Quantitative Macroeconomics Term Paper: Agents Heterogeneity in a Small Open Economy

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Abstract

This is a model of idiosyncratic and aggregate risk which includes some features of the big class of Open Macroeconomics Models. The model is solved using the Krusell and Smith (1999) algorithm and the Parametrized Expectation Approach.

1 Introduction

The standard industry of macroeconomic model with Heterogeneous Agents is based on the so called ABHI Model, where agents do not have access to a complete set of contingent consumption claims and market are exogenously incomplete. Market incompleteness arises because in the budget constraint of households all variables are function of the state at time t but not function of the state at time t+1. Then agents face an idiosyncratic risk.

Krusell and Smith (1999) combines the features of ABHI models with the Neoclassical Growth Model with aggregate risk. The result is a model with heterogeneous agents which face both aggregate and idiosyncratic risk. This characterization of the model brings some issues that have to be taken into account, in particular the aggregate state depends on individual decisions, which depend on the shock, and on the expectation about future aggregate shock. In other words, now, a characterization of the expectations about future aggregate state is needed for individual optimization. A solution to this issue is the Parametrized Expectations Approach: agents have forecasting rules for the evolution of aggregates state and they update these rules using an iterative procedure.

Large part of these model include as idiosyncratic risk: a shock on labor income and an aggregate shock on exchange rate. What I tried to do in this work is to modify the usual setup including some extensions typical of an open economy, in particular:

- Now there are two goods, a home good produced by the representative firm and sold in the country and a foreign good. Households can buy this foreign good from an exogenous foreign firm paying the cost of the good which is affected by the exchange rate volatility;
- Utility function of households includes both goods according to a CES utility function;
- The representative firm can trade with foreign sector and the interest rate is now affected by the exchange rate;
- There still is the idiosyncratic shock on labor income and there is not productivity shock;
- The home country is assumed to be a "small country", a change in the demand for the foreign good doesn't affect its price, which is, for simplicity, assumed fixed;
- The exchange rate is assumed floated in a band, for simplicity it can only take two values, the transition matrix is assumed exogenous.

2 The Model

There is a mass one of infinitely lived consumers with utility function given by:

$$U(c,h) = E_0 \sum \beta^t [(c_H^{\frac{\sigma-1}{\sigma}} + c_F^{\frac{\sigma-1}{\sigma}})^{\frac{\sigma}{\sigma-1}} - \Gamma \frac{h^{1+\gamma}}{1+\gamma}] \tag{1}$$

Flow household budget constrain is:

$$c_{t,H} + e_t c_{t,F} + k_{t+1} = (1+r)k_t + wh \tag{2}$$

Firm production function is the usual one:

$$y = k^\alpha h^\theta, \alpha + \theta < 1 \tag{3}$$

The household problem solves:

$$v(k,\epsilon;\Gamma,e) = max_{c,k'}\{u(c) - \gamma(h) + \beta E[v(k',\epsilon';\Gamma',e']\} \tag{4}$$

subject to:

$$c + a' = r(e,\mu)a + w(\mu)h\epsilon + (1-\delta)a$$
$$\Gamma' = H(\Gamma, e, e')$$
$$a' > -B \tag{5}$$

Each worker supplices ϵh units of labor input, ϵ can be 0 or 1.

The exchange rate, e, is stochastic, it has two possible realizations: 1.2 (good) and 0.8(bad). The transition is governed by the markov chain process $\pi_{ss'}$.

Idiosyncratic and aggregate shocks are correlated and $\pi_{ss'\epsilon\epsilon'}$ is the transition probability.

Markets are incomplete.

Equilibrium factor prices for given $\bar{k}$ and $\bar{h}$ are:

$$w(\bar{k},\bar{h}) = (1-\alpha)(\frac{\bar{k}}{\bar{h}})^{\alpha} \tag{6}$$

$$r(\bar{k},\bar{h},e) = \alpha(\frac{\bar{k}}{\bar{h}})^{\alpha-1} + e \tag{7}$$

The aggregate state is (Γ, e), where Γ is the distribution of consumers over holding of capital and employment status.

The law of motion for Γ is:

$$\Gamma' = H(\Gamma, e, e') \tag{8}$$

2.1 Algorithm

The KS procedure is used to compute the law of motion for the aggregate states. Agents are assumed following the forecasting rule:

$$logK' = b_0 + b_1 logK + b_2 e \tag{9}$$

The algorithm is:

1. Guess initial coefficients: b_0, b_1, b_2 for good period and bad period, and guess initial optimal decision given these coefficients;

2. Solve the maximization problem of the household and get the optimal decision rules;

3. Simulate the economy and get individual labor income shock. Compute the decision rules and given that the average capital stock and labor supply.

4. Make a OLS regression on the forecasting rule of agents, getting a new estimation for the parameters b_0, b_1, b_2;

5. Check the convergence, otherwise go back to step 2;

6. Once the convergence is achieved, compute $R2^2$ index to check how much the approximation is good.

3 Quantitative Results

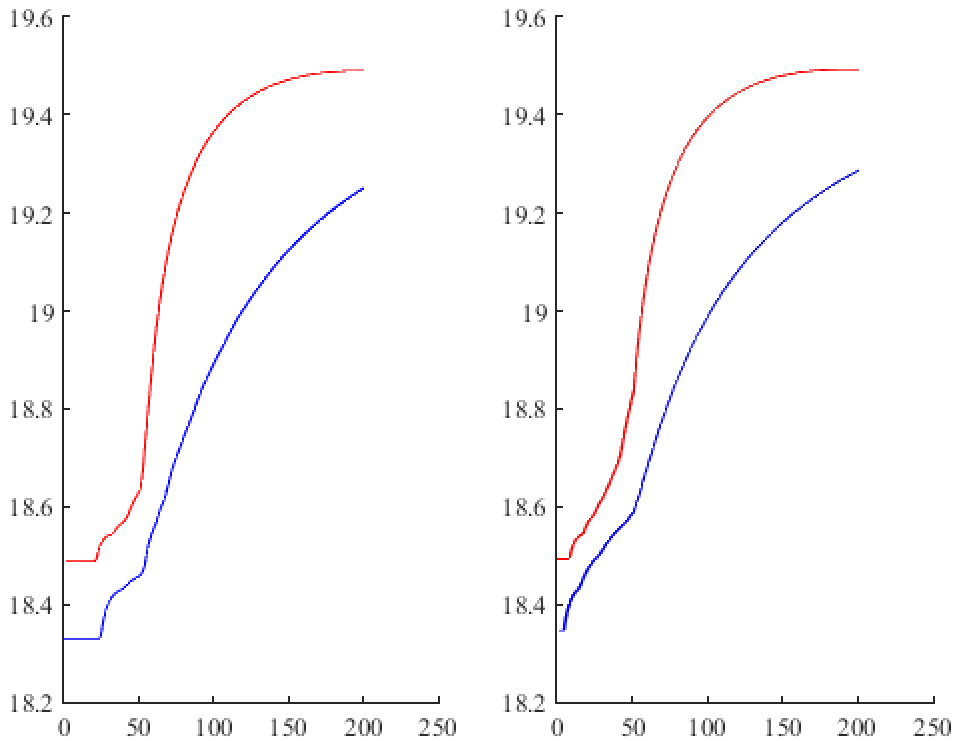

Figure 1: Value functions. LEFT: VF for unemployed with high exchange rate (red) and low exchange rate (blue). RIGHT: VF for employed with high exchange rate (red) and low exchange rate (blue).

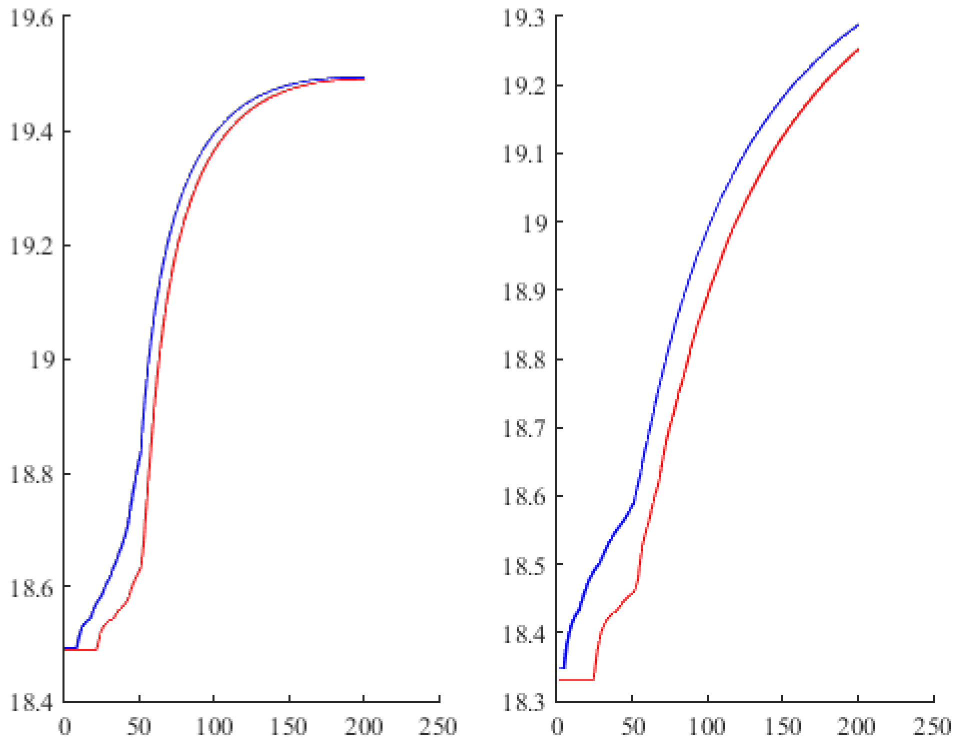

Figure 2: Value functions. LEFT: VF in high exchange rate case for unemployed (red) and employed (blue). RIGHT: VF in low exchange rate case for unemployed (red) and employed (blue).

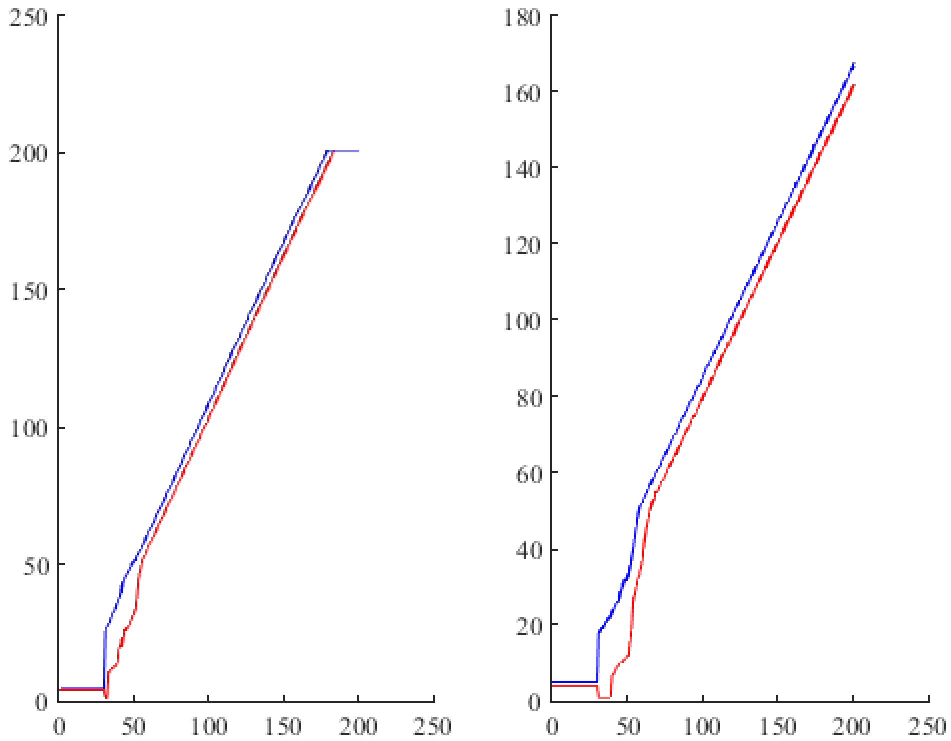

Figure 3: Optimal Policy function for Assets. LEFT: OP in high exchange rate case for unemployed (red) or employed (blue). RIGHT: OP in low exchange rate case for unemployed (red) or employed (blue).

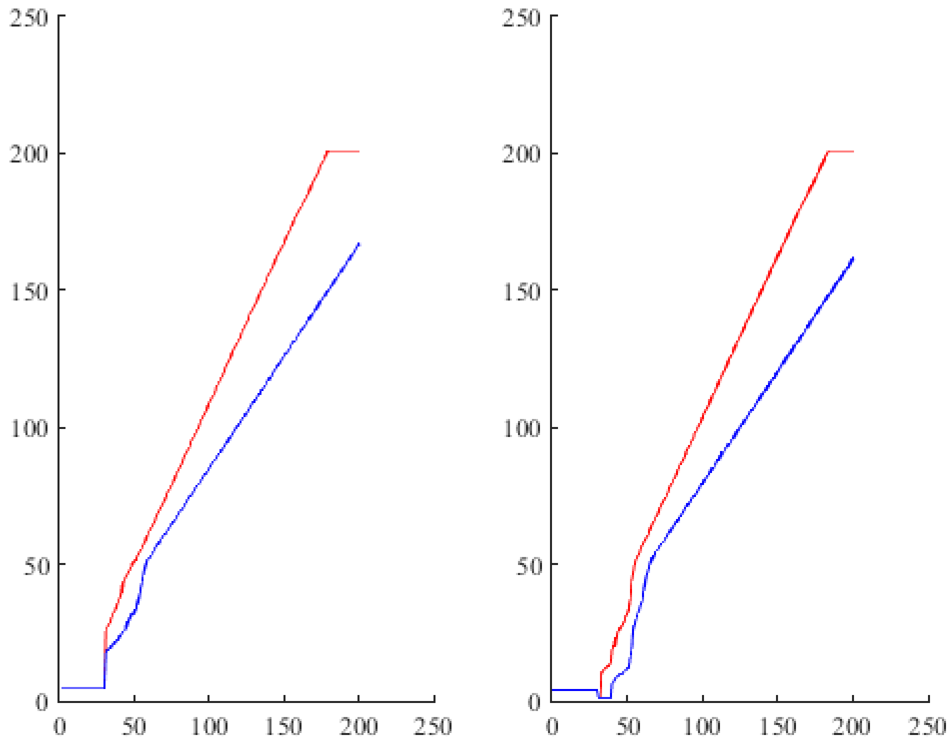

Figure 4: Optimal Policy function for Assets. LEFT: OP for employed in high exchange rate case (red) and low (blue). RIGHT: OP for unemployed in high exchange rate (red) and low(blue).

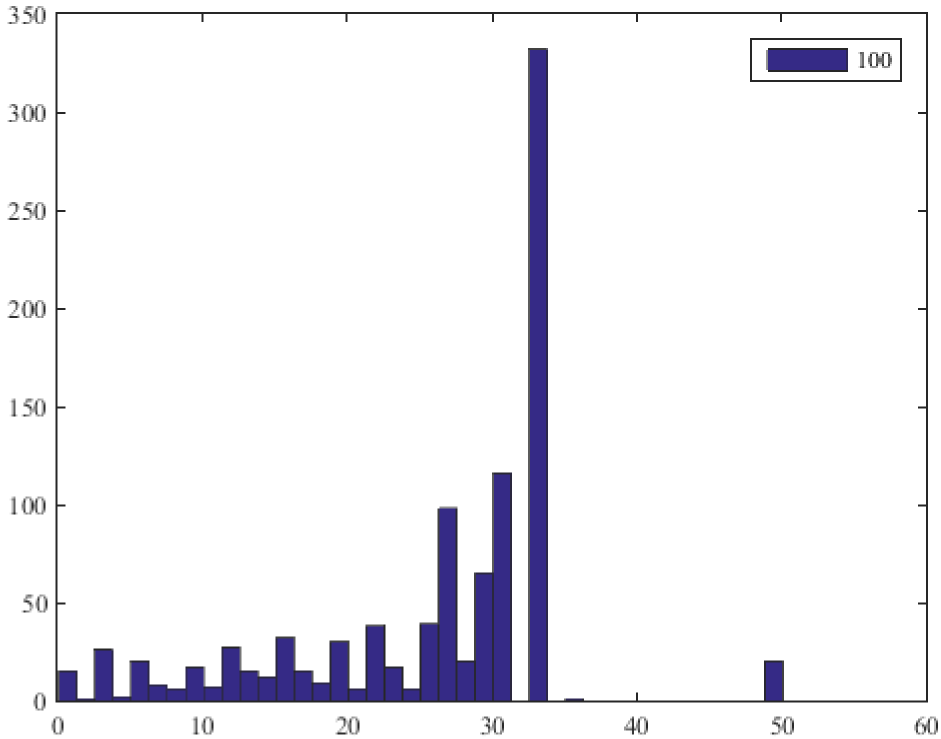

Figure 5: Evolution of Assets Distribution

3.1 Comments

In order to comment the results is needed a clarification about the exchange rate definition adopted. The definition adopted here is the indirect quotation (the home currency is the unit currency), therefore the cost of foreign good is multiplied times the exchange rate. Then a lower exchange rate implies that the domestic currency is depreciating or becoming weaker.

There is a small difference among the two Value Functions for different exchange rates given the employment status (figure 1), in both cases the VF with high exchange rate is higher than the VF with low exchange rate. There is a higher difference if we consider the differences between the VFs for different employment status given the exchange rate (figure 2): the difference between employment status is higher considering the case with low exchange rate (then where the relative cost of the foreign good is higher).

Households save more when the exchange rate is high than the case with low exchange rate (then when the relative cost of the foreign good is lower), this happen independently by the employment status (figure 3). The difference between the optimal policies with high or low exchange rate is almost equal considering the different employment status (figure 4).

Finally, the simulation of the model does not provide perfect results in term of aggregation, as in Krusell and Smith (1998), but they are still good: With high exchange rate:

$$log(\bar{k}') = 0.01 + 0.77 log \bar{k} + 0.55e$$

with

$$R^2 = 0.73 \tag{10}$$

With low exchange rate:

$$log(\bar{k}') = 0.01 + 0.67 log \bar{k} + 0.76e$$

with

$$R^2 = 0.58 \tag{11}$$

4 Conclusions

This model does not pretend to fully characterize an open economy but it is rather a starting point for further extensions of the Krusell and Smith setup.

This extension includes the presence of a foreign good, bought by households and sold by a representative firm. This scenario can be further extended including two types of firms: one "open firm" which trades with the foreign sector and another "close firm" which trades only with the internal sector. According to my knowledge this type of firms heterogeneity is in large part still unexplored by literature.

References

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