MIT International Journal of Electronics and Communication Engineering *Vol. 3, No. 1, Jan. 2013, pp. 20–24*
*ISSN No. 2230-7672 (c) MIT Publications*


# Bit Error Rate Performance of Gray Coded 8-PSK

**Manas Singhal**
Department of E and C Engineering
MIT, Moradabad, UP, INDIA
Email: manas.singhal.ec@gmail.com

**Mitali Agarwal**
Department of E and C Engineering
MIT, Moradabad, UP, INDIA
Email: mitali.mit@gmail.com

**Manish Trikha**
Department of E and I Engineering
MIT, Moradabad, UP, INDIA
Email: kshinghal@gmail.com

**Neha Sharma**
Department of E and I Engineering
MIT, Moradabad, UP, INDIA
Email: april5pankaj@gmail.com

**ABSTRACT**

The data can be modulated using Gray coded 8-PSK method. After modulation the data can be transmitted over the noisy channel. The noise is assumed to be Additive White Gaussian, i.e. the signal is passed through an AWGN channel. Now the received signal is demodulated using an 8-PSK demodulator. The performance of the system can be analysed by measuring the symbol and bit error rates. This paper shows that the bit error rate performance is better for gray coded constellation as compared with conventional signal constellation.

## I. INTRODUCTION

The techniques used to modulate digital information are different to that of analogue transmission. The data transmitted via satellite or microwave is transmitted as an analogue signal. The techniques used to transmit analogue signals are used to transmit digital signals. The problem is to convert the digital signals to a form that can be treated as an analogue signal that is then in the appropriate form to either be transmitted down a twisted cable pair or applied to the RF stage where is modulated to a frequency that can be transmitted via microwave or satellite.

The equipment that is used to convert digital signals into analogue format is a modem. The word modem is made up of the words “modulator” and “demodulator”. A modem accepts a serial data stream and converts it into an analogue format that matches the transmission medium.

There are many different modulation techniques that can be utilised in a modem.

These techniques are:

- Amplitude shift key modulation (ASK)
- Frequency shift key modulation (FSK)
- Binary-phase shift key modulation (BPSK)
- Quadrature-phase shift key modulation (QPSK)
- Quadrature amplitude modulation (QAM)
- Spread-spectrum technique

All convey data by changing some aspect of a base signal, the carrier wave (usually a sinusoid), in response to a data signal. In the case of PSK, the phase is changed to represent the data signal. The performance of all these modulation techniques can be best analysed by plotting Bit Error Rate as a function of $E_b/N_0$.

## II. PHASE SHIFT KEYING

Phase-shift keying (PSK) is a digital modulation scheme that conveys data by changing, or modulating, the phase of a reference signal (the carrier wave). Any digital modulation scheme uses a finite number of distinct signals to represent digital data. PSK uses a finite number of phases; each assigned a unique pattern of binary bits. Usually, each phase encodes an equal number of bits. Each pattern of bits forms the symbol that is represented by the particular phase. The demodulator, which is designed specifically for the symbol-set used by the modulator, determines the phase of the received signal and maps it back to the symbol it represents, thus recovering the original data. The basic PSK Techniques are described as follows:

### 2.1 Binary Phase-shift Keying (BPSK)

BPSK (also sometimes called PRK, Phase Reversal Keying, or 2PSK) is the simplest form of phase shift keying (PSK). It uses two phases which are separated by 180° and so can also be termed 2-PSK. It does not particularly matter exactly where the constellation points are positioned, and in Figure 1 they are shown on the real axis, at 0° and 180°. This modulation is the most robust of all the PSKs since it takes the highest level of noise or distortion to make the demodulator reach an incorrect decision. It is, however, only able to modulate at 1 bit/symbol and so is unsuitable for high data rate applications when bandwidth is limited.

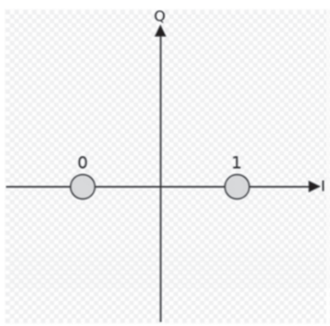


**Figure 1:** Constellation Diagram for BPSK

## 2.2 Quadrature Phase-shift Keying (QPSK)

It is also known as quaternary or quadri phase PSK, 4-PSK, or 4-QAM, QPSK uses four points on the constellation diagram, equi-spaced around a circle. With four phases, QPSK can encode two bits per symbol. Analysis shows that this may be used either to double the data rate compared to a BPSK system while maintaining the bandwidth of the signal or to maintain the data-rate of BPSK but halve the bandwidth needed.

As with BPSK, there are phase ambiguity problems at the receiver and differentially encoded QPSK is used more often in practice.

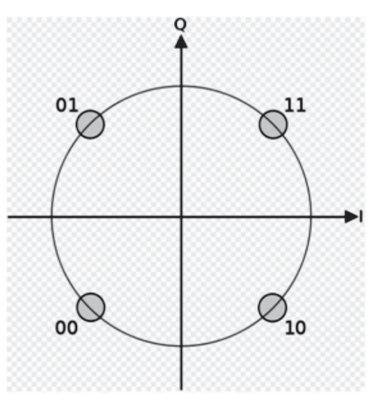


**Figure 2:** Constellation Diagram for QPSK with Gray Coding. Each Adjacent Symbol Only Differs by One Bit.

QPSK systems can be implemented in a number of ways.

### *2.2.1 Modulation*

The binary data stream is split into the inphase and quadrature-phase components. These are then separately modulated onto two orthogonal basis functions. In this implementation,

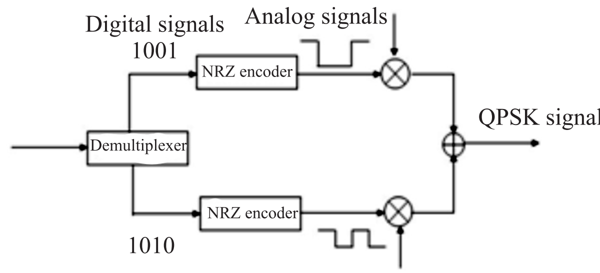


**Figure 3:** Modulator for QPSK

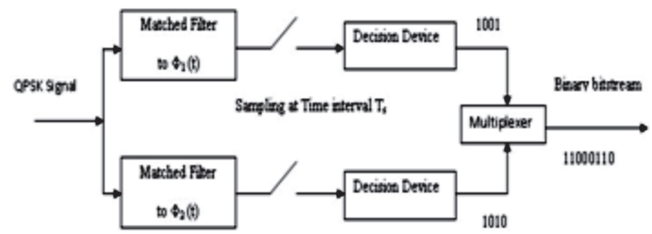


**Figure 4:** Demodulator for QPSK

two sinusoids are used. Afterwards, the two signals are superimposed, and the resulting signal is the QPSK signal. Note the use of polar non-return-to-zero encoding. These encoders can be placed before for binary data source, but have been placed after to illustrate the conceptual difference between digital and analog signals involved with digital modulation. Receiver structure for QPSK. The matched filters can be replaced with correlators. Each detection device uses a reference threshold value to determine whether a 1 or 0 is detected.

### *2.2.2 Demodulation*

For a signal that has been differentially encoded, there is an obvious alternative method of demodulation. Instead of demodulating as usual and ignoring carrier-phase ambiguity, the phase between two successive received symbols is compared and used to determine what the data must have been. When differential encoding is used in this manner, the scheme is known as differential phase-shift keying (DPSK).

Note that this is subtly different to just differentially-encoded PSK since, upon reception, the received symbols are not decoded one-by one to constellation points but are instead compared directly to one another.

Call the received symbol in the $k^{th}$ time slot $r_k$ and let it have phase $\varphi_k$. Assume without loss of generality that the phase of the carrier wave is zero. Denote the AWGN term as $n_k$. Then the decision variable for the $k-1^{th}$ symbol and the $k^{th}$ symbol is the phase difference between $r_k$ and $r_{k-1}$. That is, if $r_k$ is projected onto $r_{k-1}$, the decision is taken on the phase of the resultant complex number: where superscript * denotes complex conjugation. In the absence of noise, the phase of this is $\theta_k - \theta_{k-1}$, the phase-shift between the two received signals which can be used to determine the data transmitted.

The probability of error for DPSK is difficult to calculate in general, but, in the case of DBPSK it is: which, when numerically evaluated, is only slightly worse than ordinary BPSK, particularly at higher $E_b/N_0$ values.

Any digital modulation scheme uses a finite number of distinct signals to represent digital data. PSK uses a finite number of phases, each assigned a unique pattern of binary digits. Usually, each phase encodes an equal number of bits. Each pattern of bits forms the symbol that is represented by the particular phase. The demodulator, which is designed specifically for the symbol-set used by the modulator, determines the phase of the received signal and maps it back to the symbol it represents, thus recovering the original data. This requires the receiver to be able to compare the phase of the received signal to a reference signal — such a system is termed coherent (and referred to as CPSK).

## 2.3 Higher Order PSK (8-PSK)

Any number of phases may be used to construct a PSK constellation but 8-PSK is usually the highest order PSK constellation deployed. With more than 8 phases, the error-rate becomes too high and there are better, though more complex, modulations available such as quadrature amplitude modulation (QAM). Although any number of phases may be used, the fact that the constellation must usually deal with binary data means that the number of symbols is usually a power of 2—this allows an equal number of bits-per-symbol.

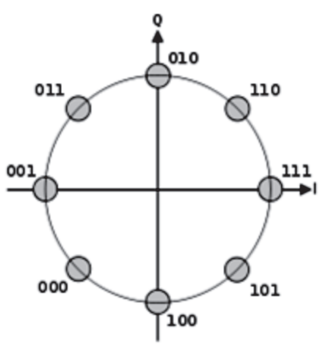


**Figure 5:** Constellation Diagram for 8-PSK with Gray Coding.

The data can be modulated using Gray coded 8-PSK method. After modulation the data can be transmitted over the noisy channel. The noise is assumed to be Additive White Gaussian, i.e. the signal is passed through an AWGN channel. Now the received signal is demodulated using an 8-PSK demodulator. The performance of the system can be analysed by measuring the symbol and bit error rates.

### *2.3.1 Modulation*

The data is modulated in the following manner:

Modulator is supplied with binary valued inputs. The value of binary sequence can vary between 0 to M-1, where M is the number of constellation points in the signal space. (M = 8 for 8-PSK).

The binary data is mapped to corresponding signal constellation point in the signal space using Gray coding.

The sequence of constellation points is then converted to the analog waveform according to the following conversion rule.

For Gray Coded ordering M the constellation is converted using

| Binary Coding | Gray Equivalent | Modulated Waveform |
|---|---|---|
| 000 | 0 | exp(0) |
| 001 | 1 | exp(jΠ/4) |
| 010 | 3 | exp(j3Π/4) |
| 011 | 2 | exp(j2Π/4) |
| 100 | 7 | exp(j7Π/4) |
| 101 | 6 | exp(j6Π/4) |
| 110 | 4 | exp(j4Π/4) |
| 111 | 5 | exp(j5Π/4) |

## 2.3 Gray Coding

Gray coding is a technique that multilevel modulation schemes often use to minimize the bit error rate. It consists of ordering modulation symbols so that the binary representations of adjacent symbols differ by only one bit.

## 2.4 Constellation Diagram

A constellation diagram is a representation of a signal modulated by a digital modulation scheme such as Quadrature Amplitude Modulation or Phase-Shift Keying. It displays the signal as a two-dimensional scatter diagram in the complex plane at symbol sampling instants. In a more abstract sense, it represents the possible symbols that may be selected by a given modulation scheme as points in the complex plane. Measured constellation diagrams can be used to recognize the type of interference and distortion in a signal.

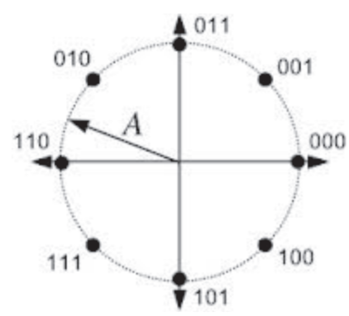


**Figure 6:** A Constellation Diagram for Gray Coded 8-PSK

By representing a transmitted symbol as a complex number and modulating a cosine and sine carrier signal with the real and imaginary parts (respectively), the symbol can be sent with two carriers on the same frequency. They are often referred to as Quadrature Carriers. A coherent detector is able to independently demodulate these carriers. This principle of using two independently modulated carriers is the foundation of Quadrature Modulation. In pure phase modulation, the phase of the modulating symbol is the phase of the carrier itself.

### *2.4.1 M-ary*

If we take 2 bits at a time, and arrange them together, we can assign each set of 2 bits to a different symbol, and then we can transmit the different symbols.

***Example:*** 4-ASK

We can use the following scheme:

"00" = +5V

"01" = +1.66V

"10" = –1.66V

"11" = –5V

We can see now that we can transmit data twice as fast using this scheme, although we need to have a more complicated receiver, that can decide between 4 different pulses instead of binary pulses this type of transmission results in reduced channel bandwidth.

## III. MATHEMATICAL ANALYSIS

For determining error-rates mathematically, some definitions will be needed:

$E_b$ = Energy-per-bit

$E_S$ = Energy-per-symbol = $nE_b$ with n bits per symbol

$T_b$ = Bit duration

$T_S$ = Symbol duration

$N_0/2$ = Noise power spectral density (W/Hz)

$P_b$ = Probability of bit-error

$P_S$ = Probability of symbol-error

The general form for BPSK follows the equation:

### 3.1 BER for BPSK

$$s_n(t) = \sqrt{\frac{2E_b}{T_b}}\cos(2p\pi f_c t + \pi(1-n)),\ n = 0, 1, \qquad (1)$$

This yields two phases, 0 and $\pi$. In the specific form, binary data is often conveyed with the following signals:

$$s_0(t) = \sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_c t + \pi) = -\sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_c t) \qquad (2)$$

for binary "0"

$$s_1(t) = \sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_c t) \qquad (3)$$

for binary "1"

where $f_c$ is the frequency of the carrier-wave.

Hence, the signal-space can be represented by the single basis function

$$\phi(t) = \sqrt{\frac{2}{T_b}}\cos(2\pi f_c t) \qquad (4)$$

where 1 is represented by $\sqrt{E_b}\phi(t)$ and 0 is represented by $-\sqrt{E_b}\phi(t)$. This assignment is, of course, arbitrary.

Q(x) will give the probability that a single sample taken from a random process with zero-mean and unit-variance Gaussian probability density function will be greater or equal to x. It is a scaled form of the complementary Gaussian error function:

$$Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^{\infty} e^{-t^2/2}dt = \frac{1}{2}\operatorname{erfc}\left(\frac{x}{\sqrt{2}}\right),\ x \ge 0 \qquad (5)$$

The bit error rate (BER) of BPSK in AWGN can be calculated as:

$$P_b = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) \quad \text{or} \quad P_b = \frac{1}{2}\operatorname{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right) \qquad (6)$$

Since there is only one bit per symbol, this is also the symbol error rate.

### 3.2 BER for M-PSK(8-PSK for M = 8)

For the general M-PSK there is no simple expression for the symbol-error probability if M > 4. It can only be obtained from:

$$P_s = 1 - \int_{-\pi/M}^{\pi/M} p_{\theta_r}(\theta_r)\, d\theta_r \qquad (7)$$

where

$$p_{\theta_r}(\theta_r) = \frac{1}{2\pi}e^{-2\gamma_s \sin^2\theta_r}\int_0^{\infty} V e^{-(V-\sqrt{4\gamma_s}\cos\theta_r)^2/2}dV \qquad (8)$$

$$V = \sqrt{r_1^2 + r_2^2} \qquad (9)$$

$$\theta_r = \tan^{-1}(r_2/r_1) \qquad (10)$$

$$\gamma_s = \frac{E_s}{N_0} \qquad (11)$$

$$r_1 \sim N(\sqrt{E_s}, N_0/2) \ \text{and} \ r_2 \sim N(0, N_0/2) \qquad (12)$$

are jointly Gaussian random variables

This may be approximated for high M and high $E_b/N_0$ by:

$$P_s \approx 2Q\left(\sqrt{2\gamma_s}\sin\frac{\pi}{M}\right) \qquad (13)$$

The bit-error probability for M-PSK can only be determined exactly once the bit-mapping is known. However, when Gray coding is used, the most probable error from one symbol to the next produces only a single bit-error and

$$P_b \approx \frac{1}{k}P_s \qquad (14)$$

(Using Gray coding allows us to approximate the Lee distance of the errors as the Hamming distance of the errors in the decoded bit stream, which is easier to implement in hardware.)

## IV. SIMULATION RESULTS

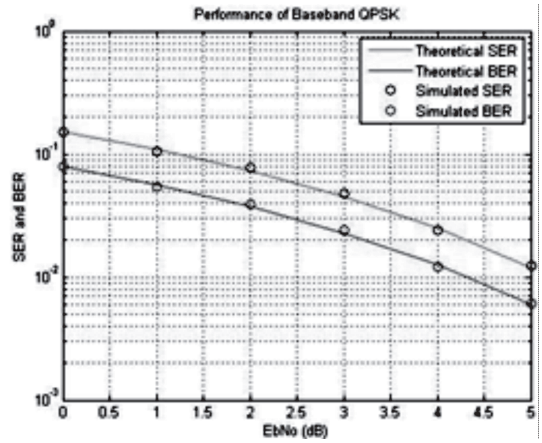


**Figure 7:** BER vs $E_b/N_0$ for Baseband QPSK

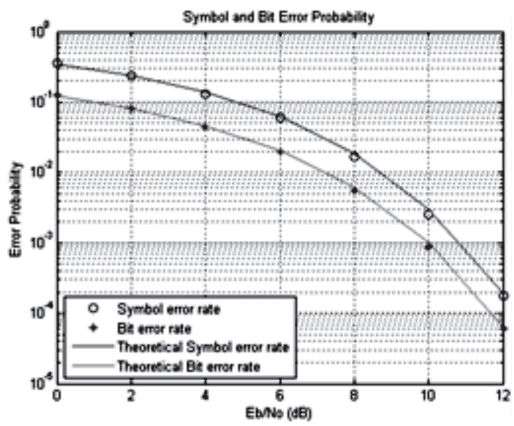


**Figure 8:** Symbol and Bit Error Probability for Baseband QPSK

## V. CONCLUSION

The graph compares the bit-error rates of QPSK Binary Coded 8-PSK and Gray Coded 8-PSK. It is seen that higher-order modulations exhibit higher error-rates; in exchange however they deliver a higher raw data-rate. The performance of Gray Coded 8-PSK and Binary Coded 8 PSK are shown in Figure 9. It is clear that the Gray Coding of higher order modulation provides better performance.

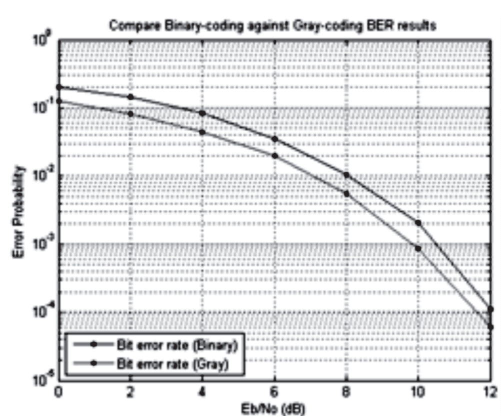


**Figure 9:** BER vs $E_b/N_0$ for Binary and Gray Coded 8-PSK

## REFERENCES

[1] M. Singhal, P. Bhardwaj, M. Trikha, "*Comparison of BER for Advance Modulation Technique using Bit Error Rate Tester*", MIT International Journal of Electronics and Communication Engineering, Vol. 2, pp. 16-19, January 2012.

[2] A. Ozozlu, J. Lima Pinto, K. Lamacraft, I. Darwazeh, "*Simulation of a Base Station Receiver for the GSM Evolution -EDGE*", Proceedings of International Conference on Telecommunications – ICT'2000, Vol. 1, Acapulco, Mexico, 22-25 May 2000, pp. 121-125.

[3] Ashkan Mashhour, "*Understanding Offset 8-PSK Modulation for EDGE*", Microwave Journal, October 1999, pp. 78-92.

[4] A. Bruce Carlson, Communication Systems: An Introduction to Signals and Noise in Electrical Communication, McGraw-Hill, 1986.

[5] J. Lima Pinto, I. Darwazeh, "*Phase Distortion and Error Vector Magnitude for 8-PSK Systems*", Proceedings of London Communications Symposium– LCS 2000, London, U.K., 14-15 September, pp. 37-40.

[6] Feher, K., Digital Communications with Microwave Applications, Prentice Hall, New Jersey, 1981.

[7] Lindsey. W.C. and Simon, M.K., Telecommunication Systems Engineering, Prentice Hall, New Jersey, 1973.

[8] Simon, M.K., "*On the Calculation in Costas Loops with Arbitrary Arm Filters*," IEEE Transactions on Communications, VOL COM-26, No. 1 (Jan. 1978) pp. 179-183.

[9] Simon, M.K., "*Tracking Performance of Costas Loops with Hard-Limited In Phase Channel*," IEEE Transactions on Communications, VOL COM-26, No, 4, April1978, pp. 420-432.

[10] Wolejsza, C.J., and Chakraborty, D., "*TDMA Modem Design Criteria*," Comsat Technical Review, Vol. 9, Number 2A, Fall 1979.

[11] Holmes, J.K., Coherent Spread Spectrum Systems, Wiley, New York, 1981.

[12] Le-Ngoc, T., and Feher, K., "*A Digital Approach to Symbol Timing Recovery Systems*," IEEE Transactions on Communications, VOL COM-28, No. 12, Dec. 1980, pp. 1993-1999.

[13] Lindsey, W.C., "*A Survey of Digital Phase Locked Loops*," Proceedings of the IEEE, Vol 69, No. 4, April 1981, pp. 410-430.

[14] Cessna, J.R., and Levy, D.M., "*Phase Noise and Transient Times for a Binary Quantized Digital Phase-Locked Loop in White Gaussian Noise*," IEEE Transactions on Communications, VOL COM-20, No. 2, April 1972, pp. 94-104.

[15] Intelsat Earth Station Standards (IESS) Document IESS-310, Rev. 1, November 30, 1998.

[16] E. Zehavi, "*Viterbi Decoder Bit Efficient Chainback Memory Method and Decoder Incorporating the Same*," US Patent 5,469,452, November 21, 1995.

[17] J.K. Wolf, "*Method and Apparatus for Resolving Phase Ambiguities in Trellis Coded Modulated Data*," US Patent 5,233,630, August 3, 1993.

[18] P.K. Singh and S. Jayasimha, "*A Low-complexity, Reduced-Power Viterbi Algorithm*," Proc. of VLSI '99, pp.61-66, IEEE Computer Society, Los Alamitos, CA.

[19] R.D. McCallister, B.A. Cochran and J.M. Liebetrou, "*Pragmatic Decoder and Method Therefor*," US Patent 6,078,625, June 20, 2000.

[20]. S. Jayasimha and T. Praveen Kumar, "Synchronizing Concatenated FECs," Proc. of National Conference on Communications (NCC-2000), pp. 13-16.