Highlights

Analytic Hierarchy Process: how to force consistency of pairwise comparisons matrix

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- generative method for forcing consistenc ;
- multi-product p-batch processing time maximization (MPBPTM) problem definition;
- linear integer programming model for the MPBPTM problem;
- exact optimization method for solving the MPBPTM problem.

Analytic Hierarchy Process: how to force consistency of pairwise comparisons matrix

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Abstract

Saaty's Analytic Hierarchy Process is an important method for assigning weight to multiple criteria. The logic of this method is not complicated. First a pairwise comparisons matrix is generated for the multiple criteria, and the normalized eigenvector of this pairwise matrix is used as the weight of the criteria. Despite this method represents an important inovation, its logic is not complicated at all. First a pairwise comparisons matrix is generated for the multiple criteria, and the normalized eigenvector of this matrix is used as the weight of criteria. But, since pairwise matrixes are usually generated manually and based only on some employee knowhow, there is a huge complexity on generating a consistent pairwise matrix. Especially when many criteria are used. This paper presents two algorithms that can be used to adjust inconsistent matrices, forcing such matrices to have a better con-

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sistency rate. The first method is a constructive method that uses the data inserted in the matrix to build a new improved one. The second method iteratively identifies inconsistencies, making minor changes in order to improve the matrix consistency rate.
Keywords: multi-product batch, processing time maximization, mathematical model, analytical solution, LINGO

1. Introduction

2. Forcing matrix consistency

Algorithm 1 function $consistencyRate()$

Require: $[a_{ij}]_{i,j=1}^{n}, RI[x]_{x=1}^{10}$

$\lambda_{max} = mainEigenvalue([a_{ij}]_{i,j=1}^{n})$ [1]

$CI = (\lambda_{max} - n)/(n-1)$

$CR \leftarrow CI/RI[n]$

return CR

2.1. Constructive algorithm

2.2. Iterative algorithm

3. Tests and results

4. Conclusions and suggestions for future works

In this paper we presented ...

Algorithm 2 function forcingConsistency()

Require: $[a_{ij}]_{i,j=1}^{n} \quad | \quad consistencyRate([a_{ij}]_{i,j=1}^{n-1}) \leq 0.1$
Ensure: $consistencyRate([b_{ij}]_{i,j=1}^{n}) \leq 0.1$

```
for j ≤ n − 1 do
    for j + 1 ≤ k ≤ n − 1 do
        if a_jk = 1 then
            if a_nj ≠ a_nk then

            end if
        else
        end if
        if a_nj > a_nk and a_jk < 0 then
        else if a_nj < a_nk and a_jk > 0 then
        end if
    end for
end for
return [b_ij]_{i,j=1}^n, CR([b_ij]_{i,j=1}^n)
```

5. CRediT authorship contribution statement

T.B. Fraga: Conceptualization, Project administration, Supervision, Software, Methodology, Validation, Formal analysis, Writing – original draft, Writing – review & editing. Í.R.B. Aquino: Data curation. R.C.S. Menêzes: Data curation.

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Algorithm 3 function constructivelyForcingConsistency()

Require: $\mathrm{M}[a_{ij}]_{i,j=1}^{n}$
Ensure: $consistencyRate([b_{ij}]_{i,j=1}^{n}) \leq 0.1$
  test consistency for $\mathrm{M}[a_{ij}]_{i,j=1}^{n}$
  if $\mathrm{M}[a_{ij}]_{i,j=1}^{n}$ is consistent **then**
    $\mathrm{M}[b_{ij}]_{i,j=1}^{n} \leftarrow \mathrm{M}[a_{ij}]_{i,j=1}^{n}$
    return $\mathrm{M}[b_{ij}]_{i,j=1}^{n}$
  else
    $\mathrm{M}[b_{ij}]_{i,j=1}^{2} = M[a_{ij}]_{i,j=1}^{2}$
    $\mathrm{M}[c_{ij}]_{i,j=1}^{2} = M[a_{ij}]_{i,j=1}^{2}$
  end if
  for $3 \leq k \leq n$ **do**
    for $l \leq k$ **do**
      if $k = l$ **then**
        $c_{kl} \leftarrow 1$
      else
        $c_{kl} \leftarrow a_{kl}$
        $c_{lk} \leftarrow a_{lk}$
      end if
    end for
    test consistency for $\mathrm{M}[c_{ij}]_{i,j=1}^{k}$
    if $\mathrm{M}[c_{ij}]_{i,j=1}^{k}$ is inconsistent **then**
      find and adjust inconsistency on $M[c_{ij}]_{i,j=1}^{k}$
    end if
    $\mathrm{M}[b_{ij}]_{i,j=1}^{k} = M[c_{ij}]_{i,j=1}^{k}$
  end for
  test consistency for $\mathrm{M}[b_{ij}]_{i,j=1}^{n}$
  return $\mathrm{M}[b_{ij}]_{i,j=1}^{n}$

Algorithm 4 function iterativelyForcingConsistency()

Require: $\mathrm{M}[a_{ij}]_{i,j=1}^{n}$
Ensure: $consistencyRate([b_{ij}]_{i,j=1}^{n}) \leq 0.1$
  return $\mathrm{M}[b_{ij}]_{i,j=1}^{n}$

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