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# Multi-objective optimisation of the operation of a water distribution network

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## Abstract

The aim was to move water through the reservoir network in such a way as to meet consumer demands and level constraints, minimise the cost of electricity, and minimise the loss of chlorine. This was to be achieved by choosing the switching intervals of reservoir inlet pumps and valves, at the same time complying with the allowed minimum interval size of each device. Flows were balanced by Linear Programming. The genetic algorithm gave confidence in the near-optimality of its solutions, through well-defined Pareto fronts between the competing objectives. The method was applied to a 16-reservoir water distribution system in Durban, South Africa. Comparison with an equivalent "dead-band" control showed a 30% improvement in a weighted objective.

**Keywords**: NSGA-II, genetic, Pareto, scheduling, LP balance

## 1. Introduction

Previous attempts at the operational optimisation of the Durban potable water distribution network include Biscos et al. (2002) by mixed integer non-linear programming (MINLP) and Purdon et al. (2010) by non-linear programming (NLP). The main difficulties experienced with these single objective approaches concerned the binary nature of the controls, and the minimum interval sizes. Solution times were long, and often only relaxed solutions were possible.

In other water distribution optimisations, López-Ibáñez (2009) used a SPEA-2 multi-objective genetic algorithm, Cui et al. (2011) used a NSGA-II multi-objective genetic algorithm and Pianosi et al. (2011) used NSGA-II in combination with an artificial neural net model. The *NSGA-II* (fast Non-dominated Sorting Genetic Algorithm II), of Deb et al. (2002) has several advantages: It has a low computational demand, preserves diversity and is elitist – ie. it has a mechanism for eliminating dominated solutions. The extension "II" to NSGA refers to a method for reducing "crowding" of the non-dominated solutions, to get a better definition of the Pareto front.

## 2. Method

### *2.1. Stream flow switch sequences*

The problem to be addressed here is the determination of an optimal sequence of binary switchings of a number of streams in the distribution network. Each such switch determines a characteristic maximum flow in that pipe connection. One "individual" candidate in the optimisation is completely determined by its switching sequence of all switchable streams in the system (Figure 1).

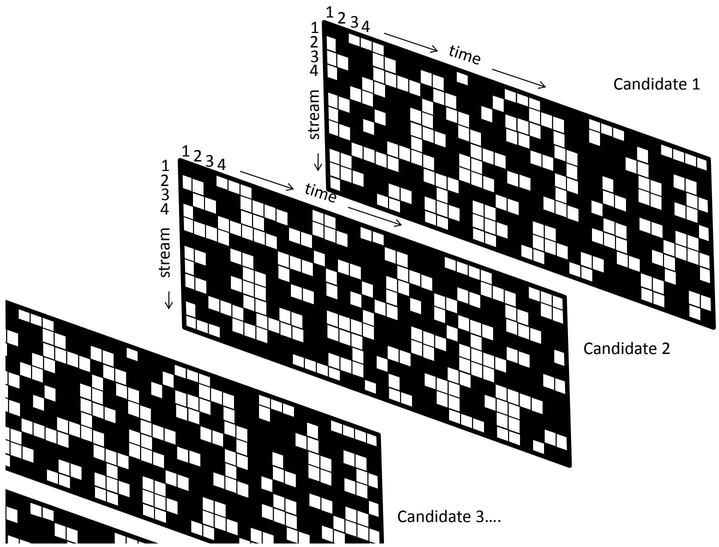


Fig. 1 Series of individual candidates, each with a proposed switching pattern over *time* (horizontal) for each pump/valve *stream* (vertical)

### *2.2. Minimum switching interval*

Random interval sizes $n$ between switchings were obtained from

$$n = round\left\{\varepsilon^{b}\left[n_{\mathrm{h}} - n_{\min}\right]\right\} + n_{\min} \tag{1}$$

Here $n_{\mathrm{h}}$ is the horizon size, as a number of steps of the basic interval (1 h), $n_{\min}$ is the minimum number of steps and $\varepsilon$ is a random number drawn from a uniform distribution over the range 0 to 1. The exponent $b$=2 creates a bias in the distribution.

### *2.3. Node volume balances by Linear Programming*

On each basic $\Delta t$ time-step, from index $t-1$ to $t$, a pipe capacity vector is set for the $M$ streams as $\boldsymbol{c}_{\mathrm{t}} = (c_{1\mathrm{t}},\ c_{2\mathrm{t}}, \ldots\ldots\ c_{\mathrm{Mt}})^{\mathrm{T}}$. In the cases of fixed flow profiles, as in consumer demands, these capacities are set to the actual flow required at any time $t$.

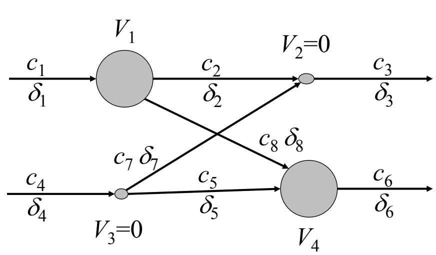


Fig. 2 Example network

Consider the example network in Figure 2. A stream connection matrix is constructed using the capacities $c_{\mathrm{jt}}$ :

$$\boldsymbol{A}_t = \begin{bmatrix} +c_1 & -c_2 & 0 & 0 & 0 & 0 & 0 & -c_8 \\ 0 & +c_2 & -c_3 & 0 & 0 & 0 & +c_7 & 0 \\ 0 & 0 & 0 & +c_4 & -c_5 & 0 & -c_7 & 0 \\ 0 & 0 & 0 & 0 & +c_5 & -c_6 & 0 & +c_8 \end{bmatrix}_t \tag{2}$$

The actual flow in a pipe section $j$ is then considered to be $f_{jt} = c_{jt}\ \delta_{jt}$. Only a selection of the $\delta_{jt}$ represent actual pump or valve switches, but as will be seen, the concept is useful as a *fractional flow* for all pipes - the $\delta_{it}$ become the variables to be optimised by LP:

Maximise $\boldsymbol{\delta}_t^T \boldsymbol{c}_t$ (3)

Such that

$0 \le \delta_{jt} \le 1$ for $j = 1,\ldots,M$ streams (4)

$\delta_{jt} = 1$ for $j$ = fixed flows (5)

$\delta_{jt} = 0$ for $j$ = pipes where valve/pump (Fig. 1) is off (6)

$\boldsymbol{\Delta}_t = \boldsymbol{A}_t \boldsymbol{\delta}_t$ (all flow imbalances at nodes) (7)

$\Delta_{it} = 0$ for i = all nodes which are junctions (ie. not reservoirs) (8)

*2.4. Integration for volume*

As seen above, it is only the streams with a pump or valve that are randomly allocated switching sequences in Figure 1. A switch $\delta_{jt}$ is forced to 0 at time $t$ if $j$ is allocated an *off*, else it is free to find its own level. Once the LP solution $\boldsymbol{\delta}_t$ is obtained, volume variations can be integrated for the $\Delta t$ period, index $t$-1 to $t$:

$$\boldsymbol{V}_t = \boldsymbol{V}_{t-1} + \Delta t\, \boldsymbol{A}_t\, \boldsymbol{\delta}_t \tag{9}$$

*2.5. Chlorine balances and Integration*

Once the stream flows have been established in the LP solution above, the best approach is to convert them to *chlorine* flows $b_{jt}$, using stream chlorine concentrations $x_{j,t\text{-}1}$ known from the previous time-step. Then a similar integration is performed for the chlorine mass-balance in reservoirs:

$$b_{jt} = x_{j,t-1} c_{jt} \quad \text{for } j = 1,\ldots,M \ \ \text{(streams)} \tag{10}$$

$$\boldsymbol{B}_t = \begin{bmatrix} +b_1 & -b_2 & \cdots \\ 0 & +b_2 & \cdots \\ \vdots & \vdots & etc \end{bmatrix}_t \tag{11}$$

$$\left[1+\tfrac{1}{2}k\Delta t\right]\boldsymbol{C}_t = \left[1-\tfrac{1}{2}k\Delta t\right]\boldsymbol{C}_{t-1} + \Delta t\, \boldsymbol{B}_t\, \boldsymbol{\delta}_t \quad \text{(reservoirs)} \tag{12}$$

$$X_{it} = C_{it} / V_{it} \quad \text{for } i = 1,\ldots,N \quad \text{(reservoirs)} \tag{13}$$

Here $k$ (=0.05 $h^{-1}$from van der Walt, 2002 ) is the first-order decay rate constant for the chlorine, and $\boldsymbol{C}_t$ is a vector of reservoir chlorine inventories at time $t$. The main difficulty is to interpret the nodal chlorine concentrations $X_{it}$ as stream concentrations $x_{jt}$

for the next time-step. Firstly $x_{jt} = X_{it}$ for streams leaving reservoirs. Next, the $X_{it}$ for pipe junctions ($V_i$=0) are evaluated by dividing chlorine inflow by volume inflow. Finally, the concentrations in streams leaving a junction $i$ can be set as $x_{jt} = X_{it}$ .

*2.6. Objective functions for minimization*

All objective values are scaled up by a factor of 100 for plotting purposes.

Volume:

$$f_v = 100\sum_{i=1}^{N}\frac{w_i}{R_i}\left\{\underbrace{\alpha n_h\left|V_{in_h} - V_{SPi}\right|}_{(\text{end deviation})} + \sum_{t=1}^{n_h}\left[\underbrace{\left|V_{it} - V_{SPi}\right|}_{(\text{setpoint deviation})} + \underbrace{\beta\left\{\max\left(0, V_{it} - V_{\max i}\right) + \max\left(0, V_{\min i} - V_{it}\right)\right\}}_{(\text{exceeding of constraints})}\right]\right\} \quad (14)$$

where $R_i = V_{\max i} - V_{\min i}$ is the range of reservoir $i$. Values were taken as follows: $n_h$=24 ($\Delta t$=1h steps), $w_i$=1 (all $i$), $\alpha$=3, $\beta$=100 (to discourage violation of constraints)

Chlorine:

The objective here is to minimise the total chlorine inventory, in order to minimise the total loss by decay.

$$f_c = \frac{100}{n_h C_{MAX}}\sum_{i=1}^{N}\sum_{t=1}^{n_h} C_{it} \quad (15)$$

where

$$C_{it}=X_{it}V_{it} \quad (16)$$

is the amount of chlorine in reservoir $i$ at time $t$, and

$$C_{MAX} = x_{FEED}V_{TOT} \quad (17)$$

where $V_{\text{TOT}}$ is the sum of maximum reservoir volumes.

Electricity:

$$f_e = \frac{100\,\tau_{DAY}}{n_h V_{\text{MAX}}}\sum_{t=1}^{n_h}\left[T_t\sum_{j=1}^{M} f_{jt}\right] \quad (18)$$

where $f_{jt} = c_{jt}\delta_{jt}$ , the flow-rate of stream $j$ during the interval $t$. An electrical power tariff diurnal variation $T_t$ was obtained from Biscos (2003).

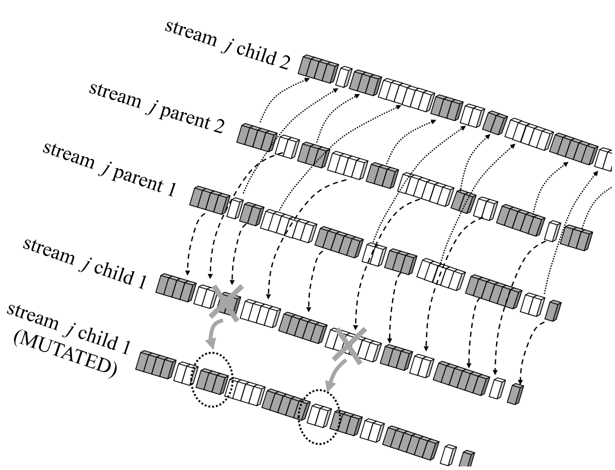


Fig. 3 Crossover and Mutation of the on/off intervals for pumps and valves

### *2.7. Genetic algorithm*

In the NSGA-II method, candidates are ranked according to the non-dominated “layer” in which they lie in the objective space. Up to 50% of the top-ranked individuals are randomly paired to swop alternate intervals (Figure 3). The two children produced by this *crossover* replace the equivalent number of individuals in the bottom fraction – so the parents are retained. For *mutation*, a specified fraction of the children (eg.20%) are subjected to a random change of a specified fraction (eg. 30%) of their intervals.

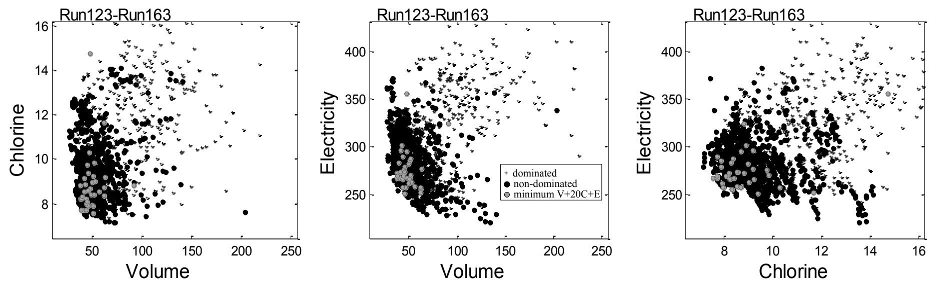


Fig.4 Three-way optimisation Pareto plots: 41 solutions superimposed, each with 50 points.

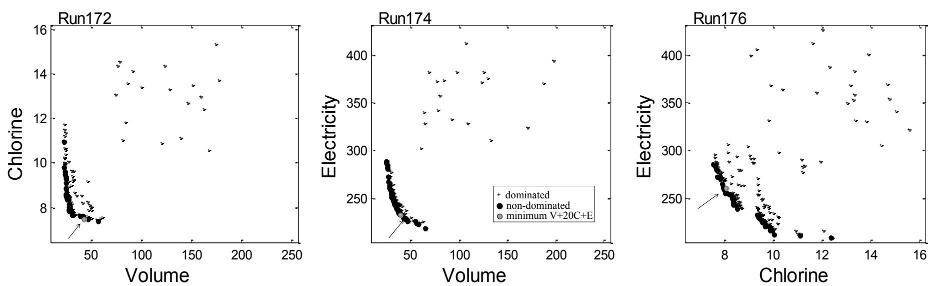


Fig.5 Two-way optimisation Pareto plots: 3 separate solutions, each with 200 points

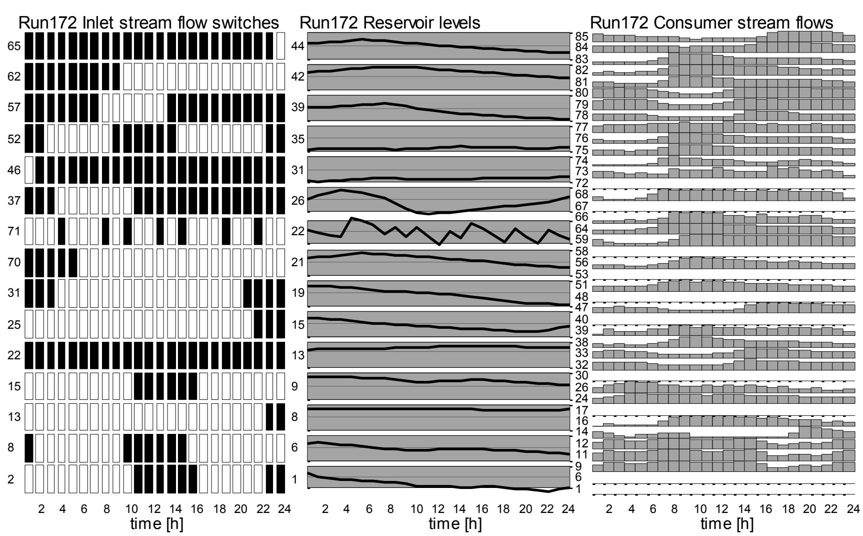


Fig. 6 Valve/pump switching (black=ON), reservoir levels and consumer flows for Run 172 (● dot on Volume-Chlorine plot, Fig 5)

## 3. Application

Optimisations were conducted with *all three* objectives (Volume, Chlorine and Electricity) as in Figure 4, and then with *selected pairs* of the three objectives as in Figure 5. The highlighted dot ● is the point with the best value of an objective sum: V+20C+E in each solution run. In Figure 4, a total of 41 separate three-objective solutions, each with 50 points, have been superimposed. Conversely, each plot in Figure 5 is a single, separate solution of 200 points, optimising only the two axis variables. The usefulness of the multi-objective analysis in highlighting objective conflicts is illustrated in Figure 5. In the Volume-Chlorine solution, the inclusion of Electricity in the V+20C+E criterion for the "best" point doubles the V penalty with little decrease in the C penalty. This particular solution, the dot ● of Run172, is plotted out in Figure 6.

## 4. Conclusion

The regular Pareto fronts in Figures 4 and 5 suggest that the points are close to the optimal non-dominated solutions. This is quite remarkable, since the system had around $10^{70}$ switching interval permutations (16 streams×5 slots av.×8 interval sizes av.).

Interesting aspects of this work included: (a) the method of ensuring minimum switching intervals, by random selection from an appropriate range; (b) the balancing of junction flows by a linear programming solution on each step; and (c) an early heuristic rejection of individuals from the population which was based on predicted trajectories towards constraints.

A weighted sum of the three simultaneous objective values, based on Volume Control, Chlorine Loss and Electricity Cost, produced a nominal "best point" in a large number of solutions. These points were clustered near the *ideal point* of the three-objective Pareto front (Figure 4), and produced an objective value 30% lower than an equivalent dead-band control.

## Acknowledgement

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