Finding Planck's constant using LEDs

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1 Introduction

You must be familiar with light emitting diodes or LEDs for short: they're the little red, blue, and green indicator lights in just about everything and are basically *the* lighting source in our modern lives. Your phone's flashlight for example is an LED. In this lab, we'll do a physics-style study of LEDs.

An LED is a semiconductor PN junction that emits light when a voltage is applied across it. It is inherently quantum device. The wavelength of the light emitted is determined by a quantum band-gap manufactured into the LED itself. The applied voltage causes an electron at the top of the band-gap to fall through it, as it combines with a positive charge at the bottom of the band-gap. A photon is emitted when this occurs. The photon wavelength is related to the size of the band-gap, with $\lambda \approx hc/E_g$, where E_g is the size (in energy) of the band-gap. A band-gap is the bulk material analog of two discrete electron levels in an atom. In the atom, an electron emits light of a definite wavelength when it falls from the higher energy level to the lower. The same thing happens with a band-gap.

The P in PN refers to a semiconductor that is electron deficient (i.e. net positive), while an N semiconductor has an excess of electrons and is net negative. The electron deficient regions are called "holes" in the P material. Here are the basics.

In Fig. 1(a), a P and N semiconductor are initially brought together. Afterwards in (b) the holes (or positive) and negative charges diffuse across the PN barrier until an equilibrium is reached. The length d_0 is called the depletion zone. At equilibrium in (c), a net electric field E_0 exists across the depletion zone due to the diffused charges. Note the y-axis in (c) is voltage. Thus, the "shelf" structure in (c) is indicative of a potential difference now set between the P and N junctions. This shelf is a potential barrier, keeping the negative and positive charges from further flowing across the junction.

Extending this idea, we now look closely at an LED, which is a PN junction optimized to emit light, as shown in Fig. 2.

In Fig. 2(a) the electrons (black dots) in the N material sit in their conduction band (CB). The holes (white dots) sit in their valence band (VB) on the other side of the barrier. The y-axis is now an energy axis, and note the VB energy is less that that of the CB. In (a) the shelf-height eV_0, keeps the two charges separate across the barrier. However, in (b), when an external voltage is applied V_{app} as shown, $V_0 \rightarrow V_0 - V_{app}$. Thus the barrier is lowered as shown in (b), and the electrons can now flow into the P material.

Once electrons flow into the P material, (re)combination with them is now possible. Electrons

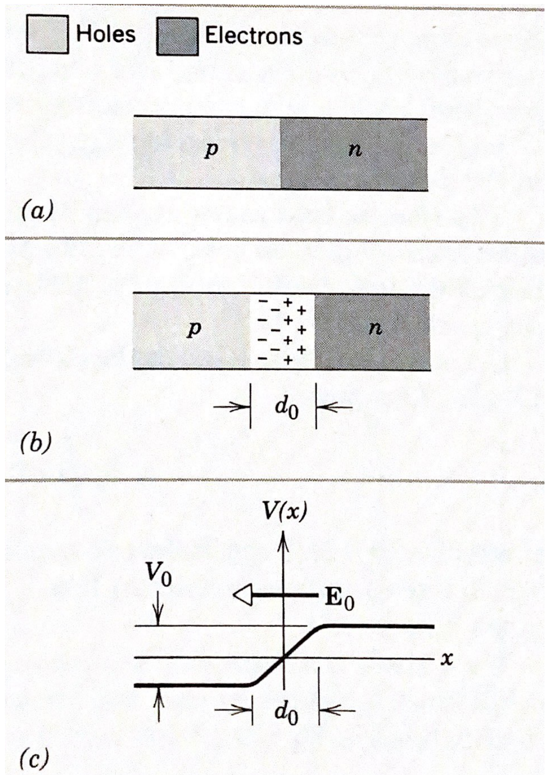


Figure 1: Basics of a PN junction. (From Halliday and Resnick, Part 2, Extended edition, p. 1249.)

combine with holes by falling through the band-gap emitting a photon of energy E_g as they do so. This is analogous to to an electron de-exciting in an atom.

Thus an LED is a PN junction that emits light when biased with an external voltage. The photon energy is directly related to the band-gap E_g (g for gap). The band-gap is an an energy difference between the high and low boundaries of the gap. So the energy of the photon the LED emits is E_g, so for the emitted photon, $E_g = h\nu$.

This is a DIY lab, you'll build your own LED circuits, take some data from it, and arrive at an approximation for Planck's constant using LEDs with a variety of light emission wavelengths.

2 Theory

You know that for a photon that

$$E = hf, \tag{1}$$

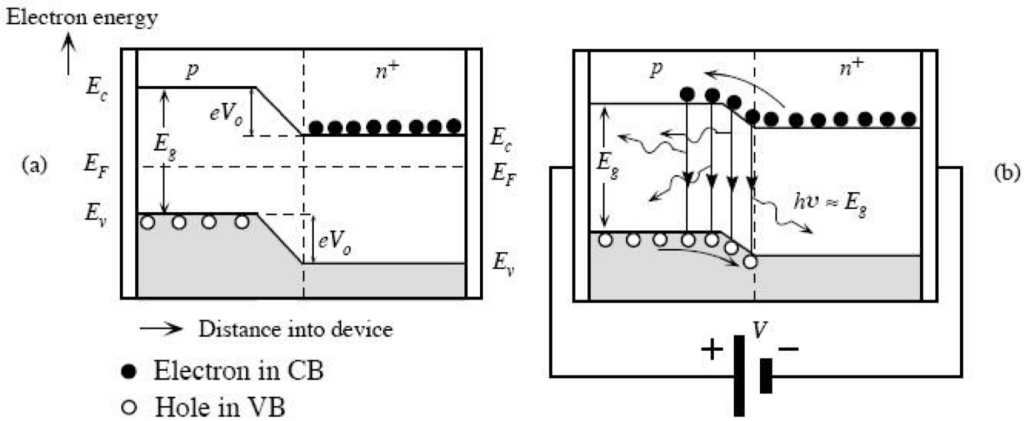


Figure 2: Basis of an LED. From `https://www.ele.uri.edu/~vijay/ELE432_Report_LED.pdf`.

where h is Planck's constant, f is the photon frequency and E the photon energy. Substituting $f = c/\lambda$ we get

$$E = \frac{hc}{\lambda}. \tag{2}$$

You are likely well in command of this equation from your Modern Physics class. It allows you to compute the energy of a photon of wavelength λ. And in doing so, the constants h and c are needed.

Applying this to an LED for $E = E_g$ we get

$$E_g = \frac{hc}{\lambda}. \tag{3}$$

Suppose though, if we measured λ emitted from the LED and we also had a way of measuring E_g? The equation would provide a correspondence between E_g and λ via the constants c and h. Here we'll assume we know c and try to determine h.[1] So what about *measuring* E_g?

We'll have more to say about this later, but for now suppose that for an LED, E_g is related to V_b, which is the threshold of the applied voltage at which the LED just begins to emit photons. The "b" stands for "barrier potential." In Fig. 2, it's the applied voltage that lowers the barrier voltage just enough to allow electrons to flow into the P material and subsequently recombine with holes to emit photons.

So, supposing $E_g = eV_b$, the last relation can become

[1]Note that h and c are both fundamental constants. They cannot be written in terms of anything else more fundamental. See `https://physics.nist.gov/cuu/Constants/introduction.html`.

LED Color	λ
Green	523.63 nm
Red	632.19 nm
Blue	460.91 nm
Yellow	589.85 nm
White[†]	445.169 nm

Table 1: Central wavelengths of a given LED emission, ±10 nm. [†]Note that white LEDs are blue LEDs at their core. A fluorescent coating (having nothing to do with the core LED emission) produces more broadband (white) light from the blue.

$$eV_b = \frac{hc}{\lambda}. \tag{4}$$

This is an equation that can be linearized to determine h if we can measure V_b and λ. Here λ is the emission wavelength of the LED and will be measured using a spectrometer (see Table 1). So measuring V_b will be our focus.

2.1 Issues with $E_g = eV_b$

We note that there are physics-related problems in declaring that $E_g = eV_b$. It should be more $E_g \approx eV_b$ and even this has some issues, but we'll go with it for now and refine our thinking as the lab unfolds.

In short, if you look back at Fig 2, it is not clear why the V_b, the voltage V_0 needs to be lowered by to commence photon emission, is not only related to the band-gap energy E_g, but *equal* to it. It is hard to ignore though, because this experiment does such a good job at delivering an experimental value for h. We'll discuss this all more as we go, but let's look at some data first.

3 Measurement of V_b

So we need to find a way of measuring V_b. We'll do so in two different ways. Both involving finding the lowest voltage at which the LED begins to emit photons.

How will you determine V_b? Do you turn up a voltage across the LED and try to see the first photons coming out? No, but we have two methods for doing so. Both will require you to know the wavelengths of LEDs used. They were measured using an OceanOptics USB2000 spectrometer, and are given in Table 1.

3.1 Method I: I-V curve

What you'll do is take an I-V curve for an LED. This means, apply a voltage across it, and measure the current it consumes. Taking an I-V curve is a very common technique for assessing an electrical device.

If you do this for a diode, you'll get a typical diode curve that looks like Fig 3.

Figure 3: Sketch of current vs. voltage for a diode. Note the rapid turn on region.

Here as you increase the voltage applied across the LED (V), it draws almost no current until suddenly it begins to, and very rapidly consumes more and more. To find V_b, you'll fit a line to the *linear portion* of the rising current, then extrapolate it down to the x-axis. The voltage where it intersects the x-axis is V_b, as shown here in the circle.

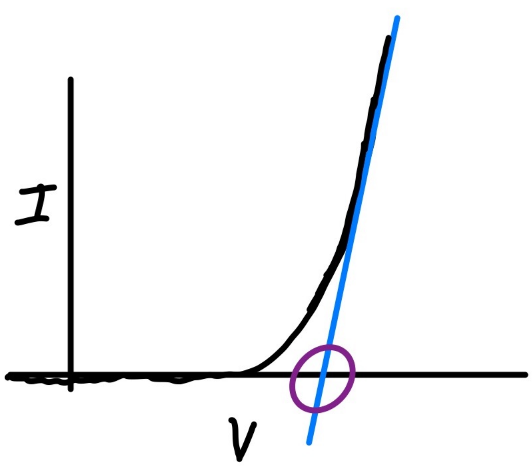
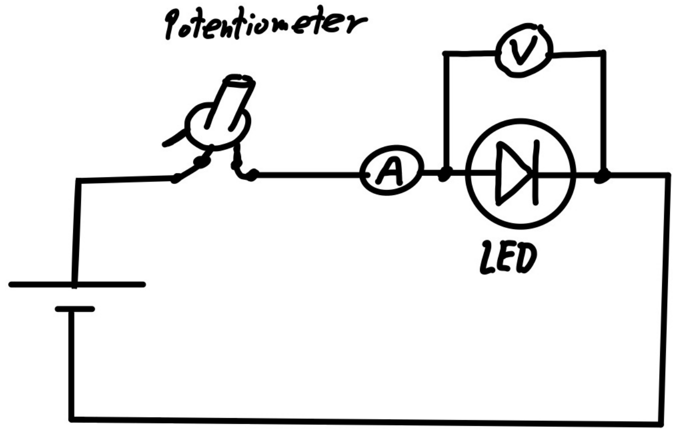


Figure 4: Fitting the linear region of a diode "on" curve and extrapolating down to the x or voltage axis. The intersection of the fit and the x-axis (see circle) is V_b.

Hopefully you can see how the extrapolation point more-or-less sets where the rapid LED turn-on begins. So your data taking plan is to find V_b for a variety of LEDs (hence values of λ), then use Eqn. 4 in some kind of curve fit to find h.

3.1.1 Experiment

Here's the circuit you need to build to measure the I-V curve. Hints for building it with the supplies your purchased can be found here: `https://youtu.be/LCZ7djKlMrE`.



Note that Ⓐ is one of your meters in current-mode and Ⓥ is one of your meters is voltmeter mode. This is why you need two "voltmeters." The potentiometer serves as a current limiter to the LED. Be sure you start with the potentiometer set at about its midpoint. When you wire everything up, the LED should be off (the resistance of the potentiometer is too high). As you lower its resistance slowly, you'll start seeing a current develop in the circuit as the LED turns on.

Take an I-V curve for as many LEDs as you know λ for (see the Table 1). You'll have to swap each LED in and out of your circuit, measuring I vs V for each one at a time.

3.2 Method II: Capacitor discharge halt

Here's another method for measuring V_b that may give better results for it. Suppose you built the circuit shown in Fig. 5.

If you touched and held the free wire to point A (for a few seconds) the voltage source would charge up the capacitor, C. If you then touched the free wire to point B, the capacitor would discharge through the resistor, R. The discharge time constant would be $\tau = RC$. The discharge would continue until the capacitor was depleted of its charge, leading to a voltage across it and the resistor of 0V.

Now suppose an LED was put in series with the resistor as shown in Fig. 6.

The capacitor can again be charged by connecting the free wire to A. Afterwards, if it was connected to B, the capacitor would discharge through the R and the LED. Think for how long the discharge would continue. Would it continue until the capacitor was depleted?

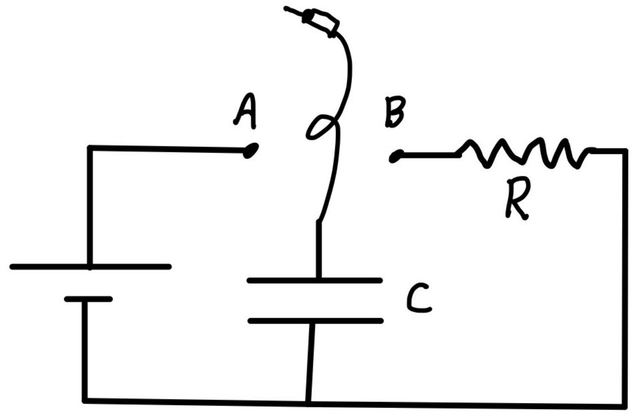


Figure 5: A charge/dischage RC circuit. Holding the free wire at A would charge the capacitor. Holding it to B would discharge it through the resistor.

No. In this circuit, the discharge would continue until the LED stopped conducting in the forward direction. In other words, the capacitor would have a discharge path until the LED stopped conducting at its V_b. The LED will in effect "halt" the capacitor's discharge, just as it reaches its V_b. In this circuit, by discharging the capacitor through R and the LED, one can measure the voltage across the LED at a time of $\approx 10RC$ after the discharge was commenced. This voltage across the LED would be V_b.

Build this circuit using a $C = 1000\mu$F and $R = 1000\Omega$ resistor. Hints on building this circuit given the supplies in your kit can be found here: `https://youtu.be/RcLXMX6g7tE`.

When done, charge the capacitor, then discharge it one-by-one through different LEDs. After about $10RC$, measure the voltage across the LED. This will be the LEDs V_b.

4 Analysis

As mentioned, use your V_b vs λ data and some linearized form of Eqn. 4 to find h. Be sure you include error analysis in your work and report Planck's constant with its uncertainty.

5 Theory leading to an additional measurement

If you reflect on the LED structure from the introduction, it is not clear why these experiments give a reasonable value for h. Why would the voltage needed to lower eV_0 in Fig. 2, as in $eV_0 \to e(V_0 - V_{app})$ be related to the band-gap energy E_g of the LED?

Evidently they are related. You've just seen in your data that the values of V_b collected

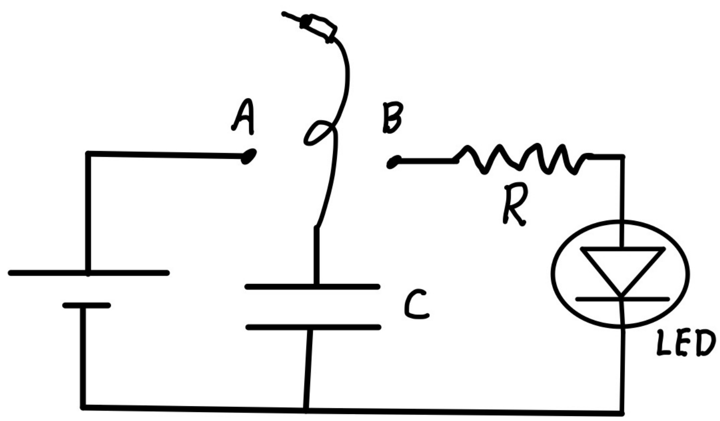


Figure 6: A charge/dischage RC circuit. Holding the free wire at A would charge the capacitor. Holding it to B would discharge it through the resistor and the LED. The discharge would stop when the LED ceases its forward conduction at V_b.

varied from LED to LED. In other words, each LED did not begin emitting photons at the same V_b. In fact loosely, V_b appeared somewhat inverse to λ (meaning the higher the LED's emission λ, the lower V_b was). Since λ is related to an LEDs E_g, then V_b must be related as well, it's just not clear how.

But why would $eV_b = E_g$, allowing is to write Eqn. 4? (Hint: no one knows why this would or could be true, other than it "kind of works here for LEDs.")

5.1 The real LED theory

The real current equation for LEDs is

$$I(V) = I_0 \left(e^{\frac{V}{V_t}} - 1 \right), \tag{5}$$

where I is the current consumed at voltage V, given a reverse current I_0. Diodes don't conduct when reverse biased, and I_0 is this small reverse current (it won't be strictly zero). Here $V_t = \eta kT/e$, where η is an LED specific parameter, k is the Boltzmann constant, T is the temperature and e the charge on an electron. There simply is no Planck's constant to be found in Eqn. 5, but it does lead to another measurement possibility.

We claimed LED current starts to flow when an applied V lowers the shelf in Fig. 2 so that electrons may cross the barrier. We stated that if $V_0 \to V_0 - V_b$ with a sufficient V_b, current can flow and light will be emitted. So Eqn. 5, would become

$$I(V) = I_0 \left(e^{\frac{V-V_b}{V_t}} - 1 \right). \quad (6)$$

Now, η for LED is typically between 1 and 3, so typical values for V_t are around 0.04V or so at room temperature. If $V - V_b >> V_t$, then $e^{\frac{V-V_b}{V_t}} >> 1$ and Eqn. 6 can become

$$I(V) = I_0 e^{\frac{V-V_b}{V_t}}. \quad (7)$$

6 Method III: Constant current

Guided by Eqn. 7, let's consider making measurements of each LED, while injecting the *same current* through each. In this case, $I(V)$ in Eqn. 7 would be the same constant for each LED, which would mean $V - V_b$ would also have to be a constant for each LED. Since each emits a different wavelength, this forces the meaning to V_b to more likely mirror E_g, for what else could one change about an LED to get a different λ from each?

Suppose you could design a circuit that could send a constant current though all of your LEDs. This can be done by wiring all LEDs in series across a power source, as shown in Figs. 7 and 8.

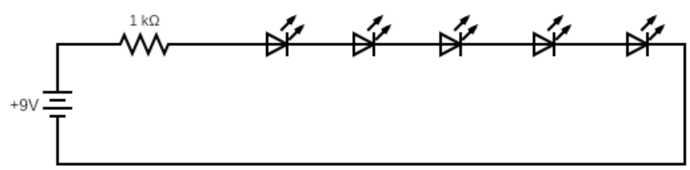


Figure 7: All LEDs in series would have the same current passing through each.

Be sure the flat edges of all LEDs point toward the negative side of the battery. Note you'll need the 9V battery if you want all LED to turn on. The 1k resistor coming from the +9V rail and feeding the circuit protects the LEDs against drawing too much current. The black and blue leads going down off the bottom of the breadboard go to an ammeter to give the current. When all LEDs are on and a current is measured, use your 2nd voltmeter to measure the voltage across each LED one by one. Let's call $V_s^{(n)}$ the voltage across the n^{th} series wired LED.

6.1 Constant current analysis

From Eqn. 7, if $I_n = I_0 e^{\frac{V-V_s^{(n)}}{V_t}}$, and we hypothesize that $eV_s^{(n)} = E_g^{(n)}$ then this becomes

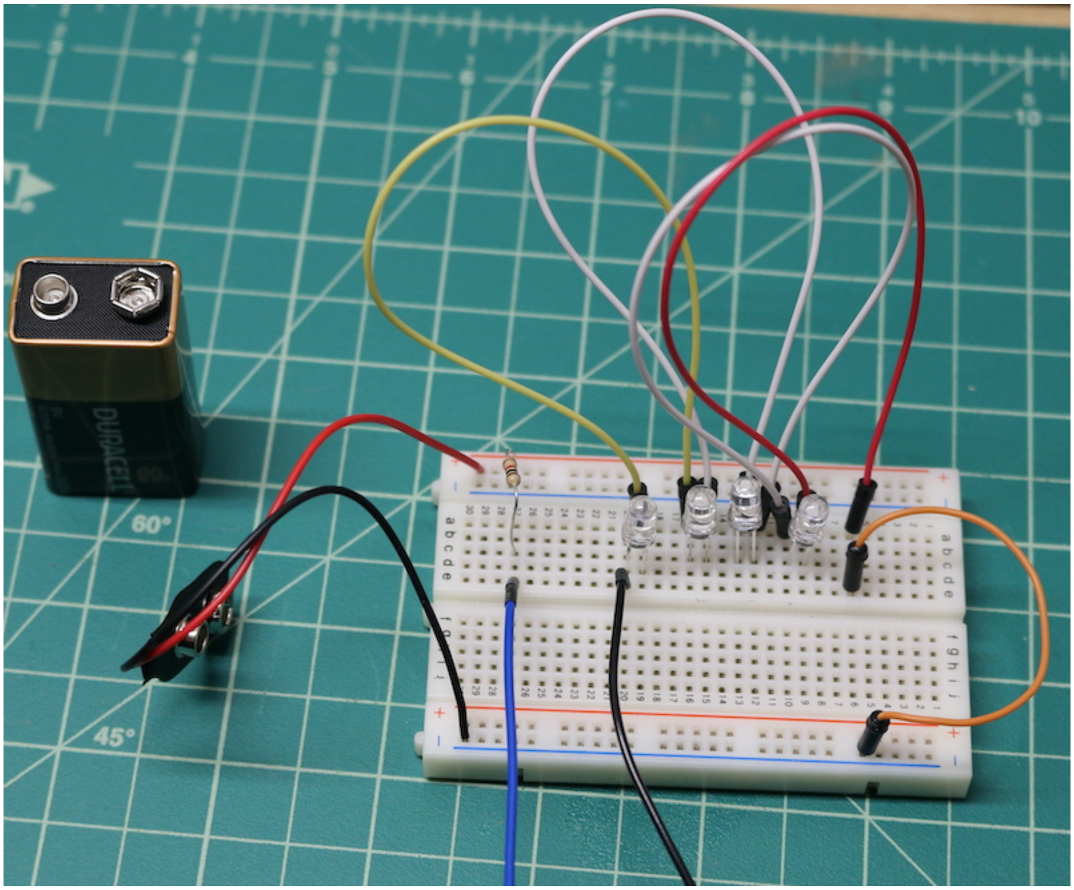


Figure 8: All LEDs in series, to pass the same current through all of them. Note a 9V battery is needed to turn all of the LEDs on at once, and the 1k resistor in series with them all for protection against drawing too much current. The black and blue leads going down off the bottom of the breadboard go to an ammeter to give the current. The voltage across each LED is also measured.

$$I = I_0 e^{\frac{V - E_g^{(n)}/e}{V_t}}. \tag{8}$$

Again with $E_g^{(n)} = hc/\lambda^{(n)}$, this becomes

$$I_n = I_0 e^{\frac{V - \frac{hc}{e\lambda_n}}{V_t}}. \tag{9}$$

Let's linearize this now, to guide our analysis. First, solve for $I/_n I_0$ to get

$$\frac{I_n}{I_0} = e^{\frac{V - \frac{hc}{e\lambda_n}}{V_t}}. \tag{10}$$

The ln of both sides gives

$$V_t \ln\left(\frac{I_n}{I_0}\right) = V - \frac{hc}{e\lambda_n}. \tag{11}$$

Finally, solving for V and letting $V \to V_n$ to reflect the voltage across the n^{th} series LED we get

$$V_n = \frac{hc}{e\lambda_n} + V_t \ln\left(\frac{I_n}{I_0}\right). \tag{12}$$

So our analysis now is to plot V_n vs $1/\lambda_n$, or the voltage across the n^{th} series LED vs the wavelength it emits. The slope will be hc/e, allowing us to determine Planck's constant yet again. Note how all of the hard parameters of the LED, namely the η in V_t and I_0 will be bunched up in the y-intercept.

6.2 A hint at why the V_b experiments worked

In Methods I and II, we weren't sure why h came out from determining V_b. The analysis here provides a clue. In each of the I-V curves, recall we set $I = 0$ and solved for the voltage at which this occurred, and called it V_b. In Eqn. 12, we could do the same, in setting $I_n = 0$, but this leads to a problem evaluating the ln. Suppose we set $I = I_0$. I_0 is the reverse current in an LED and is incredibly small, on the order of 10^{-14} and smaller. Setting I_n equal to I_0 is close to setting it to zero, making the ln vanish. Thus, V_n becomes

$$V_n = \frac{hc}{e\lambda_n}, \tag{13}$$

which is Eqn. 4!

Thus, we hypothesis that Method I worked because the V_b was determined by the slope of the I-V curve for the LED. And, as you know, the slope of a line is independent of its y-intercept, or constant along the vertical (or current axis). Thus the technique worked, but it took us this long to provide some logic as to why.

There are still problems with $eV_b = E_g$ (see "Experiments with light-emitting..." below). Usually h can only be found to within 10% of the accepted value. Also, photons energies emitted are typically $7\% - 20\%$ smaller than a given LEDs E_g. Also, values of V_b typically come in lower than E_g/e. Table 1 also quotes the peak wavelength emitted by an LED. Is this the correct wavelength to use, or is there are ± 10 nm uncertainty in any wavelength?

7 Conclusions

We hope you enjoyed determining Planck's constant using about $20 in common electronic parts (minus the $2000 spectrometer used to find the λs). Funny what a quantum world we live in with our technology huh?

References

This lab is a thorough reflection of the considerations, suggestions and techniques in the following papers.

1. One of the first attempts: `https://aapt.scitation.org/doi/abs/10.1119/1.2344608`
2. Another early attempt: `https://aapt.scitation.org/doi/10.1119/1.2350479`
3. The debate begins on why the initial experiments seem to work: `https://aapt.scitation.org/doi/abs/10.1119/1.18404`
4. And continues: `https://aapt.scitation.org/doi/abs/10.1119/1.19034`
5. Paper on the discharge halting method: `https://aapt.scitation.org/doi/abs/10.1119/1.2981288`
6. Experiments with light-emitting diodes: `https://doi.org/10.1119/1.3599072`
7. Charms and pitfalls of LEDs: `https://aapt.scitation.org/doi/10.1119/1.2344644`
8. The LED as the "ultimate lamp": `https://aapt.scitation.org/doi/10.1119/1.1301966`
9. What could go wrong with LEDs? `https://iopscience.iop.org/article/10.1088/1361-6552/ab525d`
10. Simple experimental verification of the relation between the band-gap energy and the energy of photons emitted by LEDs: `https://iopscience.iop.org/article/10.1088/0143-0807/28/3/010`