Quantitative Macroeconomics - PS III

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1 Transitions in a Representative Agent Economy

Representative agent maximizes

$$max \sum \beta^t (log c_t)$$

$$s.t.: k_{t+1} = i_t + (1-\delta) k_t$$

$$c_t + i_t = k_t^{1-\theta}(z_t * h_t)^\theta \tag{1}$$

Given the FOCs and the transversality condition the steady state is

$$1 = \beta(1 + (1-\theta)k^{-\theta}(zh)^\theta - \sigma)$$

$$c + k = k^{1-\theta}(zh)^\theta + (1-\sigma)k \tag{2}$$

Then:

$$k^{ss} = (zh)[\frac{1-\theta}{\beta^{-1}-1+\delta}]^{\frac{1}{\sigma}} \tag{3}$$

$$c^{ss} = (zh)[(\frac{1-\theta}{\beta^{-1}-1+\delta})^{\frac{1-\theta}{\theta}} - \delta(\frac{1-\theta}{\beta^{-1}-1+\delta})^{\frac{1}{\theta}}] \tag{4}$$

Given the parameters, the z which matches an annual capital-output ratio of 4 and an annual investment ratio of 0.25 is z=4.08.

Considering a permanent doubling in z the new steady state is:

$$k^{ss} = (2zh)[\frac{1-\theta}{\beta^{-1}-1+\delta}]^{\frac{1}{\sigma}} \tag{5}$$

$$c^{ss} = (2zh)[(\frac{1-\theta}{\beta^{-1}-1+\delta})^{\frac{1-\theta}{\theta}} - \delta(\frac{1-\theta}{\beta^{-1}-1+\delta})^{\frac{1}{\theta}}] \tag{6}$$

1.1 Transition from the steady states

In order to compute the transition, z and z' are set respectively to 1 and 2.

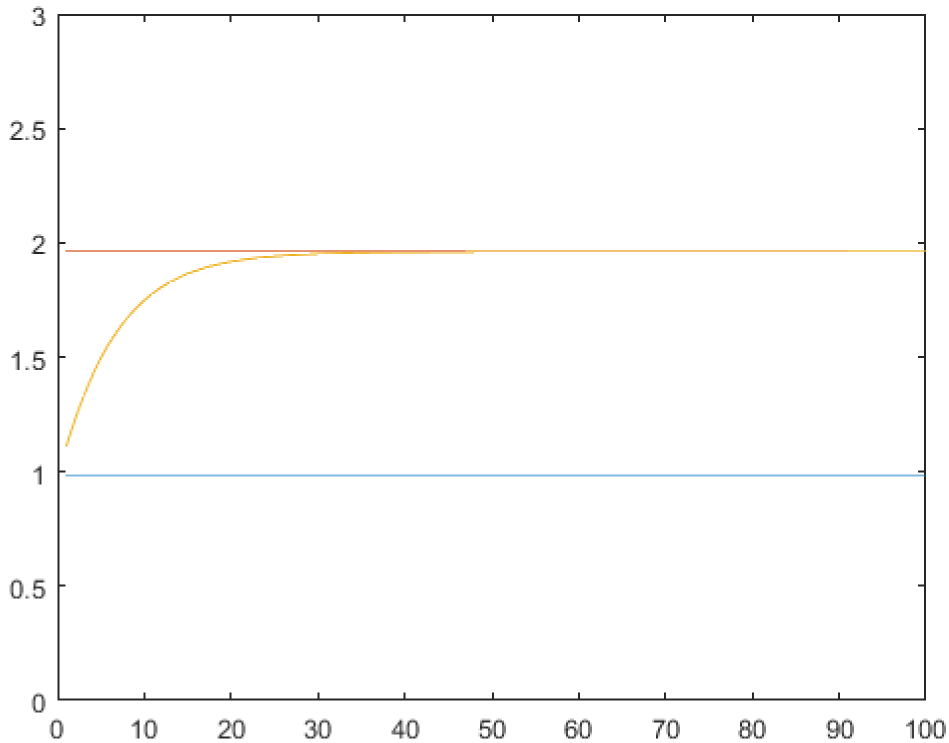

Figure 1: Transition for Capital

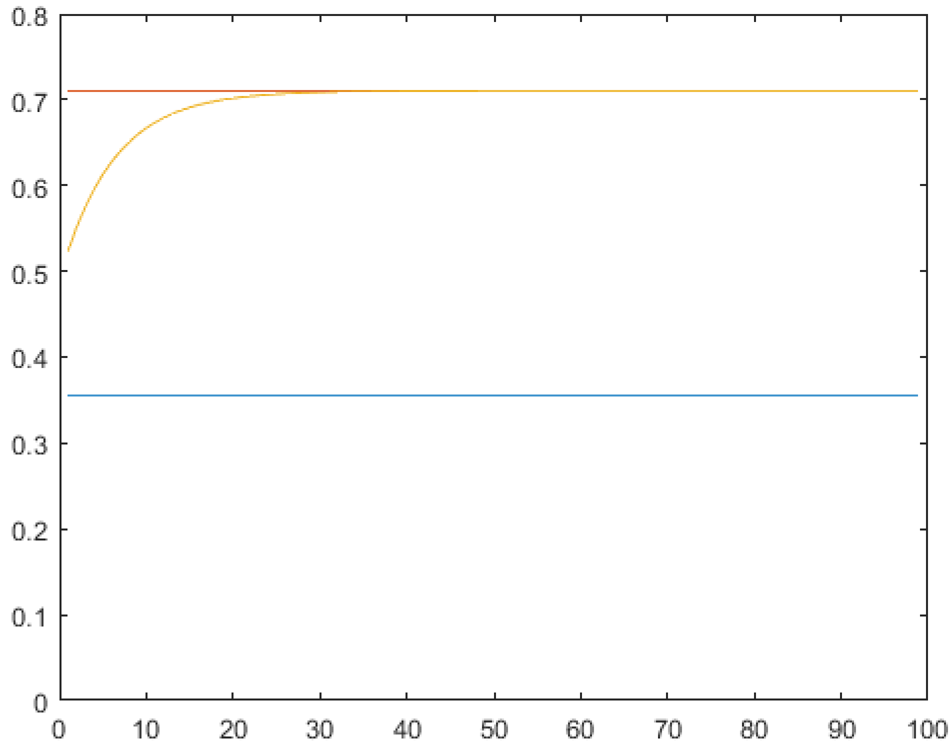

Figure 2: Transition for Consumption

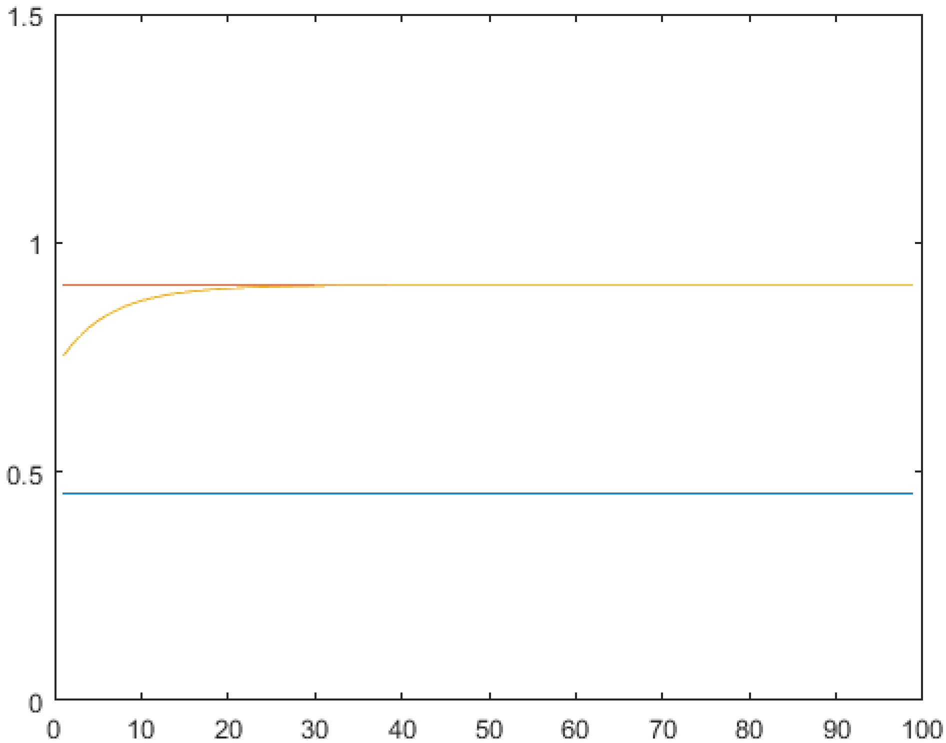

Figure 3: Transition for Output

1.2 Unexpected countershock

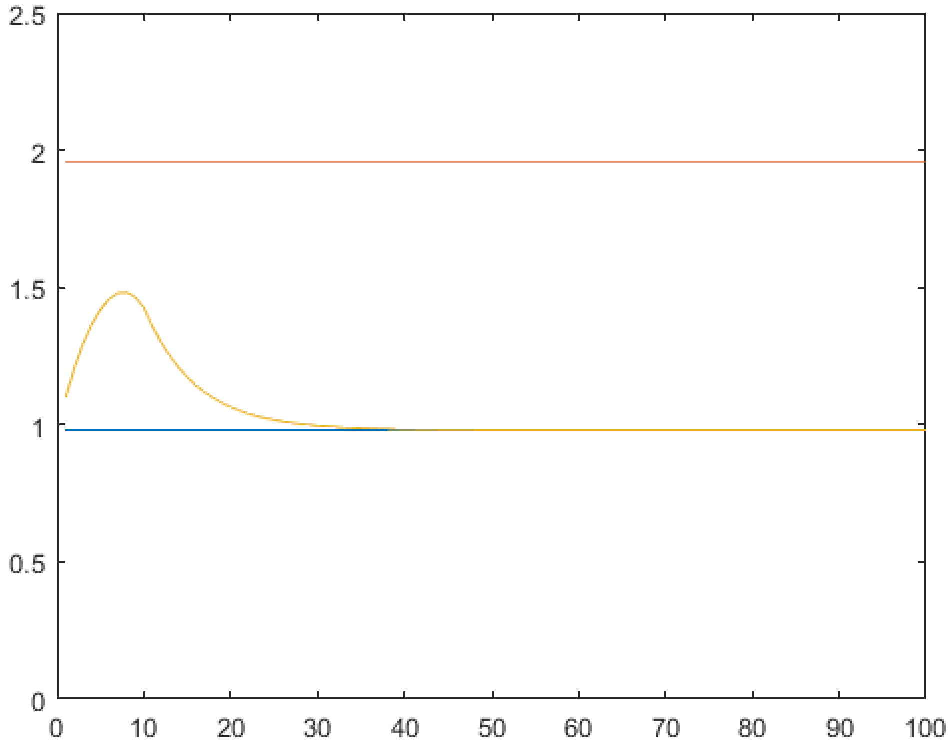

Figure 4: Transition for Capital

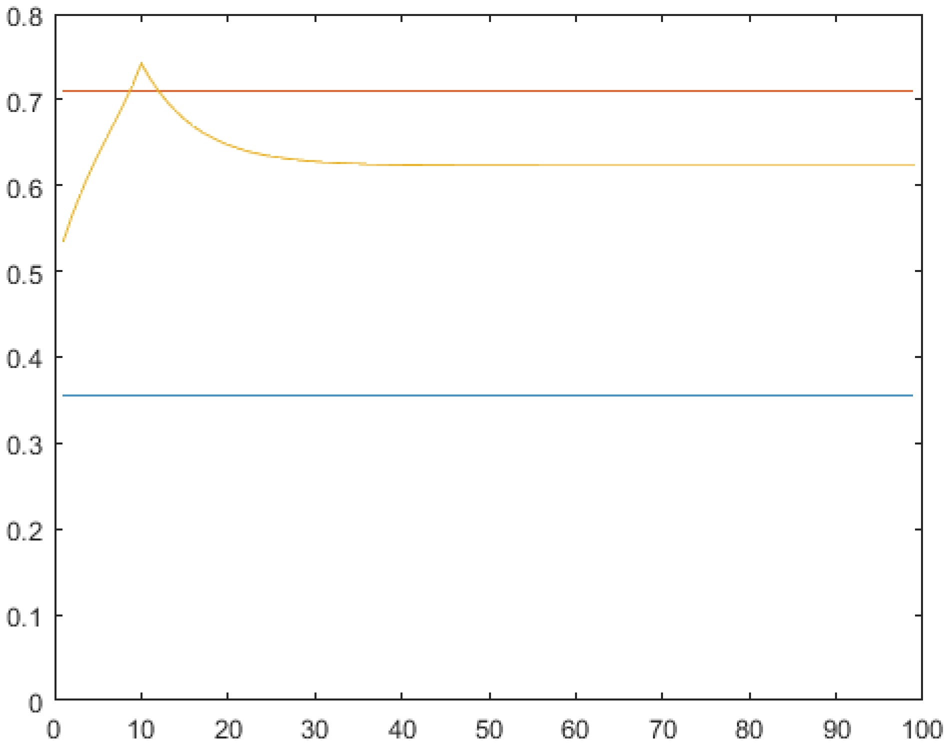

Figure 5: Transition for Consumption

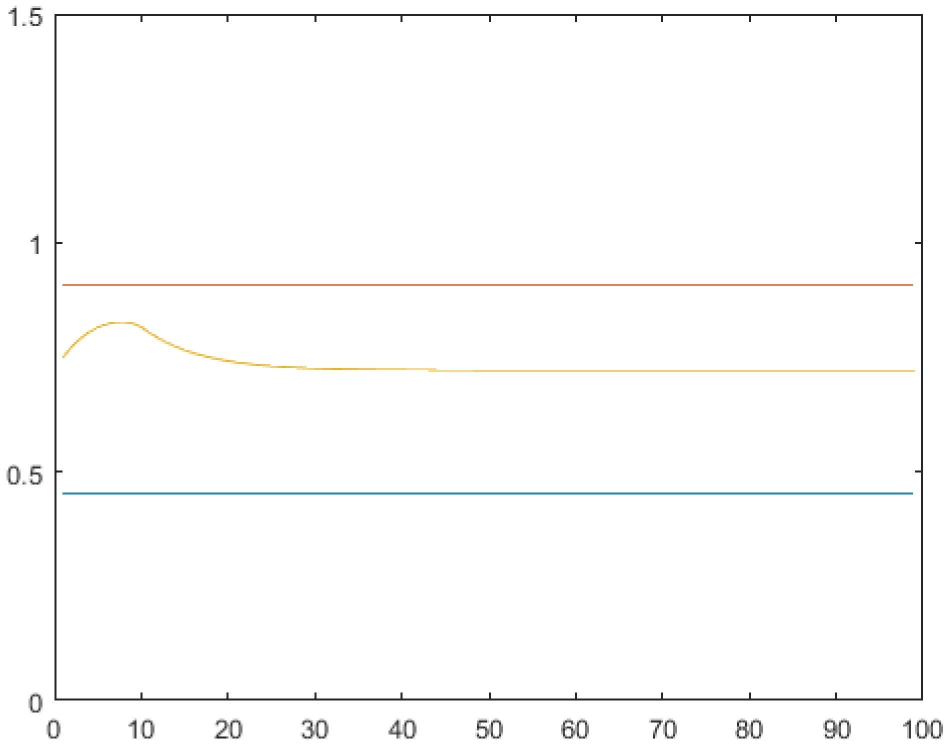

Figure 6: Transition for Output

2 General Equilibrium with Labor Supply, Uncertainty, and Progressive Labor Income Tax

2.1 Case without taxes

The General Equilibrium for this economy consists of prices r, w, value function V(y_0, η), optimal decisions rules: $g_a(y_0,\eta), g_c(y_0,\eta), g_h(y_0,\eta)$, type distribution of agents and labor supply, such that:

- Consumers maximization problem;
- (firms optimization problem);
- Consistency of distributions with optimal decision rules and Markov Chain;
- Market clearing conditions for capital, labor, assets.

In order to compute the equilibrium we solve:

$$\begin{aligned} L &= u(y_0 + wh - a, h) + \beta u(w'h' + (1+r)a, h') \\ \frac{\Delta}{\Delta a} &= -u_c'(c,h) + (1+r)\beta u_c'(c',h') = 0 \\ \frac{\Delta}{\Delta h} &= wu_c'(c,h) + u_h'(c,h) = 0 \\ \frac{\Delta}{\Delta h'} &= w'u_c'(c',h') + u_h'(c',h') = 0 \end{aligned} \tag{7}$$

Then we have a system of four equations, given by the 3 FOCs and the borrowing constraint:

$$\frac{1}{2}([[(\eta_y + 0.05)h + (1+r)a]^{-\sigma}(1+r)] + \frac{1}{2}([[(\eta_y - 0.05)h + (1+r)a]^{-\sigma}(1+r)] \tag{8}$$

$$[\eta_y h + y_{0-a]^{-\sigma}} = \kappa h^{\frac{1}{v}} \tag{9}$$

$$\frac{1}{2}([(\eta_y+0.05)h'+(1+r)a]^{-\sigma}](\eta_y+0.05)-\kappa h'^{\frac{1}{nu}}+\frac{1}{2}([(\eta_y-0.05)h'+(1+r)a]^{-\sigma}](\eta_y-0.05)-\kappa h'^{\frac{1}{nu}} \tag{10}$$

$$a = -\frac{1}{1+r}(y_1 - \epsilon_{y)} \tag{11}$$

Substituting the sequence of y_0 and the four possible productivity shocks we obtain the optimal solutions.

2.2 Proportional Tax

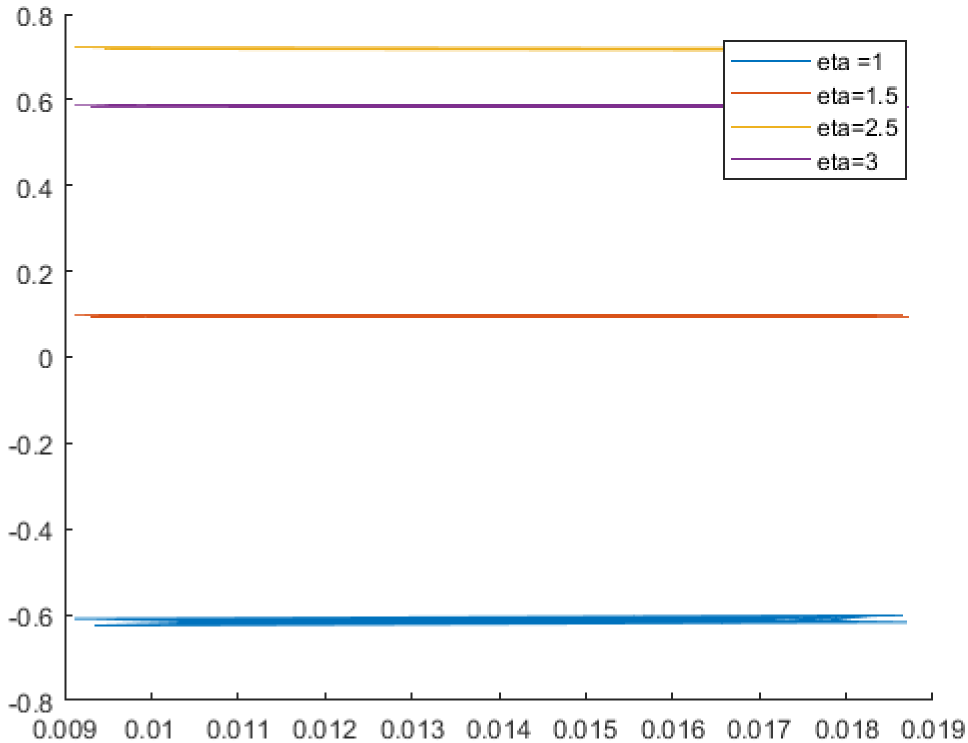

Figure 7: Optimal Saving

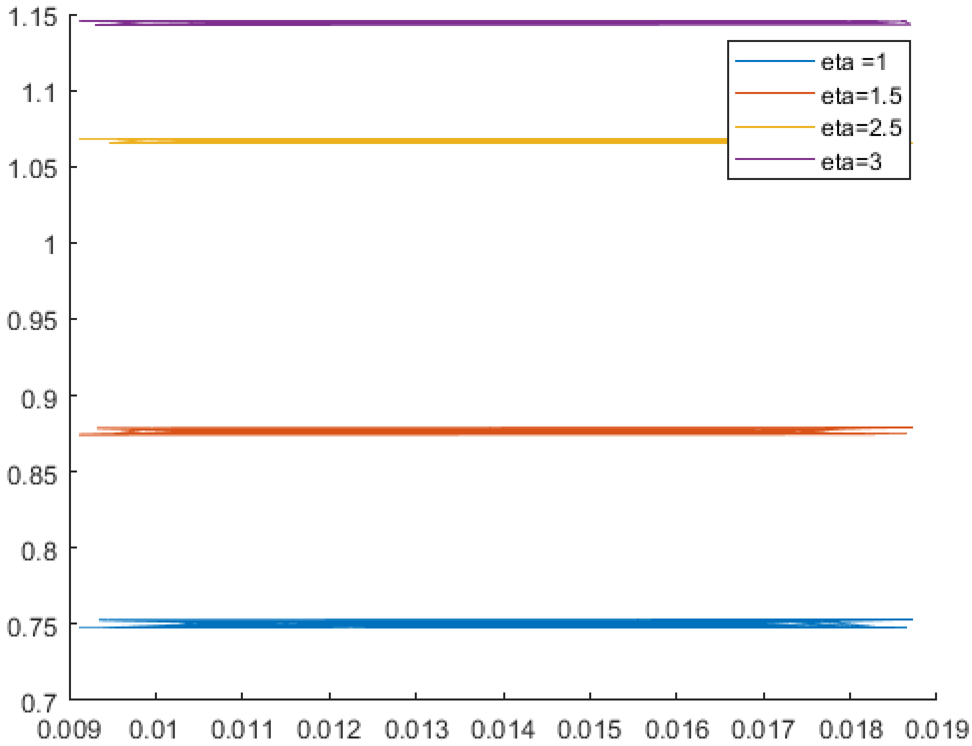

Figure 8: Optimal consumption

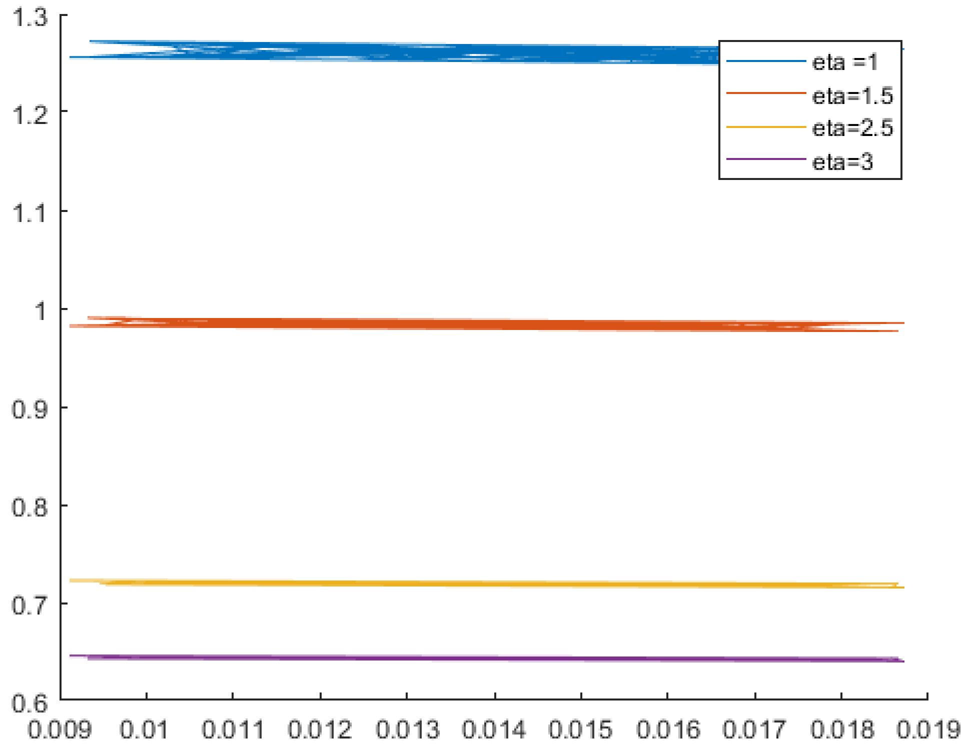

Figure 9: Optimal hours worked next period

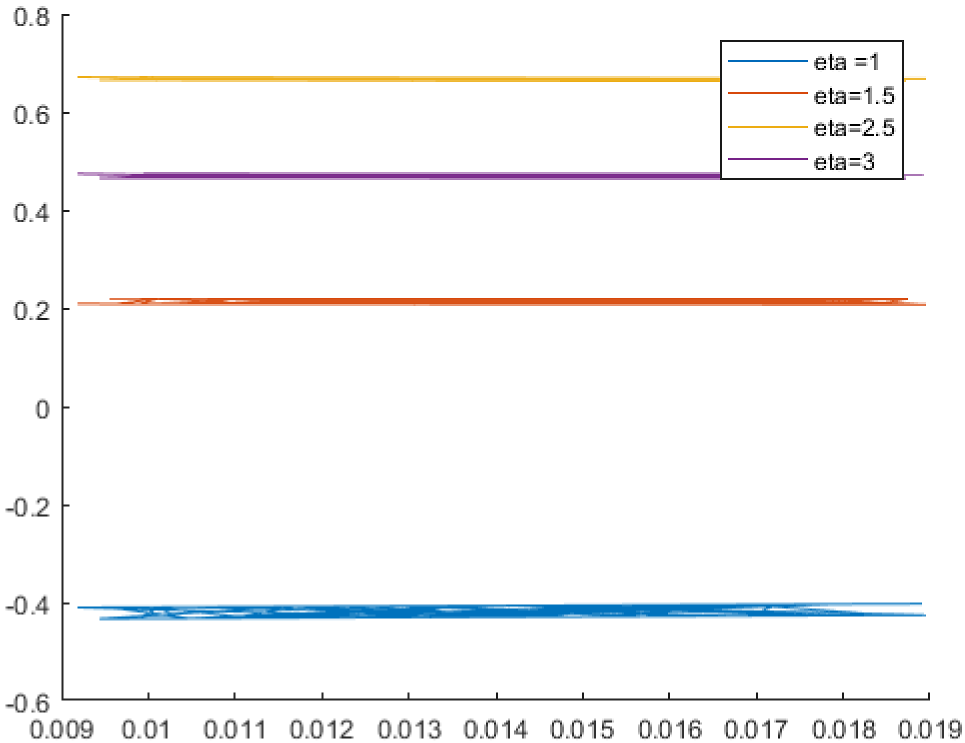

Figure 10: Optimal saving

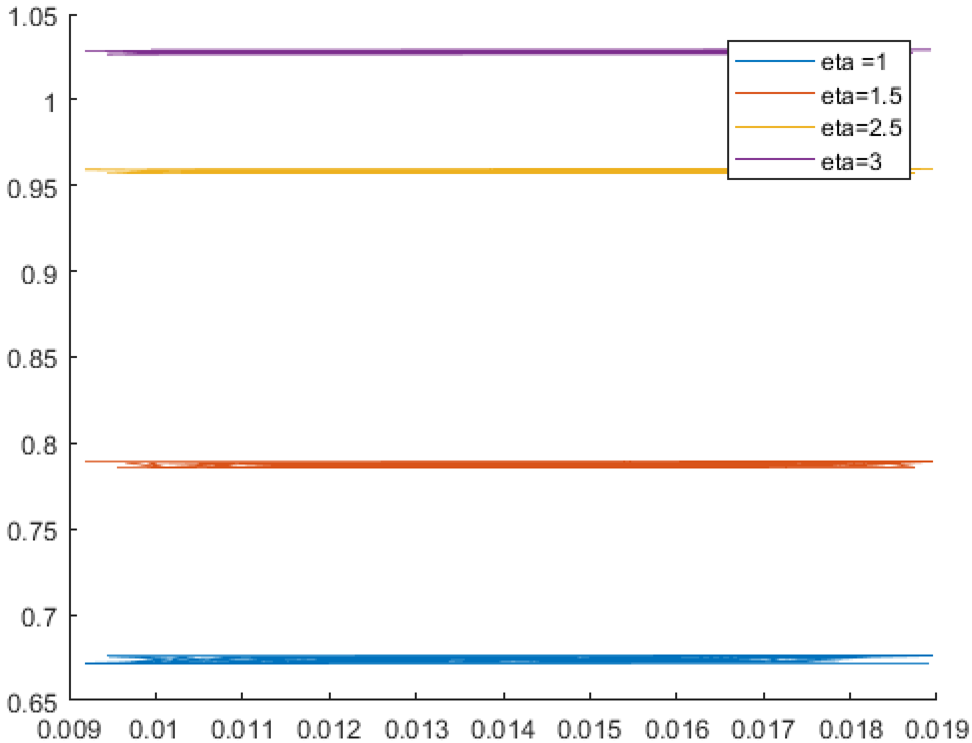

Figure 11: Optimal consumption

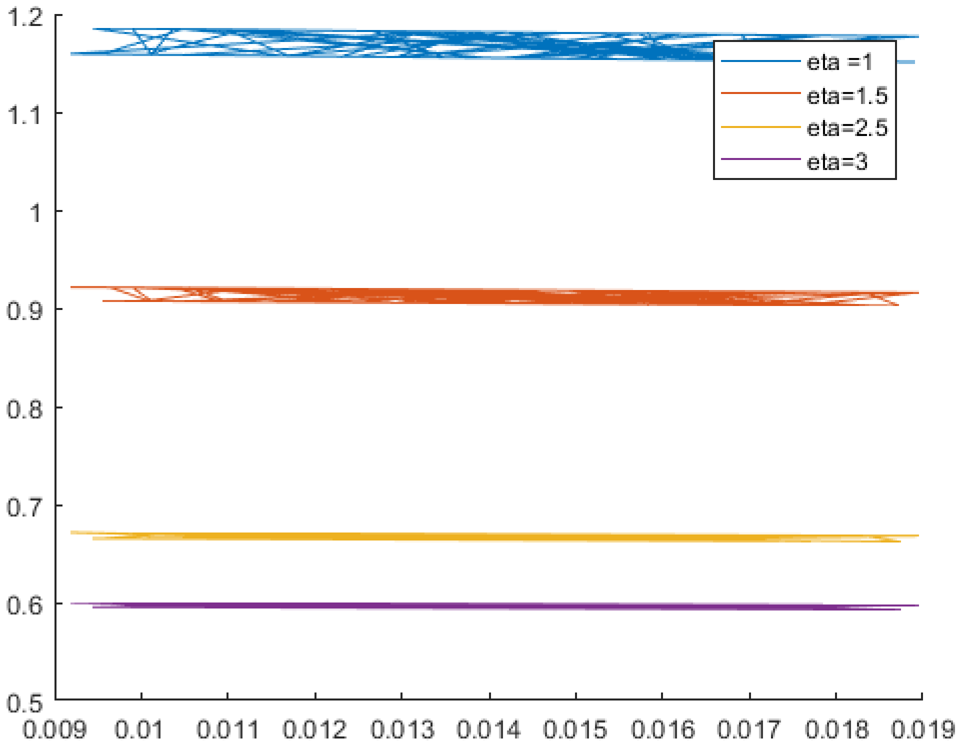

Figure 12: Optimal hours worked next period