Variational Bayesian Methods (in Neuroscience)

Tyler Benster & Aaron Andalman
Deisseroth (Tyler & Aaron) and Druckmann (Tyler) Labs

Computational Neuroscience Journal Club
Stanford University

July 10, 2019

2019-07-15

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Roadmap

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Introduction: Why care about the distribution of data?

Problem: Analyzing high dimensional data is hard

Solution: Variational Methods

Example: variational autoencoder

Discussion: Neuroscience applications

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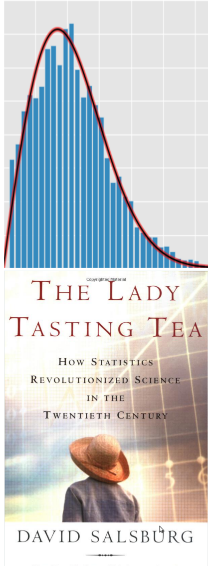
The probability distribution revolution

- Karl Pearson (1857-1936) came with the idea that scientific measurements should be conceived as coming from probability distributions.
- Scientific measurements are just random reflections of the underlying truth that is the distribution.
- “A great book on the history of statistics” $\rightarrow$ Aaron



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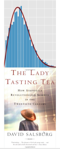

The power of probability distributions

Distributions allow scientists to:

- Understand scientific measurement
- Predict the probability of specific data
- Test specific hypothesis (p-values)
- Produce generative models
- Better conceptual understanding data.

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Estimating distributions from data

Low-Dimensional:

- Great tools to fit and understand the underlying probability distribution of data.

High-Dimensional:

- In some cases, classical statistical tools are insufficient.
- Problematic for modern neuroscience:
- Thousands of electrodes.
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Variational Bayesian Methods

Estimate the probability distribution of high-dimensional neural data.

- Compute the probability of observing a particular neural state.
- Sample neural states/trajectory from the estimate.
- Generate statistics / test hypotheses
- Estimate latent factors/states that drive the observations.
- Reduce dimensionality (with advantages over other methods, e.g. PCA, T-SNE)

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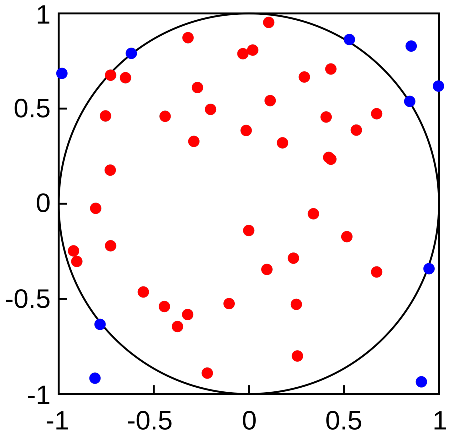
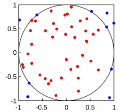
Approximating the area of a circle with Monte Carlo



40/50 samples in circle gives area of $4 * 0.8 = 3.2 \approx \pi$. $\sim 2\%$ error

CCO 1.0, wikimedia, MonteCarloIntegrationCircle.svg



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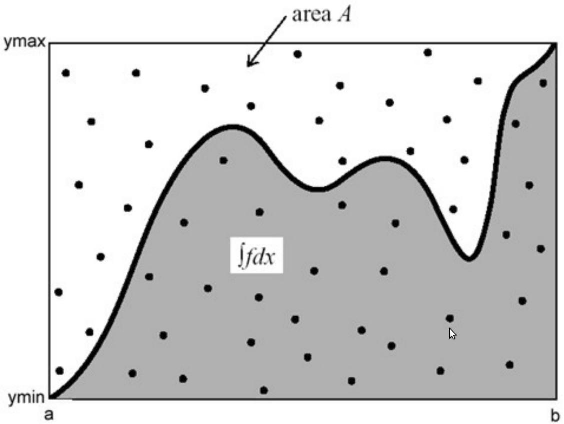
Useful for analytically intractable integrals



Robert Lin - Monte Carlo Integration

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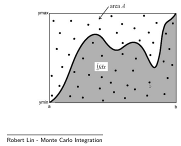
└─Useful for analytically intractable integrals



Extension to unit hypercube

We can approximate a high-dimensional integral using a Monte Carlo approximation:

$$\int_0^1 \cdots \int_0^1 g(x_1, \ldots, x_n) dx_1, \ldots, dx_n \approx \frac{1}{N} \sum_{j=1}^{N} g(\bar{x}_j)$$

where $\bar{x}_1, \ldots, \bar{x}_N \sim \mathcal{U}(0,1)$ is the i^{th} random sample

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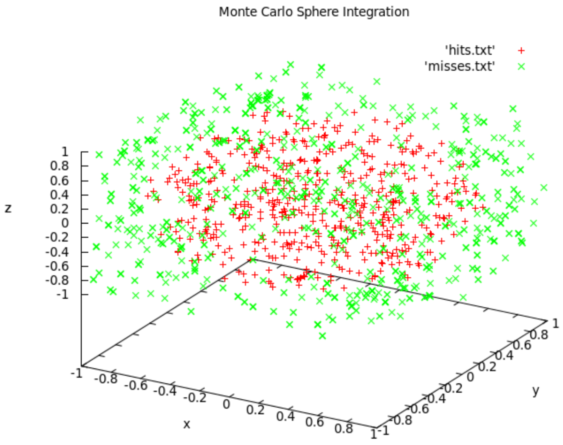
Performance scales poorly with number of dimensions



1000 samples, estimate $= 4.072$, actual $= 4.189$, $\sim 3\%$ error

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http://www2.hawaii.edu/ yuxian/phys305/a6/

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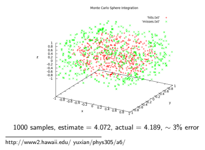

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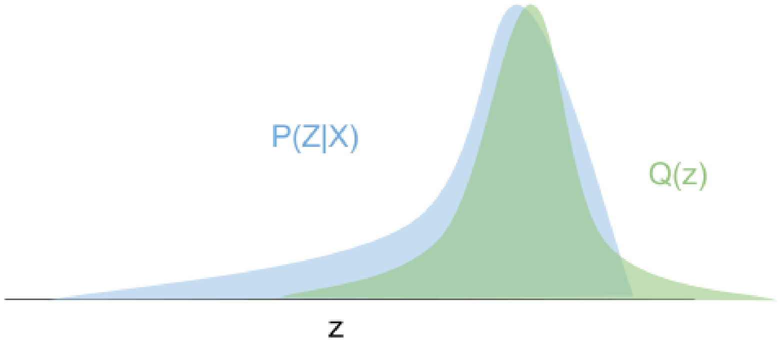
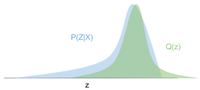
Alternate approach: Variational Bayes





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Demo

https://bit.ly/2LcEhow

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Summary

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Example: LFADS

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