

Quantitative Macroeconomics - PS I

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*Part of this code is shared with Valerio Pieroni

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1 Show that σ is the ES and compute the labor share

$$f(k, h) = [(1 - \alpha)k^{\frac{\sigma-1}{\sigma}} + \alpha h^{\frac{\sigma-1}{\sigma}}]^{\frac{\sigma}{\sigma-1}} \quad (1)$$

$$\frac{\Delta f(k, h)}{\Delta k} = \frac{\sigma}{\sigma-1} [(1 - \alpha)k^{\frac{\sigma-1}{\sigma}} + \alpha h^{\frac{\sigma-1}{\sigma}}]^{\frac{1}{\sigma-1}} \frac{(1 - \alpha)(\sigma-1)}{\sigma} k^{-\frac{1}{\sigma}} \quad (2)$$

$$\frac{\Delta f(k, h)}{\Delta h} = \frac{\sigma}{\sigma-1} [(1 - \alpha)k^{\frac{\sigma-1}{\sigma}} + \alpha h^{\frac{\sigma-1}{\sigma}}]^{\frac{1}{\sigma-1}} \frac{h(\sigma-1)}{\sigma} h^{-\frac{1}{\sigma}} \quad (3)$$

Dividing the two equations

$$\frac{\frac{\Delta f(k, h)}{\Delta k}}{\frac{\Delta f(k, h)}{\Delta h}} = \frac{(1 - \alpha)h^{\frac{1}{\sigma}}}{\alpha k^{\frac{1}{\sigma}}} = \frac{1 - \alpha}{\alpha} \left(\frac{h}{k}\right)^{\frac{1}{\sigma}} = \frac{mpk}{mph} \quad (4)$$

$$\log\left(\frac{mpk}{mph}\right) = \log\left(\frac{1 - \alpha}{\alpha}\right) \frac{1}{\sigma} \log\left(\frac{h}{k}\right) \quad (5)$$

Taking the inverse of the derivative of (5) wrt $\log(\frac{k}{h})$ we got σ which is the ES. Labor share is $\frac{H}{Y}$ and for this economy is given by:

$$\frac{H}{Y} = \alpha \frac{h^{\frac{\sigma-1}{\sigma}}}{y} \quad (6)$$

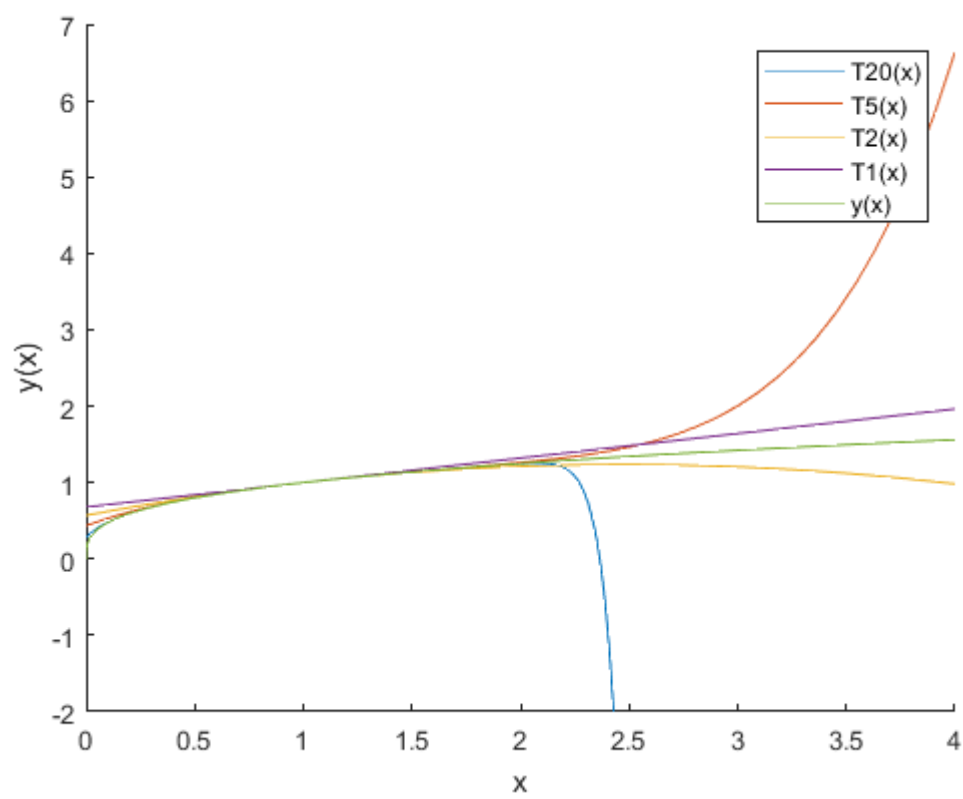


Figure 1: Taylor approximation of $y = x^{0.321}$

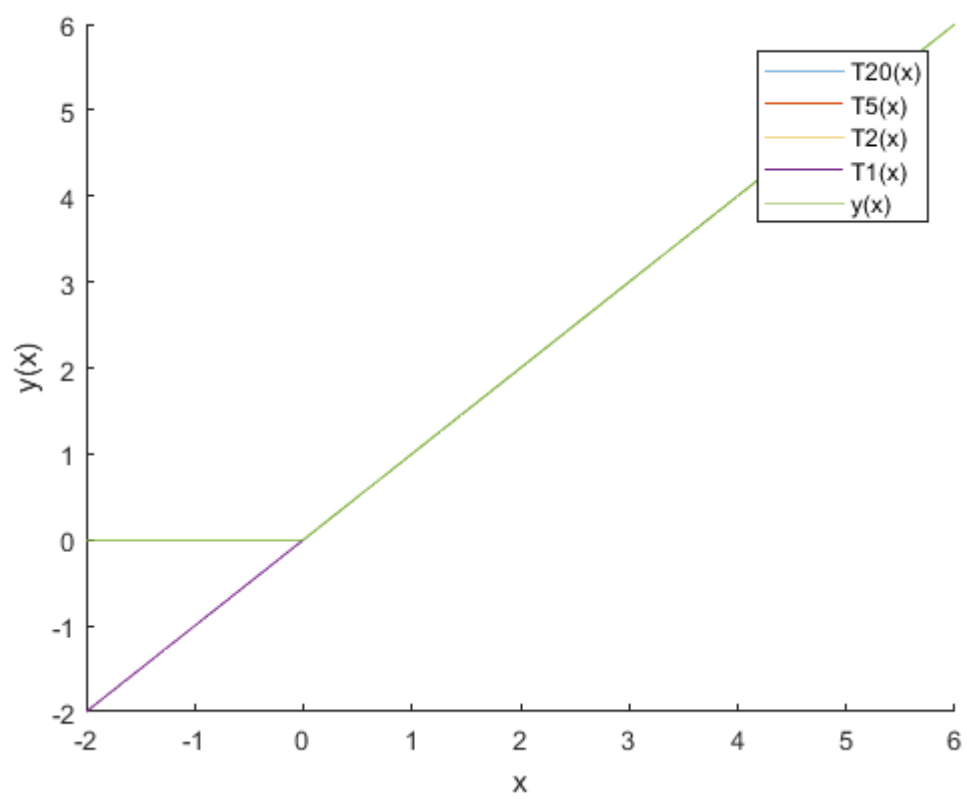


Figure 2: Taylor approximation of the ramp function

evenly spaced interpolation nodes, cubic polynomials, monomials of order 5 and

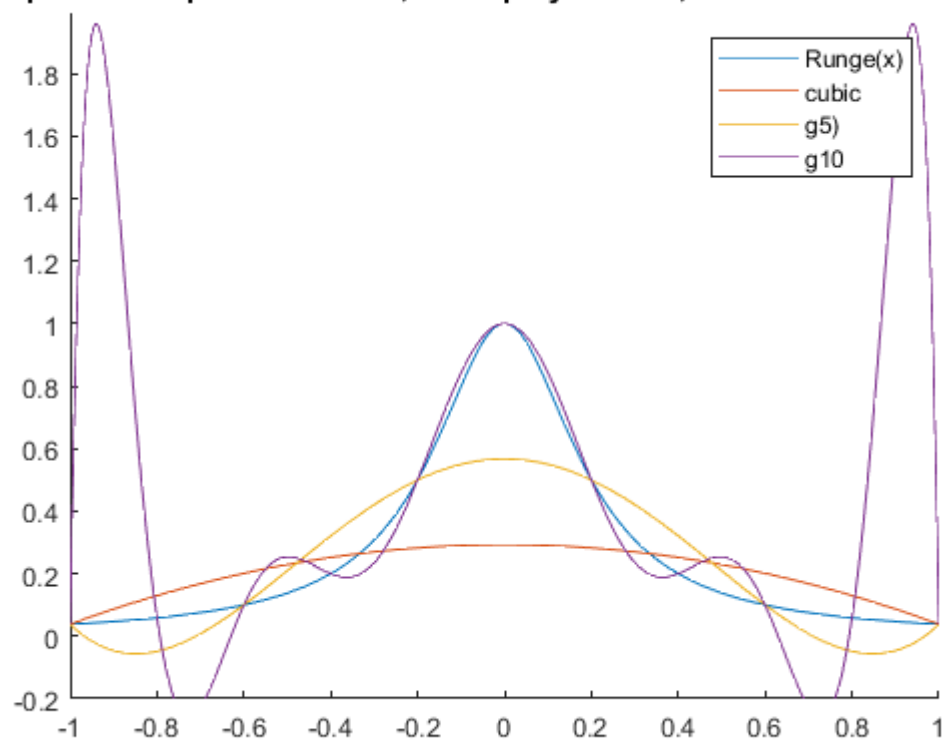


Figure 3: Runge function evenly spaced

Runge function approximation errors: evenly spaced and monomials

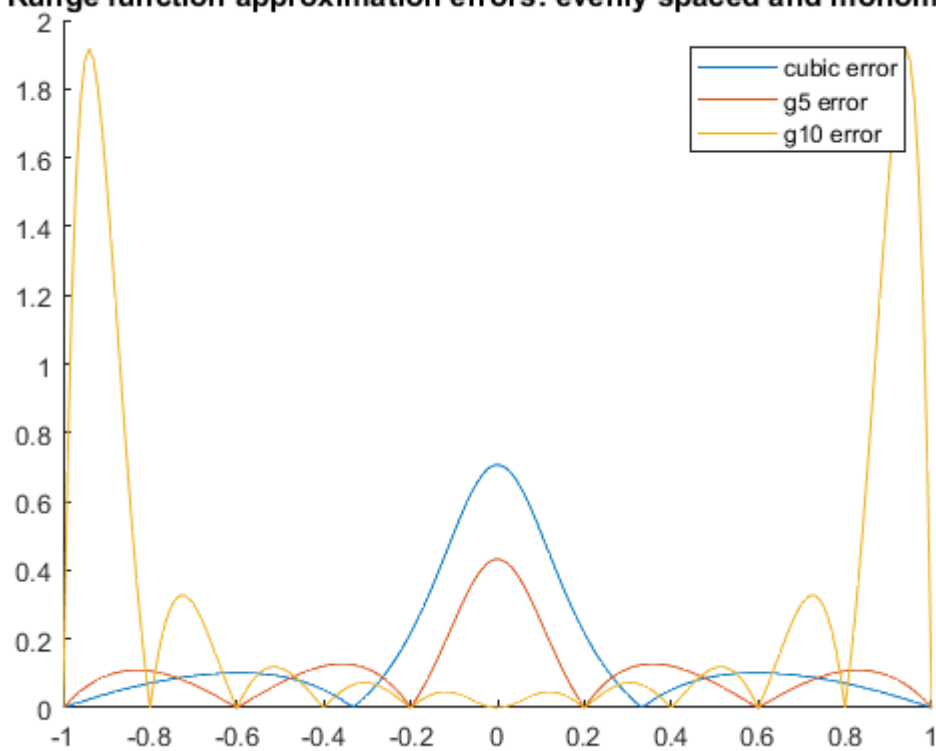


Figure 4: Runge function evenly spaced errors

venly spaced interpolation nodes, cubic polynomials, monomials of order 5 an

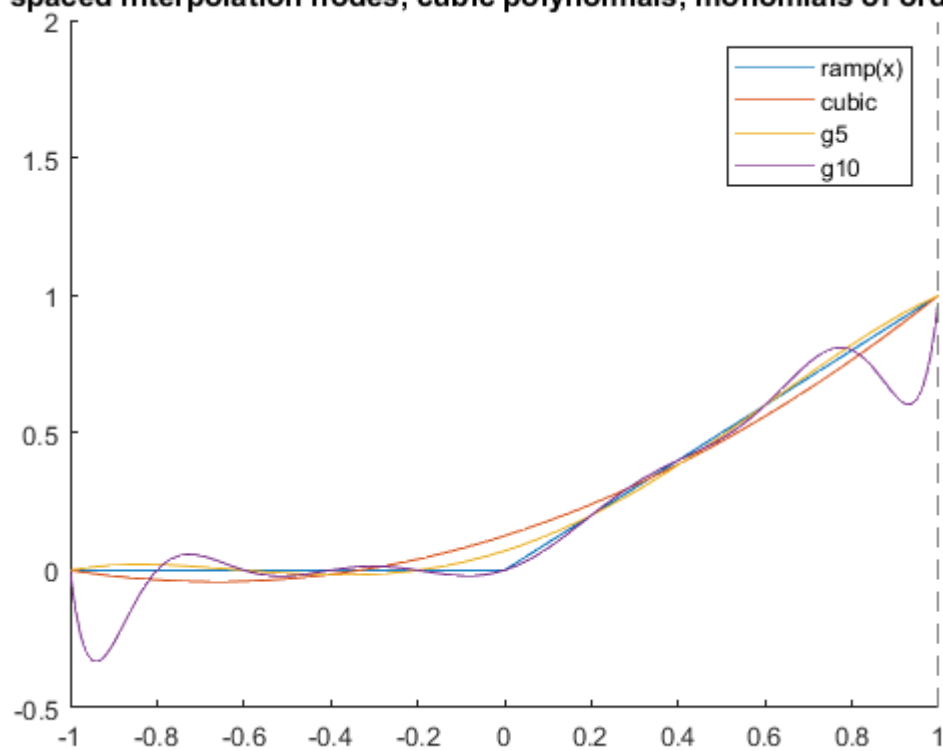


Figure 5: Ramp function evenly spaced

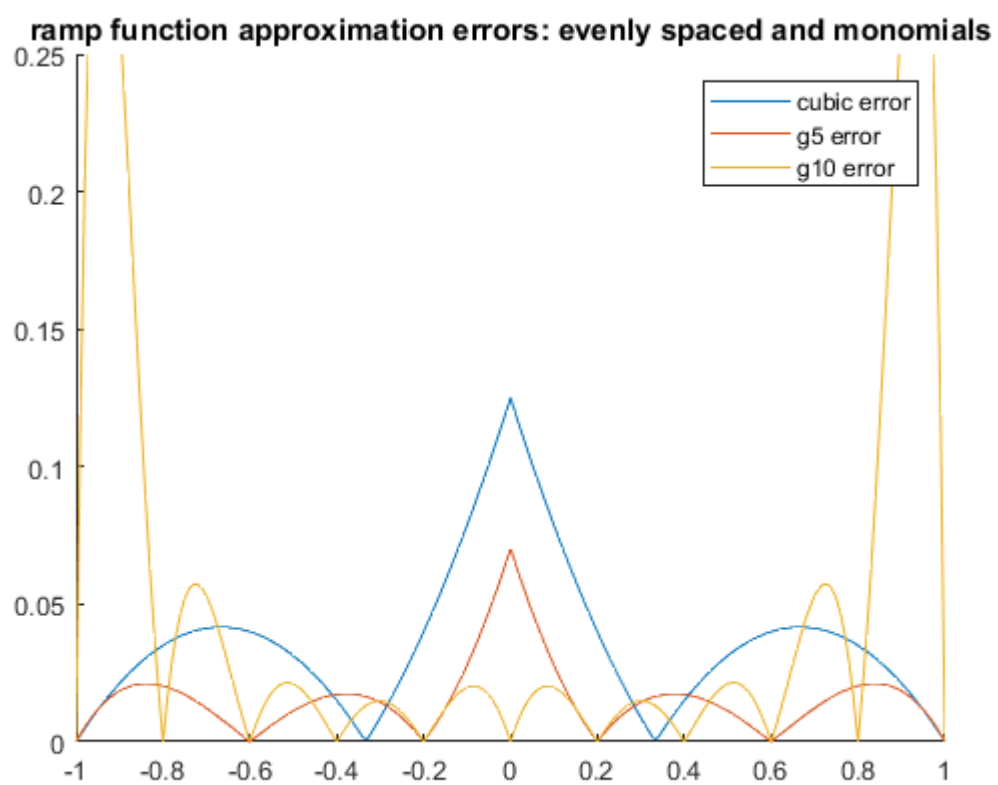


Figure 6: Ramp function evenly spaced errors

venly spaced interpolation nodes, cubic polynomials, monomials of order 5 an

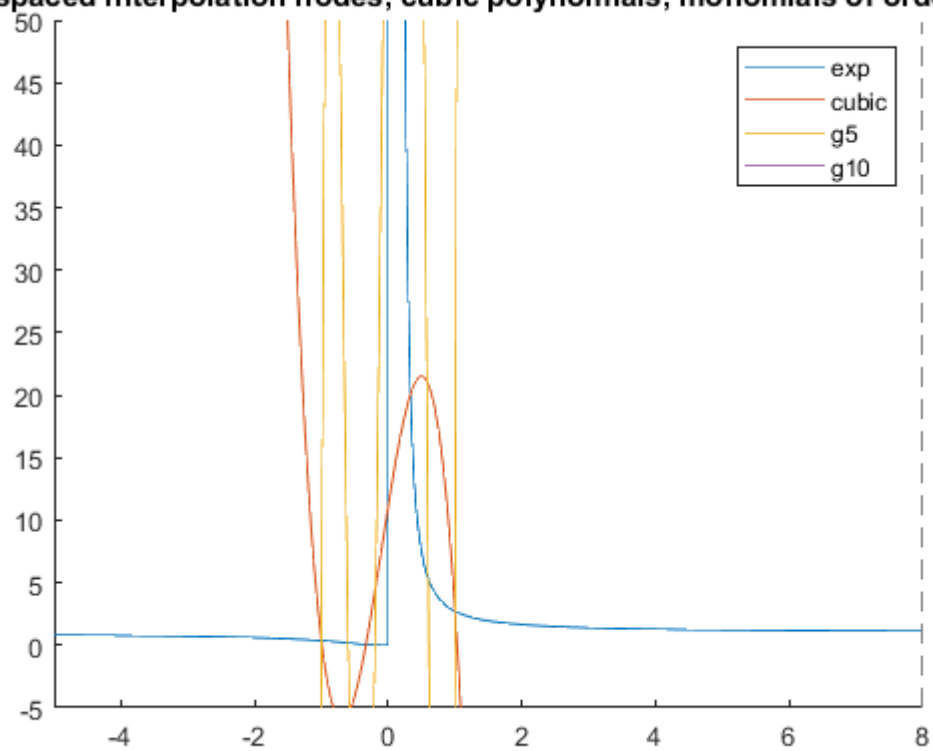


Figure 7: Exponential function evenly spaced

exponential function approximation errors: evenly spaced and monomials

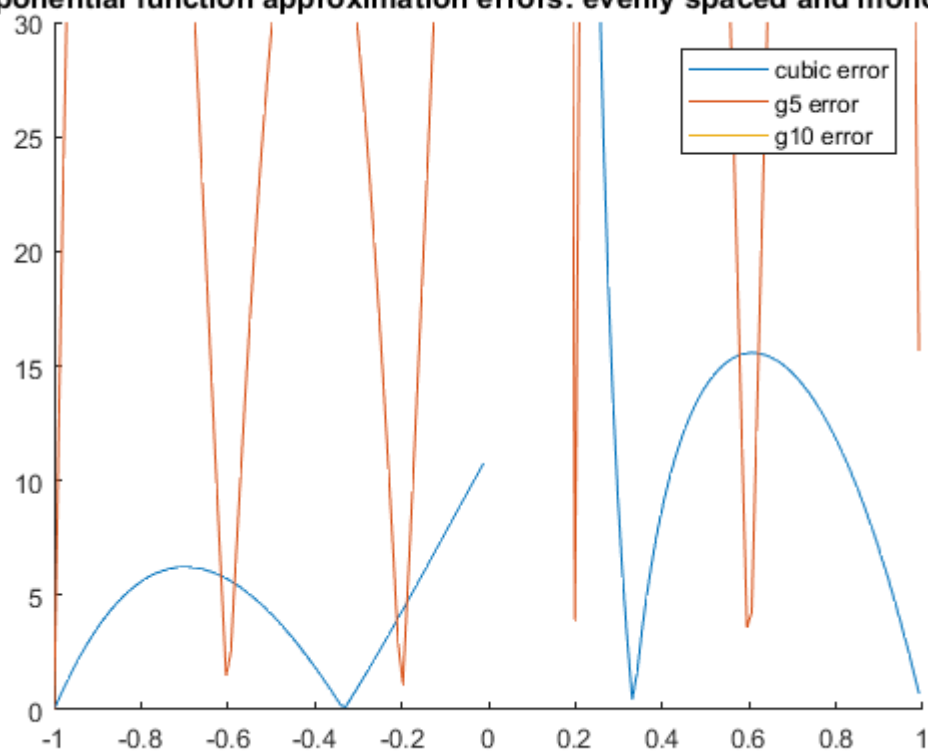


Figure 8: Exponential function evenly spaced errors

Chebyshev interpolation nodes, cubic polynomials, polynomials of order 5 and

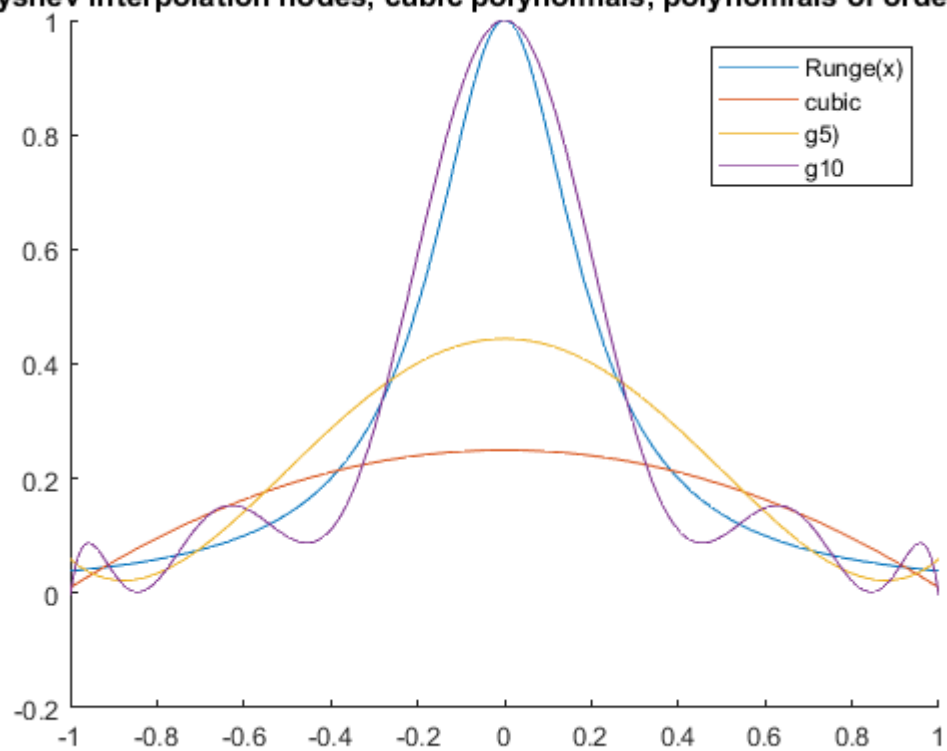


Figure 9: Runge function cheb nodes and monomials

runge function approximation errors: evenly spaced and monomials

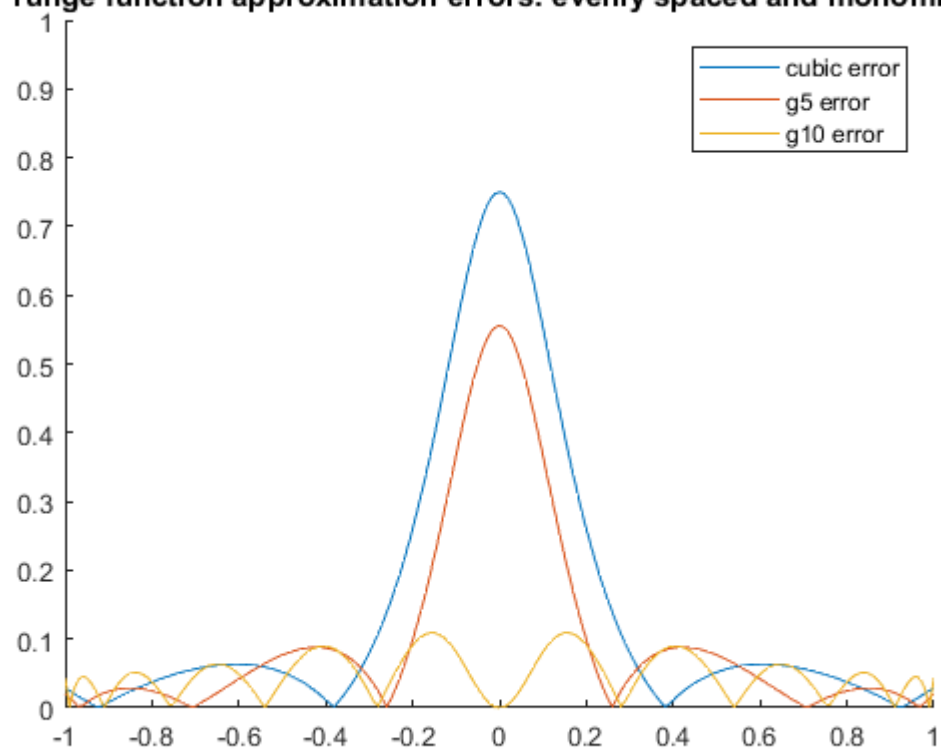


Figure 10: Runge function cheb nodes and monomials error

Chebyshev interpolation nodes, cubic polynomials, monomials of order 5 and

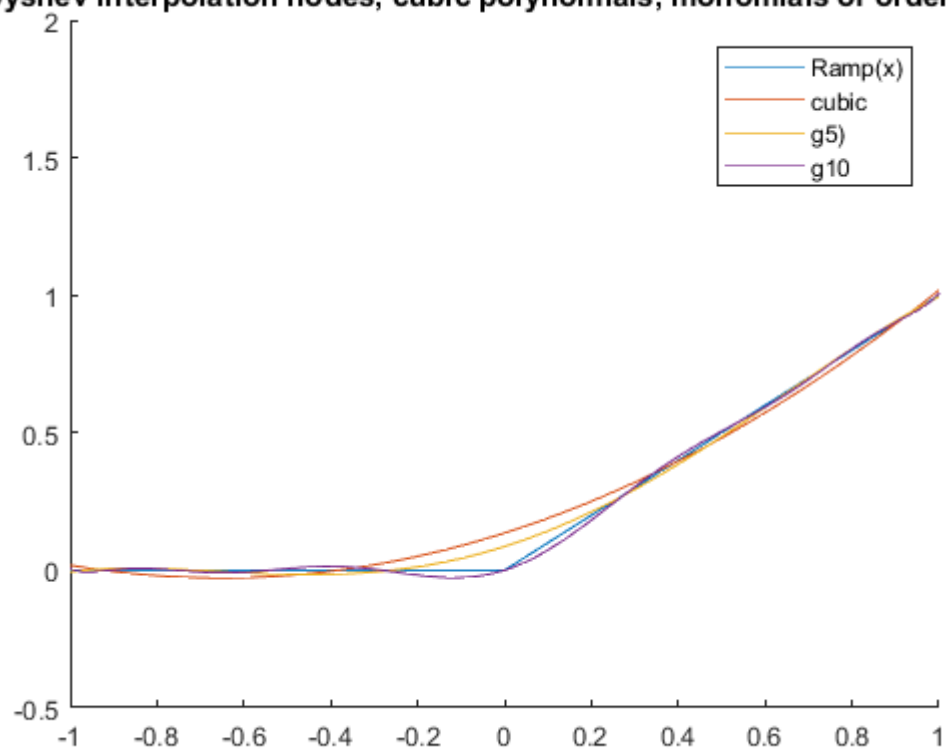


Figure 11: Ramp function cheb nodes and monomials

ramp function approximation errors: chebyshev nodes and monomials

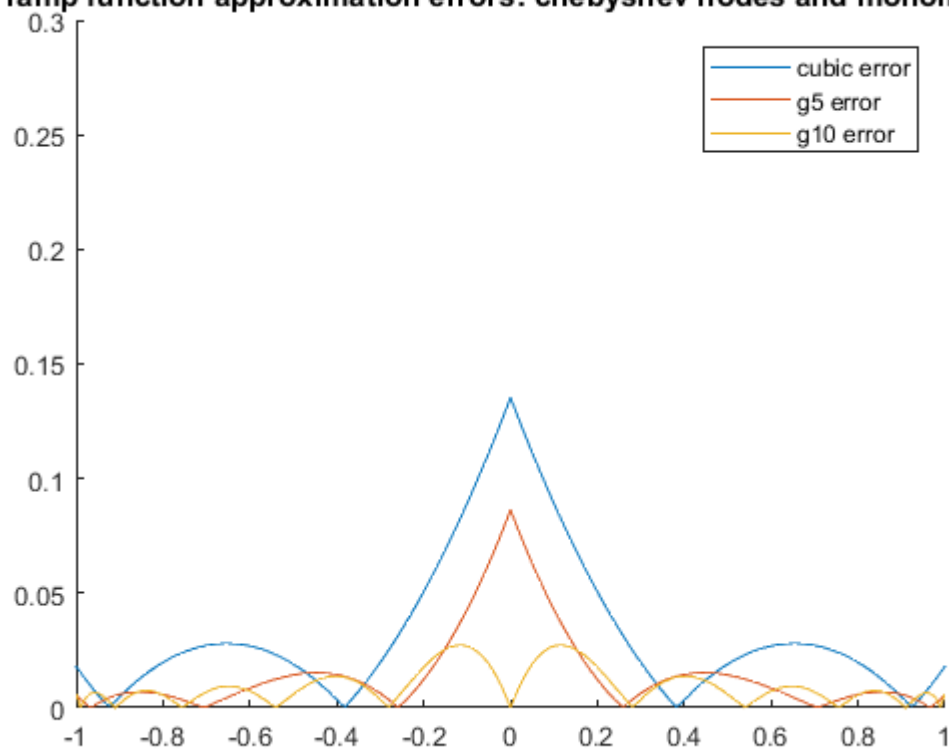


Figure 12: Ramp function cheb nodes and monomials error

Chebyshev interpolation nodes, cubic polynomials, monomials of order 5 and

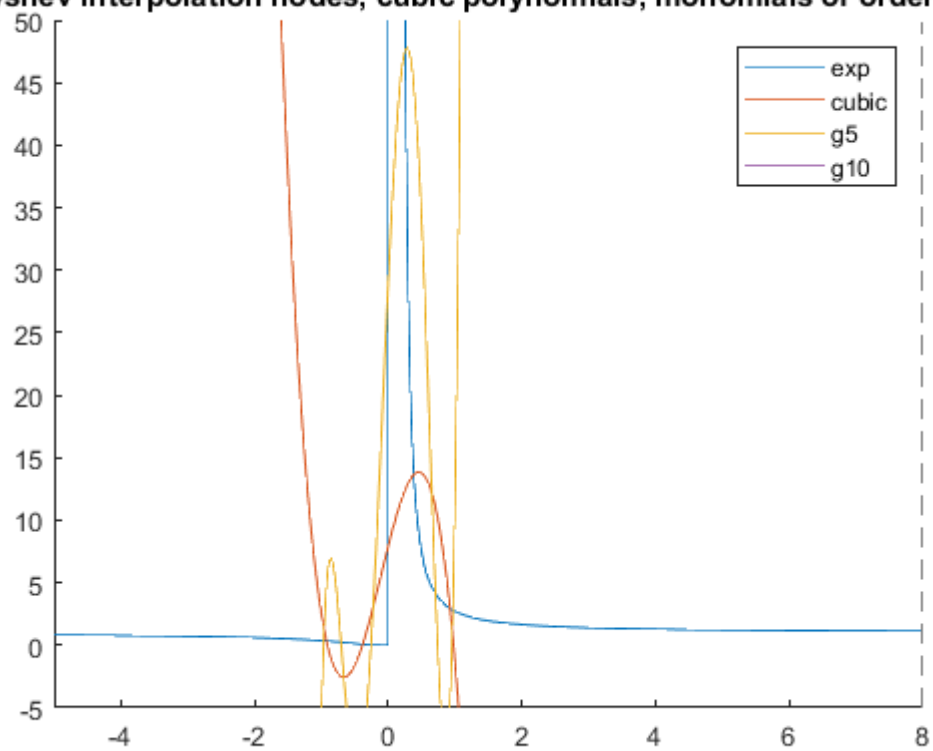


Figure 13: Exponential function cheb nodes and monomials

Exponential function approximation errors: evenly chebyshev nodes and monomials

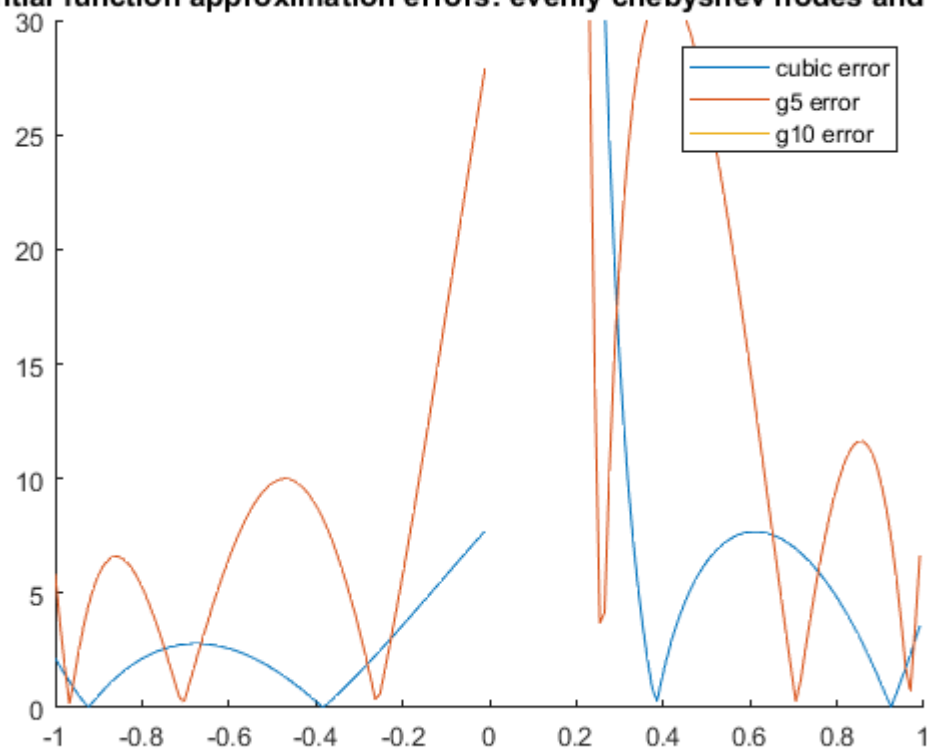


Figure 14: Exponential function cheb nodes and monomials errors

Chebyshev interpolation nodes, chebyshev polynomials of order 3, 5 and 10

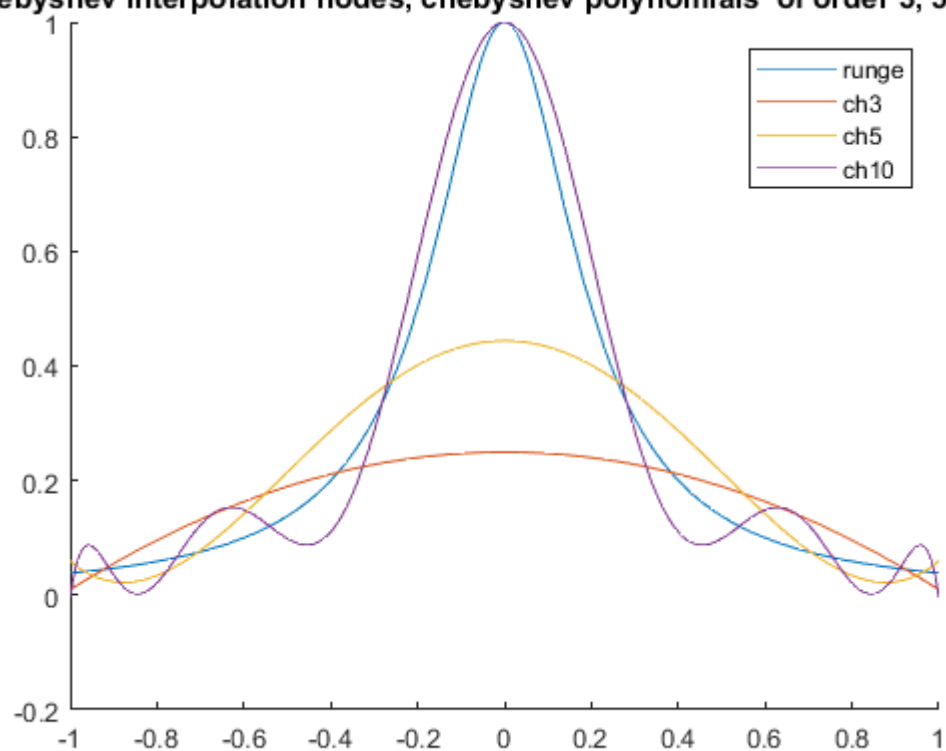


Figure 15: Runge function cheb nodes and cheb polys

runge function approximation errors: chebyshev nodes and chebyshev polys

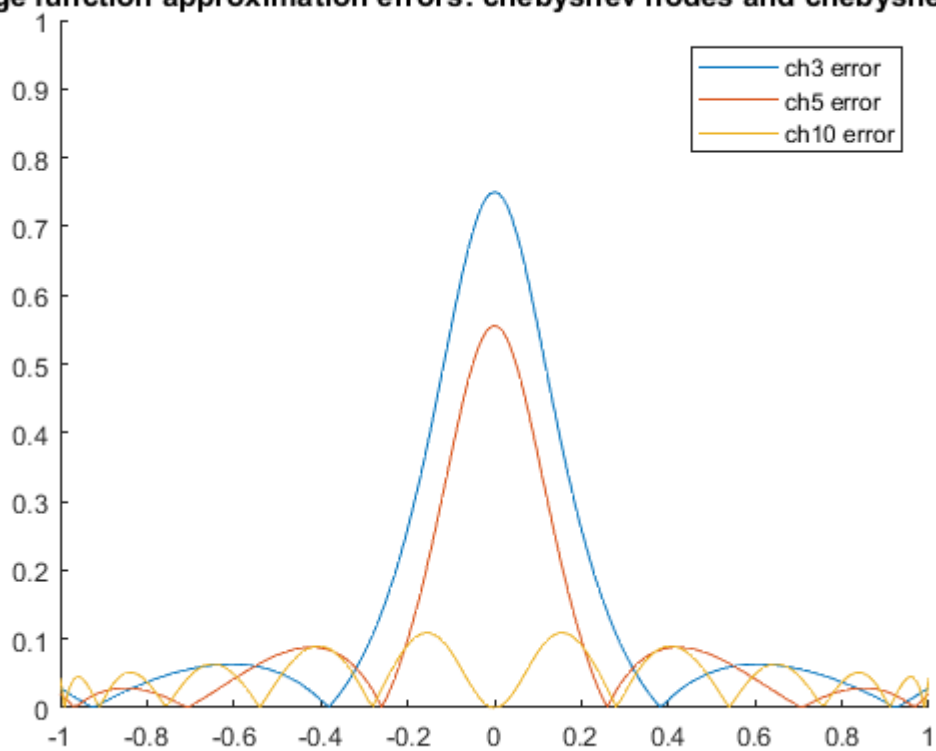


Figure 16: Runge function cheb nodes and cheb polys errors

Chebyshev interpolation nodes, chebyshev polynomials of order 3, 5 and 10

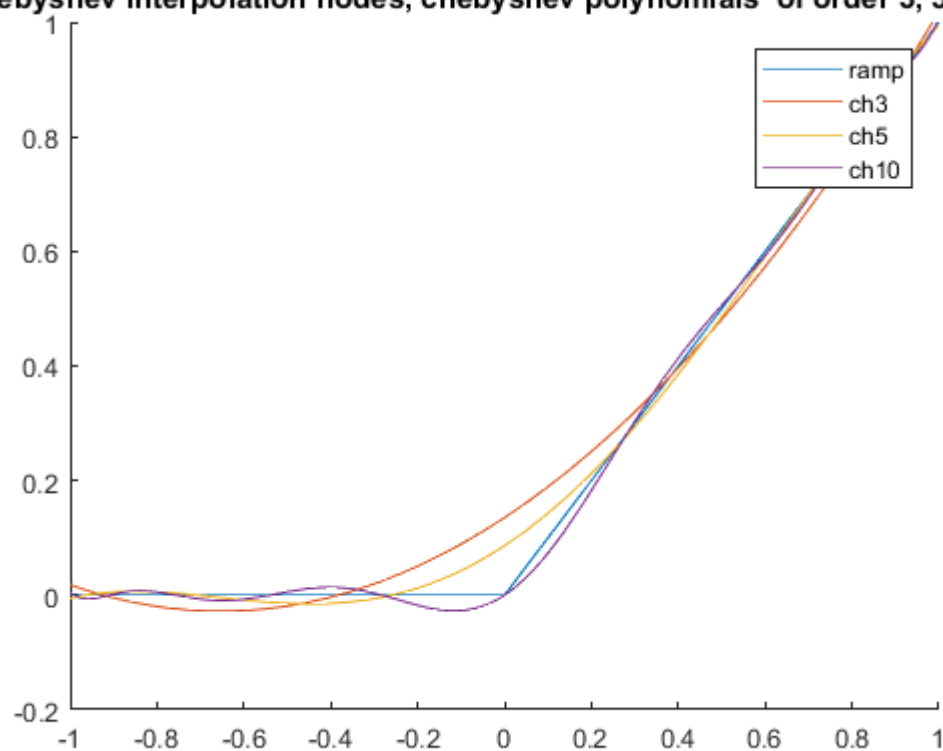


Figure 17: Ramp function cheb nodes and cheb polys

ramp function approximation errors: chebyshev nodes and chebyshev polys

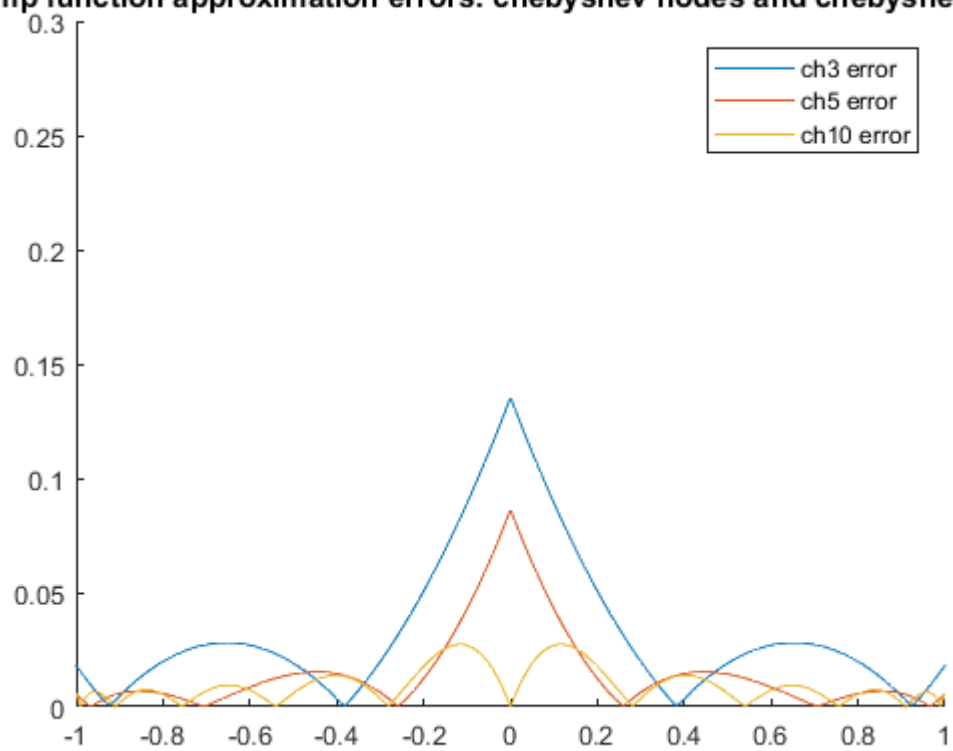


Figure 18: Ramp function cheb nodes and cheb polys errors

Chebyshev interpolation nodes, chebyshev polynomials of order 3, 5 and 10

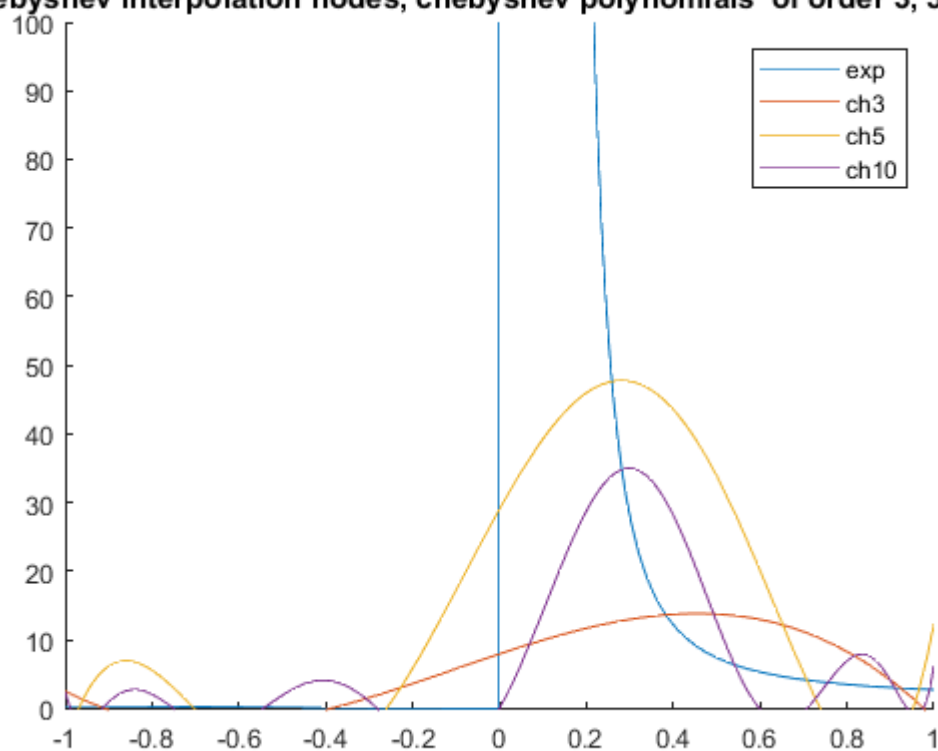


Figure 19: exponential function cheb nodes and cheb polys

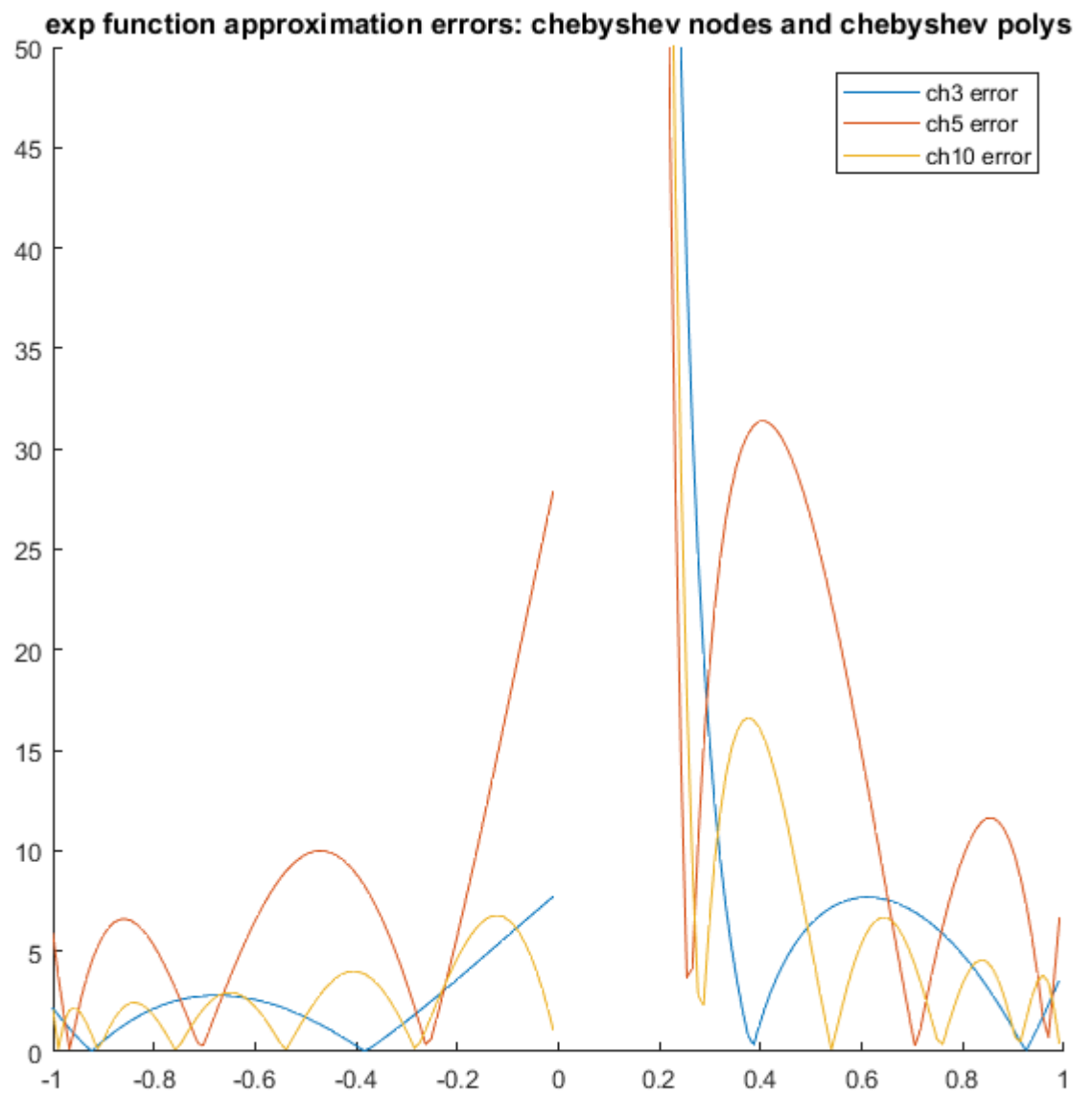
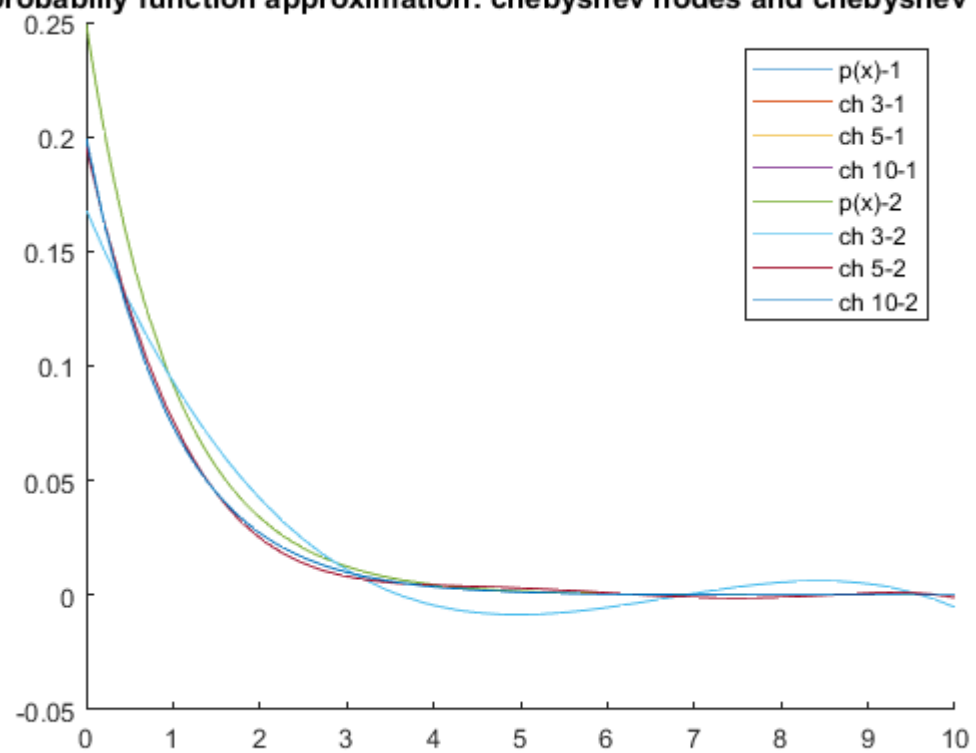
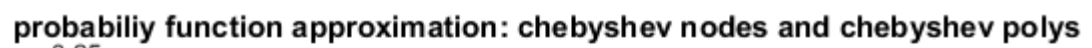


Figure 20: exponential function cheb nodes and cheb polys errors



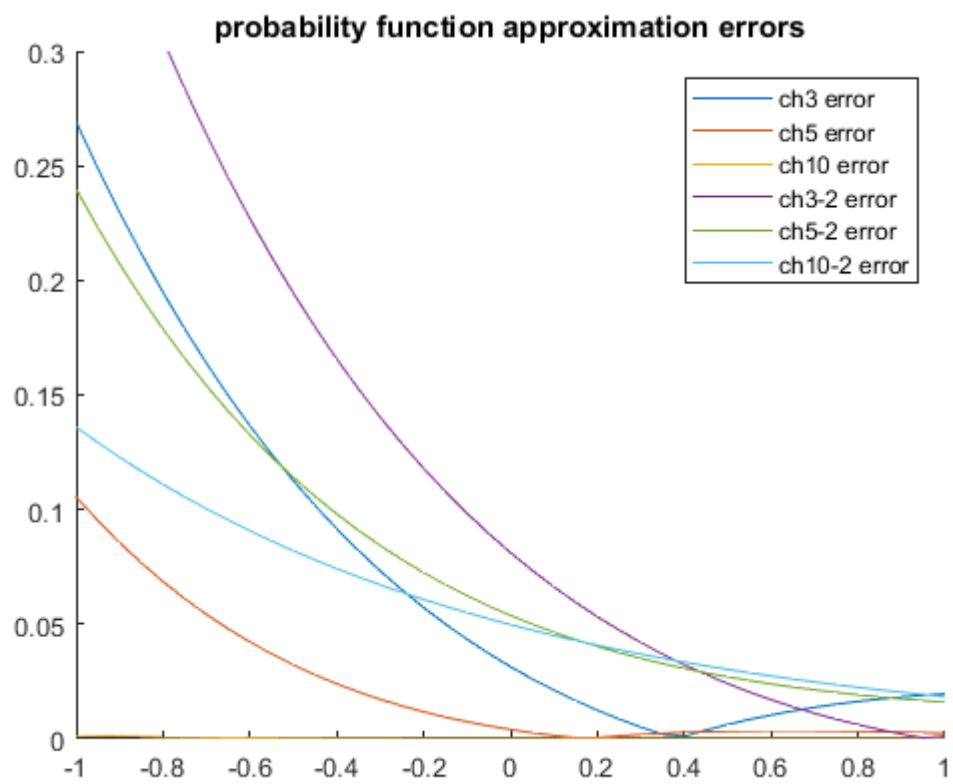


Figure 22: probability function cheb nodes and cheb polys errors

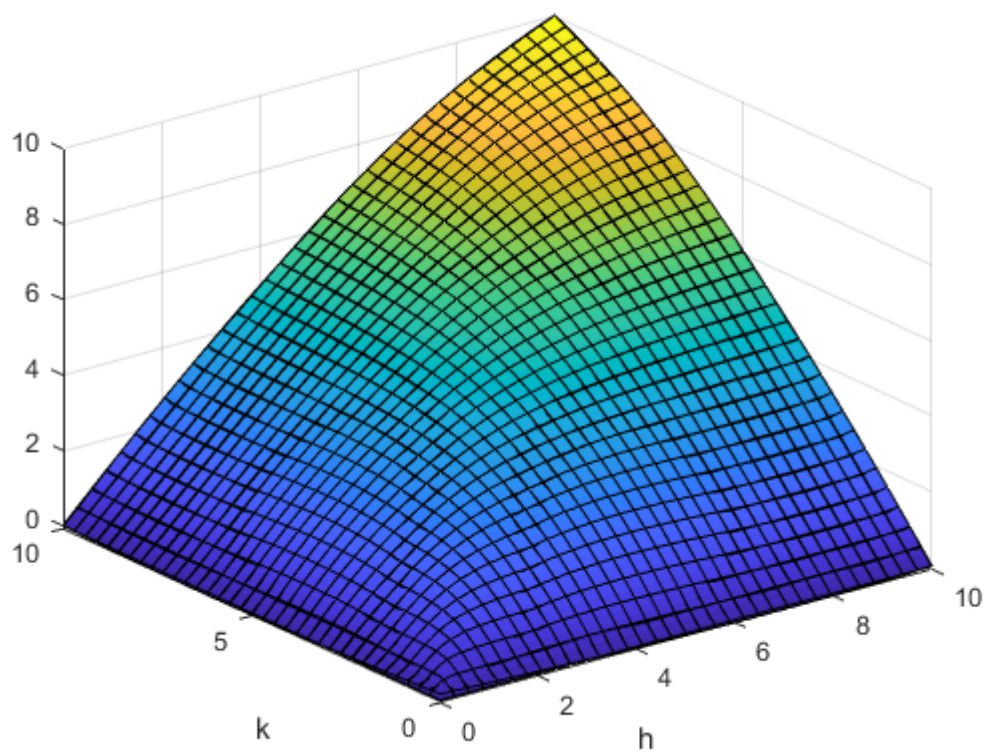


Figure 23: CES production function and isoquants

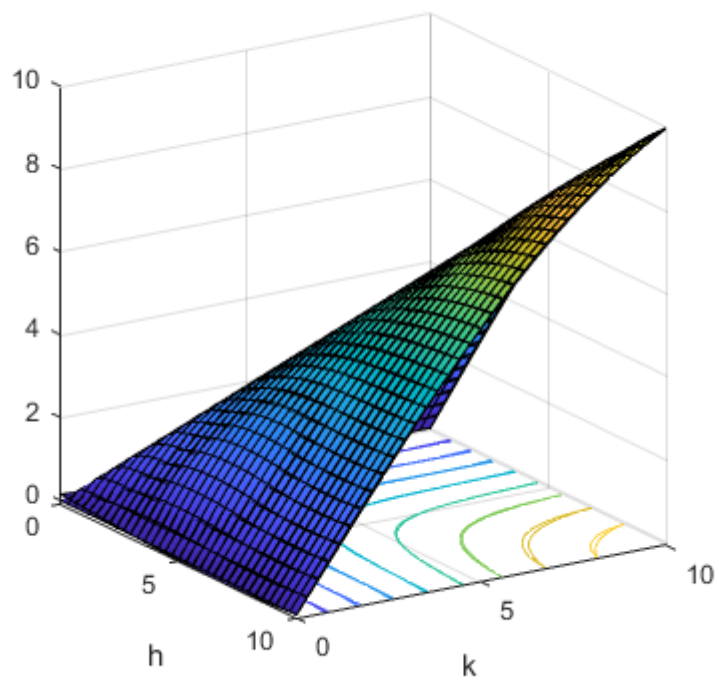


Figure 24: CES production function and isoquants

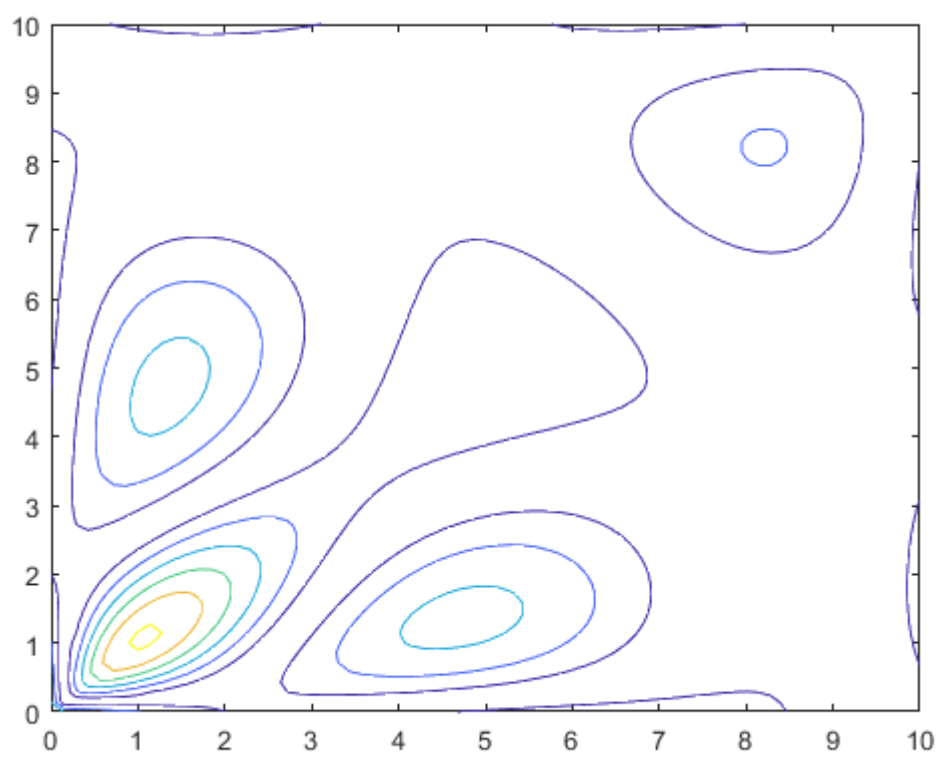


Figure 25: CES: errors of the approximation

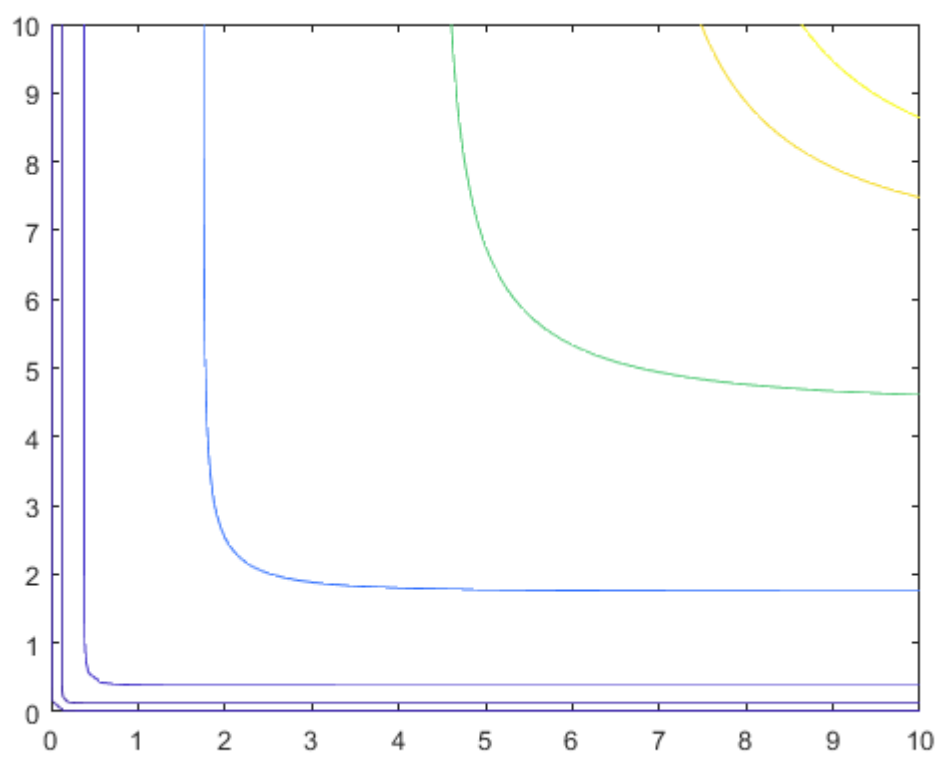


Figure 26: CES isoquants

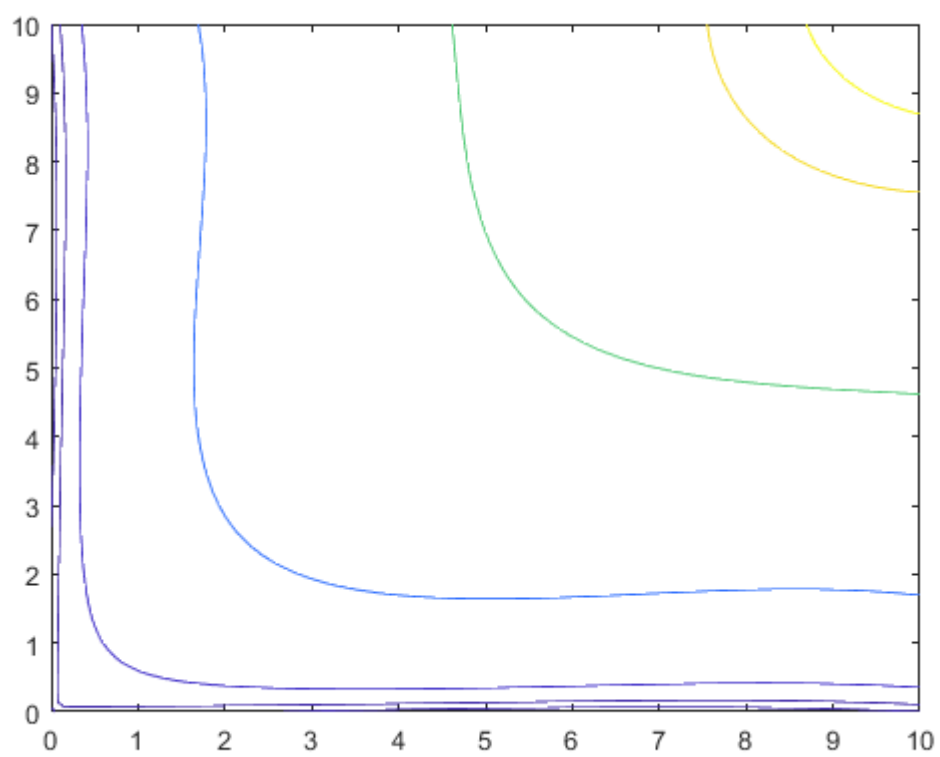


Figure 27: CES production function and isoquants

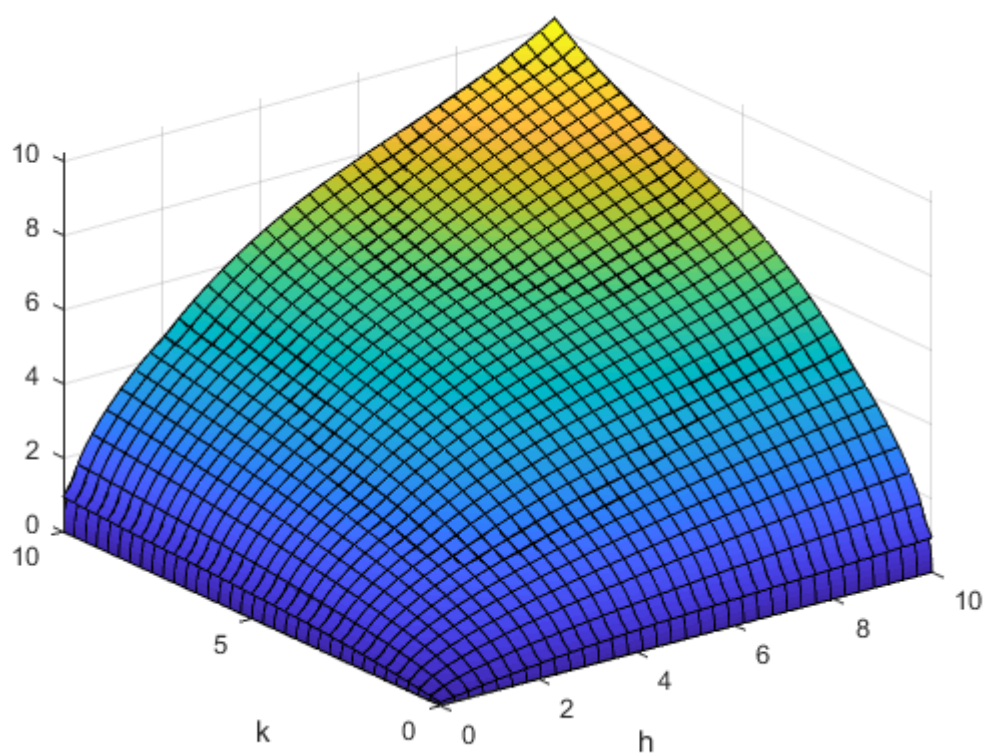


Figure 28: CES production function and isoquants

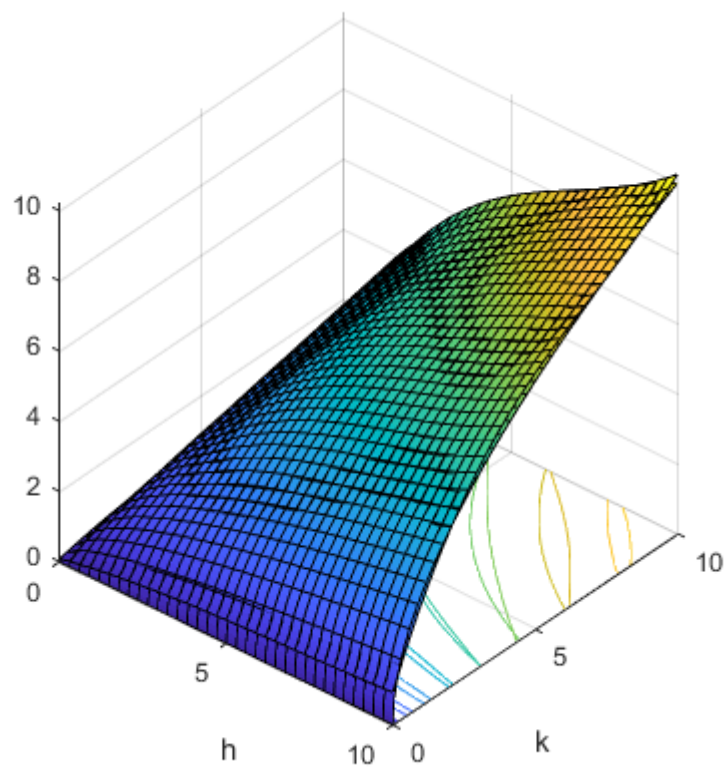


Figure 29: CES: production function and isoquants

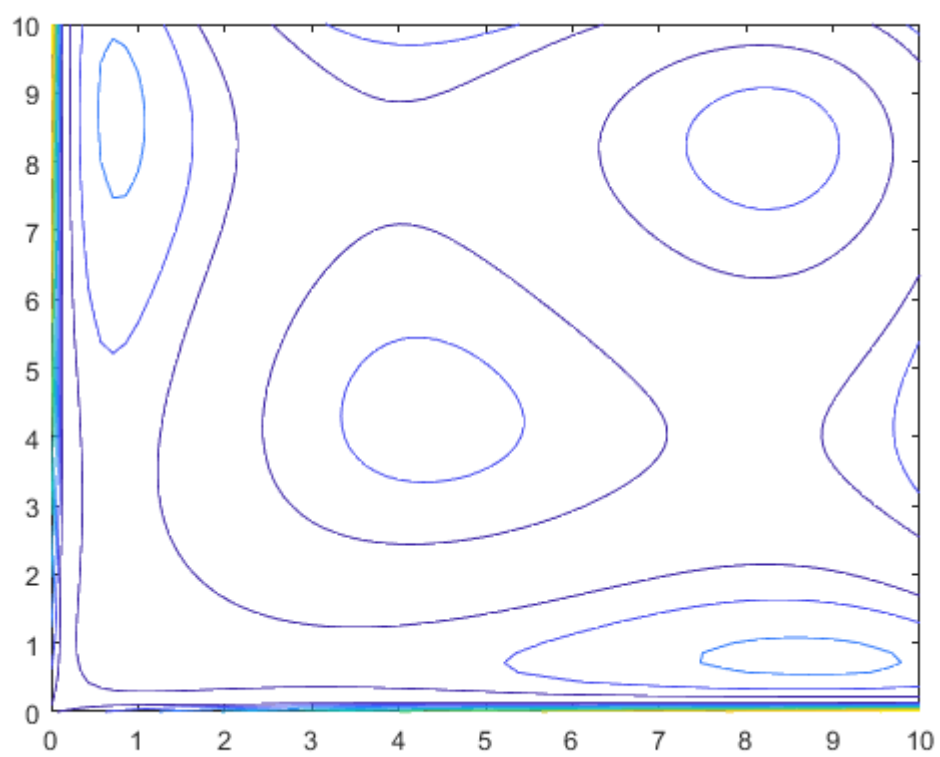


Figure 30: CES errors of the approximation

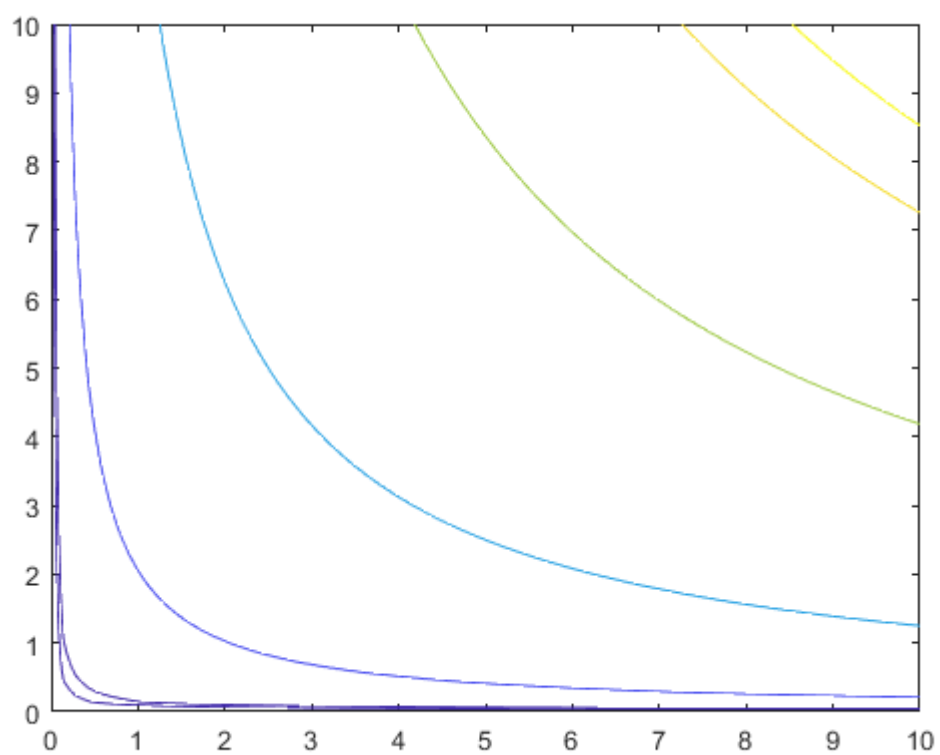


Figure 31: Isoquants

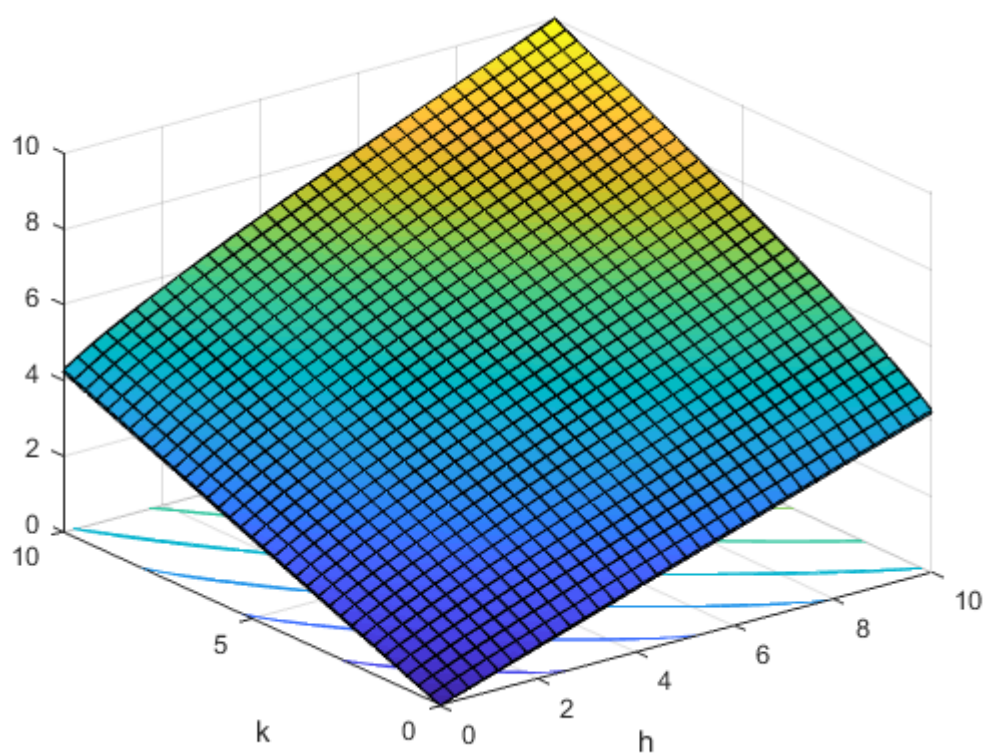


Figure 32: CES production function and isoquants

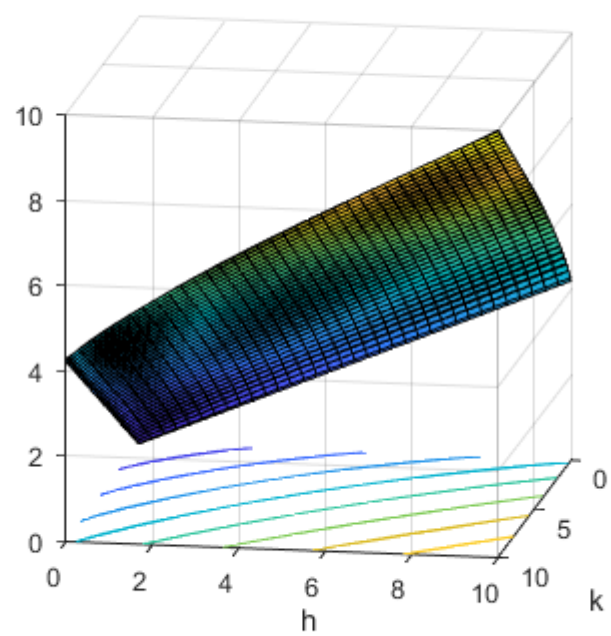


Figure 33: CES production function and isoquants

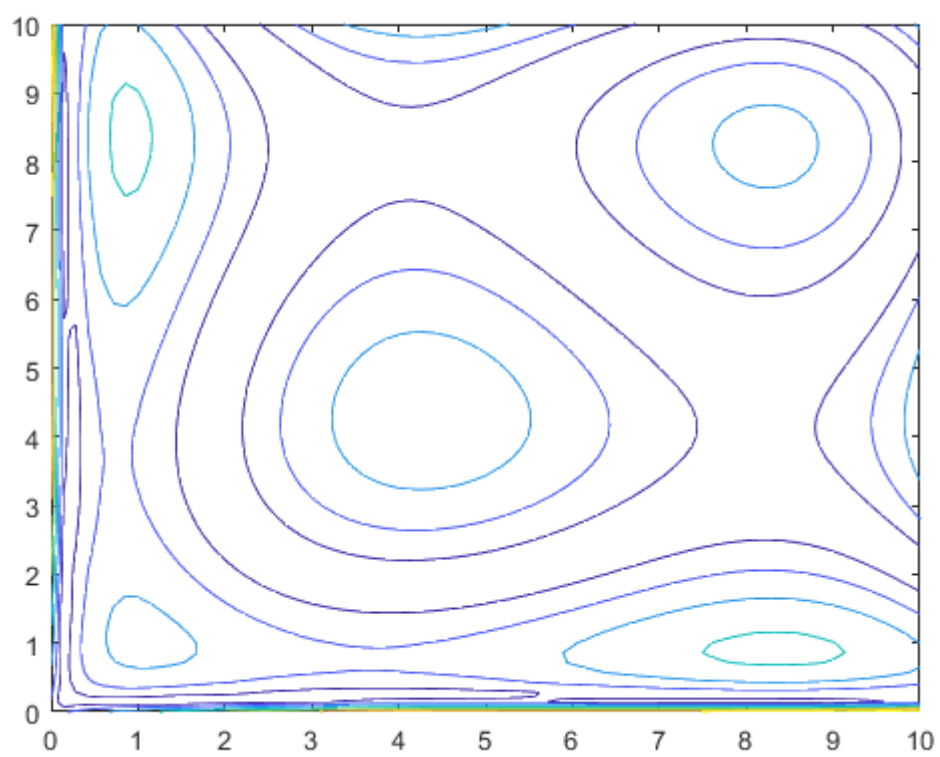


Figure 34: CES errors of the approximation

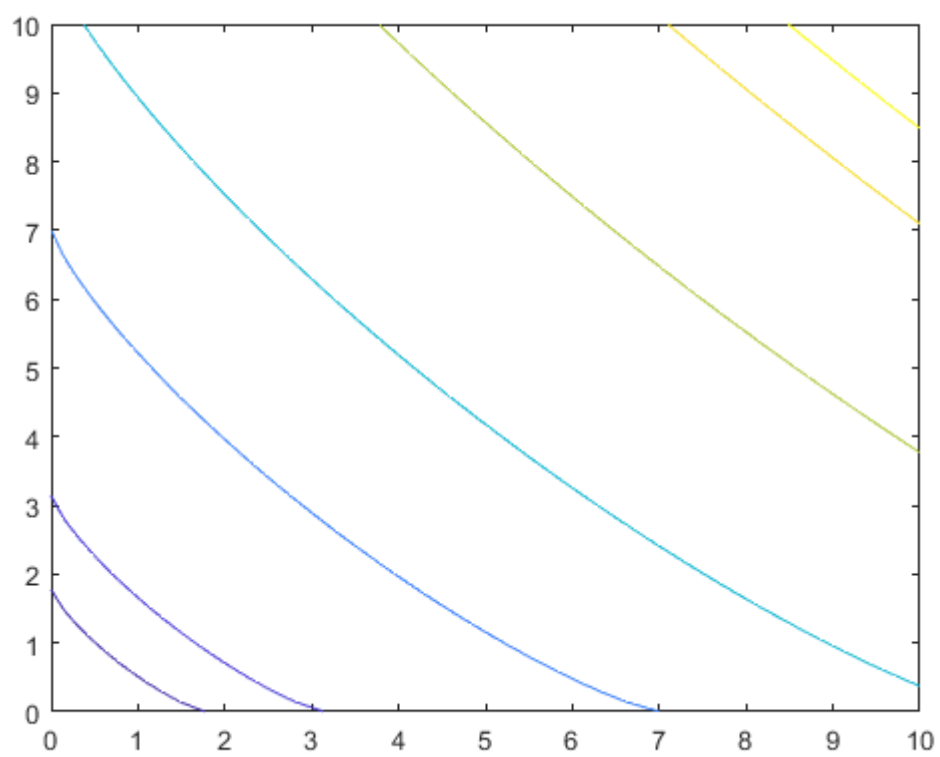


Figure 35: Isoquants