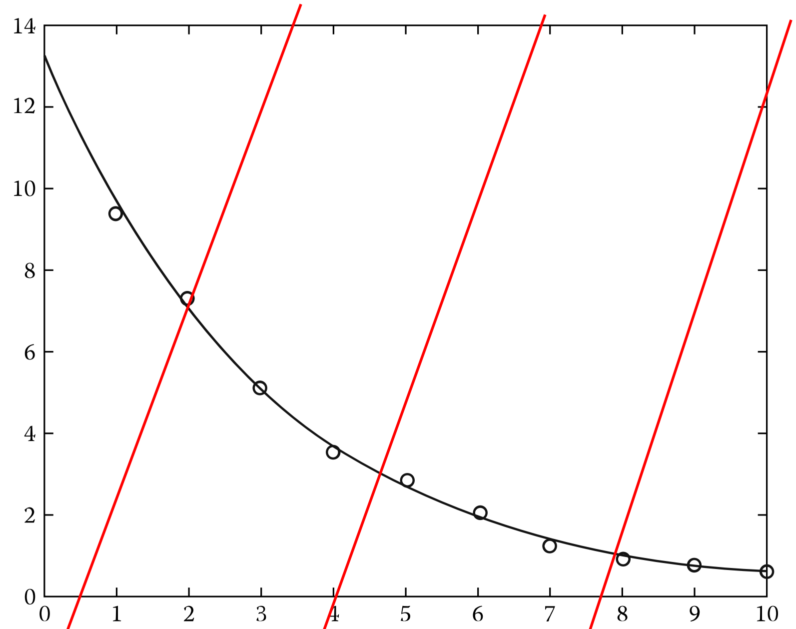

FIGURE 2.40
Simulation results of capacitor selection problem.

The following program written in a MATLAB script file determines the best fit exponential function to the data points, determines C and V_0, and plots the points and the fitted function.

```
R = 2000;
t = 1:10;
v = [9.4 7.31 5.15 3.55 2.81 2.04 1.26 0.97 0.74 0.58];
p = polyfit(t,log(v),1);
C = -1/(R*p(1))
V0 = exp(p(2))
tplot = 0:0.1:10;
vplot = V0*exp(-tplot./(R*C));
plot(t,v,‘o’,tplot,vplot)
```

The program creates also the plot shown in Figure 2.40.

3

Example 2.14: Flight of Model Rocket

The flight of a model rocket can be developed as follows. During the first 0.15 s the rocket is propelled up by the rocket engine with a force of 16 N. The rocket then flies up while slowing down under the force of gravity. After it reaches its peak, the rocket starts to fall back. When its down velocity reaches 20 m/s a parachute opens (assumed to open instantly) and the rocket continues to move down at a constant speed of 20 m/s until it hits the ground. Write a program that calculates and plots the speed and altitude of the rocket as a function of time during the flight.

Solution

The rocket is assumed to be a particle that moves along a straight line in the vertical plane. For motion with constant acceleration along a straight line, the velocity and position as a function of time are given by

$$v(t) = v_0 + at \quad \text{and} \quad s(t) = s_0 + v_0 t + \frac{1}{2}at$$

where v and s are the initial velocity and position, respectively. In the computer program the flight of the rocket is divided into three segments. Each segment is calculated in a while loop. In every pass the time increases by an increment.

Segment 1: The first 0.15 s when the rocket engine is on. During this period, the rocket moves up with a constant acceleration. The acceleration is determined by drawing a free body and a mass acceleration diagrams. From Newton's second law, the sum of the forces in the vertical direction is equal to the mass times the acceleration (equilibrium equation):

$$\sum F = F_E - mg = ma$$

Solving the equation for the acceleration gives

$$a = \frac{F_E - mg}{m}$$

The velocity and the height as a function of time are

$$v(t) = 0 + at \quad \text{and} \quad h(t) = 0 + 0 + \frac{1}{2}at^2$$

where the initial velocity and the initial position are both zero. In the computer program this segment starts when $t = 0$, and the looping continues as long as $t < 0.15$ s. The time, velocity, and height at the end of this segment are t_1, v_1, and h_1.

Segment 2: The motion from when the engine stops until the parachute opens. In this segment the rocket moves with a constant deceleration g. The speed and height of the rocket as a function of time are given by

$$v(t) = v_1 - g(t - t_1) \quad \text{and} \quad h(t) = h_1 + v_1(t - t_1) - \frac{1}{2}g(t - t_1)^2$$

In this segment the looping continues until the velocity of the rocket is –20 m/s (negative since the rocket moves down). The time and height at the end of this segment are t_2 and h_2.

Segment 3: The motion from when the parachute opens until the rocket hits the ground. In this segment the rocket moves with constant velocity (zero acceleration). The height as a function of time is given by $h(t) = h_2 - v_{chute}(t - t_2)$, where v_{chute} is the constant velocity after the parachute opens. In this segment the looping continues as long as the height is greater than zero.
A program in a script file that carries out the calculation is shown below.

```
m=0.05; g=9.81; tEngine=0.15; Force=16; vChute=-20; Dt=0.01;
clear t v h
n=1;
t(n)=0; v(n)=0; h(n)=0;
% Segment 1
a1=(Force-m*g)/m;
while t(n)<tEngine &n< 50000
    n=n+1;
    t(n)=t(n-1)+Dt;
    v(n)=a1*t(n);
    h(n)=0.5*a1*t(n)^2;
end
```

```
v1=v(n); h1=h(n); t1=t(n);
% Segment 2
while v(n)>=vChute &n<50000
   n=n+1;
   t(n)=t(n-1)+Dt;
   v(n)=v1-g*(t(n)-t1);
   h(n)=h1+v1*(t(n)-t1)-0.5*g*(t(n)-t1)^2;
end
v2=v(n); h2=h(n); t2=t(n);
% Segment 3
while h(n)>0 & n<50000
   n=n+1;
   t(n)=t(n-1)+Dt;
   v(n)=vChute;
   h(n)=h2+vChute*(t(n)-t2);
end
subplot(1,2,1)
plot(t,h,t2,h2, 'o')

subplot(1,2,2)
plot(t,v,t2,v2, 'o')
```

The accuracy of the result depends on the magnitude of the time increment Dt. An increment of 0.01 s appears to give good results. The conditional expression in the while commands also includes a condition for *n* (if *n* is larger than 50,000 the loop stops). This is done as a precaution to avoid an infinite loop in case there is an error in the statements inside the loop. The plots generated by the program are shown in Figure 2.41.

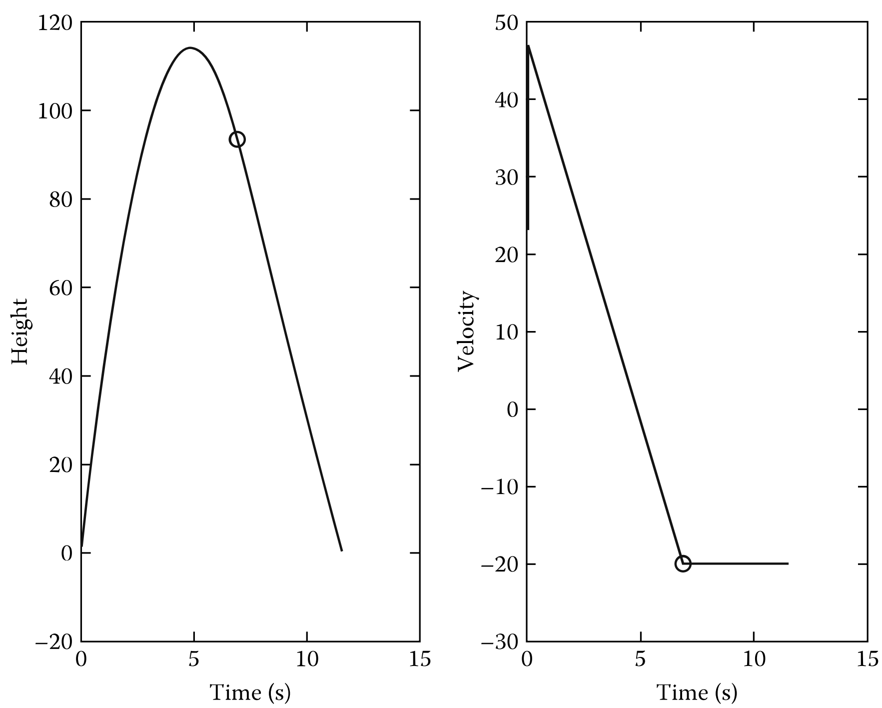

FIGURE 2.41
Simulation results of parachute system.