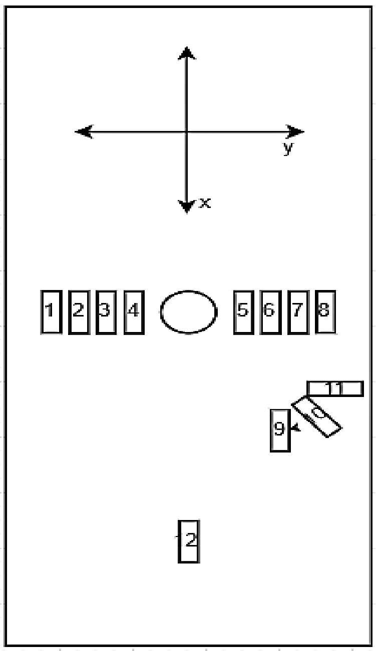
Strain Analysis Lab: Steel Plate with a Hole in the Center

By: Tyler Billings

Introduction

The purpose of this experiment is to examine the stress properties of a rectangular plate with a hole on the center under a uniform tension force in the x direction. Linear strain gages will be used to first measure a general tension force, to confirm the method of attachment then to measure strain on desired points on the plate with a hole on it. The linear strain data of the gages will be compared to appropriate methods of determining theoretical stress throughout the plate.

Experimental Setup



Note: Origin located at the center of the hole, hole equidistant from sides, equidistant from top and bottom. Axes shown.
Key Dimensions:

- Hole radius: 7.975mm (caliper)
- Width of plate (y direction): 100mm (ruler)
- Length of plate (x direction): 460mm (ruler)
- Width of gages: 5mm (caliper)
- Length of gages: 9.5mm (caliper)
- Edge of hole to gage 4/5: .8mm (caliper)
- Separation of gages 1-4, 5-8: .6mm (caliper)
- Distance to center of Rosette gage from origin: 30.145mm
- Angle to center of Rosette gage from x- axis: .48 radians
- Distance to gage 12 from origin (along x axis): 96.05mm
- Sheet thickness: .8mm

Note: measurements taken with a caliper have an error value of ± .5mm, and measurements taken with a ruler have an error value of ±1mm. Some values required more than one measurement, increasing the error accounted for.

[Figure 1] Steel plate with hole in center. Strain gages labeled 1-12 with relevant dimensions noted on the side. Tension force applied from top and bottom of rectangle.

Experimental Procedure

The installation of the Micro Measurements Strain Gage requires a carefully followed step by step procedure to ensure the adhesive has secured the gage fully and in the correct orientation with the steel plate. Testing of general strain was conducted to ensure that the gages attachment and wiring was satisfactory.

The strain gages are set up on a rectangular plate with a hole in the middle in the configuration shown in [Figure 1] (not to scale). The tension force applied at the top and bottom of the plate is reported (along the x-axis), and the strain is displayed on the strain gage monitor.

Theory

The individual strain gages (1-8, 12 [Figure 2]) measure the linear strain vertically as the copper wire within the gages is stretched proportionally to the strain at its location. The strain gages report a resistance due to the elongation of the copper wire, which is a result of an induced decrease in the cross-sectional

[1] Anon, (2017). *A Summary of Error Propogation*. [online] Available at: http://ipl.physics.harvard.edu/wp-uploads/2013/03/PS3_Error_Propagation_sp13.pdf [Accessed 14 Oct. 2017].
[2] Engineering.union.edu. (2017). *Calculating Principal Strains using a Rectangular Strain Gage Rosette*. [online] Available at: http://www.engineering.union.edu/~curreyj/MER-214_files/Analysis%20of%20a%20Strain%20Gage%20Rosette [Accessed 15 Oct. 2017].
[3] Timoshenko, S. and Goodnier, J. (1951). *Theory of Elasticity*. The Maple Press Company. [Accessed 12 Oct. 2017].

area of the wire. A Wheatstone bridge is incorporated to increase the precision of the resistance measurements. The resistance measured can be used to calculate the strain across the gages, along with the gages' gage factor.

The tangential and radial stresses on the surface of a large rectangular plate with a circular hole in the center can be calculated using the following equations:[3]

$$\sigma_\theta = \frac{\sigma_x}{2}\left(1+\frac{a^2}{r^2}\right) - \frac{\sigma_x}{2}(1+3\frac{a^4}{r^4})\cos 2\theta \qquad \text{[Equation 4]}$$

$$\sigma_r = \frac{\sigma_x}{2}\left(1+\frac{a^2}{r^2}\right) - \frac{\sigma_x}{2}\left(1+3\frac{a^4}{r^4}-4\frac{a^2}{r^2}\right)\cos 2\theta \qquad \text{[Equation 5]}$$

Where (σ_x) is the stress applied to the plate along the x-axis, (a) is the radius of the hole, r is the radial distance of the point of measurement from the center of the hole (origin) and (θ) is the angle of the point of measurement from the x-axis. For measurements taken away from the hole, and along the y-axis, the tangential stress can be simplified because ($\cos 2\theta = -1$) when ($\theta = \frac{\pi}{2}$).[3]

$$\sigma_\theta = \frac{\sigma_x}{2}\left(2+\frac{a^2}{r^2}+3\frac{a^4}{r^4}\right) \qquad \text{[Equation 6]}$$

This equation can be used to calculate the stress that would theoretically be measured by gages 1-8 [Figure 1]. The nominal stress of a steel plate is defined as:

$$\sigma_x = \frac{F_N}{A} \qquad \text{[Equation 7]}$$

[Equation 7] can be used to find the stress applied along the x axis at the edges of the plate by dividing the force applied normal to a cross section of material (F_N) by the cross-sectional area (A) of said cross section. Engineering stress can be calculated from the measured strain.

$$\sigma = E\varepsilon \qquad \text{[Equation 8]}$$

(E) represents Young's ratio, and it is given without uncertainty that Young's ratio for the material used is 210 GPa. (ε) describes the strain measured by a gage.

Gages 9-11 are set up in a Rosette gage, whose three axes are aligned 45° apart from each other and the outer two perpendicular to each other. The Rosette gage can be used to find the principle axis (φ), or angle of greatest strain at the center of the Rosette Gage, as well as the principle strains (ε_1, ε_2), which describe the maximum strain along the principle angle and the strain perpendicular to that angle.[2]

$$\varepsilon_1 = \frac{1}{2}(\varepsilon_A+\varepsilon_C)+\frac{1}{2}\sqrt{(\varepsilon_A-\varepsilon_C)^2+(2\varepsilon_B-\varepsilon_A-\varepsilon_C)^2} \qquad \text{[Equation 9]}$$

$$\varepsilon_2 = \frac{1}{2}(\varepsilon_A+\varepsilon_C)-\frac{1}{2}\sqrt{(\varepsilon_A-\varepsilon_C)^2+(2\varepsilon_B-\varepsilon_A-\varepsilon_C)^2} \qquad \text{[Equation 10]}$$

$$\varphi = \frac{1}{2}\tan^{-1}\frac{2\varepsilon_B-\varepsilon_A-\varepsilon_C}{\varepsilon_A-\varepsilon_C} \qquad \text{[Equation 11]}$$

The strains measured by the three gages within the Rosette gage ($\varepsilon_A, \varepsilon_B, \varepsilon_C$ for gages 9, 10, 11, respectively) are used to find the principle angle and strains. From the principle strain values, the principle stresses (σ_1, σ_2) can be calculated using the following equations.

$$\sigma_1 = \frac{E}{(1-\nu^2)}(\varepsilon_1+\nu\varepsilon_2) \qquad \text{[Equation 12]}$$

$$\sigma_2 = \frac{E}{(1-\nu^2)}(\varepsilon_2+\nu\varepsilon_1) \qquad \text{[Equation 13]}$$

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Where ν is Poisson's ratio. The error of a sum or difference of measured values can be found using the following equation[1].

$$\delta Q = \sqrt{(\delta a)^2 + (\delta b)^2 + (\delta c)^2 \ldots} \qquad \text{[Equation 14]}$$

If $Q = a \pm b \pm c \ldots$

The error of a product or quotient of measured elements can be found using the following equation[1].

$$\delta Q = |Q|\sqrt{x\left(\frac{\delta a}{a}\right)^2 + \left(\frac{\delta b}{b}\right)^2 + \left(\frac{\delta c}{b}\right)^2 + \cdots}$$
[Equation 15]

If $Q = \frac{a^x * b}{c}$, or any combination of elements a, b, c, and x.[1]

Results/Discussion

Using [Equation 7], the applied stress is on the top and bottom of our plate is found to be: $\sigma_x = 6.25$MPa.

To simplify calculations, forces of each strain measurement were centralized at 500N and strain values were adjusted accordingly. The stresses of gages 1-8 are listed below along with their distances from the origin and expected stress values. Each distance measurement was taken to approximately to the middle of each gage from the dimensions noted with [Figure 1]. The theoretical tangential stress values were calculated using [Equation 5]. It is important to note that this equation acts under the assumption that r>>a, which should not apply to gages 4, and 5. It is only important to compare the tangential stresses because along the y-axis the tangential stress equates to the linear gage measurement values that are recorded. The error values are calculated using a combination of [Equation 14] and [Equation 15] for all values calculated using positioning measurements.

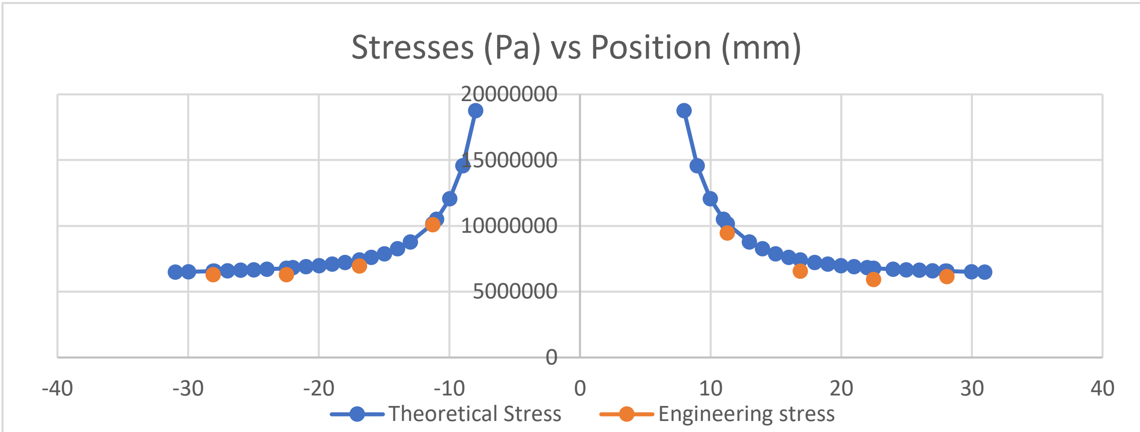
Gage	Radius (mm)	Engineering Tangential Stress (MPa)	Theoretical Tangential Stress (MPa)
1	28.075 ± 0.707	6.139	6.563 ± 0.405
2	22.475 ± 1	5.927	6.792 ± 0.740
3	16.875 ± 1.225	6.562	7.416 ± 1.318
4	11.275 ± 1.414	9.469	10.160 ± 3.122
5	11.275 ± 1.414	10.10	10.160 ± 3.122
6	16.875 ± 1.225	6.944	7.416 ± 1.318
7	22.475 ± 1	6.287	6.792 ± 0.740
8	28.075 ± 0.707	6.287	6.563 ± .405

[Table 1] Position and Engineering stress for gages 1-8, as well as the Theoretical stress at the position of the gages. Units of (Mpa) and (mm) used for viewing purposes, all calculations done using SI units.

The Engineering tangential stress shown above was calculated using [Equation 8], and although in theory the strain readings of the gages have uncertainty, this information was not available prior to calculating the stress on each gage. The theoretical tangential stresses calculated should correspond to the linear strains measured by the gages on the y-axis. The values of uncertainty for the theoretical tangential stress overlap the theoretical stresses with the engineering stresses for all but gages 2, and 7. Because the uncertainty of the strain was not accounted for, the accuracy of the strain measurements could have contributed to imprecise date if the uncertainties were significant.

[1] Anon, (2017). *A Summary of Error Propogation*. [online] Available at: http://ipl.physics.harvard.edu/wp-uploads/2013/03/PS3_Error_Propagation_sp13.pdf [Accessed 14 Oct. 2017].
[2] Engineering.union.edu. (2017). *Calculating Principal Strains using a Rectangular Strain Gage Rosette*. [online] Available at: http://www.engineering.union.edu/~curreyj/MER-214_files/Analysis%20of%20a%20Strain%20Gage%20Rosette [Accessed 15 Oct. 2017].
[3] Timoshenko, S. and Goodnier, J. (1951). *Theory of Elasticity*. The Maple Press Company. [Accessed 12 Oct. 2017].

The following graph illustrates the trends of the types of stresses plotted against the position values, with additional theoretical stress values incorporated for points along the y-axis.



[Figure 2]. Graph of Theoretical and Engineering Stress vs Position for gages 1-8.

As shown in the graph, the engineering stress appears to follow the theoretical trend with acceptable accuracy and precision (as supported by the error values from the table).

For gage 12 in the [Figure 1], the strain value was converted to engineering strain once again, and the theoretical radial value was used as comparison since the gage is located along the x-axis of the plate. Using [Equation 5], the theoretical radial stress at gage 12 was calculated to be:

$$\sigma_r = 6.143 \pm .157 \text{ MPa}.$$

The engineering stress was calculated to be 5.962 MPa, which does not fall within the error, but considering the lack of error accounted for on the strain measurement and the fact that there was only one measurement taken by the ruler to locate gage 12, low, one sided error is expected. This gage represents the area far from the hole along the x-axis, and the fact that its values fall below the applied stress value from the tension force of 6.25 MPa indicates that the hole is relieving stress along the axis of tension (x).

The Rosette Gage setup (gages 9-11) was used to calculate an angle of maximum stress from about the location of the gage, as well as the values of maximum and minimum strains at the location of the gage. Using [Equation 11] the principle angle was found to be:

$$\varphi = -.165 \text{ radians}$$

The principle strain values were calculated using [Equation 9] and [Equation 10]. From these strain values the principle stresses were calculated using [Equation 12] and [Equation 13].

$$\sigma_1 = 6.2679 \text{ MPa}$$

$$\sigma_2 = -.702 \text{ MPa}$$

Notice, the principle stress is comparable to our applied stress, meaning the hole is not relieving nor reducing tension at the location of the Rosette Gage. The error values of the principle stresses depend heavily on the uncertainty in our strain measurements from the gages, which were not recorded.

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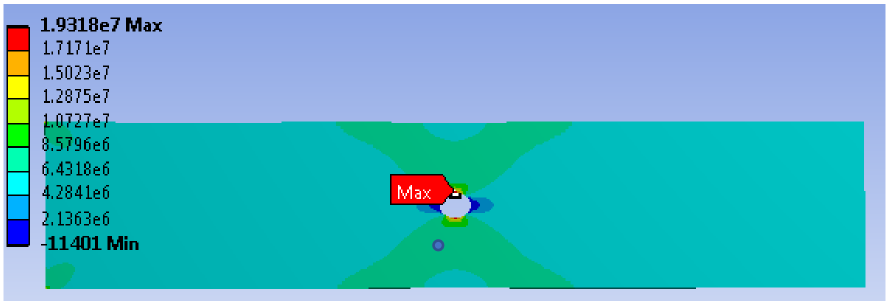
The principle angle fits the condition[3]: $-\frac{\pi}{2} < \varphi < 0$ when $\varepsilon_B < \frac{(\varepsilon_A+\varepsilon_C)}{2}$.



[Figure 3]. ANSYS Workbench simulation of steel mesh configuration with dimensions, properties, and applied force observed from plate in lab. The blue dot represents the approximate position of the Rosette Gage. Max/Min values on left given in (Pa)

The maximum stress on the piece of metal should theoretically occur at the edge of the hole along the y axis. The theoretical maximum value of stress should be three times greater than the stress applied to the plate along the x axis. This is shown by plugging r = a into [Equation 5].

$$\sigma_\theta = \frac{\sigma_x}{2}\left(2 + \frac{a^2}{a^2} + 3\frac{a^4}{a^4}\right) = 3\sigma_x = 3(6.25MPa) = 18.75 \pm 2.15\ MPa$$

Comparing this to the maximum stress value from [Figure 3], 19.318MPa, therefore the measured result from the simulation falls within the theoretical uncertainty of the maximum stress value. The elements of the mesh are half a millimeter in length, small enough to reduce differences in properties between the simulation and our plate. On the point at which the Rosette Gage would be located if it had been plotted in the simulation, the maximum strain reading is 6.4362MPa. Comparing this value to the principle stress value of 6.2679, the uncertainty of the measurement on the simulation is undefined, but would be significant as the point plotted is a probe located approximately at the Rosette's location. Further, the lines at which the color switch represent lines of equal maximum principle stress values. Like contour lines, the path of greatest change is perpendicular to those lines. We don't have enough lines of equal maximum principle stress to fully validate our principle angle measurement, but we can at least conclude that it is correctly between $-\frac{\pi}{2} < \varphi < 0$. The simulation also considers only a stress generated on one side of the plate, with a fixed side opposing it. This causes a slight buildup of stress at the fixed end.

Conclusion

The engineering stresses calculated from strain gages 1-8 were mostly within the error of the theoretically calculated tangential stresses at the gages' locations. The engineering stress of gage 12 was outside of the error calculated for theoretical radial stress at its location, although only by less than 2% of its value. The principle axis of the Rosette Gage satisfied the condition of its direction, and appeared plausible upon observation with the maximum stresses displayed on the ANSYS simulation. The Principle strain maximum compares closely to the maximum of a point closest to the coordinates of the Rosette from the simulation, but lack of uncertainty information prevented the value from being validated through error analysis.

[1] Anon, (2017). *A Summary of Error Propogation*. [online] Available at: http://ipl.physics.harvard.edu/wp-uploads/2013/03/PS3_Error_Propagation_sp13.pdf [Accessed 14 Oct. 2017].
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