

Multi100 Project

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1 Introduction

This document reports on a contribution (ID: YMESM) to the **Multi100 Project** of the Center for Open Science. The re-analysis concerns the following paper:

Cohen, J., Dupas, P., & Schaner, S. (2015). Price subsidies, diagnostic tests, and targeting of malaria treatment: evidence from a randomized controlled trial. *American Economic Review*, 105(2), 609-45.

The study was a randomized controlled trial on the effect of price subsidies on access and uptake of a particular malaria treatment, artemisinin combination therapies (ACTs), between households. The focal claim to be assessed is: ‘... a very high subsidy ... increases access [to antimalarials]’ (p. 609.).

In this document, I first spell out the statistical approach of my re-analysis (Section 2), report key results (Section 3), and a brief conclusion (Section 4). Finally, I report supplementary plots (Section 5) and all packages and dependencies used (Section 6).

2 Statistical model

Since the outcome – self-reported treatment of an illness episode with an ACT – is binary (either an ACT was used or it wasn’t), we model the data with an aggregated binomial likelihood. That is, each row in the data set is a household. We fit two Bayesian multilevel regression models that are identical except that one collapses the ACT subsidies into a single variable and the other treats the subsidy levels as separate. We refer to the first model as **pAfit** and the second as **pBfit**, based on Table 2, Panel A and B, of the original study. Our choice of covariates is fully informed by the original study.

In formal notation, our models take the following form, including prior distributions¹:

$$\begin{aligned}y_i &\sim \text{Binomial}(n_i, p_i) \\ \text{logit}(p_i) &= \alpha_{\text{STRATUM}[i]} + \beta_{\text{STRATUM}[i]} A_i + \gamma H_i \\ H_i &\sim \text{Normal}(\nu, \sigma_H) \\ \alpha &\sim \text{Normal}(0, 1.5) \\ \beta &\sim \text{Normal}(0, 1) \\ \gamma &\sim \text{Normal}(0, 1) \\ \nu &\sim \text{Normal}(0, 1) \\ \sigma_H &\sim \text{Exponential}(1)\end{aligned}$$

where y_i is the binary outcome, and p_i is the probability of taking an ACT for the i th household. Since some households reported more than one illness episodes, we allow the number of illness episodes n (i.e., the number

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¹Note that we use a ‘centered’ parameterization in notation, but **brms** by default employ a ‘non-centered’ variant for computational efficiency.

‘trials’ in the binomial model) to vary by household i . In the multilevel linear model, each randomization stratum gets its own varying intercept α_{STRATUM} and varying slopes for subsidy level β_{STRATUM} . We also include age of household head as a fixed effect γ , while we impute missing values in household head age H with draws from a Normal distribution with the mean ν and standard deviation σ_H of the sample, which are themselves given weakly regularizing priors. The prior for the average intercept α and for the average slope of subsidy level β are both given weakly regularizing priors, as are the fixed effect of household head age, which is standardized and hence centered on zero.

The varying effects for randomization strata ensure *partial pooling* across strata, such that each stratum is simultaneously and proportionally *informing* and *informed by* the average intercept and slope estimates (cf., McElreath 2020). This is facilitated via a variance-covariance matrix with multivariate Gaussian priors, where \mathbf{S} is a diagonal matrix of intercept α and slope β , and \mathbf{R} is their correlation matrix with a prior distribution of LKJcorr(4) (Lewandowski, Kurowicka, and Joe 2009). The standard deviations among intercepts σ_α and slopes σ_β are both given an Exponential(1) prior.

$$\begin{bmatrix} \alpha_{\text{STRATUM}} \\ \beta_{\text{STRATUM}} \end{bmatrix} \sim \text{Multivariate Normal} \left(\begin{bmatrix} \alpha \\ \beta \end{bmatrix}, \mathbf{S} \right)$$

$$\mathbf{S} = \begin{pmatrix} \sigma_\alpha & 0 \\ 0 & \sigma_\beta \end{pmatrix} \mathbf{R} \begin{pmatrix} \sigma_\alpha & 0 \\ 0 & \sigma_\beta \end{pmatrix}$$

Analyses are written in the popular open-source programming language R (R Core Team 2021a, version 4.1.2) using the **brms** package (Bürkner 2017a, 2018a, 2021a), an interface to the probabilistic programming language **Stan** (Carpenter et al. 2017; Stan Development Team 2021a). Data wrangling and plotting are primarily facilitated by the **tidyverse** (H. Wickham et al. 2019), **ggplot2** (H. Wickham 2016), **tidybayes** (Kay 2022b), and **bayesplot** (Gabry and Mahr 2021a; Gabry et al. 2019a) packages². The report is written in R **Markdown** (Allaire et al. 2021; Xie, Allaire, and Golemund 2018a).

3 Results

Here, we report results. Our target quantity of interest is *the contrast in posterior predicted probabilities of taking an ACT between the price subsidy interventions compared to no subsidy*. This contrast is computed (in percentage) as

$$\frac{\mathbb{E}(Y = 1 | \text{Subsidy} = 1) - \mathbb{E}(Y = 1 | \text{Subsidy} = 0)}{\mathbb{E}(Y = 1 | \text{Subsidy} = 0)} \times 100.$$

A positive contrast thus means that the subsidies increase the likelihood of taking an ACT. Further, the predictions marginalize over the age range of the sample as well as over the randomization strata.

3.1 Any subsidy level & across subsidy levels

Figure 1 shows the computed contrast as a posterior predicted density for any amount of subsidy (**pAfit** model). Zero (the dotted line) represents equal chance of taking an ACT between the two groups. The density is normalized and the gradient reflects posterior mass. The point represents the median of the distribution with a 95% interval. The median and 95% interval of the posterior predicted contrast between the treatment and the control group is thus 106.64% [-22.08, 352.24], or roughly a doubling with the bulk of the posterior mass being above zero. This is a substantial effect given that the posterior predicted median baseline of taking ACT in the control group is 13.7% [2.7, 41.2].

Likewise, Figure 2 illustrates the computed contrast in posterior predicted probabilities across subsidy levels. The inference is very similar: there is roughly a doubling of chance of taking an ACT between the control and intervention groups, and the bulk of the posterior masses is above zero. Again, this is a substantial effect. While there is little difference between the three subsidy levels overall, perhaps unsurprisingly, the highest

²The package **renv** (Ushey 2022) ensures a reproducible package environment. See Section 6 for a list of R packages, their dependencies, and version number used for this project, created using **grateful** (Rodríguez-Sánchez, Jackson, and Hutchins 2022).

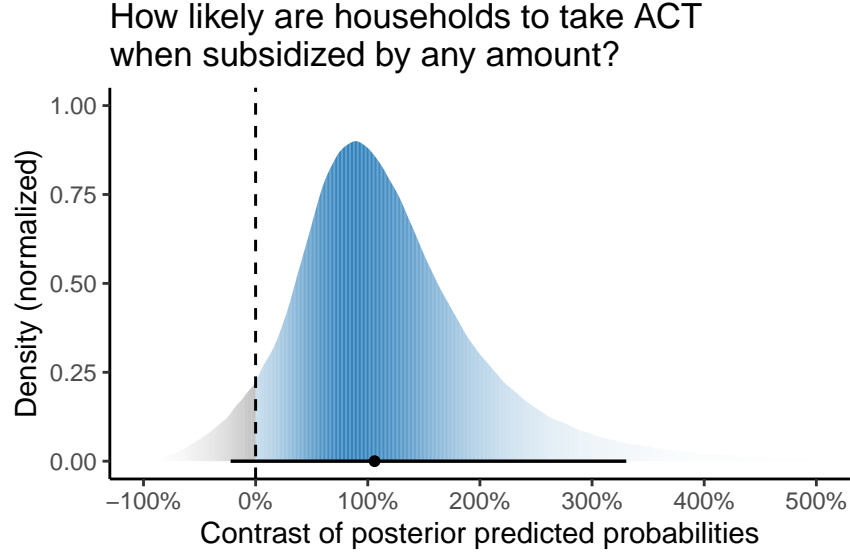


Figure 1: Results from `pAfit` model. Contrast in predicted probabilities of taking an ACT between the price subsidy intervention (any amount) compared to no subsidy. Zero (the dotted line) represents equal chance of taking an ACT between the two groups. The density is normalized and the gradient reflect posterior mass. The point represents the median of the distribution with a 95% interval.

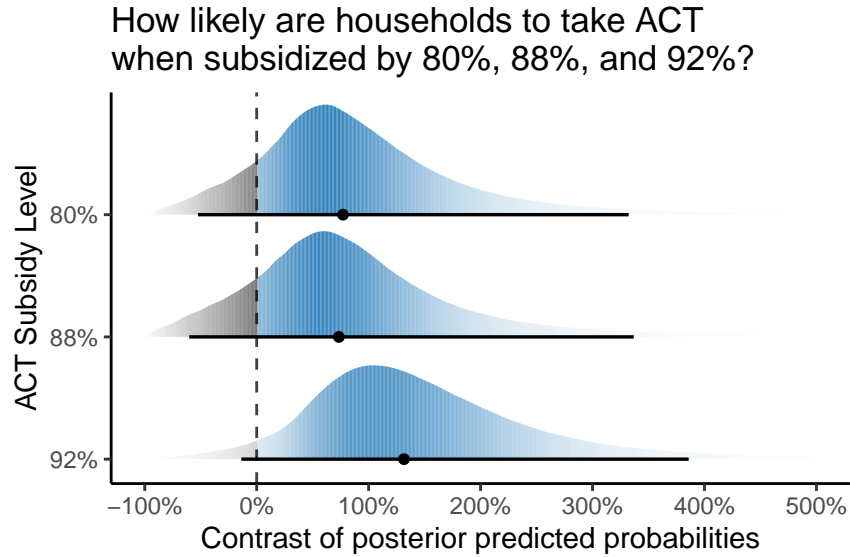


Figure 2: Results from `pBfit` model. Contrast in predicted probabilities of taking an ACT between the price subsidy intervention (for each subsidy level) compared to no subsidy. Zero (the dotted line) represents equal chance of taking an ACT between the treatment and control groups. The densities are normalized and the gradients reflect posterior mass. The points represent the median of the distributions with a 95% interval.

subsidy level (92%) has the highest median posterior predicted change, 133.67% [-13.09, 458.17], with the effects of the 80% price subsidy (78.12% [-52.18, 378.88]) and the 88% price subsidy (74.66% [-60.02, 399.59]) being almost indistinguishable.

4 Conclusion

In conclusion, this re-analysis finds substantial evidence for the claim of the original study that ‘... *a very high subsidy ... increases access [to antimalarials]*’ (p. 609.). Providing households with high price subsidies roughly doubles the chances of taking an ACT for an illness event with malaria-like symptoms, with a baseline likelihood of around 14%.

This inference largely matches the original authors’ findings, namely that ‘[s]ubsidies of 80 percent or more increase the likelihood that an illness is treated with an ACT by 16-23 percentage points (an 85-118 percent increase) [...]’ (p. 628). With that said, the present re-analysis yields a larger degree of uncertainty in inference. This is at least partly due to the fact that the re-analysis marginalizes over the age range of the sample as well as the randomization strata, whereas the original study’s inferences are conditional on specific reference values of the sample. Excluding the random effects of strata (i.e., getting predictions for an average stratum and ignoring stratum-specific variance) reduces the uncertainty such that all 95% intervals no longer (or only barely) include zero (i.e., a ‘statistically significant’ difference; see Figures 3 and 4).

Note, finally, that given the overlap between the presently obtained results and those of the original study (and also due to time and timing), I did not attempt to replicate any follow-up analyses reported in the original study, such as the ones reported in their Panel A and B, Figure 4, where the self-reported data were cross-checked with observational and behavioral data. For the same reason, I also did not attempt to replicate their baseline summary statistics (their Table 1).

5 Supplementary plots

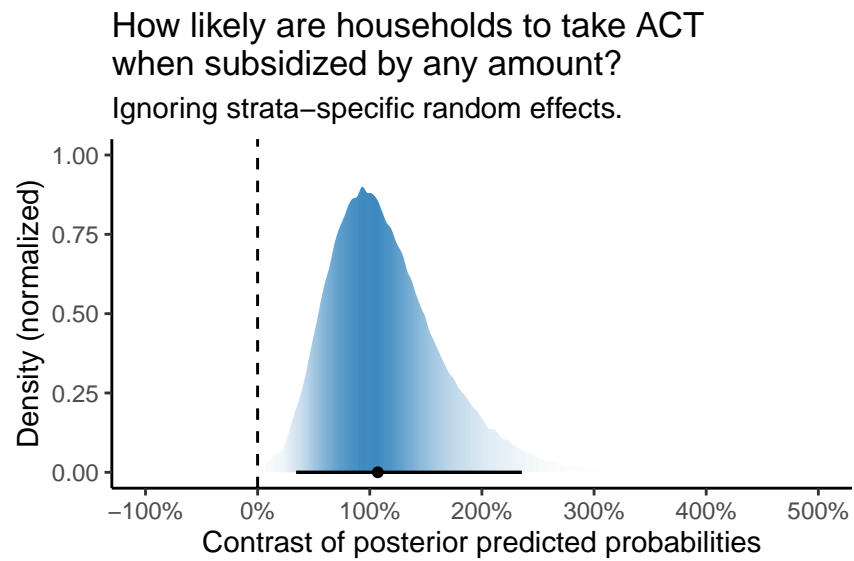


Figure 3: Same as Figure 1, but ignoring strata-specific random effects.

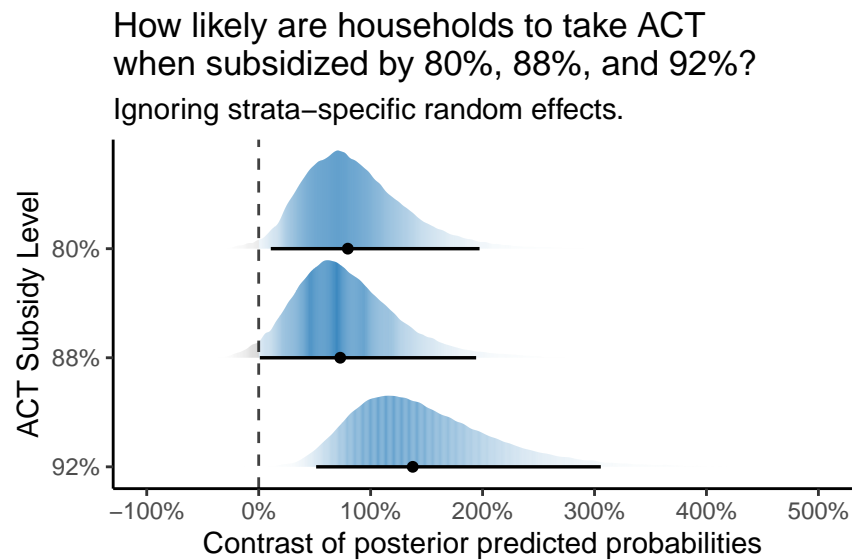


Figure 4: Same as Figure 2, but ignoring strata-specific random effects.

6 R packages and dependencies

We used R version 4.1.2 (R Core Team 2021b) and the following R packages: `abind` v. 1.4.5 (Plate and Heiberger 2016), `arrayhelpers` v. 1.1.0 (Beleites 2020), `backports` v. 1.4.1 (Lang and R Core Team 2021), `base64enc` v. 0.1.3 (Urbanek 2015), `bayesplot` v. 1.8.1 (Gabry et al. 2019b; Gabry and Mahr 2021b), `BH` v. 1.78.0.0 (Eddelbuettel, Emerson, and Kane 2021), `bit` v. 4.0.4 (Oehlschlägel and Ripley 2020), `bit64` v. 4.0.5 (Oehlschlägel and Silvestri 2020), `bridgesampling` v. 1.1.2 (Gronau, Singmann, and Wagenmakers 2020), `brms` v. 2.17.0 (Bürkner 2017b, 2018b, 2021b), `Brodingnag` v. 1.2.7 (Hankin 2007), `bslib` v. 0.3.1 (Sievert and Cheng 2021a), `cachem` v. 1.0.6 (Chang 2021a), `callr` v. 3.7.0 (Csárdi and Chang 2021a), `checkmate` v. 2.0.0 (Lang 2017), `clipr` v. 0.8.0 (Lincoln 2022), `coda` v. 0.19.4 (Plummer et al. 2006), `colorspace` v. 2.0.3 (Zeileis, Hornik, and Murrell 2009; Stauffer et al. 2009; Zeileis et al. 2020), `colourpicker` v. 1.1.1 (Attali 2021a), `commonmark` v. 1.7 (Ooms 2018), `cpp11` v. 0.4.2 (Hester and François 2021), `crosstalk` v. 1.2.0 (Cheng and Sievert 2021), `desc` v. 1.4.0 (Csárdi, Müller, and Hester 2021), `digest` v. 0.6.29 (Antoine Lucas et al. 2021), `distributional` v. 0.3.0 (O’Hara-Wild, Kay, and Hayes 2022), `DT` v. 0.20 (Xie, Cheng, and Tan 2021), `dygraphs` v. 1.1.1.6 (Vanderkam et al. 2018), `ellipsis` v. 0.3.2 (H. Wickham 2021), `evaluate` v. 0.15 (H. Wickham and Xie 2022), `fansi` v. 1.0.2 (Gaslam 2022), `farver` v. 2.1.0 (Pedersen, Nicolae, and François 2021), `fastmap` v. 1.1.0 (Chang 2021b), `fontawesome` v. 0.2.2 (Iannone 2021), `fs` v. 1.5.2 (Hester, Wickham, and Csárdi 2021), `future` v. 1.24.0 [®], `generics` v. 0.1.2 (H. Wickham, Kuhn, and Vaughan 2022), `ggdist` v. 3.1.1 (Kay 2022a), `ggridges` v. 0.5.3 (Wilke 2021), `globals` v. 0.14.0 (Bengtsson 2020), `glue` v. 1.6.1 (Hester and Bryan 2022), `grateful` v. 0.1.11 (Rodríguez-Sánchez, Jackson, and Hutchins 2022), `gridExtra` v. 2.3 (Auguie 2017), `gttable` v. 0.3.0 (H. Wickham and Pedersen 2019), `gtools` v. 3.9.2 (Warnes, Bolker, and Lumley 2021), `HDInterval` v. 0.2.2 (Meredith and Kruschke 2020), `highr` v. 0.9 (Xie and Qiu 2021), `htmltools` v. 0.5.2 (Cheng et al. 2021), `htmlwidgets` v. 1.5.4 (Vaidyanathan et al. 2021), `httpuv` v. 1.6.5 (Cheng and Chang 2022), `igraph` v. 1.2.11 (Csardi and Nepusz 2006), `inline` v. 0.3.19 (Sklyar et al. 2021), `isoband` v. 0.2.5 (Wilke and Pedersen 2021), `jqueryl` v. 0.1.4 (Sievert and Cheng 2021b), `knitr` v. 1.39 (Xie 2014, 2015, 2022a), `labeling` v. 0.4.2 (Justin Talbot 2020), `later` v. 1.3.0 (Chang and Cheng 2021), `lazyeval` v. 0.2.2 (H. Wickham 2019), `lifecycle` v. 1.0.1 (Henry and Wickham 2021), `listenv` v. 0.8.0 (Bengtsson 2019), `loo` v. 2.4.1 (Vehtari, Gelman, and Gabry 2017; Yao et al. 2017; Vehtari et al. 2020), `markdown` v. 1.1 (Allaire et al. 2019), `matrixStats` v. 0.61.0 (Bengtsson 2021a), `mime` v. 0.12 (Xie 2021a), `miniUI` v. 0.1.1.1 (Cheng 2018), `munsell` v. 0.5.0 (C. Wickham 2018), `mvtnorm` v. 1.1.3 (Genz and Bretz 2009; Genz et al. 2021), `nleqslv` v. 3.3.2 (Hasselman 2018), `numDeriv` v. 2016.8.1.1 (Gilbert and Varadhan 2019), `parallelly` v. 1.30.0 (Bengtsson 2021b), `pkgbuild` v. 1.3.1 (H. Wickham, Hester, and Csárdi 2021), `pkgconfig` v. 2.0.3 (Csárdi 2019), `plyr` v. 1.8.6 (H. Wickham 2011), `posterior` v. 1.2.0 (Vehtari et al. 2021; Bürkner et al. 2022), `prettyunits` v. 1.1.1 (Csardi 2020), `processx` v. 3.5.2 (Csárdi and Chang 2021b), `progress` v. 1.2.2 (Csárdi and FitzJohn 2019), `promises` v. 1.2.0.1 (Cheng 2021), `ps` v. 1.6.0 (Loden et al. 2021), `R6` v. 2.5.1 (Chang 2021c), `rappdirs` v. 0.3.3 (Ratnakumar, Mick, and Davis 2021), `RColorBrewer` v. 1.1.2 (Neuwirth 2014), `Rcpp` v. 1.0.8 (Eddelbuettel and François 2011; Eddelbuettel 2013; Eddelbuettel and Balamuta 2018), `RcppEigen` v. 0.3.3.9.1 (Bates and Eddelbuettel 2013), `RcppParallel` v. 5.1.5 (Allaire, Francois, et al. 2022), `renv` v. 0.15.4 (Ushey 2022), `reshape2` v. 1.4.4 (H. Wickham 2007), `rmarkdown` v. 2.14 (Xie, Allaire, and Golemund 2018b; Xie, Dervieux, and Riederer 2020; Allaire, Xie, et al. 2022), `rprojroot` v. 2.0.2 (Müller 2020), `rstan` v. 2.21.3 (Stan Development Team 2021b), `rstantools` v. 2.1.1 (Gabry, Goodrich, and Lysy 2020), `sass` v. 0.4.1 (Cheng et al. 2022), `scales` v. 1.1.1 (H. Wickham and Seidel 2020), `shiny` v. 1.7.1 (Chang et al. 2021), `shinyjs` v. 2.1.0 (Attali 2021b), `shinytan` v. 2.5.0 (Gabry 2018), `shinythemes` v. 1.2.0 (Chang 2021d), `sourcetools` v. 0.1.7 (Ushey 2018), `StanHeaders` v. 2.21.0.7 (Stan Development Team 2020), `stringi` v. 1.7.6 (Gagolewski 2021a, 2021b), `svUnit` v. 1.0.6 (Grosjean 2021), `tensorA` v. 0.36.2 (van den Boogaart 2020), `threejs` v. 0.3.3 (Lewis 2020), `tidybayes` v. 3.0.2 (Kay 2022c), `tidyselect` v. 1.1.2 (Henry and Wickham 2022), `tidyverse` v. 1.3.1 (H. Wickham et al. 2019), `tinytex` v. 0.38 (Xie 2019, 2022b), `tzdb` v. 0.2.0 (Vaughan 2021), `utf8` v. 1.2.2 (Perry 2021), `vctrs` v. 0.3.8 (H. Wickham, Henry, and Vaughan 2021), `viridisLite` v. 0.4.0 (Garnier et al. 2021), `vroom` v. 1.5.7 (Hester, Wickham, and Bryan 2021), `withr` v. 2.4.3 (Hester et al. 2021), `xfun` v. 0.29 (Xie 2021b), `xtable` v. 1.8.4 (Dahl et al. 2019), `xts` v. 0.12.1 (Ryan and Ulrich 2020), `yaml` v. 2.3.5 (Garbett et al. 2022), `zoo` v. 1.8.9 (Zeileis and Grothendieck 2005).

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