Graphical Abstract

Multiproduct Batch Processing Time Maximization Problem

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Highlights

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- Research highlight 1
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Abstract

Keywords: multiproduct batch, processing time maximization

1. Introduction

2. Multiproduct batch processing time maximization problem

The multiproduct batch processing time maximization problem arises when a set of different products are processed simultaneously in the same production batch. In this problem, it is considered that the quantity produced of each product is directly proportional to the processing time, however, with a different production rate (quantity/unit of time) for each product. In addition, there is a maximum quantity allowed for the production of batch products, defined both individually and for the set. The maximum production quantity of each product is defined according to the demand for the product. However, it is still possible to stock the products and/or send them to the outlets. In both cases, there is a stocking/shipping limit for each product and a stocking/shipping limit for the set of products in the batch. Also, there is a time limit available for processing the batch. The problem consists of defining the maximum processing time for the batch, respecting the limitations related to the quantities produced. For a better understanding of the problem, an example is presented below.

Example: A certain machine must process a batch containing 2 different products: A and B. The production rate of A is 60 g/min while the production rate of B is 40 g/min. The factory has free stock for a maximum of 3000 g of any product, and, according to the maximum stock allowed for each product, an additional 3000 g of product A and 2000 g of product B may be stocked at the factory. There is a demand for 1000 g of product A and 500 g of product B. The factory has an outlet that has free space in stock of 1000 g, which can receive a maximum of 600 g of each product. A maximum time of 100 minutes of this machine can be allocated for processing this batch. What is the maximum possible time for processing this batch ?

3. Mathematical model

Given that:

UD_i is the demand for the product i;

I is the maximum quantity allowed for additional factory storage of all products in the batch;

UI_i is the maximum quantity allowed for stocking the product i in the factory;

O is the maximum quantity allowed for shipment of all products to outlets;

UO_i is the maximum amount of product i that can be shipped to outlets;

p_i is the production rate of product i;

Z is the timeout for batch processing;

P_i is the amount of product i produced;

D_i is the amount of product i delivered for the demand;

O_i amount of product i shipped to factory outlets;

I_i is the amount of product i that will be stored at the factory;

T is the batch processing time.

We have the problem:

$$max \quad T \tag{1}$$

s.t.

$$P_i - \mathrm{p}_i * T = 0 \quad \forall i \tag{2}$$

$$P_i - D_i - O_i - I_i = 0 \quad \forall i \tag{3}$$

$$D_i \leq \mathrm{UD}_i \quad \forall i \tag{4}$$

$$O_i \leq \mathrm{UO}_i \quad \forall i \tag{5}$$

$$\sum_i O_i \leq \mathrm{O} \tag{6}$$

$$I_i \leq \mathrm{UI}_i \quad \forall i \tag{7}$$

$$\sum_i I_i \leq \mathrm{I} \tag{8}$$

$$T \leq \mathrm{Z} \tag{9}$$

$$D_i, O_i, I_i \in \mathbb{Z}^+ \quad \forall i \tag{10}$$

where:

Constraints in (2) relate the quantity produced, P_i, to batch processing time T. Constraints in (3) calculate the quantity produced, P_i, as a function of the primary variables, D_i, O_i and I_i. Constraints in (4), (5), and (7) state that the quantity delivered to demand, the quantity shipped to the autlets, and the factory-stocked quantity of each product must be less than their respective known limits. Constraints (6) and (8) state that both the sum of product quantities sent to the autlets and the sum of product quantities stored in the factory must be less than their respective maximum allowed values. The restriction in (9) establishes that there is a batch processing time limit, Z, that must be respected. And finally, the constraints in (10) inform the nature of the decision variables.

4. Analytical solution

It is possible to consider the factory stock and the outlets stock as single stock, so we have:

$$E_i = O_i + I_i \tag{11}$$

where E_i is the sum of the quantity stored at the factory and the quantity sent to the outlets of the product i.

So that:

$$E_i \leq \mathrm{UO}_i + \mathrm{UI}_i \tag{12}$$

and

$$\sum_i E_i \leq \mathrm{O} + \mathrm{I} \tag{13}$$

It is also possible to split the batch processing time into two time slots:

$$T = T' + T'' \tag{14}$$

and consider that T' is the maximum processing time used only for production that will meet the demand. Thus, we can find T', solving the reduced problem:

$$max \quad T' \tag{15}$$

s.t.

$$D_i - \mathrm{p}_i * T' = 0 \quad \forall i \tag{16}$$

$$D_i \leq \mathrm{UD}_i \quad \forall i \tag{17}$$

$$D_i \in \mathbb{Z}^+ \quad \forall i \tag{18}$$

This reduced problem can be rewritten in the form:

$$max \quad T' \tag{19}$$

s.t.

$$T' \leq \mathrm{UD}_i/\mathrm{p}_i \quad \forall i \tag{20}$$

So we have that T' will be the smallest of the ratios $\mathrm{UD}_i/\mathrm{p}_i$ of all products.

Once we find the value of T', we can calculate the value of unmet demand for each product after T', S_i, through the equations in (22).

$$S_i = \mathrm{UD}_i - \mathrm{p}_i * T' \quad \forall i \tag{21}$$

Now we consider that the time interval T'' will be used for the production of the quantities that will be stored (in the factory and in the outlets), as well as of the demand not met by the production in the first time interval, S_i.

In this case, we can find T''' by solving the second reduced problem:

$$max \quad T'' \tag{22}$$

s.t.

$$E_i - \mathrm{p}_i * T'' = 0 \quad \forall i \tag{23}$$

$$E_i - S_i \leq \mathrm{UO}_i + \mathrm{UI}_i \quad \forall i \tag{24}$$

$$\sum_i E_i - S_i \leq \mathrm{O} + \mathrm{I} \tag{25}$$

$$E_i \in \mathbb{Z}^+ \quad \forall i \tag{26}$$

Again, using a little algebra, we can rewrite this reduced problem in the form:

$$max \quad T'' \tag{27}$$

s.t.

$$T'' \leq (\mathrm{UO}_i + \mathrm{UI}_i + S_i)/\mathrm{p}_i \quad \forall i \tag{28}$$

$$T'' \leq (\mathrm{O} + \mathrm{I} + \sum_i S_i)/\sum_i \mathrm{p}_i \tag{29}$$

So, being N the number of products in the batch, we will have N + 1 inequalities that limit the value of T'' by constants and again T'' will be defined by the smallest value.

So the solution method can be decomposed into 4 steps:

step 1: find T', where:

$$T' = \min_{\forall i}\{\mathrm{UD}_i/\mathrm{p}_i\} \tag{30}$$

step 2: calculate S_i for all products.

step 3: find T'', where:

$$T'' = \min\{\min_{\forall i}\{(\mathrm{UO}_i + \mathrm{UI}_i + S_i)/\mathrm{p}_i\}, (\mathrm{O} + \mathrm{I} + \sum_i S_i)/\sum_i \mathrm{p}_i\} \tag{31}$$

step 4: calculate T, where:

$$T = T' + T'' \tag{32}$$

$$T = \min\{T, Z\} \tag{33}$$

It is probably possible to extend this thinking to the solution of a class of linear optimization problems. However, we will work on this concept in an upcoming paper.

Solution for the example presented before

Applying the method for solving the example presented in section 2, we have:

$T' \leq 1000g/60g/min$

$T' \leq 16,67min$ for A

$T' \leq 500g/40g/min$

$T' \leq 12,50min$ for B

so:

$T' = 12$

Then:

$S_A = 1000g - 12min * 60g/min = 280g$

$S_B = 500g - 12min * 40g/min = 20g$

And so we have:

$T'' \leq (3000g + 600g + 280g)/60g/min$

$T'' \leq 64,67min$ for A

$T'' \leq (2000g + 600g + 20g)/40g/min$

$T'' \leq 65,50min$ for B

$T'' \leq (1000g + 3000g + 300g)/100g/min$

$T'' \leq 43min$ for A and B

So that:

$T'' = 43min$

So we have:

$T = 55min$

As $T <$ Z=100, then this will certainly be the optimal solution.

In this case we will have:

$P_A = 3300g$ e $P_B = 2200g$

$D_A = 1000g$ e $D_B = 500g$

$E_A = 2300g$ e $E_B = 1700g$

Note that it is not important to know the values of O_i and I_i $\forall i$ however these values can be found by taking a solution from the indeterminate system of inequations:

$$\sum_i O_i \leq \text{O} \tag{34}$$

$$\sum_i I_i \leq \text{I} \tag{35}$$

5. Tests and results

To test the developed model and analytical solution method, we created a solver in C++ and a solver in LINGO (both available at github.com). We tested the solvers developed for the benchmarks presented below.

i	1	2	total
p$_i$	60	40	
UD$_i$	1000	500	
UO$_i$	600	600	1000
UI$_i$	3000	2000	3000
		Z	100

Table 1: Benchmark MBPTMP 001

i	1	2	3	total
p_i	60	40	50	
UD_i	1000	500	800	
UO_i	600	600	600	1500
UI_i	3000	2000	1000	3500
			Z	100

Table 2: Benchmark MBPTMP 002

i	1	2	3	4	5	6	7	8	9	10	total
p_i	60	40	50	40	30	50	60	10	20	40	
UD_i	1000	500	800	500	400	500	2000	300	500	1000	
UO_i	600	600	600	1500	300	200	500	800	0	200	3000
UI_i	3000	2000	1000	800	3000	1000	400	300	200	0	5000
										Z	100

Table 3: Benchmark MBPTMP 003

For building larger benchmarks, we used the following functions:

$$\mathrm{p}_i = rand()\%30 + 10 \tag{36}$$

$$\mathrm{UD}_i = rand()\%3000 + 800; \tag{37}$$

$$\mathrm{O} = rand()\%3000 + 500; \tag{38}$$

$$\mathrm{UO}_i = rand()\%(\mathrm{O} - 500) + 500; \tag{39}$$

$$\mathrm{I} = rand()\%5000 + 1000; \tag{40}$$

$$\mathrm{UI}_i = rand()\%(\mathrm{I} - 1000) + 1000; \tag{41}$$

$$Z = 100; \tag{42}$$

Seeking to enable the reproduction of the results, in the computational construction of the benchmarks we used the function srand((unsigned) source), where source is a defined value. To build the results presented in this work, we used source=0.

Table X presents a comparison of the results obtained by applying the analytical method and the LINGO solver for the solution of the previously presented benchmarks.

6. Contributions

The work for the elaboration of this paper is being carried out by three professors, as described below. Professor Fraga, T.B., was responsible for identifying the problem, mathematical modeling, proposing an analytical solution, developing a solver in C ++ for the analytical solution, developing benchmarks, and testing the analytical method and the solver in C++, as well as surveying bibliography and elaboration of the paper. Professor Henrique, M. is developing a solver in LINGO for testing and validating the model. Professor Menezes, R. is contributing to the bibliographic survey and to writing the bibliographic review presented in the paper.

References