Analysis of Longitudinal model of a B747-100/200

The purpose of this coursework is to analyse the longitudinal flight control of a B747-100/200 using Simulink modelling and uncertainty and sensitivity analysis through MATLAB. The model take the initial conditions for height, pitch rate linear velocity, and pitch angle, as well as the elevator, stabilizer, and thrust settings of the aircraft. The model can be used to describe the flight in all input aspects in two dimensions, and can be commanded through MATLAB to produce sensitivity and uncertainty analysis plots to provide an understanding of how each parameter affects the behaviour of the system.

Question 1: Creating the model

The initial Simulink Model used to describe the aircraft system can be called to run the simulation as long as all initial conditions are specified.

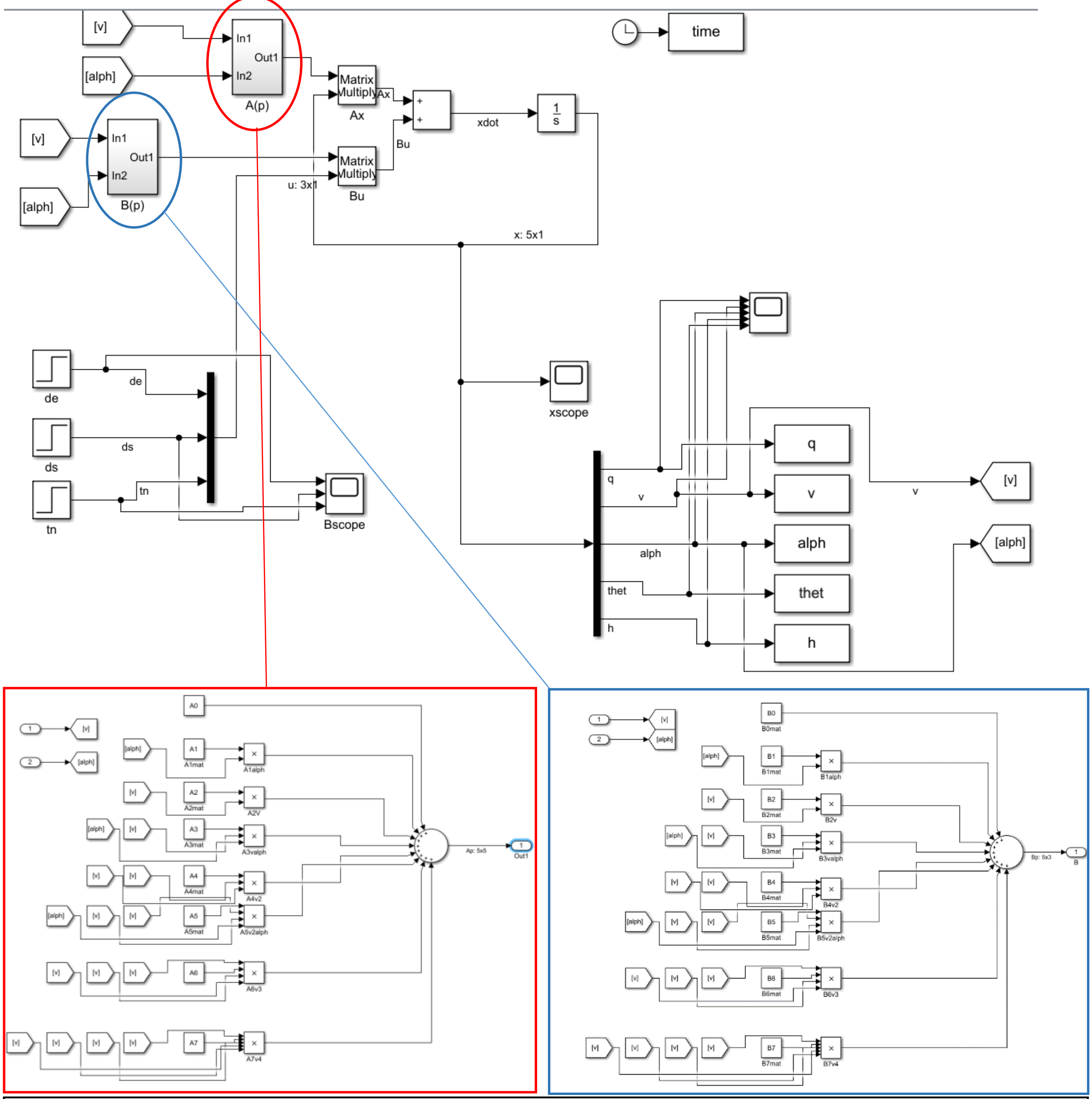


Figure 1. Diagram of the setup for the Aircraft model simulation. Subsystem A is circled in red and displayed in the bottom left. Subsystem B is circled in blue and displayed in the bottom right.

As expected, when we plotted all outputs with all input initial conditions at 0, we got all zero outputs.

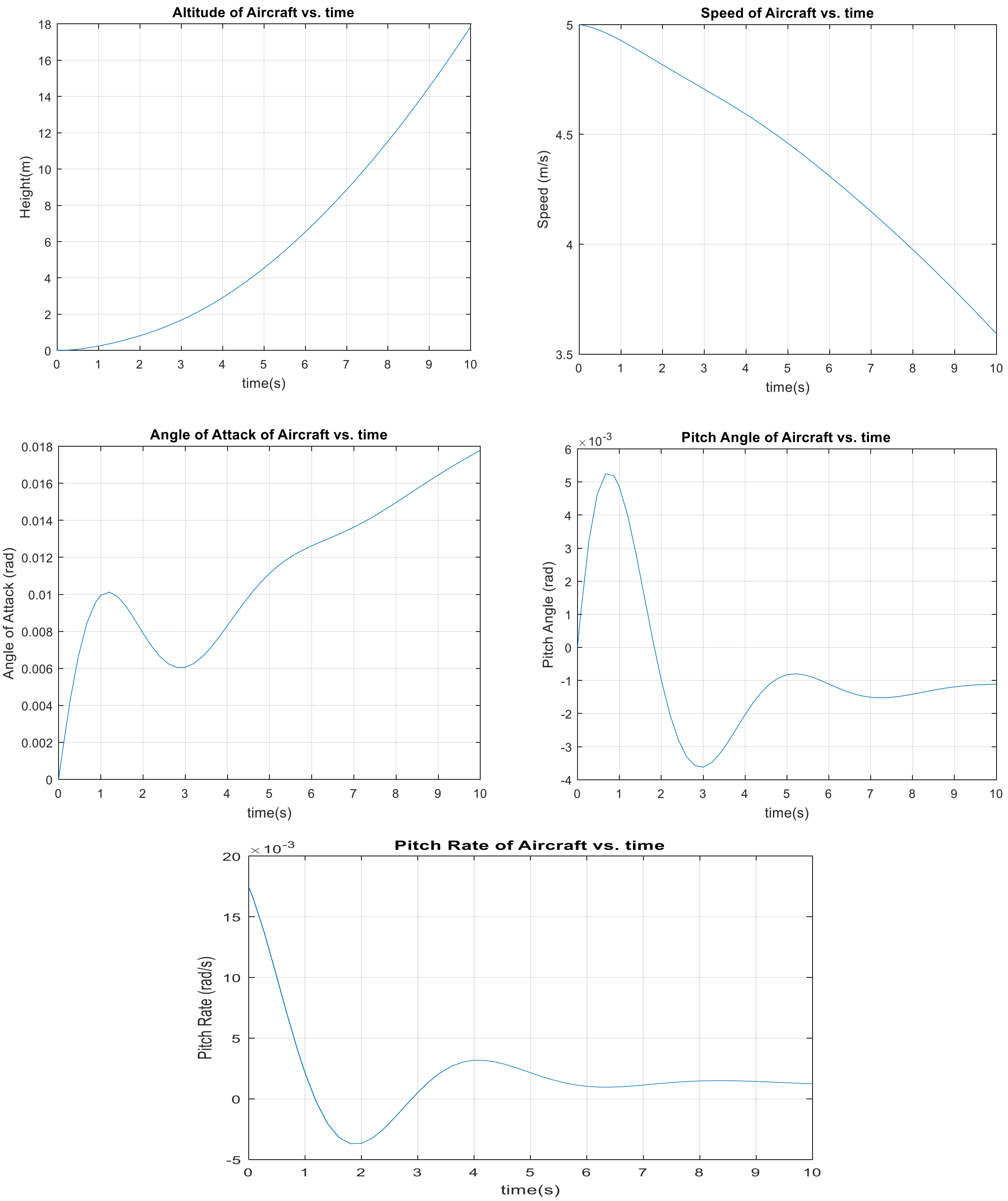
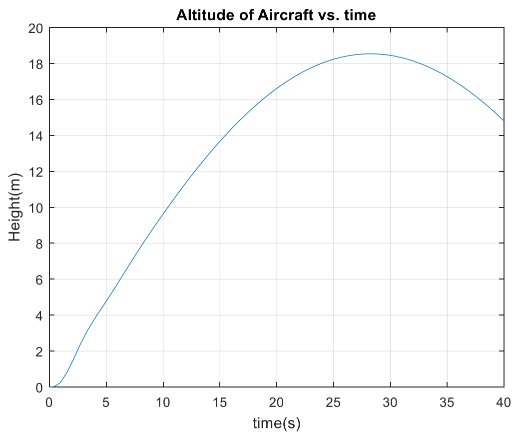


Figure 2. Plots of the 5 states of the aircraft over time interval of 10s. Initial pitch rate set to 1 rad/s, initial velocity set to 5m/s. All other initial conditions set to 0

To check the validity of the model, the pitch rate was set to 1 and the simulation ran for 10 seconds at the specified conditions above. The plots in Figure 2 show the values for the altitude, linear velocity, pitch rate, pitch angle and angle of attack were each plotted over the 10 second given interval. As expected, the altitude of our aircraft increased and the velocity decreased both at faster rates than the trials conducted with all initial conditions set to 0. The angle of attack seems to approach linearity around 7 seconds into the simulation, and at about the same time that the pitch rate and pitch angle begin to stabilize about near zero values. When the simulation is run over a longer time period with an initial speed of 0, the relationship between altitude and speed becomes clearer.



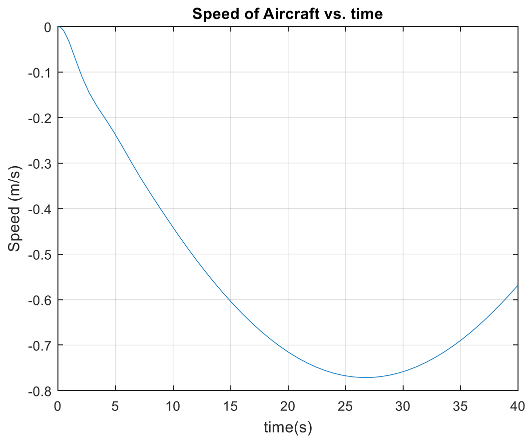


Figure 3. Comparison of the altitude and speed of the aircraft with initial conditions: pitch rate 1 deg/s, all other initial conditions including velocity set to 0

When there is no initial velocity of the aircraft, the speed dips to a negative value while the height increases until about 28s, at which point the opposite occurs. For all trials run, while keeping all initial conditions besides pitch rate and speed equal to 0, the speed and height of the aircraft appear to have inverse relationships. The behaviour of the aircraft indicates that the Simulink model is properly designed.

Code Description:

Each parameter initialized (converted to radians if angle based), x initialized as a 1x5 vector which was then used as the initial condition of the integrator used to integrate the derivative function of x to x. Each plot created using plot function:

```
figure(#)
plot(time, [parameter variable])
grid on;
xlabel('time(s)')
ylabel('(Label of Parameter)(units)')
title('(Label of Parameter) of Aircraft vs. time')
```

Question 2: Sensitivity analysis using one-factor-at-a-time method

Parameter	Variation type	Minimum Value	Maximum Value
Elevator	Input range	-1 degrees	1 degrees
Stabiliser	Input range	-1 degrees	1 degrees
Thrust	Input range	-100000 Newtons	100000 Newtons
Attack angle	Uncertainty	-1 degrees	1 degrees
Speed	Uncertainty	-5 meters/second	5 meters/second

Figure 4. Table of parameter variations applied in Sensitivity/Uncertainty analysis.

From the values shown in [Figure 4], the following MATLAB script was run to create tornado plots to display the sensitivity the altitude, speed, angle of attack, pitch rate. Each plot displays the sensitivity of the specified control parameter when varying (one at a time) the parameters in [Figure 4] to their minimum and maximum values.

Code Description:

The uncertainties and variation of the input parameters listed above are altered using if statements within a for-loop. The first run through of the loop had all initial conditions set to nominal values of 0. The program was initialized the same way as the code in Question 1, but with additional vectors that will be filled with the variance of each parameter. Each run of the loop represents a different varied parameter with all other parameters set to the nominal values. The if statements allow our nominal values to act as default without being altered, while just one parameter is increased or decreased to its maximum or minimum. The system is simulated within the for-loop, so that the variance of each variable in each run is stored in the variance vectors. Variance values were calculated using:

```
Sigvariable = rms([variable]);
```

For each parameter in each run through of the loop. Outside of the initial loop used to get the variance, additional non-nested loops are used to fill vectors of the distance from the mean values from using the variance vectors defined for each parameter. After initializing each difference vector, the following format is applied to the for loop for each variable under sensitivity analysis.

```
for jj=2:11
    variabledif=[variabledif sigvariablevec(jj)-sigvariablevec(1)];
end
```

The sigvariablevec vectors are the vectors built from the rms values of each varied initial condition. The first trial is subtracted from each variance value, which represents a magnitude of each sensitivity. For each of these five for-loops we used the following code to plot double sided bar graphs, or tornado plots, for the altitude, speed, pitch rate, angle of attack, and pitch angle.

```
figure(#)
barh([-[parameter]dif(2) -[parameter]dif(4) -[parameter]dif(6) -
[parameter]dif(8) -[parameter]dif(10)], 'r');
hold on
barh([[parameter]dif(11) [parameter]dif(3) [parameter]dif(5)
[parameter]dif(7) [parameter]dif(9)], 'g');
grid on
labels = {'vary alpha', 'vary velocity', 'vary de', 'vary ds', 'vary tn'};
set(gca, 'YTickLabel', labels)
title('Velocity SA')
xlabel('v (m/s)')
```

The script repeated this code (outside of all loops) for each of the five parameters of the x vector. The SimuLink model was not altered from the original, shown in [Figure 1].

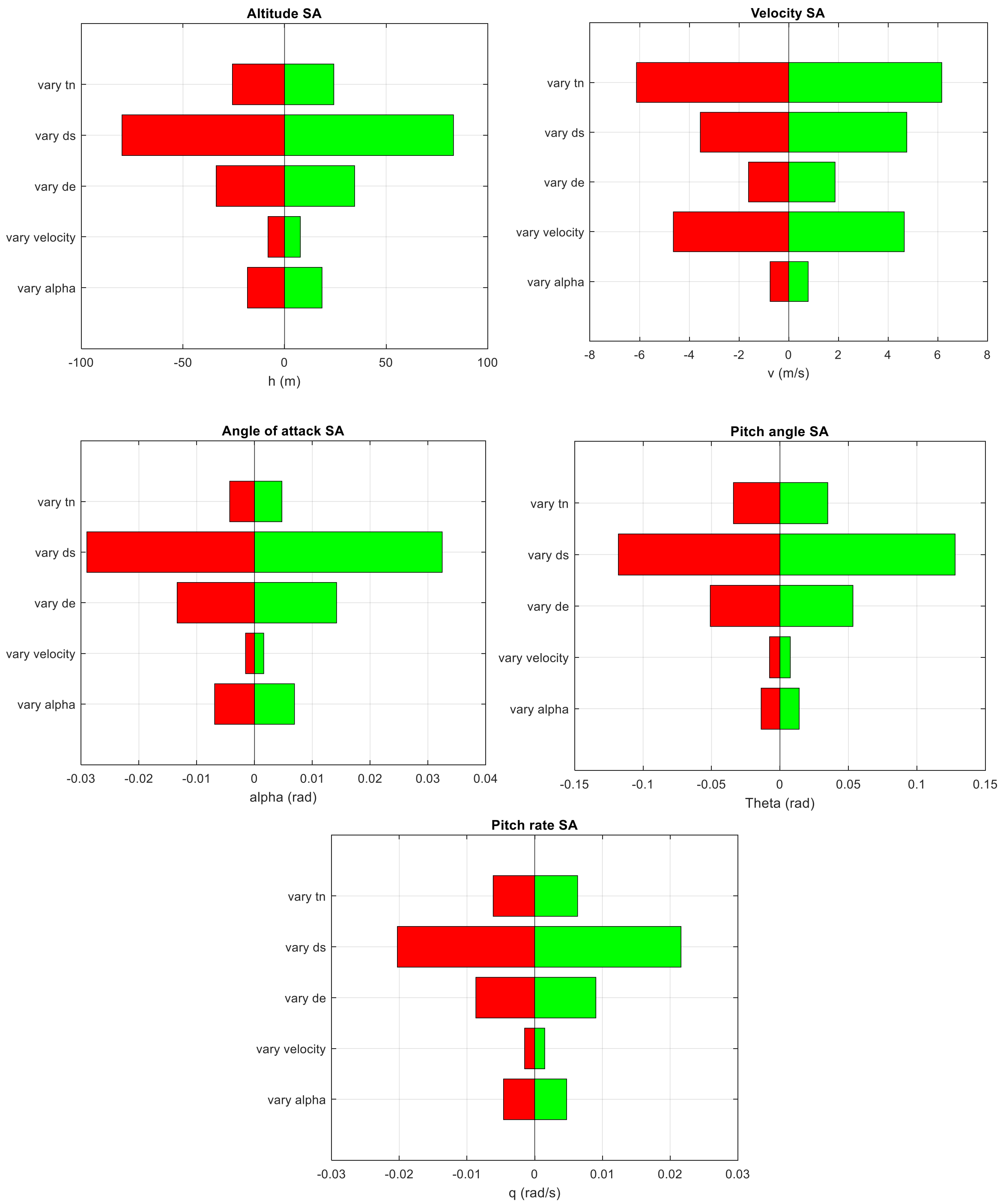


Figure 5. Tornado Graphs depicting Sensitivity/Uncertainty Analyses of all parameters of x. All nominal values set to 0.

The tornado graphs in [Figure 5] for altitude, pitch rate, pitch, and angle of attack all indicate that the respective variables are most sensitive to changes in the initial stabiliser value of the aircraft, as the variance value displayed on the graph was greatest. The speed tornado graph indicates that the speed of the aircraft is most sensitive to variations in the initial thrust of the aircraft, as well as the initial speed of the aircraft. This means that having the stabiliser at a nonzero value prevents the other control parameters of the aircraft from stabilising, and as shown results in high variance values for all states of the model (in x). The initial speed and initial angle of attack caused the least variation in all states (except for speed). These observations all come as expected of a flying aircraft under each set of conditions, further verifying the validity of the model.

Question 3: Implement PID Controllers

The set up to our model will be the same as the original with modifications to the input vector u(t). the modifications include creating a signal for gamma from the equation:

$$\gamma = \theta - \alpha \qquad \text{[Equation 1]}$$

The model within the new subsystem for u now takes in step functions for gamma and v at the requested step amplitudes, and combines them over intervals of 10 seconds for each variable staggered 10 seconds apart from one another.

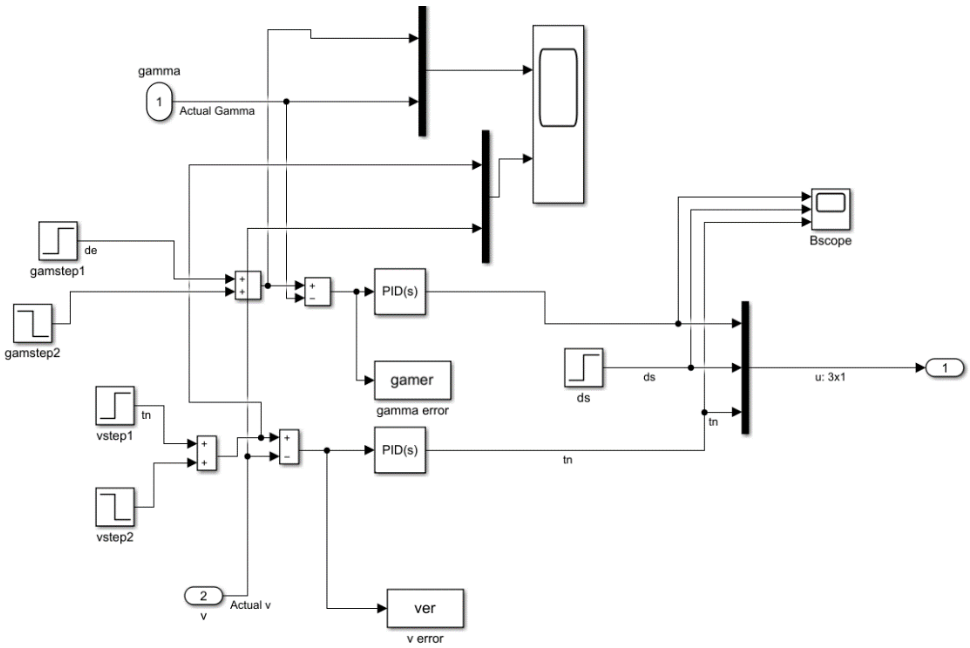


Figure 5. Diagram of new setup for u(t) within original system setup. Current gamma dealt with as the difference of alpha and theta signals from original diagram, represented by input 1 of the subsystem. Current speed input from simulation in input 2 bubble.

The PID controller is implemented to take in the error of each run through of the Simulink model and attempt to minimize it for the upcoming run. Error found by subtracting the current values of gamma and velocity from the system form their respective step inputs. The PID controllers are each tuned to take in the given performance specifications, first for initial gamma, the flight path angle of the aircraft, then for the initial speed. The initial conditions are applied for 10 seconds at a time staggered to not overlap over the course of the simulation. The tuning of the signals is done in an alternating fashion, running a trial between each period of tuning, because coupling between the parameters will be present.

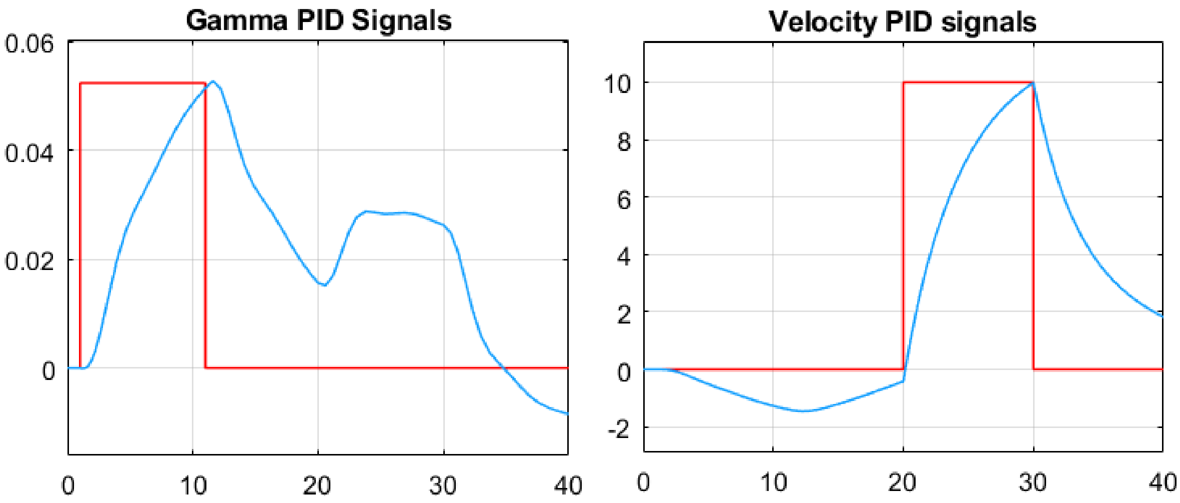
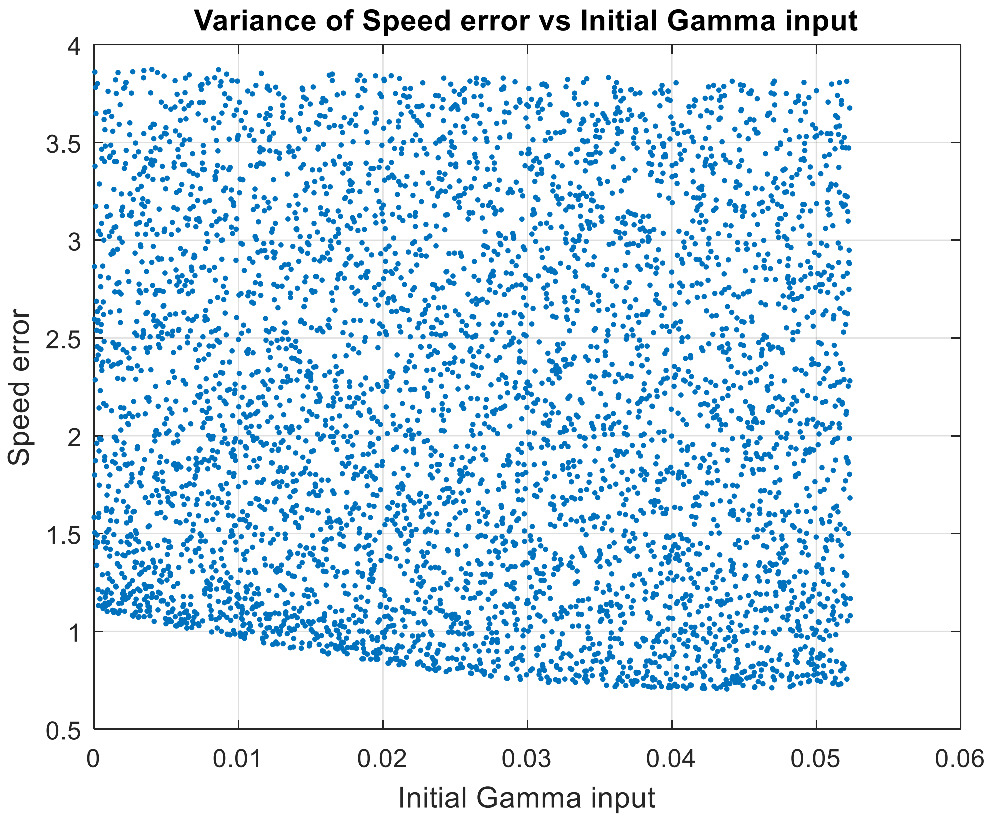
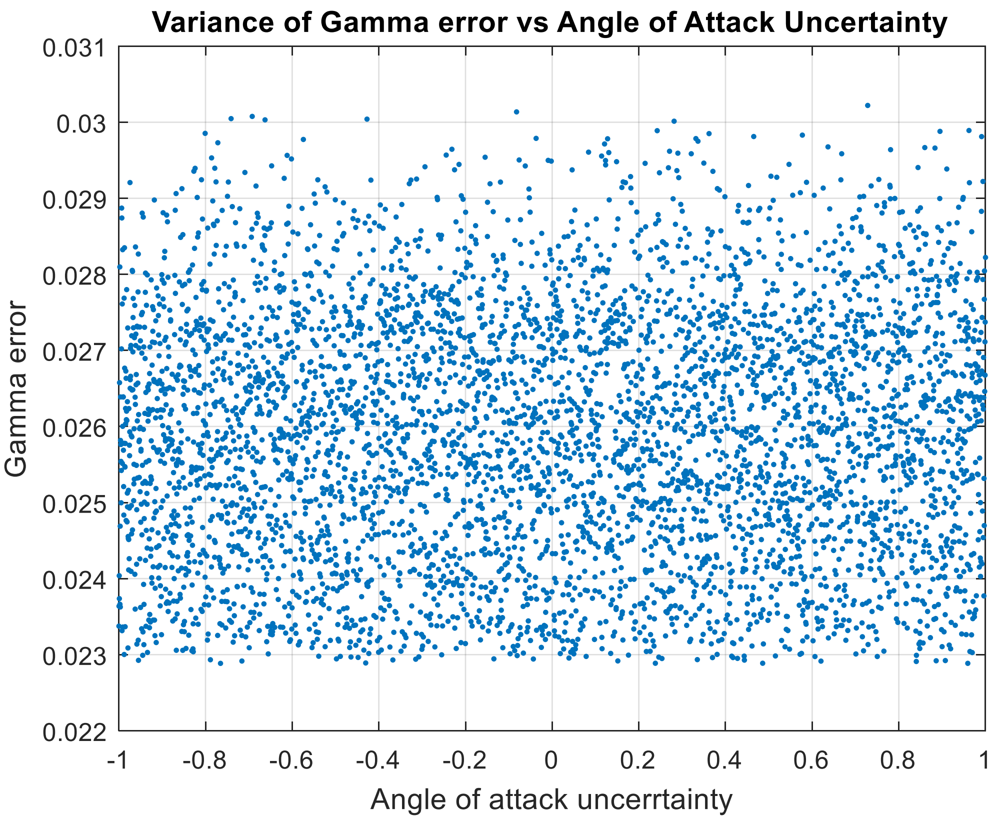


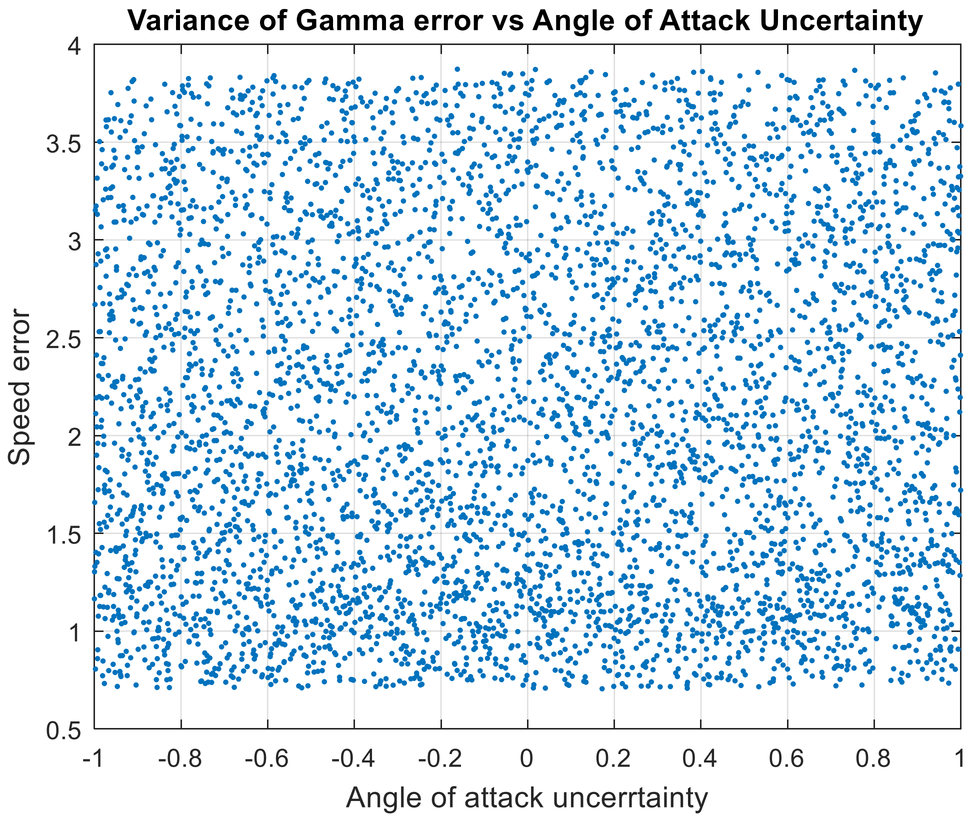
Figure 5. Signals plotted in red indicate the input signals over the given time interval for gamma and speed. Signals plotted in blue indicate the actual signals being plotted over the time interval.

The coupling affect is shown by the scope when the signal for speed spikes as the flight path angle of the aircraft enters its step function for the input.

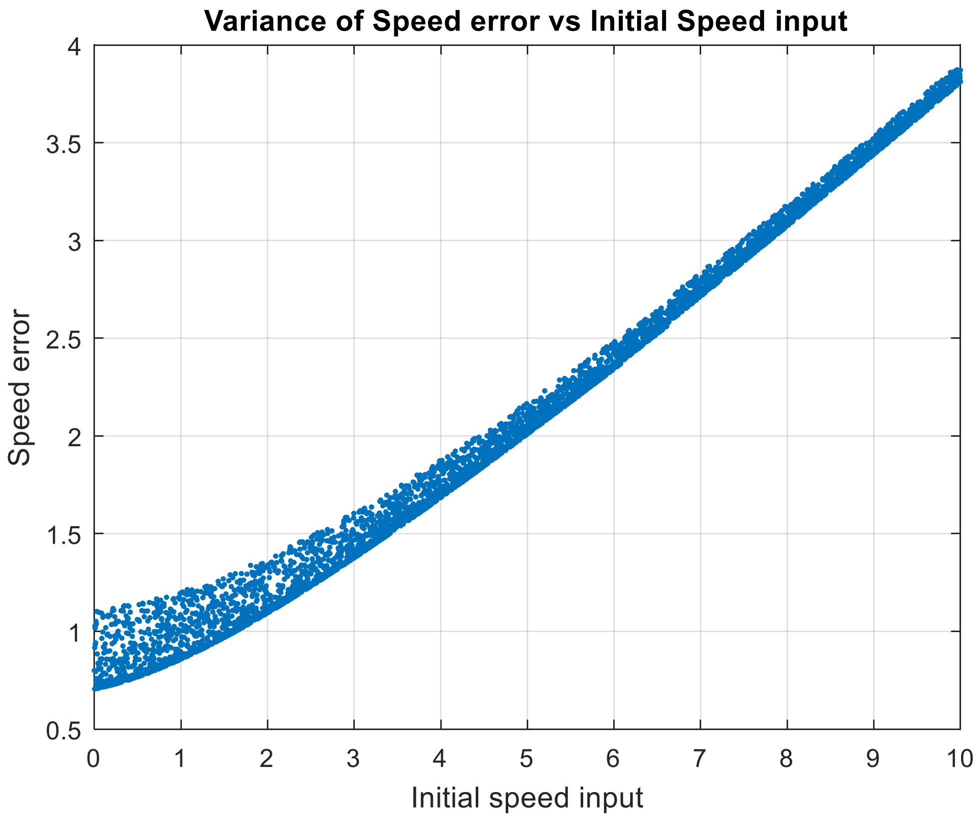
PLOTS for Part 4


Variance of Speed error vs Initial Gamma input
Speed error
Initial Gamma input
0.5
1
1.5
2
2.5
3
3.5
4
0
0.01
0.02
0.03
0.04
0.05
0.06

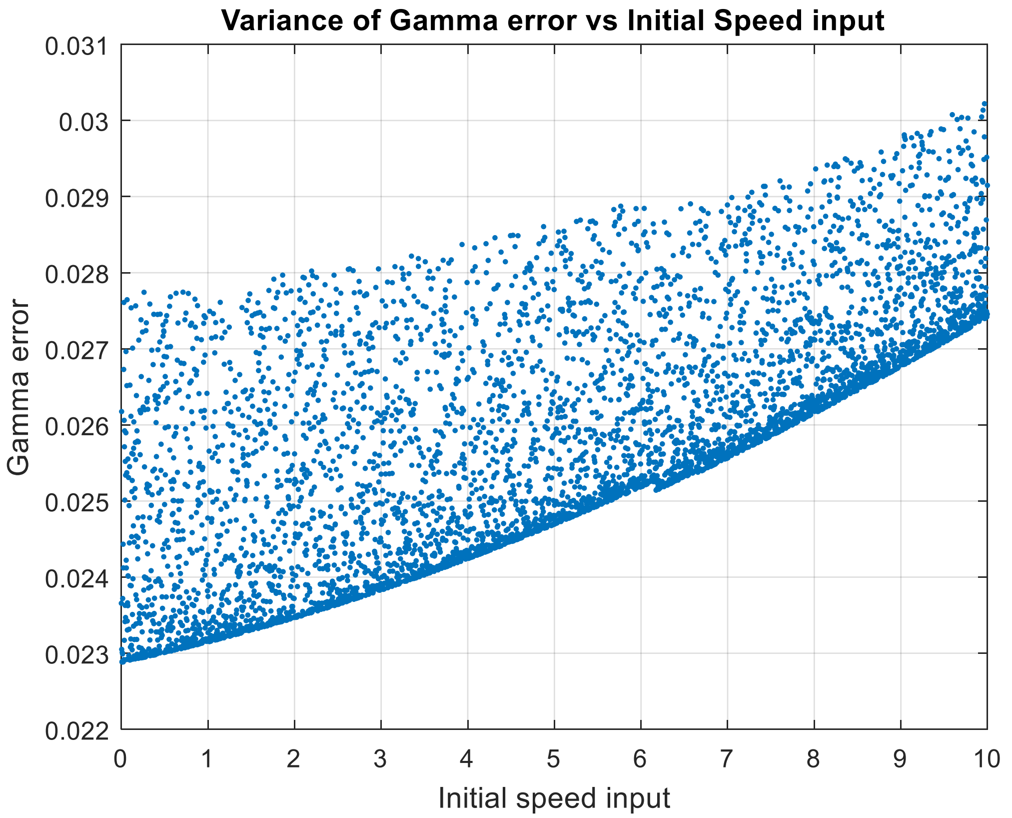

Variance of Gamma error vs Angle of Attack Uncertainty
Gamma error
Angle of attack uncerrtainty
0.022
0.023
0.024
0.025
0.026
0.027
0.028
0.029
0.03
0.031
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1

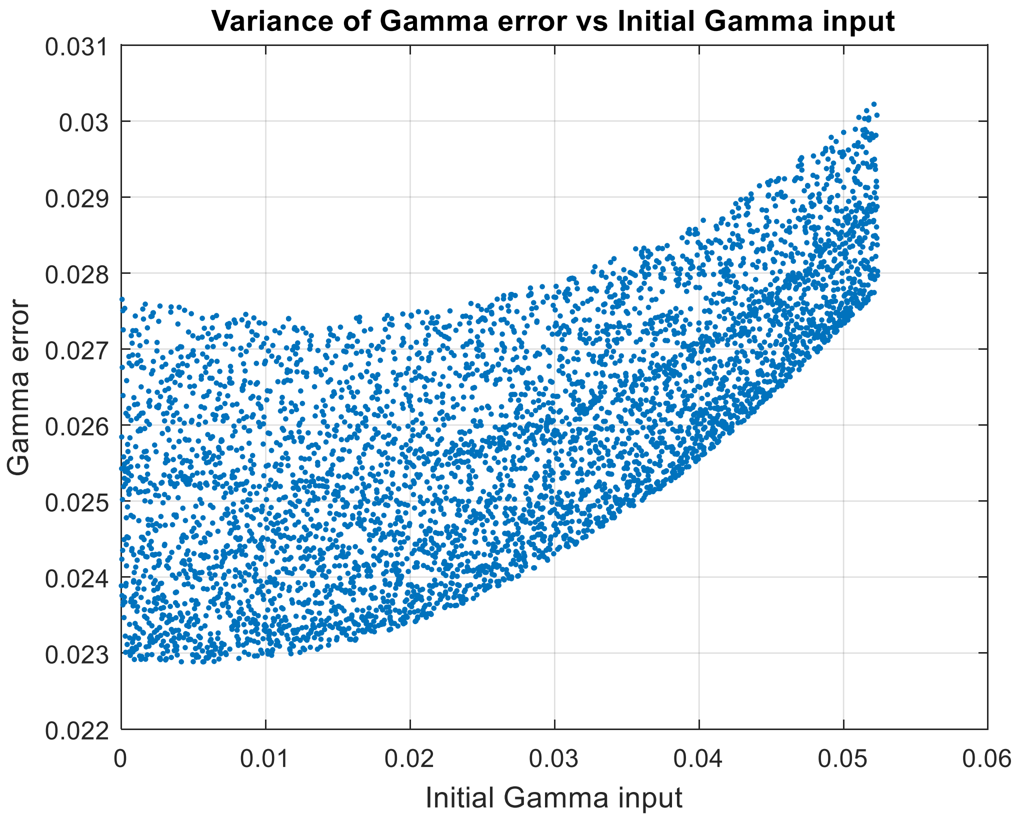


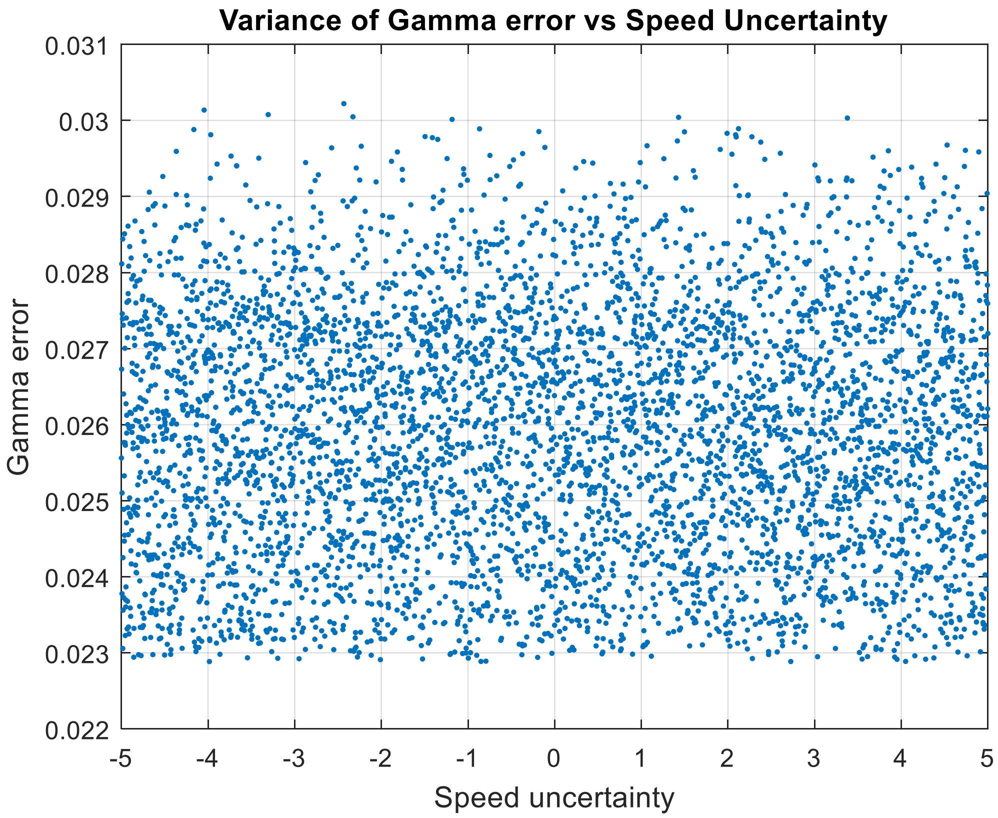
Variance of Gamma error vs Angle of Attack Uncertainty
Speed error
Angle of attack uncerrtainty
4
3.5
3
2.5
2
1.5
1
0.5
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1

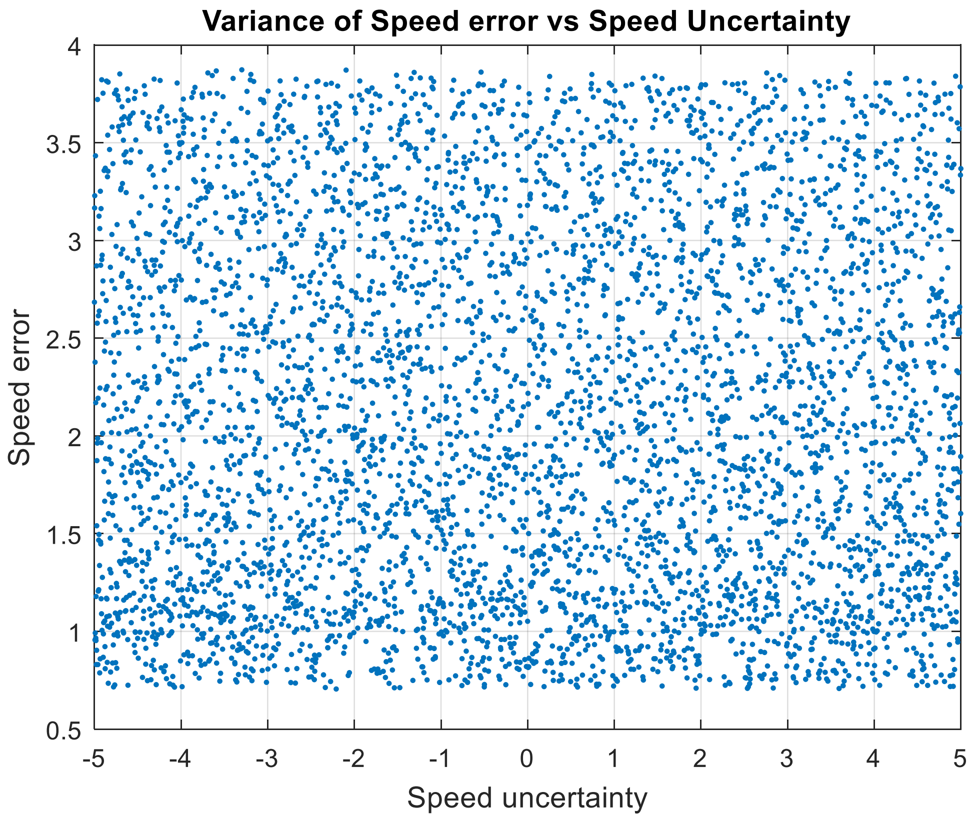


Variance of Speed error vs Initial Speed input
Speed error
Initial speed input
4
3.5
3
2.5
2
1.5
1
0.5
0
1
2
3
4
5
6
7
8
9
10


Variance of Gamma error vs Initial Speed input
Gamma error
Initial speed input
0.031
0.03
0.029
0.028
0.027
0.026
0.025
0.024
0.023
0.022
0
1
2
3
4
5
6
7
8
9
10


Variance of Gamma error vs Initial Gamma input
Gamma error
Initial Gamma input
0.031
0.03
0.029
0.028
0.027
0.026
0.025
0.024
0.023
0.022
0
0.01
0.02
0.03
0.04
0.05
0.06


Variance of Gamma error vs Speed Uncertainty
Gamma error
0.031
0.03
0.029
0.028
0.027
0.026
0.025
0.024
0.023
0.022
-5
-4
-3
-2
-1
0
1
2
3
4
5
Speed uncertainty


Variance of Speed error vs Speed Uncertainty
Speed error
4
3.5
3
2.5
2
1.5
1
0.5
-5
-4
-3
-2
-1
0
1
2
3
4
5
Speed uncertainty