Elaboration of Water Distribution Schedules in Periods of Scarcity

Tatiana Balbi Fraga

Núcleo de Tecnologia, Universidade Federal de Pernambuco

1 Introduction

Water scarcity is a global problem that has been growing over the years due to factors such as population growth, climate variations and living standards (Huang et al., 2021). Specifically in Brazil, in semi-arid regions (such as some regions of Northeast Brazil), this problem is aggravated due to the high variability of rainfall, with rainfall concentrated over time and little spatial distribution (Silva et al., 2019). Such aggravation ends up generating conflicts for the use of water resources, and water distribution becomes a priority concern for public management.

Recent studies have sought to consider the issue of water scarcity as a possible condition that must be analyzed when determining the parameters for the design or operation of water distribution networks (Sankar et al., 2015). In our work, we focus on a specific understudied class of water distribution problems described in detail by Kurian et al. (2018), considering, however, conceptual differences related to the definition of the problem.

As a contribution to this domain, we present a detailed description of the Water Distribution System (WDS) in the city of Catende - PE (Brazil) and a new mathematical model for the problem of elaborating water distribution schedules that was specifically developed based on the studied system. The mathematical model was built in parallel to the development of a solver in Lingo. Our objective was to build a model that could be applied with this solver, generating optimal solutions quickly, in order to offer practical support to the water distribution planning of the municipality. As result, we were able to develop a new general mathematical model whose main scientific contribution is its simplicity. This model has the following features: a) it seeks to determine the status of the valves (ON/OFF) and the ideal values for the flows; b) the aim of the problem is to achieve a distribution as equitable as possible; c) the valves are located at the entrance of reservoirs/zones served; d) parameters are incorporated into the model to define priority rules; e) complex calculations are replaced by the definition of maximum and minimum values for the flows; and

f) flexibility variables were proposed (time of availability of water per shift), which allows the 'ideal' planning to be performed in practice.

In the next sections we present: a detailed description of the WDS of Catende; the mathematical model developed; a summary of results of the tests performed using actual company data; a qualitative discussion about aspects of the model and the solver; a brief assessment of the limitation of the solver developed; and finally, conclusions and suggestions for future works.

2 Catende Water Distribution System

The city of Catende is located in the interior of Pernambuco and has a total area of 207,244 km. According to IBGE data obtained from the large Census carried out in 2010 (IBGE, 2021), the population of the city of Catende on that date was 37,820 inhabitants. Of these, 28,861 were located in the urban area. In this city, the municipality is responsible for supplying the population with treated water. The next sections present in detail the process of collecting, treating and distributing water in the city of Catende, as well as issues related to demand.

2.1 Water collection and treatment

The collection of water distributed in Catende is carried out in the Santa Rita and Bálsamo das Freiras weirs. The Santa Rita weir is located in the district of Roçadinho, with a maximum water storage volume of 2,500,000 m^3 . The Bálsamo das Freiras weir is located on the border between Catende and the Municipality of Jaqueira, with a maximum storage volume of 1,500,000 m^3.

Water treatment is carried out at the Catende Water Treatment Plant (WTP) (Figure 1) in a conventional and mechanical manner (Figure 2). The collection of water is carried out in the weirs arriving at the treatment plant by gravity. With an average flow of 67 l/s, the water from the Santa Rita reservoir goes to conventional treatment, and with a flow of 35 l/s, the water from the Bálsamo das Freiras reservoir is directed to the mechanical treatment plant.

After the adduction, the water captured at the Santa Rita weir, goes to the treatment phases, being received at the WTP through channels, and passing through a flow meter called Calha Pashall. In this trough aluminum sulphate and chlorine are added (Figure 3). These two chemicals help in the coagulation step by reacting with particles that are suspended in raw water. At this stage, the water goes through shares called flocculators, where the added reagents transform the impurities into larger particles to be removed during decantation.

The water, then flocculated, goes to another phase, passing through larger layers called decanters, where the flakes formed are deposited in tubular modules similar to a beehive, making the water almost suitable for consumption. In these stages of treatment, almost all contamination factors are eliminated, filtration is then carried out to further improve the decanted water which, by gravity, passes through filters made of a porous medium with the ability to retain and remove impurities even existing in the water. In disinfection, physical or chemical agents

Figura 1: Catende Water Treatment Plant.

are used, such as chlorine, used for the destruction of pathogenic organisms that cause diseases, ending with the correction of pH, where products are also added so that the treated water has a suitable pH for consumption. After being treated, the water is deposited in a supported type reservoir with a capacity of 500,000 m^3 of storage (Central Reservoir). Treatment is carried out in continuous flow.

The water collected from the Bálsamo das Freiras Reservoir is treated in mechanical filters, where the addition of chemical products is carried out automatically. The disposal of the water treated in these filters is also carried out in the Central Reservoir.

2.2   Water distribution in Catende

For the planning of water distribution, the city of Catende was divided into 6 zones (Figure 4).

In order to allow water resources to reach consumer units in the most distant neighborhoods from the WTP, capacity studies were carried out, considering the population increase. Based on these studies, the distribution managers identified the need to install reservoirs that serve as water redistribution points.

The water treated at the WTP - Catende is deposited in a central elevated reservoir (named Central Reservoir), which has a capacity of 500,000 m^3 . From this reservoir, it is distributed to another elevated type reservoir with a capacity of 400,000 m^3, both located in the WTP. The Elevated Reservoir is responsible for distributing water to the Nova Catende and Jaqueira districts. This reservoir receives water from the Central Reservoir through pumping, but distributes to

Figura 2: Conventional (left) and mechanical (right) treatment plants.

the aforementioned neighborhoods through gravitational force.

The Central Reservoir, in addition to supplying water to the Elevated Reservoir (in the WTP), also supplies three more reservoirs of the elevated type in the Panelas Piranji, Canaã and Oxifan neighborhoods, with storage capacities of 100,000 m^3, 180,000 m^3 and 20,000 m^3, respectively. The first two mentioned are supplied with the help of two pairs of pumps while the third reservoir is also supplied by gravitational force. The city center, on the other hand, receives water from the central reservoir through gravitational force, as it is directly linked to the WTP.

Figure 6 shows a representation of the water supply network described above. The supplier managers regulates the water supply in Catende, by opening or closing the valves that control the distribution flow. Currently, the company leaves these valves permanently open, maintaining a continuous flow of this resource, with the supply being interrupted only to maintain the treatment filters or the distribution network.

2.3   Demand from consumer units

Based on data obtained from the UN (United Nations), Paganine (2013) informs that 110 liters of water per day are enough to meet a person's basic needs. However, in interviews conducted with WTP administrators in Catende, it was informed that the average consumption per inhabitant in the city was around 200 liters per day. Considering the average consumption reported and data from IBGE (2021) obtained from the 2010 Census, it was estimated that an average of 7,473,960.00 liters of water per day is needed to serve the population of Catende.

Through the visits carried out at the WTP in Catende in 2016, it was also possible to obtain information on the daily production capacity and the number of households served, separated by the supply zones. Table 1 shows the water demand for each zone (neighborhood), where the necessary amount of water per household was defined considering that each household resides on average

Figura 3: Parshall gutter, addition of chlorine and aluminum sulphate.

3.9 inhabitants (average established by IBGE (2021) through the last large demographic census of 2010). Information on the number of households/zones and the average water consumption per inhabitant were provided by the WTP managers, considering the average use of 200 liters of water per inhabitant. It was also informed that almost 100% of households have reservoirs ranging between 500 and 1000 liters. These are activated when water is not supplied, either due to maintenance in the distribution network, or in periods when the water flow for treatment is low, thus limiting the daily supply.

Although Catende is located in a region that does not suffer so much from the lack of water, it has been observed over the years, a considerable drop in its water sources, alerting the population and managers about the need to save this resource. The production capacity of treated water, when the abstraction in water sources is at its maximum limit, is 288,000 liters of water per hour, in

Figura 4: Catende map by supply zones. Source: Adapted from the Archives of the Municipality of Catende, consulted in 2016
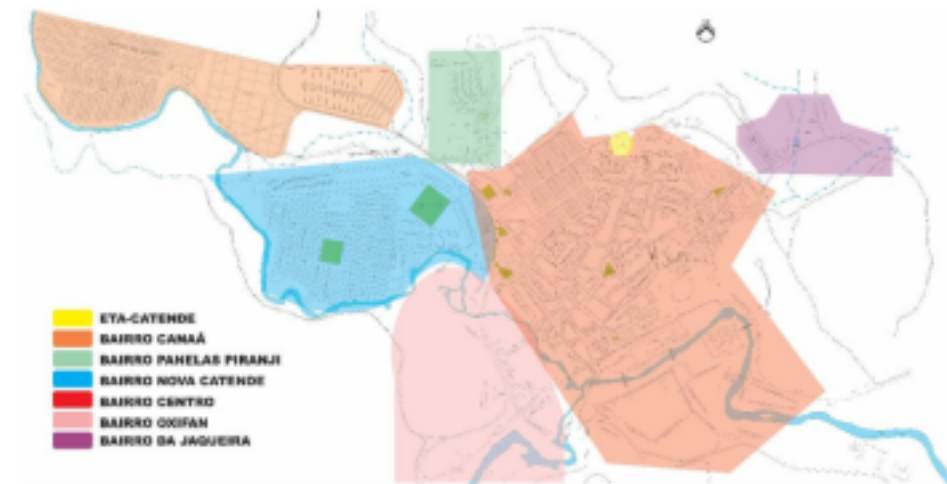
this way, in 24 hours it is possible to fill 13.8 reservoirs of 500,000 m^3. Observing the total daily demand required, it is concluded that the productive capacity does not meet the daily needs of the population, which highlights the need for good management of water distribution.

Based on this issue, the water distribution problem was then defined, as discussed in the next section.

3 Problem of Elaborating Water Distribution Schedules in Periods of Scarcity

This water distribution problem arises in times of scarcity, in which it becomes necessary to draw up a distribution schedule for the different zones of the city, based on the limitation of this precious resource. Such schedule informs the shifts in which the inhabitants will be able to acquire treated water, generating their reserves for the periods in which there will be no supply. The following section presents a detailed definition and mathematical modeling for the problem considered in this work.

3.1 Mathematical modeling

3.1.1 Decision variables

As informed in this work, the municipality carries out the distribution planning for the various zones and reservoirs considering distribution shifts. On each shift, water release is controlled by opening or closing valves. Thus, in each

Figura 5: Reservoirs raised (left) and elevated (right).

supply sector	number of households	average consumption (liters/day)
Centro	5431	4.236.180,00
Nova Catende	1043	813.540,00
Jaqueira	976	761.280,00
Panelas Piranji	459	358.020,00
Canaã	1538	1.199.640,00
Oxifan	135	105.300,00
Total	9582	7.473.960,00

Tabela 1: Demand by sector - Catende (data from 2016)

considered shift, there are two possible states for each valve - open (when it allows the transfer of water from one point to another) and closed (when it does not allow this transfer of water). Therefore, the water distribution problem basically consists of defining for each valve and for each shift, whether the valve will be open or closed, so that the primary decision variables considered in the model can be defined by the binary variables X_{dsl}, where:

$$X_{dsl} = \begin{cases} 1 & \text{whether water is supplied to location } l \text{ in the } s\text{-th shift of the day } d \\ 0 & \text{if no water is supplied to location } l \text{ in the } s\text{-th shift of the day } d \end{cases} \quad (1)$$
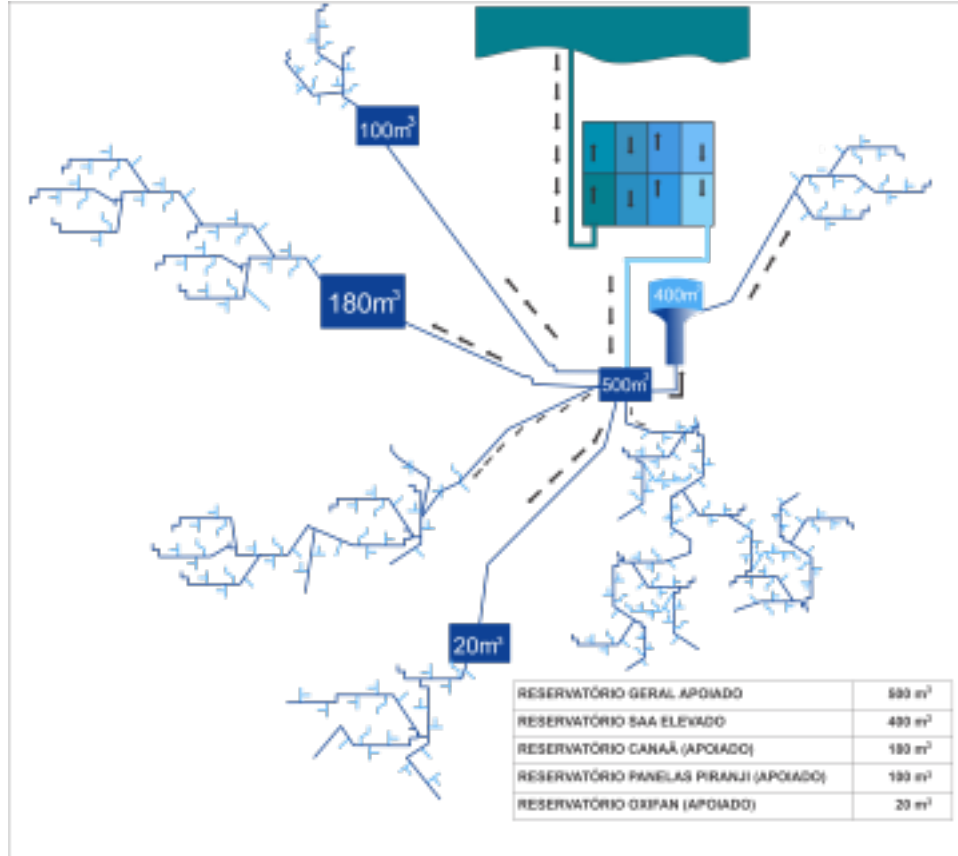
$$d = 1, ..., \text{ND}, \quad s = 1, ..., \text{NS}, \quad l = 1, ..., \text{NL}$$

in which:
ND represents the number of days considered in the planning horizon;
NS represents the number of shifts considered for each day;

Figura 6: Water Distribution Scheme in Catende.

RESERVATÓRIO GERAL APOIADO	500 m³
RESERVATÓRIO SAA ELEVADO	400 m³
RESERVATÓRIO CANAÃ (APOIADO)	180 m³
RESERVATÓRIO PANELAS PIRANJI (APOIADO)	100 m³
RESERVATÓRIO OXIPAN (APOIADO)	20 m³

and NL represents the number of locations that are part of the water supply system (either zones, reservoirs or WTPs).

In some cases the valves are replaced by pumps. In these cases, the pumps can be compared to open valves, when activated, and closed valves, when not activated. Therefore, the simplification adopted (considering only valves) does not generate any implications for the developed model.

3.1.2 Objective function

The objective of the problem considered here is, at the same time: 1) to maximize the volume distributed to the zones; 2) to make the distribution of water between the different zones as fair as possible; and 3) to minimize the idle capacity of the reservoirs.

$$maximize \quad Fitness = V_{\text{dist}} - \text{RF}_{\text{fair}} * \sum_{d=1}^{\text{ND}} \sum_{s=1}^{\text{NS}} \sum_{z=\text{NR}+1}^{\text{NL}} |\text{P}_{\text{dem}z} - P_{\text{dist}dsz}| \quad (2)$$

$$+\text{RF}_{\text{valves}} * \sum_{d=1}^{\text{ND}} \sum_{s=1}^{\text{NS}} \sum_{l=1}^{\text{NL}} X_{dsl} - \text{IrF}_{\text{idleness}} * C_{\text{idleEWSum}}$$

where RF_{fair} and $\text{RF}_{\text{valves}}$ are parameters used as relevance factors, for the fair distribution and adjustment of the valves, respectively. The irrelevance factor for idle capacity, $\text{IrF}_{\text{idleness}}$, is used in the objective function to ensure that the algorithm prioritizes meeting demand and that the remaining volume in the reservoirs becomes a secondary concern. In this work we adopt $\text{IrF}_{\text{idleness}} = 0.000005$.

The other parameters, variables and terms of the objective function are detailed in the restrictions section presented below.

3.1.3   Restrictions

Note that once the values of the primary decision variables are defined, it is possible to calculate the volume of water received daily and per shift at each location, $V_{\text{in}dsl}$, through the equations:

$$V_{\text{in}dsl} = V_l * X_{dsl}, \quad d = 1, ..., \text{ND}, \quad s = 1, ..., \text{NS}, \quad l = 1, ..., \text{NL} \quad (3)$$

where V_l represents the ideal volume of water distributed to the location l, if the corresponding valve remains open:

$$V_l = InFR_l * \text{T}, \quad d = 1, ..., \text{ND}, \quad s = 1, ..., \text{NS}, \quad l = 1, ..., \text{NL} \quad (4)$$

with T being the time per shift, and $InFR_l$ the ideal inflow, which is limited by the maximum inflow of the pipeline that reaches the reservoir at the location l, $\text{InFR}_{\text{max}l}$:

$$InFR_l <= \text{InFR}_{\text{max}l}, \quad l = 1, ..., \text{NL}. \quad (5)$$

and by a desired minimum inflow rate for this same location, $\text{InFR}_{\text{min}l}$:

$$InFR_l >= \text{InFR}_{\text{min}l}, \quad l = 1, ..., \text{NL}. \quad (6)$$

Based on the equations and inequalities presented in (3) to (6) we can then determine the daily outflow of each location l in each shift s, $V_{\text{out}dsl}$.

For the case of reservoirs, the volume of water supplied by the reservoir r at the end of the s-th shift of the day d, $V_{\text{out}dsr}$, can be defined by:

$$\text{V}_{\text{out}dsr} = \sum_{l=1}^{\text{NL}} (V_{\text{in}dsl} * \text{Y}_{rl}), \quad (7)$$

$$d = 1, ..., \text{ND}, \quad s = 1, ..., \text{NS}, \quad r = 1, ..., \text{NR}$$

where the binary variable Y_{rl} informs the physical structure of the water distribution system:

$$\mathrm{Y}_{rl} = \begin{cases} 1 & \text{if water is supplied directly from the reservoir } r \text{ to the location } l \\ 0 & \text{if water is not supplied directly from the reservoir } r \text{ to the location } l \end{cases} \quad (8)$$
$$r = 1, ..., \mathrm{NR}, \quad l = 1, ..., \mathrm{NL}$$

Furthermore, the inflows and outflows in the network are related through the flow equations in (9) and (10):

$$V_{\mathrm{rem}dsl} = V_{\mathrm{rem}(d-1)(\mathrm{NS})l} + V_{\mathrm{in}dsl} - V_{\mathrm{out}dsl}, \quad (9)$$
$$d = 1, ..., \mathrm{ND}, \quad s = 1, \quad l = 1, ..., \mathrm{NL}$$

and

$$V_{\mathrm{rem}dsl} = V_{\mathrm{rem}d(s-1)l} + V_{\mathrm{in}dsl} - V_{\mathrm{out}dsl}, \quad (10)$$
$$d = 1, ..., \mathrm{ND}, \quad s = 2, ..., \mathrm{NS}, \quad l = 1, ..., \mathrm{NL}$$

where $V_{\mathrm{rem}dsl}$ represents the volume of water remaining in the reservoir at location l at the end of the s-th shift of the day d, being $V_{\mathrm{rem}0(\mathrm{NS})l}$ a constant defined by the initial reservoir volume at location l, $\mathrm{V}_{\mathrm{ini}l}$:

$$V_{\mathrm{rem}dsl} = \mathrm{V}_{\mathrm{ini}l}, \quad d = 0, \quad s = \mathrm{NS}, \quad l = 1, ..., \mathrm{NL} \quad (11)$$

It is also necessary to consider that the remaining volumes in the reservoirs must not be negative and must always be less than their respective capacities, C_l.

$$V_{\mathrm{rem}dsl} \geq 0, \quad d = 1, ..., \mathrm{ND}, \quad s = 1, ..., \mathrm{NS}, \quad l = 1, ..., \mathrm{NL} \quad (12)$$

and

$$V_{\mathrm{rem}dsl} \leq \mathrm{C}_l, \quad d = 1, ..., \mathrm{ND}, \quad s = 1, ..., \mathrm{NS}, \quad l = 1, ..., \mathrm{NL} \quad (13)$$

In the case of the zones, the total daily volume distributed to each zone will be limited by the daily demand of that zone, D_z:

$$\sum_{s=1}^{\mathrm{NS}} V_{\mathrm{out}dsz} \leq \mathrm{D}_z, \quad d = 1, ..., \mathrm{ND}, \quad z = \mathrm{NR}+1, ..., \mathrm{NL} \quad (14)$$

The total amount of water distributed by the zones, V_{dist}, is then defined by the sum:

$$V_{\mathrm{dist}} = \sum_{z=\mathrm{NR}+1}^{\mathrm{NL}} V_{\mathrm{out}z} \quad (15)$$

where $V_{\text{out}z}$ represents the total outflow of zone z:

$$V_{\text{out}z} = \sum_{d=1}^{\text{ND}} \sum_{s=1}^{\text{NS}} V_{\text{out}dsz}, \quad z = \text{NR} + 1, ..., \text{NL} \tag{16}$$

and $V_{\text{out}dsz}$ represents the outflow of zone z at the end of the s-th shift of the day d.

Regarding the second term of the objective function, we consider in this work that, in a fair distribution, the volume proportions distributed in each shift by each zone z are as close as possible to its demand proportion, $P_{\text{dem}z}$:

$$\text{P}_{\text{dem}z} = \text{D}_{\text{shift}z}/\text{D}_{\text{total}} \tag{17}$$

being D_{total} the total demand:

$$\text{D}_{\text{total}} = \sum_{z=\text{NR}+1}^{\text{NL}} \text{D}_z \tag{18}$$

and $\text{D}_{\text{shift}z}$ the demand per shift of zone z:

$$\text{D}_{\text{shift}z} = \text{D}_z/(\text{ND} * \text{NS}), \quad z = \text{NR} + 1, ..., \text{NL} \tag{19}$$

where the daily demand of zone z, D_z, is defined by:

$$\text{D}_z = \text{C}_{\text{average}} * \text{HC} * \text{NH}_z, \quad z = \text{NR} + 1, ..., \text{NL} \tag{20}$$

such that $\text{C}_{\text{average}}$ represents the average daily consumption per inhabitant, HC represents the housing coefficient, and NH_z the number of households in the zone z.

The proportions of volume distributed in each shift by each zone, $P_{\text{dist}dsz}$, are defined by:

$$P_{\text{dist}dsz} = V_{\text{out}dsz}/V_{\text{dist}}, \quad z = \text{NR} + 1, ..., \text{NL} \tag{21}$$

The third term of the objective function seeks to ensure that the flow is distributed among the shifts, except when this distribution is bounded by flow restrictions.

Finally, the fourth term deals with another important issue to be considered: the idle capacity of the reservoirs. At the end of the last shift of each day, it is desired that the remaining reservoir volumes, $V_{\text{rest}dsl}$, are as close to their respective capacities, C_l, as possible.

Once the remaining volumes in the reservoirs have been defined, it is then possible to determine the idle capacities of the latter in each shift, $C_{\text{idle}dsl}$, through the Eqs. in (22):

$$C_{\text{idle}dsl} = \text{C}_l - V_{\text{rem}dsl}, \quad d = 1, ..., \text{ND}, \quad s = 1, ..., \text{NS}, \quad l = 1, ..., \text{NL} \tag{22}$$

being the exponentially weighted sum of idle capacities, computed in the last shifts of each day, $C_{\text{idleEWSum}}$, defined by:

$$C_{\text{idleEWSum}} = \sum_{d=1}^{\text{ND}} \sum_{l=1}^{\text{NL}} (C_{\text{idle}ds(\text{NS})})^{\text{RF}_l} \tag{23}$$

where RF_l represents the importance attributed to maintaining water in each reservoir, since there is a surplus of water to fill the reservoirs. It is important that the values of these parameters are greater than and close to 1.00. In this work, we adopted: 1.13 for the WTP ($r = 1$) and for the Central Reservoir ($r = 2$); 1.12 for the other reservoirs; and 1.00 for the zones.

Note that, in the case of zones, each zone z is considered to have a single reservoir, whose capacity is defined by the sum of the capacities of the various housing reservoirs in the zone.

$$\text{C}_z = \text{C}_{\text{averReserv}z} * \text{NH}_z, \quad z = \text{NR} + 1, ..., \text{NL} \tag{24}$$

where $\text{C}_{\text{averReserv}z}$ represents the average of the reservoir capacities of the households in the zone z.

3.1.4   Distribution time adjustment

In order to guarantee a fair distribution, in specific cases where the actual flows are greater than the ideal flows informed in the initial model solution, and cannot be reduced in practice, it is necessary to limit the water distribution time for certain locations. In this case, the proper distribution time per shift for local l, $T_{\text{adeq}l}$, can be defined according to equations in (25). In practice, once delivery times are adequately limited, water delivery will proceed as planned.

$$T_{\text{adeq}l} = \text{T} * (FR_l / \text{FR}_{\text{max}l}), \quad l = 1, ..., \text{NL} \tag{25}$$

3.2   Tests and results

To validate the proposed model, a LINGO code was developed and ran in the 'global solver' mode, using the parameter values recommended by the software LINGO (2021). The tests were carried out based on the considerations presented in the sections 3.2.2, 3.2.3 and 3.2.4 and using the data collected during this work, as summarized in the section 3.2.1. The results found as well as an analysis of these results are presented along the following sections together with the considerations studied.

3.2.1   Data collected

Tables 2, 3, 4 and 5 present a synthesis of the data referring to the weirs, reservoirs, zones and the network of studied water distribution, respectively. The conceptual and mathematical model presented in this paper, as well as the solver used to validate the model, were developed through a recent effort.

weir	name	distribution flow rate (liters/hour)
1	Santa Rita weir	241.200,00
2	Bálsamo das Freiras weir	126.000,00
Total		367.200,00

Tabela 2: Weirs data - Catende (data from 2016)

reservoir (r)	name	capacity (C_r) (m^3)	Maximum inflow rate informed ($\mathrm{InFR_{max}}_r$) (liters/hour)
1	WTP	2.500.000	367.200,00
2	Central	500.000	288.000,00
3	Elevated	400.000	não informado
4	Panelas Piranji	100.000	não informado
5	Canaã	180.000	não informado
6	Oxifan	20.000	não informado

Tabela 3: Reservoirs data - Catende (data from 2016)

However, the data presented in this section were collected in 2016 as part of an extension project carried out by the authors of this article, through which a detailed study of the water distribution process presented in the section 2 was carried out (Blinded, 2016).

number of locations (NL) = 12 (6 zones)
number of reservoirs (NR) = 6 (1 WTP)
housing coefficient (HC) = 3.9 inhabitants / household
average daily inflow per inhabitant (C_{average}) = 200 liters / inhabitant * day
Note: each weir supplies water to an WTP (conventional or mechanical). However, both supply water to the Central Reservoir. For this reason, in this work we consider only a single WTP.

3.2.2 Initial considerations

Figure 7 presents the optimal solution for the data summarized in the section 3.2.1, calculated with the aid of the 'LibreOffice Calc' software, considering the maximum daily distribution value (in a 24-hour shift), limited by the outflows of the weirs and the WTP (case 01).

As we can see in this figure, the optimal daily water distribution volume per inhabitant, considering a daily 24-hour distribution period, is 184.96 liters / (day * inhabitant). Therefore, we can conclude that, in fact, the flow of the

zone (z)	name	Number of households (NH$_z$)
7	Centro	5.431
8	Nova Catende	1.043
9	Jaqueira	976
10	Panelas Piranji	459
11	Canaã	1.538
12	Oxifan	135

Tabela 4: Zones data - Catende (data from 2016)

(Y$_{rl}$)	1	2	3	4	5	6	7	8	9	10	11	12
1	0	1	0	0	0	0	0	0	0	0	0	0
2	0	0	1	1	1	1	1	0	0	0	0	0
3	0	0	0	0	0	0	0	1	1	0	0	0
4	0	0	0	0	0	0	0	0	0	1	0	0
5	0	0	0	0	0	0	0	0	0	0	1	0
6	0	0	0	0	0	0	0	0	0	0	0	1

Tabela 5: Physical distribution structure - Catende (data from 2016)

pipeline that transports water from the WTP ($r = 1$) to the Central Reservoir ($r = 2$) is not sufficient to meet the average daily consumption of the population informed by the company (200 liters / (day * inhabitant)).

Another important piece of information that we observe in this figure are the ideal inflow rates, which would result in a fair distribution for the population of Catende. These values are basically defined as a function of water availability, demands, and initial volumes in the reservoirs. Such values were identified using the following calculation logic:

1) as the part of the inflow received by the WTP and not distributed is completely used to fill its reservoir and, as the inflow rate of the Central Reservoir is lower than the inflow rate of the WTP, so $V_{\text{in}dsr}$, for the WTP and the Central Reservoir, are calculated according to Eqs. in 26:

$$V_{\text{in}dsr} = \text{InFR}_{\max_r} * \text{T}, \quad d = 1, \quad s = 1, \quad r = 1, 2 \qquad (26)$$

2) the total availability of water, released by the Central Reservoir, which directly or indirectly supplies all other reservoirs and zones, is less than the total demand. Therefore, the volume of water received by the Central Reservoir is completely distributed to the zones (and not to fill the reservoirs), with the fair distribution being proportional to the number of households in each zone:

$$V_{\text{in}dsz} = V_{\text{in}dsr} * (\text{ND}_z/\text{ND}_{\text{total}}), \qquad (27)$$
$$d = 1, \quad s = 1, \quad r = 2, \quad z = \text{NR} + 1, ..., \text{NL}$$

and, 3) as the volumes received by the reservoirs are completely distributed:

$$\text{V}_{\text{in}dsr} = \text{V}_{\text{out}dsr} \quad d = 1, \quad s = 1, \quad r = 3, ..., \text{NR} \qquad (28)$$

being the values of $V_{\text{out}dsr}$ calculated according to the Eqs. in (7) recursively, and the ideal rates (not informed) then calculated based on the Eqs. in (4).

After performing the calculations shown in Figure 7, we run the solver in Lingo, using the same inputs shown in this figure, along with other data reported in the 3.2.1 section. The values 367,200.00 and 288,000.00 were used as the values of the maximum inflow rates informed for the WTP ($r = 1$) and for the Central Reservoir ($r = 2$), respectively. No limit values for inflow rates were informed for the other reservoirs and zones. Table 6 displays the values found by the solver. As we can see in this table, the solver found the calculated optimal values very quickly and with a very small margin of error.

Very close values were found for fitness and volumes distribution when the solver in Lingo was run considering 3 shifts of 8h (case 03). In this case, the inflow is distributed between the shifts, keeping the same ideal inflow rates. The computational effort increased considerably: computational time = 9.82 sec; and total number of iterations = 5,709.

3.2.3 Practical aspects

It would be ideal if the company could regulate the flow rates for water distribution, adjusting these rates according to each ideal solution found by the solver for the different cases evaluated by the planners. However, in practice this becomes very difficult. The following situations will be faced when looking for solutions to real problems:

- (1) real flow rates of the pipelines smaller than the calculated ideal flow rates;

- (2) actual pipelines flow rates greater than the calculated ideal flow rates;

- and (3) very small ideal flow rates, making the supply unsatisfactory.

Situations (1) and (3) can be easily solved by indicating maximum and minimum values, respectively, for the inflow rates calculated by the solver. However, care must be taken not to set values that make the problem unresolvable. The planner must always keep in mind that the water distribution network and the flow rates form a single system, with values that are essentially integrated.

In the case of situation (1), as the solver seeks to maximize the volume of water distributed to the zones, the volume of distribution (limited by the inflow rate) in a given location will be transferred to other locations seeking to meet

the demand and, later, distributed to filling the idle capacity of the reservoirs. For example, Table 8 presents the results found by the solver for case 22, in which the same data used in case 06 were considered, except that a maximum flow rate was informed for zone Z1 ($z = 7$), $\text{InFR}_{\text{max7}} =$130,000.00 liters/hour (case 06 is just like case 03 but with 2 days).

For situation (3), once a minimum value is indicated for the inflow of a given location, the solver will propose solutions in which the inflow of some locations will be null or concentrated in one or a few shifts. For example, Table 9 presents the results found by the solver for case 23, in which the same data used in case 06 were considered, except that a minimum flow rate was reported for the reservoir R5 ($r = 6$), $\text{InFR}_{\text{min6}} =$5,000.00 liters/hour. As we can see in this table, the inflow of reservoir R5 and zone Z6 received value 0 for the first shift of the first day. The solver presented a result for case 23 after setting the parameter value $\text{RF}_{\text{valves}} = 10,000$.

As for situation (2), the developed solver does not present a good answer. The value found by the solver will always be an ideal value, based on the informed restrictions. If the ideal value is lower than the real value, the respective zone will have an excess amount of water. And there will be no sure way to prevent the volume consumed by that zone from being the desired volume. Except limiting distribution time. In this case, the ideal distribution time (within the period of a shift) can be calculated using the equations in (25).

3.2.4 Solver and model limitations

Lingo is an incredible software, indispensable for mathematical modeling of any real problem. With the help of Lingo, it is possible to notice a series of initial errors in the model, which could easily go unnoticed. Additionally, the elaborated mathematical model strongly influences the solver efficiency. This forces the modeler to think about issues that limit the model's solutions, and to work with functions that make the model robust and functional.

However, Lingo is not a magic tool, Lingo's processing power is limited, due to limitations associated with exact solving techniques for optimization problems, especially when defined as non-linear and NP-Hard. For most problems, the computational effort increases as the number of decision variables increases, according to functions that are not polynomials. In other words, there is an explosion in the number of possible solutions, and current exact methodologies have not yet found a way to deal with this, in the case of problems with larger dimensions.

In this section, we present an initial discussion about the limitations of the developed solver and mathematical model. It is important to emphasize that many of the limitations presented here could be mitigated through adjustments to the model, or even through a better use of the Lingo software. However, this requires a hard work. It may seem like it, but building a robust and adequate mathematical model is not a simple task and Lingo offers a wide range of different application possibilities through various parameter adjustments, and we are still familiarizing with all these possibilities.

Table 10 presents some tests carried out to analyze the efficiency of the developed model and solver. We run the solver for case 01, and for similar cases where only the number of days and shifts have been changed. We used an interval between 1 and 7 days and, for each possibility, we considered 1, 2 and 3 shifts. The relevance / irrelevance factors referring to fair distribution, valve status and reservoir idle capacity were defined based on an extensive study. Such definition had the following purposes: to define as priorities meeting the demand, then fair distribution and finally filling the reservoirs; in order to test the ruggedness of the model, we also had the clear intention of distributing the flow among all shifts, allowing the valves to be turned off only when flow restrictions impose this condition.

As can be seen in this table, we adopted $RF_{valves} = 2000$ for cases 12, 16 and 21. This is because we identified that the developed solver had difficulty in finding optimal solutions for these three cases using $RF_{valves} = 1000$. In case 12, the solver found a solution that claimed to be a global optimum, but that was not. In cases 16 and 21, the solver could not find a viable solution to the problem within the stipulated time limit (10 minutes). An interesting aspect of the set of real data collected is that it turned the studied problem into a natural poorly sized problem. This is due, in large part, to the dimension of reservoir capacities. We believe that this problem of undersizing can be easily solved, however, to date, we have not found a solution.

Another important issue to be considered is that the adjustment of the parameters RF_{fair}, RF_{valves}, and $IrF_{idleness}$ strongly influence the efficiency of the solver as well as the solution found. An important aspect of the relevance / irrelevance parameters adopted is that they allow the planner to interfere in the solution presented by the solver, so that this solution meets the planning objectives (e.g., prioritization of filling reservoirs or meeting demand). However, this requires that the planner has good experience in applying the solver and a good understanding of how solve reacts to the adjustment of these parameters. Therefore, the good performance of the developed solver will depend largely on the qualification of the planner for this purpose. We understand that it is possible to build formulas for automatic calculation of suitable values for these parameters, using quantitative and qualitative analyzes referring to the decision-making process (e.g., formulas defined according to answers given for some questionnaire). It is also possible, of course, to develop a user-friendly graphical interface. These efforts would make using the solver much simpler, however they depend on the continuity of this project, with a strong commitment of time and dedication of professionals with adequate intellectual capacities.

4   Conclusions and future works

In the present work, a conceptual and mathematical modeling for the problem of water distribution in periods of scarcity was presented. The validation of the mathematical model was performed using a solver developed with the Lingo software from Lindo Sistems. Through the tests carried out, we concluded that

the developed model and solver fit perfectly to the studied reality, and are capable of proposing solutions that make the distribution of water volumes for the different zones fair, in periods of scarcity, through the control of flows and distribution times. Such solutions take into account the restrictions of the system and present alternatives for when the channelings are to be modulated optimally (with flow rates that optimally regulate the distribution). However, it was verified that, for the solver to present good results, it must be used by a suitably qualified employee. What may become less relevant through new advanced studies for high-level calculation of relevance / irrelevance parameters adopted in the model, as well as through the construction of a friendly graphic interface. Additionally, due to the limitations of the exact methodologies developed so far for solving non-polynomial optimization problems (such as the studied problem), the developed solver naturally presented limitations related to its ability to solve larger problems. Such limitations justify the need to develop solution algorithms based on heuristics applied to optimization problems. These efforts will be the focus of our next works.

Referências

(2016) 'Blinded'.

Huang, Z., Yuan, X. and Liu, X. (2021) 'The key drivers for the changes in global water scarcity: Water withdrawal versus water availability', *Journal of Hydrology*, Vol. 601, pp.126658

IBGE. Instituto Brasileiro de Geografia e Estatística. (2021) 'Regional data about the city of Catende', https://cidades.ibge.gov.br/brasil/pe/catende/panorama (Accessed 27 November 2021)

Kurian, V., Chinnusamy, S., Natarajan A. Narasimhan, S. and Narasimhan S. (2018) 'Optimal operation of water distribution networks with intermediate storage facilities', *Computers & Chemical Engineering*, Vol. 119, pp.215–227

LINGO [online], https://www.lindo.com/index.php/products/lingo-and-optimization-modeling (Accessed 30 November 2021)

Paganine, J. (2013) 'Dia Mundial da Água incentiva cooperação entre os países', *senadonotícias*, *Agência Senado*, https://www12.senado.leg.br/noticias/especiais/especial-cidadania/dia-mundial-da-agua-incentiva-cooperacao-entre-os-paises (Accessed 28 November 2021)

Sankar, G.S., Mohan Kumar, S., Narasimhan, S., Narasimhan S. and Murty Bhallamudi, S. (2015) 'Optimal control of water distribution networks with storage facilities', *Journal of Process Control*, Vol. 32, pp.127–137

Silva, J.L.B., Bezerra, A.C., Silva, T.T.F., Batista, P.H.D., Moura, G.B.A. and Lopes, P.M.O. (2019) 'Quantification Caatinga vegetable coverage and water availability by remote sensing in the Brazilian semiarid', *Journal of Hyperspectral Remote Sensing*, Vol. 9, No. 4, pp.166–176

Figura 7: Determination of optimal distribution values with 'LibreOffice Calc', considering a daily 24-hour distribution period

number of days				1	day			
number of shifts per day				1	shift			
time per shift				24	(hours)			
average capacity of reservoirs in the zones				1000	(liters)			case 01
average consumption per inhabitant				200	(liters / day)			
housing coefficient				3,9	(inhabitants / household)			
irrelevance factor				5E-06				

local	households	demand (liters)	unmet (liters)	capacity (m³)	initial vol. (m³)	IrF	idle capacity * (m³)	inflow ** (liters)
ETA				2.500.000,00	0,00	1,13	2.498.099,20	8.812.800,00
R1				500.000,00	0,00	1,13	500.000,00	6.912.000,00
R2				400.000,00	0,00	1,12	400.000,00	1.456.410,77
R3				100.000,00	0,00	1,12	100.000,00	331.100,81
R4				180.000,00	0,00	1,12	180.000,00	1.109.440,20
R5				20.000,00	0,00	1,12	20.000,00	97.382,59
Z1	5431	4.236.180,00	318.514,38	5.431.000,00	0,00	1,00	5.431,00	3.917.665,62
Z2	1043	813.540,00	61.169,30	1.043.000,00	0,00	1,00	1.043,00	752.370,70
Z3	976	761.280,00	57.239,92	976.000,00	0,00	1,00	976,00	704.040,08
Z4	459	358.020,00	26.919,19	459.000,00	0,00	1,00	459,00	331.100,81
Z5	1538	1.199.640,00	90.199,80	1.538.000,00	0,00	1,00	1.538,00	1.109.440,20
Z6	135	105.300,00	7.917,41	135.000,00	0,00	1,00	135,00	97.382,59
total	9582	7.473.960,00	561.960,00		weighted sum		55.519.842,31	weir outflow (liters) **
weir								8.812.800,00

					local	volume received per inhabitant (liters / inhab.)	inflow rate *** (liters / hour)
total volume distributed to zones		6.912.000,00			WTP		367.200,00
fitness (optimal value)		6.646.400,79			R1	****	288.000,00
					R2		60.683,78
relevance factors:					R3		13.795,87
fair dist.	100000				R4		46.226,68
valves	1000				R5		4.057,61
					Z1	184,96	163.236,07
					Z2	184,96	31.348,78
* at the end of the day					Z3	184,96	29.335,00
** per shift					Z4	184,96	13.795,87
*** average flow per shift					Z5	184,96	46.226,68
**** average volume per shift					Z6	184,96	4.057,61

location	total demand (liters)	capacity (m³)	initial vol. (m³)	idle capacity ** (m³)	total inflow (liters)
WTP		2,500,000.00	0.00	2498,099.20	8,812,800.00
R1		500,000.00	0.00	500,000.00	6,912,000.00
R2		400,000.00	0.00	400,000.00	1,456,410.77
R3		100,000.00	0.00	100,000.00	331,100.81
R4		180,000.00	0.00	180,000.00	11,094,40.20
R5		20,000.00	0.00	20,000.00	97,382.59
Z1	4,236,180.00	5,431,000.00	0.00	5,431,000.00	3,917,665.62
Z2	813,540.00	1,043,000.00	0.00	1,043,000.00	752,370.69
Z3	761,280.00	976,000.00	0.00	976,000.00	704,040.07
Z4	358,020.00	459,000.00	0.00	459,000.00	331,100.81
Z5	1,199,640.00	1,538,000.00	0.00	1,538,000.00	1,109,440.20
Z6	105,300.00	135,000.00	0.00	135,000.00	97,382.59
total	7,473,960.00	exp. weighted sum:		55,519,842.31	

shift data		location	valve status	ideal inflow rate (liters/hour)
1	day	WTP	open	367,200.00*
1	shif	R1	open	288,000.00*
24	hours/shift	R2	open	60,683.78
		R3	open	13,795.87
total zones outflow		R4	open	46,226.68
(liter/shift)		R5	open	4,057.61
		Z1	open	163,236.07
6,912,000.02		Z2	open	31,348.78
		Z3	open	29,335.00
optimal value (fitness)		Z4	open	13,795.86
		Z5	open	46,226.67
6,646,400.78		Z6	open	4,057.61

(computational time = 4.62 sec)   * values reported for maximum inflow rates
(total number of iterations = 2848)   ** at the end of the evaluated period

Tabela 6: Summary of results generated by the solver developed in Lingo for case 01

location	total demand (liters)	capacity (m³)	initial vol. (m³)	idle capacity ** (m³)	total inflow (liters)
WTP		2,500,000.00	0.00	2,498,099.20	8,812,800.00
R1		500,000.00	0.00	500,000.00	6,912,000.00
R2		400,000.00	0.00	400,000.00	1,456,411.09
R3		100,000.00	0.00	100,000.00	331,100.96
R4		180,000.00	0.00	180,000.00	1,109,438.02
R5		20,000.00	0.00	20,000.00	97,382.57
Z1	4,236,180.00	5,431,000.00	0.00	5,431,000.00	3,917,667.30
Z2	813,540.00	1,043,000.00	0.00	1,043,000.00	752,371.02
Z3	761,280.00	976,000.00	0.00	976,000.00	704,040.07
Z4	358,020.00	459,000.00	0.00	459,000.00	331,100.95
Z5	1,199,640.00	1,538,000.00	0.00	1,538,000.00	1,109,438.02
Z6	105,300.00	135,000.00	0.00	135,000.00	97,382.63
total	7,473,960.00	exp. weighted sum:		55,519,842.31	

	shift data		location	valve status T1,T2 and T3	ideal inflow rate (liters/hour)
1	day		WTP	open	367,200.00*
1	shif		R1	open	288,000.00*
24	hours/shift		R2	open	60,683.78
			R3	open	13,795.87
	total zones outflow		R4	open	46,226.68
	(liters)		R5	open	4,057.60
			Z1	open	163,236.07
	6,912,000.00		Z2	open	31,348.77
			Z3	open	29,335.00
	optimal value (fitness)		Z4	open	13,795.87
			Z5	open	46,226.68
	6,670,400.71		Z6	open	4,057.61

(computational time = 9.82 sec)   * values reported for maximum inflow rates

(total number of iterations = 5,709)   ** at the end of the evaluated period

Tabela 7: Summary of results generated by the solver developed in Lingo for case 03

location	total demand (liters)	capacity (m^3)	initial vol. (m^3)	idle capacity ** (m3)	total inflow (liters)
WTP		2,500,000.00	0.00	2,495,089.96	17,625,600.00
R1		500,000.00	0.00	500,000.00	12,715,560.00
R2		400,000.00	0.00	400,000.00	3,149,640.00
R3		100,000.00	0.00	100,000.00	716,040.00
R4		180,000.00	0.00	180,000.00	2,399,280.00
R5		20,000.00	0.00	20,000.00	210,600.00
Z1	8,472,360.00	5,431.00	0.00	5,431.00	6,240,000.00
Z2	1,627,080.00	1,043.00	0.00	1,043.00	1,627,080.00
Z3	1,522,560.00	976.00	0.00	976.00	1,522,560.00
Z4	716,040,00	459.00	0.00	459.00	716,040.00
Z5	2,399,280.00	1,538.00	0.00	1,538.00	2,399,280.00
Z6	210,600.00	135.00	0.00	135.00	210,600.00
total	14,947,920,00	exp. weighted sum:		110,966,229.48	

		location	valve status T1 and T2	ideal inflow rate (liters/hour)
	shift data			
2	day	WTP	open	367,200.00*
3	shift	R1	open	264,907.50
8	hours/shift	R2	open	65,617.50
		R3	open	14,917.50
	total zones outflow	R4	open	49,985.00
	(liters)	R5	open	4,387.50
		Z1	open	130,000.00*
	12,715,560.00	Z2	open	33,897.50
		Z3	open	31,720.00
	optimal value (fitness)	Z4	open	14,917.50
		Z5	open	49,985.00
	12,217,517.79	Z6	open	4,387.50

(computational time = 5.01 sec)   * values reported for maximum inflow rates
(total number of iterations = 1,030)   ** at the end of the evaluated period

Tabela 8: Summary of results generated by the solver developed in Lingo for case 22

location	total demand (liters)	capacity (m^3)	initial vol. (m^3)	idle capacity ** (m3)	total inflow (liters)
WTP		2,500,000.00	0.00	2496,198.40	17,625,600.00
R1		500,000.00	0.00	500,000.00	13,824,000.00
R2		400,000.00	0.00	400,000.00	2,912,757.37
R3		100,000.00	0.00	100,000.00	657,277.71
R4		180,000.00	0.00	180,000.00	2,218,825.95
R5		20,000.00	0.00	19,999.66	200,000.00
Z1	8,472,360.00	5,431.00	0.00	5,431.00	7,835,138.97
Z2	1,627,080.00	1,043.00	0.00	1,043.00	1,504,711.77
Z3	1,522,560.00	976.00	0.00	976.00	1,408,045.60
Z4	716,040,00	459.00	0.00	459.00	657,277.71
Z5	2,399,280.00	1,538.00	0.00	1,538.00	2,218,825.95
Z6	210,600.00	135.00	0.00	135.00	199,660,77
total	14,947,920,00	exp. weighted sum:		110,996,699.80	

	shift data		location	valve status T1/T2-T5	ideal inflow rate (liters/hour)
2	day		WTP	open/open	367,200.00*
3	shift		R1	open/open	288,000.00*
8	hours/shift		R2	open/open	60,682.45
			R3	open/open	13,693.29
	total zones outflow		R4	open/open	46,225.54
	(liters)		R5	close/open	5.000.00*
			Z1	open/open	163,232.06
	13,823,660.77		Z2	open/open	31,348.16
			Z3	open/open	29,334.28
	optimal value (fitness)		Z4	open/open	13,693.29
			Z5	open/open	46,225.54
	13,968,606.16		Z6	close/open	4,991.51

(computational time = 218.47 sec)   * values reported for maximum inflow rates
(total number of iterations = 90,978)   ** at the end of the evaluated period

Tabela 9: Summary of results generated by the solver developed in Lingo for case 23

case	ND	NS	fitness	time(s)	ierations	RF$_{fair}$	relevance factors IrF$_{idleness}$	RF$_{valves}$
01	1	1	6,646,400.78	4.62	2848	100000	0.000005	1000
02	1	2	6,658,400.79	7.47	5629	100000	0.000005	1000
03	1	3	6,670,400.71	9.82	5709	100000	0.000005	1000
04	2	1	13,293,014.91	11.38	7496	100000	0.000005	1000
05	2	2	13,317,009.22	4.52	1812	100000	0.000005	1000
06	2	3	13,341,015.05	6.95	2139	100000	0.000005	1000
07	3	1	19,939,821.57	3.40	878	100000	0.000005	1000
08	3	2	19,975,821.57	10.38	4180	100000	0.000005	1000
09	3	3	20,011,821.57	15.00	4176	100000	0.000005	1000
10	4	1	26,586,806.96	12.34	5556	100000	0.000005	1000
11	4	2	26,634,806.96	16.98	642	100000	0.000005	1000
12	4	3	26,826,806.96	27.84	6264	100000	0.000005	2000
13	5	1	33,233,971.23	17.03	6408	100000	0.000005	1000
14	5	2	33,293,970.82	35.18	8371	100000	0.000005	1000
15	5	3	33,353,912.05	90.32	15996	100000	0.000005	1000
16	6	1	39,953,314.33	24.23	7386	100000	0.000005	2000
17	6	2	39,953,314.29	22.09	5295	100000	0.000005	1000
18	6	3	40,025,314.33	244.84	37937	100000	0.000005	1000
19	7	1	46,528,835.72	58.08	95871	100000	0.000005	1000
20	7	2	46,612,835.71	24.24	3495	100000	0.000005	1000
21	7	3	46,948,761.12	96.78	12703	100000	0.000005	2000

Tabela 10: Study on the efficiency of the mathematical model and the solver