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RESEARCH ARTICLE

Numerical Adaptations of Captain Sumner's 1837 journey: a context for teaching celestial navigation

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Abstract

In an effort to add context to a classroom lesson on celestial navigation, we present a numerical adaptation of Captain T.H. Sumner's 1837 journey into St. George's Channel. The adaptation is programmed into a 'live' web-based map. This allows for a flexible and highly visual presentation that highlights two important topics in celestial navigation: the origin of the line of position and the scale of maps. Considerations driving the numerical adaptation are discussed, as is as an overview of a classroom lesson we have been using.

1. Introduction

In a previously published manuscript in this journal (Bensky, 2018), the use of Google Maps (Google Maps, 2015) was detailed as a technique for producing accurate, flexible, and appealing visuals for use in teaching celestial navigation (hereafter CN). In this paper, we present another use case of this technique, which is to reproduce Captain T.H. Sumner's 1837 journey up the Celtic Sea (Richardson, 1943; Williams, 1994; Vanvaerenbergh and Ifland, 2003; NIMA, 2015), as part of a classroom lesson on CN.

As before, our needs are to support a unit on CN in a university-level general education course that covers the history and science of navigation (Astronomy and Astrophysics, 2020). Finding compelling content and lessons on CN that hold the attention of today's university student is difficult. Most published material on CN focuses on practicing the technique with real or contrived scenarios, aimed at the soon-to-be sailor who may need to put CN skills promptly into practice (Schlereth, 2000; Karl, 2011; Burch, 2017). Such material also includes many puns justifying the need for learning classical CN only to handle a situation where technology (such as GPS) fails. Our view of CN is different: given the topic of our class (navigation), CN is presented as a success in the application of scientific and mathematical principles after the long struggle to navigate reliably up to about the year 1800. Thus our continual effort to create CN-related pedagogical materials.

When learning of Sumner's journey and his discovery of the line of position, we immediately saw great pedagogical potential, in that as a journey of record it includes both a reflective narrative from Sumner himself as well as his numerical navigational data. We thought it could provide useful context and purpose for many considerations involved with the teaching of CN, including the use of spherical trigonometry, the navigational triangle, sun shots, using a nautical almanac, hour angles, map scales and plotting (Schlereth, 2000; Karl, 2011). Also, being a successful journey, it provides a sorely needed positive note on pre-GPS navigation, after covering the unsuccessful efforts of Shovell, Anson and

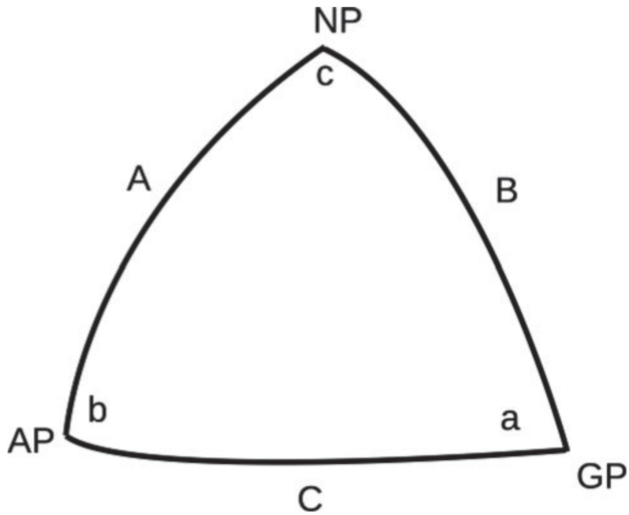

Figure 1. *Navigational triangle.*

LaSalle (Sobel, 2007; Bensky, 2013), which are used early in the class to motivate the need for progress in navigational science.

This paper proceeds with the two sections that follow. In Section 2, we show our best numerical adaptation of Sumner's journey. In Section 3, our classroom lesson is outlined, as it draws from the numerical information of this journey. This is followed by a conclusion and 'exercises' that may be offered to the student (Appendix). It is hoped this work will appeal to the reader learning or strengthening their own knowledge of celestial navigation, as well as instructors of CN in need of a classroom lesson.

2. Sumner's journey

Captain Sumner's journey is well covered in other work (Vanvaerenbergh and Ifland, 2003). Briefly, he was headed to Greenock, Scotland, from Charleston, South Carolina, USA. Due to bad weather as he approached the south coast of Ireland, he was only able to take a single altitude measurement of the sun at 10:15 am on December 17, 1837. From this altitude (alt), knowledge of the geographical position (GP) of the sun at this time, and his latitude (lat) from deduced reckoning (DR), he was able to compute his longitude (lng). (This computation was called 'longitude by chronometer,' not to be confused with the dual chronometer technique of finding longitude.) He was wary of a persistent wind from the south, so he made two further lng calculations, each assuming his DR lat was an additional 10′ further north. When all three points were plotted, he found they all fell on a straight line. He realised observers at all three points would measure the same alt of the sun simultaneously, ushering in the idea of the 'line of position', for which Captain Sumner is known. Fortuitously, his interim ground truth, Smalls lighthouse, also fell along this line. Sumner sailed NE along this line and promptly spotted the lighthouse.

2.1. Numerical representation of Sumner's journey

Representing Sumner's journey on a programmable map requires accurate numerical knowledge of his path, so we now must sift through what is known (and believed) about his journey in order to create a first visual on a programmable map, using the techniques from our previous work.

The central CN theme of Sumner's calculations uses a spherical triangle, like that represented by Figure 1.

Here, NP is the north pole, GP the geographical position of the sun, AP the ship's assumed position, c the local hour angle, b the azimuth (or angle) to GP, A the co-latitude of the AP, B the co-latitude of

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Figure 2. *Blue dot: Sumner's original DR position, yellow dot: Smalls lighthouse (LH), Red dots: Sumner's calculated positions (0, 1, and 2).*

GP, and C the co-altitude of sun. Angle a is not used. The pertinent equation for Sumner's approach is

$$\cos(c) = \frac{\cos(C) - \cos(A)\cos(B)}{\sin(A)\sin(B)}. \tag{1}$$

which links B from a nautical almanac, C from an alt measurement, and A from the DR lat. Solving for c is related to his lng via the Greenwich Hour Angle (GHA) of the sun (also from a nautical almanac). The parameter A is what Sumner varied in 10′ increments north. To begin with a numerical recreation, we pull statements and figures from a reprint of Sumner's original publication (Vanvaerenbergh and Ifland, 2003). In doing so, we immediately found a problem with the rote literature. We thus work up two possible numerical adaptations of the journey, suitable for our classroom lesson.

2.2. Initial problem

Sumner took an alt measurement of the sun on 1837-12-17 at 10:15 am of 12°10′ (page A-15 of Vanvaerenbergh). On page A-14 of the same work, his DR lat was reported as 51°37′. His DR lng was not reported but can be estimated from a map plot presumably produced by Sumner (Plate III of Vanvaerenbergh) to be 6·73 W (here −6·73°).

His DR lat gives A in Equation (1) and his alt measurement gives C. Side B comes from use of a nautical almanac to find the declination (decl) of the sun at the time of his measurement. Computing lng requires the sun's GHA, also from a nautical almanac. We used online calculators for this, setting them to 10:15 am on 1837-12-17 (183 years ago). We found a decl of 23·22°S (NASA, 2020; NOAA, 2020) and a GHA of 334·67° (Umland, 2020). With these numbers in place, we were ready for our first numerical recreation at Sumner's journey.

Using Equation (1) as described, we get his 'longitude from chronometer' location to be (51·62°, −4·71°), and his two 'what-if' points (each 10′ further north) to be (51·79°, −4·07°) and (51·95°, −3·39°).

A plot of these points using our mapping technique is shown in Figure 2. In this figure, the blue point labelled 'DR' is his supposed DR position and the yellow point labelled 'LH' is Smalls lighthouse. The three connected red points produce a line through the three computed positions (here labelled 0, 1 and 2), the plot of which immediately exposes a problem.

Sumner claimed that his first calculation (i.e. our point 0) placed his position 9 Nautical Miles (NMi) east of his DR, and the second two (points 1 and 2) 27 and 54 NMi ENE of his DR (hereafter the '9 + 27 + 54 check'), respectively. In Figure 2, the first point is 75 NMi and the other two are 100 and 126 NMi from his DR. Also, the line containing the three computed positions supposedly contained Smalls lighthouse.

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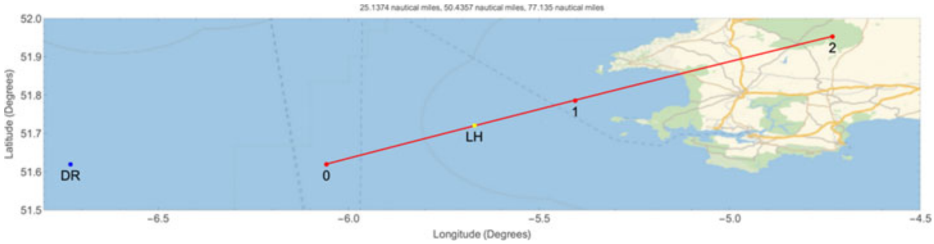

Figure 3. *Sumner's computed positions with modified GHA and declination values.*

Figure 4. *Adaptation #1 of Sumner's calculations. The blue dot (original DR) is 9 NMi from the leftmost red dot, and 34 and 62 NMi from the more rightward red dots.*

The entire line is obviously shifted too far to the east, does not contain the lighthouse and is mostly on land near the mainland U.K. We must therefore re-examine the parameters involved prior to bringing this journey into the classroom. We do so in two adaptations presented below, a process made manageable using our programmable mapping technique.

2.3. Adaptation #1

In this adaptation, we assume that Sumner's altitude measurement of 12·1° is reliable, as is his DR lat. We wonder, though, if extrapolating back 183 years for the GHA and decl gave correct values. We did our due diligence in this regard, consulting the finest numerical sources we could find (NASA, 2020; NOAA, 2020; PyEphem, 2020; Umland, 2020; Wolfram, 2020), but still had lingering doubts (and realised that implementing our own ephemeris computations is beyond the scope here). We thus allow slight manual variation in these parameters with the goal of having Smalls lighthouse fall on the line, while point 0 moves closer to the DR position. (This is one instance of the value of Sumner's narrative.) Our eventual GHA and decl came out to be 333·2° and 23·25° (each changed by <1% of those originally stated) to obtain the triple of (51·62°,–6·06°), (51·79°,–5·41°) and (51·95°,–4·73°) for points 0, 1 and 2, respectively, resulting in Figure 3.

As pointed out by a reviewer, this is equivalent to considering that sun may have been observed approximately 6 min later than the time reported by Sumner. We find this small time difference acceptable given the well-known historical hardship of keeping accurate time at sea. This places the lighthouse on the same line as the computed points, but still leaves his original DR position 25 NMi from the first point 0. We thus adjust his DR lng to −6·3°, making the distance 9 NMi, 34 NMi and 62 NMi, comfortably close to the 9 + 27 + 54 check, as shown in Figure 4.

We can proceed in our lesson using the points in Figure 4, but note that it still differs from Sumner's account (Plate III of Vanvaerenbergh), in which Smalls lighthouse falls between points 1 and 2 in the triple, not between 0 and 1 as shown in Figure 4. This suggests Sumner's stated DR lng was too far west by 0·43°.

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2.4. Adaptation #2

Adaptation #1 allowed for a small and manual changes to the GHA, decl and lng used in Sumner's calculations. While yielding results closer to Sumner's narrative, it still did not produce a close match to it. Manual adjustment of the parameters was difficult, as small changes to one required changes to others, all while attempting to nudge the system closer to Sumner's narrative. This naturally led us to wonder if allowing for small changes in *all* of Sumner's navigational parameters would lead to another, more optimised adaptation.

We decided on a two-fold goal for a computer-driven minimisation algorithm: (1) minimise the sum of distances to the $9+27+54$ check, and (2) minimise the distance of Smalls lighthouse to the line containing the three computed points. The driving equations are as follows.

First, Equation (1) is solved for c, giving

$$c(A,B,C) = \frac{180}{\pi}\left(\cos^{-1}\left[\frac{\cos(C)-\cos(A)\cos(B)}{\sin(A)\sin(B)}\right]+\frac{\pi}{4}\right), \tag{2}$$

with the $\pi/4$ making the angle determination map friendly, and the $180/\pi$ converting to degrees. Next, a distance formula between two *(lat, lng)* points is defined as (Stackoverflow, 2020)

$$d(lat_1, lng_1, lat_2, lng_2) = \\ 6925\,\sin^{-1}\sqrt{\frac{1}{2}-\frac{1}{2}\cos\left[\frac{\pi}{180}(lat_2-lat_1)\right]+\frac{1}{2}\cos\left[\frac{\pi}{180}lat_1\right]\cos\left[\frac{\pi}{180}lat_2\right]\left(1-\cos\left[\frac{\pi}{180}(lng_2)\right]\right)}, \tag{3}$$

which returns a distance (in NMi) between points (lat_1,lng_1) and (lat_2,lng_2).

We now define a function to generate a *(lat,lng)* point based on Sumner's longitude by chronometer technique or

$$p(n, decl, alt, gha) = \left(51.62+\frac{10n}{60}, c\left(90-\left(51.62+\frac{10n}{60}\right), 90+decl, 90-alt\right)+gha\right). \tag{4}$$

This is a function of the navigational parameters *decl*, *alt* and *gha*, and *n*, which is an integer from 0.2, representing the calculated point. Note $n=0$ reduces p to

$$p(0, decl, alt, gha) = (51.62, c(90-51.62, 90+decl, 90-alt)+gha), \tag{5}$$

which is Sumner's first 'longitude by chronometer' calculation. Values of $n=1$ and $n=2$ give his what-if calculations of being $10'$ then $20'$ further north.

Lastly, we define a function that returns the distance between a point and a line to be

$$g(x_0, y_0, a, b, c) = \frac{|ax_0+by_0+c|}{\sqrt{a^2+b^2}}, \tag{6}$$

where (x_0,y_0) is a single point and the line that the form $ax+by+c=0$. In use, the point will be the location of Smalls lighthouse, and the line will be the one containing all three of Sumer's computed points.

The function to minimise M sums the distances from Sumner's DR to each of the $n=0$, 1 and 2 points, offsetting each term to its respective part in the $9+27+54$ check. It also contains the function

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Figure 5. *Adaptation #2 of Sumner's journey.*

$g(x_0,y_0,a,b,c)$ from Equation (6) to be

$$\begin{aligned} &M(lng, decl, alt, gha) \\ &\quad = |d(p(0, 25{\cdot}23 + \Delta decl, 12{\cdot}16 + \Delta alt, 333{\cdot}2 + \Delta gha), (51{\cdot}62, -6 + \Delta lng)) - 9| \\ &\qquad + |d(p(1, 25{\cdot}23 + \Delta decl, 12{\cdot}16 + \Delta alt, 333{\cdot}2 + \Delta gha), (51{\cdot}62, -6 + \Delta lng)) - 27| \\ &\qquad + |d(p(2, 25{\cdot}23 + \Delta decl, 12{\cdot}16 + \Delta alt, 333{\cdot}2 + \Delta gha), (51{\cdot}62, -6 + \Delta lng)) - 54| \\ &\qquad + g(51{\cdot}72, -5{\cdot}670, a, b, c), \end{aligned} \tag{7}$$

which written more compactly becomes

$$\begin{aligned} &M(lng, decl, alt, gha) \\ &\quad = -90 + g(51{\cdot}72, -5{\cdot}670, a, b, c) \\ &\qquad + \sum_{n=0}^{2} |d(p(n, 25{\cdot}23 + \Delta decl, 12{\cdot}16 + \Delta alt, 333{\cdot}2 + \Delta gha), (51{\cdot}62, -6 + \Delta lng))|. \end{aligned} \tag{8}$$

Here a, b and c are computed in real time based on a linear fit to Sumer's three computed points as the minimiser runs. Note the (lat,lng) of Smalls lighthouse is known, and a 'weighting coefficient' can be placed in front of the g function to vary the importance of this term (we used 2·0). We hypothesise that if M can be minimised relative to small changes in the navigation parameters ($\Delta decl$, Δalt, Δgha and Δlng), another suitable adaptation of Sumner's journey may be found.

The 'Minimise' function is used in Mathematica (Wolfram, 2020), which gives $\Delta lng = -0{\cdot}488$, $\Delta decl = 0{\cdot}280$, $\Delta alt = 0{\cdot}03$, and $\Delta gha = 0{\cdot}959$. The computed points 0, 1 and 2 come out to be (51·62, −6·022), (51·79, −5·48) and (51·95, −4·92) and are plotted in Figure 5.

We note a nice correspondence with the 9 + 27 + 54 check, here coming in at 9 NMi, 31 NMi, and 54 NMi. Additionally, Smalls lighthouse is very close to the line containing the calculated points. Sumner's supposed DR lng is still further west than that found here at −6·26°. This value is similar to that found in Adaptation #1.

We are cautious with our work to this point, as we are proposing a correction to historical data, to make a better match to Sumner's narrative, also noting how our final result is close to that in Bowditch (NIMA, 2015). Our belief in Sumner's altitude measurement (i.e. his sextant skills) in Adaptation #1 is justified, seeing here only a small correction to the altitude parameter. We also note that in the literature, Smalls lighthouse fell between calculated points 1 and 2, not 0 and 1 as arrived at here. We were unable to find a model that replicates this part of his journey.

We now proceed with our classroom lesson, given that we have a model consistent with the description of Sumner's journey. We note again the importance of developing pedagogical lessons in CN.

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3. The in-class lesson

By the time we come to learning Sumner's story in our class, the ideas and mechanics behind CN have been presented. Having a story such as Sumner's to transition into is a welcome relief from the highly technical and mathematical content used in introducing CN. Our lesson proceeds as follows. We begin with some manual map plotting, then gradually introduce the live programmable map.

3.1. Introduction

We naturally start by telling the story, as outlined at the beginning of Section 2 of this work, and on page 9 of Vanvaerenbergh. This is kept as a general overview, including a thorough visual using a standard world map, which alone is an opportunity to expose students to geography beyond their own familiar localities. The sun's visibility at only 10:15 am demonstrates a need for CN beyond the dual chronometer technique, which is typically remembered by students as a technique that only works with the sun at noon (since the sun's apex and thus local time are so easy to discern).

3.2. Initial map plotting

We begin with a manual map plotting exercise, for Sumner's DR position and Smalls lighthouse. Blanks maps are given to the students with axes extending from $-4°$ to $-8°$ in longitude and $48°$ to $53°$ in latitude. Such a map roughly exposes the southern coast of Ireland, St. George's Channel and the Pembrokeshire Coast National Park. (To the reader: We quietly note reversing the logic from our numerical adaptations, for our pedagogy: we claim to know the modified DR longitude at the *start* of the lesson, that came to us only at the *end* of a numerical adaptation to make the $9 + 27 + 54$ check work.)

3.3. Computing 'longitude by chronometer'

We now approach Sumner's first longitude by chronometer calculation with a close look at Equation (1), which is a formidable one for general education students (but alas we are at a 'technical' university). Thanks to Sumner's story, each parameter has a definite meaning. We take careful inventory of what Sumner knew and demonstrate how the entire right-hand side of Equation (1) is known. We work together in class (with calculators) to arrive at c. A targeted discussion now ensues about the relationship between c (the LHA), GHA and longitude. Sumner's story brings a strong sense of purpose to these parameters, as they involve finding his longitude, so ill-fated in the history of navigation. A discussion is often accompanied by the addition of one more arc drawn from the NP in Figure 1, which is that for the Prime Meridian at Greenwich (most conveniently drawn to the east of the NP–GP line shown in Figure 1). With the longitude computed, this point is plotted on the map, with a reference (and check) to the 9 NMi distance between the DR and this computed position, as per the story.

We next return to the general world map, highlighting the Celtic Sea into St. George's Channel, and ask what one would do if they were suspicious of a persistent current from the south. This naturally leads to assuming one's latitude is further north, and that suspicion in one's DR needs no justification, as it is inherently error prone. Some discussion is warranted on the meaning of Sumner's $10'$ further north (10 NMi of due north travel on a great circle given the 60 NMi/° rule of thumb). This leads to two more calculations of c involving Equation (1), and two more points plotted on the map. The points are shown to fall on the same line, which requires a pause, as an extremely important idea on CN is emerging: the line of position.

3.4. Live map visual

We now bring out the live programmable map into our lesson. The reader may access the map as well (Bensky, 2020). Note that being a 'live' map, the usual zoom and panning controls are both functional.

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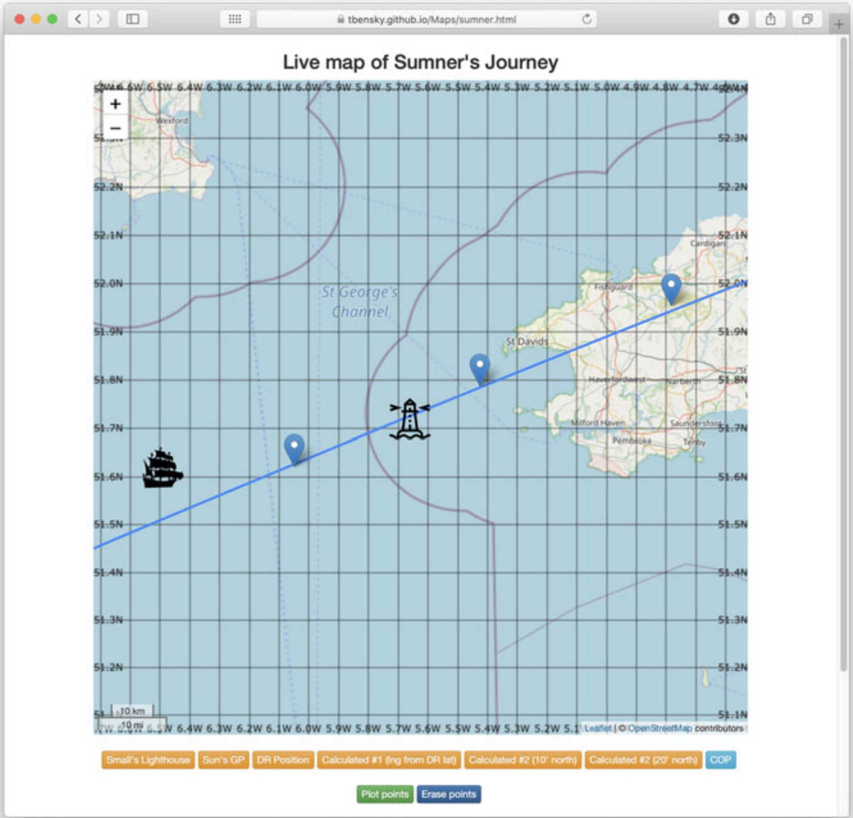

Figure 6. *A live map of Sumner's journey. All map controls (zooming and panning) work, and the user is able to choose which aspects of Sumner's journey to plot. Note here the zoom has made the visible portion of the circle of position appear straight.*

The map implementation of Sumner's journey is straightforward: it plots all needed points, including Sumner's DR, Smalls lighthouse, the computed points, the sun's GP and the circle of position (COP). Shown in Figure 6, the user clicks on buttons of points they wish to include on the map. When this map is brought up and projected in class for the students, its 'live' nature affords the instructor many pedagogical possibilities.

We initially show Sumner's DR and the sun's GP, which was in Botswana (southern Africa) at the time of Sumner's sighting. We then show the COP, with which the students are already familiar, but this application re-enforces its meaning.

3.5. The line of position

We have found the line of position difficult to get at pedagogically. Circles of position have been presented earlier in the class as an outcome of a sextant shot. We proceed with the COP on the live map as follows.

Using the live map, we display Sumner's calculated positions one by one, but show how we must zoom into the Celtic Sea to see the individual points. This is a very revealing process, as repeated clicking on the map's zoom control causes the GP to eventually leave the map, while the visible portion of the COP in the Celtic Sea begins to straighten as the individual calculated points become distinct.

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There are two important concepts in CN that the live map illustrates nicely. First, plotting the DR and the GP on the same map is discussed as forcing some reckoning with issues involving the scale of the map (Schlereth, 2000; Karl, 2011). Using the live map makes this clear, as the thickness of the line used initially to render the COP on a map that includes both the DR and GP positions is as wide as the state of Florida, USA. Second, seeing the COP become straight at high map zoom into a more local swath of ocean clearly shows the origin of a LOP.

The astute student will recognise the emergence of a LOP from a previously learned topic in a calculus course: the LOP is a line tangent to the COP and perpendicular to its radius, which is also the azimuth to the GP. Williams's humour (Williams, 1994, page 112) on this revelation serves as an anecdotal point to ponder about the historical development of CN.

3.6. Conclude the lesson

We now discuss Sumner's revelation of the LOP and how he sailed along it to promptly sight the lighthouse, noting that the lighthouse also being on his LOP was only fortuitous. We take time and allow students to ensure their hand-drawn maps correspond with the programmable map, and invariably take some 'what-if' questions involving the live map. We note the map will also plot any arbitrary *(lat,lng)* point by keying it into the text entry box.

Lastly, a comparison between Sumner's journey and the technique of St. Hilaire (Vanvaerenbergh and Ifland, 2003) is appropriate. We highlight how both focused on the utility of Equation (1). Sumner's inputs were C and B, while St. Hilaire's were A, c and B. Sumner varied A, while St. Hilaire placed more utility in his DR position *(A,c)* as his 'assumed position.' Sumner wanted to compute c, and St. Hilaire C. While Sumner plotted many computed points, St. Hilaire compared observed and computed (sun) altitudes. Sumner's (mathematical) plots will reveal a LOP while St. Hilaire will use his comparison to move a (sketched) LOP perpendicular to the azimuth, toward or away from the GP. Sumner assumes he is on the computed LOP, and St. Hilaire looks for the intersection of multiple LOPs.

4. Conclusions

Acknowledging the importance of developing modern pedagogical tools for use in teaching CN, we presented numerical adaptations of Sumner's 1837 journey into St. George's Channel. Our results are disseminated mostly into a live browser-based map (Open Street Maps, 2020), suitable for classroom use and student exploration.

Given the data available, there seems to be a discrepancy between Sumner's narrative and his actual locations, which this work aims to rectify. At the onset, Sumner's location was found to be extremely sensitive to the GHA, a parameter we varied by $<1\%$, which made calculations on Sumner's position mostly consistent with his narrative.

We find Sumner's journey to be a welcome pedagogical tool, presented after a sequence of technical and mathematical lessons introducing the tools and techniques of celestial navigation. This numerical adaptation served as the foundation for a live browser-based map, allowing for a flexible and highly visual presentation highlighting many issues of celestial navigation, most notably those of the origin of the line of position and issues involving the scaling of maps. A comparison between the navigational approaches of both Sumner and St. Hilaire is presented. There have been many lively and context-rich class sessions due to this work. We hope instructors of celestial navigation might also benefit. Potential questions for a student of CN, in the context of this work are given in the Appendix.

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Appendix

This section contains exercises that may be presented to the student of celestial navigation, as a result of the lesson presented here.

1. Describe all parameters in Equation (1). Do so both in an exact and less-exact manner. For example, A is the co-latitude of the DR, but to simplify conversations, can you discuss A as just the DR's 'latitude,' since the co-relation is so trivial?
2. Express a relationship between c in Equation (1), a ship's longitude, and the GHA of the sun, in the case where the GHA is east of Greenwich. Repeat for a GHA west of Greenwich. Figure 1 may help.
3. Given the sun's GHA of 333·2° and declination of 23·25°S, an altitude measurement from a ship is found to be 12·16°. Compute the longitude of the ship.
4. A line of position points NE. Describe how the orientation of this line would change if it were based on a more westward GP. More eastward?
5. A longitude is computed from Equation (1) in part using the GHA of the sun. What would happen to the computed longitude if the GHA were further west? Further east?
6. A line of position is computed in part from an altitude measurement of the sun. The line runs NE. If the altitude used was larger, how would this affect the line's position? Altitude was smaller?
7. Sketch a line of position, in any orientation. Now, draw a dotted line that would contain the GP of the object used to find the line of position.
8. In terms of celestial navigation, what is the significance of Smalls lighthouse also being on Sumner's computed line of position?

9. A map shows both the DR position of a ship and the GP of the sun, which is 5000 miles from the DR position. Use a pencil to draw the COP of the ship. Discuss map scaling issues, relative to the thickness of your pencil line.
10. Suppose a map shows the LOP for a ship drawn with a pencil. The extent of the map is 10′ on each side and is near the equator. Discuss map scaling issues, relative to the thickness of your pencil line.
11. Draw a neat, accurate circle with a 6 cm radius using a pencil and drawing compass. Now draw a line tangent to the circle. Estimate visually what percentage of the circle overlaps with the line. What zoom level on the circle would you need before all you would see is this overlap portion?
12. Access the live map. Plot both Sumner's DR, the GP of the sun, and the COP. Compare the width of the line used to render the COP to some other worldwide feature, and estimate its thickness (in meters). Now plot Sumner's additional three computed points. Next, zoom in, so that the three additional points become distinct. Re-evaluate the thickness of the line used to render the COP, and comment on its curvature. Discuss the issue of map scale as you transition from seeing the COP to just an LOP.
13. Access the live map and find the city in which you live. Zoom in on the map until only your city is visible. Estimate the NS and EW size of your city in terms of degrees of longitude and latitude.
14. In Section 2.2, it was noted that changing the GHA from 334·67 to 333·2 corresponded to a 6 min time difference in the sun's motion. Justify the 6 min.

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