

Dear editor and reviewers,

we are enormously grateful for your important contributions. Below is a summary of the changes made, according to the suggestions presented:

*first reviewer*

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*second reviewer*

- In order to clarify the symbols used in the model, we created the Table 2 (page 18).
- For clarity, the symbols used have been adjusted, and the text throughout the paper has been modified accordingly.
- The problem with the numerical scale was solved by adjusting the values used for the model parameters. All tests were redone and the results presented in the paper were changed.
- In order to explain the computational challenges related to the proposed model, we have included the sentence: 'The model presented in this work is a non-linear, mixed-integer mathematical programming model. Nonlinearity happens due to the objective function and the constraints in (2), (16) and (18). The non-polynomial nature of the model arises due to the binary decision variables  $X_{dsl}$ .' (page 15).
- 'remove impurities even existing in the water.' was replaced by 'remove impurities still in the water.' (page 4).
- 'capacity of treated water, when the abstraction in water sources is at its maximum limit, is 288,000' was replaced by 'capacity of treated water is 288,000' (page 7).
- We have improved the quality of all figures.

*other modifications*

- We improved the mathematical model by removing some unnecessary calculations and correcting some errors.
- Section 3 was completely redone so that the mathematical model was formally presented.
- We modified the words LINGO and LINDO by writing it with capital letters.
- We also fixed the measurement units used.
- Some information was presented in the form of footnotes.

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## **Elaboration of Water Distribution Schedules in Periods of Scarcity**

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**Abstract:**

Scarcity of treated water is a global problem that directly impacts the quality of life and, therefore, brings to light the need for more careful management of water resources in locations that are affected. Although there is a vast literature on the problem of design and operation of treated water distribution networks, few studies consider the issue of scarcity. In the present paper, we carry out a study on the real water distribution problem of a city located in Northeastern Brazil. In this study, we conceptually and mathematically model the problem addressed as a new specific water distribution optimization problem, and implement the model developed using LINGO software from LINDO Systems. We conclude the study showing that the developed solver is a practical, effective and efficient tool, which can be easily used by a suitably qualified employee.

**Keywords:** Mathematical Modeling; Water Distribution Schedules; Water Distribution System; Water Scarcity; LINGO.

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### **1 Introduction**

Water scarcity is a global problem that has been growing over the years due to factors such as population growth, climate variations and living standards (Huang et al., 2021). Specifically in Brazil, in semi-arid regions (such as some regions of Northeast Brazil), this problem is aggravated due to the high variability of rainfall, with rainfall concentrated over time and little spatial distribution (Silva et al., 2019). Such aggravation ends up generating conflicts for the use of water resources, and water distribution becomes a priority concern for public management.

Several scientific studies address the water distribution problem, including the design, calibration, simulation, operation and control of water distribution systems. According to Sankar et al. (2015), when it comes to distribution operation and control, the main objectives considered are: minimizing the cost of energy required for pumping; regulating pressures for preventing/reducing leakages; and maximizing the quality of water supplied to the users by reducing the residence time of water in the network. Few studies have sought to consider the issue of water scarcity. In this scenario, the problem of water distribution is usually restricted to the operation / control of an intermittent water distribution system with storage facilities. According to Sankar et al. (2015), in the case of water deficient situations it is not possible to meet the demand, so it is also necessary to identify the optimal outflow from demand nodes and it becomes important that the control strategy allocates water so as to minimize the shortfall in supply, and supplies the available water in as equitable a manner as possible. Based on that purpose, the authors present a nonlinear model predictive control (NMPC) technique that determines the optimal control valve settings, and so the optimal outflow rates from demand nodes. However, usually, in practice, the problem of water deficiency is solved with the adoption of distribution schedules and not only through valve adjustment. Kurian et al. (2018) present a mathematical model and a three step decomposition approach

to solve the valves and pumps ON/OFF operation problem. The model proposed by the authors determines distribution schedules seeking to minimize energy consumption and meet demand. In our work, we focus on the specific understudied class of water distribution problems described in detail by Kurian et al. (2018), considering, however, conceptual differences related to the definition of the problem.

As a contribution to this domain, we present a detailed description of the Water Distribution System (WDS) in the city of Catende - PE (Brazil) and a new mathematical model for the problem of elaborating water distribution schedules that was specifically developed based on the studied system. The mathematical model was built in parallel to the development of a solver in **LINGO**. Our objective was to build a model that could be applied with this solver, generating optimal solutions quickly, in order to offer practical support to the water distribution planning of the municipality. As result, we were able to develop a new general mathematical model whose main scientific contribution is its simplicity. This model has the following features: a) it seeks to determine the status of the valves (ON/OFF) and the ideal values for the flows; b) the aim of the problem is to achieve a distribution as equitable as possible; c) the valves are located at the entrance of reservoirs/zones served; d) parameters are incorporated into the model to define priority rules; e) complex calculations are replaced by the definition of maximum and minimum values for the flows; and f) flexibility variables were proposed (time of availability of water per shift), which allows the 'ideal' planning to be performed in practice.

In the next sections we present: a detailed description of the WDS of Catende; the mathematical model developed; a summary of results of the tests performed using actual company data; a qualitative discussion about aspects of the model and the solver; a brief assessment of the limitation of the solver developed; and finally, conclusions and suggestions for future works.

## 2 Catende Water Distribution System

The city of Catende is located in the interior of Pernambuco and has a total area of 207,244 km. According to IBGE data obtained from the large Census carried out in 2010 (IBGE, 2021), the population of the city of Catende on that date was 37,820 inhabitants. Of these, 28,861 were located in the urban area. In this city, the municipality is responsible for supplying the population with treated water. The next sections present in detail the process of collecting, treating and distributing water in the city of Catende, as well as issues related to demand.

### 2.1 Water collection and treatment

The collection of water distributed in Catende is carried out in the Santa Rita and Bálsamo das Freiras weirs. The Santa Rita weir is located in the district of Roçadinho, with a maximum water storage volume of 2,500,000 m<sup>3</sup>. The Bálsamo das Freiras weir is located on the border between Catende and the Municipality of Jaqueira, with a maximum storage volume of 1,500,000 m<sup>3</sup>.

Water treatment is carried out at the Catende Water Treatment Plant (WTP) (Figure 1) in a conventional and mechanical manner (Figure 2). The collection of water is carried out in the weirs arriving at the treatment plant by gravity. With an average flow of 67 l/s, the water from the Santa Rita reservoir goes to conventional treatment, and with a flow of 35

l/s, the water from the Bálsamo das Freiras reservoir is directed to the mechanical treatment plant.

**Figure 1** Catende Water Treatment Plant.



**Figure 2** Conventional (left) and mechanical (right) treatment plants.



After the adduction, the water captured at the Santa Rita weir, goes to the treatment phases, being received at the WTP through channels, and passing through a flow meter called Calha Pashall. In this trough aluminum sulphate and chlorine are added (Figure 3). These two chemicals help in the coagulation step by reacting with particles that are suspended in raw water. At this stage, the water goes through shares called flocculators, where the added reagents transform the impurities into larger particles to be removed during decantation.

The water, then flocculated, goes to another phase, passing through larger layers called decanters, where the flakes formed are deposited in tubular modules similar to a beehive,

**Figure 3** Parshall gutter, addition of chlorine and aluminum sulphate.

making the water almost suitable for consumption. In these stages of treatment, almost all contamination factors are eliminated, filtration is then carried out to further improve the decanted water which, by gravity, passes through filters made of a porous medium with the ability to retain and **remove impurities still in the water**. In disinfection, physical or chemical agents are used, such as chlorine, used for the destruction of pathogenic organisms that cause diseases, ending with the correction of pH, where products are also added so that the treated water has a suitable pH for consumption. After being treated, the water is deposited in a supported type reservoir with a capacity of 500,000 m<sup>3</sup> of storage (Central Reservoir). Treatment is carried out in continuous flow.

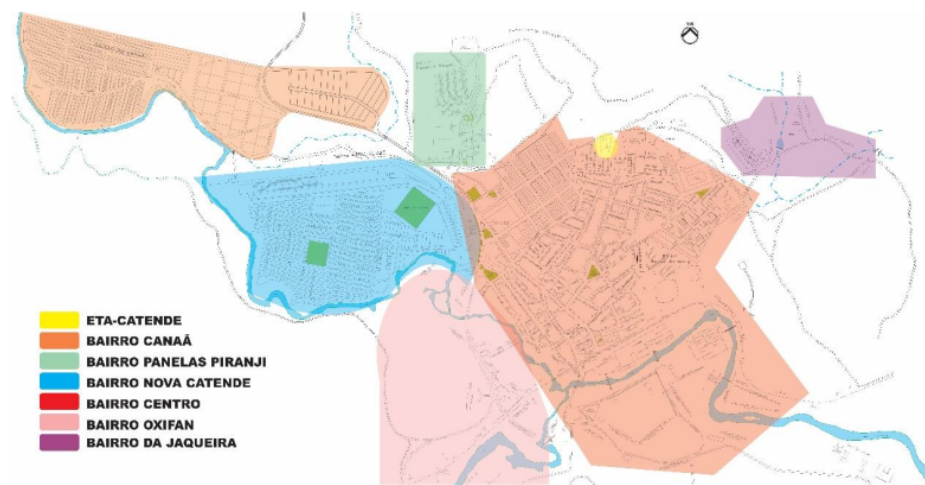
The water collected from the Bálsamo das Freiras Reservoir is treated in mechanical filters, where the addition of chemical products is carried out automatically. The disposal of the water treated in these filters is also carried out in the Central Reservoir.

## 2.2 *Water distribution in Catende*

For the planning of water distribution, the city of Catende was divided into 6 zones (Figure 4).



**Figure 4** Catende map by supply zones. Source: Adapted from the Archives of the Municipality of Catende, consulted in 2016 (subtitles in Portuguese - ETA Catende is the WTP and the neighborhoods are the distribution zones).



In order to allow water resources to reach consumer units in the most distant neighborhoods from the WTP, capacity studies were carried out, considering the population increase. Based on these studies, the distribution managers identified the need to install reservoirs that serve as water redistribution points.

The water treated at the WTP - Catende is deposited in a central elevated reservoir (named Central Reservoir), which has a capacity of 500,000 m<sup>3</sup>. From this reservoir, it is distributed to another elevated type reservoir with a capacity of 400,000 m<sup>3</sup>, both located in the WTP. The Elevated Reservoir is responsible for distributing water to the Nova Catende and Jaqueira districts. This reservoir receives water from the Central Reservoir through pumping, but distributes to the aforementioned neighborhoods through gravitational force.

**Figure 5** Reservoirs raised (left) and elevated (right).



The Central Reservoir, in addition to supplying water to the Elevated Reservoir (in the WTP), also supplies three more reservoirs of the elevated type in the Panelas Piranji, Canaã



consumption reported and data from IBGE (2021) obtained from the 2010 Census, it was estimated that an average of 7,473,960.00 liters of water per day is needed to serve the population of Catende.

Through the visits carried out at the WTP in Catende in 2016, it was also possible to obtain information on the daily production capacity and the number of households served, separated by the supply zones. Table 1 shows the water demand for each zone (neighborhood), where the necessary amount of water per household was defined considering that each household resides on average 3.9 inhabitants (average established by IBGE (2021) through the last large demographic census of 2010). Information on the number of households/zones and the average water consumption per inhabitant were provided by the WTP managers, considering the average use of 200 liters of water per inhabitant. It was also informed that almost 100% of households have reservoirs ranging between 500 and 1000 liters. These are activated when water is not supplied, either due to maintenance in the distribution network, or in periods when the water flow for treatment is low, thus limiting the daily supply.

supply sector	number of households	average consumption (liters/day)
Centro	5431	4.236.180,00
Nova Catende	1043	813.540,00
Jaqueira	976	761.280,00
Panelas Piranji	459	358.020,00
Canaã	1538	1.199.640,00
Oxifan	135	105.300,00
Total	9582	7.473.960,00

**Table 1** Demand by sector - Catende (data from 2016)

Although Catende is located in a region that does not suffer so much from the lack of water, it has been observed over the years, a considerable drop in its water sources, alerting the population and managers about the need to save this resource. The production capacity of treated water is 288,000 liters of water per hour. Observing the total daily demand required, it is concluded that the productive capacity does not meet the daily needs of the population, which highlights the need for good management of water distribution.

Based on this issue, the water distribution problem was then defined, as discussed in the next section.

### 3 Problem of Elaborating Water Distribution Schedules in Periods of Scarcity

This water distribution problem arises in times of scarcity, in which it becomes necessary to draw up a distribution schedule for the different zones of the city, based on the limitation of this precious resource. Such schedule informs the shifts in which the inhabitants will be able to acquire treated water, generating their reserves for the periods in which there will be



no supply. The following section presents a mathematical model for the problem considered in this work.

### 3.1 Mathematical modeling

The symbols used in the model are shown in the Table ( 2 ). And the mathematical model is presented as follows:

$$\text{maximize } TVD - FRF * \sum_{z=NR+1}^{NL} |PDem_z - PDist_z| \quad (1)$$

$$+VRF * \sum_{d=1}^{ND} \sum_{s=1}^{NS} \sum_{l=1}^{NL} X_{dsl} - ICIrF * IdleCEW Sum$$

subj. to

$$InF_{dsl} - V_l * X_{dsl} = 0, \quad \forall d, s, l \quad (2)$$

$$V_l - InFR_l * T = 0, \quad \forall l \quad (3)$$

$$InFR_l - MaxInFR_l \leq 0, \quad \forall l \quad (4)$$

$$InFR_l - MinInFR_l \geq 0, \quad \forall l \quad (5)$$

$$OutF_{dsr} - \sum_{l=1}^{NL} (InF_{dsl} * Y_{rl}) = 0, \quad \forall d, s, r \quad (6)$$

$$VRes_{dsl} - VRes_{(d-1)(NS)l} - InF_{dsl} + OutF_{dsl} = 0, \quad s = 1, \quad \forall d, l \quad (7)$$

$$VRes_{dsl} - VRes_{d(s-1)l} - InF_{dsl} + OutF_{dsl} = 0, \quad s = 2, \dots, NS, \quad \forall d, l \quad (8)$$

$$VRes_{dsl} - IRV_l = 0, \quad d = 0, \quad s = NS, \quad \forall l \quad (9)$$

$$VRes_{dsl} \geq 0, \quad \forall d, s, l \quad (10)$$

$$VRes_{dsr} - C_r \leq 0, \quad \forall d, s, r \quad (11)$$

$$VRes_{dsz} - AZC_z * NH_z \leq 0, \quad \forall d, s, z \quad (12)$$

$$OutF_z - D_z \leq 0, \quad \forall z \quad (13)$$

$$OutF_z - \sum_{d=1}^{ND} \sum_{s=1}^{NS} OutF_{dsz} = 0, \quad \forall z \quad (14)$$

$$TDV - \sum_{z=NR+1}^{NL} OutF_z = 0 \quad (15)$$

$$PDist_z - \frac{OutF_z}{TDV} = 0, \quad \forall z \quad (16)$$

$$IdleC_{dsl} - C_l + VRes_{dsl} = 0, \quad s = NS, \quad \forall d, l \quad (17)$$

$$IdleCEWSum - \sum_{l=1}^{NL} (IdleC_{(ND)(NS)l})^{RF_l} = 0 \quad (18)$$

$$X_{dsl} \in \{0, 1\} \quad (19)$$

$$V_l \in \mathbb{R}_+ \quad (20)$$

The objective function ( 1 ) at the same time: a) maximizes the volume distributed to the zones (first term); b) makes the distribution of water between the different zones as fair as possible (second term); and c) minimizes the idle capacity of the reservoirs (fourth term). The third term is used to force the uniqueness of the solution (or at least reduce the number of possible solutions). Using the + sign before the term makes the valves <sup>a</sup> remain open as long as possible. The parameters FRF and VRF are used to regulate the order of importance of the second and third terms, respectively. And ICIRF is used to ensure that the algorithm prioritizes meeting demand and that the remaining volume in the reservoirs becomes a secondary concern. The equations in ( 2 ) and ( 3 ) are used to, respectively, determine the water inflow and ideal inflow rate based on the evaluated solution. The constraints in ( 4 ) and ( 5 ) limit the inflow rates values. The equations in ( 6 ) guarantee that the reservoirs outflows are exactly the amount of water that each reservoir distributes to other reservoirs or zones<sup>b</sup>. The equations in ( 7 ) and ( 8 ) are the well-known flow constraints, being the equations in ( 9 ) boundary conditions. The constraints in ( 10 ), ( 11 ) and ( 12 ) limit the values of the remaining volumes of the reservoirs. The constraints in ( 13 ) ensure that the total volume distributed to each zone will not exceed its total demand. The equations in ( 14 ), ( 15 ) and ( 16 ) calculate the total volume delivered to each zone, the total volume delivered to all zones and the proportions of volume distributed to each zone, respectively. The equations in ( 17 ) and ( 18 ) calculate the idle capacities of the reservoirs in the last shift of the last day, and an exponentially weighted sum of these values, respectively. Finally, the constraints in ( 19 ) and ( 20 ) inform the nature of the primary decision variables.

<sup>a</sup>In some cases the valves are replaced by pumps. In these cases, the pumps can be compared to open valves, when activated, and closed valves, when not activated. Therefore, the simplification adopted (considering only valves) does not generate any implications for the developed model.

<sup>b</sup>Note that, in the case of zones, each zone  $z$  is considered to have a single reservoir, whose capacity is defined by the sum of the capacities of the various housing reservoirs in the zone.

### 3.2 Distribution time adjustment

In order to guarantee a fair distribution, in specific cases where the actual flows are greater than the ideal flows informed in the initial model solution, and cannot be reduced in practice, it is necessary to limit the water distribution time for certain locations. In this case, the proper distribution time per shift for local  $l$ ,  $T_l^*$ , can be defined according to equations in (21). In practice, once delivery times are adequately limited, water delivery will proceed as planned.

$$T_l^* = T * (InFR_l / \text{MaxInFR}_l), \quad \forall l \quad (21)$$

## 4 Tests and results

To validate the proposed model, a LINGO code was developed and ran in the 'global solver' mode, using the parameter values recommended by the software LINGO (2021) (with the exception of the Precision parameter, on the Interface tab, whose value was set to 11). The tests were carried out based on the considerations presented in the sections 4.2, 4.3 and 4.4 and using the data collected during this work, as summarized in the section 4.1. The results found as well as an analysis of these results are presented along the following sections together with the considerations studied.

### 4.1 Data collected

Tables 3, 4, 5 and 6 present a synthesis of the data referring to the weirs, reservoirs, zones and the network of studied water distribution, respectively. The conceptual and mathematical model presented in this paper, as well as the solver used to validate the model, were developed through a recent effort. However, the data presented in this section were collected in 2016 as part of an extension project carried out by the authors of this article, through which a detailed study of the water distribution process presented in the section 2 was carried out (Blinded, 2016).

number of locations (NL) = 12 (6 zones)

number of reservoirs (NR) = 6 (1 WTP)

housing coefficient (HC) = 3.9 inhabitants / household

average daily inflow per inhabitant (IAC) = 200 liters / inhabitant \* day

Note: each weir supplies water to an WTP (conventional or mechanical). However, both supply water to the Central Reservoir. For this reason, in this work we consider only a single WTP.

weir	name	distribution flow rate (liters/hour)
1	Santa Rita weir	241.200,00
2	Bálsamo das Freiras weir	126.000,00
Total		367.200,00

**Table 3** Weirs data - Catende (data from 2016)

reservoir ( $r$ )	name	capacity ( $C_r$ ) (liters)	maximum inflow rate ( $\text{MaxInFR}_r$ ) (liters/hour)
1	WTP	2.500.000	367.200,00
2	Central	500.000	288.000,00
3	Elevated	400.000	uninformed
4	Panelas Piranji	100.000	uninformed
5	Canaã	180.000	uninformed
6	Oxifan	20.000	uninformed

**Table 4** Reservoirs data - Catende (data from 2016)

zone ( $z$ )	name	number of households ( $\text{NH}_z$ )
7	Centro	5.431
8	Nova Catende	1.043
9	Jaqueira	976
10	Panelas Piranji	459
11	Canaã	1.538
12	Oxifan	135

**Table 5** Zones data - Catende (data from 2016)

( $Y_{rl}$ )	1	2	3	4	5	6	7	8	9	10	11	12
1	0	1	0	0	0	0	0	0	0	0	0	0
2	0	0	1	1	1	1	1	0	0	0	0	0
3	0	0	0	0	0	0	0	1	1	0	0	0
4	0	0	0	0	0	0	0	0	0	1	0	0
5	0	0	0	0	0	0	0	0	0	0	1	0
6	0	0	0	0	0	0	0	0	0	0	0	1

**Table 6** Physical distribution structure - Catende (data from 2016)

## 4.2 Initial considerations

Figure 7 presents the optimal solution for the data summarized in the section 4.1, calculated with the aid of the 'LibreOffice Calc' software, considering the maximum daily distribution value (in a 24-hour shift), limited by the outflows of the weirs and the WTP, and the following values for the parameters:  $\text{FRF} = 500,000$ ,  $\text{VRF} = 500,000$ ,  $\text{ICIrF} = 0.0001$ ,  $\text{RF}_r = 1.13$ ,  $r = 1, 2$ ;  $\text{RF}_r = 1.12$ ,  $\forall r | r \neq 1, 2$ ; and  $\text{RF}_z = 1.00$ ,  $\forall z$  (case 01).

**Figure 7** Determination of optimal distribution values with 'LibreOffice Calc', considering a daily 24-hour distribution period.

	A	B	C	D	E	F	G	H	I
1	number of days				1 day				
2	number of shifts per day				1 shift				
3	time per shift				24 (hours)				
4	average capacity of reservoirs in the zones				1000 (liters)				case 01
5	average consumption per inhabitant				200 (liters / day)				
6	housing coefficient				3.90 (inhabitants / household)				
7	irrelevance factor				1.00E-04				
8									
9									
10	local	households	demand (liters)	unmet (liters)	capacity (liters)	initial vol. (liters)	RF	idle capacity * (liters)	inflow (liters)
11	ETA				2,500,000.00	0.00	1.13	599,200.00	8,812,800.00
12	R1				500,000.00	0.00	1.13	500,000.00	6,912,000.00
13	R2				400,000.00	0.00	1.12	400,000.00	1,456,410.77
14	R3				100,000.00	0.00	1.12	100,000.00	331,100.81
15	R4				180,000.00	0.00	1.12	180,000.00	1,109,440.20
16	R5				20,000.00	0.00	1.12	20,000.00	97,382.59
17	Z1	5,431	4,236,180.00	318,514.38	5,431,000.00	0.00	1.00	5,431,000.00	3,917,665.62
18	Z2	1,043	813,540.00	61,169.30	1,043,000.00	0.00	1.00	1,043,000.00	752,370.70
19	Z3	976	761,280.00	57,239.92	976,000.00	0.00	1.00	976,000.00	704,040.08
20	Z4	459	358,020.00	26,919.19	459,000.00	0.00	1.00	459,000.00	331,100.81
21	Z5	1,538	1,199,640.00	90,199.80	1,538,000.00	0.00	1.00	1,538,000.00	1,109,440.20
22	Z6	135	105,300.00	7,917.41	135,000.00	0.00	1.00	135,000.00	97,382.59
23									
24	total	9,582	7,473,960.00	561,960.00	weighted sum			18,826,516.31	weir outflow (liters)
25									
26	weir								8,812,800.00
27									
28	total volume distributed to zones				6,912,000.00	local		average volume	inflow rate (liters / hour)
29	fitness (optimal value)				12,910,117.35			received per inhabitant	
30								(liters / day)	367,200.00
31									288,000.00
32	relevance factors:					R2		60,683.78	
33	fair distribution	500,000				R3		13,795.87	
34	valves	500,000				R4		46,226.68	
35						R5		4,057.61	
36						Z1	184.96	163,236.07	
37	* at the end of the day					Z2	184.96	31,348.78	
38						Z3	184.96	29,335.00	
39						Z4	184.96	13,795.87	
40						Z5	184.96	46,226.68	
41						Z6	184.96	4,057.61	

As we can see in this figure, the optimal daily water distribution volume per inhabitant, considering a daily 24-hour distribution period, is 184.96 liters / (day \* inhabitant). Therefore, we can conclude that, in fact, the flow of the pipeline that transports water from the WTP to the Central Reservoir is not sufficient to meet the average daily consumption of the population informed by the company (200 liters / (day \* inhabitant)).

Another important piece of information that we observe in this figure are the ideal inflow rates, which would result in a fair distribution for the population of Catende. These values are basically defined as a function of water availability, demands, and initial volumes in the reservoirs. Such values were identified using the following calculation logic:

1) as the part of the inflow received by the WTP and not distributed is completely used to fill its reservoir and, as the inflow rate of the Central Reservoir is lower than the inflow rate of the WTP, so  $InF_{dsr}$ , for the WTP ( $r = 1$ ) and the Central Reservoir ( $r = 2$ ), are calculated according to Eqs. in 22:

$$InF_{dsr} = \text{MaxInFR}_r * T, \quad d = 1, \quad s = 1, \quad r = 1, 2 \quad (22)$$

2) the total availability of water, released by the Central Reservoir, which directly or indirectly supplies all other reservoirs and zones, is less than the total demand. Therefore, the volume of water received by the Central Reservoir is completely distributed to the zones (and not to fill the reservoirs), with the fair distribution being proportional to the number of households in each zone:

$$InF_{dsz} = InF_{dsr} * \frac{D_z}{TD}, \quad d = 1, \quad s = 1, \quad r = 2, \quad \forall z \quad (23)$$

and, 3) as the volumes received by the reservoirs are completely distributed:

$$InF_{dsr} = OutF_{dsr} \quad d = 1, \quad s = 1, \quad r = 3, \dots, NR \quad (24)$$

being the values of  $OutF_{dsr}$  calculated according to the Eqs. in (6) recursively, and the ideal rates (not informed) then calculated based on the Eqs. in (2) and (3).

After performing the calculations shown in Figure 7, we run the solver in **LINGO**, using the same inputs shown in this figure, along with other data reported in the 4.1 section. The values 367,200.00 and 288,000.00 were used as the values of the maximum inflow rates informed for the WTP and for the Central Reservoir, respectively. No limit values for inflow rates were informed for the other reservoirs and zones. Table 7 displays the values found by the solver. As we can see in this table, the solver found the calculated optimal values very quickly and without any errors observed.



location	total demand (liters)	capacity (liters)	initial vol. (liters)	idle capacity ** (liters)	total inflow (liters)
WTP		2,500,000.00	0.00	599,200.00	8,812,800.00
R1		500,000.00	0.00	500,000.00	6,912,000.00
R2		400,000.00	0.00	400,000.00	1,456,410.77
R3		100,000.00	0.00	100,000.00	331,100.81
R4		180,000.00	0.00	180,000.00	1,109,440.20
R5		20,000.00	0.00	20,000.00	97,382.59
Z1	4,236,180.00	5,431,000.00	0.00	5,431,000.00	3,917,665.62
Z2	813,540.00	1,043,000.00	0.00	1,043,000.00	752,370.70
Z3	761,280.00	976,000.00	0.00	976,000.00	704,040.08
Z4	358,020.00	459,000.00	0.00	459,000.00	331,100.81
Z5	1,199,640.00	1,538,000.00	0.00	1,538,000.00	1,109,440.20
Z6	105,300.00	135,000.00	0.00	135,000.00	97,382.59
total	7,473,960.00	exp. weighted sum:		18,826,516.31	
shift data		location	valve status	ideal inflow rate (liters/hour)	
1	day	WTP	open	367,200.00*	
1	shif	R1	open	288,000.00*	
24	hours/shift	R2	open	60,683.78	
		R3	open	13,795.87	
		R4	open	46,226.68	
		R5	open	4,057.61	
		Z1	open	163,236.07	
		Z2	open	31,348.78	
		Z3	open	29,335.00	
		Z4	open	13,795.87	
		Z5	open	46,226.68	
		Z6	open	4,057.61	
	total zones outflow (liters)			6,912,000.00	
	optimal value (fitness)			12,910,117.35	
	(computational time = 0.43 sec)			* values reported for maximum inflow rates	
	(total number of iterations = 3,440)			** at the end of the evaluated period	

**Table 7** Summary of results generated by the solver developed in Lingo for case 01.

The same values were found for volumes distribution when the solver in LINGO was run considering 3 shifts of 8h (case 03). In this case, the inflow is distributed between the shifts, keeping the same ideal inflow rates.

### 4.3 Practical aspects

It would be ideal if the company could regulate the flow rates for water distribution, adjusting these rates according to each ideal solution found by the solver for the different cases evaluated by the planners. However, in practice this becomes very difficult. The following situations will be faced when looking for solutions to real problems:

- (1) real flow rates of the pipelines smaller than the calculated ideal flow rates;

- (2) actual pipelines flow rates greater than the calculated ideal flow rates;
- and (3) very small ideal flow rates, making the supply unsatisfactory.

Situations (1) and (3) can be easily solved by indicating maximum and minimum values, respectively, for the inflow rates calculated by the solver. However, care must be taken not to set values that make the problem unresolvable. The planner must always keep in mind that the water distribution network and the flow rates form a single system, with values that are essentially integrated.

In the case of situation (1), as the solver seeks to maximize the volume of water distributed to the zones, the volume of distribution (limited by the inflow rate) in a given location will be transferred to other locations seeking to meet the demand and, later, distributed to filling the idle capacity of the reservoirs. For example, Table 9 presents the results found by the solver for case 22, in which the same data used in case 06 were considered, except that a maximum flow rate was informed for zone Z1 ( $z = 7$ ),  $\text{MaxInFR}_7 = 130,000.00$  liters/hour (case 06 is just like case 03 but with 2 days).

For situation (3), once a minimum value is indicated for the inflow of a given location, the solver will propose solutions in which the inflow of some locations will be null or concentrated in one or a few shifts. For example, Table 10 presents the results found by the solver for case 23, in which the same data used in case 06 were considered, except that a minimum flow rate was reported for the reservoir R5 ( $r = 6$ ),  $\text{MinInFR}_6 = 10,000.00$  liters/hour. As we can see in this table, the R5 reservoir and the Z6 zone only receive flow in the third shift of the first and second days. The solver presented a result for case 23 after setting the parameter value  $\text{VRF} = 5,000$ . The solution found by the solver is not optimal. It would be possible to obtain a better solution, for example, by distributing the volume received by zone Z6 among all shifts.

As for situation (2), the developed solver does not present a good answer. The value found by the solver will always be an ideal value, based on the informed restrictions. If the ideal value is lower than the real value, the respective zone will have an excess amount of water. And there will be no sure way to prevent the volume consumed by that zone from being the desired volume. Except limiting distribution time. In this case, the ideal distribution time (within the period of a shift) can be calculated using the equations in (21).

#### 4.4 Solver and model limitations

LINGO is an incredible software, indispensable for mathematical modeling of any real problem. With the help of LINGO, it is possible to notice a series of initial errors in the model, which could easily go unnoticed. Additionally, the elaborated mathematical model strongly influences the solver efficiency. This forces the modeler to think about issues that limit the model's solutions, and to work with functions that make the model robust and functional.

However, LINGO is not a magic tool, LINGO's processing power is limited, due to limitations associated with exact solving techniques for optimization problems, especially when defined as non-linear and NP-Hard. For most problems, the computational effort increases as the number of decision variables increases, according to functions that are not polynomials. In other words, there is an explosion in the number of possible solutions, and current exact methodologies have not yet found a way to deal with this, in the case of problems with larger dimensions.

The model presented in this work is a non-linear, mixed-integer mathematical programming model. Nonlinearity happens due to the objective function and the constraints in (2), (16) and (18). The non-polynomial nature of the model arises due to the binary decision variables  $X_{dst}$ . In this section, we present an initial discussion about the limitations of the developed solver and mathematical model. It is important to emphasize that many of the limitations presented here could be mitigated through adjustments to the model, or even through a better use of the LINGO software. However, this requires a hard work. It may seem like it, but building a robust and adequate mathematical model is not a simple task and LINGO offers a wide range of different application possibilities through various parameter adjustments, and we are still familiarizing with all these possibilities.

Table 11 presents some tests carried out to analyze the efficiency of the developed model and solver. We run the solver for case 01, and for similar cases where only the number of days and shifts have been changed. We used an interval between 1 and 7 days and, for each possibility, we considered 1, 2 and 3 shifts. The relevance / irrelevance factors referring to fair distribution, valve status and reservoir idle capacity were defined based on an extensive study. Such definition had the following purposes: to define as priorities meeting the demand, then fair distribution and finally filling the reservoirs; in order to test the ruggedness of the model, we also had the clear intention of distributing the flow among all shifts, allowing the valves to be turned off only when flow restrictions impose this condition.

An important issue to be considered is that the adjustment of the parameters FRF, VRF, and ICrF strongly influence the efficiency of the solver as well as the solution found. An important aspect of the relevance / irrelevance parameters adopted is that they allow the planner to interfere in the solution presented by the solver, so that this solution meets the planning objectives (e.g., prioritization of filling reservoirs or meeting demand). However, this requires that the planner has good experience in applying the solver and a good understanding of how solve reacts to the adjustment of these parameters. Therefore, the good performance of the developed solver will depend largely on the qualification of the planner for this purpose. We understand that it is possible to build formulas for automatic calculation of suitable values for these parameters, using quantitative and qualitative analyzes referring to the decision-making process (e.g., formulas defined according to answers given for some questionnaire). It is also possible, of course, to develop a user-friendly graphical interface. These efforts would make using the solver much simpler, however they depend on the continuity of this project, with a strong commitment of time and dedication of qualified professionals.

The solver developed in lingo language as well as the files generated with the results are available at <https://github.com/blinded>.

## 5 Conclusions and future works

In the present work, a conceptual and mathematical modeling for the problem of water distribution in periods of scarcity was presented. The validation of the mathematical model was performed using a solver developed with the LINGO software from LINDO Systems. Through the tests carried out, we concluded that the developed model and solver fit perfectly to the studied reality, and are capable of proposing solutions that make the distribution of water volumes for the different zones fair, in periods of scarcity, through the control of flows and distribution times. Such solutions take into account the restrictions of the system and present alternatives for when the channelings are to be modulated optimally (with

flow rates that optimally regulate the distribution). However, it was verified that, for the solver to present good results, it must be used by a suitably qualified employee. What may become less relevant through new advanced studies for high-level calculation of relevance / irrelevance parameters adopted in the model, as well as through the construction of a friendly graphic interface. Additionally, due to the limitations of the exact methodologies developed so far for solving non-polynomial optimization problems (such as the studied problem), the developed solver naturally presented limitations related to its ability to solve larger problems. Such limitations justify the need to develop solution algorithms based on heuristics applied to optimization problems. These efforts will be the focus of our next works.

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*indexes*

$d$	used to designate a day, $d = 1, \dots, ND$
$s$	used to designate a shift of a day, $s = 1, \dots, NS$
$l$	used to designate a location that are part of the water supply system (either zones, reservoirs or WTPs), $l = 1, \dots, NL$
$r$	used to designate a WTP or a reservoir, $r = 1, \dots, NR$
$z$	used to designate a zone, $z = NR + 1, \dots, NL$

*parameters*

ND	number of days
NS	number of daily distribution shifts
NL	number of locations
NR	number of reservoirs (reservoirs and WTPs)
T	time per shift
$IRV_l$	initial volume of reservoir at the location $l$
$C_l$	capacity of reservoir at the location $l$
$AZC_z$	average reservoir capacities of the households in the zone $z$
$NH_z$	number of households in the zone $z$
HC	housing coefficient
IAC	represents the average daily consumption per inhabitant
$MaxInFR_l$	maximum inflow rate of the pipeline that reaches the reservoir at the location $l$
$MinInFR_l$	minimum inflow rate set for the pipeline that reaches the reservoir at the location $l$
$Y_{rl}$	binary variable that informs the physical structure of the water distribution system. $Y_{rl} = 1$ if water is supplied directly from the reservoir $r$ to the location $l$ and $Y_{rl} = 0$ if that doesn't happen
$RF_l$	importance attributed to maintaining water in reservoir at location $l$ , since there is a surplus of water to fill the reservoirs. It is important that the values of these parameters are greater than and close to 1.00.
FRF	relevance factor for the fair distribution
VRF	relevance factor for the adjustment of the valves
ICIrF	irrelevance factor for idle capacity
$D_z$	demand of zone $z$ , $D_z = IAC * NH_z * HC * ND$
TD	total demand, $TD = \sum_{z=1}^{NZ} D_z$
$PD_z$	$z$ zone percentage demand, $PD_z = \frac{D_z}{TD}$

*decision variables*

$X_{dsl}$	binary variable that receives value 1 when the inlet valve of location $l$ is open on the $s$ -th shift of day $d$ , and receives value 0 when it is closed in the same period
$V_l$	ideal volume of water distributed to the location $l$ in any shift in which the corresponding valve remains open

*secondary variables*

$InF_{dsl}$	water inflow received by location $l$ on the $s$ -th shift of day $d$
$OutF_{dsr}$	water outflow delivered by reservoir $r$ on the $s$ -th shift of day $d$
$InFR_l$	ideal inflow rate of location $l$
$VRes_{dsl}$	volume of water remaining in the reservoir at location $l$ at the end of the $s$ -th shift of the day $d$
$OutF_z$	total outflow of zone $z$
$TV_D$	total volume distributed
$PDist_z$	percentage distributed to zone $z$
$IdleC_{dsl}$	idle capacity of reservoir at location $l$ at the end of the $s$ -th shift of the day $d$
$IdleCEWSum$	exponentially weighted sum of idle capacities

**Table 2** Table of symbols used in the mathematical model.

location	total demand (liters)	capacity (liters)	initial vol. (liters)	idle capacity ** (liters)	total inflow (liters)
WTP		2,500,000.00	0.00	599,200.00	8,812,800.00
R1		500,000.00	0.00	500,000.00	6,912,000.00
R2		400,000.00	0.00	400,000.00	1,456,410.77
R3		100,000.00	0.00	100,000.00	331,100.81
R4		180,000.00	0.00	180,000.00	1,109,440.20
R5		20,000.00	0.00	20,000.00	97,382.59
Z1	4,236,180.00	5,431,000.00	0.00	5,431,000.00	3,917,665.62
Z2	813,540.00	1,043,000.00	0.00	1,043,000.00	752,370.70
Z3	761,280.00	976,000.00	0.00	976,000.00	704,040.08
Z4	358,020.00	459,000.00	0.00	459,000.00	331,100.81
Z5	1,199,640.00	1,538,000.00	0.00	1,538,000.00	1,109,440.20
Z6	105,300.00	135,000.00	0.00	135,000.00	97,382.59
total	7,473,960.00	exp. weighted sum:		18,826,516.31	
shift data			location	valve status T1,T2 and T3	ideal inflow rate (liters/hour)
1	day	WTP		open	367,200.00*
3	shif	R1		open	288,000.00*
8	hours/shift	R2		open	60,683.78
		R3		open	13,795.87
	total zones outflow	R4		open	46,226.68
	(liters)	R5		open	4,057.61
		Z1		open	163,236.07
	6,912,000.00	Z2		open	31,348.78
		Z3		open	29,335.00
	optimal value (fitness)	Z4		open	13,795.87
		Z5		open	46,226.68
	24,910,117.32	Z6		open	4,057.61

(computational time = 0.15 sec)

(total number of iterations = 524)

\* values reported for maximum inflow rates

\*\* at the end of the evaluated period

**Table 8** Summary of results generated by the solver developed in Lingo for case 03.



location	total demand (liters)	capacity (liters)	initial vol. (liters)	idle capacity ** (liters)	total inflow (liters)
WTP		2,500,000.00	0.00	0.00	16,324,000.00
R1		500,000.00	0.00	9,054.90	13,824,000.00
R2		400,000.00	0.00	20,835.03	3,528,804.97
R3		100,000.00	0.00	20,835.03	795,204.97
R4		180,000.00	0.00	20,835.03	2,558,444.97
R5		20,000.00	0.00	20,000.00	210,600.00
Z1	8,472,360.00	5,431,000.00	0.00	5,431.00	6,240,000.00
Z2	1,627,080.00	1,043,000.00	0.00	1,043.00	1,627,080.00
Z3	1,522,560.00	976,000.00	0.00	976.00	1,522,560.00
Z4	716,040.00	459,000.00	0.00	459.00	716,040.00
Z5	2,399,280.00	1,538,000.00	0.00	1,538.00	2,399,280.00
Z6	210,600.00	135,000.00	0.00	135.00	210,600.00
total	14,947,920.00		exp. weighted sum:	9,883,380.87	

shift data		location	valve status T1, T2 and T3	ideal inflow rate (liters/hour)
2	day	WTP	open	340,083.33
3	shift	R1	open	288,000.00*
8	hours/shift	R2	open	73,516.77
		R3	open	16,566.77
	total zones outflow (liters)	R4	open	53,300.94
		R5	open	4,387.50
		Z1	open	130,000.00*
	12,715,560.00	Z2	open	33,897.50
		Z3	open	31,720.00
	optimal value (fitness)	Z4	open	14,917.50
		Z5	open	49,985.00
	48,638,517.09	Z6	open	4,387.50

(computational time = 0.34 sec)

(total number of iterations = 1,015)

\* values reported for maximum inflow rates

\*\* at the end of the evaluated period

**Table 9** Summary of results generated by the solver developed in Lingo for case 22.

location	total demand (liters)	capacity (liters)	initial vol. (liters)	idle capacity ** (liters)	total inflow (liters)
WTP		2,500,000.00	0.00	0	16,324,000.00
R1		500,000.00	0.00	500,000.00	13,824,000.00
R2		400,000.00	0.00	400,000.00	2,912,821.54
R3		100,000.00	0.00	100,000.00	662,201.63
R4		180,000.00	0.00	180,000.00	2,218,880.40
R5		20,000.00	0.00	20,000.00	194,765.18
Z1	8,472,360.00	5,431,000.00	0.00	5,431,000.00	7,835,331.25
Z2	1,627,080.00	1,043,000.00	0.00	1,043,000.00	1,504,741.39
Z3	1,522,560.00	976,000.00	0.00	976,000.00	1,408,080.15
Z4	716,040.00	459,000.00	0.00	459,000.00	662,201.63
Z5	2,399,280.00	1,538,000.00	0.00	1,538,000.00	2,218,880.40
Z6	210,600.00	135,000.00	0.00	135,000.00	194,765.18
total	14,947,920.00	exp. weighted sum:		15,448,543.58	
shift data		location	valve status open	ideal inflow rate (liters/hour)	
2	day	WTP	every shift	340,083.33	
3	shift	R1	every shift	288,000.00*	
8	hours/shift	R2	every shift	60,683.78	
		R3	every shift	13,795.87	
		R4	every shift	46,226.68	
		R5	D1-T3 / D2-T3	12,172.82	
		Z1	every shift	163,236.07	
		Z2	every shift	31,348.78	
		Z3	every shift	29,335.00	
		Z4	every shift	13,795.87	
		Z5	every shift	46,226.68	
		Z6	D1-T3 / D2-T3	12,172.82	
	total zones outflow (liters)				13,824,000.00
	optimal value (fitness)				14,142,455.14

(computational time = 54.49 sec)

(total number of iterations = 464,543)

\* values reported for maximum inflow rates

\*\* at the end of the evaluated period

**Table 10** Summary of results generated by the solver developed in Lingo for case 23.

case	ND	NS	optimum	fitness	rel. error	relevance factors	
						time(s)	iterations
01	1	1	12,910,117.35	12,910,117.35	0%	0.43	3,440
02	1	2	18,910,117.35	18,910,117.35	0%	0.72	5,425
03	1	3	24,910,117.35	24,910,117.32	≈ 0%	0.15	524
04	2	1	25,822,455.15	25,822,454.86	≈ 0%	0.71	4,808
05	2	2	37,822,455.15	37,822,454.72	≈ 0%	0.21	914
06	2	3	49,822,455.15	49,822,454.59	≈ 0%	0.26	789
07	3	1	38,734,455.15	38,734,454.72	≈ 0%	0.11	517
08	3	2	56,734,455.15	56,734,454.51	≈ 0%	0.24	771
09	3	3	74,734,455.15	74,734,454.32	≈ 0%	0.65	1,139
10	4	1	51,646,455.15	51,646,455.14	≈ 0%	0.29	1511
11	4	2	75,646,455.15	75,646,454.30	≈ 0%	0.43	976
12	4	3	99,646,455.15	99,646,454.04	≈ 0%	0.94	1,295
13	5	1	64,558,455.15	64,558,454.43	≈ 0%	0.30	916
14	5	2	94,558,455.15	94,558,454.10	≈ 0%	0.59	1,130
15	5	3	124,558,455.15	124,558,453.76	≈ 0%	1.00	1,016
16	6	1	77,470,455.15	77,470,454.29	≈ 0%	0.24	738
17	6	2	113,470,455.15	113,470,453.89	≈ 0%	0.97	1,191
18	6	3	149,470,455.15	149,470,455.13	≈ 0%	1.63	1,332
19	7	1	90,382,455.15	90,382,454.14	≈ 0%	0.50	1,480
20	7	2	132,382,455.15	132,382,453.68	≈ 0%	2.27	2,059
21	7	3	174,382,455.15	174,382,453.21	≈ 0%	2.51	1,445

**Table 11** Study on the efficiency of the mathematical model and the solver.