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Optimal control of water distribution networks with storage facilities

Gokul Siva Sankar[a], S. Mohan Kumar[a], Sridharakumar Narasimhan[a,*], Shankar Narasimhan[a], S. Murty Bhallamudi[b]

[a] Department of Chemical Engineering, Indian Institute of Technology Madras, India
[b] Department of Civil Engineering, Indian Institute of Technology Madras, India

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ABSTRACT

Optimal operation of water distribution networks (WDNs) is concerned with meeting consumer demands at desired pressures in an efficient and equitable manner while conserving resources. This can be achieved by implementing advanced control schemes such as model predictive control (MPC). If sufficient water is available, the control objective is to meet consumer demands while preventing wastage. On the other hand, if the available water is insufficient or inadequate to meet consumer demands at the required pressures, equitable distribution of the available resource is of primary importance. In this contribution, a nonlinear model predictive controller is proposed for optimal operation of WDNs that can deal with both the above situations. The proposed approach takes into account availability of storage facilities at the source and demand points. In addition, the control algorithm can account for plant-model mismatch. Performance of the proposed model based control strategy is illustrated through numerical simulations of an illustrative WDN operating under various water availability scenarios. In the water sufficient scenario, the proposed MPC strategy is able to meet the consumer requirements while minimizing the excess amount of water supplied. In the water deficient scenario, the MPC algorithm is able to exploit the available storage facilities at consumer end to reduce the daily supply deficit by about 20%. Using a longer prediction horizon in MPC results in a further reduction of about 40% in the daily supply deficit.

1. Introduction

A water distribution network (WDN) is an important component of civic infrastructure used for providing water for domestic as well as industrial uses. Research in the field of water distribution networks has been concerned mainly with the simulation, calibration and optimal design of networks. Since water scarcity is a major problem in many countries, utilities that operate WDNs are interested in increasing the efficiency and reliability of operation while conserving water. As a result, there has been increased focus on optimal control of WDNs for improving the operational performance of the system. Optimal operation and control of water distribution networks deals with the problem of taking appropriate operational decisions to achieve certain performance goals or objectives. The objectives that have been considered in the past include (i) minimizing the cost of energy required for pumping, (ii) regulating pressures for preventing/reducing leakages, and (iii) maximizing the quality of water supplied to the users by reducing the residence time of water in the network.

Under the assumption that the required demands of customers can be met, one of the important objectives considered in the control of a WDN is the minimization of operating costs, a significant proportion of which is the cost of pumping [1–3]. Lansey and Awumah [4] used dynamic programming in a simulation–optimization framework for determining optimal pump operations for a fixed horizon, while limiting the number of pump switches between on and off status. In the above methods, the decision variables were the status of pumps which were treated as discrete binary (0 or 1) variables. Klempous et al. [5] proposed two algorithms: one for simulation of the network and the other for determining the actual number of working pump units and regulating valve positions to minimize energy costs, taking into account the varying prices for electricity. Ertin et al. [6] determined a pump-schedule that control pumps at a booster station to meet water demands while maintaining reservoirs at acceptable levels. Yu et al. [7] considered both constant rates and variable rates for cost of electricity. They considered variable speed pumps and thus the decision variables are pump static pressure heads and water levels in tanks. Rao and Salomons [8] proposed an artificial

* Corresponding author. Tel.: +91 4422574177.
E-mail address: sridharkrn@iitm.ac.in (S. Narasimhan).

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Nomenclature

A_R	cross-sectional area of the reservoir (m^2)
A_t	cross-sectional area of the storage tank (m^2)
a_{ij}	pipe-node incidence coefficients
C_v	valve coefficient
d_i	outflow rate at demand node i (m^3/s)
d_i^{sp}	desired outflow rate at demand node i (m^3/s)
D_j	diameter of the pipe j (m)
Δd_i	cumulative deviation between demand outflow rate and the requirement at demand node i (m^3/s)
E	number of pipes in the WDN
E_v	number of control valves
J	objective function of the control problem
G_L	specific gravity of the fluid
Δh_j	head loss in pipeline element j (m)
Δh_{v_j}	head loss across control valve j (m)
k_i	resistance offered to outflow at demand node i
h_i	pressure head at node i (m)
$h_{i\,\min}$	specified minimum pressure head to be maintained at demand node i (m)
H_r	water level in reservoir (m)
H_{t_i}	water level in storage tank at demand node i (m)
$H_{t_i\,\max}$	maximum storage height of the storage tank at demand node i (m)
$\hat{H}_{t_i}$	estimate of water level in storage tank at demand node i (m)
$\hat{H}_{t_i}^{pmm}$	estimate of plant-model mismatch in terms of water level in storage tank at demand node i (m)
L_j	length of the pipeline j (m)
M	control horizon length
N	number of nodes in the network
N_d	number of demand nodes
P	prediction horizon length
Q_{in_i}	sum of inflow rates into a reservoir/tank at node i (m^3/s)
Q_j	flow rate in pipeline/valve j (m^3/s)
Q_{out_i}	sum of outflow rates from a reservoir/tank at node i (m^3/s)
Δt	sampling time period (s)
u	vector of manipulated inputs
V	fraction of valve open
z_i	elevation of node i (m)
λ	Hazen–Williams coefficient
τ	valve rangeability

neural network (ANN) model as a surrogate for hydraulic simulation model, and used a genetic algorithm (GA) as the optimization technique to minimize energy consumption.

The other aspect that has been considered in the operation and control of a WDN is the quality of water supplied. Cembrano et al. [9] considered cost of water supply and treatment, and costs related to pressure and flow regulation in their objective function. The decision variables are optimal number of pumps to operate, and control valve positions. Ostfeld and Shamir [10,11] developed models for optimal operation of a multi-quality water supply system under steady and unsteady state conditions. The objective was to minimize the total cost of water treatment and pumping, while delivering water to all consumers at acceptable qualities and pressure. Constans et al. [12] developed a new control-oriented computational model for chlorine concentration in distribution networks operating under varying demand conditions. They solved a constrained linear programming optimization problem which minimizes the difference between the permissible and actual chlorine concentration levels.

All the above studies on optimal operation and control of WDNs only solve the "open loop problem", viz the offline optimization problem of determining the optimal operating policy for pumps and valves so as to satisfy the chosen objectives. Active online control of the WDN based on feedback from measured variables of the network is not incorporated in these formulations. The feedback information from the system is important for dealing with uncertainties in the model and in the operation. Only the methods proposed by [13] and [9] mention the use of incorporating feedback from the measurements in their formulation and the method for achieving this is clearly explained only by [13], under the assumption that the controlled variables are all measured.

Miyaoka and Funabashi [13] developed a two-level control scheme for the optimal control of WDNs such that the pressures at all nodes and inflows at the supply nodes are close to the specified values. This is to minimize the wastage of water due to excess withdrawal at the consumption nodes. In the first level, the optimal operating point which minimizes the above objective function is determined. A linear quadratic regulator (LQR), based on observed flows and pressures, is designed at the second level to determine the optimal valve settings that regulate the WDN operation at the optimal state determined in the first level. Jowitt and Xu [14] proposed a method for reducing the leakage from the network by optimally controlling the pressures at the outflow nodes by manipulation of flow control valves. A pressure dependent outflow model of WDN was used in their formulation.

Biscos et al. [15] proposed a method for generating control strategies ahead of time, using predictive techniques, to achieve certain objectives such as maximization of the use of low cost power (overnight pumping) and the maintenance of target chlorine concentration. Constraints of the problem are defined on valve openings, reservoir levels, and chlorine concentration. A model predictive control (MPC) technique is used to solve the problem. Tu et al. [16] developed a multi-commodity flow model to optimize water distribution and water quality in a water supply system with blending requirements, perfect mixing and two way flow conditions.

The above-described MPC techniques use a single, monolithic controller for network control. Alternatively, Trnka et al. [17] have demonstrated different types of distributed control schemes. Javalera et al. [18] showed that the solution of multi-agent control based on negotiations are within acceptable degree of accuracy as compared to a centralized controller. Given the large, complex and heterogeneous nature of WDNs, distributed MPC (DMPC) is an attractive alternative to centralized MPC. For control of large WDNs, Leirens et al. [19] proposed the use of DMPC based on linearized models of the system. Ocampo-Martinez et al. [20] derived a suboptimal DMPC that allowed hierarchical controllers to control each sub-network.

In all the above methods for optimal operation/control, the objectives chosen are applicable to situations in which sufficient water is available to meet the demand requirements at all the nodes. However, with the alarming depletion of water resources and increasing population, it may not always be possible to meet customer demands at all times and in all places. In certain situations, even if adequate quantity of water is available, the pressure in the balancing reservoirs may not be sufficient to deliver the required flow rate of water to customers. These situations are collectively referred to in this work as water deficient. In water deficient situations, it is important that the control strategy allocates water so as to minimize the shortfall in supply, and supplies the available water in as equitable a manner as possible. Even if sufficient water is available, it is necessary to conserve this precious resource by preventing wastage through excess withdrawal. This makes the management and control of a WDN an interesting and challenging problem [21].

Apart from balancing reservoirs that are used as pressure regulators in WDNs, in several developing countries such as India, storage facilities in the form of underground sumps and overhead tanks are available at the consumption nodes of the distribution networks for storing water. Storage units facilitate intermittent operation of WDNs, providing flexibility in the operation of WDNs. These tanks serve as intermediate storage elements (buffer units) when demand requirements cannot be met, especially in water deficient situations. Only in a recent paper [22] has the availability of storage tanks at consumer nodes been modelled and taken into account in the hydraulic simulation of a WDN. The control of WDNs which also take into account storage facilities available at the demand nodes was first proposed by Sankar et al. [23], where limited results for water sufficient scenarios were reported. An appropriate control objective has to be chosen such that the flexibility offered by the storage units is utilized in the optimal operation of WDNs with storage facilities, to meet the hourly needs of the consumers in both water sufficient and water deficient situations.

In this work, we propose a nonlinear model predictive control (NMPC) technique that can be used to operate and control a WDN under sufficient and deficient conditions of water supply. Since a WDN is inherently a coupled nonlinear system, and a change in a control valve position can have an effect on the outflows at several nodes, the proposed NMPC will be able to effectively deal with the interaction between different controlled variables. The proposed method also takes into account storage facilities available at demand nodes, and can handle the supply side and storage constraints that need to be satisfied. The performance of the NMPC technique is demonstrated by numerical simulation studies on an illustrative network.

2. Mathematical modelling of WDNs

Any model predictive controller for a system requires three key components: (i) a model for simulating the dynamics of the system, (ii) a strategy for estimating the current state of the system from available measurements (observer), and (iii) an algorithm i.e., controller for determining the optimal values of the manipulated (decision) variables. The dynamic model of a WDN used for control purposes is developed in this section.

As WDNs operate on a 24 h basis, for controlling the operations of a WDN, it is sufficient to take control actions (such as changing control valve positions, status of on–off valves and status/speed of pumps) once every hour. A quasi-steady state model can be used to describe the behaviour of flows in a WDN fairly accurately, since the fluid is incompressible and transient changes introduced by the controller or by external disturbances are quickly transmitted and dissipated. In a quasi-steady state model, it is assumed that the flows and pressures are steady during each time interval between control actions, and can be described using a steady state WDN model. The reservoir levels and water levels in tanks are updated at the end of each time period based on the inflows to and outflows from the reservoirs and tanks during the time period. The steady state calculations for the next time period are based on the updated reservoir levels and the current position/status of valves and/or pumps provided by the controller. This approach is similar to performing extended period simulations in analysis of water distribution networks.

In literature [24], two types of steady state models have been developed to determine the flows and pressures in a WDN. On one hand, in a demand driven model, the source pressures at reservoirs and outflow rates from all demand nodes are assumed to be given, and the pipe flows and nodal pressures are computed using the hydraulic model. On the other hand, a pressure dependent outflow model assumes that the outflow rates from demand nodes are dependent on the pressures at these nodes. The demand driven flow model is an appropriate choice, if the objective of control is to minimize the energy requirements, under the assumption that the demand requirements of consumers are met exactly. The outflow rates at demand nodes in such cases are equal to the specified demand requirements. However, this model is unsuitable in deficient situations, where it might not be possible to meet the consumer demands. In such cases, the optimal outflows from demand nodes have to be determined as part of the control strategy to meet the desired objectives. As this work is concerned with the latter case, a pressure dependent outflow model is used. Using this model, the controller determines the optimal control valve settings which affect the pressures at the demand nodes, thereby determining the optimal outflow rates from demand nodes.

The pressure dependent outflow model for a WDN is described by the following three sets of equations: (i) continuity equation at each node, (ii) correlations for head loss in a pipe and a control valve, as a function of flow rate, and (iii) an equation for outflow rate from a demand node as a function of the pressure at that node. The continuity equation at any node is given as,

$$\sum_{j=1}^{E} a_{ij} Q_j(k) - d_i(k) = 0, \quad i = 1, \ldots, N \tag{1}$$

where at time k, Q_j is the flow through the pipeline j, d_i is the supply rate (or inflow rate if the node is a source node) at the node i, N and E are the total number of nodes and pipes in the network, respectively. The coefficients a_{ij} describe the incidence of different pipes on the nodes as follows:

$$a_{ij} = \begin{cases} 1 & \text{if flow in pipe } j \text{ is directed towards node } i \\ -1 & \text{if flow in pipe } j \text{ is directed away from node } i \\ 0 & \text{if flow in pipe } j \text{ is not incident on node } i \end{cases} \tag{2}$$

The head loss in a pipeline is given by the empirical Hazen–Williams equation,

$$\Delta h_j(k) = \frac{\gamma L_j \,\mathrm{sign}\,(Q_j(k)) |Q_j(k)|^{1.85}}{C_j^{1.85} D_j^{4.87}}, \quad j = 1, \ldots, E \tag{3}$$

where C_j is the Hazen–Williams roughness coefficient, L_j is the length of the pipeline j, D_j is the internal diameter of the pipe j, and γ is a factor depending on the dimensional system used. $\gamma = 10.7$ when SI system of units is used (flow rate, Q_j is expressed in m^3/s, length and diameter of the pipelines in m, and Δh in m of water).

Several different relations have been proposed to relate the outflow rate to the nodal pressure [25]. In this work, the following relation is used to relate the outflow rate at each demand node i to the corresponding nodal pressure.

$$h_i(k) - z_i = R_i d_i^2(k), \quad i = 1, \ldots, N_d \tag{4}$$

where N_d is the total number of demand nodes, h_i is the pressure head at the node i and z_i is the elevation and, R_i is the resistance offered to outflow at demand node i.

In addition, pressure drop across non-pipe elements in the network such as control valves have to be modelled. Since the manipulated variables are valve settings (in percentage opening), it is necessary to model the pressure drop as a function of valve openings. The following equation as in [26] is used to predict the pressure drop across any valve:

$$\Delta h_v(k) = \frac{Q^2(k) G_L}{(C_v((e^{\ln \tau V(k)})/\tau))^2} \tag{5}$$

where Δh_v is the head loss across the valve, Q is the flow rate through the valve, C_v is the valve flow coefficient when valve is fully

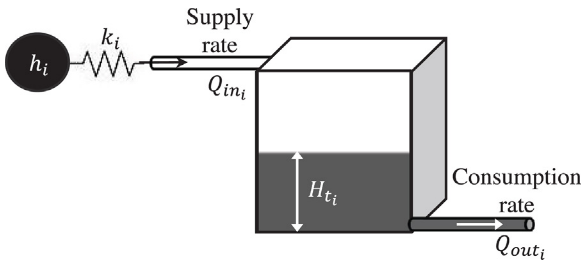

Fig. 1. Storage tank linked to a demand node of a WDN.

open, τ is the valve rangeability, V is a fraction of valve opening (0 when closed and 1 when fully open), and G_L is the specific gravity of the fluid flowing through the valve. The term in the denominator of above equation models the change in valve coefficient as a function of valve opening.

During each time period, the reservoir water levels are treated as fixed quantities and the corresponding steady state solution is obtained. This steady state solution is used to update the reservoir levels to be used for the following time period. Essentially, the differential equation relating the reservoir level or the water level in the storage tanks (as the case may be), is approximated as follows:

$$H_i(k) = H_i(k-1) + \frac{\Delta t}{A_i}\left(Q_{\text{in}_i}(k) - Q_{\text{out}_i}(k)\right) \qquad (6)$$

where $H_i(k)$ is the water level in the ith reservoir/storage tank during time period k, $Q_{\text{in}_i}(k)$ and $Q_{\text{out}_i}(k)$ are the total inflow and outflow rates, respectively, Δt is the sampling time and A_i is the cross sectional area of the reservoir/tank. In WDNs with storage facilities at demand nodes, the demand node is linked to the storage tank as shown in Fig. 1 and the inflow to the storage tank, Q_{in_i} at the node i is equal to the outflow from demand node, d_i (i.e., $Q_{\text{in}_i} = d_i$).

The consumption rate from storage tank in the network is typically uncertain and depends on the consumer behaviour. Generally a projected estimate of the consumer demand rate is available (referred to as the demand set point rate), which is used as a basis for operation of the WDN. The actual consumption rate may be more or less than the demand set point rate due to different reasons. We need a model of the consumer behaviour in order to estimate the water level in the storage tank as part of the model predictive control strategy. In our model, we assume that the average rate of water consumption from the storage tank by a consumer in any period is equal to the demand set point rate, if sufficient or excess water is supplied. If the water supplied is insufficient to meet the demand set point rate, then consumption rate is limited by the amount of water supplied during the period together with the amount of water already available in the storage tank. Under this assumption, the consumption rate from the storage tank is given by

$$Q_{\text{out}_i}(k) = \min\left(d_i^{sp}(k),\ \frac{H_i(k-1)\times A_i}{\Delta t} + Q_{\text{in}_i}(k)\right), \quad i = 1, \ldots, N_d \qquad (7)$$

where $d_i^{sp}(k)$ is the demand set-point, and $H_i(k-1)$ is the height of the water level in the storage tank at demand node i during the time period $k-1$.

Eqs. (1)–(7) are solved simultaneously to determine the steady state flow rates in pipelines, pressure heads at all nodes, outflow rates at all demand nodes in the network, and water levels in storage tanks at the demand nodes, given the minimum specifications: water levels in the reservoirs (source pressure heads), control valve stem positions and the network characteristics that include network topology, pipeline, valve and pump characteristics. This steady state model is used in extended period simulations for predicting the dynamic behaviour of the network to be used by the control algorithm.

3. Model predictive control of WDN

3.1. MPC strategy

Model predictive control (MPC) is an advanced control technique that has been widely accepted by the chemical [27] and biochemical industries [28,29] due to its multivariable formulation and its constraint handling abilities. Qin and Badgwell [30] have given a comprehensive overview of industrial model predictive control technology. The basic idea behind the MPC technique is shown in Fig. 2.

A key requirement for using an MPC technique is a dynamic model of the process which can be used to predict the behaviour of the process states over a finite time horizon, given the values of the manipulated variables over that time period [31]. Using this dynamic model, at each time step k, an optimization problem is solved. The objective function is typically chosen as the weighted sum of differences between predicted state variables and their desired set point profiles. The objective may also include a penalty for abrupt changes in the manipulated variable values by adding a term containing the weighted sum of the changes in the manipulated variable profiles. The time period for prediction contains P time steps and is known as the prediction horizon. The manipulated variable values are chosen over a control horizon of M time steps (usually, $M \le P$) after which they are held constant over the remaining $P - M$ time steps. Although M moves are optimized, only the first move $u(k)$ is implemented where, $u(k)$ is the vector of manipulated variables. After $u(k)$ is implemented, the measurements, $\bar{y}(k)$, are obtained.

Due to plant-model mismatch (mismatch between the process model and the actual process) and other unaccounted disturbances that may have occurred during the kth time interval, the measured outputs, $\bar{y}(k)$, will not be equal to the value predicted using the model. Therefore, a correction for the model error is performed. A new optimization problem is then solved over a predicted horizon of P steps adjusting M control moves. This approach is known as receding horizon control. The MPC technique can be summarized as follows [32]:

- Explicit use of the model to predict the process output at future time instants (prediction horizon)
- Calculation of control sequence over the control horizon, minimizing an objective function; and
- Receding strategy, so that at each instant the horizon is displaced towards the future, which involves the implementation of the first control signal of the sequence calculated at each step.

It should be noted that unlike open loop methods that determine the optimal control policy based on a model and directly implement the policy, MPC is a closed loop technique developed for online control purposes. This implies that the response of the process to the control actions implemented at any time step, as indicated by the measurements, is fed back to the MPC controller. This feedback occurs in two ways. At any time step, the current estimate of the state of the process is required, using which the future prediction of the process behaviour is obtained. The estimate of the current state is obtained from the available measurements using a state estimator, as discussed later. A second feedback path is through the estimate of the model error, including unaccounted disturbances, $\hat{H}^{pmm}$. This is also obtained from the current measurements and state estimates as described in the following subsections.

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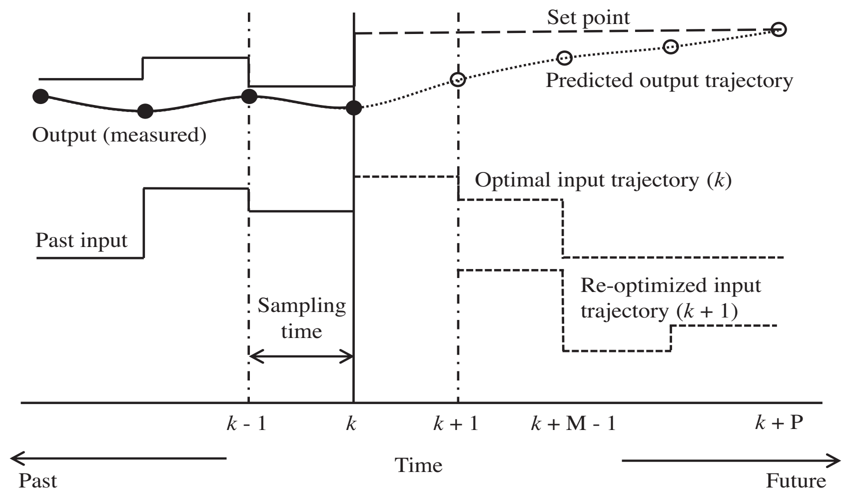

Fig. 2. MPC theory.

3.2. MPC problem formulations

The quasi-steady state model of the system described in Section 2 is used to predict the future state trajectory of the WDN in the MPC framework. The states of the system are flow rates in pipelines, pressure heads at the nodes, outflow rates at demand nodes and water level in storage tanks. The control actions are effected by manipulations of stem positions of continuous control valves in the network. Typically, for control of WDNs, demand node outflow rates and water level in storage tanks at demand nodes (for WDNs with storage facility) are chosen to be the controlled outputs of the system.

WDNs are operated on a 24 h basis and it is sufficient to provide control actions every hour. Physical limitations of the system such as storage capacity of the storage tanks (output constraints), and limitation on the control valve opening (input constraints) are the constraints to be satisfied while providing a control action. In addition, adequate pressures at the demand nodes of the network (state constraints) also have to be maintained. An appropriate choice of demand set point profile for each demand node is made, such that the temporal and spatial variations in the water usage pattern by the consumers are captured. The proposed MPC structure for control of a WDN is shown in Fig. 3.

Without considering the storage facilities at demand nodes, Kumar [26] used an objective function that minimizes the instantaneous deviation of the outflow rate from its desired set point as given below,

$$\min_{U(k)} J = \sum_{i=1}^{N_d}\sum_{j=0}^{P-1}\left(d_i^{sp}(k+j) - \hat{d}_i(k+j)\right)^2 \tag{8}$$

subject to

$$0 \le u(k+j|k-1) \le 1, \quad j = 0, \ldots, P-1 \tag{9}$$

$$u(k+j|k-1) = u(k+M-1|k-1), \quad \text{for } j = M, \ldots, P-1, \quad \text{if } M < P-1 \tag{10}$$

$$h_i(k+j) \ge h_{i,\min}, \quad i = 1, \ldots, N_d \text{ and } j = 0, \ldots, P-1 \tag{11}$$

Hydraulic model, Eqs. (1)–(6).

Where P is the prediction horizon and M is the control horizon, $u(k+j|k-1), j = 0, \ldots, P-1$ are the fractional openings of the control valves (i.e., parameter $V(k)$ in Eq. (5)),

$U(k) = [u(k|k-1)^T \ldots u(k+P-1|k-1)^T]^T$ is the set of manipulated variables, h_i is the pressure head at node i and $h_{i,\min}$ is the minimum pressure head to be maintained at the node i, and $\hat{d}_i(k+j)$ is the predicted outflow rate from the demand node i at time $k+j$. It may be noted that the MPC described above is a nonlinear model predictive controller (NMPC) because the dynamic model for a WDN is nonlinear.

If in every time period there is sufficient water to meet all consumer demands at the required pressure, the above control objective will meet the demand rate of consumers in each period exactly, because oversupply is penalized. If the available water is insufficient to meet the demand rate in any period, then the shortfall for that period will be more or less equitably distributed. The above objective does not take advantage of storage facilities at consumer locations to supply water in advance during periods of plenty, to make up for any shortfall that may occur in subsequent time periods. In order to exploit the flexibility offered by storage facilities at demand nodes, the control objective is reformulated in this study.

If there is no storage facility at demand nodes, the outflow rate from demand nodes (also referred to as supply rates) must be controlled to be nearly equal to the demand rate. However, if storage facility is available, then the supply rate need not match the demand rate. If the supply rate in any time period is greater than the demand rate, the excess water can be stored, subject to limitations of storage. On the other hand, if the supply rate is less than the demand rate, the deficit can be met using the water available in the storage facility. In order to account for the effect of storage tanks, the cumulative excess or deficit in water consumed by customers as compared to the required demand over a time horizon is kept track of in every time period, and the proposed control objective attempts to minimize the sum squared deviations of cumulative excess/deficit over the horizon for all demand nodes.

The cumulative difference at time period k between the consumption rate and the demand rate for node i is given by

$$\Delta\hat{d}_i(k) = \sum_{j=1}^{k}\left(Q_{\text{in}_i}(j) - d_i^{sp}(j)\right) - H_i(k)\frac{A_i}{\Delta t} \tag{12}$$

The cumulative difference can also be written in recursive form as

$$\Delta\hat{d}_i(k) = \Delta\hat{d}_i(k-1) + Q_{\text{in}_i}(k) - d_i^{sp}(k) - \left(H_i(k) - H_i(k-1)\right)\frac{A_i}{\Delta t} \tag{13}$$

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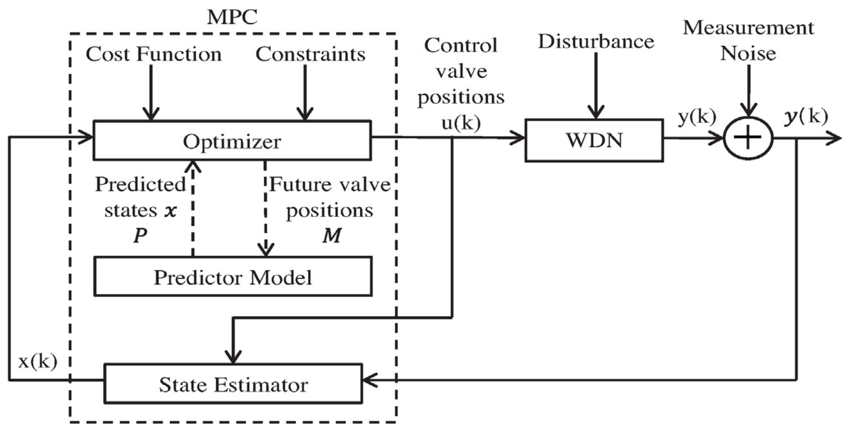

Fig. 3. Block diagram representation of MPC structure used in the control of WDNs.

It may be noted the actual consumption rate of water in any period is inferred from the storage tank level measurements at the beginning and the end of the time period. The proposed MPC control problem is formulated using the cumulative difference between consumption rate and demand rate over the prediction horizon as follows:

$$\min_{U(k)} J = \sum_{i=1}^{N_d}\sum_{j=0}^{P-1}[W_1\hat{H}_i(k+j|k-1) + W_2\Delta\hat{d}_i^2(k+j|k-1)] \tag{14}$$

subject to

$$0 \le u(k+j|k-1) \le 1, \quad j = 0, \ldots, P-1 \tag{15}$$

$$u(k+j|k-1) = u(k+M-1|k-1), \quad \text{for}\, j = M, \ldots, P-1, \quad \text{if}\, M < P-1 \tag{16}$$

$$h_i(k+j) \ge h_{i,\min}, \quad i = 1, \ldots, N_d \text{ and } j = 0, \ldots, P-1 \tag{17}$$

$$0 \le \hat{H}_i(k+j|k-1) \le H_{i,\max}, \quad i = 1, \ldots, N_d \text{ and } j = 0, \ldots, P-1 \tag{18}$$

Hydraulic model, Eqs. (1)–(6).

Where $\hat{H}_i(k+j|k-1)$ are the estimated tank heights at the ith demand node during time intervals $k+j$ based on measurements available until $k-1$, $\Delta\hat{d}_i(k)$ is the predicted cumulative deviation between the demand requirement and the consumption rate at node i, and W_1 and W_2 are the appropriate weighting factors.

The tank heights are estimated based on the consumption model given by Eq. (7).

$$\hat{H}_i(k+j|k-1) = \max\left((Q_{\text{in}_i}(k+j) - d_i^{sp}(k+j))\frac{\Delta t}{A_i} + \hat{H}_i(k+j-1|k-1),\ 0\right) \tag{19}$$

$$\hat{H}_i(k-1|k-1) = H_i(k-1) \tag{20}$$

The cumulative deviation term in Eq. (14) forces the controller to allocate water so as to match the outflow rate from a demand node with the demand requirement during the prediction horizon, taking into account excess or deficit water supplied during preceding periods. Also, by minimizing water levels in the tanks simultaneously with the cumulative deviation, accumulation of water in the tanks is kept as low as possible. As discussed earlier, the constraints for the control problem include minimum pressure constraints at all the outflow nodes for each of the P time periods, upper and lower bounds on the control valve positions, and storage limitations of the tanks at the demand points.

It may be noted that the above choice of the objective function can be used for both water sufficient and deficient situations. On one hand, in water sufficient case, the objective function will minimize excess withdrawal (potential wastage) of water by regulating the pressure at demand nodes and maintaining sufficient water levels in the storage tanks just enough to cater to the needs of consumers at the current time period. On the other hand, when the available water is insufficient to meet the hourly demands, the above objective function will minimize the shortage in supply of water to the consumers by utilizing the flexibility provided by the storage tanks.

4. State estimation

Typically, due to high instrumentation cost and scale of the network, WDNs are poorly instrumented. Hence, only a subset of the system state variables may be measured directly and these are corrupted by sensor noise. The remaining state variables have to be estimated using the available measurements, measurement error characteristics and a model of the process.

The state estimator provides estimates of all system variables, including any unknown source pressures and outflow rates at demand nodes, using all the available flow rate, pressure and tank level measurements, by solving a constrained weighted least squares optimization problem. In this optimization problem, the constraints are the governing equations, and the objective function is the squared sum of deviations between all the measured and corresponding estimated quantities. The noise in the measurements is taken into account by appropriately choosing the weights. Kumar et al. [33] has developed an efficient estimator based on graph theoretic concepts for estimating the state of a WDN. Graph theoretic concepts were used to derive a reduced optimization problem by suitably choosing a set of independent variables (chord flows) and eliminating all the equations other than loop balance equations. Although, the above method uses a demand driven model of WDNs, it can be extended to the pressure dependent outflow model of the system. The only modification required is inclusion of Eq. (4) in addition to mass balance equation for each node, and head loss equation for each element in the network. Eq. (4) is similar to the valve equation, and is therefore handled in the same manner as the head-flow relation for other elements. The state estimator is used to obtain the current estimates of all the variables in the

WDN during each time period, using the available measurements from the network and specified values of the manipulated variables implemented by the control algorithm.

In a WDN without storage facilities, an estimator as described above can be used to provide estimates of unmeasured quantities including actual consumption at the demand nodes. However, while storage tanks at demand nodes provide a buffering action, they also decouple the network, i.e., the consumption cannot be inferred unless the water levels in the storage tanks are measured and/or the consumption is directly measured. In this work, we assume that water levels in all storage tanks are measured.

5. Handling uncertainties

Measurement errors are a source of uncertainty which are handled using an estimator as discussed above. In addition, the assumed demand set profile for ith demand node represented by d_i^{sp}, may not match the consumers' actual consumption of water from the tanks (denoted by Q_{out_i}). Therefore, at each time step $k-1$, an estimate of the disturbance/mismatch term, $\hat{H}_i^{pmm}(k-1)$, is obtained and used to update the set point for the succeeding period k. The estimate of the mismatch $\hat{H}_i^{pmm}(k-1)$ is obtained as follows:

$$\hat{H}_i^{pmm}(k-1) = H_i(k-1) - \hat{H}_i(k-1|k-2) \tag{21}$$

where $\hat{H}_i(k-1|k-2)$ is the predicted water level in the tank at time $k-1$ using the extended period simulator at $(k-2)$th time instant.

MPC determines the optimal valve positions for time $k-1$ using the estimate, $\hat{H}_i(k-1|k-2)$. However, after implementation of this control move, based on the response of the WDN, the actual values of water levels in storage tanks are obtained. Thus the difference between these two quantities can be attributed to both the uncertainties in the model as well or other disturbances that may have occurred during the time interval from $k-2$ to $k-1$.

An update of the nominal demand set point profile for the succeeding time instant, $d_{i_{\text{nom}}}^{sp}(k)$, is made in accordance with the uncertainty in the consumption pattern as given below.

$$d_i^{sp}(k) = d_{i_{\text{nom}}}^{sp}(k) + \frac{A_{t_i}}{\Delta t}\hat{H}_i^{pmm}(k-1) \tag{22}$$

If the consumer consumes less water than the demand set point in the preceding time period, the above set point update procedure can be viewed as providing the leverage to the consumer to utilize the water that he/she has saved to his/her comfort. On the other hand, if the consumption rate exceeds the demand set point then the consumer is penalized for over consumption by reducing the supply in the next time period. The set point update procedure, when $P>1$, is carried out only on the succeeding time period rather than updating the set points of P time steps throughout the prediction horizon. This is because, the set point update procedure done over the prediction horizon will be equivalent to rewarding/penalizing the consumer P times instead of once, as cumulative deviation (given in Eq. (13)) is minimized in the objective function of MPC. Using the above-proposed technique of estimation of process-model mismatch and a corresponding set point update procedure, the MPC technique can be applied for control of WDN to handle the uncertainty in the water consumption pattern.

6. Results and discussion

Application of the proposed NMPC algorithm is illustrated using a sample network containing 11 nodes, 10 pipes and 2 continuous control valves as shown in Fig. 4. The network has a reservoir as a source node at node 1 and two demand nodes at 5 and 11. Storage facilities are available at both the demand nodes. Cross sectional area of the storage tanks at nodes 5 and 11 are 12 m^2 and 16 m^2, respectively, and heights of the tanks are considered to be 2.5 m and 3 m, respectively. Elements 11 and 12 are continuous control valves. The pipe and valve data are given in Tables 1 and 2, respectively. The elevation at all nodes is taken as zero. The head at the source node is taken as 27 m and the minimum pressure head to be maintained at any demand node is 2 m of water.

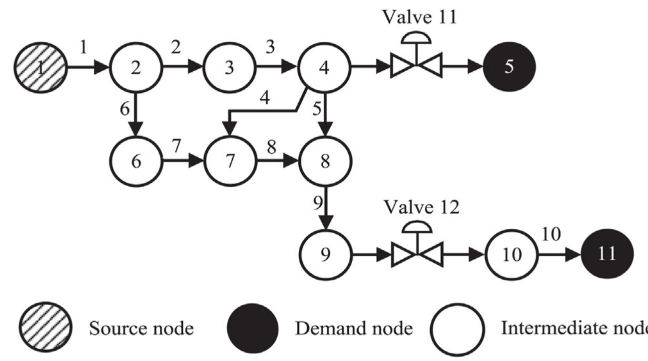

Fig. 4. Schematic of the WDN used in the demonstration of NMPC technique.

It is assumed that flow rate measurement is available for the pipeline 1, pressure measurements are available at nodes 1 and 8, and water level in both the tanks are measured on an hourly basis. It may be noted that outflow rates at the demand nodes 5 and 11 are not measured. The synthetic measured data at each time instant are generated as explained below.

1. For specified water levels in the reservoirs, valve openings and demand node resistances, a pressure driven steady state solver is used to obtain the true pressures at nodes and flow rates in the pipelines and the demand node outflow rates in the network.
2. Synthetic measurements at the specified locations are obtained by adding noise to the corresponding true values.
3. The measurements are used in the state estimator to obtain estimates of all state variables.

It is also assumed that the water distribution network operates on a daily cycle and it is sufficient to provide control actions once every 1 h. For this reason, all the results of the control algorithm are presented here for 24 h period with implementation of control

Table 1
Pipeline data for the sample network.

Pipe No.	Node		Length (m)	Diameter (m)
	From	To		
1	1	2	15.7	0.1079
2	2	3	28	0.1016
3	3	4	25	0.0889
4	4	7	18.2	0.0889
5	4	8	39.2	0.0762
6	2	6	18	0.1016
7	6	7	37.2	0.0889
8	7	8	30.7	0.0889
9	8	9	10	0.0889
10	10	11	11.3	0.0889

Table 2
Characteristics of the control valves used in the sample network.

Valve No.	Node		C_v	Valve rangeability
	From	To		
11	4	5	100	50
12	9	10	150	50

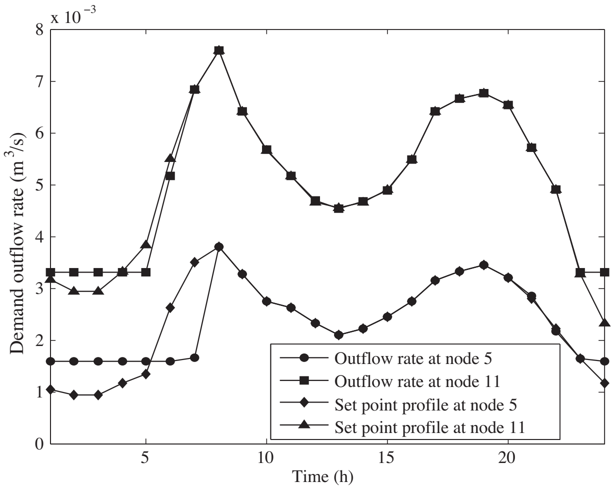

Fig. 5. Comparison of demand set point and supply (or outflow) rate in water sufficient situation.

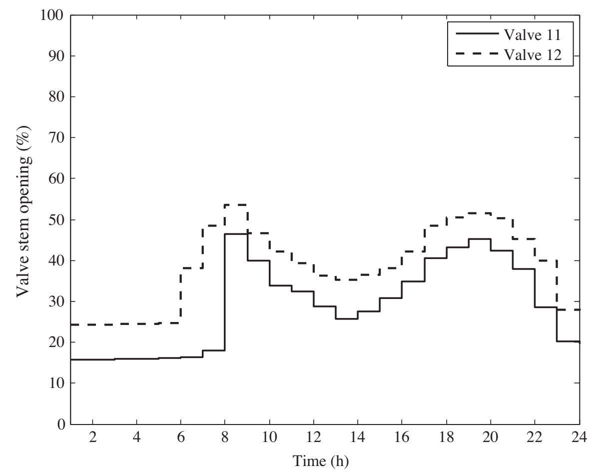

Fig. 6. Optimal control valve settings.

moves generated for every 1 h. The daily demand set point profile is assumed to be the same from one day to the next. The performance of the control algorithm is evaluated for two water availability scenarios. In the first scenario, sufficient water is assumed to be available to meet the required demands, while in the second scenario, available pressure head in the reservoir (source node) is insufficient to meet the specified peak demands.

In order to assess the performance of the controller, either the deviation of the total amount of water supplied from the amount demanded during the 24 h period may be considered, or the instantaneous deviation of the supplied outflow rate from the required rate may be considered. In this study, the total volume of water supplied in a day is compared with the actual daily requirement because sufficient storage capacity exists at each consumer node and the excess amount supplied in any time period can be stored and utilized later.

6.1. Water sufficient scenario

The performance of the control algorithm is evaluated by considering the first scenario where sufficient water is available to meet the consumers' demand requirements. The lengths of the control and prediction horizons are both chosen to be equal to 1. The synthetic data are generated without any addition of process disturbance, whereas, a zero mean Gaussian measurement noise with standard deviation of 5% of the corresponding true values is added to the measured flow rates and pressures. When sufficient water is available to meet all the demands, and when there is no process-model mismatch, the controller should be able to determine the optimal valve settings for satisfying the set-point demand profiles as closely as possible.

MPC computes the optimal valve openings for specified water levels in the reservoirs. The comparison of the demand set point profiles at demand nodes 5 and 11 with the corresponding outflow rates as computed by the control algorithm is shown in Fig. 5. It can be observed that the outflow rates computed by the controller closely match with the set point demand profiles for most time periods, implying that the controller is able to determine the optimal valve settings necessary to meet the demands exactly in the absence of process-model mismatch. Nevertheless, there is an initial oversupply at both the demand nodes. The oversupply during the periods 1–4 is due to the minimum pressure constraint of 2 m of water that has to be met. This is observed in Fig. 7, which shows that the pressures at the two demand nodes during the periods 1–4 are equal to the minimum stipulated pressure. Since the consumer consumes only the required quantity of water in every time period, the excess water supplied accumulates in the storage tanks. If storage tanks were not present at the demand nodes, the excess water is a potential waste [23]. However, here the controller takes into account the water available in the tanks and the oversupply is compensated by undersupply in the periods 5–7. The MPC scheme allocates water so as to meet the integral requirement over the 24 h period by accounting for initial water level in tanks, earlier under/oversupplied, thus ensuring a sustainable operation of the WDN in water sufficient scenario. At the 24th time period, again, there is an oversupply to satisfy the minimum pressure constraint. The volume of water demanded is compared with volume of water supplied and consumed over the 24 h period at the demand nodes and the corresponding under/over consumption in Table 3. It should be noted that since there is no mismatch between assumed and actual consumption by the consumer, over consumption is not possible. From Table 3, it can be noted that the under/over consumption by consumers at both demand nodes are 0 and the excess water supplied is accumulated in the storage tank at the final time period (24 h).

The optimal settings of the control valves 11 and 12, determined by the controller to achieve the above solution is shown in Fig. 6. In addition, the pressure variation at nodes 5 and 11 shown in Fig. 7, indicate that the pressures at these nodes meet the minimum pressure requirement at all time periods.

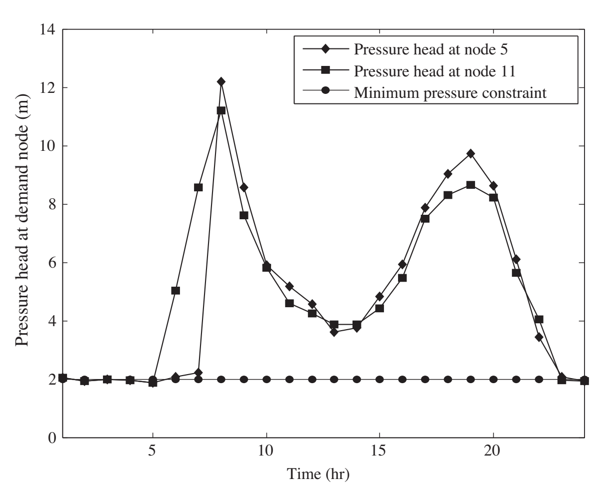

Fig. 7. Pressure head profile at both the demand nodes of the network.

Table 3
Integral quantity of water demanded, supplied and consumed in water sufficient case.

Node	Initial water level in tank (m)	Volume of water demanded (m^3)	Volume of water supplied (m^3)	Volume of water consumed (m^3)	Final water level in tank (m)
5	0.15	204.33	204.04	204.33	0.125
11	0	432.78	436.42	432.78	0.227

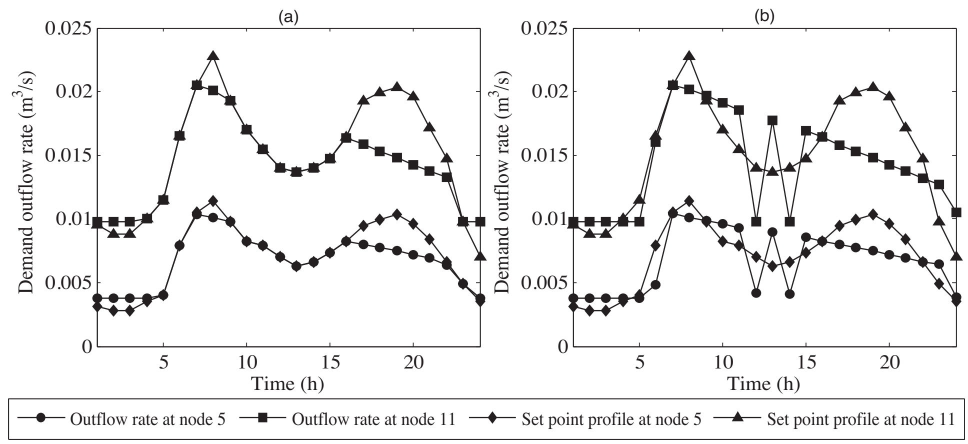

Fig. 8. Demand outflow rate profile with $P = 1$ and $M = 1$: (a) without storage tank [26] and (b) with storage tank.

6.2. Water deficient scenario

For the simulation of water deficit situation, the demand requirements of the consumers are increased 3-fold as compared to the demand requirements assumed in the previous scenario. The pressure head available at the source node is insufficient to meet the peak demand requirements which results in inability of the network to deliver the demanded flow rate for all nodes at peak periods thereby causing instantaneous deficits in the amount of water supplied. However, the required integral quantity of water is available at the source node. For comparison, we first analyze the case when storage tanks are not available at the demand nodes, by using the objective function for the control problem defined by Eq. (8). Fig. 8(a) shows the demand node outflow rate profile at the two demand nodes and their corresponding set point profiles for this case.

6.2.1. Performance of the control algorithm

Fig. 8(a) shows that the maximum shortage of water occurred at demand node 11 and is about 5.9%. In contrast, when the proposed control algorithm was used which accounts for storage facilities in the formulation, the undersupply during the peak time period 8 and periods 18–22, were compensated by oversupply in the other periods as shown in Fig. 8(b). When considered over the 24 h period, the shortage in supply was reduced to 4.57% at the demand node 11 (Table 4). The shortage can be further minimized by choosing a longer prediction horizon as described below.

6.2.2. Parameter selection: choice of prediction and control horizons

A far-sighted policy (larger value of prediction horizon) leads to better controller performance than a shorter prediction horizon, specifically in water deficient situations where the available water is insufficient to meet the peak demands. This is due to the fact that for a longer prediction horizon, the controller takes into account future demand requirements and it is therefore able to better anticipate the future shortage or surplus that can arise and uses the available storage facility at demand nodes effectively, while making the decision to supply more or less water than the demand rate in each time interval.

MPC was applied with $P = 5$ and $M = 1$. The demand node outflow rate profile obtained can be seen in Fig. 9. At first sight, the controller may seem to perform poorly as the demand node outflow rates are not tracking their set points. However, on analyzing the water level profile in Fig. 10, it can be noticed that water gets accumulated before the peak occurs and the consumers satisfy their water needs by consuming the water from the storage tank even if there is an undersupply during periods of peak demand. Water levels at the end of each sampling time after consumption are indicated in Fig. 10. Furthermore, it may be noted that both the tanks are empty during the time periods 18–22 but the consumers face scarcity of water only during the periods 19–22 as shown in actual consumption plots in Fig. 9, rather than the periods 8 and 17–22.

Results show that the percentage under consumption at nodes 5 and 11 are 6.12% and 6.35% (equivalently, 37.49 m3 and 82.51 m^3), respectively, for $P = 1$ and $M = 1$, whereas, for $P = 5$ and $M = 1$, the percentage under consumption at nodes 5 and 11 are reduced to 4.86% and 4.66%, respectively and in terms of volume, the under

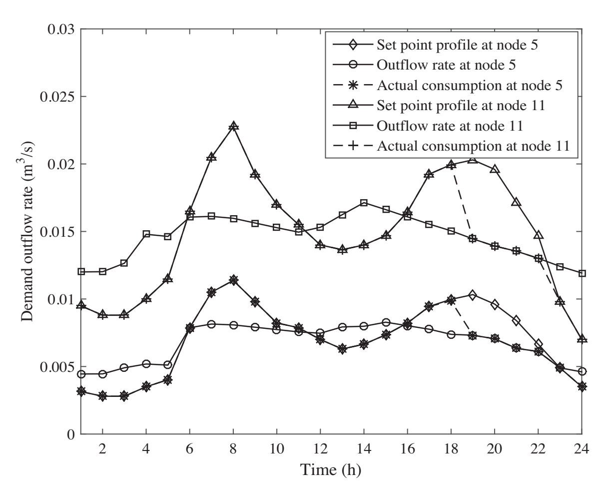

Fig. 9. Demand outflow rate profile for water deficient case with $P = 5$ and $M = 1$.

Table 4
Integral quantity of water demanded, supplied and consumed in water deficient case with $P = 1$ and $M = 1$.

Node	Initial water level in tank (m)	Volume of water demanded (m^3)	Volume of water supplied (m^3)	Volume of water consumed (m^3)	Final water level in tank (m)
5	0.15	612.99	580.34	575.50	0.553
11	0	1298.34	1238.97	1215.83	1.446

Table 5
Integral quantity of water demanded, supplied and consumed in water deficient case with $P = 5$ and $M = 1$.

Node	Initial water level in tank (m)	Volume of water demanded (m^3)	Volume of water supplied (m^3)	Volume of water consumed (m^3)	Final water level in tank (m)
5	0.15	612.99	585.35	583.18	0.33
11	0	1298.34	1264.93	1237.78	1.29

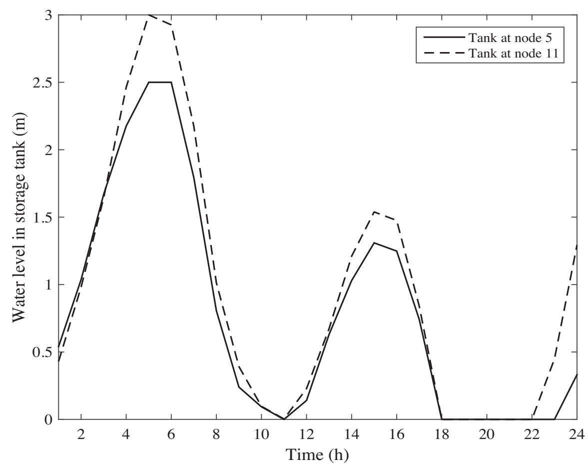

Fig. 10. Water level in storage tanks at demand nodes in water deficient situation.

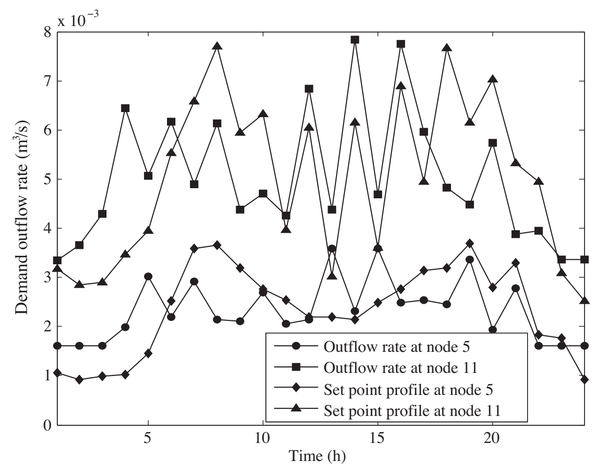

Fig. 11. Demand outflow rate profile with $P = 5$ and $M = 1$ for the case with external disturbance.

consumption at both the demand nodes have dropped to 29.81 m^3 and 60.56 m^3 as seen in Table 5. The proposed control strategy managed to reduce the shortage in supply as well as satisfy the hourly demand requirements for most periods, achieving at least 20% reduction in the net under consumption over the 24 h period compared to the previous case. Based on the performance of the controller for different values of M and P, and the computational requirements, a choice of $M = 1$, and $P = 5$ is considered suitable for this system, and hence these values are taken for all further simulation results reported in this work. It should be noted that an increase in the control horizon M, increases the computational requirements, since the number of optimization variables for the control problem is proportional to M.

6.3. *Effect of external disturbance*

In the following case study, the effect of unmeasured external disturbances on the performance of the controller is evaluated. In a real situation, this may be due to error in valve positioning, or errors in the parameters used in the model, or variation in the consumers' consumption pattern. In this work, effects of all the uncertainties are modelled as random deviations from nominal demand set point profile. The demand requirements of the consumers in this case are assumed to be the same as in the sufficient water availability scenario. Plant-model mismatch caused by variation in the consumers' consumption pattern from the demand set profile, is simulated by adding a random deviation to the amount of water withdrawn from the storage tank at each time period. In this case, a Gaussian disturbance with zero mean and standard deviation equal to 5% of the outflows is added to the true outflows. This implies that the water levels in the storage tanks estimated using the outflow rates and the demand set point profile, will not match with the actual water levels.

The outflow rates computed by the MPC with $P = 5$ and $M = 1$, at nodes 5 and 11 are shown in Fig. 11. It may be noted that Eq. (22) has been included in the control algorithm in order to account for the model mismatch caused by external disturbance. Water levels in both the tanks at the end of each time instant are shown in Fig. 12. From the figure, it can be noted that, both the tanks at the demand nodes 5 and 11 never run dry throughout the 24 h period. This indicates that the controller is able to effectively reject the

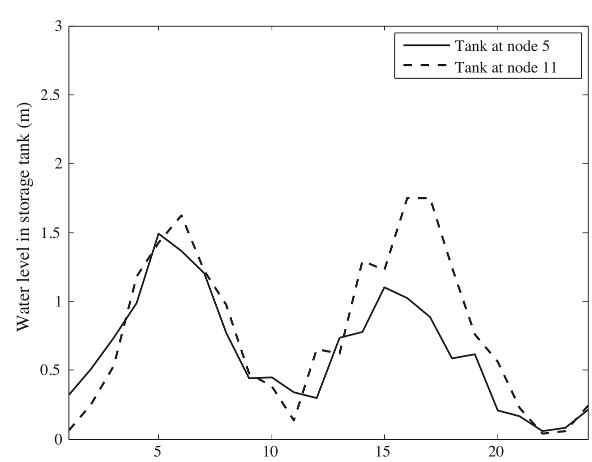

Fig. 12. Water level in storage tanks at demand nodes in the presence of external disturbance.

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Table 6
Integral quantity of water demanded, supplied and consumed in water sufficient case with plant-model mismatch.

Node	Initial water level in tank (m)	Volume of water demanded (m^3)	Volume of water supplied (m^3)	Volume of water consumed (m^3)	Final water level in tank (m)
5	0.15	200.71	201.50	200.71	0.22
11	0	429.44	433.35	429.44	0.24

unmeasured disturbances and satisfy the hourly demand requirements of the consumers in spite of the uncertainty in the consumption pattern. Water levels in the storage tanks at the end of 24 h period are presented in Table 6. This case study shows the robustness of the proposed methodology in providing effective control logic that accounts for unmeasured disturbances.

7. Conclusions

In this work, a nonlinear model predictive control technique has been developed for the optimal operation of water distribution networks with storage facilities. Through simulation studies on an illustrative network, it is demonstrated that, in a water sufficient situation, the proposed MPC technique meets exactly the demand requirements of the consumers. The proposed control technique, with an appropriate choice of prediction and control horizons, was also able to achieve equitable distribution of the available resource as well as satisfy the hourly needs of the consumers for most periods, in the water deficient situation. By including an estimate of the plant-model mismatch term in the control objective, the controller is also able to account for deviations in consumption from the nominal consumption pattern, and meet the hourly demand requirements of the consumers.

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