Multi100 Project

Theiss Bendixen*

2022-05-20

1 Introduction

This document reports on a contribution (ID: YMESM) to the **Multi100 Project** of the Center for Open Science. The re-analysis concerns the following paper:

Cohen, J., Dupas, P., & Schaner, S. (2015). Price subsidies, diagnostic tests, and targeting of malaria treatment: evidence from a randomized controlled trial. *American Economic Review*, *105*(2), 609-45.

The study was a randomized controlled trial on the effect of price subsidies on access and uptake of a particular malaria treatment, artemisinin combination therapies (ACTs), between households. The focal claim to be assessed is: *'... a very high subsidy ... increases access [to antimalarials]'* (p. 609.). In this document, I first spell out the statistical approach of my re-analysis (Section 2), report key results (Section 3), and a brief conclusion (Section 4). Finally, I report supplementary plots (Section 5) and all `R` (R Core Team 2021, version 4.1.2) packages and dependencies used (Section 6).

2 Statistical model

Since the outcome – self-reported treatment of an illness episode with an ACT – is binary (either an ACT was used or it wasn't), we model the data with an aggregated binomial likelihood. That is, each row in the data set is a household. We fit two Bayesian multilevel regression models that are identical except that one collapses the ACT subsidies into a single variable and the other treats the subsidy levels as separate. We refer to the first model as `pAfit` and the second as `pBfit`, based on Table 2, Panel A and B, of the original study. We subset the data similarly to the original study, such that we 1) only focus on the first illness episode reported by each household subsequent to voucher distribution and 2) exclude households that were also provided with a subsidy for rapid diagnostic malaria tests (p. 627). Our choice of covariates is likewise fully informed by the original study.

In formal notation, our models take the following form, including prior distributions[1]:

$$y_i \sim \text{Binomial}(n_i, p_i) \tag{1}$$
$$\text{logit}(p_i) = \alpha_{\text{STRATUM}[i]} + \beta_{\text{STRATUM}[i]} A_i + \gamma H_i \tag{2}$$
$$H_i \sim \text{Normal}(\nu, \sigma_H) \tag{3}$$
$$\nu \sim \text{Normal}(0, 1) \tag{4}$$
$$\sigma_H \sim \text{Exponential}(1) \tag{5}$$
$$\alpha \sim \text{Normal}(0, 1.5) \tag{6}$$
$$\beta \sim \text{Normal}(0, 1) \tag{7}$$
$$\gamma \sim \text{Normal}(0, 1) \tag{8}$$

*Aarhus University, Denmark. Contact: tb@cas.au.dk

[1]Note that we use a 'centered' multilevel parameterization in notation, but `brms` by default employ a 'non-centered' variant for computational efficiency.

where y_i is the binary outcome, and p_i is the probability of taking an ACT for the ith household (line 1). Since some households reported more than one illness episodes, we allow the number of illness episode n (i.e., the number 'trials' in the binomial model) to vary by household i. In the multilevel linear model (line 2), each randomization stratum gets its own varying intercept α_{STRATUM} and varying slopes for subsidy level β_{STRATUM}. We also include age of household head as a fixed effect γ, while we (line 3) impute missing values in household head age H with draws from a Normal distribution with mean ν and standard deviation σ_H estimated from the sample, which are themselves given weakly regularizing priors (line 4-5). The prior for the average intercept α and for the average slope of subsidy level β are both given weakly regularizing priors (line 6-7), as are the fixed effect of household head age, which is standardized and hence centered on zero (line 8).

The varying effects for randomization strata ensure *partial pooling* across strata, such that each stratum is simultaneously and proportionally *informing* and *informed by* the average intercept and slope estimates (cf., McElreath 2020). This reduces the risk of over-fitting and improves computational efficiency. Partial pooling is facilitated via a variance-covariance matrix with multivariate Gaussian priors, where $\mathbf{S}$ is a diagonal matrix of intercept α and slope β, and $\mathbf{R}$ is their correlation matrix with a prior distribution of LKJcorr(4) (Lewandowski, Kurowicka, and Joe 2009). The standard deviations among intercepts σ_α and slopes σ_β are both given an Exponential(1) prior.

$$\begin{bmatrix} \alpha_{\text{STRATUM}} \\ \beta_{\text{STRATUM}} \end{bmatrix} \sim \text{Multivariate Normal}\left(\begin{bmatrix} \alpha \\ \beta \end{bmatrix}, \mathbf{S} \right)$$

$$\mathbf{S} = \begin{pmatrix} \sigma_\alpha & 0 \\ 0 & \sigma_\beta \end{pmatrix} \mathbf{R} \begin{pmatrix} \sigma_\alpha & 0 \\ 0 & \sigma_\beta \end{pmatrix}$$

Analyses are conducted with the `brms` package (Bürkner 2017a, 2018a, 2021a), an interface to the probabilistic programming language `Stan` (Carpenter et al. 2017; Stan Development Team 2021a). MCMC diagnostics and posterior predictive checks were reasonable. Data wrangling and plotting are primarily facilitated by the `tidyverse` (H. Wickham et al. 2019), `ggplot2` (H. Wickham 2016), and `tidybayes` (Kay 2022b) packages[2]. The report is written in `R Markdown` (Allaire et al. 2021; Xie, Allaire, and Grolemund 2018a).

3 Results

Here, we report results. Our target quantity of interest is *the contrast in posterior mean predicted probabilities of taking an ACT between the price subsidy interventions compared to no subsidy.* This contrast is akin to an average treatment effect and we compute it (in percentage) as

$$\frac{\mathbb{E}(Y = 1|\text{Subsidy} = 1) - \mathbb{E}(Y = 1|\text{Subsidy} = 0)}{\mathbb{E}(Y = 1|\text{Subsidy} = 0)} \times 100$$

for each draw of the posterior distribution (in this case, 4000 post-warmup draws), which we then in turn summarize by its mean and 95% interval. A positive contrast thus means that the subsidies increase the likelihood of taking an ACT. Further, the contrast marginalizes over[3] the distribution of age as well as the randomization strata by simulating a counter-factual population, wherein all households were assigned to both the treatment and the control group while retaining covariates as observed[4].

3.1 Any subsidy level & across subsidy levels

Figure 1 shows the computed contrast as a posterior predicted density for any amount of subsidy (`pAfit` model). Zero (the dotted line) represents equal chance of taking an ACT between the two groups. The density is normalized and the gradient reflects posterior mass. The point represents the mean of the distribution with a 95% interval.

[2]The package `renv` (Ushey 2022) ensures a reproducible package environment. See Section 6 for a list of `R` packages, their dependencies, and version number used for this project, created using `grateful` (Rodríguez-Sánchez, Jackson, and Hutchins 2022).

[3]Also sometimes referred to as *g-computation* or *standardisation*.

[4]For the households with missing values in age of household head, we get predictions for the posterior mean of the imputed values.

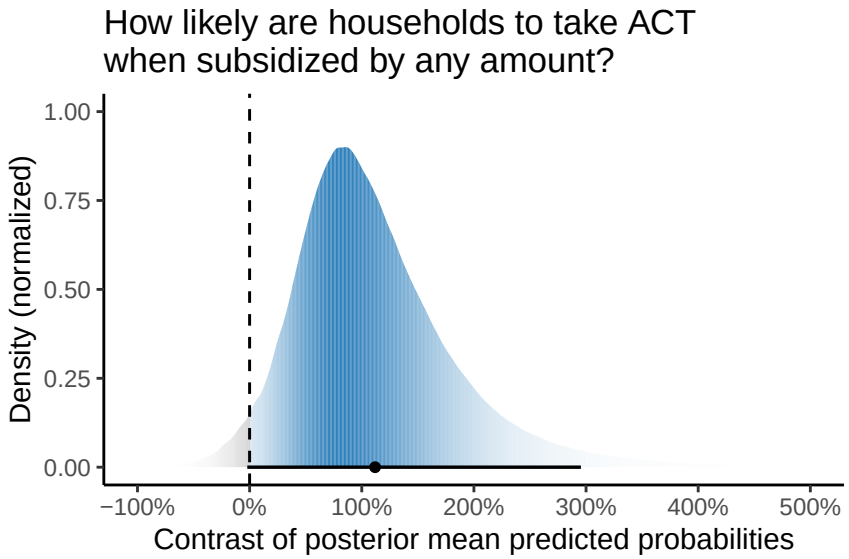

Figure 1: Results from `pAfit` model. Contrast in predicted probabilities of taking an ACT between the price subsidy intervention (any of the subsidy levels) compared to no subsidy. Zero (the dotted line) represents equal chance of taking an ACT between the two groups. The density is normalized and the gradient reflect posterior mass. The point represents the mean of the distribution with a 95% interval.

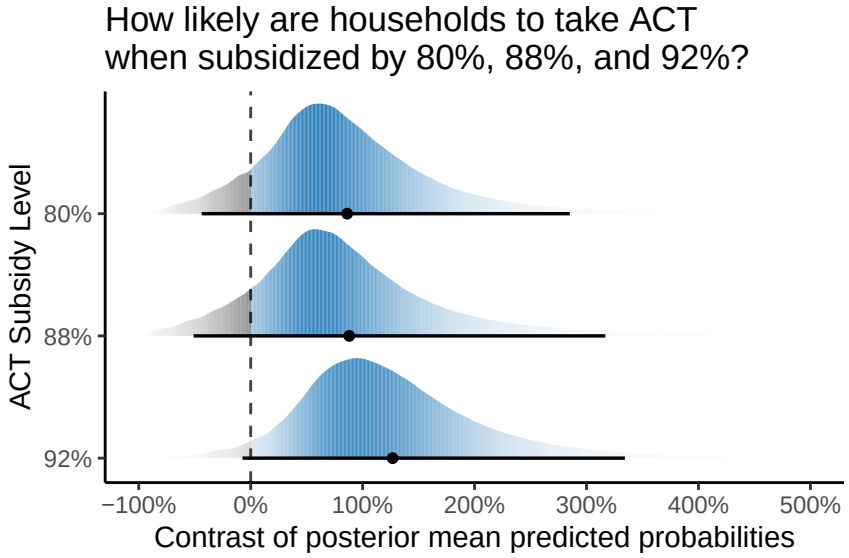

Figure 2: Results from `pBfit` model. Contrast in predicted probabilities of taking an ACT between the price subsidy intervention (for each subsidy level) compared to no subsidy. Zero (the dotted line) represents equal chance of taking an ACT between the treatment and control groups. The densities are normalized and the gradients reflect posterior mass. The points represent the mean of the distributions with a 95% interval.

The mean[5] and 95% interval of the posterior predicted contrast between the treatment and the control group is thus 113.36% [-2.03, 303.51], or roughly a doubling with the bulk of the posterior mass being above zero. This is a substantial effect given that the posterior predicted mean baseline of taking ACT in the control group is 21.7% [5, 52.1].

Likewise, Figure 2 illustrates the computed contrast in posterior mean predicted probabilities across subsidy levels. The inference is very similar: there is roughly a doubling of chance of taking an ACT between the control and intervention groups, and the bulk of the posterior masses is above zero. Again, this is a substantial effect. While there is little difference between the three subsidy levels overall, perhaps unsurprisingly, the highest subsidy level (92%) has the highest mean posterior predicted change (130.45% [-7.08, 356.09]), with the effects of the 80% price subsidy (88.21% [-43.7, 296.49]) and the 88% price subsidy (93.51% [-50.79, 353.67]) being almost indistinguishable.

4 Conclusion

In conclusion, this re-analysis finds substantial evidence for the claim of the original study that *'... a very high subsidy ... increases access [to antimalarials]'* (p. 609.). Providing households with high price subsidies roughly doubles the chances of taking an ACT for an illness event with malaria-like symptoms, with a baseline likelihood of around 22%. This inference largely matches the original authors' findings, namely that '[s]ubsidies of 80 percent or more increase the likelihood that an illness is treated with an ACT by 16-23 percentage points (an 85-118 percent increase) [...]' (p. 628).

With that said, the present re-analysis yields a large degree of uncertainty in inference. This is at least partly due to the fact that the re-analysis marginalizes over the distribution of covariates. Excluding the random effects of strata (i.e., getting predictions for an average stratum and ignoring stratum-specific variance) reduces the uncertainty such that all 95% intervals no longer (or only barely) include zero (i.e., a 'statistically significant' difference; see Figures 3 and 4).

Note, finally, that given the overlap between the presently obtained results and those of the original study (and also due to time and timing), I did not attempt to replicate any follow-up analyses reported in the original study, such as the ones reported in their Panel A and B, Figure 4, where the self-reported data were cross-checked with observational and behavioral data. For the same reasons, I also did not attempt to replicate their baseline summary statistics (their Table 1). A reasonable balance between control and treatment groups is essential for the assumption of conditional exchangeability that underlies our average treatment effect estimates.

[5]Note that, due to some skewness in the distributions, the mean does not perfectly indicate the values with highest posterior mass.

5 Supplementary plots

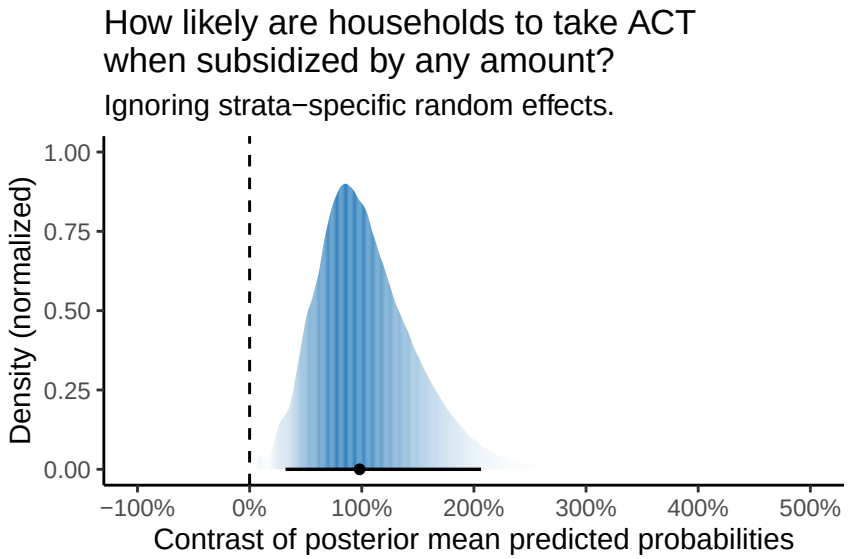

Figure 3: Same as Figure 1, but ignoring strata-specific random effects.

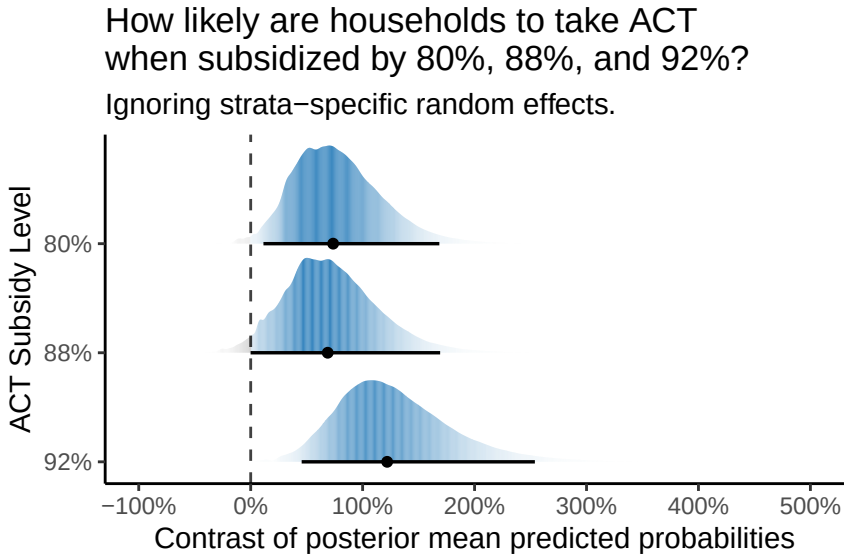

Figure 4: Same as Figure 2, but ignoring strata-specific random effects.

6 R packages and dependencies

We used R version 4.1.2 (R Core Team 2021) and the following R packages: abind v. 1.4.5 (Plate and Heiberger 2016), arrayhelpers v. 1.1.0 (Beleites 2020), backports v. 1.4.1 (Lang and R Core Team 2021), base64enc v. 0.1.3 (Urbanek 2015), bayesplot v. 1.8.1 (Gabry et al. 2019; Gabry and Mahr 2021), BH v. 1.78.0.0 (Eddelbuettel, Emerson, and Kane 2021), bit v. 4.0.4 (Oehlschlägel and Ripley 2020), bit64 v. 4.0.5 (Oehlschlägel and Silvestri 2020), bridgesampling v. 1.1.2 (Gronau, Singmann, and Wagenmakers 2020), brms v. 2.17.0 (Bürkner 2017b, 2018b, 2021b), Brobdingnag v. 1.2.7 (Hankin 2007), bslib v. 0.3.1 (Sievert and Cheng 2021a), cachem v. 1.0.6 (Chang 2021a), callr v. 3.7.0 (Csárdi and Chang 2021a), checkmate v. 2.0.0 (Lang 2017), clipr v. 0.8.0 (Lincoln 2022), coda v. 0.19.4 (Plummer et al. 2006), colorspace v. 2.0.3 (Zeileis, Hornik, and Murrell 2009; Stauffer et al. 2009; Zeileis et al. 2020), colourpicker v. 1.1.1 (Attali 2021a), commonmark v. 1.7 (Ooms 2018), cpp11 v. 0.4.2 (Hester and François 2021), crosstalk v. 1.2.0 (Cheng and Sievert 2021), desc v. 1.4.0 (Csárdi, Müller, and Hester 2021), digest v. 0.6.29 (Antoine Lucas et al. 2021), distributional v. 0.3.0 (O'Hara-Wild, Kay, and Hayes 2022), DT v. 0.20 (Xie, Cheng, and Tan 2021), dygraphs v. 1.1.1.6 (Vanderkam et al. 2018), ellipsis v. 0.3.2 (H. Wickham 2021), evaluate v. 0.15 (H. Wickham and Xie 2022), fansi v. 1.0.2 (Gaslam 2022), farver v. 2.1.0 (Pedersen, Nicolae, and François 2021), fastmap v. 1.1.0 (Chang 2021b), fontawesome v. 0.2.2 (Iannone 2021), fs v. 1.5.2 (Hester, Wickham, and Csárdi 2021), future v. 1.24.0 [@], generics v. 0.1.2 (H. Wickham, Kuhn, and Vaughan 2022), ggdist v. 3.1.1 (Kay 2022a), ggridges v. 0.5.3 (Wilke 2021), globals v. 0.14.0 (Bengtsson 2020), glue v. 1.6.1 (Hester and Bryan 2022), grateful v. 0.1.11 (Rodríguez-Sánchez, Jackson, and Hutchins 2022), gridExtra v. 2.3 (Auguie 2017), gtable v. 0.3.0 (H. Wickham and Pedersen 2019), gtools v. 3.9.2 (Warnes, Bolker, and Lumley 2021), HDInterval v. 0.2.2 (Meredith and Kruschke 2020), highr v. 0.9 (Xie and Qiu 2021), htmltools v. 0.5.2 (Cheng et al. 2021), htmlwidgets v. 1.5.4 (Vaidyanathan et al. 2021), httpuv v. 1.6.5 (Cheng and Chang 2022), igraph v. 1.2.11 (Csardi and Nepusz 2006), inline v. 0.3.19 (Sklyar et al. 2021), isoband v. 0.2.5 (Wilke and Pedersen 2021), jquerylib v. 0.1.4 (Sievert and Cheng 2021b), knitr v. 1.39 (Xie 2014, 2015, 2022a), labeling v. 0.4.2 (Justin Talbot 2020), later v. 1.3.0 (Chang and Cheng 2021), lazyeval v. 0.2.2 (H. Wickham 2019), lifecycle v. 1.0.1 (Henry and Wickham 2021), listenv v. 0.8.0 (Bengtsson 2019), loo v. 2.4.1 (Vehtari, Gelman, and Gabry 2017; Yao et al. 2017; Vehtari et al. 2020), markdown v. 1.1 (Allaire et al. 2019), matrixStats v. 0.61.0 (Bengtsson 2021a), mime v. 0.12 (Xie 2021a), miniUI v. 0.1.1.1 (Cheng 2018), munsell v. 0.5.0 (C. Wickham 2018), mvtnorm v. 1.1.3 (Genz and Bretz 2009; Genz et al. 2021), nleqslv v. 3.3.2 (Hasselman 2018), numDeriv v. 2016.8.1.1 (Gilbert and Varadhan 2019), parallelly v. 1.30.0 (Bengtsson 2021b), pkgbuild v. 1.3.1 (H. Wickham, Hester, and Csárdi 2021), pkgconfig v. 2.0.3 (Csárdi 2019), plyr v. 1.8.6 (H. Wickham 2011), posterior v. 1.2.0 (Vehtari et al. 2021; Bürkner et al. 2022), prettyunits v. 1.1.1 (Csardi 2020), processx v. 3.5.2 (Csárdi and Chang 2021b), progress v. 1.2.2 (Csárdi and FitzJohn 2019), promises v. 1.2.0.1 (Cheng 2021), ps v. 1.6.0 (Loden et al. 2021), R6 v. 2.5.1 (Chang 2021c), rappdirs v. 0.3.3 (Ratnakumar, Mick, and Davis 2021), RColorBrewer v. 1.1.2 (Neuwirth 2014), Rcpp v. 1.0.8 (Eddelbuettel and François 2011; Eddelbuettel 2013; Eddelbuettel and Balamuta 2018), RcppEigen v. 0.3.3.9.1 (Bates and Eddelbuettel 2013), RcppParallel v. 5.1.5 (Allaire, Francois, et al. 2022), renv v. 0.15.4 (Ushey 2022), reshape2 v. 1.4.4 (H. Wickham 2007), rmarkdown v. 2.14 (Xie, Allaire, and Grolemund 2018b; Xie, Dervieux, and Riederer 2020; Allaire, Xie, et al. 2022), rprojroot v. 2.0.2 (Müller 2020), rstan v. 2.21.3 (Stan Development Team 2021b), rstantools v. 2.1.1 (Gabry, Goodrich, and Lysy 2020), sass v. 0.4.1 (Cheng et al. 2022), scales v. 1.1.1 (H. Wickham and Seidel 2020), shiny v. 1.7.1 (Chang et al. 2021), shinyjs v. 2.1.0 (Attali 2021b), shinystan v. 2.5.0 (Gabry 2018), shinythemes v. 1.2.0 (Chang 2021d), sourcetools v. 0.1.7 (Ushey 2018), StanHeaders v. 2.21.0.7 (Stan Development Team 2020), stringi v. 1.7.6 (Gagolewski 2021a, 2021b), svUnit v. 1.0.6 (Grosjean 2021), tensorA v. 0.36.2 (van den Boogaart 2020), threejs v. 0.3.3 (Lewis 2020), tidybayes v. 3.0.2 (Kay 2022b), tidyselect v. 1.1.2 (Henry and Wickham 2022), tidyverse v. 1.3.1 (H. Wickham et al. 2019), tinytex v. 0.38 (Xie 2019, 2022b), tzdb v. 0.2.0 (Vaughan 2021), utf8 v. 1.2.2 (Perry 2021), vctrs v. 0.3.8 (H. Wickham, Henry, and Vaughan 2021), viridisLite v. 0.4.0 (Garnier et al. 2021), vroom v. 1.5.7 (Hester, Wickham, and Bryan 2021), withr v. 2.4.3 (Hester et al. 2021), xfun v. 0.29 (Xie 2021b), xtable v. 1.8.4 (Dahl et al. 2019), xts v. 0.12.1 (Ryan and Ulrich 2020), yaml v. 2.3.5 (Garbett et al. 2022), zoo v. 1.8.9 (Zeileis and Grothendieck 2005).

References

Allaire, JJ, Romain Francois, Kevin Ushey, Gregory Vandenbrouck, Marcus Geelnard, and Intel. 2022. *RcppParallel: Parallel Programming Tools for 'Rcpp'*. https://CRAN.R-project.org/package=RcppParallel.

Allaire, JJ, Jeffrey Horner, Yihui Xie, Vicent Marti, and Natacha Porte. 2019. *Markdown: Render Markdown with the c Library 'Sundown'*. https://CRAN.R-project.org/package=markdown.

Allaire, JJ, Yihui Xie, Jonathan McPherson, Javier Luraschi, Kevin Ushey, Aron Atkins, Hadley Wickham, Joe Cheng, Winston Chang, and Richard Iannone. 2021. *Rmarkdown: Dynamic Documents for r*. https://github.com/rstudio/rmarkdown.

———. 2022. *Rmarkdown: Dynamic Documents for r*. https://github.com/rstudio/rmarkdown.

Antoine Lucas, Dirk Eddelbuettel with contributions by, Jarek Tuszynski, Henrik Bengtsson, Simon Urbanek, Mario Frasca, Bryan Lewis, Murray Stokely, et al. 2021. *Digest: Create Compact Hash Digests of r Objects*. https://CRAN.R-project.org/package=digest.

Attali, Dean. 2021a. *Colourpicker: A Colour Picker Tool for Shiny and for Selecting Colours in Plots*. https://CRAN.R-project.org/package=colourpicker.

———. 2021b. *Shinyjs: Easily Improve the User Experience of Your Shiny Apps in Seconds*. https://CRAN.R-project.org/package=shinyjs.

Auguie, Baptiste. 2017. *gridExtra: Miscellaneous Functions for "Grid" Graphics*. https://CRAN.R-project.org/package=gridExtra.

Bates, Douglas, and Dirk Eddelbuettel. 2013. "Fast and Elegant Numerical Linear Algebra Using the RcppEigen Package." *Journal of Statistical Software* 52 (5): 1–24. https://www.jstatsoft.org/v52/i05/.

Beleites, C. 2020. *Arrayhelpers: Convenience Functions for Arrays*. https://CRAN.R-project.org/package=arrayhelpers.

Bengtsson, Henrik. 2019. *Listenv: Environments Behaving (Almost) as Lists*. https://CRAN.R-project.org/package=listenv.

———. 2020. *Globals: Identify Global Objects in r Expressions*. https://CRAN.R-project.org/package=globals.

———. 2021a. *matrixStats: Functions That Apply to Rows and Columns of Matrices (and to Vectors)*. https://CRAN.R-project.org/package=matrixStats.

———. 2021b. *Parallelly: Enhancing the 'Parallel' Package*. https://CRAN.R-project.org/package=parallelly.

Bürkner, Paul-Christian. 2017a. "Brms: An R Package for Bayesian Multilevel Models Using Stan." *Journal of Statistical Software* 80 (1): 1–28. https://doi.org/10.18637/jss.v080.i01.

———. 2017b. "brms: An R Package for Bayesian Multilevel Models Using Stan." *Journal of Statistical Software* 80 (1): 1–28. https://doi.org/10.18637/jss.v080.i01.

———. 2018a. "Advanced Bayesian Multilevel Modeling with the R Package Brms." *The R Journal* 10 (1): 395–411. https://doi.org/10.32614/RJ-2018-017.

———. 2018b. "Advanced Bayesian Multilevel Modeling with the R Package brms." *The R Journal* 10 (1): 395–411. https://doi.org/10.32614/RJ-2018-017.

———. 2021a. "Bayesian Item Response Modeling in R with brms and Stan." *Journal of Statistical Software* 100 (5): 1–54. https://doi.org/10.18637/jss.v100.i05.

———. 2021b. "Bayesian Item Response Modeling in R with brms and Stan." *Journal of Statistical Software* 100 (5): 1–54. https://doi.org/10.18637/jss.v100.i05.

Bürkner, Paul-Christian, Jonah Gabry, Matthew Kay, and Aki Vehtari. 2022. "Posterior: Tools for Working with Posterior Distributions." https://mc-stan.org/posterior/.

Carpenter, Bob, Andrew Gelman, Matthew Hoffman, Daniel Lee, Ben Goodrich, Michael Betancourt, Marcus Brubaker, Jiqiang Guo, Peter Li, and Allen Riddell. 2017. "Stan: A Probabilistic Programming Language." *Journal of Statistical Software* 76 (1).

Chang, Winston. 2021a. *Cachem: Cache r Objects with Automatic Pruning*. https://CRAN.R-project.org/package=cachem.

———. 2021b. *Fastmap: Fast Data Structures*. https://CRAN.R-project.org/package=fastmap.

———. 2021c. *R6: Encapsulated Classes with Reference Semantics*. https://CRAN.R-project.org/package=R6.

———. 2021d. *Shinythemes: Themes for Shiny*. https://CRAN.R-project.org/package=shinythemes.

Chang, Winston, and Joe Cheng. 2021. *Later: Utilities for Scheduling Functions to Execute Later with Event*

Loops. https://CRAN.R-project.org/package=later.

Chang, Winston, Joe Cheng, JJ Allaire, Carson Sievert, Barret Schloerke, Yihui Xie, Jeff Allen, Jonathan McPherson, Alan Dipert, and Barbara Borges. 2021. *Shiny: Web Application Framework for r*. https://CRAN.R-project.org/package=shiny.

Cheng, Joe. 2018. *miniUI: Shiny UI Widgets for Small Screens*. https://CRAN.R-project.org/package=miniUI.

———. 2021. *Promises: Abstractions for Promise-Based Asynchronous Programming*. https://CRAN.R-project.org/package=promises.

Cheng, Joe, and Winston Chang. 2022. *Httpuv: HTTP and WebSocket Server Library*. https://CRAN.R-project.org/package=httpuv.

Cheng, Joe, Timothy Mastny, Richard Iannone, Barret Schloerke, and Carson Sievert. 2022. *Sass: Syntactically Awesome Style Sheets ('Sass')*. https://CRAN.R-project.org/package=sass.

Cheng, Joe, and Carson Sievert. 2021. *Crosstalk: Inter-Widget Interactivity for HTML Widgets*. https://CRAN.R-project.org/package=crosstalk.

Cheng, Joe, Carson Sievert, Barret Schloerke, Winston Chang, Yihui Xie, and Jeff Allen. 2021. *Htmltools: Tools for HTML*. https://CRAN.R-project.org/package=htmltools.

Csardi, Gabor. 2020. *Prettyunits: Pretty, Human Readable Formatting of Quantities*. https://CRAN.R-project.org/package=prettyunits.

Csardi, Gabor, and Tamas Nepusz. 2006. "The Igraph Software Package for Complex Network Research." *InterJournal* Complex Systems: 1695. https://igraph.org.

Csárdi, Gábor. 2019. *Pkgconfig: Private Configuration for 'r' Packages*. https://CRAN.R-project.org/package=pkgconfig.

Csárdi, Gábor, and Winston Chang. 2021a. *Callr: Call r from r*. https://CRAN.R-project.org/package=callr.

———. 2021b. *Processx: Execute and Control System Processes*. https://CRAN.R-project.org/package=processx.

Csárdi, Gábor, and Rich FitzJohn. 2019. *Progress: Terminal Progress Bars*. https://CRAN.R-project.org/package=progress.

Csárdi, Gábor, Kirill Müller, and Jim Hester. 2021. *Desc: Manipulate DESCRIPTION Files*. https://CRAN.R-project.org/package=desc.

Dahl, David B., David Scott, Charles Roosen, Arni Magnusson, and Jonathan Swinton. 2019. *Xtable: Export Tables to LaTeX or HTML*. https://CRAN.R-project.org/package=xtable.

Eddelbuettel, Dirk. 2013. *Seamless R and C++ Integration with Rcpp*. New York: Springer. https://doi.org/10.1007/978-1-4614-6868-4.

Eddelbuettel, Dirk, and James Joseph Balamuta. 2018. "Extending extitR with extitC++: A Brief Introduction to extitRcpp." *The American Statistician* 72 (1): 28–36. https://doi.org/10.1080/00031305.2017.1375990.

Eddelbuettel, Dirk, John W. Emerson, and Michael J. Kane. 2021. *BH: Boost c++ Header Files*. https://CRAN.R-project.org/package=BH.

Eddelbuettel, Dirk, and Romain François. 2011. "Rcpp: Seamless R and C++ Integration." *Journal of Statistical Software* 40 (8): 1–18. https://doi.org/10.18637/jss.v040.i08.

Gabry, Jonah. 2018. *Shinystan: Interactive Visual and Numerical Diagnostics and Posterior Analysis for Bayesian Models*. https://CRAN.R-project.org/package=shinystan.

Gabry, Jonah, Ben Goodrich, and Martin Lysy. 2020. *Rstantools: Tools for Developing r Packages Interfacing with 'Stan'*. https://CRAN.R-project.org/package=rstantools.

Gabry, Jonah, and Tristan Mahr. 2021. "Bayesplot: Plotting for Bayesian Models." https://mc-stan.org/bayesplot/.

Gabry, Jonah, Daniel Simpson, Aki Vehtari, Michael Betancourt, and Andrew Gelman. 2019. "Visualization in Bayesian Workflow." *J. R. Stat. Soc. A* 182: 389–402. https://doi.org/10.1111/rssa.12378.

Gagolewski, Marek. 2021a. "Stringi: Fast and Portable Character String Processing in r." *Journal of Statistical Software*.

———. 2021b. *Stringi: Fast and Portable Character String Processing in r*. https://stringi.gagolewski.com/.

Garbett, Shawn P, Jeremy Stephens, Kirill Simonov, Yihui Xie, Zhuoer Dong, Hadley Wickham, Jeffrey Horner, et al. 2022. *Yaml: Methods to Convert r Data to YAML and Back*. https://CRAN.R-project.org/package=yaml.

Garnier, Simon, Ross, Noam, Rudis, Robert, Camargo, et al. 2021. *viridis - Colorblind-Friendly Color Maps for r*. https://doi.org/10.5281/zenodo.4679424.

Gaslam, Brodie. 2022. *Fansi: ANSI Control Sequence Aware String Functions*. https://CRAN.R-project.org/package=fansi.

Genz, Alan, and Frank Bretz. 2009. *Computation of Multivariate Normal and t Probabilities*. Lecture Notes in Statistics. Heidelberg: Springer-Verlag.

Genz, Alan, Frank Bretz, Tetsuhisa Miwa, Xuefei Mi, Friedrich Leisch, Fabian Scheipl, and Torsten Hothorn. 2021. *mvtnorm: Multivariate Normal and t Distributions*. https://CRAN.R-project.org/package=mvtnorm.

Gilbert, Paul, and Ravi Varadhan. 2019. *numDeriv: Accurate Numerical Derivatives*. https://CRAN.R-project.org/package=numDeriv.

Gronau, Quentin F., Henrik Singmann, and Eric-Jan Wagenmakers. 2020. "bridgesampling: An R Package for Estimating Normalizing Constants." *Journal of Statistical Software* 92 (10): 1–29. https://doi.org/10.18637/jss.v092.i10.

Grosjean, Philippe. 2021. *SciViews-r*. MONS, Belgium: UMONS. https://www.sciviews.org/SciViews-R/.

Hankin, R. K. S. 2007. "Very Large Numbers in r: Introducing Package Brobdingnag." *R News* 7.

Hasselman, Berend. 2018. *Nleqslv: Solve Systems of Nonlinear Equations*. https://CRAN.R-project.org/package=nleqslv.

Henry, Lionel, and Hadley Wickham. 2021. *Lifecycle: Manage the Life Cycle of Your Package Functions*. https://CRAN.R-project.org/package=lifecycle.

———. 2022. *Tidyselect: Select from a Set of Strings*. https://CRAN.R-project.org/package=tidyselect.

Hester, Jim, and Jennifer Bryan. 2022. *Glue: Interpreted String Literals*. https://CRAN.R-project.org/package=glue.

Hester, Jim, and Romain François. 2021. *Cpp11: A c++11 Interface for r's c Interface*. https://CRAN.R-project.org/package=cpp11.

Hester, Jim, Lionel Henry, Kirill Müller, Kevin Ushey, Hadley Wickham, and Winston Chang. 2021. *Withr: Run Code 'with' Temporarily Modified Global State*. https://CRAN.R-project.org/package=withr.

Hester, Jim, Hadley Wickham, and Jennifer Bryan. 2021. *Vroom: Read and Write Rectangular Text Data Quickly*. https://CRAN.R-project.org/package=vroom.

Hester, Jim, Hadley Wickham, and Gábor Csárdi. 2021. *Fs: Cross-Platform File System Operations Based on 'Libuv'*. https://CRAN.R-project.org/package=fs.

Iannone, Richard. 2021. *Fontawesome: Easily Work with 'Font Awesome' Icons*. https://CRAN.R-project.org/package=fontawesome.

Justin Talbot. 2020. *Labeling: Axis Labeling*. https://CRAN.R-project.org/package=labeling.

Kay, Matthew. 2022a. *ggdist: Visualizations of Distributions and Uncertainty*. https://doi.org/10.5281/zenodo.3879620.

———. 2022b. *tidybayes: Tidy Data and Geoms for Bayesian Models*. https://doi.org/10.5281/zenodo.1308151.

Lang, Michel. 2017. "checkmate: Fast Argument Checks for Defensive r Programming." *The R Journal* 9 (1): 437–45. https://journal.r-project.org/archive/2017/RJ-2017-028/index.html.

Lang, Michel, and R Core Team. 2021. *Backports: Reimplementations of Functions Introduced Since r-3.0.0*. https://CRAN.R-project.org/package=backports.

Lewandowski, Daniel, Dorota Kurowicka, and Harry Joe. 2009. "Generating Random Correlation Matrices Based on Vines and Extended Onion Method." *Journal of Multivariate Analysis* 100 (9): 1989–2001.

Lewis, B. W. 2020. *Threejs: Interactive 3d Scatter Plots, Networks and Globes*. https://CRAN.R-project.org/package=threejs.

Lincoln, Matthew. 2022. *Clipr: Read and Write from the System Clipboard*. https://CRAN.R-project.org/package=clipr.

Loden, Jay, Dave Daeschler, Giampaolo Rodola', and Gábor Csárdi. 2021. *Ps: List, Query, Manipulate System Processes*. https://CRAN.R-project.org/package=ps.

McElreath, Richard. 2020. *Statistical Rethinking: A Bayesian Course with Examples in R and Stan*. Second. CRC Press.

Meredith, Mike, and John Kruschke. 2020. *HDInterval: Highest (Posterior) Density Intervals*. https://CRAN.R-project.org/package=HDInterval.

Müller, Kirill. 2020. *Rprojroot: Finding Files in Project Subdirectories*. https://CRAN.R-project.org/packag

e=rprojroot.
Neuwirth, Erich. 2014. *RColorBrewer: ColorBrewer Palettes*. https://CRAN.R-project.org/package=RColorBrewer.
O'Hara-Wild, Mitchell, Matthew Kay, and Alex Hayes. 2022. *Distributional: Vectorised Probability Distributions*. https://CRAN.R-project.org/package=distributional.
Oehlschlägel, Jens, and Brian Ripley. 2020. *Bit: Classes and Methods for Fast Memory-Efficient Boolean Selections*. https://CRAN.R-project.org/package=bit.
Oehlschlägel, Jens, and Leonardo Silvestri. 2020. *Bit64: A S3 Class for Vectors of 64bit Integers*. https://CRAN.R-project.org/package=bit64.
Ooms, Jeroen. 2018. *Commonmark: High Performance CommonMark and Github Markdown Rendering in r*. https://CRAN.R-project.org/package=commonmark.
Pedersen, Thomas Lin, Berendea Nicolae, and Romain François. 2021. *Farver: High Performance Colour Space Manipulation*. https://CRAN.R-project.org/package=farver.
Perry, Patrick O. 2021. *Utf8: Unicode Text Processing*. https://CRAN.R-project.org/package=utf8.
Plate, Tony, and Richard Heiberger. 2016. *Abind: Combine Multidimensional Arrays*. https://CRAN.R-project.org/package=abind.
Plummer, Martyn, Nicky Best, Kate Cowles, and Karen Vines. 2006. "CODA: Convergence Diagnosis and Output Analysis for MCMC." *R News* 6 (1): 7–11. https://journal.r-project.org/archive/.
R Core Team. 2021. *R: A Language and Environment for Statistical Computing*. Vienna, Austria: R Foundation for Statistical Computing. https://www.R-project.org/.
Ratnakumar, Sridhar, Trent Mick, and Trevor Davis. 2021. *Rappdirs: Application Directories: Determine Where to Save Data, Caches, and Logs*. https://CRAN.R-project.org/package=rappdirs.
Rodríguez-Sánchez, Francisco, Connor P. Jackson, and Shaurita D. Hutchins. 2022. *Grateful: Facilitate Citation of r Packages*. https://github.com/Pakillo/grateful.
Ryan, Jeffrey A., and Joshua M. Ulrich. 2020. *Xts: eXtensible Time Series*. https://CRAN.R-project.org/package=xts.
Sievert, Carson, and Joe Cheng. 2021a. *Bslib: Custom 'Bootstrap' 'Sass' Themes for 'Shiny' and 'Rmarkdown'*. https://CRAN.R-project.org/package=bslib.
———. 2021b. *Jquerylib: Obtain 'jQuery' as an HTML Dependency Object*. https://CRAN.R-project.org/package=jquerylib.
Sklyar, Oleg, Duncan Murdoch, Mike Smith, Dirk Eddelbuettel, Romain Francois, Karline Soetaert, and Johannes Ranke. 2021. *Inline: Functions to Inline c, c++, Fortran Function Calls from r*. https://CRAN.R-project.org/package=inline.
Stan Development Team. 2020. "StanHeaders: Headers for the R Interface to Stan." https://mc-stan.org/.
———. 2021a. "RStan: The R Interface to Stan." https://mc-stan.org/.
———. 2021b. "RStan: The R Interface to Stan." https://mc-stan.org/.
Stauffer, Reto, Georg J. Mayr, Markus Dabernig, and Achim Zeileis. 2009. "Somewhere over the Rainbow: How to Make Effective Use of Colors in Meteorological Visualizations." *Bulletin of the American Meteorological Society* 96 (2): 203–16. https://doi.org/10.1175/BAMS-D-13-00155.1.
Urbanek, Simon. 2015. *Base64enc: Tools for Base64 Encoding*. https://CRAN.R-project.org/package=base64enc.
Ushey, Kevin. 2018. *Sourcetools: Tools for Reading, Tokenizing and Parsing r Code*. https://CRAN.R-project.org/package=sourcetools.
———. 2022. *Renv: Project Environments*. https://CRAN.R-project.org/package=renv.
Vaidyanathan, Ramnath, Yihui Xie, JJ Allaire, Joe Cheng, Carson Sievert, and Kenton Russell. 2021. *Htmlwidgets: HTML Widgets for r*. https://CRAN.R-project.org/package=htmlwidgets.
van den Boogaart, K. Gerald. 2020. *tensorA: Advanced Tensor Arithmetic with Named Indices*. https://CRAN.R-project.org/package=tensorA.
Vanderkam, Dan, JJ Allaire, Jonathan Owen, Daniel Gromer, and Benoit Thieurmel. 2018. *Dygraphs: Interface to 'Dygraphs' Interactive Time Series Charting Library*. https://CRAN.R-project.org/package=dygraphs.
Vaughan, Davis. 2021. *Tzdb: Time Zone Database Information*. https://CRAN.R-project.org/package=tzdb.
Vehtari, Aki, Jonah Gabry, Mans Magnusson, Yuling Yao, Paul-Christian Bürkner, Topi Paananen, and Andrew Gelman. 2020. "Loo: Efficient Leave-One-Out Cross-Validation and WAIC for Bayesian Models."

https://mc-stan.org/loo/.

Vehtari, Aki, Andrew Gelman, and Jonah Gabry. 2017. "Practical Bayesian Model Evaluation Using Leave-One-Out Cross-Validation and WAIC." *Statistics and Computing* 27: 1413–32. https://doi.org/10.1007/s11222-016-9696-4.

Vehtari, Aki, Andrew Gelman, Daniel Simpson, Bob Carpenter, and Paul-Christian Bürkner. 2021. "Rank-Normalization, Folding, and Localization: An Improved Rhat for Assessing Convergence of MCMC (with Discussion)." *Bayesian Analysis*.

Warnes, Gregory R., Ben Bolker, and Thomas Lumley. 2021. *Gtools: Various r Programming Tools*. https://CRAN.R-project.org/package=gtools.

Wickham, Charlotte. 2018. *Munsell: Utilities for Using Munsell Colours*. https://CRAN.R-project.org/package=munsell.

Wickham, Hadley. 2007. "Reshaping Data with the reshape Package." *Journal of Statistical Software* 21 (12): 1–20. http://www.jstatsoft.org/v21/i12/.

———. 2011. "The Split-Apply-Combine Strategy for Data Analysis." *Journal of Statistical Software* 40 (1): 1–29. http://www.jstatsoft.org/v40/i01/.

———. 2016. *Ggplot2: Elegant Graphics for Data Analysis*. Springer-Verlag New York. https://ggplot2.tidyverse.org.

———. 2019. *Lazyeval: Lazy (Non-Standard) Evaluation*. https://CRAN.R-project.org/package=lazyeval.

———. 2021. *Ellipsis: Tools for Working with ...* https://CRAN.R-project.org/package=ellipsis.

Wickham, Hadley, Mara Averick, Jennifer Bryan, Winston Chang, Lucy D'Agostino McGowan, Romain François, Garrett Grolemund, et al. 2019. "Welcome to the tidyverse." *Journal of Open Source Software* 4 (43): 1686. https://doi.org/10.21105/joss.01686.

Wickham, Hadley, Lionel Henry, and Davis Vaughan. 2021. *Vctrs: Vector Helpers*. https://CRAN.R-project.org/package=vctrs.

Wickham, Hadley, Jim Hester, and Gábor Csárdi. 2021. *Pkgbuild: Find Tools Needed to Build r Packages*. https://CRAN.R-project.org/package=pkgbuild.

Wickham, Hadley, Max Kuhn, and Davis Vaughan. 2022. *Generics: Common S3 Generics Not Provided by Base r Methods Related to Model Fitting*. https://CRAN.R-project.org/package=generics.

Wickham, Hadley, and Thomas Lin Pedersen. 2019. *Gtable: Arrange 'Grobs' in Tables*. https://CRAN.R-project.org/package=gtable.

Wickham, Hadley, and Dana Seidel. 2020. *Scales: Scale Functions for Visualization*. https://CRAN.R-project.org/package=scales.

Wickham, Hadley, and Yihui Xie. 2022. *Evaluate: Parsing and Evaluation Tools That Provide More Details Than the Default*. https://CRAN.R-project.org/package=evaluate.

Wilke, Claus O. 2021. *Ggridges: Ridgeline Plots in 'Ggplot2'*. https://CRAN.R-project.org/package=ggridges.

Wilke, Claus O., and Thomas Lin Pedersen. 2021. *Isoband: Generate Isolines and Isobands from Regularly Spaced Elevation Grids*. https://CRAN.R-project.org/package=isoband.

Xie, Yihui. 2014. "Knitr: A Comprehensive Tool for Reproducible Research in R." In *Implementing Reproducible Computational Research*, edited by Victoria Stodden, Friedrich Leisch, and Roger D. Peng. Chapman; Hall/CRC. http://www.crcpress.com/product/isbn/9781466561595.

———. 2015. *Dynamic Documents with R and Knitr*. 2nd ed. Boca Raton, Florida: Chapman; Hall/CRC. https://yihui.org/knitr/.

———. 2019. "TinyTeX: A Lightweight, Cross-Platform, and Easy-to-Maintain LaTeX Distribution Based on TeX Live." *TUGboat*, no. 1: 30–32. https://tug.org/TUGboat/Contents/contents40-1.html.

———. 2021a. *Mime: Map Filenames to MIME Types*. https://CRAN.R-project.org/package=mime.

———. 2021b. *Xfun: Supporting Functions for Packages Maintained by 'Yihui Xie'*. https://CRAN.R-project.org/package=xfun.

———. 2022a. *Knitr: A General-Purpose Package for Dynamic Report Generation in r*. https://yihui.org/knitr/.

———. 2022b. *Tinytex: Helper Functions to Install and Maintain TeX Live, and Compile LaTeX Documents*. https://github.com/yihui/tinytex.

Xie, Yihui, J. J. Allaire, and Garrett Grolemund. 2018a. *R Markdown: The Definitive Guide*. Boca Raton, Florida: Chapman; Hall/CRC. https://bookdown.org/yihui/rmarkdown.

———. 2018b. *R Markdown: The Definitive Guide*. Boca Raton, Florida: Chapman; Hall/CRC. https://bookdown.org/yihui/rmarkdown.

Xie, Yihui, Joe Cheng, and Xianying Tan. 2021. *DT: A Wrapper of the JavaScript Library 'DataTables'*. https://CRAN.R-project.org/package=DT.

Xie, Yihui, Christophe Dervieux, and Emily Riederer. 2020. *R Markdown Cookbook*. Boca Raton, Florida: Chapman; Hall/CRC. https://bookdown.org/yihui/rmarkdown-cookbook.

Xie, Yihui, and Yixuan Qiu. 2021. *Highr: Syntax Highlighting for r Source Code*. https://CRAN.R-project.org/package=highr.

Yao, Yuling, Aki Vehtari, Daniel Simpson, and Andrew Gelman. 2017. "Using Stacking to Average Bayesian Predictive Distributions." *Bayesian Analysis*. https://doi.org/10.1214/17-BA1091.

Zeileis, Achim, Jason C. Fisher, Kurt Hornik, Ross Ihaka, Claire D. McWhite, Paul Murrell, Reto Stauffer, and Claus O. Wilke. 2020. "colorspace: A Toolbox for Manipulating and Assessing Colors and Palettes." *Journal of Statistical Software* 96 (1): 1–49. https://doi.org/10.18637/jss.v096.i01.

Zeileis, Achim, and Gabor Grothendieck. 2005. "Zoo: S3 Infrastructure for Regular and Irregular Time Series." *Journal of Statistical Software* 14 (6): 1–27. https://doi.org/10.18637/jss.v014.i06.

Zeileis, Achim, Kurt Hornik, and Paul Murrell. 2009. "Escaping RGBland: Selecting Colors for Statistical Graphics." *Computational Statistics & Data Analysis* 53 (9): 3259–70. https://doi.org/10.1016/j.csda.2008.11.033.