Quantitative Macroeconomics - PS VII

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1 Solution of the model without aggregate risk

1.1 Household problem

The household problem in recursive formulation is:

$$v(k,\epsilon;\Gamma,z) = max_{c,a'}((log(c) - \Gamma\frac{n^{1+\gamma}}{1+\gamma}) + \beta E[v(k',\epsilon';\Gamma',z') \mid z,\epsilon)) \tag{1}$$

under the constraints:

$$c + a' = (1+r(z,\mu))a + w(z,\mu)n\epsilon + (1-\delta)a \tag{2}$$

$$\mu' = H(\mu) \tag{3}$$

$$a' >= -B \tag{4}$$

The intratemporal condition is:

$$v'(n) = u'(c)w\epsilon \tag{5}$$

The intertemporal conditions are:

$$u'(c) = \beta E[u'(c')(1+r)] + \mu \tag{6}$$

$$a' = (1+r)a + nw\epsilon - c \tag{7}$$

From (5) we have

$$w_t = \frac{v'(n_t}{u'(c_t)\epsilon} \tag{8}$$

Substituting in the budget constraint we have:

$$c + a' = (1+r(z,\mu))a + \frac{v'(n)}{u'(c)}n + (1-\delta)a \tag{9}$$

And the Bellman equation is:

$$v(a,e;\Gamma,z) = max_{a'}(u((1+r)a + \frac{v'(n)}{u'(c)}n + (1-\delta)a - a') + \beta E[v(k',\epsilon';\Gamma',z') \mid z,\epsilon] \tag{10}$$

1.1.1 Comparison of Policy functions

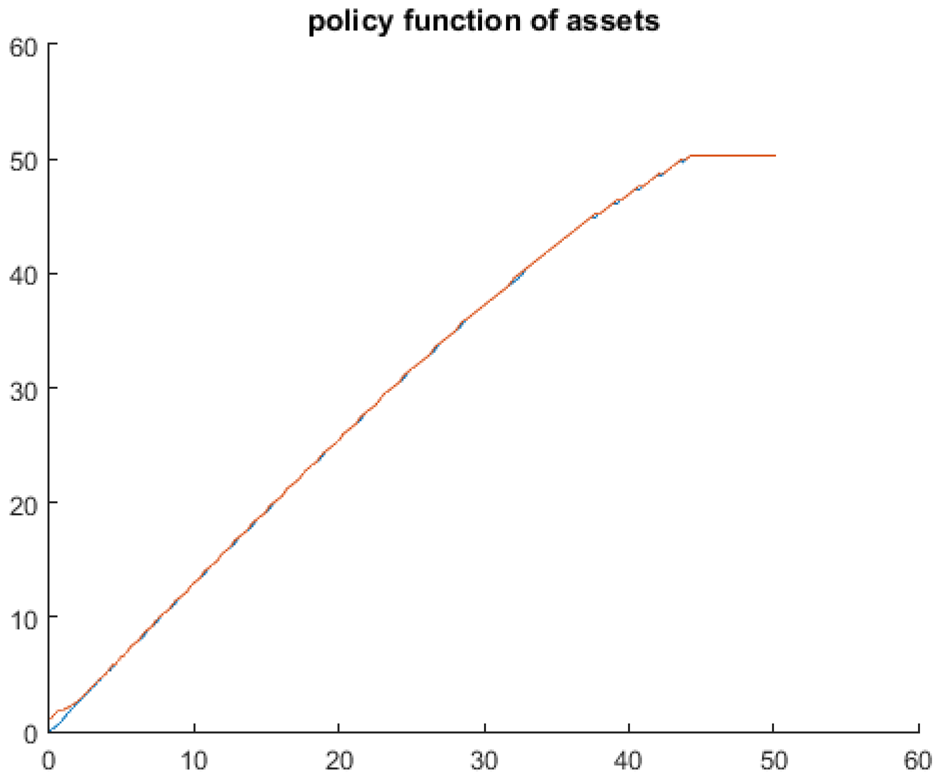

Figure 1: Policy function of asset in good periods

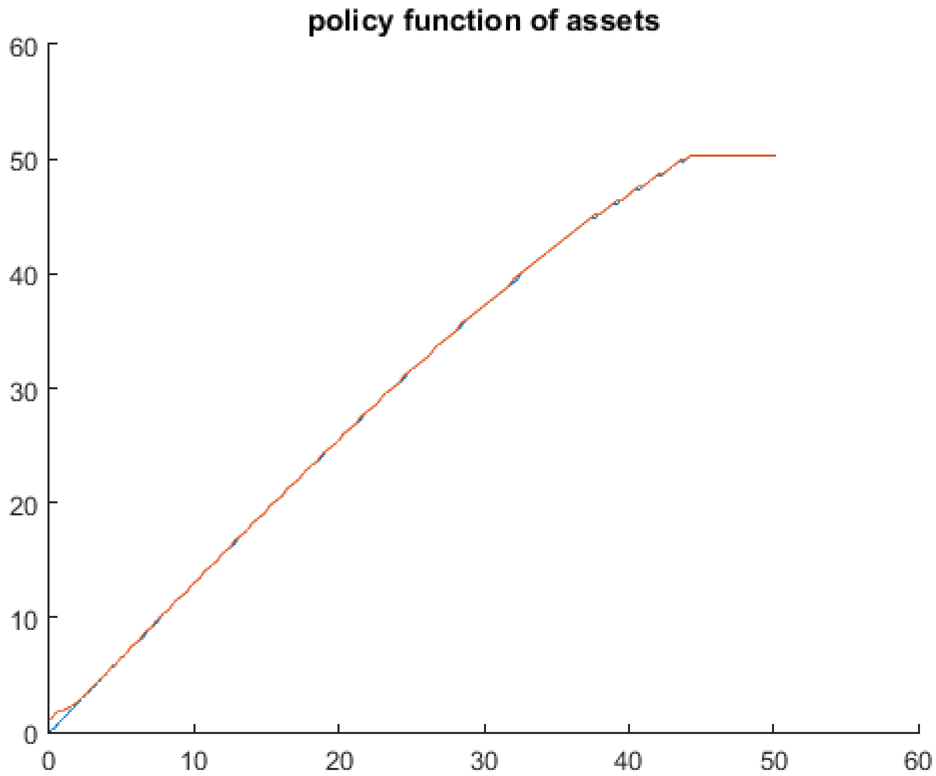

Figure 2: Policy function of asset in bad periods

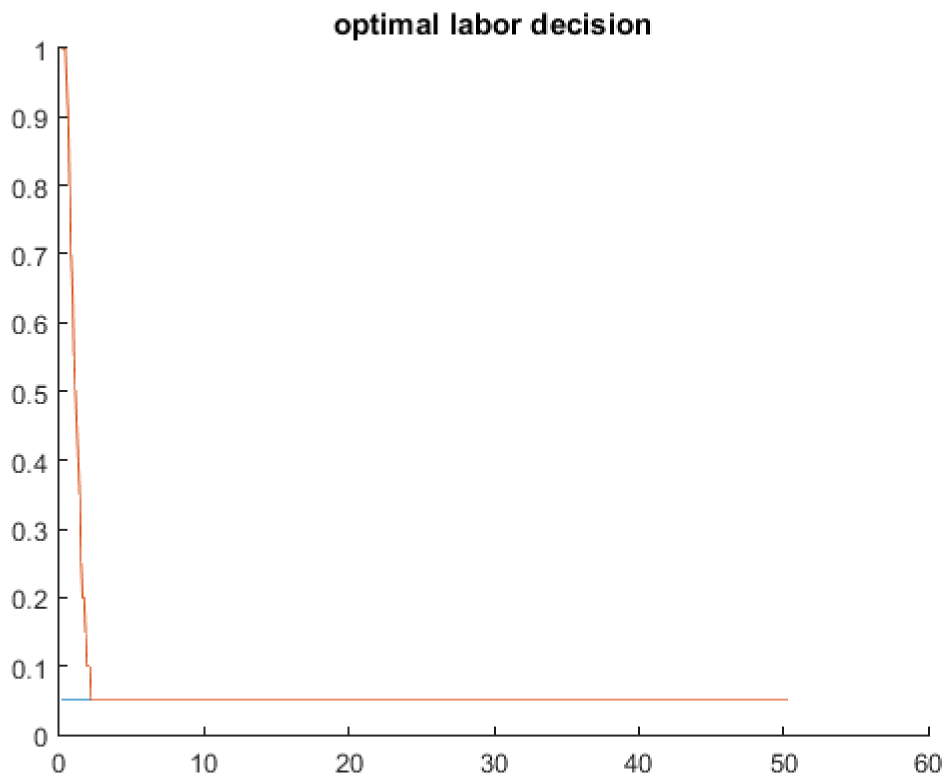

Figure 3: Optimal labor decision in good periods

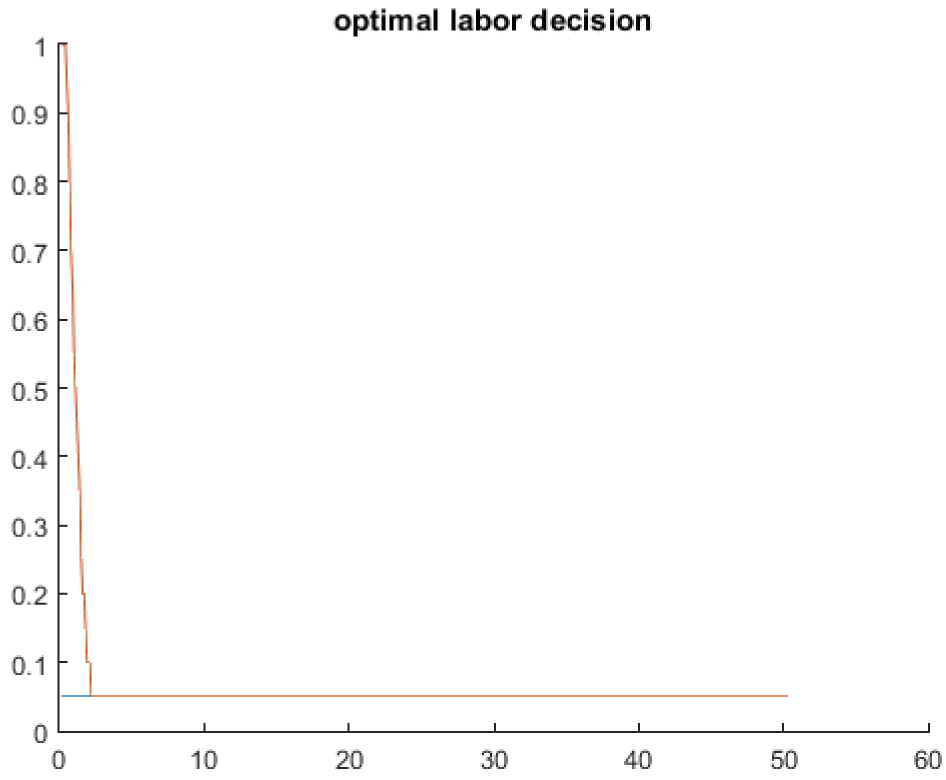

Figure 4: Optimal labor decision of asset in bad periods

1.2 Equilibrium

Equilibrium conditions are: (Labor market is also in equilibrium by Walras Law)

$$f(K,N)+(1-\delta)K = C+K' \tag{11}$$

$$K = \int \sum g(a,y)\Gamma(a,y)da \tag{12}$$

2 The solution with aggregate risk

The household problem in recursive formulation is:

$$v(\bar{k}, \epsilon, z, \mu_z) = max_{k', c, n)} = (u(c,n) + \beta \sum_z \sum_c v(k', \epsilon', z', \mu') \pi_{z', \epsilon')}) \tag{13}$$

It can be rewritten as:

$$v(\bar{k}, \epsilon, z, K) = max_{c,n,k'}(u(c,n) + \beta \sum_z \sum_\epsilon v(k', z', \epsilon', K') \pi_{z', \epsilon'} \tag{14}$$

where K'=H(K,z) and N= G(K,z)