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When Does Air Resistance Become Significant in Projectile Motion?

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In an article in this journal,[1] it was shown that air resistance could never be a significant source of error in typical free-fall experiments in introductory physics laboratories. Since projectile motion is the two-dimensional version of the free-fall experiment and usually follows the former experiment in such laboratories, it seemed natural to extend the same analysis to this type of motion. We shall find that again air resistance does not play a significant role in the parameters of interest in a traditional projectile motion experiment.

In a typical experiment in an introductory physics laboratory, a projectile (such as a steel ball) is launched from a tabletop at an angle above the horizontal. The projectile then strikes the floor a distance R (known as the range) from where it is launched, as shown in Fig. 1. Students then measure the range and sometimes the time of flight T using an electronic timer. Knowing the initial speed of the projectile (which can be measured using photogates) and the angle of launch, they theoretically calculate the range, the time of flight, and the maximum height reached by the projectile using the equations of motion for a projectile in the absence of air resistance.

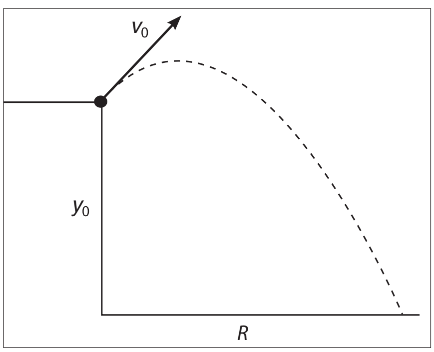

Fig. 1. Projectile launched from a tabletop.

During the motion of a projectile, however, air resistance continuously retards its motion. As a result, the deviation of the actual trajectory of the projectile from that calculated by neglecting air resistance increases as the launch speed, size of the ball, and the time of flight increases.[2] The objective of this study is to provide an estimate of the size of the errors involved in the quantities of interest in such an experiment if they are calculated by neglecting air resistance. Although there are many resources available on the internet that allow interactive simulation of the projectile motion in the presence of air resistance,[3-6] to the best of the author's knowledge, none of these resources presents quantitative information about the size of the errors in calculated values for quantities of interest that result if air resistance is neglected.

The equations of motion of a projectile with an air resistance that is proportional to its speed are exactly solvable and can be found in almost any textbook on classical mechanics.[7] However, except for very limited cases, a linear drag force is not realistic. A more realistic form of air resistance for an object moving in air is proportional to the square of the speed of the object,[1,2]

$$F_{\mathrm{air}} = cv^2, \tag{1}$$

where the coefficient of air resistance c is a constant. Then the equations of motion for the object are given by[8]

$$\frac{dv_x}{dt} = -\frac{c}{m} v v_x \tag{2}$$

$$\frac{dv_y}{dt} = -g - \frac{c}{m} v v_y, \tag{3}$$

where

$$v = \sqrt{v_x^2 + v_y^2} \tag{4}$$

and where we have taken the upward direction to be positive. Equations (2) and (3) are coupled nonlinear differential equations and, although they can be solved analytically using the homotopy analysis method,[9] the solution is not in terms of elementary functions and is beyond the level of introductory physics courses. Therefore, we have to solve them numerically.

A Fortran program incorporating the Euler-Richardson algorithm,[10] which is provided in the appendix,[11] runs on any Fortran compiler, such as the freely available g77. This program solves Eqs. (2) and (3) and calculates the time of flight (T), the range[12] (R), and the maximum height (H) for the projectile. It also calculates the same quantities by neglecting air resistance and finds the percent errors.

One of the most popular projectile launchers used in physics laboratories is made by PASCO, which comes in two models, the short range (ME-6800) and the long range (ME-6801), with maximum muzzle speeds of approximately 7.0 m/s and 8.9 m/s, respectively. They both use steel balls of 2.5-cm diameter.

For a spherical object of diameter D moving in air, the coefficient of air resistance in Eq. (1) in SI units[13] is given by $c = 0.22D^2$. Therefore, the quantity c/m in Eqs. (2) and (3) reduces to

$$\frac{c}{m} = \frac{0.22D^2}{(1/6)\pi D^3 \rho} = \frac{1.32}{\pi D \rho}, \tag{5}$$

where ρ is the density of the object. Thus, for a solid steel ball of density 7850 kg/m^3 and diameter 2.5 cm (the PASCO projectile), we find $c/m = 2.141 \times 10^{-3}$ m^{-1}. If we now specify the initial speed and the angle of launch, the program in the appendix[11] can be used to calculate the time of flight, the range, and the maximum height of the projectile. Since in introductory physics labs essentially all projectiles are launched with initial speeds that are less than 10.0 m/s, we have used initial speeds of 5.0 m/s and 10.0 m/s to calculate the values in Tables I and II, respectively. We have also used an initial height of 1 m, which is the typical height of the laboratory tables. In

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Table I. A solid steel ball of diameter 2.5 cm launched with an initial speed of 5 m/s from an initial height of 1 m. The numbers in parentheses represent percent errors in calculated values assuming no air resistance.

θ (deg)	T (s)	R (m)	H (m)
0	0.45 (–0.11)	2.25 (0.15)	1.00 (0.00)
10	0.55 (–0.07)	2.70 (0.23)	1.04 (0.00)
20	0.66 (–0.04)	3.08 (0.31)	1.15 (0.02)
30	0.77 (–0.01)	3.34 (0.38)	1.32 (0.04)
40	0.89 (0.02)	3.38 (0.43)	1.53 (0.07)
50	0.99 (0.04)	3.16 (0.45)	1.75 (0.10)
60	1.07 (0.05)	2.67 (0.45)	1.95 (0.13)
70	1.14 (0.07)	1.94 (0.45)	2.12 (0.14)
80	1.18 (0.07)	1.02 (0.41)	2.23 (0.15)
90	1.19 (0.07)	0.00 (0.00)	2.27 (0.15)

Table II. A solid steel ball of diameter 2.5 cm launched with an initial speed of 10 m/s from an initial height of 1 m. The numbers in parentheses represent percent errors in calculated values assuming no air resistance.

θ (deg)	T (s)	R (m)	H (m)
0	0.45 (–0.17)	4.50 (0.32)	1.00 (0.00)
10	0.66 (–0.08)	6.48 (0.63)	1.15 (0.03)
20	0.92 (0.07)	8.55 (1.01)	1.59 (0.18)
30	1.19 (0.19)	10.17 (1.36)	2.27 (0.38)
40	1.45 (0.30)	10.94 (1.61)	3.09 (0.57)
50	1.68 (0.38)	10.63 (1.77)	3.96 (0.72)
60	1.87 (0.43)	9.21 (1.81)	4.78 (0.82)
70	2.01 (0.45)	6.78 (1.76)	5.45 (0.88)
80	2.09 (0.46)	3.59 (1.68)	5.89 (0.90)
90	2.12 (0.46)	0.00 (0.00)	6.04 (0.91)

Tables I and II, the numbers in parentheses are percent errors in the corresponding values calculated from the well-known equations in the absence of air resistance,

$$T^* = \frac{1}{g}\left(v_{0y} + \sqrt{v_{0y}^2 + 2gy_0}\right) \tag{6}$$

$$R^* = v_{0x}T^* \tag{7}$$

$$H^* = y_0 + \frac{v_{0y}^2}{2g}, \tag{8}$$

where the star superscript indicates that air resistance is not taken into account. Note that if the projectile is launched with $v_{0y} < 0$, the maximum height is just the initial height instead of that given by Eq. (8).

Table I shows that for an initial speed of 5 m/s, the effect of air resistance in the time of flight, the range, and the maximum height is quite negligible for all launch angles. The maximum error is only about 0.45% in the value of range for launch angles between 50° and 70°. Table II shows that even with an initial speed of 10 m/s, air resistance still does not play a significant role in these quantities, with a maximum error of only about 1.8% in the value of range for the same interval of the launch angles.

Similar results are obtained for spheres launched at angles below the horizontal (negative angles), as shown in the online appendix.[11] Although we have studied the effect of air resistance for only two initial speeds, the errors given in Tables I-IV (Tables III and IV can be viewed in the online appendix[11]) can be interpolated or extrapolated to obtain rough estimates of the errors for other initial speeds. Accurate errors as well as the actual quantities, however, can be obtained by running the program in the appendix[11] or numerically solving Eqs. (2) and (3) by any other method.

The intention of this article is not to suggest that one can neglect air resistance in any projectile motion. In fact for higher launch speeds and larger projectiles, air resistance can be very important and significantly alter the trajectory of the projectile as described by Brancazio.[2] These effects may also be verified by running the computer program in the appendix.[11] The goal here is to show that in typical laboratory experiments on projectile motion, where a small ball and low launch speeds are used, air resistance is not a significant source of error. Therefore, in such experiments discrepancies between the experimental values and those obtained from Eqs. (6)-(8) assuming no air resistance should not be attributed to air resistance.

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11. The appendix can be viewed at *TPT Online*, http://dx.doi.org/10.1119/1.5025298, under the Supplemental tab.
12. In projectile motion, "range" normally refers to the horizontal distance traveled by the projectile when it returns to the same level from which it is launched. In this article, however, we extend this definition to the horizontal distance traveled by the projectile before it hits the ground regardless of its initial position.
13. See Ref. 7, p. 56.

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