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Online Model Predictive Control of Municipal Water Distribution Networks

Gokul Siva Sankar, Sridharakumar Narasimhan * and Shankar Narasimhan
Department of Chemical Engineering, Indian Institute of Technology Madras, India

Abstract

Optimal operation of municipal Water Distribution Networks (WDNs) is based on optimizing one or more performance metrics while meeting consumer demands and satisfying supply side and storage constraints. This can be achieved by implementing advanced control schemes such as Model Predictive Control (MPC). With the alarming decrease in fresh water supplies, the primary focus of online control strategies should be to conserve water. A novel control strategy that can handle both water sufficient and deficient cases is proposed for WDNs with storage facilities. Performance of the developed model based online control strategy is tested by numerical simulations of an illustrative WDN.

Keywords: water distribution network, model predictive control, optimization, storage tanks

1. Introduction

Municipal water distribution networks supply water from sources such as dams, lakes and ponds to the consumers at the demand points through a network of pipes, valves and pumps such that their demands are met at desired pressures. WDNs have to be operated efficiently and reliably to supply the guaranteed amount of water in both directly pumped (24×7) and intermittent (possible in networks with storage facilities) modes of supply. In addition, depletion of water resources, shortage in water availability for supply and growth in demand have necessitated improved and sustainable water management practices in distribution systems. Traditionally, optimal operation of WDNs is concerned with minimizing the total cost of operation while maintaining water quality, regulating pressure in the pipelines to prevent leaks, and meeting consumer demands, assuming that sufficient water is available for meeting consumer demands (Kumar, 2008). However, in several countries such as India, overhead tanks and underground sumps are available at individual households for storage of water. These tanks can serve as intermediate storage elements when peak demand requirements cannot be met. Hence, an optimal control strategy has to be devised that considers the supply side and storage constraints to meet demand requirements. We introduce a novel MPC based control strategy for WDNs which takes into account the limitations of the actuators and storage tanks to address both water deficient and water surplus scenarios.

2. Modelling Water Distribution Networks

2.1. Pipes, Nodes and Node Resistance

A WDN consists of pipes, pumps, storage tanks, and valves. It has nodal points where water is withdrawn/supplied or where two or more pipes meet (Bhave and Gupta, 2009).

*sridharkrn@iitm.ac.in

A WDN can be modelled using mass balances for all storage tanks and nodal points, and the relations between pressure drop/rise and flow rate for pipes, pumps and valves. In addition, for a pressure-driven outflow model, relations between the pressure at the demand points and outflows are also included. The model equations are as follows.

Pressure drop in pipes is given by

$$\Delta h_j = \frac{\gamma L_j \operatorname{sign}(Q_j)|Q_j|^{1.85}}{\lambda_j^{1.85} D_j^{4.87}}, \quad j = 1, ..., P \tag{1}$$

where λ_j is Hazen-William roughness coefficient of pipe j, Δh_j is the head loss in pipe j, L_j and D_j are the length and diameter of pipe j in m, Q_j is the flow through j^{th} pipeline in m^3/s and γ is the factor depending on the dimensional system used, is 10.7 when SI system of units is followed.

Mass balance at nodal points is given by

$$\sum_{j=1}^{p} a_{ij} Q_j - d_i = 0, \quad i = 1, ..., N \tag{2}$$

where d_i is the demand (or supply) at node i, N and P are the total number of nodes and pipes in the network respectively and a_{ij} is a coefficient that has a value 0 if pipe j is not linked to node i and has a value +1 or -1 otherwise, depending on whether the flow in pipe j is towards node i or away from it, respectively.

Outflow rate at a demand point is related to the pressure at that node by

$$p_i = k_i d_i^2 \;\; i = 1, ..., N \tag{3}$$

where p_i and and k_i are the pressure and resistance offered to outflow at node i.

2.2. Valves, Reservoirs and Tanks

A valve is a control device in a WDN that is used to vary the flow rate and/or pressure by closing and opening the stem. Head loss equation for a valve is given by

$$\Delta h_v = \frac{Q^2 G_L}{\left(C_v \frac{e^{ln\tau V}}{\tau}\right)^2} \tag{4}$$

where Δh_v is the head loss across the valve for a flow rate Q, C_v is the valve coefficient when the valve is 100% open, τ is the valve rangeability, V is the fraction of valve opening and G_L is the specific gravity of the fluid.

Tanks are storage units in WDNs and are associated with a minimum and maximum capacity. Quasi-steady state approximation is used here, wherein water level in all reservoirs and tanks are assumed to be constant in each time period and are updated at the end of the period. Mass balance expression for a reservoir/tank and physical storage limits of a tank are

$$H_z(k+1) = H_z(k) + \frac{\Delta t}{A_z}\left(Q'_{\text{in}}(k) - Q'_{\text{out}}(k)\right), \quad z = r \text{ or } t \tag{5}$$

$$0 \le H_t(k) \le H_{t\text{max}} \tag{6}$$

where $H_r(k)$ and $H_t(k)$ are liquid levels in reservoir and tank at the end of time period k, A_r and A_t are cross sectional areas of reservoir and tank, $Q'_{\text{in}}(k)$ and $Q'_{\text{out}}(k)$ are sum of steady state rate of inflows to and outflows from the reservoir/tank respectively, Δt is the sampling time and $H_{t\text{max}}$ represents maximum storage height.

3. Control Problem Description

Traditionally, demand outflows are the controlled variables and control can be effected by manipulations of stem positions of continuous control valves. Continuous valves provide better controllability of flow rate and/or pressure in WDNs than on-off valves. Generally, WDNs are non-square systems with one or more demand outflows being controlled by a single valve. The constraints to be satisfied represent the physical limitations of the system (limitation on the storage capacity of the tanks as in Eq. 6) and manipulated variables (valve opening in percentage can be from 0% to 100%). A suitable demand set point profile can be chosen to capture the temporal and spatial variations in the water usage pattern. To conserve water, the amount of water supplied should be just enough to meet the consumers' demand as over supply may lead to wastage of water.

3.1. Control strategy

Kumar (2008) proposed the use of MPC strategy for optimal operation of directly pumped WDNs. Control moves were calculated by minimizing the instantaneous deviation of demand outflow from the desired flow rate. However, storage facilities such as tanks or sumps, that are usually available at the demand points, were not accounted for. If it is not possible to meet peak demand requirements, water can be stored in these buffer tanks during the preceding periods and can be used to meet peak demand requirements. Thus, the use of buffer tanks allows intermittent supply and it is not necessary to meet the instantaneous demand rate. Rather, it is sufficient to schedule the supply such that the average daily requirement is met. However, in water sufficient situations, it is important not to overfill the tanks in order to conserve water. Hence, a common control strategy has to be devised which can address both these situations. The withdrawal rate from a tank is typically unknown. In the present work, it is assumed to be limited by either the amount of water in the tank or the actual demand rate, whichever is lower.

$$Q'_{\mathrm{out}_i}(k) = \min\left(d_i^{sp}(k), \frac{H_t(k)\times A_t}{\Delta t} + Q'_{\mathrm{in}_i}(k)\right), \quad i = 1, \ldots, N_d \tag{7}$$

where $d_i^{sp}(k)$ and $Q'_{\mathrm{out}_i}(k)$ are demand set-point and the amount of water withdrawn from the tank, respectively. $Q'_{\mathrm{in}_i}(k)$ is the flow rate into the tank and is equal to the outflow rate from the demand node to which it is linked. At any instant k, the assumed demand set profile may not match the consumers' actual consumption of water from the tanks and the cumulative deviation, $\Delta d_i(k)$ can be calculated as:

$$\Delta d_i(k) = \Delta d_i(k-1) + Q'_{\mathrm{in}_i}(k) - (H_i(k) - H_i(k-1))A_t/\Delta_t - d_i^{sp}(k) \tag{8}$$

The second term in Eq. 8 is the actual withdrawal rate of water inferred from the liquid levels in the tank at the beginning and end of a time period. The control objective chosen in the present work is to minimize the sum square of cumulative deviations. This will try to ensure that the amount of water supplied closely matches the required amount of water over the horizon of interest. In addition, to prevent wastage of water when it is available in excess, water level in storage tank at the end of horizon is also simultaneously minimized. Hence, the control objective chosen is to minimize a weighted sum of the two terms.

3.2. MPC Formulation

MPC is an advanced control scheme that uses a model of the system to be controlled to predict its behaviour over a finite time horizon in the future (Maciejowski, 2002). The process in the basic scheme of MPC shown in figure 1, is a simulator that uses the pressure

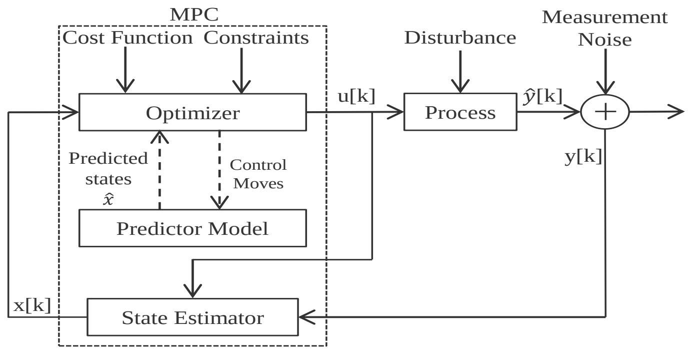

Figure 1. Basic MPC scheme

driven model to simulate the working of WDN. State estimator developed by Kumar et al. (2008) is used to estimate the current states (flow rate, pressure and outflow rate) from the corrupted measurements. The predictor model (also called extended period simulator) requires an explicit model for predicting the process output for N_p (prediction horizon) time steps ahead given N_c (control horizon) future control moves. The optimizer determines the optimal future control moves which minimizes the cost function while taking into account the constraints. Control moves for the current time period is implemented and the procedure is repeated for the next time period, based on the new set of measurements.

$$\min_{u_k,\ldots,u_{k+N_p-1}} \quad J = \sum_{i=1}^{N_d}\sum_{j=1}^{N_p} H_{t_i}^2(k+j) + \Delta d_i^2(k+j) \tag{9}$$

$$0 < u_k < 1 \tag{10}$$

$$p_j \geq p_{\min}, \quad j \in \text{demand nodes} \tag{11}$$

where u_k is the fractional control valve position at k^{th} instant, N_p is the prediction horizon, N_d is the total number of demand nodes in the network, p_j is the pressure at node j and $p_{\min}$ is the minimum pressure to be maintained.

4. Demonstration

The use of MPC strategy for online control of WDNs is tested on a sample network shown in figure 2. Elevation of all nodes is taken as zero and the source pressure is considered as 30 m of water and a minimum gauge pressure requirement of 2 m of water is imposed at the demand nodes. Cross sectional area of the storage tanks at nodes 5 and 11 are assumed to be 6 m^2 and 8 m^2 respectively, and maximum height of these tanks are taken to be 3 m. Initial water level in both tanks is assumed to be 0.1 m. The two control valves have a

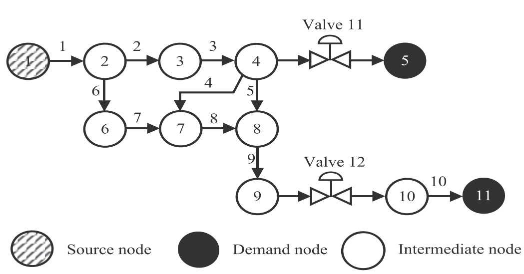

Figure 2. Schematic of the WDN used in the demonstration of MPC strategy

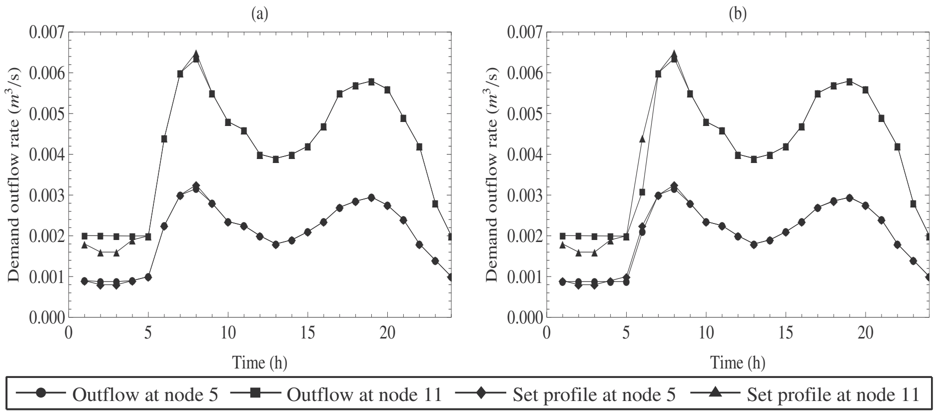

Figure 3. (a) Demand outflow profile without storage tank and (b) with storage tank

rangeability of 50 and the valve coefficient, C_v, of the valves 11 and 12 are 50 and 100 respectively. Pressure at nodes 1, 4 and 8, flow rate in pipe 1 and water level in the two tanks are measured on an hourly basis. The case when there is sufficient water to meet the need of customers is studied. MPC is applied with $N_p = 1$ and $N_c = 1$. The outflow rates at the two demand points obtained with the objective function used by Kumar (2008) without taking into account the storage tanks available at these points, are shown in 3(a). It is observed that the outflow rate matches the demand rate for most time periods, except the oversupply in the first few periods (to satisfy the pressure constraint at demand nodes). The total outflow during the 24 h period exceeds the requirement at demand node 11 by 0.97%. On the other hand, if storage facilities at the demand points and the control objective given by Eq. 9 are considered, then the solution obtained is shown in 3(b). It is observed that the initial oversupply is compensated by undersupply during the periods 5 to 8, such that the integral amount of water supplied during the 24 h period matches with the requirement. Both the tanks were empty at the end of 24 h period. In water deficient case, MPC scheme allocates water so as to meet the requirement over the 24 h period with minimal percentage undersupply. However, the results are not reported here in the interest of space.

5. Conclusions

In this work, MPC scheme was used for generating control strategies that operate WDNs with buffer tanks at demand nodes, to meet the performance goals. Sustainable operation of WDNs using a MPC control scheme was demonstrated with a sample WDN, resulting in water savings.

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