Paper Title*

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I. Introduction

II. Problem description

In the plastic bags manufacturing process, extrusion is the most important stage since it is the main responsible for products fabrication. This stage begins with the insertion of resins into a kind of funnel coupled to the machine. Such material may be polypropylene (PP), polyethylene (PE) or a mixture of both. Also, the source of the polyethylene resins can be either virgin or recycled material. Next, pigments are added to the resins according to the desired color for the plastic bags being manufactured. The added material (resins and pigments) is then heated and then passes through an air cylinder, receiving the shape of a balloon. Such a balloon rises along the machine while the material cools down gaining more resistance. After, the material passes through two other cylinders forming the bobbins of plastic bags. In the first cylinder, a single bobbin has the width defined by the machine settings. In the second cylinder, cutting is performed, dividing the single bobbin into a set of bobbins narrower, wich widths are defined according to the specifications of the desired bags. After the extrusion stage, the plastic bags bobbins will be printed (if necessary) and then cut out, forming the lots (bobbins) of plastic bags that will be sent to inventory section.

As can be seen above, one of the important issues that must be dealt with when planning the daily plastic bags production is to determine which models of bags should be processed in each production batch of each extruder. This question becomes complex since the company applies something like a build-to-order strategy and wants to make the production plan to follow the sales schedule whenever possible. So, the proposed solution must minimize the unwanted storage cost without, however, allowing delivery delays. Since the company also has factory outlets for which overproduction is usually directed, some inventory flexibility is allowed in order to avoid machines idleness. Also, since each batch consists of a set of products of the same material and colored with the same pigments, the determination of the set of products that will be processed together must be made taking into account restrictions related to both the material and the color. One should also consider the machine capacity constraints, such as those related to cylinders width and to the daily time limit assigned to operation of each extruder. The main features of this problem are specified below:

Planning

- The planning horizon is divided into equal lenth time-periods. Each time period represents a working day.
- Once producted, product lots are stored. The inventory level is updated at the end of each period time. Unit invetory cost [R$] are know and fixed. The inventory cost per product is direct related to its storage time.
- Product demands [g] for each time period are know and given in advance. Product lots sold are delivered at the end of each time period. Unitary contribution of products [R$] are know and fixed.
- In order to ensure the proper operation of the factory outlets, they must maintain adequate level of some products in their own inventory. In this case, the factory outlets will be understood as common customers and those needs will be accounted for each product demand determination.
- With respect to the factory outlets extra demand, this is restrained by two factors: 1) total space available in stores for storage of these products; and 2) the limitation related to the sales capacity of the extra quantity for each product.
- Also, there is a maximum limit for quantities in stock per time period of each produc in factory, defined based on the total space destined for the storage of the products.

Balancing

- A production batch consists of a set of bobbins (lots) of plastic bags, with the same color, material and the same total length, processed simultaneously in the same extruder.
- The total length of bobbins is defined according to the start time and the end time of the batch processing.
- The sum of the widths of the bobbins processed in the same batch can not exceed the width of the extruder cylinder.
- The choice of products that will form a batch must be made so that the production is as close as possible to sales schedule.

Scheduling

- A number of extruders operate in parallel independently during a finite processing time interval which corresponds to the period of a working day. Thus, several batches can be processed simultaneously, one in each extruder. However each batch must be processed only once, by only one extruder, and no preemption il allowed.
- The start and the end time of each batch processing must be defined in order to attend the sales schedule, but avoiding, whenever its is possible, the storage cost. Idleness of machines should also be avoided.
- The sum of the batch processing times allocated to the same extruder over a time period must not exceed the daily processing limit of the extruder. The setup time must also be considered before every new batch to be processed. This time is fixed and known in advance.

The following section presents a linear mixed integer mathematical model for the problem presented.

III. Mathematical formulation

The following notations are used in the model

Indexes

d represents each business day on time interval considered in the planning

o represents the factory outlets

k represents the extrusers

i, j represents each specifc product considered in the planning

s represents each set of products that will be processed together on some extruder (*i. e.*, production batch)

Parameters

ND number of business days in the given time interval

NO number of factory outlets

NE number of extruders

NP number of products

NS maximun number of production sets allowed for one day

Cap_k production capacity of extruder k (minutes/day)

CW_k width of the cylinder of extruder k (centimeters)

PR_k production rate of the extruder k (centimeters/minute)

CR_k operation cost rate of the extruder k (R\$/minute)

ST_k setup time of the extruder k (minutes)

SC_k setup cost of the extruder k (R\$)

MPT_k minimum processing time for the sets processed on extruder k (minutes)

M_i interger number representing the compound (material and color) of product i

WR_i weight ratio of product i (grams/centimeter)

PW_i width of product i (centimeters)

UC_i unitary contribution of product i (R\$/grams)

IC_i inventory cost of product i (R\$/grams)

D_{id} demand of product i planned for the day d (grams)

MI maximum inventory allowed in factory, due to capacity or other restrictions (grams)

MI_i maximum inventory allowed for product i in factory, due to capacity or other restrictions (grams)

q_{i0} current inventory of product i (grams) - inventory boundary condition

O_{io} maximum overstock allowed for product i in factory outlet o, due to sales restrictions (grams)

O_o total overstock allowed in factory outlet o, due to capacity or other restrictions (grams)

q_{io0} current overstock of product i in factory outlet o (grams) - overstock boundary condition

f_{od} binary parameter taking the value 1 if the factory outlet o can receive extra production on day d, and 0 otherwise

Decision variables

x_{iskd} binary decision variable taking the value 1 if the product i belongs to the set s processed by extruder k and on day d, and 0 otherwise

s_{skd} starting time of processing of set s by extruder k and on day d

f_{skd} finishing time of processing of set s by extruder k and on day d

q_{iod} overproduction of product i sent for the factory outlet o on day d

Auxiliary variables

t_{skd} the processing time (minutes) of the set of production s processed by the extruder k on day d. It can be computed as follows:

$$t_{skd} = f_{skd} - s_{skd} \qquad s = 1,...,\text{NS}, \quad k = 1,...,\text{NE}, \quad d = 1,...,\text{ND} \tag{1}$$

a_{skd} binary decision variable taking the value 1 if the set s must be processed by extruder k and on day d, and 0 otherwise. This variable is directly related with t_{skd} according to the following equations:

$$(1 - a_{skd}) * t_{skd} \leq 0 \quad s = 1,...,\text{NS}, \quad k = 1,...,\text{NE}, \quad d = 1,...,\text{ND} \tag{2}$$

$$a_{skd} \leq t_{skd} \quad s = 1,...,\text{NS}, \quad k = 1,...,\text{NE}, \quad d = 1,...,\text{ND} \tag{3}$$

and it is related with x_{iskd} according to the following equations:

$$x_{iskd} <= a_{skd} \qquad i = 1,...,\text{NP}, \quad s = 1,...,\text{NS}, \quad k = 1,...,\text{NE}, \quad d = 1,...,\text{ND} \tag{4}$$

$$a_{skd} <= \sum_{i=1}^{NP} x_{iskd} \qquad s = 1,...,\text{NS}, \quad k = 1,...,\text{NE} \quad d = 1,...,\text{ND} \tag{5}$$

p_{id} quantity processed (grams) of product i on day d. It can be computed as follows:

$$p_{id} = \sum_{s=1}^{\text{NS}} \sum_{k=1}^{\text{NE}} (\text{PR}_k \times t_{skd} \times \text{WR}_i \times x_{iskd}) \quad i = 1,...,NP, \quad d = 1,...,ND \tag{6}$$

q_{id} inventory of product i on day d. It can be computed as follows:

$$q_{id} = q_{i(d-1)} + p_{id} - \text{D}_{id} - \sum_{o=1}^{\text{NO}} q_{iod} \quad i = 1,...,\text{NP}, \quad d = 1,...,\text{ND} \tag{7}$$

o_{iod} total overstock of product i in factor outlet o on day d. It can be computed as follows:

$$o_{iod} = o_{io(d-1)} + q_{iod} \qquad i = 1,...,\text{NP}, \quad o = 1,...,\text{NO}, \quad d = 1,...,\text{ND} \tag{8}$$

The model is formulated as follows

$$maximize \qquad PL = \sum_{i=1}^{\text{NP}} \sum_{d=1}^{\text{ND}} (\text{UC}_i \times p_{id} - \text{IC}_i \times q_{id}) - \sum_{s=1}^{NS} \sum_{k=1}^{NE} \sum_{d=1}^{ND} (\text{PC}_k \times t_{skd} + \text{SC}_k \times a_{skd}) \tag{9}$$

Subject to:

$$p_{id} + q_{i(d-1)} \geq \text{D}_{id} \qquad i = 1,...,\text{NP}, \quad d = 1,...,\text{ND} \tag{10}$$

$$q_{id} \leq \text{MI}_i \qquad i = 1,...,\text{NP}, \quad d = 1,...,\text{ND} \tag{11}$$

$$\sum_{i=1}^{\text{NP}} q_{id} \leq \text{MI} \qquad d = 1,...,\text{ND} \tag{12}$$

$$o_{iod} \leq \text{O}_{io} \qquad i = 1,...,\text{NP}, \quad o = 1,...,\text{NO}, \quad d = 1,...,\text{ND} \tag{13}$$

$$\sum_{i=1}^{\text{NP}} \sum_{d=1}^{\text{ND}} o_{iod} \leq \text{O}_o \qquad o = 1,...,\text{NO} \tag{14}$$

$$(1 - f_{od}) \times q_{iod} \leq 0 \qquad i = 1,...,\text{NP}, \quad o = 1,...,\text{NO}, \quad d = 1,...,\text{ND} \tag{15}$$

$$\sum_{i=1}^{\text{NP}} (\text{PW}_i \times x_{iskd}) \leq \text{CW}_k \qquad s = 1,...,\text{NS}, \quad k = 1,...,\text{NE}, \quad d = 1,...,\text{ND} \tag{16}$$

$$(\text{M}_g - \text{M}_i) \times x_{gskd} \times x_{iskd} = 0 \qquad g = 1,...,\text{NP}, \quad i = 1,...,\text{NP}, \quad s = 1,...,\text{NS}, \quad k = 1,...\text{NE}, \quad d = 1,...,\text{ND} \tag{17}$$

$$t_{skd} \geq 0 \qquad s = 1,...,\text{NS}, \quad k = 1,...,\text{NE}, \quad d = 1,...,\text{ND} \tag{18}$$

$$t_{skd} - \text{MPT}_k \times a_{skd} \geq 0 \qquad s = 1,...,\text{NS}, \quad k = 1,...,\text{NE}, \quad d = 1,...,\text{ND} \tag{19}$$

$$f_{skd} - \text{Cap}_k \leq 0 \qquad s = 1,...,\text{NS}, \quad k = 1,...,\text{NE}, \quad d = 1,...,\text{ND} \tag{20}$$

$$\sum_{s=1}^{\text{NS}} t_{skd} \leq \text{Cap}_k \qquad k = 1,...,\text{NE}, \quad d = 1,...,\text{ND} \tag{21}$$

$$\sum_{k=1}^{\text{NE}} a_{skd} \leq 1 \qquad s = 1,...,\text{NS}, \quad d = 1,...,\text{ND} \tag{22}$$

$$s_{skd} - \text{TS}_k \times a_{skd} = 0$$
$$s = 1 \quad k = 1, ..., \text{NE}, \quad d = 1, ..., \text{ND} \qquad (23)$$

$$s_{skd} - f_{(s-1)kd} - \text{TS}_k \times a_{skd} = 0$$
$$s = 2, ..., \text{NS} \quad k = 1, ..., \text{NE}, \quad d = 1, ..., \text{ND} \qquad (24)$$

$$a_{skd} \in \{0, 1\} \qquad s = 1, ..., \text{NS}, \quad k = 1, ..., \text{NE}, \quad d = 1, ..., \text{ND} \qquad (25)$$

$$x_{sjkd} \in \{0, 1\} \qquad i = 1, ..., \text{NP}, \quad s = 1, ..., \text{NS}, \quad k = 1, ..., \text{NE}, \quad d = 1, ..., \text{ND} \qquad (26)$$

$$s_{jkd}, e_{jkd} \geq 0 \text{ e inteiros} \qquad j = 1, ..., \text{J}, \quad k = 1, ..., \text{K} \quad d = 1, ..., \text{D} \qquad (27)$$

$$q_{oid} \geq 0 \text{ e inteiros} \qquad o = 1, ..., \text{O}, \quad i = 1, ..., \text{I}, \quad d = 1, ..., \text{D} \qquad (28)$$

As show in Eq. (9), the objective function maximizes the profit, Considering both the cumulative contribution and the inventory, processing and setup costs. Constraint set (10) guarantees that the production will be adequate to meet sales planning while constraint sets (11) and (12) respectively ensures that the inventory per product and the total inventory will both respect the inventory limitations. Constraint set (16) establishes that each set of production s shall be so that the sum of the widths of the respective products shall not exceed the extruder cylinder width. Constraint sets (17) ensures that the materials and the colors of products of the same set are identical. Constraint sets (18) and (20) ensure that the processing of some production set s does not finish before it starts and even after the factory office hours. Constraint set (21) ensures that the total processing time assigned to an extruder is within its capacity. Constraints in (22) prevent a given set from being allocated to more than one extruder. Constraint set (23) and (24) prevents two distincts production batch from been simultaneously processed by the same extruder and also guarantee that the proposed solution take account of setup times. Finaly, constraint sets (26), (27) and (28) inform the nature of decision variables.

IV. Resultados e discussões

Para validação do modelo apresentado na seção anterior foi utilizado o problema teste definido a seguir:

Número de produtos (unidades) = 4;
Número de extrusoras (unidades) = 2;
Número de lojas de fábrica (unidades) = 2;
Horizonte de planejamento (dias) = 1;

Capacidade limíte para estocagem de produtos na fábrica (gramas) = 300;

Dia	produto				unidade
	1	2	3	4	
1	200	200	200	200	g

Tabela I
Informações de demanda.

	produto				unidade
	1	2	3	4	
componente	1	1	2	2	tipo
largura	40	30	50	50	cm
taxa de peso	1	2	2	1	g/cm
contribuição unitária	10.5	14.0	12.0	10.0	R$
custo unitário de estoque	2.0	3.0	1.5	4.0	R$
estoque atual	10	0	0	0	unidade
estoque máximo permitido em fábrica	100	100	100	100	unidade

Tabela II
Informações dos produtos.

	extrusora		unidade
	1	2	
capacidade	480	350	min/dia
largura do cilindro	100	100	min/dia
taxa de produção	10	15	cm/min
tempo de setup	5	10	min
custo de setup	50	100	R$

Tabela III
Informações das extrusoras.

o	1				2				unit
O_o	800				800				g/day
i	1	2	3	4	1	2	3	4	-
q_{io0}	0	30	0	0	0	0	20	0	g
O_{io}	300	300	300	300	300	300	300	300	g

Tabela IV
Informações das lojas de fábrica.

Solução encontrada

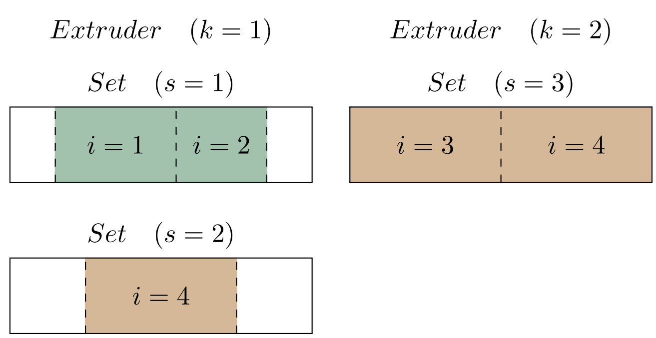

Figura 1. Batches.

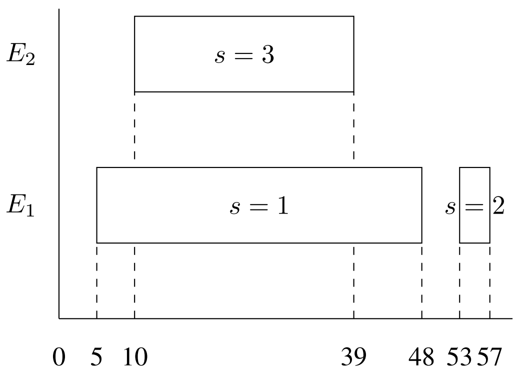

Figura 2. Gantt.