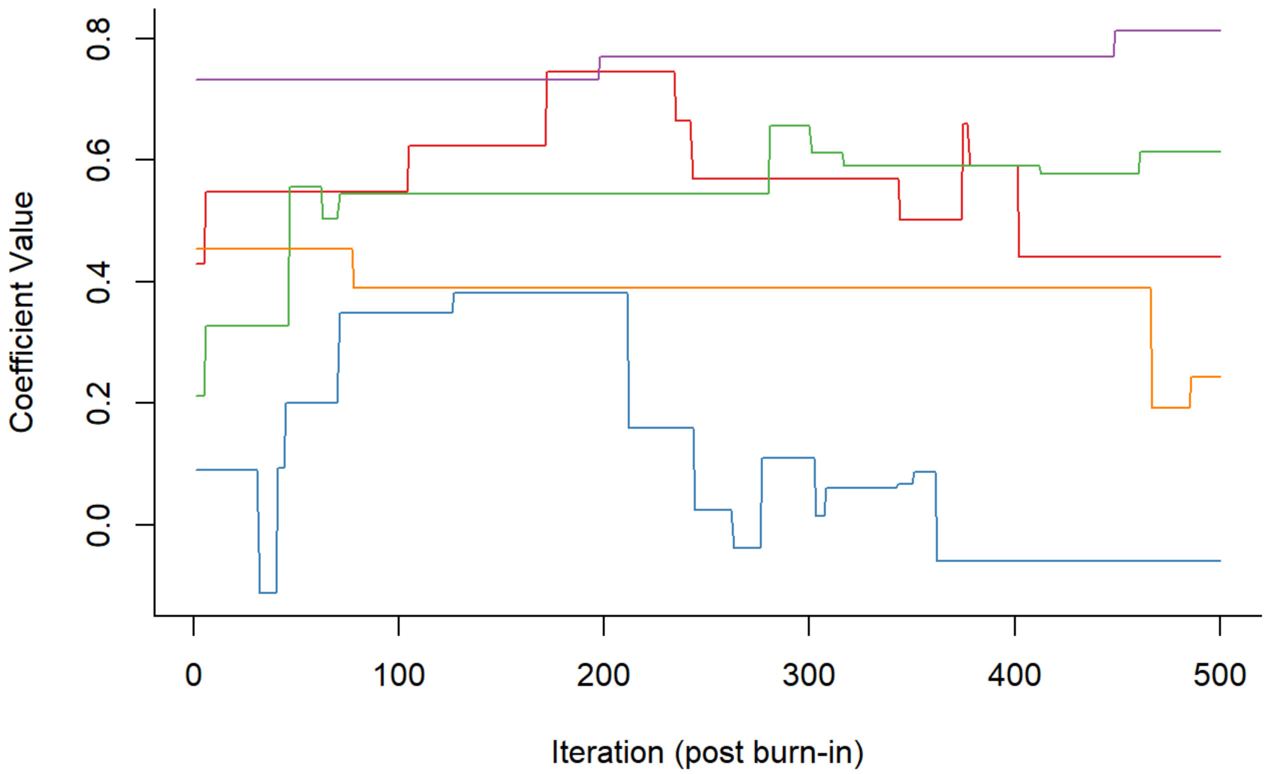

Trace Plot: 4qtr
Coefficient Value
3
2
1
0
-1
0
100
200
300
400
500
Iteration (post burn-in)


Trace Plot: home_binary
Coefficient Value
0.2
0.0
-0.2
-0.4
0
100
200
300
400
500
Iteration (post burn-in)

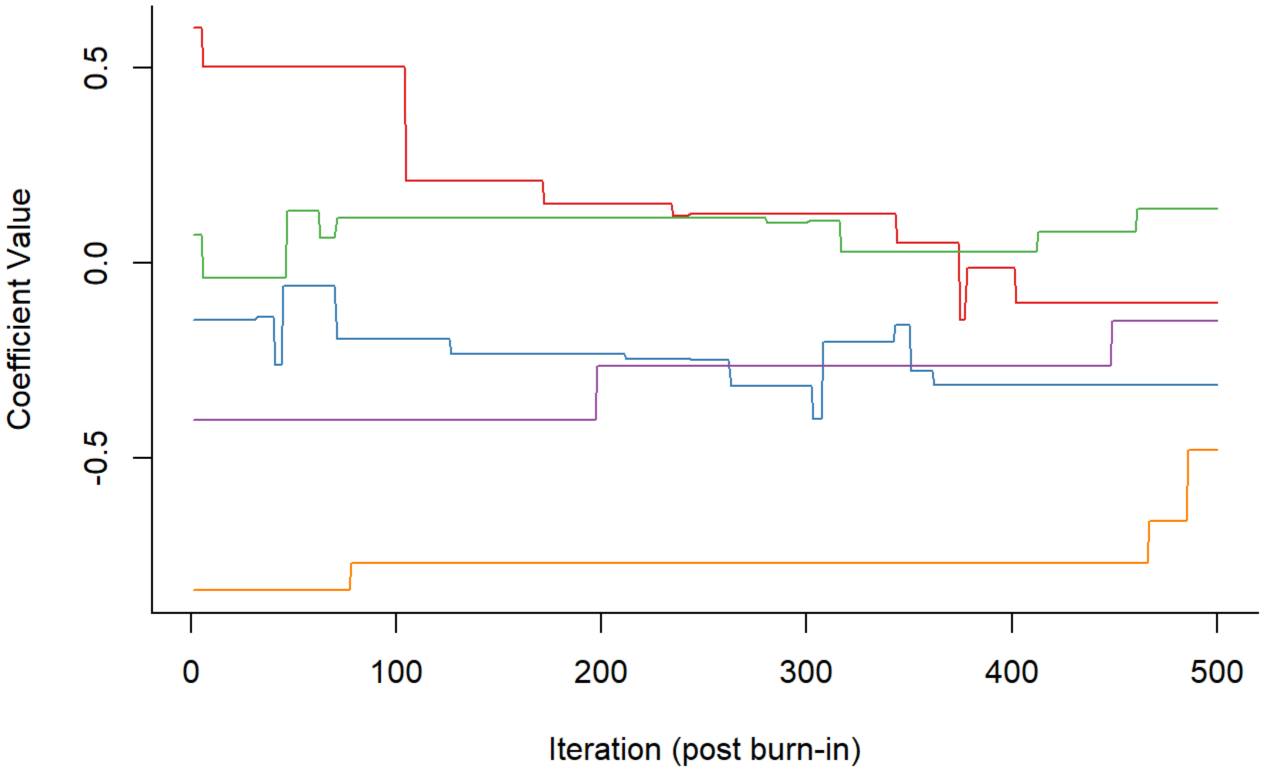
Trace Plot: prev_run



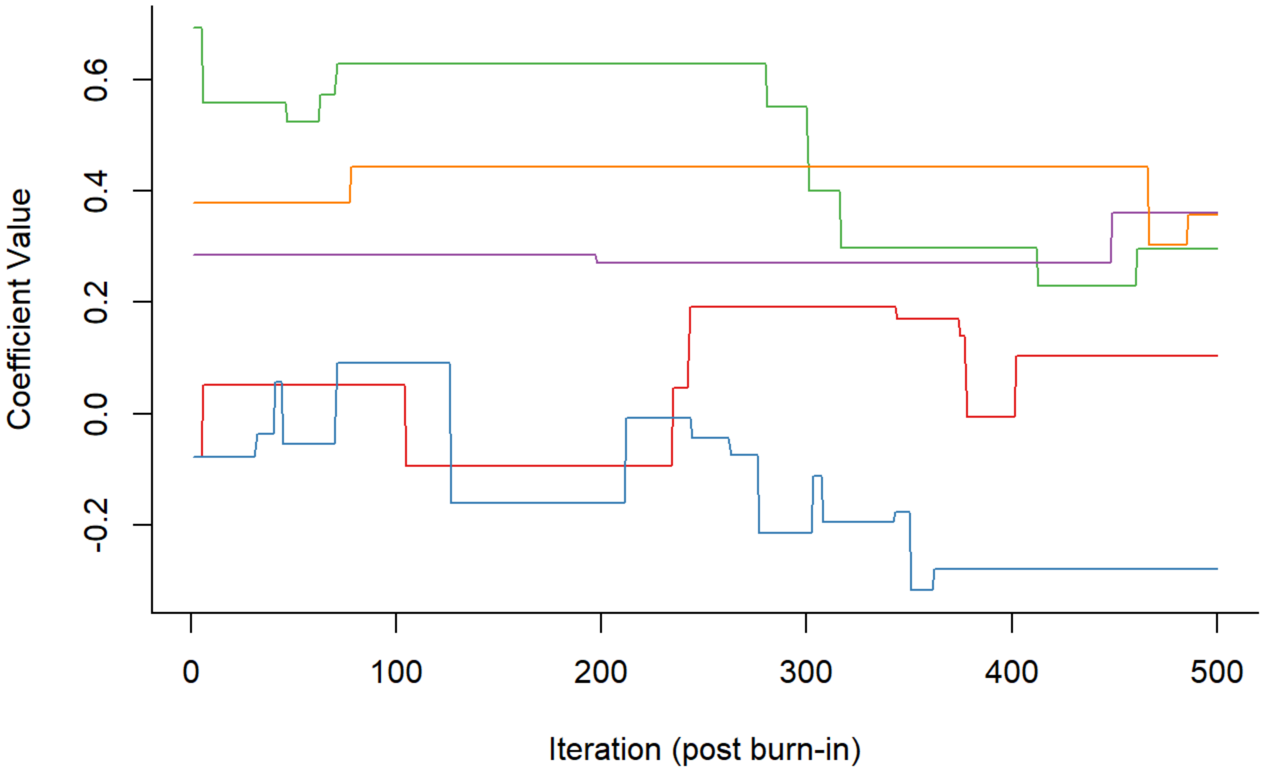
Trace Plot: prev_pass



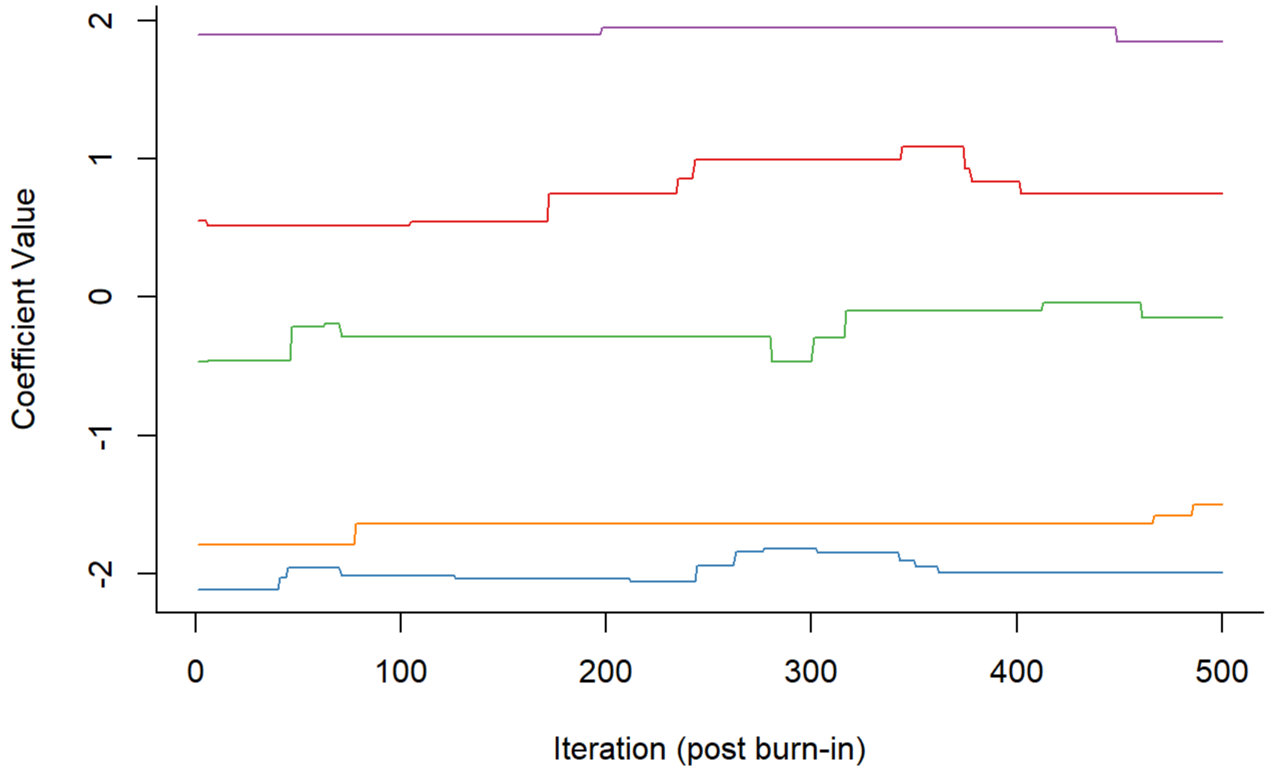
Trace Plot: prev2_run



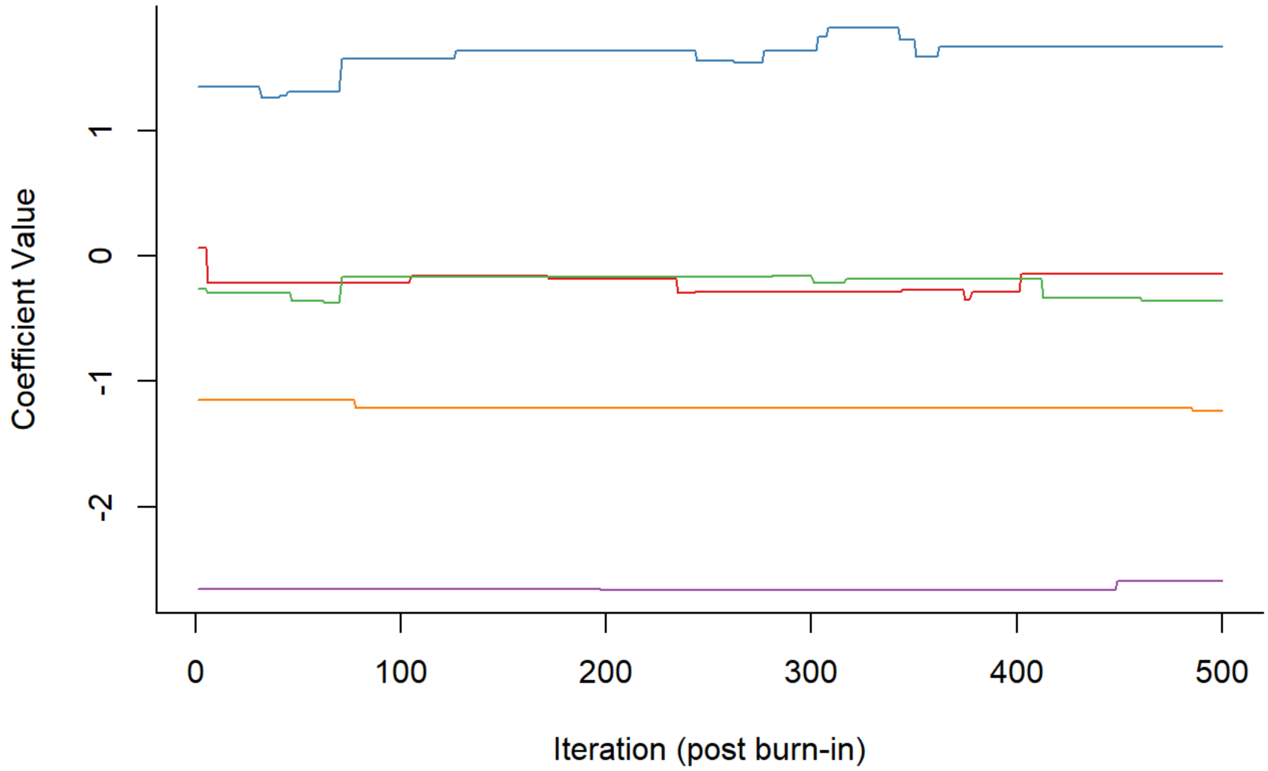
Trace Plot: prev2_pass

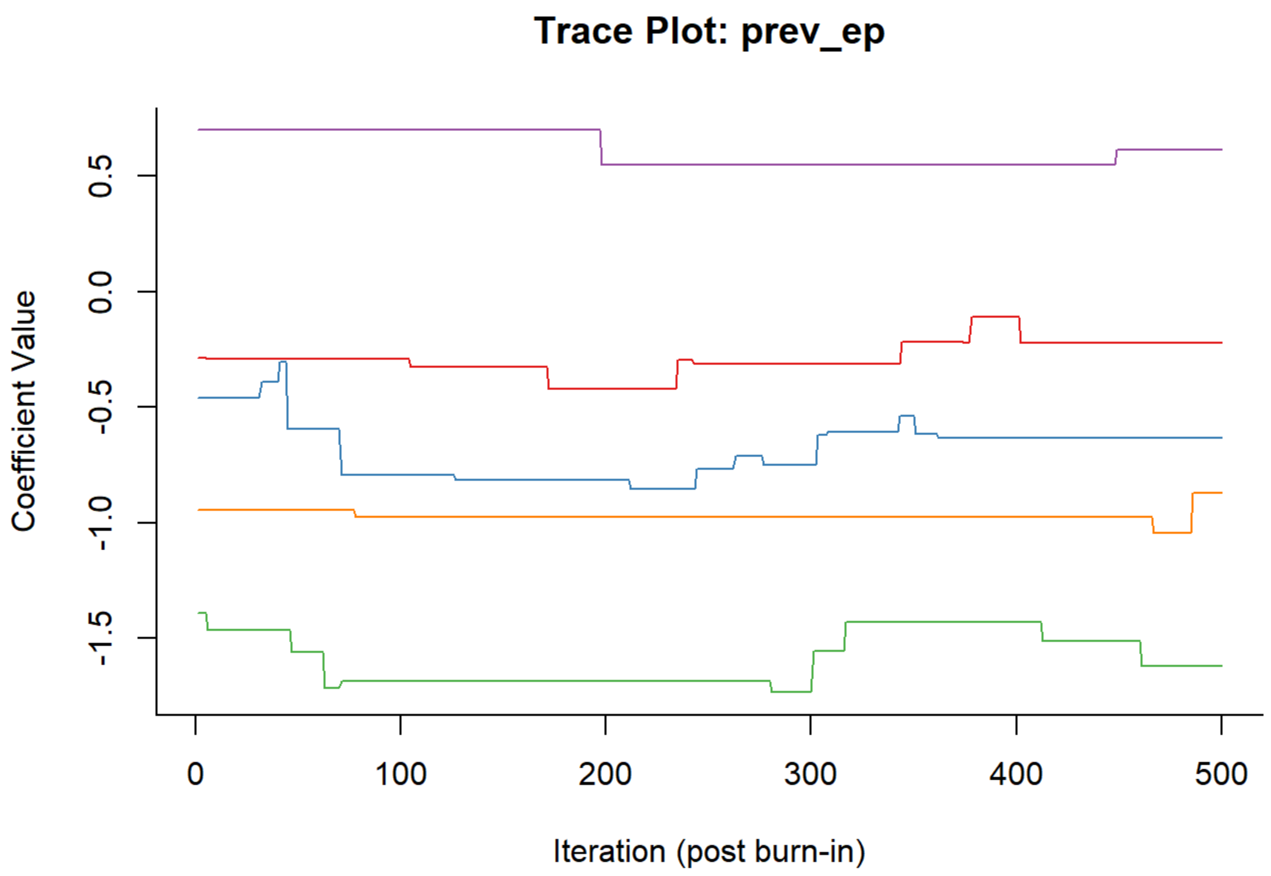


Trace Plot: prev_kickoff

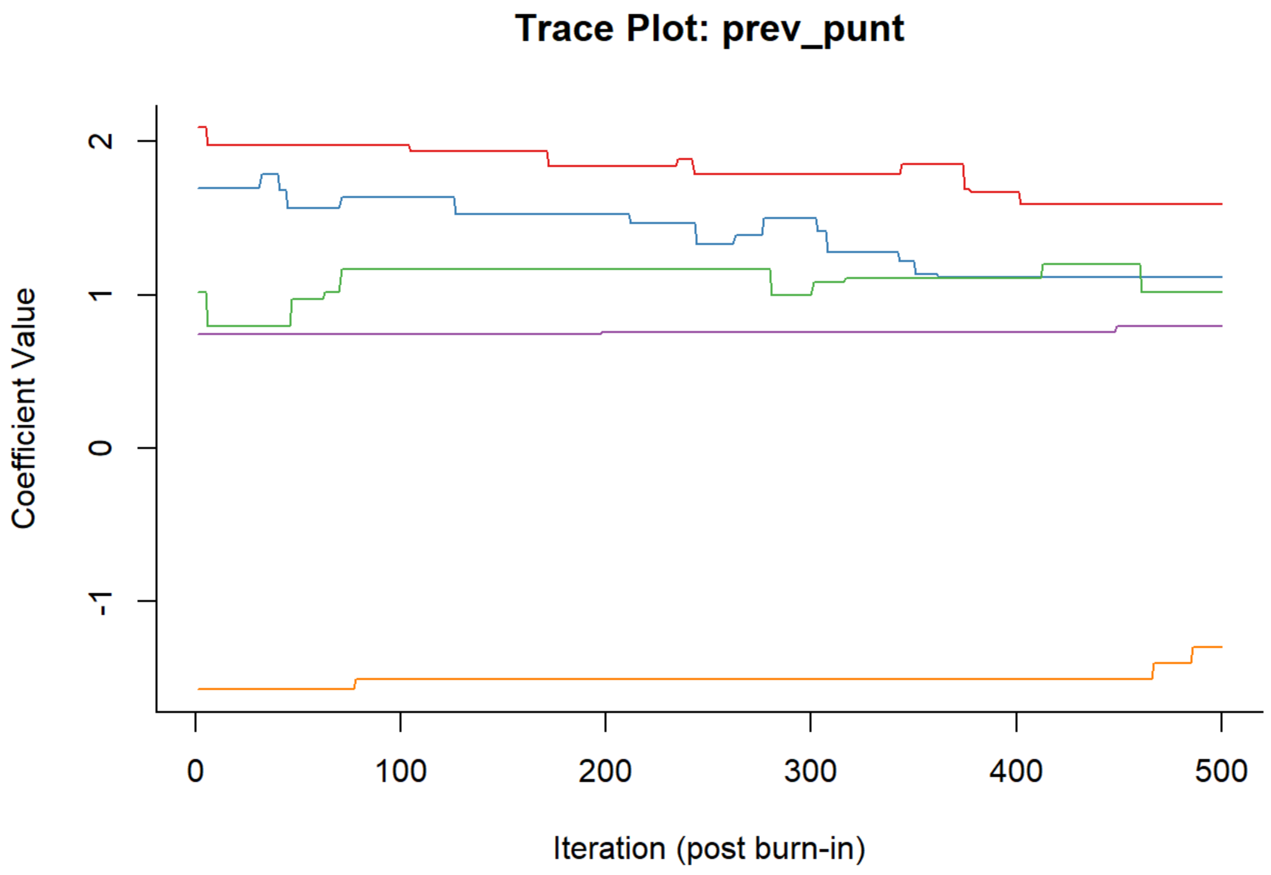


Trace Plot: prev_qb_ks

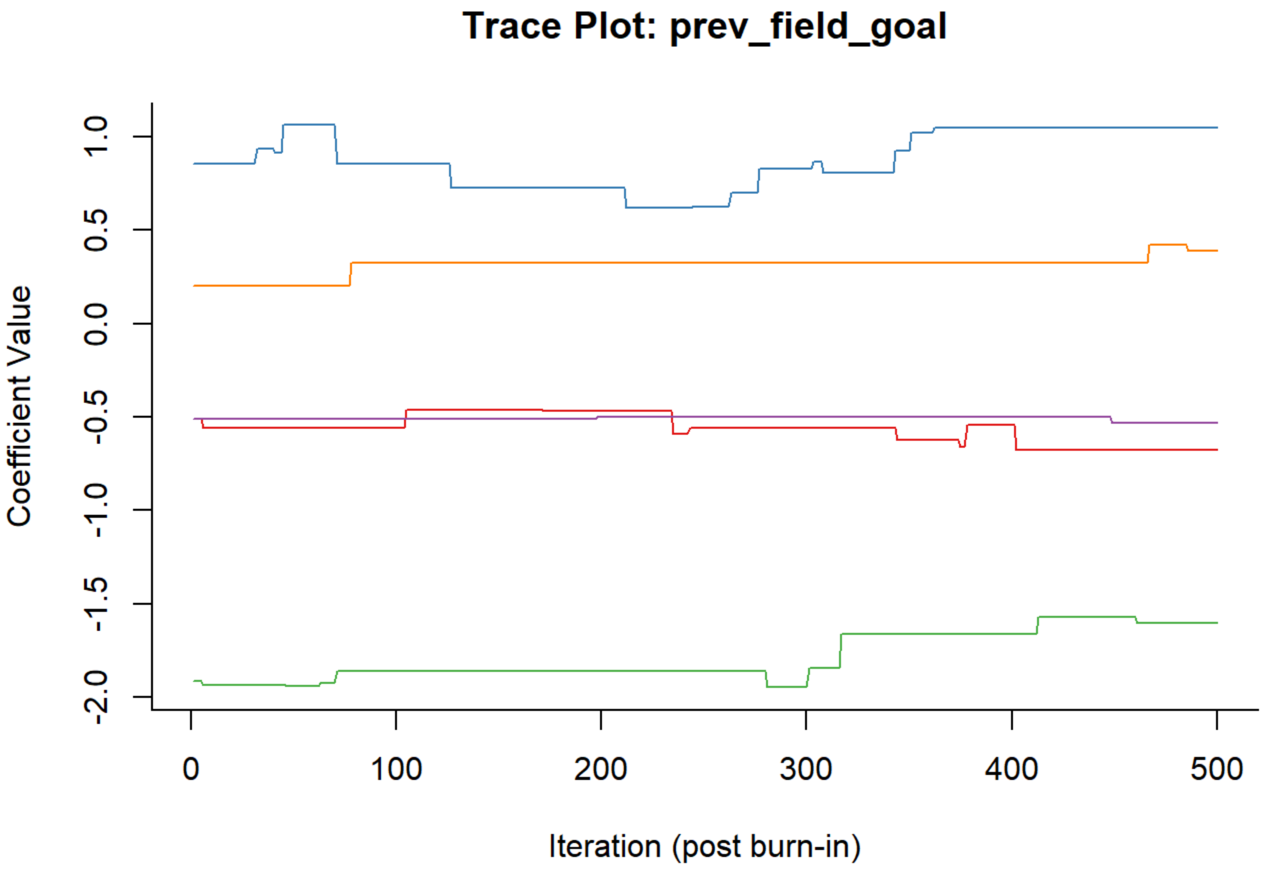




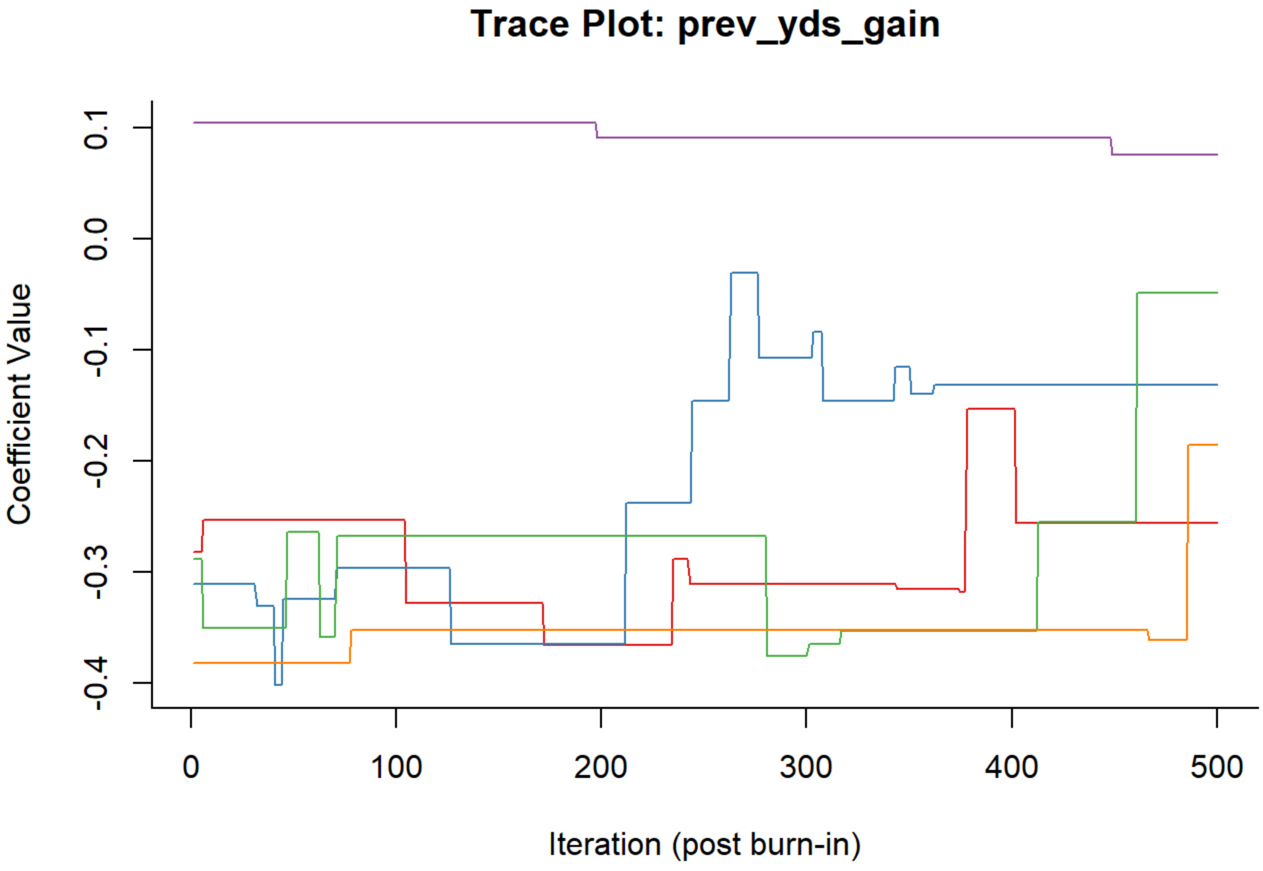
Trace Plot: prev_ep



Trace Plot: prev_punt

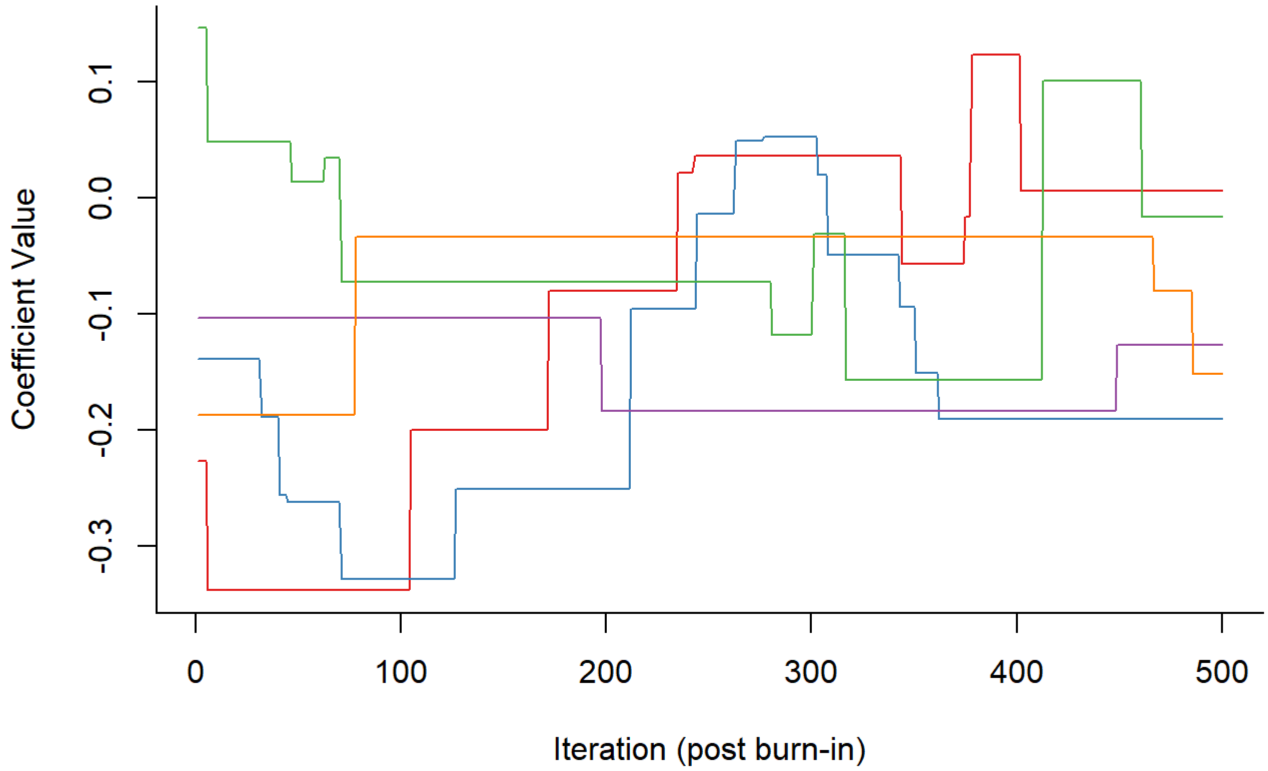


Trace Plot: prev_field_goal

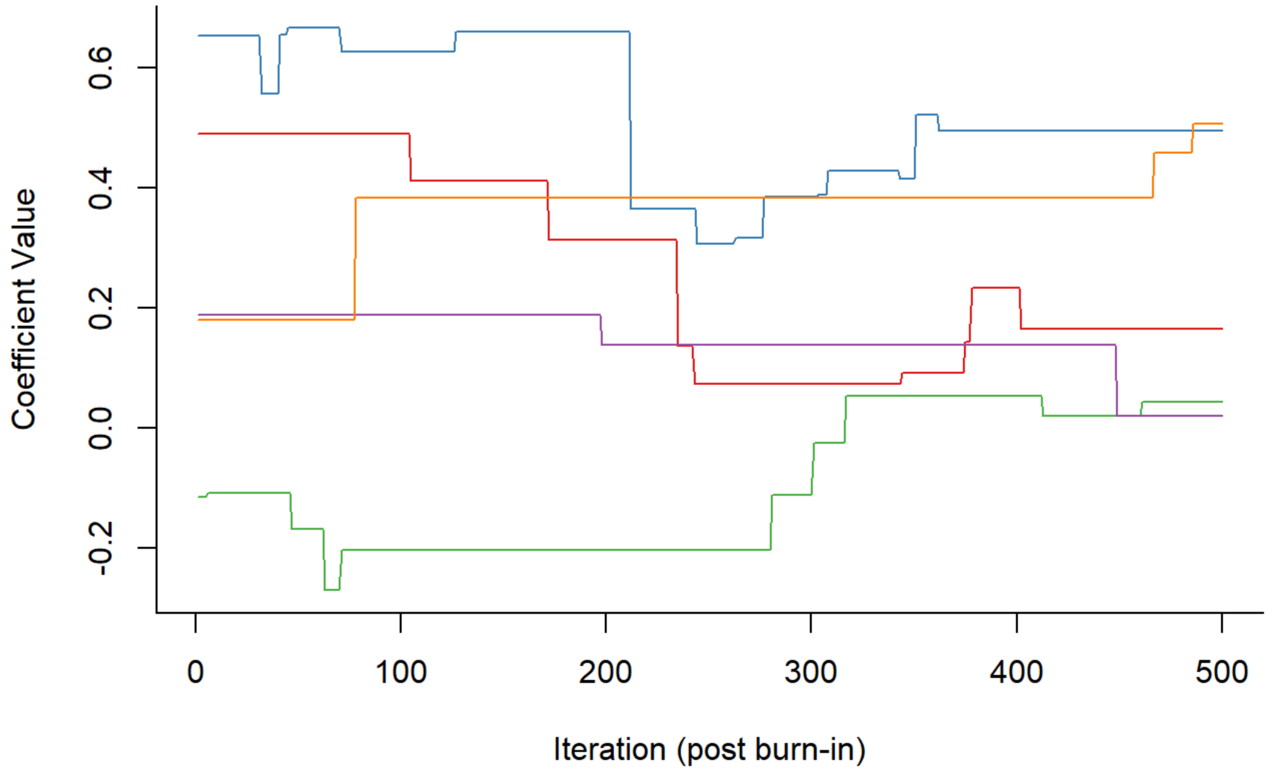


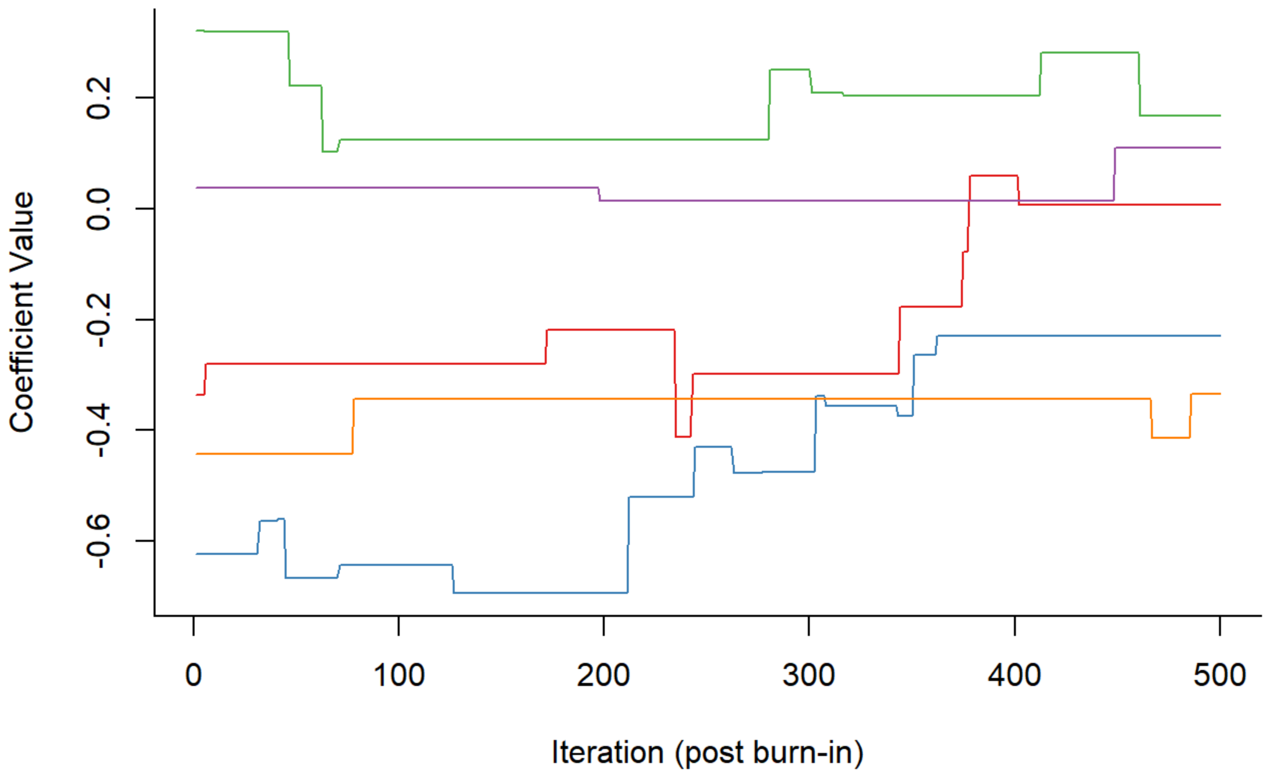
Trace Plot: prev_yds_gain

Trace Plot: ig_pass_pct

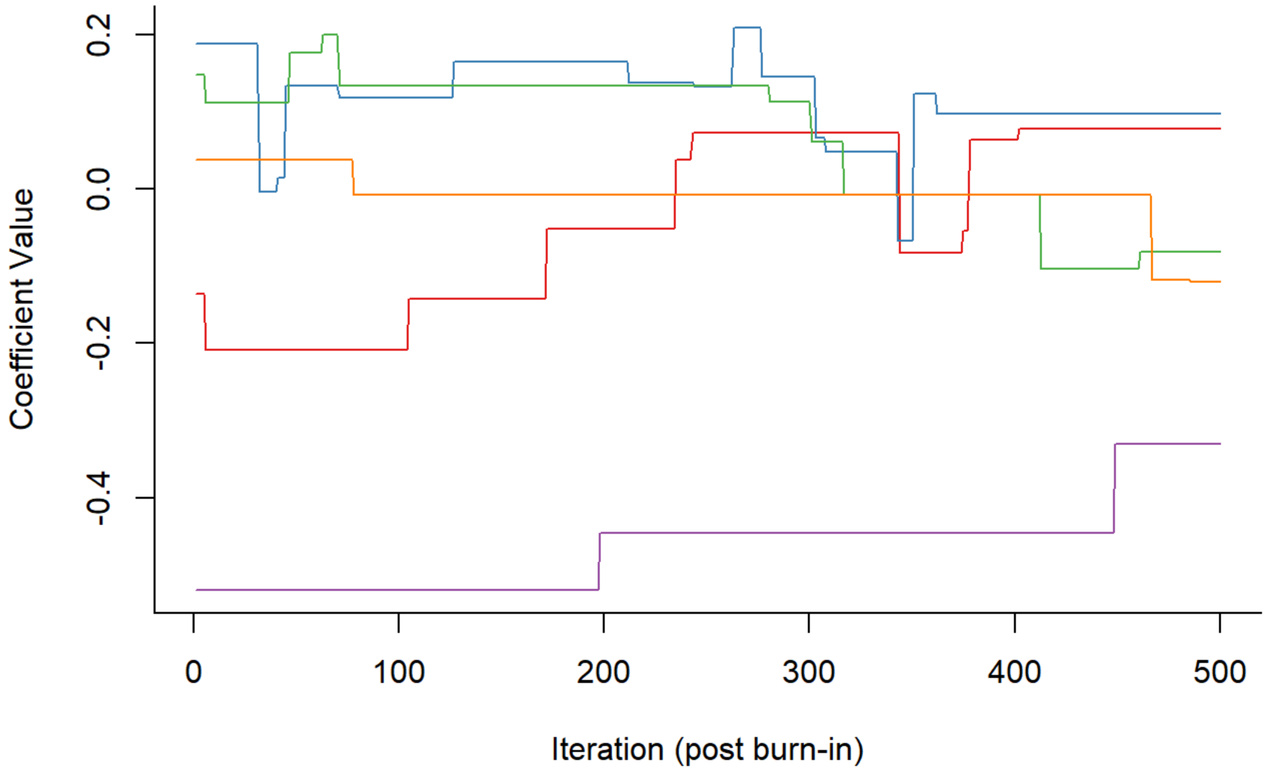


Trace Plot: ig_pass_suc_pct



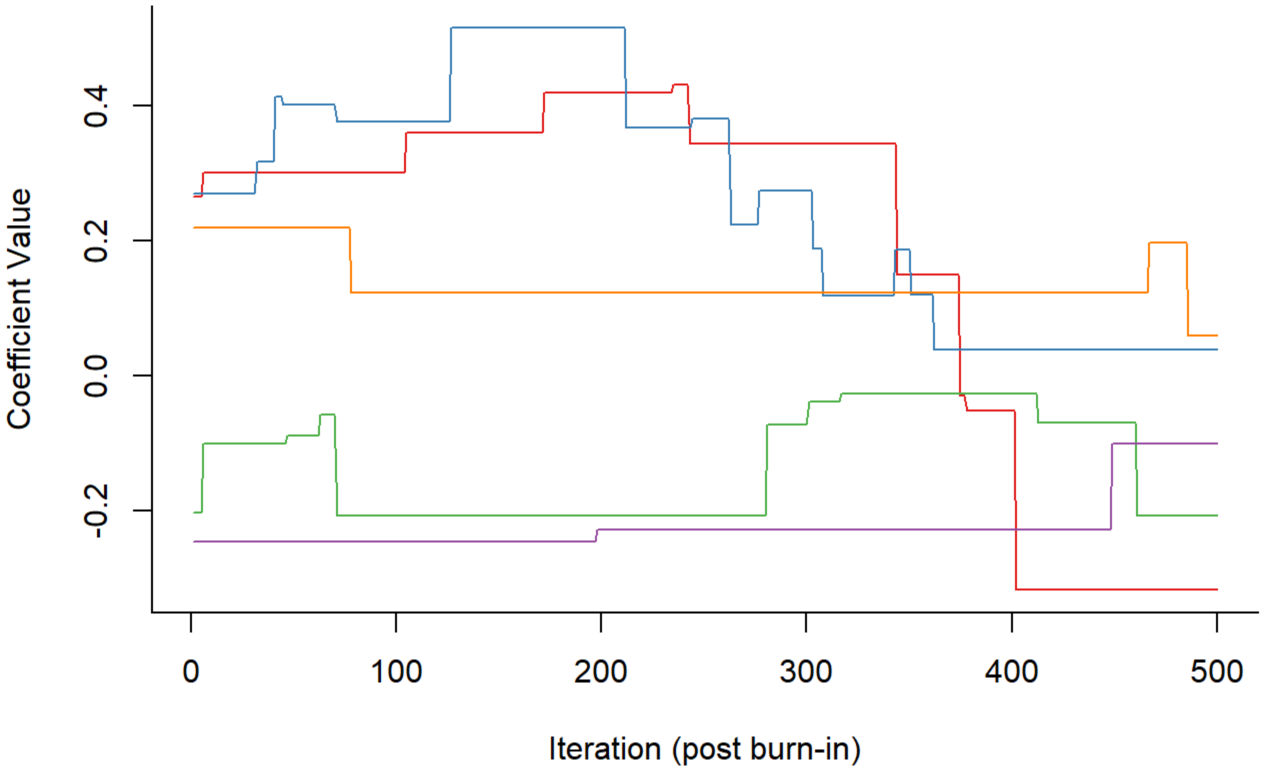


Trace Plot: ig_avg_pass_yds

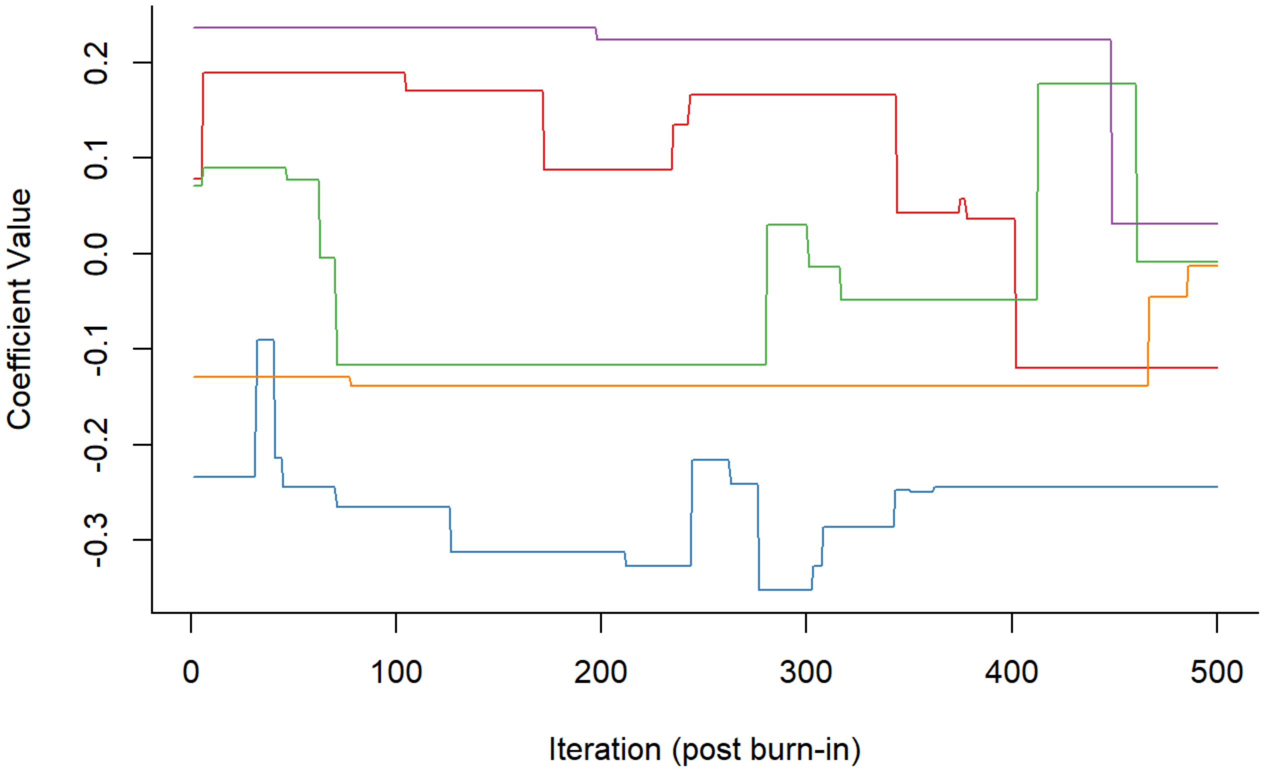


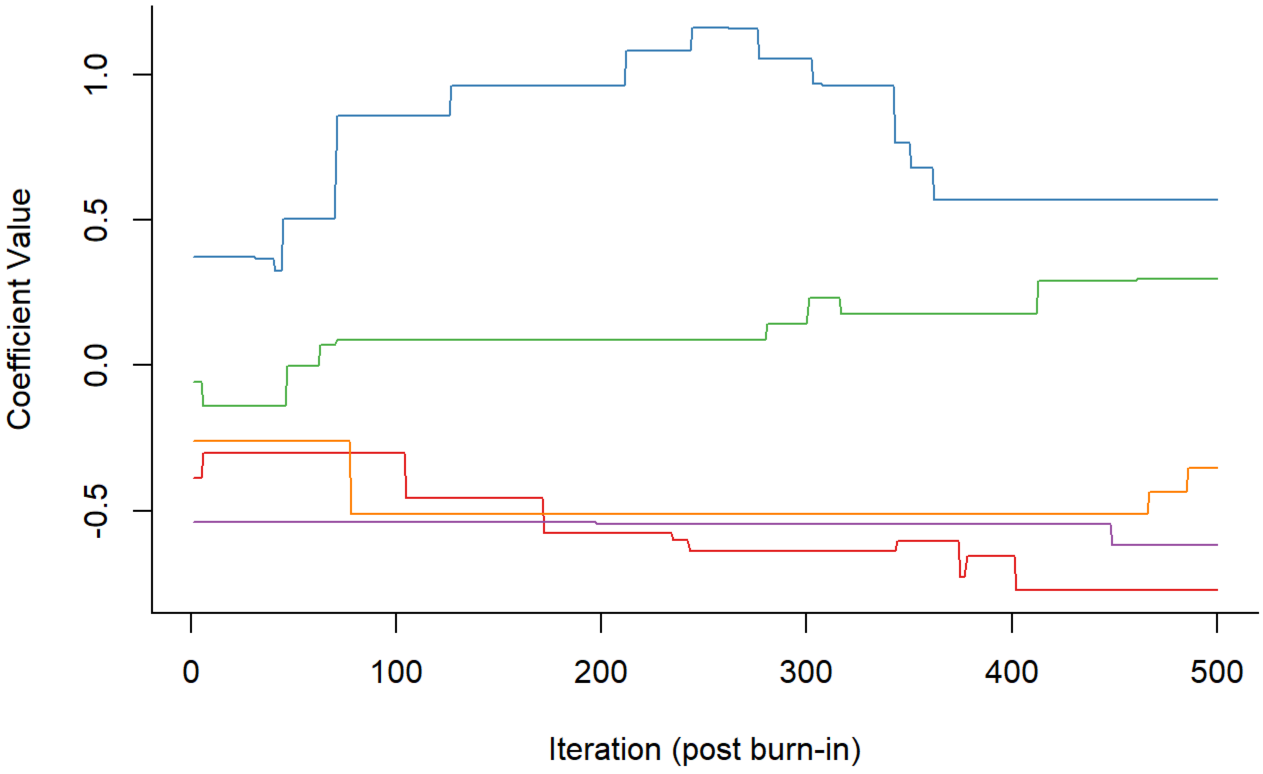
Trace Plot: ig_avg_run_yds

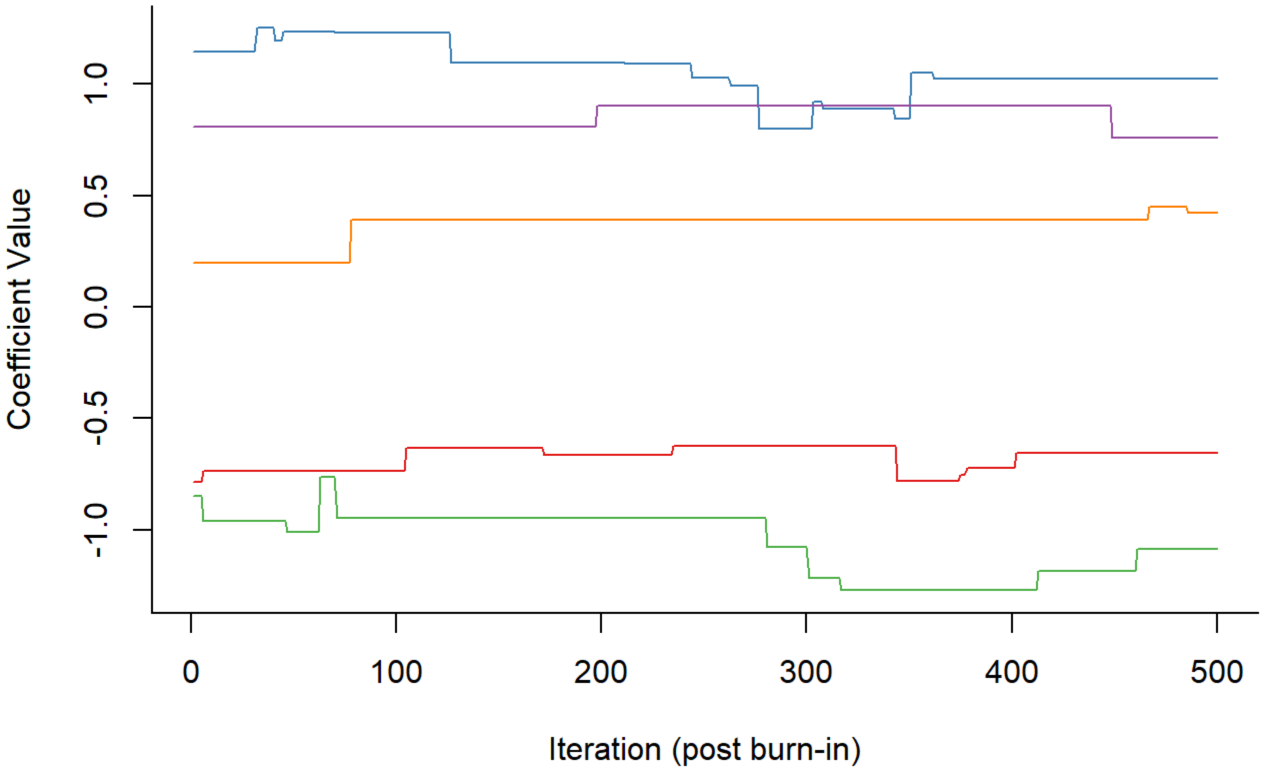
Trace Plot: pass_sd

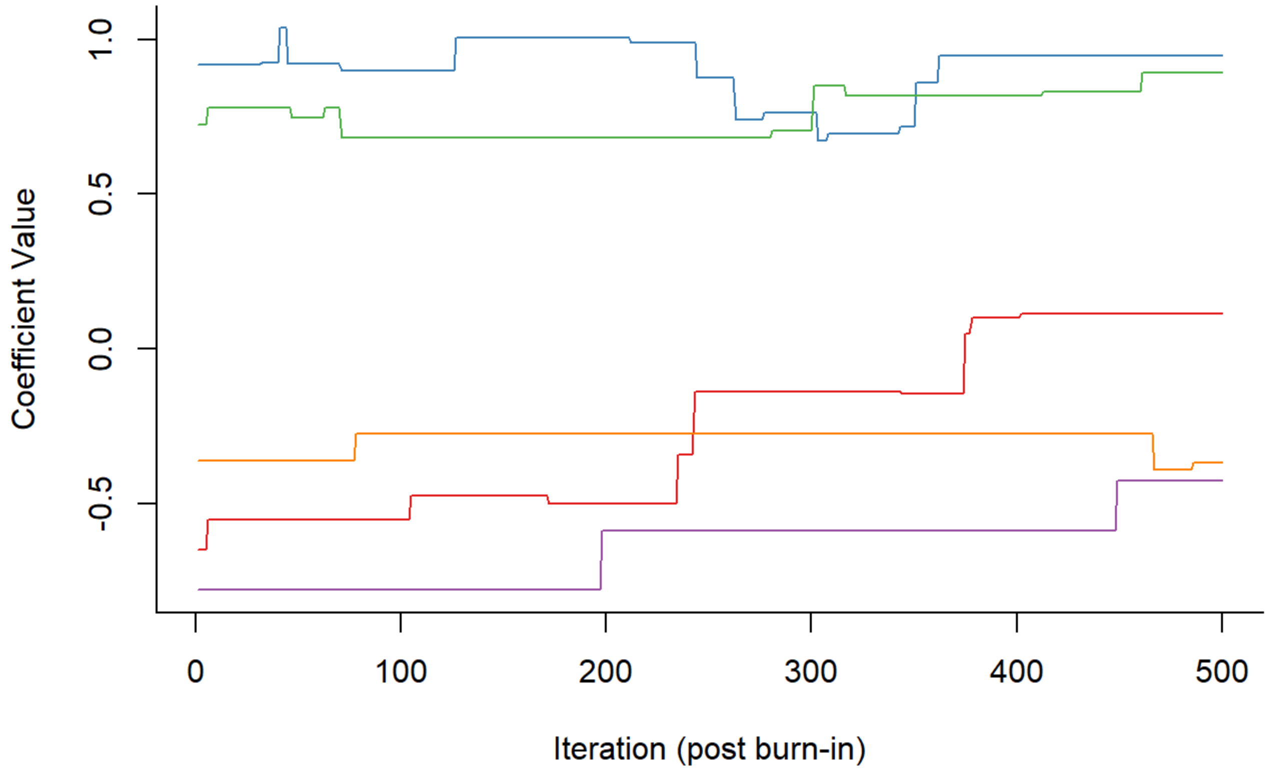


Trace Plot: run_sd

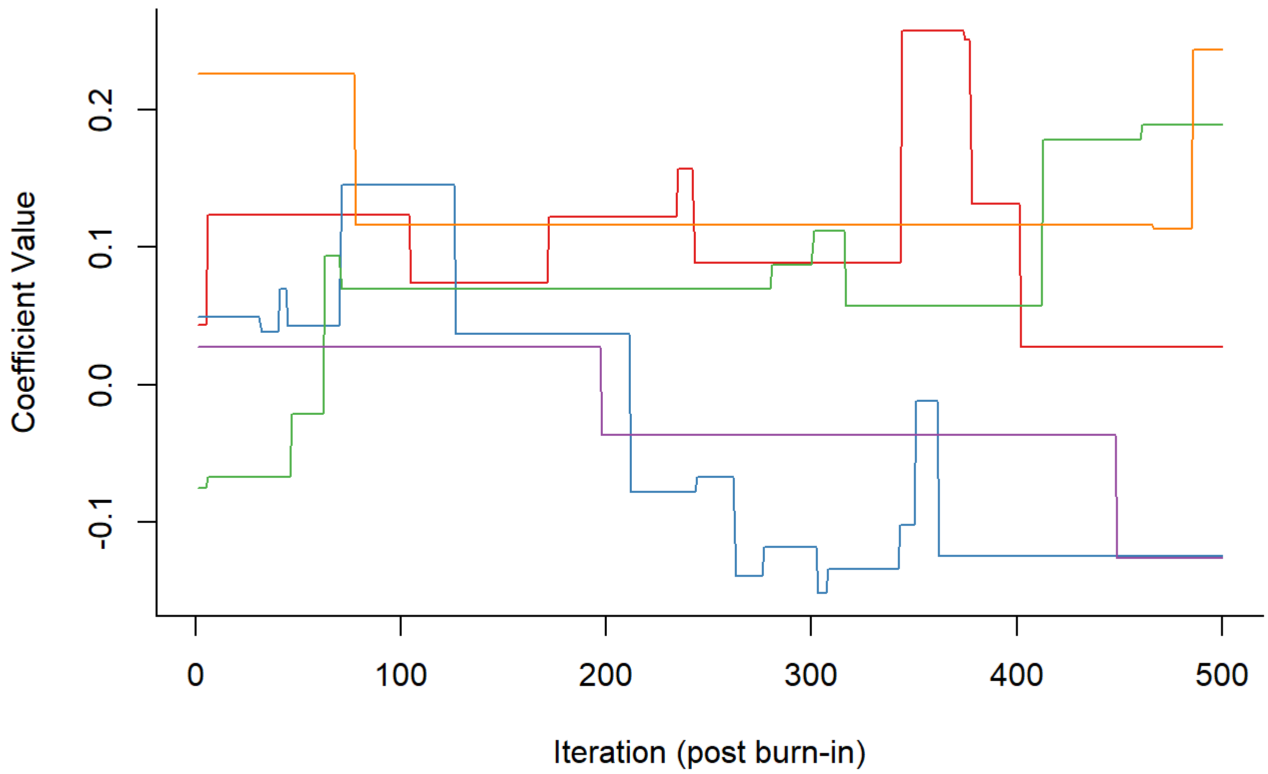



Trace Plot: fieldgoalrange

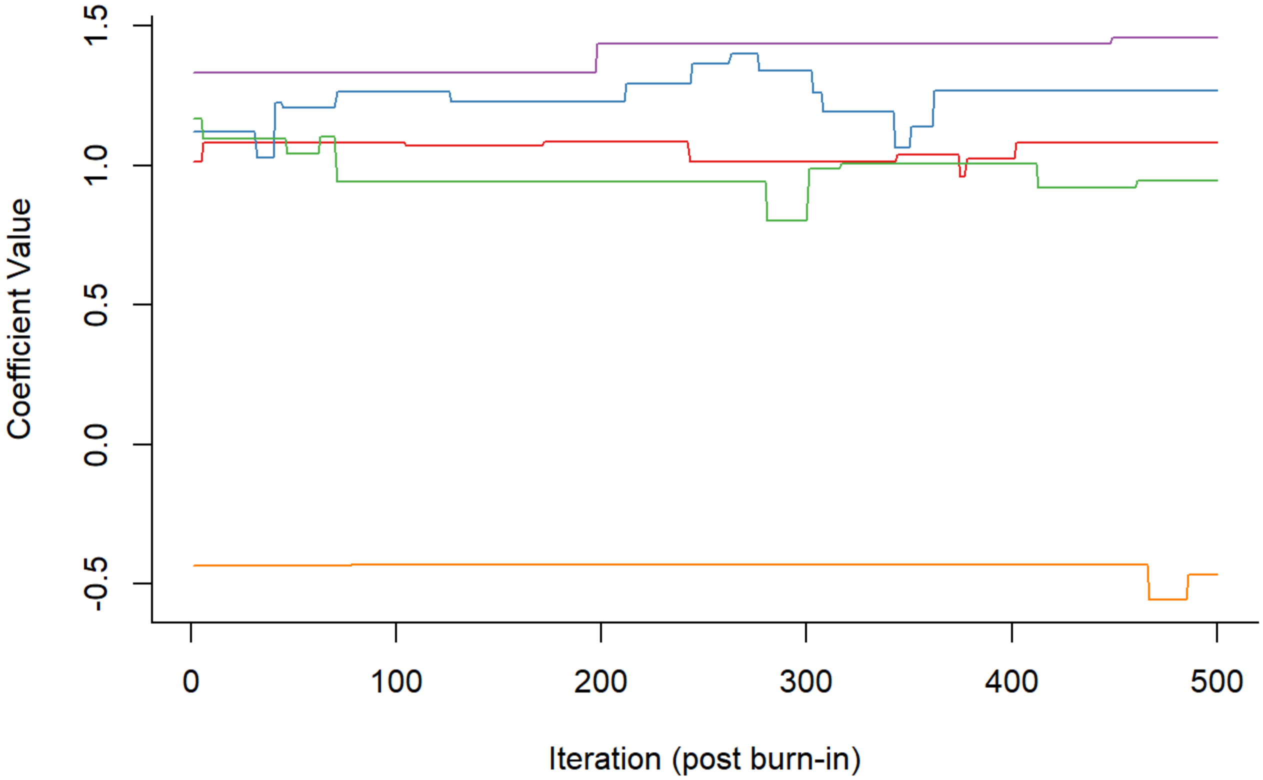

Trace Plot: redzone



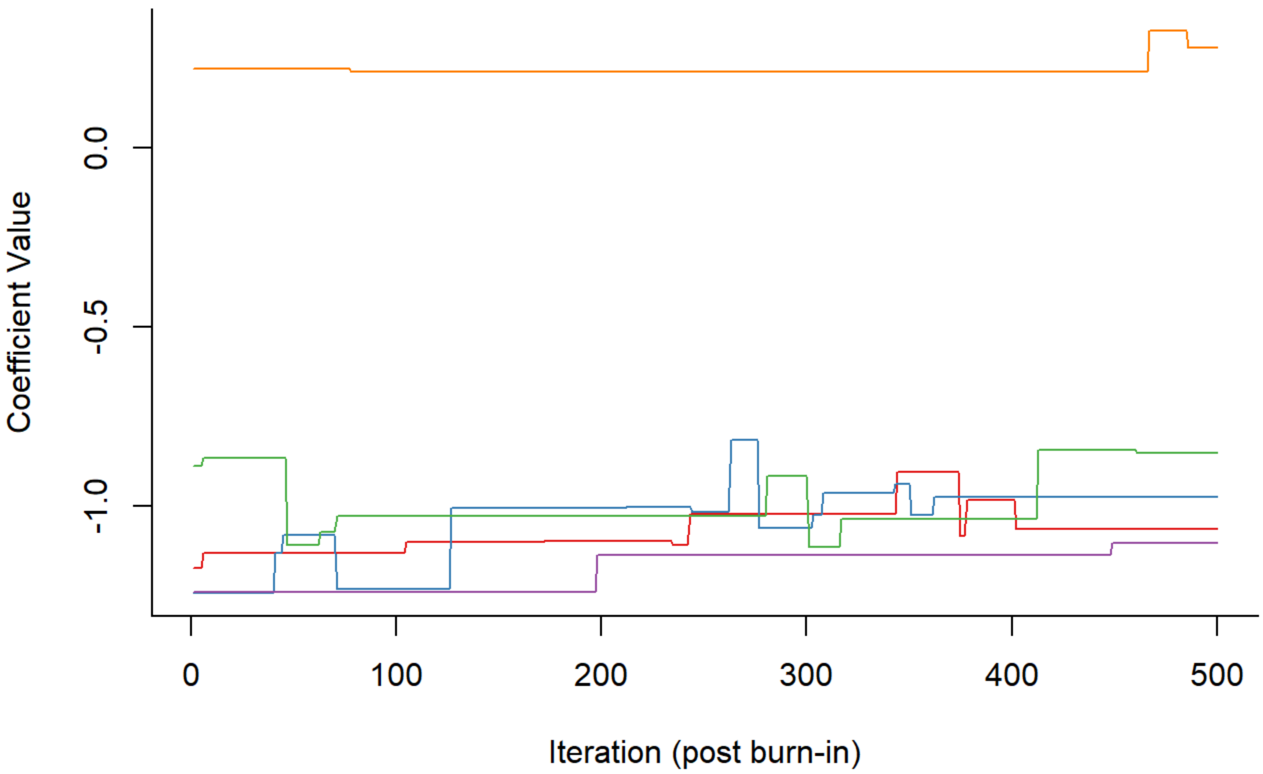
Trace Plot: lt_3min_half



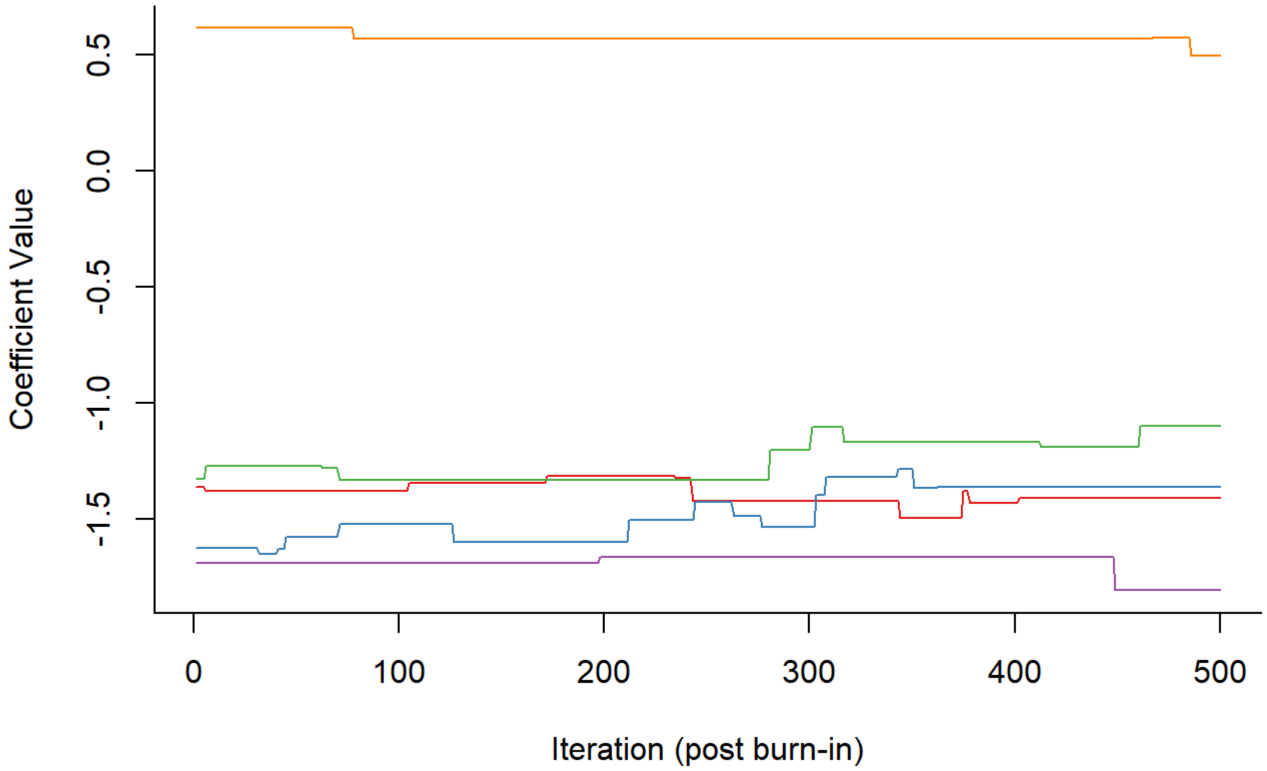
Trace Plot: ratingQB



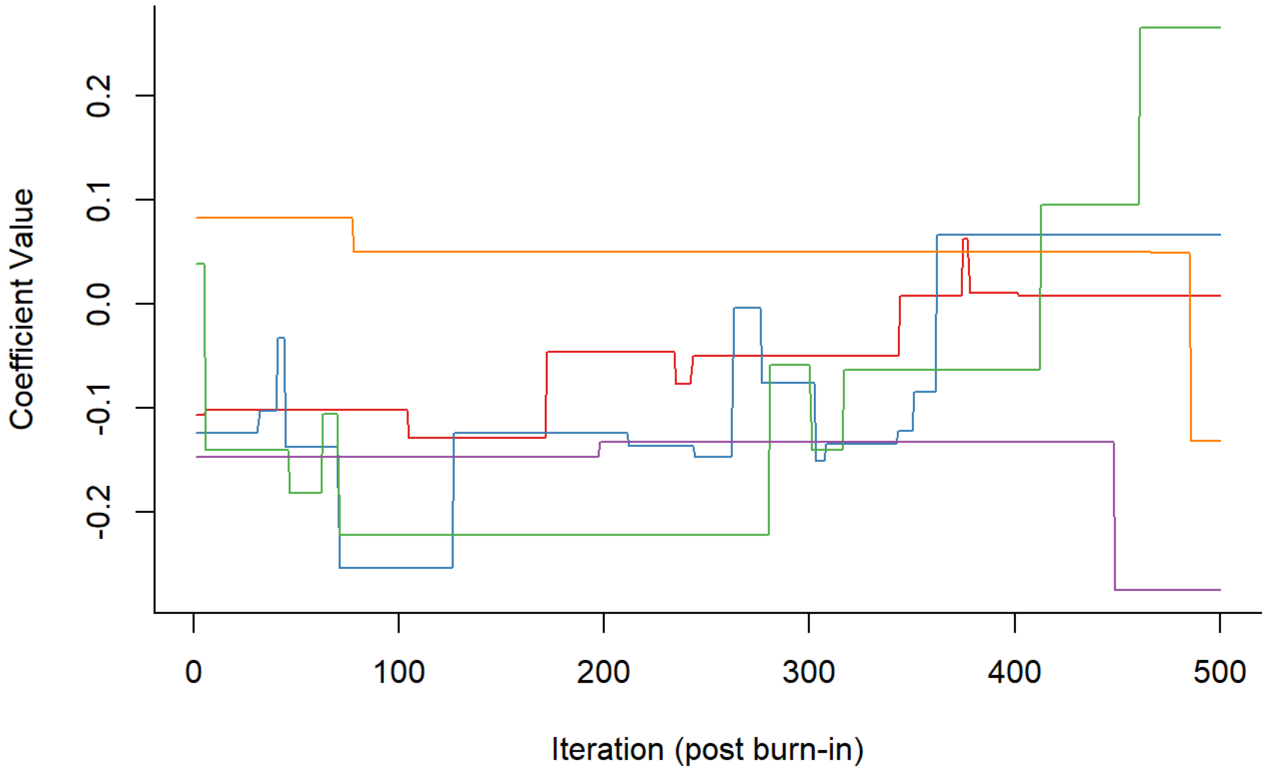
Trace Plot: ratingWR



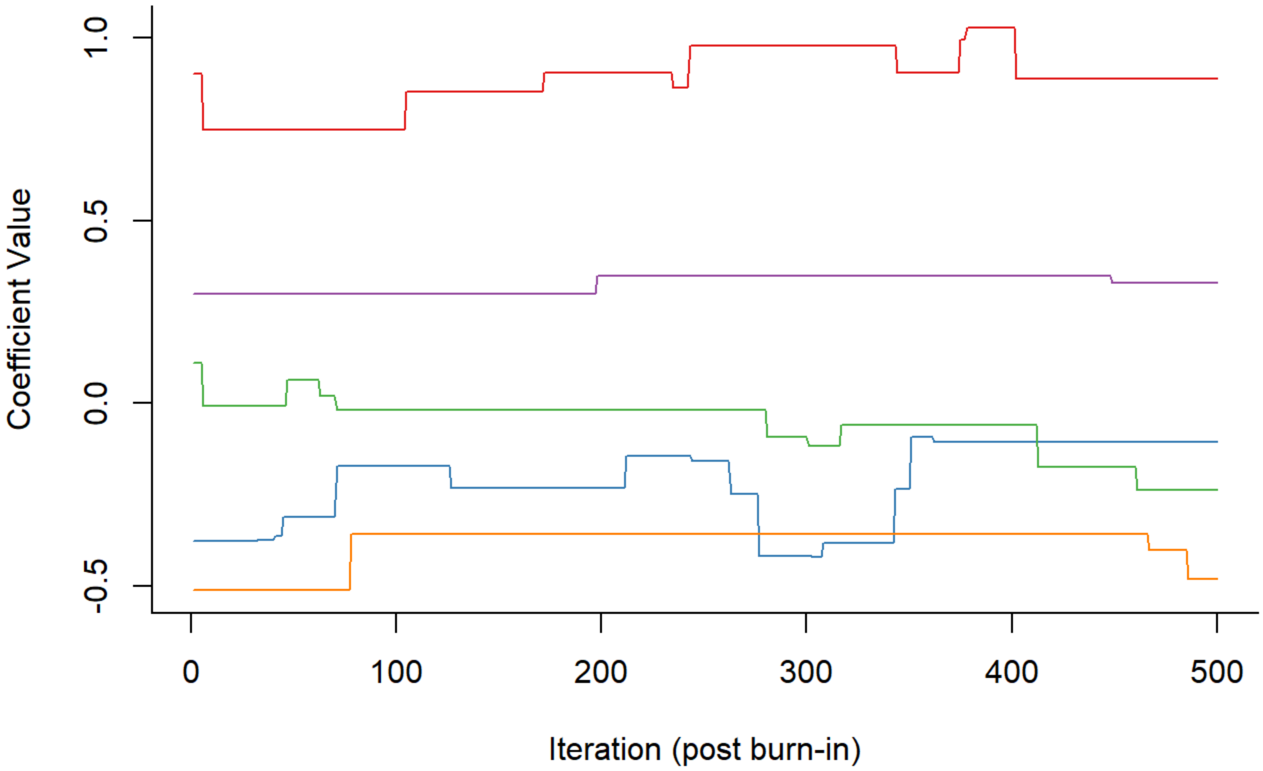
Trace Plot: ratingRB



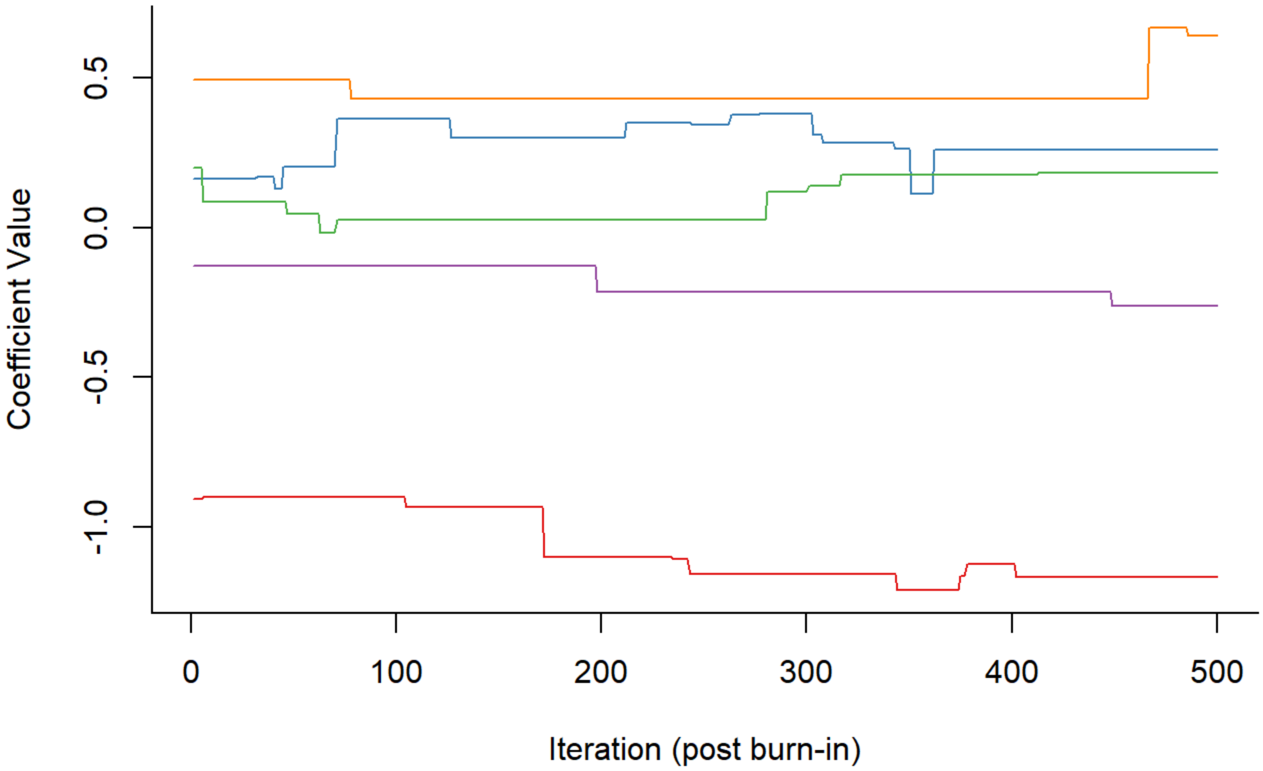
Trace Plot: WRRB



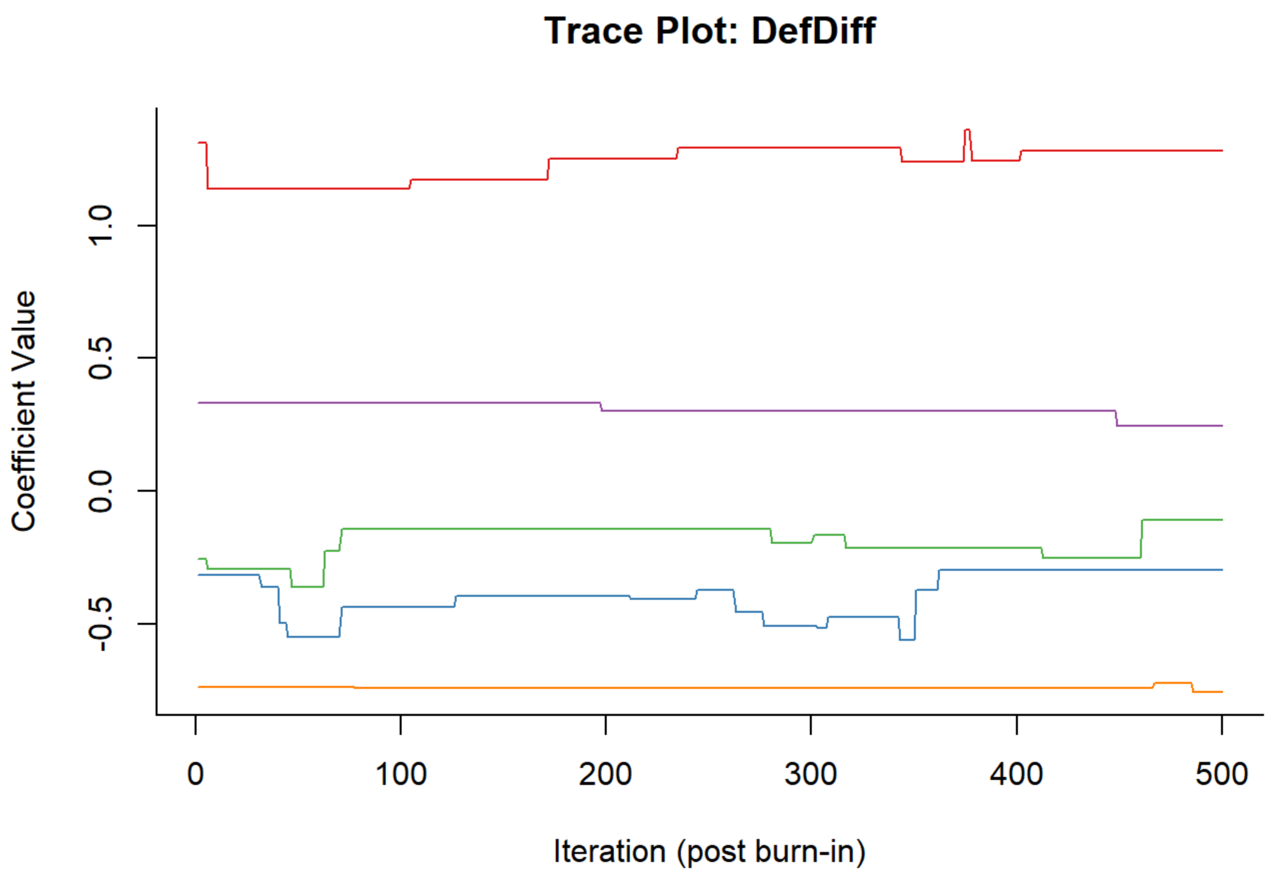
Trace Plot: ratingLB



Trace Plot: ratingDefPass



Trace Plot: ratingDefRun



6. Discussion

The MCMC diagnostics reveal that the simple Metropolis sampler struggled to converge. Several indicators point to this: large standard deviations and wide credible intervals suggest the sampler failed to concentrate around posterior modes; low ESS values indicate severe autocorrelation within chains; and high $\hat{R}$ values show substantial between-chain variance.

Despite these convergence challenges, the model achieves 72.3% accuracy, which is a meaningful improvement over the 61.2% baseline pass probability. The interpretability of the coefficient estimates allows us to understand which factors most strongly predict passing plays. For instance, shotgun formation shows a strong positive association with passing, while a first down state is associated with running plays.

The rudimentary nature of the Metropolis sampler is likely the key reason for convergence issues. More sophisticated algorithms like Hamiltonian Monte Carlo (as implemented in Stan) would likely improve mixing substantially.

7. Conclusion

This analysis demonstrates that Bayesian logistic regression can strike a balance between interpretability and predictive performance in sports analytics. The model achieved competitive accuracy (72.3% with 10,000 iterations, 70.8% with 1,000 iterations per 5 chains) while providing transparent coefficient estimates that explain the relationships between game situations and play calling tendencies.

Key findings:

- Shotgun formation is the strongest predictor of passing plays
- Red zone situations favor running plays
- Yards to go until first down positively correlates with passing

Future directions:

- Employ Hamiltonian Monte Carlo for improved convergence
- Expand the dataset to include team-specific tendencies
- Incorporate time-varying player ratings as seasons progress

The Bayesian approach offers a framework for sports prediction that complements frequentist and black-box methods, providing insights that are actionable for coaches, analysts, and engaged fans.