Modeling Local Water Storages Delivering Customer Demands in WDN Models

O. Giustolisi[1]; L. Berardi[2]; and D. Laucelli[3]

Abstract: Water distribution network (WDN) models account for customer-demands as water withdrawals concentrated in nodes. Customer-demands can be assumed to be constant or varying with nodal head/pressure entailing demand-driven or pressure-driven simulation, respectively. In both cases, the direct connection of customer properties to the hydraulic system is implicitly assumed. Nonetheless, in many technical situations, the service pipe fills a local private storage (e.g., a roof tank or a basement tank) from which the water is actually delivered to customers by gravity or pumping systems. In such contexts, the service pipe fills the local tank by means of a top orifice. Consequently, what is really connected to the hydraulic system is a tank, which is subject to a filling/emptying process while supplying water to customers. Therefore, since modeling this technical situation in WDN analyses is necessary, the paper develops a formulation for nodal water withdrawals in WDN models accounting for the filling/emptying process of inline tanks between the hydraulic network and customers. The formulation is also introduced in a widely used method for steady-state WDN modeling, the global gradient algorithm, and its effectiveness to increase the hydraulic accuracy of results is discussed using a simple case study and a small network. **DOI:** 10.1061/(ASCE)HY.1943-7900.0000812. *© 2014 American Society of Civil Engineers.*

Author keywords: Tank modeling; Water distribution networks; Hydraulic network modeling; Water distribution network simulation; Extended period simulation.

Introduction

The mathematical problem related to steady-state water distribution network (WDN) modeling is governed by energy balance equations along each pipe and mass balance equations in each node where the head is unknown.

The first algorithm to solve WDN modeling was proposed by Cross (1936). Cross transformed the mathematical problem to a loop analysis (loop method) in order to reduce the number of unknowns and adopted a local linearization algorithm in order to avoid the need of solving systems of linear equations. Actually, the local linearization strategy makes Cross' algorithm convergence strongly dependent on the initialization of flow rates and size of the problem (number of loops). However, it is important to recognize that Cross' algorithm was conditioned by the absence of computational resources. In addition, at the beginning of the last century, the main technical problem was to analyze the hydraulic system for planning and design purposes. All these circumstances motivated the coarse skeletonization of WDN models and the a priori assumption of fixed nodal demands as obtained from predicted customer water requests.

Since the 1960s, with the advent of the first computers, many researchers have proposed global linearization algorithms that were characterized by the simultaneous solution of all the mathematical problem equations (Martin and Peters 1963; Shamir and Howard 1968; Epp and Fowler 1970; Hamam and Brammeler 1971; Kesavan and Chandrashekar 1972; Wood and Charles 1972; Collins et al. 1978; Isaacs and Mills 1980; Wood and Rayes 1981; Carpentier et al. 1987; Todini and Pilati 1988). In fact, the computational resources permitted the solution of the nonlinear mathematical problem by iterations in which, at each iteration, a system of linear equations is solved. However, the nodal demands were still considered fixed a priori in the model, and relevant WDN modeling approach is designated as demand-driven. All the global linearization algorithms iteratively solve a system of linear equations whose size and type depend on the adopted strategy. In particular, Todini and Pilati (1988) developed the global gradient algorithm (GGA), which exhibits excellent convergence characteristics and is still used in the freely available software EPANET2 (Rossman 2000). GGA requires the iterative solution of a sparse symmetric system of linear equations whose size is the number of unknown nodal heads. Todini and Rossman (2013) have recently presented a review work, which unifies WDN algorithms in a common framework.

In the development of these WDN models, the work by Collins et al. (1978) is of crucial importance since they concluded that, under a condition of monotone formulation of head losses (also internal to pumps if any), the solution of the mathematical problem exists and is unique.

In the 1980s, the general availability of computational resources made it possible to consider WDN modeling for general management purposes. Thus, Bhave (1981) first considered the dependence of demands on system pressure. Later, Germanopoulos (1985) combined a leakage term with pressure-dependent customer-demands. Similar head-demand models were proposed soon after (Wagner et al. 1988; Reddy and Elango 1989; Chandapillai 1991). In particular, the generic pressure-demand model for controlled outlets suggested by Wagner et al. (1988) is considered the most feasible to predict WDN pressure-deficient conditions with respect to customer water requests (Gupta and Bhave 1996).

[1]Full Professor, Technical Univ. of Bari, Via Edoardo Orabona 4, 70125 Bari, Italy (corresponding author). E-mail: o.giustolisi@poliba.it

[2]Research Fellow, Technical Univ. of Bari, Via Edoardo Orabona 4, 70125 Bari, Italy. E-mail: l.berardi@poliba.it

[3]Assistant Professor, Technical Univ. of Bari, Via Edoardo Orabona 4, 70125 Bari, Italy. E-mail: d.laucelli@poliba.it

Note. This manuscript was submitted on November 24, 2012; approved on July 29, 2013; published online on July 31, 2013. Discussion period open until June 1, 2014; separate discussions must be submitted for individual papers. This paper is part of the ***Journal of Hydraulic Engineering***, Vol. 140, No. 1, January 1, 2014. © ASCE, ISSN 0733-9429/2014/1-89-104/$25.00.

WDN modeling with demands depending on pressure/head is termed *pressure-driven* and allows a pressure-driven analysis. Nonheuristic algorithms for WDN pressure-driven modeling started with Todini (2003), who accounted for pressure-demand relationships in GGA considering customer-demands, while Giustolisi et al. (2008b) accounted for the pressure-background leakages relationship at the pipe level. Ang and Jowitt (2006) proposed a different approach, based on introducing a set of artificial reservoirs into the network, for analyzing water distribution systems under pressure-deficient conditions. Recently, Giustolisi and Walski (2012) presented a comprehensive work on WDN modeling and demand components discussing the demand-driven versus pressure-driven analysis.

Actually, although Wagner's pressure-demand relationship for customer-demands is hydraulically consistent, it is not everywhere differentiable (Ackley et al. 2001); thus, several methods were developed to assure the differentiability of pressure-demand relationships (Tucciarelli et al. 1999; Tanyimboh et al. 2001; Tanyimboh and Templeman 2004). For the same reason, Giustolisi et al. (2008a, b) introduced an adaptive overrelaxation parameter to pressure-driven analysis within GGA, and Piller and van Zyl (2007, 2009) developed a pressure-driven WDN model using the *content* and *co-content* proposed by Collins et al. (1978).

In the later years, water utilities increasingly adopted models developed from geographic information systems and, for this reason, the WDN models have become more detailed in terms of the topological representation. This fact increases the model size, e.g., all-mains models containing tens or even hundreds of thousands of pipes and nodes are being commonly built. In addition, in order to support management decisions, the optimization of hydraulic systems requiring multiple model runs is becoming a common practice.

Today, the challenge of WDN modeling is, on the one hand, to increase the computational efficiency and robustness of solving algorithms, and on the other hand to enhance the accuracy of modeling control devices (e.g., Piller and Bremond 2001; Deuerlein et al. 2009; Giustolisi et al. 2012a), modeling demand (e.g., Giustolisi and Walski 2012), and modeling variable tank level (e.g., Van Zyl et al. 2006; Todini 2011; Giustolisi et al. 2012b; Avesani et al. 2012).

In terms of the computational efficiency, Giustolisi and Todini (2009) enhanced the GGA (EGGA) in order to account for the evenly distributed demand along pipes. Later, Giustolisi (2010), Giustolisi et al. (2012c), and Giustolisi and Laucelli (2011) further extended EGGA to account for any pattern of connection to properties along pipes. Consequently, EGGA allows removing serial nodes/pipe-sections from the topological representation of the original network model, providing a significant improvement of computational efficiency, while preserving energy and mass balance equations at all original hydraulic elements and not forfeiting the hydraulic accuracy of the model.

Finally, Giustolisi et al. (2012b) generalized the steady-state WDN modeling accounting for variable tank levels in order to increase model stability by performing the mass balance at tank nodes inside the steady-state runs, instead of updating tank levels after each hydraulic snapshot.

This work proposes the demand modeling accounting for local storages, i.e., inline tanks, actually supplying water to customers (by gravity or pumping systems, according to the position of the water storage, on the roof or in the basement, respectively). Such local water storage volumes are actually very common in those countries (e.g., in the Mediterranean area) where water supply is intermittent and not reliable. In addition, there are some regions (e.g., in southern Italy) where private water distribution systems are traditionally fed by private inline water storages whose filling/emptying process is likely to affect the overall system behavior. Some examples are the filling process of inline tanks in intermittent distributions (Criminisi et al. 2009; De Marchis et al. 2011); i.e., the water is not continuously supplied over time, or after a pressure-deficient occurrence, and the prediction of the actual supplied customer-demands in a pressure-deficient condition accounting for the existence of inline tanks. An earlier study concerning the effects of these tanks on consumption patterns was presented. It emphasized also the high uncertainty behind the use of private storage tanks (Cobacho et al. 2008).

To date, few commercial WDN simulation packages are able to model tanks fed from the top by float valves. Nonetheless, none of them can reproduce the overall system functioning under pressure deficient conditions when tank is empty, and consequently the demand downstream of the tank cannot be fully satisfied. Moreover, in such pressure-deficient scenarios, the recent pressure-driven upgrades of the EPANET package, based on Wagner's model, is not able to correctly reproduce the behavior of inline tanks, as shown in this paper.

Consequently, the present contribution demonstrates the need for including inline tanks in WDN models in order to enhance the accuracy of WDN analyses. This, in turn, is aimed at achieving a more realistic modeling of real WDN for management purposes.

Formulation for Nodal Water Withdrawals of WDN Models Accounting for Inline Tanks

A single run of the steady-state WDN model presents a snapshot representing a real hydraulic system that is supposed to be in a steady-state condition over a time interval ΔT. This is an abstraction (Giustolisi and Walski 2012) holding under the assumption of slowly time-varying boundary conditions, such as demands, level of tanks, state of valves, and the working condition of the pumps (Todini 2011; Giustolisi et al. 2012b). Under these assumptions, the inertial and dynamic effects are neglected and are not included in the WDN model snapshot.

For this reason, consider the volume balance in a time step Δt for a tank filled by an orifice from the top and delivering a demand from the bottom, given by

$$\begin{array}{lll} V^{\text{in}}(t) = \int_t^{t+\Delta t} C(t)\sqrt{P(t) - \Delta z^{\text{orif}}}dt & \text{Inflow} \\ V^{\text{out}}(t) = \int_t^{t+\Delta t} d(t)dt & \text{Outflow} \end{array} \quad (1)$$

where $V^{\text{in}}(t)$ = inflow water volume at time t; $V^{\text{out}}(t)$ = outflow water volume at time t; $C(t)$ = outflow coefficient of the orifice filling the tank at time t; $P(t)$ = model pressure at time t in the service connection node on the main pipe; Δz^{orif} = difference in elevation between the orifice and the connection node and $d(t)$ = customer demand at time t. The term $P(t) - \Delta z^{\text{orif}}$ in the first line of Eq. (1) is the residual pressure at the orifice. It is preferred to keep explicit Δz^{orif} without defining the variable *residual pressure* in order to distinguish the WDN model variable, $P(t)$, from the orifice elevation. In the first line of Eq. (1), the head losses along the service pipe have been neglected, although it is always possible to account for the service pipe in WDN modeling.

The demand $d(t)$ in the second line of Eq. (1) is actually a stationary average value in Δt, consistently with steady-state modeling assumption (Giustolisi and Walski 2012); consequently, it is a constant value over Δt.

Then, assuming that the model pressure $P(t)$ is constant in Δt, Eq. (1) becomes

J. Hydraul. Eng. 2014.140:89-104.

$$\begin{aligned} &V^{\text{in}}(t)=\sqrt{P(t)-\Delta z^{\text{orif}}}\int_t^{t+\Delta t}C(t)dt \quad \text{Inflow}\\ &V^{\text{out}}(t)=d(t)\Delta t \quad \text{Outflow}\end{aligned} \tag{2}$$

Assuming a constant model pressure $P(t)$ in Δt means to neglect the change in the network flow rates and head losses due to the variation of the outflow coefficient $C(t)$ in Δt. Actually, $P(t)$ variation cannot be neglected if the local variation of the nodal outflow considerably influences the head losses in the hydraulic system. In this case, the time step of the simulation (Δt_s) needs to be lower than Δt to refine the analysis.

The mass balance at the tank allows computation of the final water volume, $V(t+\Delta t)$, for a given initial water volume, $V(t)$, inflow in the tank and outflow $d(t)$. In fact, it is easy to obtain

$$\begin{aligned} &V(t+\Delta t)-V(t)=V^{\text{in}}(t)-V^{\text{out}}(t)\\ &\Rightarrow V(t+\Delta t)=\sqrt{P(t)-\Delta z^{\text{orif}}}\int_t^{t+\Delta t}C(t)dt-d(t)\Delta t+V(t)\end{aligned} \tag{3}$$

Finally, modeling the inflow/outflow process of the tank requires considering the occurrence of a null water volume when the tank is empty and the closure of the orifice when the tank is completely filled. Hence

$$0\le V(t+\Delta t)\le V^{\max} \tag{4}$$

where $V^{\max}$ = maximum water volume of the tank.

In order to analyze the effect of inline tanks in WDN models, two different valve controls are considered in the following, namely the ON/OFF and the linear volume-controlled opening of the orifice.

Actually, the orifice feeding inline tanks are controlled by floating valves which follow non-linear behavior depending on the valve type. Nonetheless, this work is not aimed at discussing all possible valve behaviors but, rather, to demonstrate the need for accounting of inline tanks in WDN models. Thus, from a WDN modeling perspective, assuming a linear opening of the orifice with volume entails an average behavior of the valve which can be acceptable considering all other uncertainties surrounding modeling of real WDN [e.g., demand, pipe roughness, and inline tanks (Cobacho et al. 2008)]. Moreover, the linear assumption permits the achievement of significant results providing also an easily verifiable formulation.

Nevertheless, the interested reader is referred to Appendix where the main conclusions achieved under the linear assumption are demonstrated to hold also for any (also nonlinear) relationship between the outflow orifice coefficient and tank volume.

ON/OFF Control of the Orifice: Case of Constant C

In the case where the orifice of the tank has two states only, closed or fully open, i.e., an ON/OFF control is implemented, $C(t)$ is constant in Δt if $V^{\max}$ is not reached. Thus, the mass balance in Eq. (3) allows writing

$$\begin{aligned} &V(t+\Delta t)=d^{\text{fill}}(t)\Delta t-d(t)\Delta t+V(t)\\ &d^{\text{fill}}(t)=C^{\max}\sqrt{P(t)-\Delta z^{\text{orif}}}\end{aligned} \tag{5}$$

where $d^{\text{fill}}(t)$ = average flow rate filling the tank during Δt; and $C^{\max}$ = maximum outflow coefficient of the orifice.

However, Eq. (5) needs to be specified for the two cases related to the inequalities in Eq. (4): the tank becomes empty, $V(t+\Delta t)<0$, or full, $V(t+\Delta t)>V^{\max}$, and $V(t+\Delta t)$ needs to be assumed null or equal to $V^{\max}$, respectively.

In the case of a tank becoming empty during Δt, $V(t+\Delta t)$ computed by Eq. (5) is lower than zero, and the assumption $V(t+\Delta t)=0$ in the first of Eq. (5) allows that

$$d^{\text{act}}(t)\Delta t=d^{\text{fill}}(t)\Delta t+V(t)=C^{\max}\sqrt{P(t)-\Delta z^{\text{orif}}}\Delta t+V(t) \tag{6}$$

where $d^{\text{act}}(t)$ = actual average demand supplied to customers during Δt. Eq. (6) states that the tank completely empties during Δt, although the orifice is fully opened, because the inflow summed to the initial water volume is insufficient to satisfy the customer water requests.

In the case of a tank becoming completely full during Δt, $V(t+\Delta t)$ computed by Eq. (5) is greater than the maximum volume, and the assumption $V(t+\Delta t)=V^{\max}$ in the first line of Eq. (5) allows

$$\begin{aligned} &C^{\max}\sqrt{P(t)-\Delta z^{\text{orif}}}\ge d(t)+\frac{V^{\max}-V(t)}{\Delta t}\\ &\Rightarrow d^{\text{fill}}(t)=d(t)+\frac{V^{\max}-V(t)}{\Delta t}\end{aligned} \tag{7}$$

Thus, in the WDN modeling, it is possible to assume a fixed flow rate filling the tank over Δt given by Eq. (7), because $d^{\text{fill}}(t)$ is known. Actually, the orifice status changes during Δt, from opened to closed, thus two steady-states should be computed. However, it is possible to neglect the dynamic effect of the orifice closure and the changes which are eventually induced in the network status. This way, consistently with the steady-state approximation, the WDN model represents a snapshot of the hydraulic system assuming the average tank inflow, $d^{\text{fill}}(t)$, during Δt of Eq. (7).

Finally, it is possible to estimate Δt_x (time interval to completely fill the tank), using

$$\Delta t_x=\frac{V^{\max}-V(t)}{d^{\text{fill}}(t)-d(t)}\le\Delta t \tag{8}$$

Volume-Controlled Orifice: Case of Linear Variation of C with Water Volume

In the case where the orifice is controlled by the water volume (i.e., level) into the tank, the outflow coefficient varies in time, and the mass balance in Eq. (3) allows

$$\begin{aligned} &V(t+\Delta t)=d^{\text{fill}}(t)\Delta t-d(t)\Delta t+V(t)\\ &d^{\text{fill}}(t)=\frac{\sqrt{P(t)-\Delta z^{\text{orif}}}\int_t^{t+\Delta t}C(t)dt}{\Delta t}\end{aligned} \tag{9}$$

It is here assumed that the outflow coefficient is a linear decreasing function from $C^{\max}$ to 0 of the water volume into the tank from 0 to $V^{\max}$, hence

$$\begin{cases} C(V)=0 & V>V^{\max}\\ C(V)=\frac{C^{\max}}{V^{\max}}(V^{\max}-V) & 0<V\le V^{\max}\\ C(V)=C^{\max} & V\le 0\end{cases} \tag{10}$$

thus

J. Hydraul. Eng. 2014.140:89-104.

$$\int_t^{t+\Delta t} C(t)dt = \int_t^{t+\Delta t} \frac{C(V)}{dV/dt} dV = \frac{\Delta t}{[V(t+\Delta t) - V(t)]}$$
$$\times \int_{V(t)}^{V(t+\Delta t)} \frac{C^{\max}}{V^{\max}} (V^{\max} - V) dV$$
$$= [2V^{\max} - V(t+\Delta t) - V(t)] \frac{C^{\max}}{2V^{\max}} \Delta t \tag{11}$$

It is worth noting that in Eq. (11), a constant filling/emptying rate over Δt is assumed, i.e., $dV/dt = [V(t+\Delta t) - V(t)]/\Delta t$. This approximation makes the average value of C in Δt linear with volume instead of time. The effect of this approximation will be discussed in the first case study and in Appendix [for any function $C = C(V)$].

Accordingly, the value of $d^{\text{fill}}(t)$ in the second of Eq. (9) is

$$d^{\text{fill}}(t) = (2V^{\max} - V(t) - V(t+\Delta t)) \frac{C^{\max}}{2V^{\max}} \sqrt{P(t) - \Delta z^{\text{orif}}}$$
$$= \frac{2V^{\max} - V(t) - V(t+\Delta t)}{\Delta t^{\text{fill}}(t)}$$
$$\Delta t^{\text{fill}}(t) = \left(\frac{C^{\max}}{2V^{\max}} \sqrt{P(t) - \Delta z^{\text{orif}}} \right)^{-1} \tag{12}$$

where $\Delta t^{\text{fill}}(t)$ = filling time of the tank, from empty $[V(t) = 0]$ to full $[V(t+\Delta t) = V^{\max}]$, for $P(t)$ model pressure at time t and $d(t) = 0$.

The final water volume, $V(t+\Delta t)$, can be obtained from $d^{\text{fill}}(t)$ in Eq. (12) and the mass balance in the first of Eq. (9)

$$V(t+\Delta t) = \frac{(2V^{\max} - V(t)) \frac{\Delta t}{\Delta t^{\text{fill}}(t)} - d(t)\Delta t + V(t)}{1 + \frac{\Delta t}{\Delta t^{\text{fill}}(t)}} \tag{13}$$

Accordingly, using Eq. (13) in Eq. (12), the formulation of $d^{\text{fill}}(t)$ without the explicit dependence on $V(t+\Delta t)$ is obtained

$$d^{\text{fill}}(\Delta t^{\text{fill}}(t), d(t), V(t)) = \frac{2(V^{\max} - V(t)) + d(t)\Delta t}{\Delta t^{\text{fill}}(t) \left(1 + \frac{\Delta t}{\Delta t^{\text{fill}}(t)}\right)} \tag{14}$$

Similarly to previous section, let us consider the two inequalities in Eq. (4).

If the tank becomes completely empty, i.e., $V(t+\Delta t)$ computed by Eq. (13) is lower than zero, it is assumed $V(t+\Delta t) = 0$ in Eq. (12) and in the first of Eq. (9) to obtain

$$d^{\text{act}}(t)\Delta t = d^{\text{fill}}(t)\Delta t + V(t) = (2V^{\max} - V(t)) \frac{\Delta t}{\Delta t^{\text{fill}}(t)} + V(t) \tag{15}$$

In the case $V(t) = V^{\max}$, i.e., the tank is completely filled, there is a condition which cannot be maintained if $d(t)$ is not null. In fact, when the orifice starts opening, the outflow coefficient is infinitesimal and the instantaneous flow rate filling the tank is infinitesimal. Thus, the orifice needs to open and reach an equilibrium value of the water volume (V^{eq}) into the tank for which the inflow, through the outflow coefficient, equals the customer-demand $d(t)$.

According to Eq. (10), from Eq. (12) it is possible to write

$$\frac{C^{\max}}{V^{\max}} (V^{\max} - V^{eq}) \sqrt{P(t) - \Delta z^{\text{orif}}} = d(t)$$
$$\Rightarrow V^{eq}(t) = V^{\max} - \frac{d(t)\Delta t^{\text{fill}}(t)}{2} \tag{16}$$

where $V^{eq}(t)$ = equilibrium water volume at time t for which the flow rate filling the tank equals the outflow due to the customer-demand.

If $V(t+\Delta t)$, computed over the time step Δt, is greater than the maximum volume $V^{\max}$ (tank becoming full), Eq. (13) allows

$$(V^{\max} - V(t)) \left(\frac{\Delta t}{\Delta t^{\text{fill}}(t)} - 1 \right) - d(t)\Delta t \geq 0 \tag{17}$$

Eq. (17) states that if $\Delta t \leq \Delta t^{\text{fill}}(t)$ and $d(t) > 0$, the value $V^{\max}$ cannot be reached. Nonetheless, the condition $\Delta t \leq \Delta t^{\text{fill}}(t)$ is necessary to avoid volume oscillations around $V^{eq}(t)$ during the simulations in consequence of the approximation of constant dV/dt in Eq. (11), as will be shown and discussed in the first case study. Thus, the tank with a volume-controlled orifice delivering a demand cannot be completely filled.

Finally, it is worth noting that if $V(t) = V^{eq}(t)$, the value $V^{\max}$ cannot be reached, independently on the condition $\Delta t \leq \Delta t^{\text{fill}}(t)$.

Appendix demonstrates that such major conclusions hold also for any (also nonlinear) relationship $C = C(V)$.

Generalizing Steady-State WDN Models

This section explicates the introduction of the demand component accounting for the filling/emptying process of inline tanks in steady-state WDN modeling.

The hydraulic model of a WDN is based on the following two equations which represent the energy and mass conservation laws for the network pipes and nodes, respectively:

$$H_j - H_i + R_k Q_k |Q_k|^{n-1} = 0$$
$$\sum_s \pm Q_{is} - d_i(H) = 0 \tag{18}$$

The pipe energy balance system in Eq. (18) does not consider the existence of a pumping system or a minor head loss (Giustolisi et al. 2012a), without loss of generality for the present section.

In Eq. (18), H_i and H_j = unknown nodal heads at the upstream (ith) and downstream (jth) terminal nodes of the kth pipe according to the assumed positive direction for the unknown flow rate Q_k from node i to j; R_k = kth pipe hydraulic resistance; $d_i(H)$ = demand at the ith node and Q_{is} = flow rates of the s pipes joining in the ith node. The sign of Q_{is} depends on the assumed pipe positive direction for the sth pipe with respect to flow entering the node. The term $d_i(H)$ is a nodal demand which might be a sum of demand components defined by means of different pressure-demand relationships (Giustolisi and Walski 2012). In this general case, the model and analysis are named *pressure/head-driven* (Giustolisi and Walski 2012). In the particular case of fixed demands, the nodal demand is constant and formally moved to the right hand of the mass balance equation; the model and analysis are named demand-driven.

In the case where the ith node represents a tank (i.e., the nodal head is assumed to be known) the energy balance equation is modified by moving the known head, e.g., $H_0 = H_i$, to the right-hand side

$$H_j + R_k Q_k |Q_k|^{n-1} = H_0 \tag{19}$$

and the mass balance equation disappears in the model.

The model of a hydraulic network of n_p pipes, n_n demand nodes (i.e., internal nodes) and n_0 tank or reservoir nodes (i.e., known heads) is represented in a matrix form by

J. Hydraul. Eng. 2014.140:89-104.

$$\begin{aligned} \mathbf{A}_{pp}\mathbf{Q}_p + \mathbf{A}_{pn}\mathbf{H}_n &= -\mathbf{A}_{p0}\mathbf{H}_0 \\ \mathbf{A}_{np}\mathbf{Q}_p + \mathbf{d}_n(\mathbf{H}) &= \mathbf{0} \end{aligned} \tag{20}$$

where $\mathbf{Q}_p = [n_p, 1]$ column vector of unknown pipe flow rates; $\mathbf{H}_n = [n_n, 1]$ column vector of unknown nodal heads; $\mathbf{H}_0 = [n_0, 1]$ column vector of known nodal heads; $\mathbf{d}_n = [n_n, 1]$ column vector of demand components; $\mathbf{A}_{pn} = \mathbf{A}_{np}^T$ and $\mathbf{A}_{p0}$ = topological incidence sub-matrices of size $[n_p, n_n]$ and $[n_p, n_0]$, respectively; and $\mathbf{A}_{pp}\mathbf{Q}_p = [n_p, 1]$ column vector of the evenly distributed pipe head losses eventually containing the terms related to internal head loss of pump systems and minor losses. The GGA solution of the system in Eq. (20) is reported in several works (e.g., Todini 2003; Giustolisi and Walski 2012; Giustolisi et al. 2012a) and can be obtained by iteratively solving the following equations:

$$\begin{aligned} \mathbf{F}_n^{\text{iter}} &= (\mathbf{A}_{np}\mathbf{Q}_p^{\text{iter}} - \mathbf{C}_n) - \mathbf{A}_{np}(\mathbf{D}_{pp}^{\text{iter}})^{-1}(\mathbf{A}_{pp}^{\text{iter}}\mathbf{Q}_p^{\text{iter}} + \mathbf{A}_{p0}\mathbf{H}_0) \\ \mathbf{H}_n^{\text{iter}+1} &= [\mathbf{A}_{np}(\mathbf{D}_{pp}^{\text{iter}})^{-1}\mathbf{A}_{pn} + \mathbf{D}_{nn}^{\text{iter}}]^{-1}\mathbf{F}_n^{\text{iter}} \\ \mathbf{Q}_p^{\text{iter}+1} &= \mathbf{Q}_p^{\text{iter}} - (\mathbf{D}_{pp}^{\text{iter}})^{-1}(\mathbf{A}_{pp}^{\text{iter}}\mathbf{Q}_p^{\text{iter}} + \mathbf{A}_{p0}\mathbf{H}_0 + \mathbf{A}_{pn}\mathbf{H}_n^{\text{iter}+1}) \end{aligned} \tag{21}$$

$$\begin{aligned} &\mathbf{C}_n = \mathbf{d}_n \quad \text{and} \quad \mathbf{D}_{nn}^{\text{iter}} = \mathbf{0}_{nn} && \text{for demand-driven analysis} \\ &\mathbf{C}_n = [\mathbf{d}_n(\mathbf{H})]^{\text{iter}} - \mathbf{D}_{nn}^{\text{iter}}\mathbf{H}_n^{\text{iter}} && \text{for pressure-driven analysis} \end{aligned}$$

where iter is a counter of the iterative solving algorithm; $\mathbf{D}_{pp}$ is a diagonal matrix whose elements are the derivatives of the head loss components function (minor losses, internal losses of pump systems and uniformly distributed losses) with respect to pipe flows; and $\mathbf{D}_{nn}$ (different from zero in pressure-driven simulation only) is a diagonal matrix whose elements are the derivatives of each demand component (in $\mathbf{d}_n$) with respect to nodal pressures or heads.

Once $V(t+\Delta t)$ is computed at each iteration, e.g., by the first line of Eqs. (5) or (13), the demand component for inline-tanks during the iterations is given for the ON/OFF controlled orifices by

$$d^{\text{fill}}(t) = \begin{cases} d(t) + \frac{V^{\max} - V(t)}{\Delta t} & V(t+\Delta t) > V^{\max} \\ C^{\max}\sqrt{P(t) - \Delta z^{\text{orif}}} & V(t+\Delta t) \leq V^{\max} \end{cases} \tag{22}$$

and for the linear volume-controlled orifices by

$$d^{\text{fill}}(t) = \begin{cases} \dfrac{2(V^{\max} - V(t)) + d(t)\Delta t}{\Delta t^{\text{fill}}(t)\left(1 + \dfrac{\Delta t}{\Delta t^{\text{fill}}(t)}\right)} & 0 < V(t+\Delta t) \leq V^{\max} \\ \dfrac{2V^{\max} - V(t)}{\Delta t^{\text{fill}}(t)} & V(t+\Delta t) \leq 0 \end{cases} \tag{23}$$

It is worth noting that the condition of V greater than $V^{\max}$ is not reported in Eq. (23), since it does not occur as previously explained.

The derivative of d^{fill} with respect to P for $\mathbf{D}_{nn}$ can be easily achieved from Eqs. (22) and (23).

Finally, when $V(t+\Delta t) = 0$ at a given iteration iter of the GGA, the Eq. (13) for the prediction of $V(t+\Delta t) < 0$ at iter + 1 corresponds to evaluation of the sign of its numerator being the denominator positive, then Eq. (13) can be always used.

Demand-driven and Pressure-Deficient Analyses

The classic demand-driven analysis assumes fixed the customer-demands, $d(t)$, as given in steady-state WDN modeling when the hydraulic capacity of the system is sufficient to deliver the requested customer-demand at each node of the network (Giustolisi and Walski 2012).

In the case of ON/OFF controlled orifices of inline tanks, when the hydraulic system finds a tank completely full, $V(t) = V^{\max}$ and from Eq. (5) the following condition holds:

$$C^{\max}\sqrt{P(t) - \Delta z^{\text{orif}}} - d(t) \geq 0 \tag{24}$$

It is plausible to assume that the orifice closes and opens during Δt and the average inflow $d^{\text{fill}}(t)$ equals $d(t)$. In WDN modeling, a demand-driven analysis is then applicable, because $d^{\text{fill}}(t)$ is known from $d(t)$ and, consequently, can be fixed.

In the case of volume-controlled orifices, a demand-driven analysis is applicable when the tank maintains the equilibrium water volume, $V^{eq}(t)$, because only in this case $d^{\text{fill}}(t) = d(t)$. However, Eq. (16) states that this condition requires a constant $d(t)$ and $\Delta t^{\text{fill}}(t)$, i.e., a constant pressure value $P(t)$ at the orifice. In the most general case of varying $d(t)$ and $\Delta t^{\text{fill}}(t)$ over time, e.g., in an extended period simulation (EPS), $V^{eq}(t)$ varies among the steady-state modeling snapshots and this fact causes a filling/emptying process, which does not allow demand-driven analysis, as will be shown and discussed in the second case study. As a consequence, demand-driven analysis is not actually applicable and pressure-driven analysis (PDA) needs to be used.

In case the required customer-demands cannot be satisfied [i.e., the tank is empty, $V(t+\Delta t) = 0$], in the classic WDN modeling pressure-deficient analysis (Giustolisi and Walski 2012), Eqs. (6) (for ON/OFF control) and (15) (for linear volume-controlled orifices) allow

$$\begin{aligned} d^{\text{act}}(t) &= C^{\max}\sqrt{P(t) - \Delta z^{\text{orif}}} + \frac{V(t)}{\Delta t} && \text{(ON/OFF orifices)} \\ d^{\text{act}}(t) &= \frac{2V^{\max} - V(t)}{\Delta t^{\text{fill}}(t)} + \frac{V(t)}{\Delta t} && \text{(volume controlled orifices)} \end{aligned} \tag{25}$$

Therefore, the formulation of actually supplied customer-demands in a pressure-deficient condition accounting for inline tanks depends on $\Delta t^{\text{fill}}(t)$ [i.e., $P(t)$, $V^{\max}$, and $V(t)$]. In addition, it is very different from the Wagner's model (Wagner et al. 1988), which is usually adopted for modeling WDN under pressure deficient conditions, as will be shown in the second and third case study. Such difference is due to the mass balance at inline tanks, which is completely neglected in Wagner's model.

Case Study: Filling Process and Selection of a Feasible Simulation Step Size

The simple schematic system in Fig. 1 is used here. The values of $\Delta z^{\text{orif}} = 0$ and $V(t=0) = 0$ are assumed, and the filling process considering both ON/OFF and linear volume-controlled functioning of the orifice are simulated.

The parameters of the simulation are the following: $\Delta t = 480$ min; $d = 25$ L/s, and $V^{\max} = 45$ m^3, i.e., the maximum water storage meets one half hour of the customer-demand request. Such values reflect multiple connections to private properties which are lumped at a single node, as usually assumed in WDN modeling practices; nonetheless, the same conclusions can be obtained considering a single household connection.

In addition, the value of $C^{\max}$ is assumed equal to $d/H_0^{1/2}$ ($= 0.00456$ m$^{5/2}$/s) and $2d/H_0^{1/2}$ ($= 0.00912$ m$^{5/2}$/s) in the case of ON/OFF and volume-controlled orifice, respectively, where $H_0 = 30$ m is the level of the reservoir. The value of $C^{\max}$ was doubled in the case of volume-controlled orifice in order to have

J. Hydraul. Eng. 2014.140:89-104.

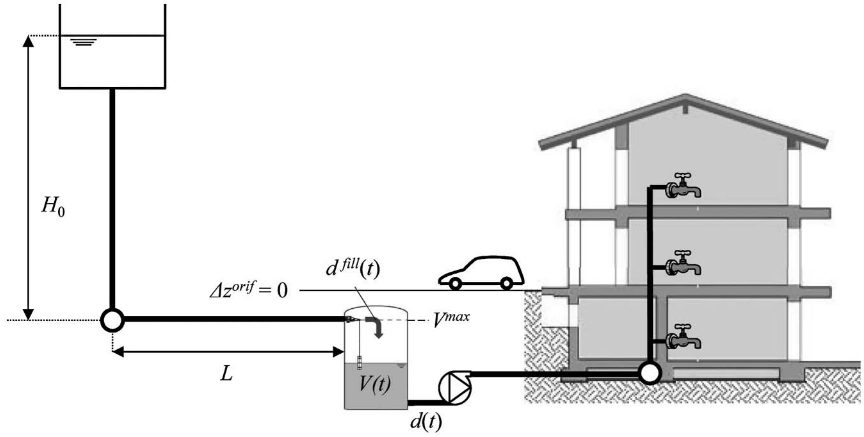

Fig. 1. Hydraulic scheme used for the cases of study

the same average coefficient of the ON/OFF case considering its variation with the volume.

The pipe hydraulic resistance per unit length K is assumed equal to 250 s^2/m^6 [i.e., in Eqs. (18) and (19), $R = K \cdot L$ using the Darcy-Weisbach formula: $n = 2$] and the pipe length L equal to 0.01 m in order to consider negligible the service pipe head loss, i.e., less than 1 cm; consequently, $P(t) \approx H_0 = 30$ m, $\Delta t^{\mathrm{fill}} =$ 30 min and Eq. (16) gives $V^{eq} = 22.5$ m^3 (i.e., $V^{\max}/2$).

Fig. 2 shows the 8-h (480 min) simulation using two step sizes, $\Delta t_s = \{60, 15\}$ min, for both the type of orifices being modeled. The comparison between the low (15-min) and high (60-min) simulation step sizes for the case of the volume-controlled orifice evidences a different pattern of the filling process curve, although the equilibrium value V^{eq} (22.5 m^3) is asymptotically reached in both the cases. The different pattern is caused by the assumption of a constant dV/dt in Eq. (11).

Using $\Delta t_s = 60$ min, $V(\Delta t_s) = 30$ m^3 of the first simulation step is greater than V^{eq} ($= 22.5$ m^3). Therefore, the assumption of a constant dV/dt over Δt_s, as required to achieve Eq. (11), is too coarse with respect to the value of V^{eq}. In fact, the empty tank cannot be filled more than V^{eq}, thus, an anomalous oscillation occurs around it.

Using $\Delta t = 15$ min, $V(\Delta t_s) = 15$ m^3 of the simulation first step is lower than V^{eq}. Therefore, the simulation reproduces the filling process which asymptotically reaches the value of 22.5 m^3 without exceeding it.

The previous numerical experiment allows to state that, in general, $V(t + \Delta t_s) - V(t) \leq V^{eq}(t)$ is a condition to select a feasible simulation step size Δt_s in order to avoid the oscillation because of the approximation on dV/dt in Eq. (11).

Eqs. (13) and (16) allow to achieve the feasible simulation step size, Δt_s, condition by assuming $V(t + \Delta t_s) - V(t) \leq V^{eq}(t)$

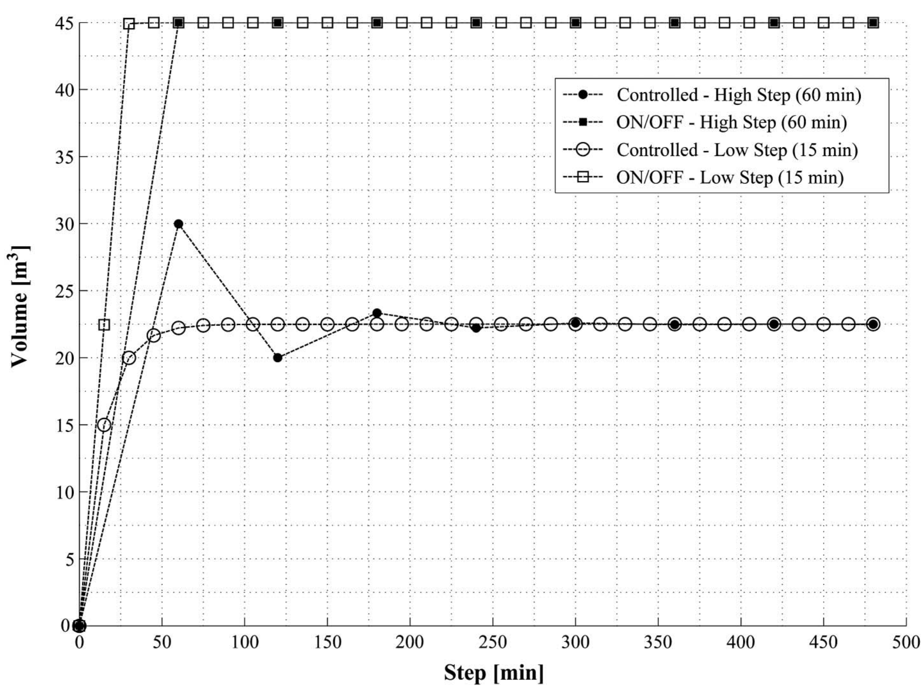

Fig. 2. Simulation of the filling process from an empty tank considering two step sizes (Δt_s)

J. Hydraul. Eng. 2014.140:89-104.

$$\Delta t_s \leq \frac{V^{eq}(t) - V(t)}{\frac{2V^{\max} - V(t)}{\Delta t^{\text{fill}}(t)} - d(t) - \frac{V^{eq}(t)}{\Delta t^{\text{fill}}(t)}} = \Delta t^{\text{fill}}(t) \tag{26}$$

Eq. (26) states that the feasible Δt_s depends on the velocity of filling process $[\Delta t^{\text{fill}}(t)]$, i.e., on the prior known characteristics of the tank (maximum water storage, outflow coefficient of the orifice, and its elevation) and $P(t)$, which could be maximized, e.g., considering the level of reservoirs. This is a relevant result in order to choose a feasible Δt_s in WDN modeling by minimizing $\Delta t^{\text{fill}}(t)$ in Eq. (26).

Case Study: EPS and Demand-Driven versus Pressure-Driven Analysis

The simple system in Fig. 1 is here used to perform an EPS using the pattern reported in Fig. 3 for required customer-demands $d(t)$ over time. Each of the 24 steady-state WDN simulations represents the hydraulic system snapshot in $\Delta t = 60$ min (Giustolisi and Walski 2012; Giustolisi et al. 2012b).

Four EPS have been performed considering both ON/OFF and a volume-controlled orifice functioning and starting from $V(t = 0) = 0$ and $V(t = 0) = V^{\max}$. In addition, a fifth EPS considers a pressure-deficient analysis using Wagner's (1988) model with the minimum pressure for a full service equal to 30 m and a null minimum pressure for any service (Giustolisi and Walski 2012).

The parameters are as reported in the previous section excluding $V^{\max}$, which is here set to 180 m^3, i.e., the maximum water storage meets 2 h of maximum required customer-demand. Consequently, as in the previous case study, the maximum $V^{eq}(t)$ is equal to 90 m^3 and the minimum $\Delta t^{\text{fill}}(t)$ is equal to 120 min; thus, the assumption $\Delta t_s = \Delta t = 60$ min satisfies the condition in Eq. (26). Finally, three service pipe lengths ($L = \{0.001, 125, 250\}$ m) have been tested in order to show three different hydraulic system behaviors with respect to inline tanks.

Case $L = 0.001$ m

In this case the service pipe head loss is negligible and the pressure $P(t)$ at the orifice is always equal to H_0 (= 30 m).

Fig. 4 shows the patterns of tank volumes which evidence a transient phase during which the two curves, corresponding to ON/OFF or volume-controlled cases respectively, are separate, and a successive phase where those curves coincide. This fact indicates that the initial condition of tanks determines only a transient during which the hydraulic system fills or empties the tank until an equilibrium level is reached, as is clear in the volume-controlled case.

In addition, Fig. 4 shows that in ON/OFF case, after a filling transient for the tank starting empty, the volume is always equal to $V^{\max}$. Furthermore, Fig. 5 shows that, after the transient, the inflow pattern of the tank $d^{\text{fill}}(t)$ follows the known required customer-demand pattern of Fig. 3. This fact means that, after the transient, a demand-driven analysis is applicable in ON/OFF case using the known demand pattern, $d(t)$, of Fig. 3 as fixed tank inflow. On the contrary, Fig. 4 shows that in the volume-controlled case a filling/emptying process occurs because the value $V^{eq}(t)$ varies over time. Thus, the inflow curves in Fig. 5 are different from those of Fig. 3; consequently; the demand-driven analysis cannot be performed because $d^{\text{fill}}(t)$ needs to be calculated by the model.

Finally, as the head losses are negligible the fifth EPS is trivial because the minimum pressure for a full service is always satisfied, i.e., the pressure-deficient condition does not occur.

Case $L = 125$ m

In this case the service pipe head loss is not negligible and the pressure $P(t)$ at the orifice is always lower than H_0 (30 m).

Similarly to the case of Fig. 4, Fig. 6 shows a transient phase of the two curves corresponding to the volume-controlled orifice. Fig. 6 also shows the ON/OFF orifice functioning the occurrence of a filling/emptying process. This is caused by the fact that the outflow coefficient $C^{\max}$ at $P(t) < 30$ m is insufficient to fulfill some of the required demand $d(t)$ over time (Fig. 3), and at that time the tank empties to deliver $d(t)$. This fact is also evident from Fig. 7 showing that the system delivers water to the tank with the maximum flow rate (close to 20 L/s) allowed by the hydraulic equilibrium.

Fig. 8 confirms that the required customer-demand is fully delivered in the case of inline tanks (the two curves of ON/OFF and controlled cases coincide), while also showing that this is not true in the case of the fifth simulation using Wagner's model, because $P(t)$ is always lower than the minimum pressure for a full service (30 m). This fact is of relevance because it demonstrates that, in

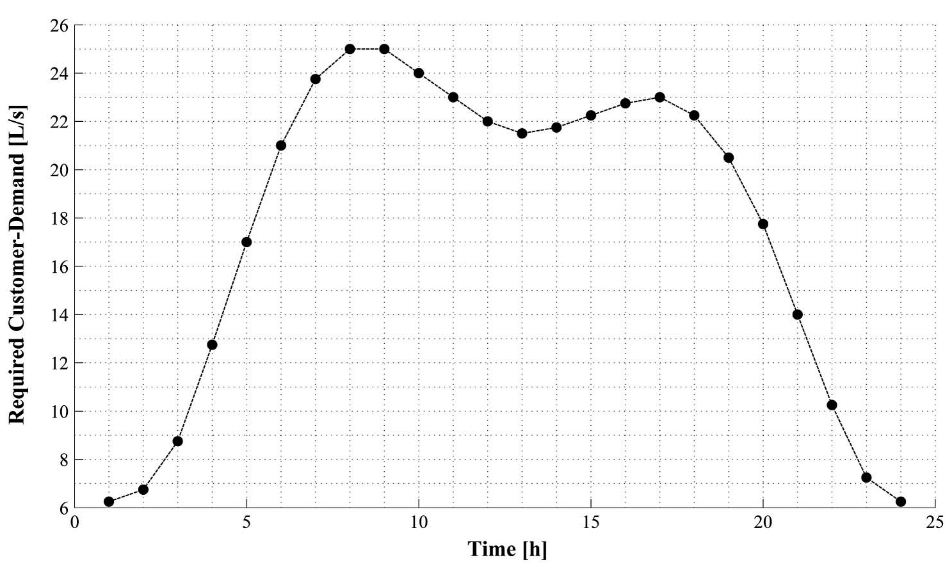

Fig. 3. Pattern of required customer-demands

J. Hydraul. Eng. 2014.140:89-104.

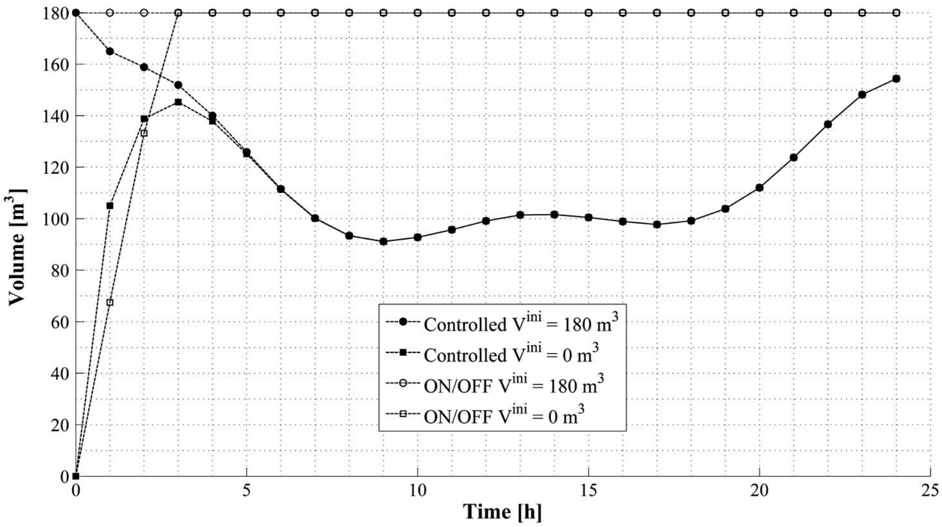

Fig. 4. Tank volumes: Case $L = 0.001$ m

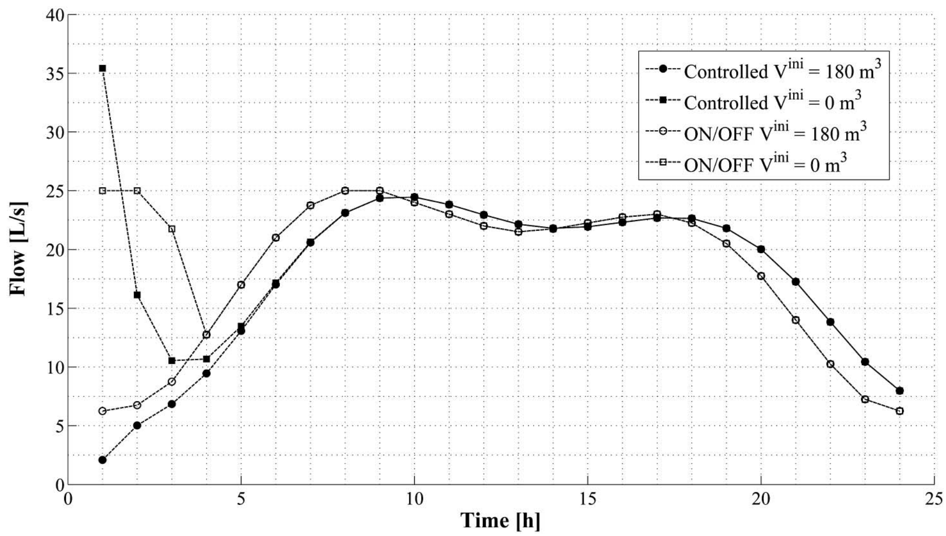

Fig. 5. Tank inflows (WDN model nodal demands): Case $L = 0.001$ m

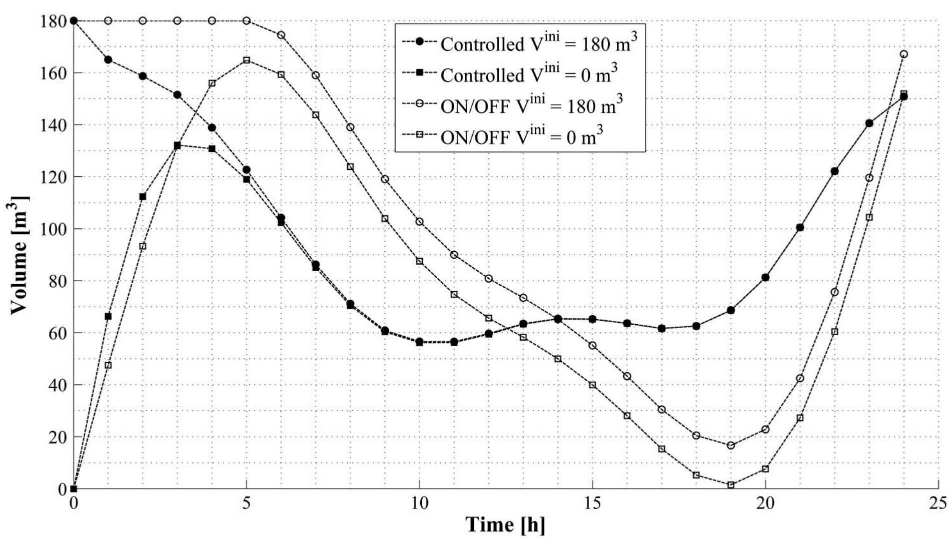

Fig. 6. Tank volumes: Case $L = 125$ m

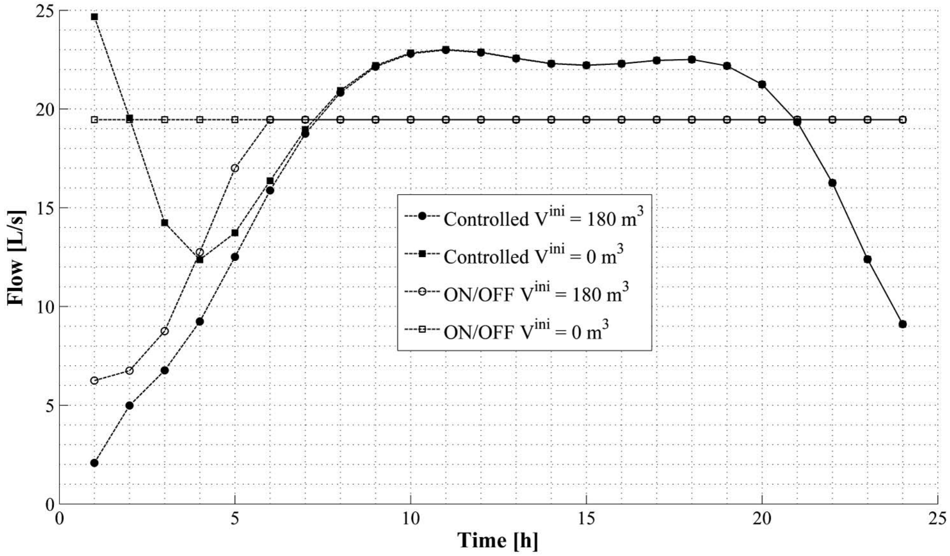

Fig. 7. Tank inflows (WDN model nodal demands): Case $L = 125$ m

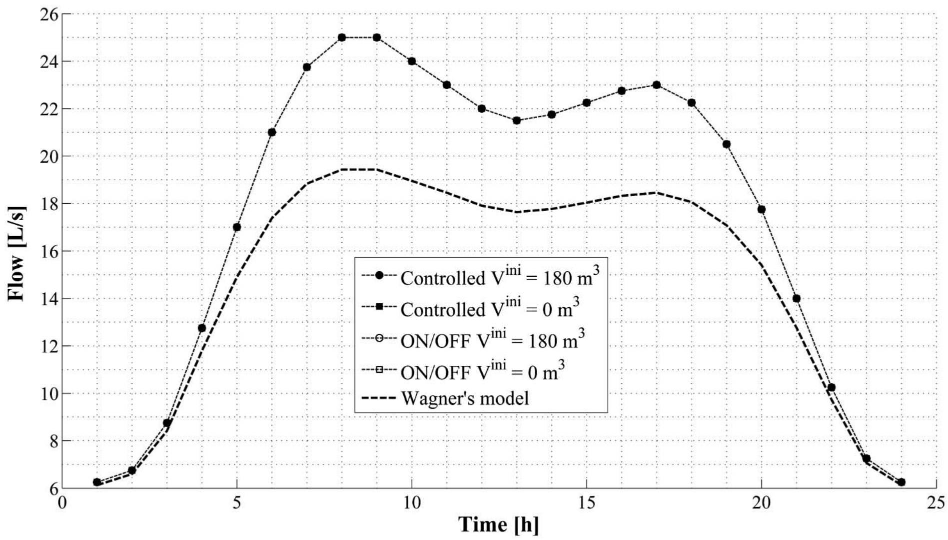

Fig. 8. Actually supplied customer-demands: Case $L = 125$ m (the two curves of ON/OFF and controlled cases coincide)

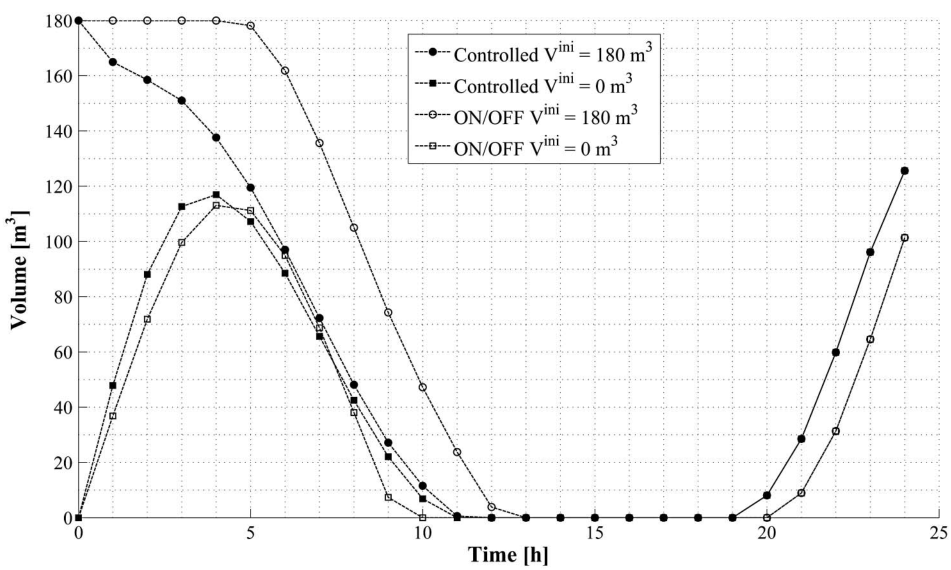

Fig. 9. Tank volumes: Case $L = 250$ m

J. Hydraul. Eng. 2014.140:89-104.

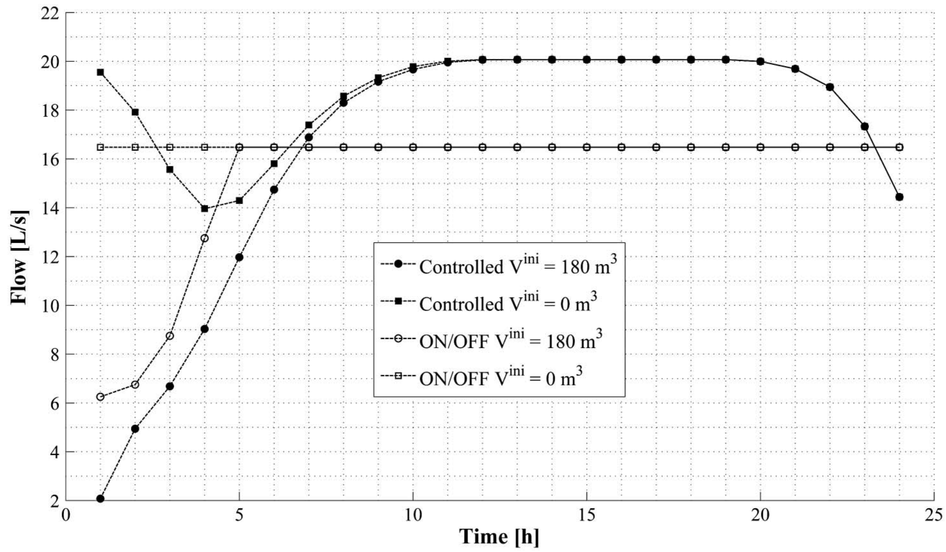

Fig. 10. Tank inflows (WDN model nodal demands): Case $L = 250$ m

pressure-deficient conditions, using Wagner's model instead of describing the filling/empting processes of inline-tanks leads to a wrong prediction of the actually supplied customer-demand.

Case $L = 250$ m

In this case, the service pipe head loss is doubled with respect to the previous section in order to generate a complete emptying of the tanks during EPS, as evidenced in Fig. 9.

Fig. 10 shows that for the volume-controlled orifice, a constant inflow occurs when the tank is empty. The constant value is greater than that corresponding to the ON/OFF orifice functioning because the maximum outflow coefficient is two times greater.

Finally, Fig. 11 shows the patterns of actually supplied customer-demands when the filling/emptying process, i.e., the tank water storage, is insufficient to assure the required customer demands at some hours. Those patterns significantly differ from that using Wagner's model, further confirming the need of properly modeling inline tanks in order to get hydraulically consistent results.

Case Study: EPS of a Real WDN with Inline Tanks

The WDN reported in Fig. 12 has been used to analyze the effect of modeling inline tanks over an EPS. The small size of such example network permits to analyze the behavior of each node without impairing the validity of results for larger systems. Table 1 reports network data in terms of pipe length and unit hydraulic resistance, node elevation ($\Delta z^{\mathrm{orif}} = 0$) and peak required customer-demands. For the sake of the example, it is assumed here that, for all nodes, the time pattern of required customer demands is that reported in Fig. 3, null minimum pressure for any service, 10 m pressure for correct water supply service, and total head at node 24 (i.e., reservoir) is 20 m. The latter conditions induces a pressure deficit in few nodes of the network, as exemplified below.

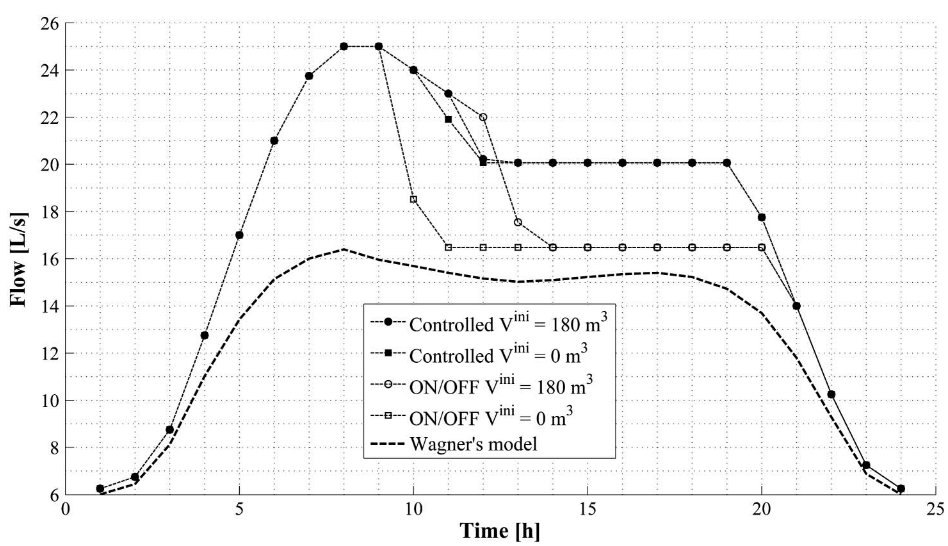

Fig. 11. Actually supplied customer-demands: Case $L = 250$ m

J. Hydraul. Eng. 2014.140:89-104.

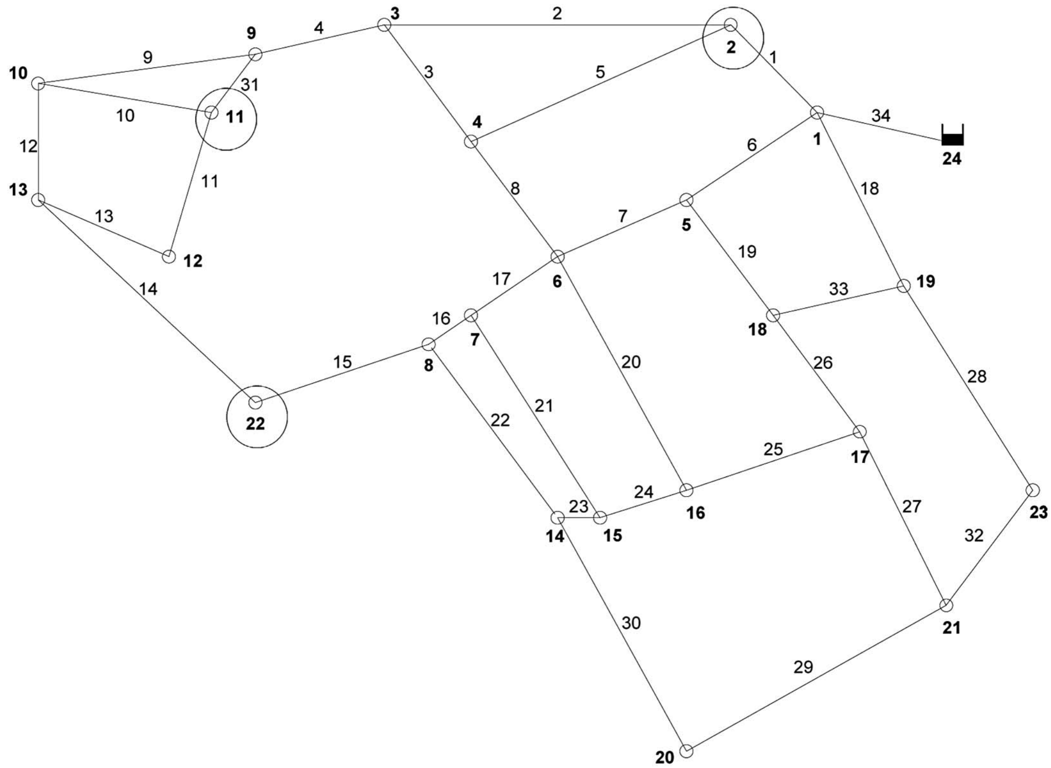

Fig. 12. Layout of Apulian network

Table 1. Data of Apulian Network

Pipe ID	L_k (m)	K_k (s^2/m^6)	Node ID	Z_i (m)	d_i (L/s)
1	348.5	0.460	1	6.4	8.72
2	955.7	0.867	2	7	13.64
3	483	265.147	3	6	11.92
4	400.7	0.867	4	8.4	11.40
5	791.9	265.147	5	7.4	8.08
6	404.4	0.247	6	9	12.24
7	390.6	0.460	7	9.1	7.32
8	482.3	265.147	8	9.5	8.44
9	934.4	265.147	9	8.4	9.76
10	431.3	9.882	10	10.5	11.68
11	513.1	265.147	11	9.6	7.20
12	428.4	9.882	12	11.7	6.04
13	419	265.147	13	12.3	12.16
14	1,023.1	265.147	14	10.6	10.88
15	455.1	18.565	15	10.1	7.40
16	182.6	0.867	16	9.5	8.96
17	221.3	0.867	17	10.2	9.16
18	583.9	18.565	18	9.6	8.64
19	452	3.068	19	9.1	11.76
20	794.7	265.147	20	13.9	10.68
21	717.7	265.147	21	11.1	11.72
22	655.6	1.639	22	11.4	9.6
23	165.5	265.147	23	10	8.28
24	252.1	265.147	24 (H)	20	0
25	331.5	265.147	—	—	—
26	500	5.629	—	—	—
27	579.9	18.565	—	—	—
28	842.8	265.147	—	—	—
29	792.6	265.147	—	—	—
30	846.3	9.882	—	—	—
31	164	1.639	—	—	—
32	427.9	265.147	—	—	—
33	379.2	265.147	—	—	—
34	158.2	0.247	—	—	—

It is also supposed that each customer-demand node is provided by a local inline tank which is fed by an orifice whose maximum outflow coefficient (i.e., when the tank is empty) is $C^{\max} = 0.00316\ \mathrm{m}^{5/2}/\mathrm{s}$ (i.e., delivering 10 L/s at 10-m nodal pressure).

For brevity, only the volume-controlled functioning of the orifice is analyzed in this case study. The following configurations are assumed: (1) the volume of all inline tanks is sufficient to provide 1 h of the maximum water requests; and (2) the volume of all inline tanks is sufficient to fulfill 2 h of the maximum water requests. For each configuration, three EPS are performed starting from: (1) all tanks empty, (2) all tanks full, and (3) all tanks containing a random percentage of the maximum volume. It is worth noting that the time step of analysis $\Delta t_s = 60$ min was verified to be always lower than $\Delta t^{\mathrm{fill}}(t)$ under all network configurations.

Fig. 13 shows the customer-demand supplied at nodes 2, 11, and 22 by assuming the two configurations of storage volumes, starting from all tanks empty; in addition, the demand supplied to customers by using Wagner's model are reported. Nodes 2, 11, and 22 have been selected to be representative of three different possible behaviors, with nodes 11 and 22 experiencing pressure deficient conditions.

Node 2 (circles in Fig. 13) is close to the reservoir and the pressure is always sufficient to provide the required customer-demand. Nonetheless, at peak hours the flow filling the tank is lower than the required demand, thus the tank empties and only a fraction of customers' request (i.e., as from mass balance in the tank) can be satisfied. Increasing the maximum storage (i.e., from 60 min to 120 min of maximum request) permits an increase in delivered flow from 6 h to 8 h.

Node 22 (triangles in Fig. 13) shows the same behavior as the tank (i.e., it empties during the peak hours), although the delivered demand is always larger than that returned by applying Wagner's

J. Hydraul. Eng. 2014.140:89-104.

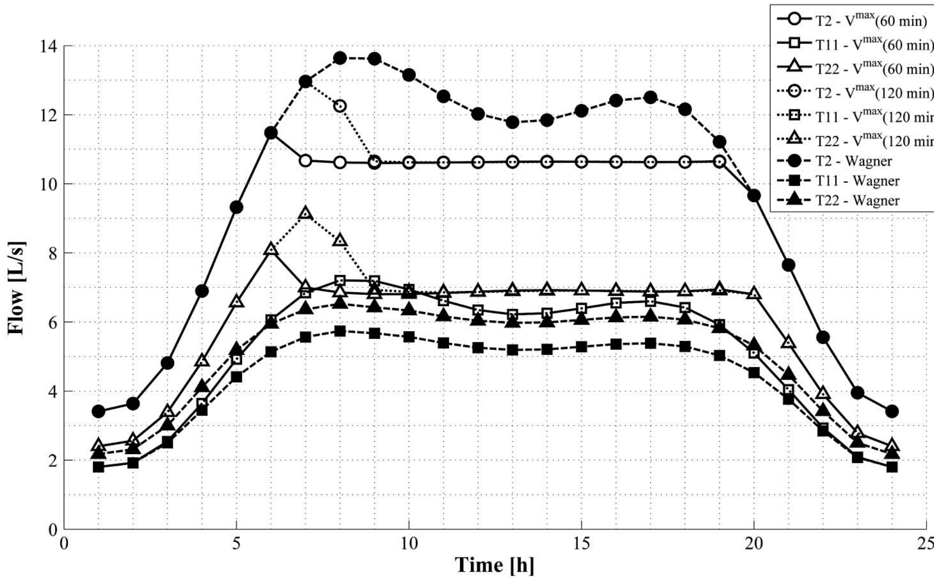

Fig. 13. Actually supplied customer-demands at nodes 2, 11, and 22; case $V(t = 0) = 0$

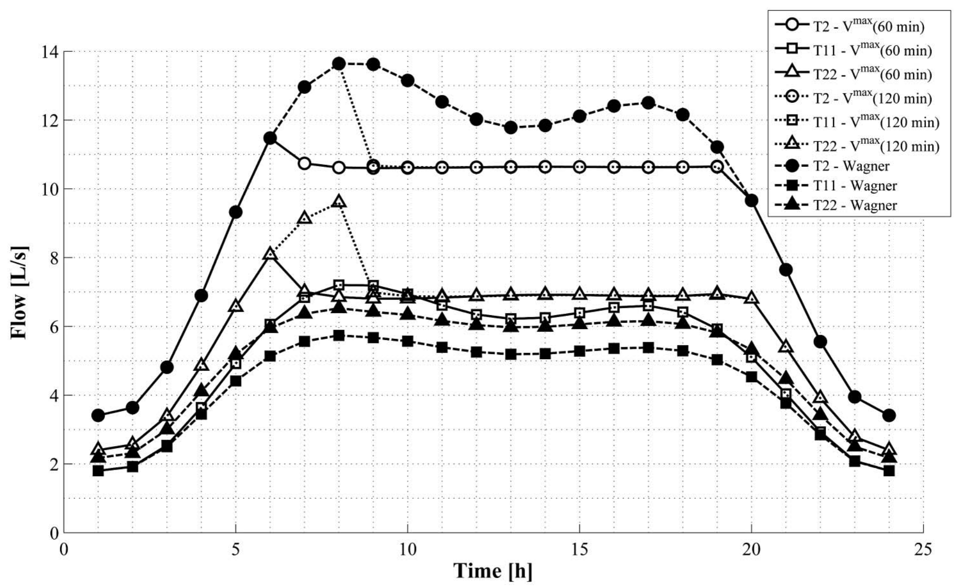

Fig. 14. Actually supplied customer-demands at nodes 2, 11, and 22; case $V(t = 0) = V^{max}$

model. This is due to the pressure regime induced by the inline tanks (i.e., volume–controlled orifices) through the network which is different from that using Wagner's model.

Finally, also Node 11 (i.e., squares in Fig. 13) delivers more water than Wagner's case but it is able to fulfill customers-demand without emptying (under both maximum storage configurations); this is due to joint effect of lower nodal demand and adequate pressure to fill up the inline tank.

Figs. 14 and 15 shows quite similar behaviors, although they are obtained starting from all full tanks and randomly filled tanks, respectively. The only difference is the maximum supplied customer-demands at peak hour, thus confirming that the initial filling conditions of tanks affects only a transient, as reported in the previous case study. Finally, Fig. 16 shows that accounting for inline tanks in WDN models results into quite different total water volume supplied at each node over the 24-h operating cycle, with respect to results achieved by using Wagner's model. Overall, using Wagner's model in the example network would result into a daily water volume which is on average about 5% and 7% lower than accounting for inline tanks, with 60 and 120 min maximum storage volumes, respectively. Thus, using Wagner's model when customers are fed by inline tanks may result into underestimating (e.g., at nodes 11 and 22) or overestimating (e.g., at nodes 2 and 6) actual supplying capacity of the network.

Conclusions

In many technical situations, the service pipes deliver water to local storage, inline tanks, from which required customer-demands are really supplied. The formulation of the specific nodal demand component (compulsory for steady-stated WDN modeling) accounting

J. Hydraul. Eng. 2014.140:89-104.

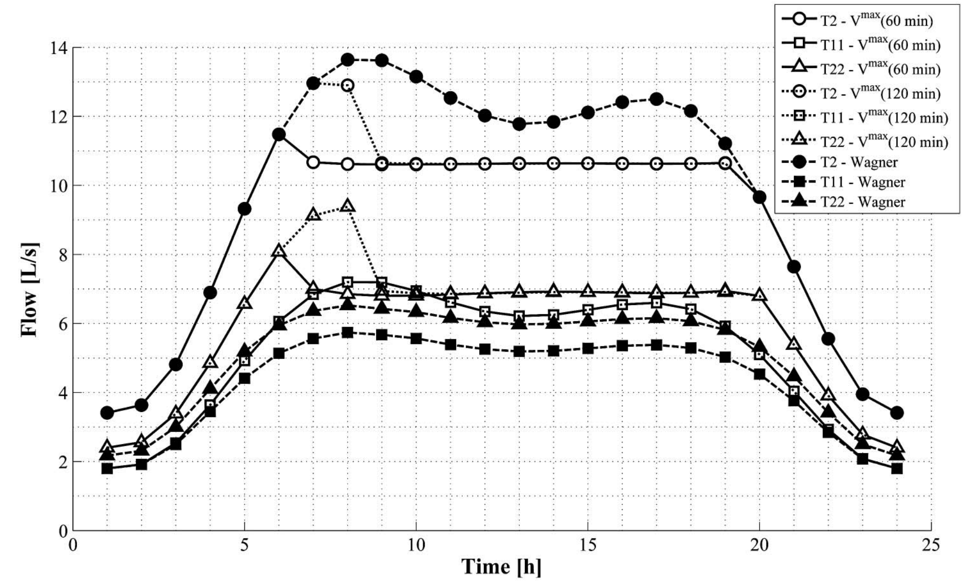

Fig. 15. Actually supplied customer-demands at nodes 2, 11, and 22; case $V(t=0)$ as a random fraction of $V^{\max}$

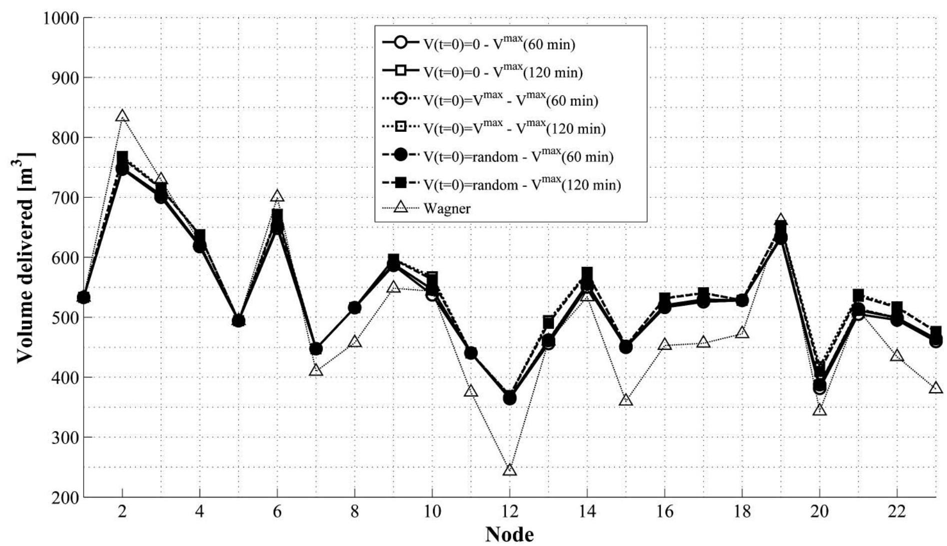

Fig. 16. Daily water volume supplied to customers at each node

for the existence of inline tanks between the hydraulic network and customers has been presented.

Two types of functioning of filling orifices on top of tanks have been considered: ON/OFF and linear volume-controlled. Nevertheless, the main conclusions are demonstrated to hold for any $C(V)$ relationship.

The achieved formulation and the two simple case studies demonstrate that demand-driven analysis is applicable to inline tanks whose orifice is characterized by an ON/OFF functioning as a special case when the local storage is fully filled and the hydraulic capacity of the network is sufficient to supply the required customer-demands.

However, pressure-driven analysis is mostly mandatory in order to model the filling/emptying process of tanks which determines the actual nodal withdrawals of the model.

Finally, the comparison between the proposed model and the Wagner's pressure-demand model for pressure-deficient conditions further demonstrates that modeling inline tanks is advisable for technicians to avoid misleading conclusions about actually supplied demand. In fact, Wagner's model does not account for mass balance at inline tanks, thus leading to noticeable inconsistencies with water volumes actually delivered to customers over the operating cycle.

Appendix. Validity of Relationships for Nonlinear $C(V)$

This appendix demonstrates the validity of the main results achieved by assuming a linear relationship $C = C(V)$ also in the case of any nonlinear $C(V)$.

Fig. 17 shows a generic relationship between the orifice outflow coefficient and tank volume as a decreasing continuous function $C = C(V)$ ranging from $C^{\max}$, for fully open orifice at $V = 0$, to $C = 0$ for closed orifice at $V = V^{\max}$.

J. Hydraul. Eng. 2014.140:89-104.

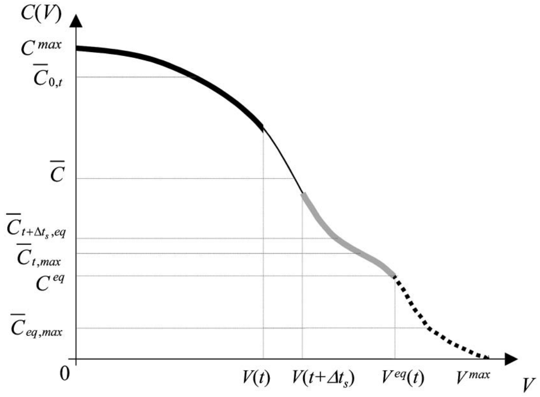

Fig. 17. Plot of nonlinear relationship $C(V)$ between the orifice outflow coefficient and inline tank volume

Once a function $C(V)$ is assigned, the second Eq. (9) permits the solution of the filling flow d^{fill} for any t by solving the integral of $C(V)$ between $V(t)$ and $V(t+\Delta t)$. Eq. (9) permits obtaining the time step Δt^{fill} to make the tank completely full $[V(t+\Delta t^{\text{fill}}) = V^{\max}]$ from empty $[V(t)=0]$, under nodal pressure $P(t)$ and $d(t)=0$

$$V^{\max} = \sqrt{P(t)-\Delta z^{\text{orif}}}\,\frac{\Delta t^{\text{fill}}}{V^{\max}}\int_0^{V^{\max}} C(V)dV$$
$$\Rightarrow \Delta t^{\text{fill}}(t) = \left(\frac{\bar{C}}{V^{\max}}\sqrt{P(t)-\Delta z^{\text{orif}}}\right)^{-1} \quad (27)$$

where $\bar{C}$ is the integral mean of $C(V)$ between $V=0$ (at $t=0$), and $V=V^{\max}$ (at $t=\Delta t^{\text{fill}}$) which can be computed before the simulation starts (it is also reported in Fig. 17).

Once $C=C(V)$ is assigned, the mass balance in Eq. (9) permits getting the volume $V(t+\Delta t)$, although it is also the upper bound of the integral, as reported in Eq. (28)

$$V(t+\Delta t) = \Delta t\frac{\sqrt{P(t)-\Delta z^{\text{orif}}}}{V(t+\Delta t)-V(t)} \times \int_{V(t)}^{V(t+\Delta t)} C(V)dV - d(t)\Delta t + V(t) \quad (28)$$

Considering that $0 \le V(t+\Delta t) \le V^{\max}$, as reported in Eq. (4), the following general results hold.

If the tank becomes empty over Δt [i.e., $V(t+\Delta t) < 0$] then $V(t+\Delta t) = 0$ is imposed and the first Eq. (9) and Eq. (27) obtains the supplied demand

$$d^{\text{act}}(t) = \frac{\sqrt{P(t)-\Delta z^{\text{orif}}}}{V(t)}\int_0^{V(t)} C(V)dV + \frac{V(t)}{\Delta t}$$
$$\Rightarrow d^{\text{act}}(t) = \frac{V^{\max}}{\Delta t^{\text{fill}}(t)}\frac{\bar{C}_{0,t}}{\bar{C}} + \frac{V(t)}{\Delta t} \quad (29)$$

where a constant filling/emptying rate is assumed over Δt, i.e., $dV/dt = [V(t+\Delta t)-V(t)]/\Delta t$ and $\bar{C}_{0,t}$ is the integral mean of $C(V)$ between $V=0$ and $V(t)$ (i.e., the curve portion in solid black in Fig. 17), which can be computed since $V(t)$ and function $C(V)$ are known at each steady-state simulation snapshot. As said about Eq. (25), Eq. (29) also confirms that d^{act} assuming inline tanks with any $C=C(V)$ is quite different from Wagner's model.

If the tank is completely full at time t [i.e., $V(t)=V^{\max}$], this condition cannot be actually maintained if $d(t)$ is not null and the equilibrium volume $V^{eq}(t)$ is such that

$$C^{eq}(t)\sqrt{P(t)-\Delta z^{\text{orif}}} = d(t) \Rightarrow C^{eq}(t) = d(t)\Delta t^{\text{fill}}(t)\frac{\bar{C}}{V^{\max}} \quad (30)$$

which depends on $d(t)$ and $P(t)$ [i.e., $\Delta t^{\text{fill}}(t)$ in Eq. (27)], as for Eq. (16), thus the equilibrium volume $V^{eq}(t)$ cannot be maintained in the general case of varying $P(t)$ and $d(t)$ among successive steady-state modeling snapshots (e.g., EPS), and demand-driven modeling is actually inapplicable.

If the tank becomes completely full over Δt [i.e., $V(t+\Delta t) > V^{\max}$] then $V(t+\Delta t) = V^{\max}$ is imposed and, using Eq. (27) and (28), the following condition is obtained:

$$\Delta t\frac{\sqrt{P(t)-\Delta z^{\text{orif}}}}{V^{\max}-V(t)}\int_{V(t)}^{V^{\max}} C(V)dV - d(t)\Delta t + V(t) \ge V^{\max}$$
$$\Rightarrow (V^{\max}-V(t))\left(\frac{V^{\max}}{V^{\max}-V(t)}\frac{\Delta t}{\Delta t^{\text{fill}}(t)}\frac{\bar{C}_{t,\max}}{\bar{C}} - 1\right)$$
$$- d(t)\Delta t \ge 0 \Rightarrow \Delta t > \underbrace{\frac{\bar{C}}{\bar{C}_{t,\max}}\frac{V^{\max}-V(t)}{V^{\max}}\Delta t^{\text{fill}}(t)}_{A(t)} \quad (31)$$

where $\bar{C}_{t,\max}$ is the integral mean of $C(V)$ between $V(t)$ and $V^{\max}$. Eq. (31) demonstrates that, if $d(t) > 0$, the value $V^{\max}$ could be reached only if $\Delta t > A(t)$, but such condition cannot be matched in order to avoid oscillation, as discussed in the following.

It is worth noting that if $V(t) = V^{eq}(t)$, the value $V^{\max}$ cannot be reached independently on the condition $\Delta t > A(t)$; in fact, considering Eq. (30), Eq. (31) provides

$$(V^{\max}-V^{eq}(t))\left(\frac{V^{\max}}{V^{\max}-V^{eq}(t)}\frac{\Delta t}{\Delta t^{\text{fill}}(t)}\frac{\bar{C}_{eq,\max}}{\bar{C}} - 1\right)$$
$$- d(t)\Delta t \ge 0 \Rightarrow d(t)\left(\frac{\bar{C}_{eq,\max}}{C^{eq}(t)} - 1\right)\Delta t - (V^{\max}-V^{eq}(t)) \ge 0 \quad (32)$$

which is impossible because $V^{\max} > V^{eq}(t)$ by definition, and the integral mean of $C(V)$ between V^{eq} and $V^{\max}$ (i.e., dotted line in Fig. 17) is $\bar{C}_{eq,\max} < C^{eq}$ for decreasing $C(V)$.

In addition, as reported in the first case study, a condition to not have model instabilities around $V^{eq}(t)$ during the filling process is obtained by imposing $V(t+\Delta t_s) - V(t) \le V^{eq}(t) \to V(t+\Delta t_s) - V^{eq}(t) \le V(t)$; Eq. (28) provides

$$V(t+\Delta t_s) - V^{eq}(t) = \Delta t_s\frac{\sqrt{P(t)-\Delta z^{\text{orif}}}}{V(t+\Delta t)-V^{eq}(t)} \times \int_{V^{eq}(t)}^{V(t+\Delta t_s)} C(V)dV - d(t)\Delta t_s \le V(t)$$
$$\Rightarrow \Delta t_s\bar{C}_{t+\Delta t_s,eq}\sqrt{P(t)-\Delta z^{\text{orif}}} - d(t)\Delta t_s \le V(t) \quad (33)$$

where $\bar{C}_{t+\Delta t,eq}$ is the integral mean of $C(V)$ between $V(t+\Delta t_s)$ and $V^{eq}(t)$ [i.e., the solid gray portion of curve $C(V)$ in Fig. 17]. Using Eq. (29), the above condition provides the time step Δt_s to avoid instabilities

J. Hydraul. Eng. 2014.140:89-104.

$$\Delta t_s \bar{C}_{t+\Delta t_s,eq} \frac{V^{\max}}{\bar{C}\Delta t^{\text{fill}}(t)} - d(t)\Delta t_s \leq V(t)$$
$$\Rightarrow \Delta t_s \leq \underbrace{\frac{\bar{C}}{\bar{C}_{t+\Delta t_s,eq} - C^{eq}(t)} \frac{V(t)}{V^{\max}} \Delta t^{\text{fill}}(t)}_{B(t)} \tag{34}$$

It can be easily demonstrated that $A(t) \geq B(t)$; in fact

$$\frac{\bar{C}}{\bar{C}_{t,\max}} \frac{V^{\max} - V(t)}{V^{\max}} \Delta t^{\text{fill}}(t) \geq \frac{\bar{C}}{\bar{C}_{t+\Delta t_s,eq} - C^{eq}(t)} \frac{V(t)}{V^{\max}} \Delta t^{\text{fill}}(t)$$
$$\Rightarrow \frac{V^{\max}}{V(t)} \geq \frac{\bar{C}_{t,\max} + \bar{C}_{t+\Delta t_s,eq} - C^{eq}(t)}{\bar{C}_{t+\Delta t_s,eq} - C^{eq}(t)} \geq 1 \tag{35}$$

which is true for any $V(t)$. Indeed, the last inequality holds since, during the filling process, $\bar{C}_{t+\Delta t,eq} - C^{eq}(t) \geq 0$, because the function $C(V)$ is decreasing and $V(t+\Delta t_s) \leq V^{eq}(t)$ in order to fill the tank without instabilities around $V^{eq}(t)$. Thus, in order to avoid model instabilities due to the assumption of constant filling/emptying rate dV/dt over Δt, the equation should be $\Delta t_s \leq B(t) \leq A(t)$ and, from Eq. (30), the tank cannot be completely filled since the condition $\Delta t > A(t)$ is not satisfied.

Notation

The following symbols are used in this paper:

$\mathbf{A}_{pn}$, $\mathbf{A}_{np}$, $\mathbf{A}_{p0}$ = topological incidence sub-matrices in the WDN model;
$\mathbf{A}_{pp}$ = diagonal matrix in the WDN model;
$\bar{\mathbf{A}}_{pn}$ = general topological matrix in the WDN model;
$\mathbf{B}_{pp}$ = diagonal matrix used in the GGA;
$C(t)$ = outflow coefficient of the orifice filling the tank at time t;
$C^{\max}$ = maximum outflow coefficient of the orifice filling the tank;
$\mathbf{D}_{nn}$ = derivative of pressure-driven demands with respect to $\mathbf{H}_n$;
$\mathbf{D}_{pp}$ = derivative of head losses with respect to $\mathbf{Q}_p$;
$\mathbf{d}_n$ = vector of nodal demand components in WDN model;
$d(t)$ = required customer-demand at time t (average in Δt);
$d^{\text{act}}(t)$ = actual demand supplied to customers at time t (average in Δt);
$d_i(H)$ = nodal demand at the ith node;
$d^{\text{fill}}(t)$ = flow rate filling the tank at time t (average in Δt);
$\mathbf{F}_n$ = temporary matrix used in the GGA;
$\mathbf{H}_n$ = vector of total network heads;
$\mathbf{H}_0$ = vector tank heads;
H_i = unknown head of the ith node;
K = unit pipe hydraulic resistance;
L = pipe length;
$P(t)$ = model pressure at time t in the node of the service connection on the pipe;
$\mathbf{Q}_p$ = vector of pipe flows/discharges;
Q_{is} = flow rates of the s pipes joining in the ith node;
Q_k = unknown flow rate of the kth pipe;
R_k = hydraulic resistance of the kth pipe;
$V(t)$ = initial water volume in the tank at time t;
$V(t+\Delta t)$ = final water volume in the tank after a time interval Δt
$V^{eq}(t)$ = water volume at time t for which inflow equals outflow of customer-demands;
$V^{\text{in}}(t)$ = inflow water volume at time t;
$V^{\max}$ = maximum water volume of the tank;
$V^{\text{out}}(t)$ = outflow water volume at time t;
Δt = time interval of the steady-state snapshot;
$\Delta t^{\text{fill}}(t)$ = filling time of the tank, from empty to filled, for a given $P(t)$ and $d(t) = 0$;
Δt_s = simulation step size;
Δt_x = time interval to completely fill the tank; and
Δz^{orif} = difference in elevation between the connection node and the orifice feeding the tank.

References

Ackley, J. R. L., Tanyimboh, T. T., Tahar, B., and Templeman, A. B. (2001). "Head-driven analysis of water distribution systems." *Proc., Computer and Control in Water Industry, Water software systems: theory and applications*, B. Ulanicki, ed., Research Studies Press, Baldock, U.K., 183–192.

Ang, W. K., and Jowitt, P. W. (2006). "Solution for water distribution systems under pressure-deficient conditions." *J. Water Resour. Plann. Manage.*, 10.1061/(ASCE)0733-9496(2006)132:3(175), 175–182.

Avesani, D., Righetti, M., Righetti, D., and Bertola, P. (2012). "The extension of EPANET source code to simulate unsteady flow in water distribution networks with variable head tanks." *J. Hydroinf.*, 14(4), 960–973.

Bhave, P. R. (1981). "Node flow analysis of water distribution systems." *J. Transp. Eng.*, 107(4), 457–467.

Carpentier, P., Cohen, G., and Hamam, Y. (1987). "Water network equilibrium, variational formulation and comparison of numerical algorithms." *Proc., Computer Application in Water Supply*, Wiley, New York.

Chandapillai, J. (1991). "Realistic simulation of water distribution system." *J. Transp. Eng.*, 10.1061/(ASCE)0733-947X(1991)117:2(258), 258–263.

Cobacho, R., Arregui, F., Cabrera, E., and Cabrera, E., Jr. (2008). "Private water storage tanks: Evaluating their inefficiencies." *Water Pract. Tech.*, 3(1).

Collins, M., Cooper, L., Helgason, R., Kenningston, J., and LeBlanc, L. (1978). "Solving the pipe network analysis problem using optimization techniques." *Manage. Sci.*, 24(7), 747–760.

Criminisi, A., Fontanazza, C. M., Freni, G., and La Loggia, G. (2009). "Evaluation of the apparent losses caused by water meter under-registration in intermittent water supply." *Water Sci. Technol.*, 60(9), 2373–2382.

Cross, H. (1936). "Analysis of flow in networks of conduits or conductors." *Bulletin No. 286*, Univ. of Illinois Engineering Experimental Station, Urbana, IL, 1–29.

De Marchis, M., Fontanazza, C. M., Freni, G., La Loggia, G., Napoli, E., and Notaro, V. (2011). "Analysis of the impact of intermittent distribution by modelling the network-filling process." *J. Hydroinf.*, 13(3), 358–373.

Deuerlein, J. W., Simpson, A. R., and Dempe, S. (2009). "Modeling the behavior of flow regulating devices in water distribution systems using constrained nonlinear programming." *J. Hydraul. Eng.*, 10.1061/(ASCE)HY.1943-7900.0000108, 970–982.

Epp, R., and Fowler, A. G. (1970). "Efficient code for steady-state flows in networks." *J. Hydr. Div.*, 96(1), 3–56.

Germanopoulos, G. (1985). "A technical note on the inclusion of pressure dependent demand and leakage terms in water supply network models." *Civ. Eng. Syst.*, 2(3), 171–179.

Giustolisi, O. (2010). "Considering actual pipe connections in WDN analysis." *J. Hydraul. Eng.*, 10.1061/(ASCE)HY.1943-7900.0000266, 889–900.

Giustolisi, O., Berardi, L., and Laucelli, D. (2012a). "Accounting for directional devices in WDN modeling." *J. Hydraul. Eng.*, 10.1061/(ASCE)HY.1943-7900.0000585, 858–869.

Giustolisi, O., Berardi, L., and Laucelli, D. (2012b). "Generalizing WDN simulation models to variable tank levels." *J. Hydroinf.*, 14(3), 562–573.

Giustolisi, O., Berardi, L., Laucelli, D., and Savic, D. A. (2012c). "A computationally efficient modeling method for large size water network

analysis." J. Hydraul. Eng., 10.1061/(ASCE)HY.1943-7900.0000517, 313–326.
Giustolisi, O., Kapelan, Z., and Savic, D. A. (2008a). "An algorithm for automatic detection of topological changes in water distribution networks." J. Hydraul. Eng., 10.1061/(ASCE)0733-9429(2008)134:4(435), 435–446.
Giustolisi, O., and Laucelli, D. (2011). "Water distribution network pressure-driven analysis using EGGA." J. Water Resour. Plann. Manage., 10.1061/(ASCE)WR.1943-5452.0000140, 498–510.
Giustolisi, O., Savic, D. A., and Kapelan, Z. (2008b). "Pressure-driven demand and leakage simulation for water distribution networks." J. Hydraul. Eng., 10.1061/(ASCE)0733-9429(2008)134:5(626), 626–635.
Giustolisi, O., and Todini, E. (2009). "Pipe hydraulic resistance correction in WDN analysis." Urban Water J., 6(1), 39–52.
Giustolisi, O., and Walski, T. M. (2012). "Demand components in water distribution network analysis." J. Water Resour. Plann. Manage., 10.1061/(ASCE)WR.1943-5452.0000187, 356–367.
Gupta, R., and Bhave, P. R. (1996). "Comparison of methods for predicting deficient network performance." J. Water Resour. Plann. Manage., 10.1061/(ASCE)0733-9496(1996)122:3(214), 214–217.
Hamam, Y. M., and Brammeler, A. (1971). "Hybrid method for the solution of piping networks." Proc., Inst. Electr. Eng., 118(11), 1607–1612.
Isaacs, L. T., and Mills, K. G. (1980). "Linear theory method for pipe network analysis." J. Hydr. Div., 106(7), 1191–1201.
Kesavan, H. K., and Chandrashekar, M. (1972). "Graph-theoretic models for pipe network analysis." J. Hydr. Div., 98(2), 345–364.
Martin, D. W., and Peters, G. (1963). "The application of Newton's method to network analysis by digital computers." J. Inst. Water Eng., 17(2), 115–129.
Piller, O., and Bremond, B. (2001). "Modeling of pressure regulating devices: A problem now solved." Proc., 3rd Annual Symp. on Water Distribution Systems Analysis, ASCE, Reston, VA.
Piller, O., and Van Zyl, J. E. (2007). "A unified framework for pressure-driven network analysis." Proc., Water Management Challenges in Global Change: Proc., Computer and Control in Water Industry (CCWI2007), B. Ulaniki, K. Vairavamoorthy and D. Butler, eds., Taylor and Francis, London, 25–30.
Piller, O., and Van Zyl, J. E. (2009). "Pressure-driven analysis of network sections via high-lying nodes." Proc., Computer and Control in Water Industry, J. Boxal and C. Maksimovic, eds., Taylor and Francis, London, 257–262.
Reddy, L. S., and Elango, K. (1989). "Analysis of water distribution networks with head dependant outlets." Civ. Eng. Syst., 6(3), 102–110.
Rossman, L. A. (2000). Epanet2 users manual, U.S. Environmental Protection Agency, Cincinnati, OH.
Shamir, U., and Howard, C. D. D. (1968). "Water distribution network analysis." J. Hydr. Div., 94(HY1), 219–234.
Tanyimboh, T. T., Tabesh, M., and Burrows, R. (2001). "Appraisal of source head methods for calculating reliability of water distribution networks." J. Water Resour. Plann. Manage., 10.1061/(ASCE)0733-9496(2001)127:4(206), 206–213.
Tanyimboh, T. T., and Templeman, A. B. (2004). "A new nodal outflow function for water distribution networks." Proc., 4th Int. Conf. on Engineering Computation Technology (CD-ROM), Civil-Comp Press, Stirling, U.K.
Todini, E. (2003). "A more realistic approach to the 'extended period simulation' of water distribution networks." Proc., Advances in Water Supply Management, C. Maksmovic, D. Butler and F. A. Memon, eds., A. A. Balkema Publishers, Lisse, Netherlands, 173–184.
Todini, E. (2011). "Extending the global gradient algorithm to unsteady flow extended period simulations of water distribution systems." J. Hydroinf., 13(2), 167–180.
Todini, E., and Pilati, S. (1988). "A gradient method for the solution of looped pipe networks." Proc., Computer Applications in Water Supply, Wiley, New York, 1–20.
Todini, E., and Rossman, L. (2013). "Unified Framework for Deriving Simultaneous Equation Algorithms for Water Distribution Networks." J. Hydraul. Eng., 139(5), 511–526.
Tucciarelli, T., Criminisi, A., and Termini, D. (1999). "Leak analysis in pipeline systems by means of optimal valve regulation." J. Hydraul. Eng., 10.1061/(ASCE)0733-9429(1999)125:3(277), 277–285.
Van Zyl, J., Savic, D. A., and Walters, G. A. (2006). "Explicit integration method for extended-period simulation of water distribution systems." J. Hydraul. Eng., 10.1061/(ASCE)0733-9429(2006)132:4(385), 385–392.
Wagner, J. M., Shamir, U., and Marks, D. H. (1988). "Water distribution reliability: simulation methods." J. Water Resour. Plann. Manage., 10.1061/(ASCE)0733-9496(1988)114:3(276), 276–294.
Wood, D. J., and Charles, C. O. A. (1972). "Hydraulic network analysis using linear theory." J. Hydr. Div., 98(7), 1157–1170.
Wood, D. J., and Rayes, A. G. (1981). "Reliability of algorithms for pipe network analysis." J. Hydr. Div., 107(10), 1145–1161.

J. Hydraul. Eng. 2014.140:89-104.