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# Production planning problem in extruders

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**Abstract** Insert your abstract here.

## 1 Introduction

## 2 Production allocation and balancing problem in extruders

In the plastic bags manufacturing process, extrusion is the most important stage since it is the main responsible for products fabrication. This stage begins with the insertion of resins into a kind of funnel coupled to the machine. Such material may be polypropylene (PP), polyethylene (PE) or a mixture of both. Also, the source of the polyethylene resins can be either virgin or recycled material. Next, pigments are added to the resins according to the desired color for the plastic bags being manufactured. The added material (resins and pigments) is then heated and then passes through an

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air cylinder, receiving the shape of a balloon. Such a balloon rises along the machine while the material cools down gaining more resistance. After, the material passes through two other cylinders forming the bobbins of plastic bags. In the first cylinder, a single bobbin has the width defined by the machine settings. In the second cylinder, cutting is performed, dividing the single bobbin into a set of bobbins narrower, wich widths are defined according to the specifications of the desired bags. After the extrusion stage, the plastic bags bobbins will be printed (if necessary) and then cut out, forming the lots (bobbins) of plastic bags that will be sent to inventory section. As can be seen above, one of the important issues that must be dealt with when planning the daily plastic bags production is to determine which models of bags should be processed in each production batch of each extruder. This question becomes complex since the company applies something like a build-to-order strategy and wants to make the production plan to follow the sales schedule whenever possible. So, the proposed solution must minimize the unwanted storage cost without, however, allowing delivery delays. Since the company also has factory outlets for which overproduction is usually directed, some inventory flexibility is allowed in order to avoid machines idleness. Also, since each batch consists of a set of products of the same material and colored with the same pigments, the determination of the set of products that will be processed together must be made taking into account restrictions related to both the material and the color. One should also consider the machine capacity constraints, such as those related to cylinders width and to the daily time limit assigned to operation of each extruder. The main features of this problem are specified below:

*Planning*

- The planning horizon is divided into equal lenth time periods. Each time period represents a working day.
- Once producted, product lots are stored. The inventory level is updated at the end of each period time. Unit invetory cost are know and fixed. The inventory cost per product is direct related to its storage time.
- Product demands for each time period are know and given in advance. Product lots sold are delivered at the end of each time period. Unitary contribution of products are know and fixed.
- In order to ensure the proper operation of the factory outlets, they must maintain adequate level of some products in their own inventory. In this case, the factory outlets will be understood as common customers and those needs will be accounted for each product demand determination.
- With respect to the factory outlets extra demand, this is restrained by two factors:
  1. total space available in stores for storage of these products;
  2. and the limitation related to the sales capacity of the extra quantity for each product.
- Also, there is a maximum limit for quantities in stock per time period of each produc in factory, defined based on the total space destined for the storage of the products.

*Balancing*

- A production batch consists of a set of bobbins (lots) of plastic bags, with the same color, material and the same total length, processed simultaneously in the same extruder.
- The total length of bobbins is defined according to the start time and the end time of the batch processing.
- The sum of the widths of the bobbins processed in the same batch can not exceed the width of the extruder cylinder.
- The choice of products that will form a batch must be made so that the production is as close as possible to sales schedule.

*Scheduling*

- A number of extruders operate in parallel independently during a finite processing time interval which corresponds to the period of a working day. Thus, several batches can be processed simultaneously, one in each extruder. However each batch must be processed only once, by only one extruder, and no preemption il allowed.
- The start and the end time of each batch processing must be defined in order to attend the sales schedule, but avoiding, whenever its is possible, the storage cost. Idleness of machines should also be avoided.
- The sum of the batch processing times allocated to the same extruder over a time period must not exceed the daily processing limit of the extruder. The setup time must also be considered before every new batch to be processed. This time is fixed and known in advance. The follow-

ing section presents a linear mixed integer mathematical model for the problem presented.

## 3 Mathematical model

The symbols used in the model are shown in Table 1. And the mathematical model is presented as follows:

$$\begin{aligned} \mathit{maximize} \quad & \sum_{i=1}^{\mathrm{NP}} \sum_{d=1}^{\mathrm{ND}} \{\mathrm{UC}_i * P_{id} - \mathrm{IC} * Q_{id} - \mathrm{UC} * U_{id}\} \\ & - \sum_{e=1}^{\mathrm{NE}} \sum_{d=1}^{\mathrm{ND}} \sum_{b=1}^{\mathrm{NB}} \{\mathrm{CR}_e * A_{bed} * T_b + \mathrm{SC}_e * A_{bed}\} \end{aligned} \tag{1}$$

*subj. to*

$$P_{id} - \sum_{b=1}^{\mathrm{NB}} \sum_{e=1}^{\mathrm{NE}} \{B_{ib} * A_{bed} * \mathrm{WR}_i * \mathrm{PW}_i * \mathrm{PR}_e * T_b\} = 0, \quad \forall i, d \tag{2}$$

$$\sum_{e=1}^{\mathrm{NE}} \sum_{d=1}^{\mathrm{ND}} A_{bed} \leq 1 \quad \forall b \tag{3}$$

$$\sum_{b=1}^{\mathrm{NB}} \{A_{bed} * (T_{bd} + \mathrm{ST})\} - \mathrm{C}_e \leq 0 \quad \forall e, d \tag{4}$$

$$Q_{id} - Q_{i(d-1)} - P_{id} + D_{id} + \sum_{o=1}^{\mathrm{NO}} Q_{iod} = 0, \quad \forall i, d \tag{5}$$

$$Q_{i0} - \mathrm{Q}_i = 0, \quad \forall i \tag{6}$$

$$Q_{id} - \mathrm{MI}_i \leq 0, \quad \forall i, d \tag{7}$$

$$\sum_{i=1}^{\mathrm{NP}} \sum_{d=1}^{\mathrm{ND}} Q_{id} - \mathrm{MI} \leq 0 \tag{8}$$

$$Q_{iod} - \mathrm{O}_{io} \leq 0, \quad \forall i, o, d \tag{9}$$

$$\sum_{i=1}^{\mathrm{NP}}\sum_{d=1}^{\mathrm{ND}} Q_{iod} - \mathrm{O}_o \leq 0, \quad \forall o \tag{10}$$

$$U_{id} - U_{i(d-1)} - \mathrm{D}_{id} + D_{id} = 0, \quad \forall i, d \tag{11}$$

$$U_{i0} = 0, \quad \forall i \tag{12}$$

$$\sum_{i=1}^{\mathrm{NP}} \{B_{ibd} * A_{bed} * \mathrm{PW}_i\} - \mathrm{W}_e \leq 0 \quad \forall b, e, d \tag{13}$$

$$\sum_{i=1}^{\mathrm{NP}}\sum_{j=i+1}^{\mathrm{NP}} \{B_{ibd} * B_{jbd} * \mathrm{K}_{ij}\} - \frac{(\sum_{i=1}^{\mathrm{NP}} B_{ibd})!}{2! * ((\sum_{i=1}^{\mathrm{NP}} B_{ibd}) - 2)!} = 0 \quad \forall b, d \tag{14}$$

$$B_{ibd} \in \{0, 1\} \quad \forall i, b, d \tag{15}$$

$$A_{bed} \in \{0, 1\} \quad \forall b, e, d \tag{16}$$

$$T_{bd} \geq 0, \quad \forall b, d \tag{17}$$

$$Q_{iod}, \geq 0, \quad \forall i, o, d \tag{18}$$

The objective function (1) maximizes the profit, considering the cumulative contribution and the inventory, processing and setup costs. Eqs. in (2) calculate the quantity produced of each product. Eqs. in (**??**) are used for daily calculation of setup times in each extruder. Eqs. in (3) informs that, daily, each batch must be processed in a single extruder. Constraints in (4) ensure that the sum of processing and setup times of all batches processed daily in the same extruder does not exceed the capacity of the same extruder. Eqs. in (5) relate the quantity produced, with the quantity in stock, the demand and the quantity sent to the outlets, being the Eqs. in (6) boundary conditions. The restrictions in (7) and (8) guarantee, respectively, that, in the factory, the stock limit for each product and the general stock limit will be respected. The restrictions in (9) and (10), respectively,

**Fig. 1** Please write your figure caption here

guarantee that these same limits will also be respected in each outlet. Constraints in (13) guarantee that the sum of the widths of the products of the same batch is not greater than the width of the extruder in which this same batch will be processed. Eqs. in (14) ensure that the products processed in the same batch are always of the same color and the same material. Finally, the constraints in (15), (16), (17) and (18) inform the nature of the decision variables.

## 4 Tests and results

and [1]

### *4.1 Benchmark*

To test the developed model, we consider the following benchmark:

Planejamento para 3 dias.

Temos a seguinte demanda:

as required. Don't forget to give each section and subsection a unique label (see Sect. 2).

## References

1. Author, Journal **Volume,** (year) page numbers.
2. Author, *Book title* (Publisher, place year) page numbers

**Fig. 2** Please write your figure caption here

| | |
|---|---|
| *indexes* | |
| $d$ | used to designate a day, $d = 1, ..., \mathrm{ND}$ |
| $o$ | used to designate a factory outlet, $o = 1, ..., \mathrm{NO}$ |
| $e$ | used to designate a extruser, $e = 1, ..., \mathrm{NE}$ |
| $i, j$ | used to designate each specifc product considered |
| | in the planning, $i, j = 1, ..., \mathrm{NP}$ |
| $b$ | used to designate a production batch, $b = 1, ..., \mathrm{NB}$ |
| *parameters* | |
| $\mathrm{ND}, \mathrm{NO}$ | number of days and number of factory outlets, respectively |
| $\mathrm{NE}, \mathrm{NP}$ | number of extruders and number of products, respectively |
| NB | maximun number of batchs |
| $\mathrm{C}_e$ | daily production capacity of extruder $e$ (time/day) |
| $\mathrm{W}_e$ | width of the cylinder of extruder $e$ |
| $\mathrm{PR}_e, \mathrm{CR}_e$ | production rate and operation cost rate of the extruder $e$, |
| | respectively |
| $\mathrm{ST}, \mathrm{SC}_e$ | setup time and setup cost of the extruder $e$, respectively |
| $\mathrm{MPT}_e$ | min processing time for the batch processed on extruder $e$ |
| $\mathrm{WR}_i, \mathrm{PW}_i, \mathrm{IC}_i$ | weight ratio, width and inventory costof product $i$ |
| $\mathrm{UC}_i$ | unitary contribution of product $i$ |
| ST | setup time |
| $\mathrm{K}_{ij}$ | binary parameter taking the value 1 if the products $i$ and $j$ |
| | has the same colour and material, and 0 otherwise |
| $\mathrm{D}_{id}$ | demand of product $i$ planned for the day $d$ |
| $\mathrm{MI}, \mathrm{MI}_i$ | maximum inventory mand aximum inventory for product $i$ |
| | allowed in factory, due to capacity or other restrictions |
| $\mathrm{Q}_i$ | current inventory of product $i$ - inventory boundary cond. |
| $\mathrm{O}_o, \mathrm{O}_{io}$ | overstock and maximum overstock for product $i$ allowed |
| | in factory outlet $o$, due to sales restrictions |

**Table 1** Symbols used in the mathematical model (indexes and parameters)

| | |
|---|---|
| *decision variables* | |
| $B_{ibd}$ | binary decision variable taking the value 1 if the product $i$ belongs to the batch $b$ on day $d$, and 0 otherwise |
| $A_{bed}$ | binary decision variable taking the value 1 if batch $b$ must be processed by extruder $e$ and on day $d$, and 0 otherwise |
| $T_{bd}$ | time of processing of batch $b$ on day $d$ |
| $Q_{iod}$ | overproduction of product $i$ sent for the factory outlet $o$ on day $d$ |
| *secondary variables* | |
| $T_{bd}$ | processing time of batch $b$ processed on day $d$ |
| $P_{id}$ | quantity processed of product $i$ on day $d$ |
| $Q_{id}$ | inventory of product $i$ at the end of day $d$ |
| $ST_{ed}$ | total setup time of machine $e$ on day $d$ |

**Table 2** Symbols used in the mathematical model (primary and secondary variables)

| day / product | P1 | P2 | P3 | P4 | P5 | P6 | P7 | P8 | P9 | P10 | P11 | P12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| D1 | 10 | 15 | 20 | 5 | 7 | 30 | 0 | 0 | 0 | 20 | 5 | 10 |
| D2 | 5 | 0 | 0 | 15 | 15 | 0 | 10 | 10 | 15 | 5 | 15 | 20 |
| D3 | 0 | 5 | 10 | 5 | 7 | 0 | 0 | 8 | 0 | 20 | 0 | 10 |

**Table 3** Demand)

**Table 4** Please write your table caption here

| first | second | third |
|---|---|---|
| number | number | number |
| number | number | number |