Graphical Abstract

Multi-period and Multi-product Batch Processing Time Maximization Problem

Tatiana Balbi Fraga

Highlights

Multi-period and Multi-product Batch Processing Time Maximization Problem

Tatiana Balbi Fraga

- ...;
- ...;
- ...;
- ....

Multi-period and Multi-product Batch Processing Time Maximization Problem

Tatiana Balbi Fraga[a,*]

[a]*Agreste Academic Center - Federal University of Pernambuco, Avenida Marielle Franco, Nova Caruaru, Caruaru, 55014-900, PE, Brazil*

Abstract

...

Keywords: multiproduct batch, processing time maximization, C++, LINGO

1. Introduction

T....

As scientific contributions,....

In the next sections, a presentation of problem is made along with an application example. Section 3 presents an interger linear mathematical model for the problem and section **??** presents an analytical method for its solution. Section 5 presents the tests and results obtained by a solver in C++ applying the analytical solution and a solver developed in LINGO to new proposed benchmarks. In sections 7 and 8, the contributions to this work are informed and acknowledgments are given, respectively. Finally, section 6 presents the paper's conclusions and suggestions for future works.

2. Multi-period and multi-product batch processing time maximization problem

In MPMPBPTM problem we consider a planning horizon with ND days, (ND $\geq$ 1), and a daily demand (UD$_{id}$ $\geq$ 0) during this period for each product of the batch. Additionally, there is already a prior production plan,

*corresponding author

Email address: tatiana.balbi@ufpe.br (Tatiana Balbi Fraga)

so that it is possible that there is already a previous production program for each product of the batch. Thus the availability for factory storage and shipping to outlets will be subject to change according to prior production planning. The problem is to determine a maximum processing time for the batch, taking into account the restrictions considered. For a better understanding of the problem, an example is presented below.

Example: A certain machine must process a batch containing 2 different products: A and B. The production rate of A is 60 g/min while the production rate of B is 40 g/min. The factory has free stock for a maximum of 3000 g of any product, and, according to the company's inventory policy, a maximum of 3000 g of product A and 2000 g of product B can be stocked at the factory. For the first day, there is a demand for 1000 g of product A and 500 g of product B, and for the second day, there is a demand for 500 g of product A and 300 g of product B. The factory has an outlet that has free space in stock of 1000 g, which can receive a maximum of 600 g of each product. There is a previous production planning for 1000 units of product A for the first day and 1500 units of product B for the second day. A maximum time of 100 minutes of this machine can be allocated for processing this batch. What is the maximum possible time for processing this batch ?

The following section presents a mathematical model for this problem.

3. Mathematical model

Given that:

UD_{id} is the demand for the product i on day d;
PO_{id} is the planned production of the product i on day d;
O is the maximum quantity allowed for shipment of all products to outlets;
UO_i is the maximum amount of product i that can be shipped to outlets;
p_i is the production rate of product i;
Z is the timeout for batch processing;

UI_{id} is the maximum quantity allowed for stocking in the factory the product i on day d;
being UI_{i0} the maximum quantity allowed for P_i;
I_d is the free factory storage of all products on the end of the day d;

being I_0 the initial free inventory for all products on factory on the start of the (*first*) planning day ($d = 1$);

P_i is the amount of product i to be produced on batch on the current date (first planning day);
D_{id} is the amount of product i delivered for the demand on day d;
O_{id} amount of product i shipped to factory outlets;
T is the batch processing time.

We have the problem:

$$max \quad T \tag{1}$$

s.t.

$$P_i - \mathrm{p}_i * T = 0 \quad \forall i \tag{2}$$

$$P_i + UI_{i1} - D_{i1} - O_{i1} = \mathrm{UI}_{i0} - \mathrm{PO}_{i1} \tag{3}$$

$$UI_{id} - UI_{i(d-1)} - D_{id} - O_{id} = -\mathrm{PO}_{id} \quad \forall i, \forall (d > 1) \tag{4}$$

$$I_1 + \sum_i \{P_i - D_{i1} - O_{i1}\} = \mathrm{I}_0 - \sum_i \mathrm{PO}_{i1} \tag{5}$$

$$I_d - I_{d-1} + \sum_i \{P_i - D_{id} - O_{id}\} = -\sum_i \mathrm{PO}_{id} \quad \forall (d > 1) \tag{6}$$

$$\sum_d O_{id} \leq \mathrm{UO}_i \quad \forall i \tag{7}$$

$$\sum_i \sum_d O_{id} \leq \mathrm{O} \tag{8}$$

$$D_{i1} \leq \mathrm{UD}_{i1} \quad \forall i \tag{9}$$

$$D_{id} \leq \sum_{k=1}^{d} \mathrm{UD}_{ik} - \sum_{k=1}^{d-1} D_{ik} \quad \forall i, \forall d > 1 \tag{10}$$

$$T \leq \mathrm{Z} \tag{11}$$

$$UI_{id}, I_d, O_{id}, D_{id} \in \mathbb{Z}^+ \quad \forall i, d \tag{12}$$

$$I_d \in \mathbb{Z}^+ \quad \forall d \tag{13}$$

where:

Constraints in (2) relate the quantity produced, P_i, to batch processing time T. Constraints in (3) calculate the quantity produced, P_i, as a function of the primary variables, D_i, O_i and I_i. Constraints in (4), (5), and (7) state that the quantity delivered to demand, the quantity shipped to the autlets, and the factory-stocked quantity of each product must be less than their respective known limits. Constraints (6) and (8) state that both the sum of product quantities sent to the autlets and the sum of product quantities stocked in the factory must be less than their respective maximum allowed values. The restriction in (9) establishes that there is a batch processing time limit, Z, that must be respected. And finally, the constraints in (10) inform the nature of the decision variables.

4. Exact optimizatin method

Uma solução ótima para este problema pode ser construída através de duas etapas, conforme explicado a seguir:

etapa 1: distribuição da produção previamente planejada

passo 1: Desconsidere a produção a ser planejada:

$$P_i = 0, \forall i \tag{14}$$

passo 2:
Calcule D_{i1} usando as equações em (15):

$$D_{i1} = \min\{\mathrm{UD}_{i1}, \mathrm{PO}_{i1}\} \quad \forall i \tag{15}$$

Calcule o valor da produção não ainda não alocada, $\mathrm{S}_{id}, \forall i$, pelas equações em (16):

$$\mathrm{S}_{i1} = \mathrm{PO}_{i1} - D_{i1}, \quad \forall i \tag{16}$$

Calcule os valores D^+_{id}, S^+_{id}, D^-_{id} e $\mathrm{S}^-_{id}, \forall i$, usando as equações de (17) à (21).

$$D^+_{i1} = D^-_{i1} = D_{i1} \tag{17}$$

$$D^+_{id} = \min\{(\sum_{k=1}^{d} \mathrm{UD}_{ik} - \sum_{k=1}^{d-1} D_{ik}), (\sum_{k=1}^{d-1} (\mathrm{PO}_{ik} - D^+_{ik}) + \mathrm{PO}_{id})\} \quad \forall i, \forall (d > 1) \tag{18}$$

$$\mathrm{S}^+_{id} = \mathrm{PO}_{id} - D^+_{id}, \quad \forall i, \forall (d > 1) \tag{19}$$

$$D^-_{id} = \min\{(\sum_{k=2}^{d} \mathrm{UD}_{ik} - \sum_{k=2}^{d-1} D_{ik}), (\sum_{k=1}^{d-1} (\mathrm{PO}_{ik} - D^-_{ik}) + \mathrm{PO}_{id})\} \quad \forall i, \forall (d > 1) \tag{20}$$

$$\mathrm{S}^-_{id} = \mathrm{PO}_{id} - D^-_{id}, \quad \forall i, \forall (d > 1) \tag{21}$$

Queremos encontrar D_{id}, $D^-_{id} \leq D_{id} \leq D^+_{id}, \forall i, (d > 1)$, tal que as restrições em (22) sejam respeitadas:

$$D_{id} = \min x \in \{D^-_{id}, D^+_{id} \quad | \quad UI_{id} \geq 0\} \quad \forall i, \forall (d > 1) \tag{22}$$

Contudo, para solução do problema, iremos adotar:

$$D_{id} = D^-_{id} \quad \forall i, \forall (d > 1) \tag{23}$$

$$\mathrm{S}_{id} = \mathrm{S}^-_{id} \quad \forall i, \forall (d > 1) \tag{24}$$

passo 3: Calcule os valores $O_{id}, \forall i, d$, usando as equações em (25):

$$O_{id} = \min\{\mathrm{S}_{id}, (\mathrm{UO}_i - \sum_{k=1}^{d-1} O_{ik}), (\mathrm{O} - \sum_{j=1}^{i-1} \sum_{k=1}^{d-1} O_{jk})\} \quad \forall i, d \tag{25}$$

passo 4:

calcule os valores para $UI_{id}, \forall i, d$, usando as equações em (3) e (4)).
calcule os valores para $I_i, \forall i$, usando as equações em (5) e (6).

Step 2: MPBPTM problem construction

passo 1: Calcular a quantidade limite para estocagem em fábrica para cada produto, UI_i.

$$\max \mathrm{UI}_i \tag{26}$$

sj. to:

$$\mathrm{UI}_i \leq UI_{i1} \tag{27}$$

$$\mathrm{UI}_i \leq UI_{id} + \sum_{k=2}^{d} \{\mathrm{UD}_{ik} - D_{ik}\} \tag{28}$$

Para compreender a solução proposta, vamos considerar um exemplo:
para 3 dias de planejamento, temos:
dia −¿ estoque disponível −¿ demanda não atendida no dia −¿ soma da demanda não atendida
d1 −¿ 40
d2 −¿ 20 −¿ 10 −¿ 10
d3 −¿ 15 −¿ 5 −¿ 15
d4 −¿ 10 −¿ 0 −¿ 15
considerando apenas o limite de estoque para o primeiro dia, a parcela da produção que pode ser estocada em fábrica é 40.

Contudo, esse estoque ficará em fábrica durante todo o período de planejamento considerado, exceto se houver saída para atender à demanda.

Assim, no segundo dia o estoque será igual ao estoque do primeiro dia menos o valor que sairá para atender à demanda acumulada para o dia.

o limite de recebimento de estoque do segundo dia será o limite de estoque mais a demanda não atendida.

Então das 40 unidades estocadas em fábrica no primeiro dia, dez unidades serão entregues à demanda e 30 unidades ficarão estocadas em fábrica.

Como o limite de estoque em fábrica para o segundo dia é de 20 unidades, então isso significa que há 10 unidades excedentes em estoque de fábrica. Ou seja, o estoque do primeiro dia, e portanto a produção devem ser diminuídos em 10 unidades.

Ou seja, temos um limite de estoque em fábrica para o primeiro dia deve ser de 30 unidades.

Então estocando 30 unidades em fábrica no primeiro dia, e no segundo dia, como 10 unidades serão entregues para atender à demanda, teremos 20 unidades estocadas.

No terceiro dia, mais 5 unidades serão entregues à demanda e o estoque ficará com 15 unidades.

Já no quato dia, nada será entregue a demanda de forma que teremos um estoque de 15 unidades. Contudo o limite de estocagem para o quarto dia é de 10 unidades, assim teremos 5 unidades excedentes. Ou seja, é necessário que o estoque do primeiro dia seja reduzido em 5 unidades.

A partir do raciocínio apresentado, podemos verificar a limitação do estoque em fábrica no primeiro dia deve levar em consideração às limitações de recebimento de estoque para cada dia seguinte, respeitando às retrições

$$\mathrm{UI}_i \leq UI_{i1} \tag{29}$$

$$\mathrm{UI}_i \leq UI_{id} + \sum_{k=2}^{d} \{\mathrm{UD}_{ik} - D_{ik}\} \tag{30}$$

Aqui temos, para o produto p:

$$RL_{i1} \leq 40 \tag{31}$$

$$RL_{i2} \leq 20 + 10, RL_{i2} \leq 30 \tag{32}$$

$$RL_{i3} \leq 15 + (10 + 5), RL_{i3} \leq 30 \tag{33}$$

$$RL_{i4} \leq 10 + (10 + 5), RL_{i4} \leq 25 \tag{34}$$

então temos que:

$$\mathrm{UI}_i = \min\{UI_{i1}, \min_d\{UI_{id} + \sum_{k=2}^{d} \{\mathrm{UD}_{ik} - D_{ik}\}\}\} \tag{35}$$

para o estoque total podemos utilizar o mesmo raciocínio, sendo que

$$\mathrm{I} = \min\{I_1, \min_d\{I_d + \sum_i \sum_{k=2}^{d} \{\mathrm{UD}_{ik} - D_{ik}\}\}\} \tag{36}$$

So, based on calculated values, we built the MPBPTM problem using the equations:

$$PL_i d = UI_{i(d)} - UD_i \quad \forall i \tag{37}$$

$$\mathrm{UD}_i = \mathrm{UD}_{i1} - D_{i1} \quad \forall i \tag{38}$$

$$\mathrm{I} = \min\{I_1, \min_d\{I_d + \sum_i \sum_{k=2}^{d} \{\mathrm{UD}_{ik} - D_{ik}\}\}\} \tag{39}$$

$$\mathrm{UI}_i = \min\{UI_{i1}, \min_d\{UI_{id} + \sum_{k=2}^{d} \{\mathrm{UD}_{ik} - D_{ik}\}\}\} \tag{40}$$

$$\mathrm{O} = \mathrm{O} - \sum_i \sum_d O_{id} \tag{41}$$

$$\mathrm{UO}_i = \mathrm{UO}_i - \sum_d O_{id}, \quad \forall i \tag{42}$$

Being p_i and Z defined according to the MPMPBPTM problem presented.

Obs: Note that some values of UI_{id} and I_d can be negative. This means that, in prior planning, some demand of the first day will be met by some previous planned production performed in a future period and that cannot be modified, i.e. $D_{id} > D^{-}_{id}$ for at least one ordained pair (i, d). In this case, considering that prior planning was a viable planning, the negative values of the UI_{id} and I_d will be offset by the positive surplus values of UD_i, so this will not result in change in the optimal solution of the problem through the method used in the next step.

The possible negative maximum values for variables are defined by the following equations. If this equations are not met, it means that initial planning was not a viable planning.

$$\max_{d}\{UI_{id}\} \geq \mathrm{UD}_{i1} - D_{i1}, \quad \forall i \tag{43}$$

$$\max_{d}\{I_{d}\} \geq \sum_{i}\{\mathrm{UD}_{i1} - D_{i1}\} \tag{44}$$

Step 3: MPBPTM solution by Fraga's analytical method

Once the MPBPTM problem is built, it can be solved by the analytical method proposed by Fraga (Fraga et al., 2023). This method can be resumed into two steps, as follows:

step 1: find T^*, where:

$$T^* = \lfloor \min\{\min_{\forall i}\{(\mathrm{UO}_i + \mathrm{UI}_i + \mathrm{UD}_i)/\mathrm{p}_i\}, (\mathrm{O} + \mathrm{I} + \sum_{i}\mathrm{UD}_i)/\sum_{i}\mathrm{p}_i\}\rfloor \tag{45}$$

step 2: calculate T, where:

$$T = \min\{T^*, \mathrm{Z}\} \tag{46}$$

Solution for the example presented before

Applying the method for solving the example presented in section 2, we have:

Como planejamento prévio de produção, temos:
$\mathrm{P}_{11} = 1000$, $\mathrm{P}_{12} = 0$, $\mathrm{P}_{21} = 0$, $\mathrm{P}_{22} = 1500$.

Como demanda, temos:
$\mathrm{UD}_{11} = 1000$, $\mathrm{UD}_{12} = 500$, $\mathrm{UD}_{21} = 500$, $\mathrm{UD}_{22} = 300$.

Distribuindo a produção para atender à demanda de acordo com as restrições em (9) e (10), temos:

$D_{11} = 1000$, $D_{12} = 0$, $D_{21} = 0$, $D_{22} = 300$.

Não há nenhuma sobra relativa a produção de A. E há uma sobra de produção de 1200 unidades de B no segundo dia.

Como estoque livre nos outlets, temos:
$\mathrm{UO}_1 = 600$; $\mathrm{UO}_2 = 600$; $\mathrm{O} = 1000$;

Assim, distribuindo a sobra da produção para os outlets de acordo com as restrições em (7) e (8), temos:
$O_{11} = 0$, $O_{12} = 0$, $O_{21} = 0$, $O_{22} = 600$.

Com relação ao estoque livre em fábrica, temos:
$\mathrm{UI}_{10} = 3000$, $\mathrm{UI}_{20} = 2000$, $\mathrm{I}_0 = 3000$.

Portanto, calculando os valores disponíveis para produção de acordo com as equações em (3), (4) e (5), temos:
$UI_{11} = 3000 - 1000 + 1000 + 0 = 3000$, $UI_{12} = 3000 - 0 + 0 + 0 = 3000$, $UI_{21} = 2000 - 0 + 0 + 0 = 2000$, $UI_{22} = 2000 - 1500 + 300 + 600 = 1400$.

$I_1 = 3000 - 1000 + 1000 + 0 = 3000$, $I_2 = 3000 - 1500 + 300 + 600 = 2400$.

Portanto temos agora as informações referentes aos limites estocagem em fábrica diário para cada produto assim como o limite de estocagem em fábrica diário para todos os produtos.

Definindo o problema MPBPTM, temos:

$\mathrm{UD}_1 = 1000 - 1000 = 0$, $\mathrm{UD}_2 = 0$.

$\mathrm{I} = \min_d\{3000, 2400\} = 2400$.

$\mathrm{UI}_1 = \min_d\{3000, 3000\} = 3000$, $\mathrm{UI}_2 = \min_d\{2000, 1400\} = 1400$.

$\mathrm{O} = 1000 - 600 = 400$.

$\mathrm{UO}_1 = 600 - 0 = 600$, $\mathrm{UO}_2 = 600 - 600 = 0$.

$p_1 = 60$, $p_2 = 40$.

$Z = 100$.

Então, aplicando o método de partição do tempo proposto pro Fraga, temos:

$T' = 0$ min

Then:

$S_A = 0$ g

$S_B = 0$ g

And so we have:

$T'' \leq \lfloor (3000 \text{ g} + 600 \text{ g} + 0 \text{ g})/(60 \text{ g/min}) \rfloor$

$T'' \leq 60$ min   for A

$T'' \leq \lfloor (1400 \text{ g} + 0 \text{ g} + 0 \text{ g})/(40 \text{ g/min}) \rfloor$

$T'' \leq 35$ min   for B

$T'' \leq \lfloor (400 \text{ g} + 2400 \text{ g} + 0 \text{ g})/(100 \text{ g/min}) \rfloor$

$T'' \leq 28$ min   for A and B

So that:

$T'' = 28$ min

So we have:

$T = 28$ min

As $T <$ Z, then $T = 28$ min will certainly be the optimal solution.

In this case we will have:

$P_A = 1,680$ g e $P_B = 1,120$ g

$D_A = 0$ g e $D_B = 0$ g

$E_A = 1,680$ g e $E_B = 1,120$ g

5. Tests and results

To test the developed model and analytical solution method, we created a solver in C++ and a solver in LINGO (https://github.com/tbfraga/COPSolver). The tests were performed on a notebook with an Intel i7 processor. We tested the solvers developed for the benchmarks presented on Tables 1, 2 and 3, and for randon benchmarks generated by the following functions:

$$\mathrm{p}_i = \mathrm{rand}()\%30 + 10 \tag{47}$$

$$\mathrm{UD}_i = \mathrm{rand}()\%3000 + 800; \tag{48}$$

$$\mathrm{seed1} = \mathrm{rand}()\%3000 + 500; \tag{49}$$

$$\mathrm{seed2} = \mathrm{rand}()\%5000 + 1000; \tag{50}$$

$$\mathrm{O} = \mathrm{N}/2 * \mathrm{seed1}; \tag{51}$$

$$\mathrm{UO}_i = \mathrm{rand}()\%(\mathrm{seed1} - 500) + 500; \tag{52}$$

$$\mathrm{I} = \mathrm{N}/2 * \mathrm{seed2}; \tag{53}$$

$$\mathrm{UI}_i = \mathrm{rand}()\%(\mathrm{seed2} - 1000) + 1000; \tag{54}$$

$$Z = 100; \tag{55}$$

i	1	2	total
p_i	60	40	
UD_i	1000	500	
UO_i	600	600	1000
UI_i	3000	2000	3000
		Z	100

Table 1: Benchmark MPBPTMP 001

i	1	2	3	total
p_i	60	40	50	
UD_i	1000	500	800	
UO_i	600	600	600	1500
UI_i	3000	2000	1000	3500
			Z	100

Table 2: Benchmark MPBPTMP 002

Randomly generated benchmarks were named RMPBPTMP N, being N the number of products. Seeking to enable the reproduction of the results, in the computational construction of the benchmarks we used the function srand((unsigned) source), where source is a defined value. To build the results presented in this work, we used source=0. The benchmarks used for the tests performed can be consulted at github.com/blinded.

Table 4 presents the results obtained by applying the analytical method (C++ solver) and the LINGO solver for the solution of the previously presented benchmarks.

It is important to point out that the LINGO solver was developed with the purpose of validating the results found by the proposed analytical method.

i	1	2	3	4	5	6	7	8	9	10	total
p_i	60	40	50	40	30	50	60	10	20	40	
UD_i	1000	500	800	500	400	500	2000	300	500	1000	
UO_i	600	600	600	1500	300	200	500	800	0	200	3000
UI_i	3000	2000	1000	800	3000	1000	400	300	200	0	5000
										Z	100

Table 3: Benchmark MPBPTMP 003

problem	LINGO solver	time (s)	analytical method	time (s)
MPBPTMP 1	55	0.04	55	0.001
MPBPTMP 2	48	0.06	48	0.001
MPBPTMP 3	30	0.09	30	0.001
RMPBPTMP 20	100	0.08	100	0.001
RMPBPTMP 50	98	0.12	98	0.001
RMPBPTMP 100	98	0.12	98	0.002
RMPBPTMP 1,000	78	3.10	78	0.002
RMPBPTMP 10,000	70	168.87	70	0.006

Table 4: Results obtained with the LINGO solver and the analytical method

As the analytical method is a polynomial time complexity method, we did not intend to compare computational costs, however it is possible to verify that both solvers are capable of finding optimal solutions for the proposed benchmarks very quickly, even for very large problems.

We have considered here a one-day period problem, so in a future work we will study the complexities of solving a multi-period problem and try to propose new solution methods if needed.

6. Conclusions and suggestions for future works

In this paper we presented the Multi-product Batch Processing Time Maximization problem, as well as a mathematical model and an analytical solution method for this problem. The mathematical model and analytical method were tested, respectively, by a solver developed with the LINGO software, from LINDO Systems, and with a solver developed in C++ language.

Optimum results were found for all proposed benchmarks, very quickly, even in the case of very large problems, which demonstrates the efficiency of the proposed method. As in this paper we considered a planning period of one day, in a future work we will study the complexities that arise when considering the same problem in a multi-period scenario. We will also verify if there is the possibility of extending the proposed analytical method to solve a class of linear integer programming problems with similar characteristics to the studied problem.

7. CRediT authorship contribution statement

T.B. Fraga: Conceptualization, Project administration, Supervision, Software, Methodology, Validation, Formal analysis, Writing – original draft, Writing – review & editing. Í.R.B. Aquino: Data curation.

8. Acknowledgments

We are enormously grateful to Coordenação de Aperfeiçoamento de Pessoal de Nível Superior (CAPES) and to Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq) for the financial support provided to our projects. We also thank LINDO systems team for the LINGO software license, without which this work would not have been possible and the to the owner of the company in the plastics sector, who allowed us to learn about his company's production process. Finally, we would like to thank Pró-reitoria de Extensão e Cultura da UFPE (PROExC) and the Research Director of Propesqi (Pró-reitoria de Pesquisa e Inovação da UFPE) for their support and recognition of our work, and dear Professors Antônio José da Silva Neto and João Flávio Vieira de Vasconcellos from IPRJ/UERJ, who contributed significantly to the formation of essential skills for the development of our projects.

References

Eilon. (1985). Multi-product batch production on a single machine - A problem revisited. *OMEGA Int. J. of Mgmt Sci.*, Vol. 13 (5), pp. 453–468.

Kim, M., Jung, J. H. and Lee, I. (1996). Intelligent scheduling and monitoring for multi-product networked batch processes. *Computers chem. Engn*, Vol. 20 (Suppl.), pp. 1149–1154.

Li, C., Wang, F., Gupta, J.N.D., Chung, T. (2022). Scheduling identical parallel batch processing machines involving incompatible families with different job sizes and capacity constraints. *Computers & Industrial Engineering*, Vol. 169, pp. 108115.

Liu, G., Li, F., Yang, X., and Qiu. S. (2020). The multi-stage multi-product batch-sizing problem in the steel industry. *Applied Mathematics and Computation*, Vol. 369, 124830.

Méndez, C.A., Henning, G.P., Cerdá, J. (2000). Optimal scheduling of batch plants satisfying multiple product orders with different due-dates. *Computers and Chemical Engineering*, Vol. 24, pp. 2223–2245.

OMEGA Journal. (1993). Single Machine Multi-product Batch Scheduling: Testing Several Solution Methods. *OMEGA Int. J. of Mgmt Sci.*, Vol. 21 (6), pp. 709–711.

Ravemark, D. E., and Rippin, D. W. T. (1998). Optimal design of a multi-product batch plant. *Computers chem. Engng*, Vol. 22 (1-2), pp. 177–183.

Shi, B., Qian, X., Sun, S., Yan, L. (2017). Rule-based scheduling of multi-stage multi-product batch plants with parallel units. *Chinese Journal of Chemical Engineering*, in press.