$\begin{array}{c} \text{MORTGAGE LOAN VALUE PREDICTION WITH MACHINE} \\ \text{LEARNING} \end{array}$

by

Thomas R. Billman III

A paper submitted in partial fulfillment of the requirements to complete Honors in the Department of Mathematics and Statistics.

| Examining Committee: | Approved By: | | | | | | | |
|---|---|--|--|--|--|--|--|--|
| | Dr. Yishi Wang Faculty Supervisor | | | | | | | |
| Dr. Zhuan Ye | | | | | | | | |
| Dr. Joseph Farinella | | | | | | | | |
| | Dr. Zhuan Ye Chair, Mathematics and Statistics | | | | | | | |
| Honors Council Representative | | | | | | | | |
| Director of the Honors Scholars College | | | | | | | | |

University of North Carolina Wilmington

Wilmington, North Carolina

April, 2017

TABLE OF CONTENTS

| ABS | STRAC' | Ti | ii | | | | |
|-----|---------------|-----------------------------|----|--|--|--|--|
| 1 | Introd | uction | 1 | | | | |
| 2 | Litera | ture Review | 1 | | | | |
| 3 | Our D | Pataset | 2 | | | | |
| 4 | Net P | resent Value | 7 | | | | |
| | 4.1 | Our Analysis | 9 | | | | |
| | 4.2 | Prepaid and Current Loans | 9 | | | | |
| | 4.3 | Default | 0 | | | | |
| | 4.4 | Application | .1 | | | | |
| 5 | Geogr | aphic Mapping | 2 | | | | |
| 6 | High I | Performance Computing | 5 | | | | |
| 7 | Linear | Regression | 5 | | | | |
| | 7.1 | Initial Data Cleaning | 5 | | | | |
| | 7.2 | Cook's Distance | 6 | | | | |
| | 7.3 | Multi-collinearity | 7 | | | | |
| | 7.4 | Significant Covariates | 8 | | | | |
| | 7.5 | Data Transformation | 9 | | | | |
| 8 | Rando | om Forest | 0 | | | | |
| | 8.1 | Regression | 0 | | | | |
| | 8.2 | Classification | 1 | | | | |
| 9 | Rando | om Generalized Linear Model | 2 | | | | |
| | 9.1 | Regression | 2 | | | | |
| | 9.2 | Classification | 3 | | | | |
| 10 | Conclu | usion | 3 | | | | |
| BEF | REFERENCES 24 | | | | | | |

| Acknowledgments | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | 2 | 26 |
|-----------------|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|---|----|
|-----------------|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|--|---|----|

ABSTRACT

This project investigates relationships between mortgages' net present values (NPV) and their related covariates through public datasets from Freddie Mac. These datasets contain loan records for single family houses with 30 year fixed interest rates. To our knowledge, this is the first effort of such investigations on this complexly structured dataset. Given the size of the datasets as well as the complexity of the problem, our investigation begins with cleaning and calculating NPVs based on each loans records, both effectively and efficiently on high performance computing clusters provided by Texas Advance Computing Center. Classical statistical methods and contemporary machine learning algorithms are deployed for regression and classification. Computation results suggest that machine learning algorithms outperform classical regression and classification methods.

1 Introduction

Our research primarily focuses on using contemporary machine learning techniques to predict mortgage values more accurately than traditional linear methods. Previous research has focused on classifying loans at time of origination as prepaid, paid as planned, or default[5]. Through literature review, we find related work, but nothing using the Net Present Value (NPV). While it is important to predict the status of a loan, banks may have more interest in profitability prediction. As a proxy for profit, we compute the NPV of each loan which adjusts all payments to the time the loan was issued. This is different way to compare the financial impact of loans as opposed do to end state classification methods. Additionally, once we calculate NPVs we predict them using data available at time of loan origination.

2 Literature Review

A mortgage is a loan that is secured by real estate [11]. If a borrower stops making payments on their loan it goes into default, and the bank can foreclose on the property. Through the process of foreclosure, the bank that wrote the mortgage claims the property from the borrower. The bank then sells the property and uses the proceeds to recover the rest of their outstanding balance. If the property has dropped in value, it is possible that the bank can lose large sums of money by writing mortgages, so it is important to make sure banks select borrowers who are likely to make their payments. Due to the high financial stakes, the ability to determine which borrowers are mortgage-worthy is very important and the subject of much research.

One project used contemporary machine learning techniques to model whether loans will carry out as planned, end in default, or end prepaid [5]. This project considered methods such as Binary Logit, Multinomial Logit, K-Nearest Neighbors, K-fold

Cross Validation, and Random Forest. Of the models considered, the most accurate model was random forest (RF) classification. This model could classify loans with 93% accuracy. Given that loans ending in default can have large differences in financial impact, the NPV analysis helps with this issue. Due to the proven accuracy of RF classification, we decided to use random forest modeling in our analysis[5]. Another project involved an unprecedented dataset of 120 million prime and subprime mortgages from 1995 to 2014[12]. After adding local economic metrics to their dataset, neural networks were used to predict how many loans would be end as either prepaid or default within random portfolios of thousands of loans. Their research showed that neural networks considerably outperformed similar analysis using traditional logit techniques. This is particularly impactful for agencies that package and sell mortgage-backed securities as it can drastically improve the methods of choosing loans for their products. This is also a good indicator for our project, that machine learning algorithms will yield better predictions of NPV as compared to linear regression.

3 Our Dataset

Our dataset was obtained from the Federal Home Loan Mortgage Corporation, better known as Freddie Mac, which is a public government-sponsored enterprise. We used their Single Family Loan Level Dataset, which lists origination and performance data for loans based on financial quarter of origination ¹. The dataset is composed of two files where the first file lays out the details of each loan's origination. It contains 391,419 observations of 26 variables, which are:

| Credit Score | A number summarizing the borrower's |
|--------------|--|
| Credit Score | creditworthiness and prepared by third parties |

¹The dataset can be found at http://www.freddiemac.com/research/datasets/sf_loanlevel_dataset.html, and only registered accounts can download it

| Date of the first scheduled payment | | | | | | | | |
|--|--|--|--|--|--|--|--|--|
| Indicates if an individual is 1) Purchasing the | | | | | | | | |
| mortgaged property, 2) will reside in the | | | | | | | | |
| property as primary residence, 3) does not have | | | | | | | | |
| ownership interests in other residential | | | | | | | | |
| properties | | | | | | | | |
| Date of the final scheduled payment | | | | | | | | |
| Similar to Zip Codes, but for large metropolitan | | | | | | | | |
| areas containing 2.5 million people or more. | | | | | | | | |
| These are defined by the US Census | | | | | | | | |
| Percentage of loss coverage on the loan, to be | | | | | | | | |
| paid to Freddie Mac in the event of a default | | | | | | | | |
| Number of properties covered by this mortgage | | | | | | | | |
| Denotes whether the home is owner occupied, a | | | | | | | | |
| second home, or investment property | | | | | | | | |
| Original mortgage loan amount plus possible | | | | | | | | |
| second mortgage amount divided by initial | | | | | | | | |
| property value | | | | | | | | |
| Borrower monthly income divided by monthly | | | | | | | | |
| mortgage payment | | | | | | | | |
| Initial amount loaned in the mortgage note | | | | | | | | |
| initial amount loaned in the mortgage note | | | | | | | | |
| Initial mortgage loan amount divided by initial | | | | | | | | |
| property value | | | | | | | | |
| Original rate indicated on mortgage note | | | | | | | | |
| | | | | | | | | |

| Channel | What type of organization sold Freddie Mac |
|------------------------------|--|
| - " | this loan (Retail, Broker, etc.) |
| Duan arms ant Dan alter Elag | Indicates if the borrower is penalized for |
| Prepayment Penalty Flag | prepayment |
| Product Type | All entries are Fixed Rate Mortgages |
| Property State | The U.S. State the property is located in |
| Duopouty Type | Indicates property type (Single-Family Home, |
| Property Type | Condo, Co-op, etc) |
| Postal Code | First three numbers of the property's Zip Code |
| Loan Sequence Number | Unique identifier assigned to each loan |
| I D | Indicates if the loan is to purchase the house, or |
| Loan Purpose | refinance the property |
| Original Loan Term | Number of payments calculated from First |
| Original Loan Term | Payment Date and Maturity Date |
| Number of Borrowers | Number of Borrowers obligated to repay the |
| Number of Borrowers | mortgage $(1 \text{ or } > 1)$ |
| Seller Name | Entity who sold the loan to Freddie Mac |
| Servicer Name | Entity who is currently servicing the loan on |
| Servicer Ivaine | Freddie Mac's behalf |
| Super Conforming Flag | Loans that exceed conforming loan limits |
| Pre-HARP Loan Sequence | Links a HARP loan to its pre-HARP origination |
| Number | data |
| | |

This is the data we will be using to predict NPV, as it is all collected during the loan selection process.

The second file of the dataset contains monthly performance data for each loan where each loan has an entry for every month regarding the status of the loan. This file has 2,311,802 observations of 23 variables such as:

| I C N I | Same number found in origination file and |
|-----------------------------|---|
| Loan Sequence Number | used to link the two |
| Monthly Reporting Period | Current Month of entry |
| | Mortgage ending balance for the monthly |
| Current Actual UPB | reporting period. It includes scheduled and |
| | unscheduled principal reductions |
| Current Loan Delinquency | Continuous number of months since Due |
| Status | Date of Last Paid Installment (DDLPI) |
| Loop Age | Number of months since the origination of |
| Loan Age | the loan |
| Remaining Months to Legal | The remaining number of months until the |
| Maturity | mortgage Maturity Date |
| Repurchase Flag | This indicates loans that have been |
| Repurchase Flag | repurchased or made whole |
| Modification Flor | This indicates that the loan has been |
| Modification Flag | modified |
| | A code indicating why the loan's balance |
| Zero Balance Code | was reduced to zero $(1 = Prepaid/Matured)$ |
| | Voluntarily, $3 = \text{Foreclosure, etc.}$ |
| Zero Balance Effective Date | The month in which the event triggering |
| Zero Darance Effective Date | the Zero Balance Code took place |
| Current Interest Rate | The current interest rate on the mortgage |
| Ourrent Interest Nate | after any modifications |

| | Current amount of non-interest bearing | | | | | | | | |
|------------------------------|--|--|--|--|--|--|--|--|--|
| Current Deferred UPB | UPB (Only occurs in the event of some loan | | | | | | | | |
| | modifications) | | | | | | | | |
| | The date that the loan's scheduled interest | | | | | | | | |
| Due Date of Last Paid | and principal payments were paid through, | | | | | | | | |
| Installment (DDLPI) | regardless of when last payment was | | | | | | | | |
| | actually made | | | | | | | | |
| MI Danamina | Proceeds received from mortgage insurance | | | | | | | | |
| MI Recoveries | in the event of default | | | | | | | | |
| Not Color Duocoda (NCD) | Amount received from sale of property less | | | | | | | | |
| Net Sales Proceeds (NSP) | selling expenses | | | | | | | | |
| Non-MI Recoveries | Other proceeds such as tax, insurance, etc. | | | | | | | | |
| Non-IVIT Recoveries | paid to Freddie Mac | | | | | | | | |
| | Expenses Freddie Mac bears in the event of | | | | | | | | |
| | foreclosure. This is an aggregation of Legal | | | | | | | | |
| Expenses | Costs, Maintenance and Preservation Costs, | | | | | | | | |
| | Taxes and Insurance, and Miscellaneous | | | | | | | | |
| | Expenses | | | | | | | | |
| | Legal costs associated with sale of property | | | | | | | | |
| Legal Costs | (not included in NSP) in the event of | | | | | | | | |
| | foreclosure | | | | | | | | |
| Maintenance and Preservation | Costs associated with maintaining property | | | | | | | | |
| Costs | during foreclosure | | | | | | | | |
| Taxes and Insurance | Cost of taxes and insurance incurred with | | | | | | | | |
| Takes and mountaine | sale of property | | | | | | | | |
| | | | | | | | | | |

| Miggellen gang Eym angag | Other expenses associated with sale of | | | | | |
|--------------------------|---|--|--|--|--|--|
| Miscellaneous Expenses | property | | | | | |
| | Default UPB - NSP + Delinquent Accrued | | | | | |
| | Interest - Expenses - Recoveries where | | | | | |
| Actual Loss | Delinquent Accrued Interest is the interest | | | | | |
| | owed on payments missed since DDLPI | | | | | |
| Modification Cost | Costs associated with a rate modification | | | | | |
| | event | | | | | |

The Actual Loss variable is particularly relevant to our research, as it gives us a comprehensive overview of losses suffered by the bank holding the loan in the event of default. Between the loan origination and performance data, we can accurately asses how valuable each loan was for the bank at the time it was written and associate that with data collected at origination. It is also important to note that given the time restrictions of our research we only used the first quarter of 1999 as our dataset. We chose first quarter of 1999 because it was the oldest set. This gives it a larger proportion of loans that are already settled as compared to other sets which will have more loans which are still being paid off.

4 Net Present Value

The value of money is driven by the fact that it can be used to purchase goods and services. Individuals must be compensated to delay consumption. In addition, due to inflation the purchasing power of money decreases with time. Intuitively, we know that fifty years ago a dollar bill could purchase an entire meal and today the same dollar bill may only buy a drink. This concept is known as the time

value of money. When evaluating investment, it is necessary to consider the fact that cash flows in different periods have different values. Investors calculate the net present value of the cash flows to determine if an investment is acceptable. The net present value is defined as the present value of the inflows minus the present value of the outflows. A positive net present value indicates that the investment should be accepted since it generates a profit for the firm and a negative net present value indicates that an investment should be rejected since it is not profitable.

To express this mathematically, we let R_t represent a cash flow at time t. If money is received, R_t is positive and if an amount is paid, R_t will be negative. (1+i) represents amount an investment should appreciate to compensate the lender for the time value of their money for one unit of time t and i represented the effective interest rate for one unit of time. A financial asset consisted of n different cash flows could have its net present value calculated with the following formula:

$$NPV_{total} = \sum_{t=1}^{n} \frac{R_t}{(1+i)^t}$$

Because all the cash flows associated with an asset are included, and brought to the present, NPV_{total} represents what an asset is worth at the present. It is also worth noting that we assume that if the bank did not invest in this loan, they would invest in a 30 year bond instead, as the most comparable financial asset. Because of this, we set our interest rate i to be the monthly London Inter-bank Offering Rate (LIBOR) from January of 1999. We chose LIBOR as our index because it represents the rate at which a large bank could loan money as a 30 year commitment. Since the LIBOR rate was 2.93% yearly, our monthly rate came out to around .241%.

4.1 Our Analysis

To calculate the NPVs of these loans, we first need to determine whether or not they ended as paid off or with a foreclosure. We began by associating each loan with its corresponding performance entries. Our programs used the value of each loan's Zero Balance Code to determine if the loan was paid off in good standing or foreclosure. These values and their meanings can be found in the data set user guide². If the Zero Balance Code is not present or NA, it is assumed that the loan is still current and assigns the value "Current". If the Zero Balance Code is 1, that means that the loan is prepaid, and it is marked "Prepaid". Finally, if the Zero Balance Code is 3 or 9, the loan ended in foreclosure and it is marked "Default". Once the loan has been classified, it gets put into one of two NPV calculation formulas.

4.2 Prepaid and Current Loans

If a loan is either current or prepaid the NPV is calculated with the same function. Due to the fact that many people do not make level payments on their loans, we use the performance data to compute each payment made separately. The theory behind this is that each payment has two parts; one part of the payment compensates the bank for the time value of holding the borrower's outstanding balance, and the other part pays down the outstanding balance. This can be referred to as the interest and principal portions of the payment, respectively. Additionally, since this is the only cash flow for the bank at this time, it is represented by R_t in our NPV calculations. So:

$$R_t = Payment_t = Interest_t + Principal_t$$

We define the outstanding principal, or Current Unpaid Balance, at time t as $CUPB_t$. Additionally, since the interest rate r is quoted as a yearly percent in

²Data set guide: http://www.freddiemac.com/research/pdf/user_guide.pdf

our data set (3 for 3% instead of .03) we have to divide by 1200 to determine the monthly interest rate charged by the bank. It follows that the interest owed in a given monthly payment is the previous CUPB multiplied by the monthly interest rate. Additionally, by checking the difference in CUPB we find the amount the principal was paid down. Mathematically:

$$Interest_t = CUPB_{t-1} * (r/1200)$$

$$Principal_t = CUPB_{t-1} - CUPB_t$$

Therefore:

$$R_{t} = CUPB_{t-1} * (r/1200) + CUPB_{t-1} - CUPB_{t}$$

$$NPV_{payments} = \sum_{t=1}^{n} \frac{CUPB_{t-1} * (r/1200) + CUPB_{t-1} - CUPB_{t}}{(1+i)^{t}}$$

However, to get the total NPV of the loan we need to subtract the original amount lent or Original Unpaid Balance (OUPB). This does not need to be adjusted for time as it was lent at time of origination, yielding us a final:

$$NPV_{total} = NPV_{payments} - OUPB$$

Our analysis will be refer to this NPV_{total} as just NPV. Our R code reflects this formula for NPV calculation.

4.3 Default

In the event that a loan ended in foreclosure, a different NPV formula was required. To find the NPV here, we take a similar approach as in the previous case with one major difference. A defaulted loan has a remaining outstanding balance that was not paid off at the end of their loan. This balance has to be adjusted

using all the expenses associated with foreclosing on a home and the net proceeds received by selling the home.

$$NPV_{total} = \sum_{t=1}^{n} \frac{CUPB_{t-1} * (r/1200) + CUPB_{t-1} - CUPB_{t}}{(1+i)^{t}} - OUPB + \frac{CUPB_{T} + AL}{(1+i)^{T}}$$

 $CUPB_T$ is the current unpaid balance at time of account closure, and AL represents Actual Loss, which is given in the dataset. T represents the number of months from loan origination to loan default. Since AL is listed as a negative number, the cash flow the bank receives at time of account closure would be CUPB + AL. Since this occurs many years into the mortgage, it is important to adjust it back to time of origination. In this case we assume account closure happens T months after origination. In our code we find this by taking the date of the last payment and adding the number of months it took until account closure. This can be calculated using First Payment Date and Zero Balance Effective Date. This gives us the total number of months between origination and foreclosure, or T.

4.4 Application

Due to the fact that this data is comprised of two files, it was imperative to find a way to match the performance data for each loan to its respective origination data before we could compute each loan's NPV. This was challenging because the number of performance entries for each origination entry is variable. Additionally, due to the size of our data set we had to solve this problem in an efficient manner. Our first solution that worked utilized a for loop and took roughly 30 minutes to match performance data to 1000 origination entries. Once we switched to an sapply() method, the time was cut to around 10 minutes. Finally, by using matrix operations we could match 1000 origination entries to their performance counterparts in around 6 seconds. We determined that this was fast enough to precess the full dataset in

a reasonable amount of time. The R code for this can be found in my Github Repository³, but an outline of the process is as follows:

| Step 1: Read in the datasets | Read both with read.delim() |
|------------------------------------|--|
| Step 2: Look for when the | Subtract each sequence number from the |
| Sequence Number Changes | previous entry |
| Step 3: Determine which have | T I. d d |
| differences | Isolate nonzero entries |
| Step 4: Partition performance | Use lapply() with our list of sequence |
| data into sets by origination file | number changes |

Finally, once we had our loans classified and developed a formula to compute the NPV in either case, we computed them all. It is also worth noting that we did not compute NPVs of loans that only have one performance file or were marked as repurchased prior to property disposition. These were all minor cases, and not useful for prediction. We added the full list of NPVs as another column to the origination file, and this was used for our regression and classification. Once we had all the performance data for each origination entry, we can calculate the NPV and attach it to the origination entry for our data set. This is what will be used for analysis.

5 Geographic Mapping

Given that our data had the first three numbers of each loan's zip code, we decided to look at how these NPVs look across the country. To do this, we used R packages such as ggplot2, evaluate, mapproj, fiftystater, zipcode, ggmap, and tidyverse. Our code followed this process:

³GitHub Repository: https://github.com/tbillman/Wang499

| Step 1: Load libraries | library("ggplot2"), etc. |
|---|--|
| Step 2: Read data | read_csv("File Location") |
| Step 3: Find representative Zip Code for all leading 3 digits of Zip Codes in our dataset | $00200 \rightarrow 00210, 00500 \rightarrow 00501 \dots 99800 \rightarrow$ $99801,99900 \rightarrow 99901$ |
| Step 4: Find representative states for all leading 3 digits of Zip Codes in our dataset | $00200 \rightarrow \text{NH}, 00500 \rightarrow \text{NY} \dots 99800 \rightarrow$ AK, $99900 \rightarrow \text{AK}$ |
| Step 5: Match each entry's Zip Code to it's respective state with our data frame | Entry 1 has Zip Code 19300, is in PA, and has NPV \$-12,008.92 |
| Step 6: Compute the mean NPV for each state | $AK \rightarrow $19,284.79, AL \rightarrow $18,721.94 \dots$ |
| Step 7: Compute the standard deviation of NPV for each state | $AK \rightarrow \$13,673.22, AL \rightarrow \$15,694.98 \dots$ |
| Step 8: Compute the ratio of mean of NPV and standard deviation of NPV for each state | $\mathrm{AK} \rightarrow 1.4104,\mathrm{AL} \rightarrow 1.1927\dots$ |
| Step 9: Graph data with ggplot2() | Figures 1, 2 and 3 |

The three graphs we plotted were the average NPV (Figure 1), standard deviation of NPVs (Figure 2), and the ratio between the two (Figure 3). Figure 3 is useful for banks looking for the best risk adjusted loan opportunities.

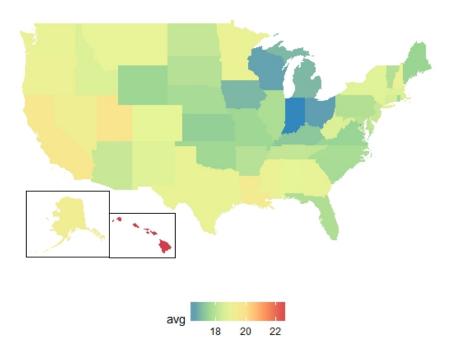


Figure 1: Average NPV by state (average scaled by \$1000)

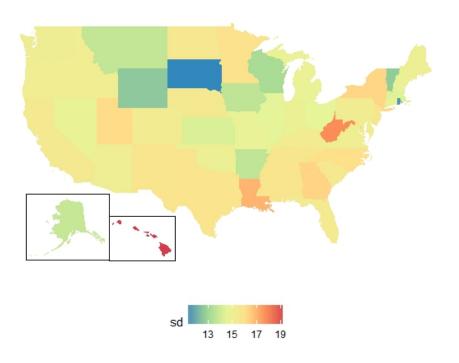


Figure 2: NPV standard deviation by state (SD scaled by \$1000)

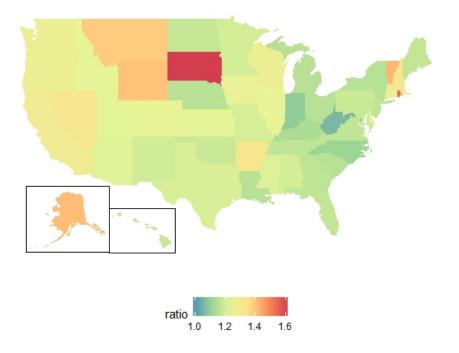


Figure 3: $\frac{AVG.NPV}{SD(NPV)}$ by state

6 High Performance Computing

Due to the large size of this data set and the computational requirements of the regression and classification methods we are implementing, access to the Stampede2 supercomputer greatly sped up our ability to run this analysis. Stampede2 was funded by the National Science Foundation, and is the flagship supercomputer at the Texas Advanced Computing Center. Due to the high computing power and memory of the Knights Landing nodes, we could run RF and RGLM analysis on our full data set in only three hours.

7 Linear Regression

7.1 Initial Data Cleaning

Initial NPV Distribution

Once we had our NPVs calculated, we began with simple linear regression to see if there was much correlation between the origination data and NPVs. After removing trivial columns with only one unique value and only keeping rows that had information in all

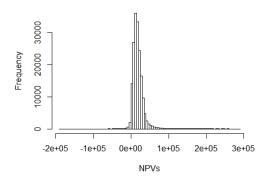


Figure 4: NPVs of complete cases

columns, we were left with a 178,058 x 25 matrix. Figure 4 shows the initial distribution of the NPVs. After running an initial Linear Regression on this data, we obtained an R_a^2 value of .008039, which is very low. Given the strong clustering in the middle, we believed that there were some outliers on both ends of the curve and that systematically removing these would boost the predictive power of our regression.

7.2 Cook's Distance

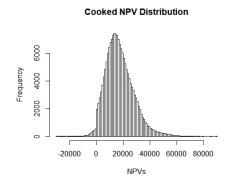


Figure 5: Entries without outlying covariates

In order to remedy our outlier problem, we calculated the Cook's Distance of each point. "Cook's distance, denoted by D_i is an aggregate influence measure, showing the effect off the *i*th case on all n fitted values":

$$D_{i} = \frac{\sum_{j=1}^{n} (\hat{Y}_{j} - \hat{Y}_{j(i)})^{2}}{pMSE}$$

Where \hat{Y}_j denotes the predicted value of the jth observation and $\hat{Y}_j(i)$ is the predicted value of the jth observation when the ith observation is removed. Additionally, p represents the number of covariate predictors in our linear regression model, and

MSE is the mean squared error of our model[9]. Removing Cook's Distance outliers is useful because our regression is aimed at predicting mortgage values of typical loans, and outlying loans can be considered on a case by case basis. A general rule of thumb is to discard points with distance greater than $\frac{4}{n}$, where n is the number of data points[2]. After points with outlying values were removed our distribution can be seen in Figure 5. It is also worth noting that in this distribution there is a large spike right around where NPV = 0. This is because there is a large number of defaulted mortgages with Actual Loss = 0. This is due to financial regulations where if the bank can recover more than their CUPB and foreclosure costs the remaining proceeds go to the borrower. This leaves many defaults that would have a positive NPV just above 0. After rerunning another linear regression, our R_a^2 value jumped to .01711. However, this is still very low, so we looked to other tactics to improve predictive power.

7.3 Multi-collinearity

We also checked for multi-collinearity between our different predictors. "A formal method of detecting the presence of multicollinearity that is widely accepted is the use of variance inflation factors [VIFs]. These factors measure how much the variances of the estimated regression coefficients are inflated as compared to when the predictor variables are not linearly related" [9]. This can be expressed quantitatively as follows:

$$(VIF)_k = (1 - R_k^2)^{-1}$$

Where R_k^2 is the R^2 value predicting the covariate k using the remaining covariates. Common tolerance limits for VIF are 10,100, and 1,000 ([9, p. 408-410]). A low VIF indicates low multicollinearity, whereas a high value indicates that certain variables are not important in our linear regression. When computing the VIF for all of our

```
Coefficients:
                  Estimate Std. Error t value Pr(>|t|)
(Intercept)
                -1.288e+05
                             9.462e+03
                                       -13.614
CreditScore
                   .620e+00
                             5.093e-01
                                        -7.107
                   133e+01
                             2.314e+00
                                        -9.219
'MI
   Percentage
                 1.244e+01
                             2.725e+00
                                         4.564
                                                5.02e-06
UPB
                 2.011e-02
                             4.814e-04
                                            776
                             2.408e+00
LTV
                 1.778e+01
                                          7.386 1.52e-13
                -1.363e+03
                             7.625e+01 -17.871
                0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Signif. codes:
Residual standard error: 10230 on 172697 degrees of freedom
Multiple R-squared: 0.01713,
                                Adjusted R-squared:
                                                      0.01709
               430 on 7 and 172697 DF,
                                         p-value: < 2.2e-16
F-statistic:
```

Figure 6: ANOVA table of Linear Regression

variables, all values were below 3 with the exception of Combined Loan to Value Ratio (CLTV) and Loan to Value Ratio (LTV). These variables had VIFs of over 2000. This is because CLTV is only different than LTV if someone refinances their mortgage, which is rare. Since LTV is more useful for loan origination, we kept that variable and removed CLTV from the regression. After removing CLTV, the VIF of LTV dropped below 3, but our R_a^2 value remained similar. This indicates that multicollinearity was not having a large effect on suppressing our R_a^2 value.

7.4 Significant Covariates

Figure 6 shows all the covariates with p values below .05 at this point. However, a few of the covariate estimates are not immediately intuitive. Particularly, an increase of Credit Score is associated with a decrease in NPV. This is likely because borrowers with high credit score are more likely to prepay their mortgages, resulting in a lower NPV of the mortgage. Additionally, although a higher interest rate increases payment size Interest Rate has a negative coefficient. We believe this is related to the fact that a higher interest rates are usually given to less qualified borrowers, who are more likely to default on their loans. The rest of the variables

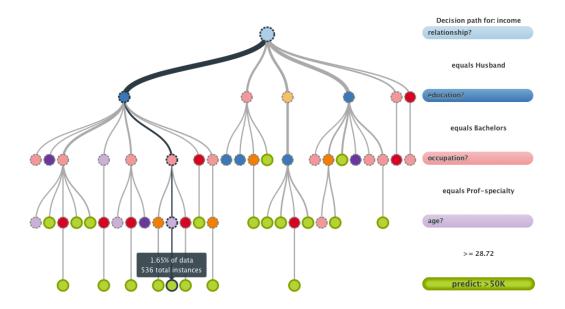
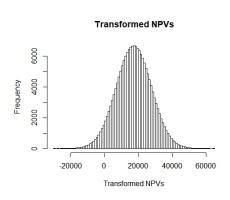


Figure 7: An example decision tree for income prediction seem to be going in an intuitive direction.

7.5 Data Transformation

The final tactic we tried was transforming the data into a normally distributed set. This was to test if the non-normality of our dataset was having an effect on suppressing our predic-



tive power. To do this, we found the percentile of each NPV value, and mapped it to a normal distribution with the same mean and standard deviation as our dataset. To do this we used the ecdf() function in R. However, even after this our R_a^2 value only rose to .01812. After this we concluded that simple linear regression would not have strong enough predictive power on this dataset to be useful.

8 Random Forest

Random forest is a contemporary machine learning technique developed by Dr. Leo Breiman at University of California at Berkeley in 2001[3]. Fundamentally, RF takes random subsets of the data and uses them to train decision trees. An example of a decision tree can be found in Figure 7. By training many of these trees on subsets of the data and taking an average of their predictions, we develop a more robust prediction model. This method can be used to predict either continuous or categorical variables. When deciding how to analyze this dataset with RF, we had a few options. Our first attempt was using the dataset to predict NPVs as a continuous variable via regression model. Another option was partitioning the NPV data into categories (Negative, Low, Medium, High) and predicting which NPV category a particular loan would fall into.

8.1 Regression

When running regression we recognized the importance of cross validation. This is a strategy of partitioning the data into a training set and testing set multiple times. For each partition a model is built using 80% of the data, then test it on the remaining 20% and check its accuracy. This is useful to prevent over-fitting in our model. We randomly assigned each entry to a number 1 through 5. Each entry consisted of the origination data of a loan as well as its NPV. We built five models, each with one index as our test data and the remaining four as our training data. This is a technique we used for all our machine learning analysis. Our R_a^2 values were all between .064 and .0661 with an average of .065. While this is significantly better than the linear regression, this still very poor predictive power. An outline of our code's process can be found in Figure 8.

| Step 1: Remove Degenerate | Columns like Product Type, which only contain | | | | | | |
|-------------------------------|--|--|--|--|--|--|--|
| Columns | one value are not useful for analysis | | | | | | |
| Step 2: Remove Incomplete | Rows with missing values are discarded, as the | | | | | | |
| Rows | machine learning models do not deal with them | | | | | | |
| Ttows | well | | | | | | |
| Step 3: Remove Cook's | Any entries with Cook's Distance greater than | | | | | | |
| Distance outliers | 4/n are removed | | | | | | |
| Step 4: Remove Variables not | Variables such as Loan Sequence Number and | | | | | | |
| useful for regression | Borrower Number are removed | | | | | | |
| Step 5: Partition data into 5 | Randomly assign a number 1-5 to each entry | | | | | | |
| sets | from a uniform distribution | | | | | | |
| Step 6: Build model on each | This gives up 5 different models of either RF or | | | | | | |
| set of 4 indexes | RGLM for either regression or classification, | | | | | | |
| Set of 4 indexes | each built on approximately 80% of the data | | | | | | |
| Step 7: Test each model on | Use the model to try and predict NPV of test | | | | | | |
| the remaining index | data with test origination data | | | | | | |
| Step 8: Verify and report | For regression, we report R_a^2 , for classification | | | | | | |
| 1 1 | we report proportion of observations that were | | | | | | |
| accuracy | correctly classified | | | | | | |

Figure 8: Machine learning pseudo-code

8.2 Classification

Due to the low predictive power of regression, we also used RF to predict the NPV of a loan in a more general sense. To do this, we split the data into the following four categories:

- Below \$0
- \$0 \$10,000
- \$10,000 \$30,000
- Above \$30,000

Once we did this, we used RF to predict which NPV category a given loan would fall into, with only the origination data as predictors. If the model had no predictive power, we would still expect a correct guess 25% of the time by chance. After

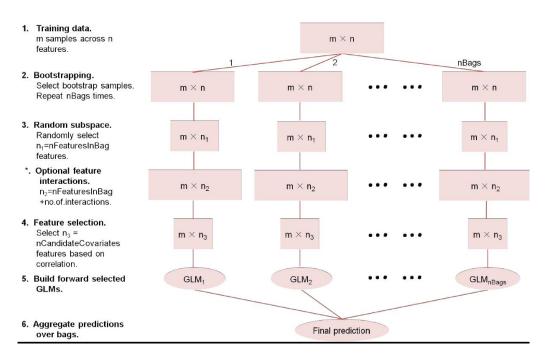


Figure 9: An overview of how RGLM works [13]

running our classification, our results had a minimum of .614, maximum of .625, and average of .620. This is significantly better than a blind guess, and a notable result.

9 Random Generalized Linear Model

Another technique we thought would be useful was using Random Generalized Linear Models. The way RGLM works is very similar to RF, however instead of training decision trees, RGLM trains generalized linear models. This is very nice because it takes the ensembling aspect of RF with a model that is easier to interpret.

9.1 Regression

One issue we encountered with RGLM is that our code could not use factor variables for regression prediction. As such, the R_a^2 value for our RGLM regression suffered. After running our analysis our five values ranged from .0174 to .0194 with an average of .0187. This is around the same as our initial linear regressions, and

does not have strong predictive power.

9.2 Classification

Due to the binary nature of GLM classification, to classify NPV with RGLM, we decided to opt for classification into NPVs above and below the median. After running our analysis in a similar way to RF, our cross validated results had a minimum accuracy of .543, maximum accuracy of .560, and average of .557. This is not very much better than random guessing, which we would assume to be around .500.

10 Conclusion

While the ability to predict mortgage loan NPV with the origination data was not as strong as we had initially thought, we did still prove that methods such as RF and RGLM outperform simple linear models. In the future, collating other quarters or data into the models as well as microeconomic and macroeconomic indicators may also help boost predictive power.

REFERENCES

- A. Liaw and M. Wiener (2002). Classification and Regression by randomForest.
 R News 2(3), 18–22.
- [2] Bollen, Kenneth A.; Jackman, Robert W. (1990). Fox, John; Long, J. Scott, eds. Regression Diagnostics: An Expository Treatment of Outliers and Influential Cases. Modern Methods of Data Analysis. Newbury Park, CA: Sage. pp. 25791. ISBN 0-8039-3366-5.
- [3] Breiman, Leo. "Random forests." Machine learning 45.1 (2001): 5-32.
- [4] D. Kahle and H. Wickham. ggmap: Spatial Visualization with ggplot2. The R Journal, 5(1), 144-161. URL http://journal.r-project.org/archive/2013-1/kahlewickham.pdf
- [5] Deng, Grace. "Analyzing the Risk of Mortgage Default" (2016)
- [6] Doug McIlroy. Packaged for R by Ray Brownrigg, Thomas P Minka and transition to Plan 9 codebase by Roger Bivand. (2017). mapproj: Map Projections. R package version 1.2-5. https://CRAN.R-project.org/package=mapproj
- [7] Hadley Wickham (2017). tidyverse: Easily Install and Load the 'Tidyverse'. R package version 1.2.1. https://CRAN.R-project.org/package=tidyverse
- [8] Jeffrey Breen (2012). zipcode: U.S. ZIP Code database for geocoding. R package version 1.0. https://CRAN.R-project.org/package=zipcode
- [9] Kutner, Michael H. Applied linear regression models. –4th ed. Michael H. Kutner, Christopher J. Nachtsheim, John Neter.

- [10] Marvin N. Wright, Andreas Ziegler (2017). ranger: A Fast Implementation of Random Forests for High Dimensional Data in C++ and R. Journal of Statistical Software, 77(1), 1-17. doi:10.18637/jss.v077.i01
- [11] Mortgage Basics. https://www.knowyouroptions.com/buy/buying-process/qualify-for-a-mortgage/mortgage-basics
- [12] Sirignano, Justin. Sadhwani, Apaar. Giesecke, Kay. "Deep Learning for Mortgage Risk" (2015)
- [13] Song L, Langfelder P, Horvath S. (2013) Random generalized linear model: a highly accurate and interpretable ensemble predictor. BMC Bioinformatics 14:5 PMID: 23323760 DOI: 10.1186/1471-2105-14-5.
- [14] Stampede 2 User Guide. (2018) https://portal.tacc.utexas.edu/user-guides/stampede2
- [15] William Murphy (2016). fiftystater: Map Data to Visualize the Fifty U.S. States with Alaska and Hawaii Insets. R package version 1.0.1. https://CRAN.Rproject.org/package=fiftystater

ACKNOWLEDGMENTS

I would like to thank Dr. Yishi Wang for advising me throughout this project. Whether it was our weekly meetings, emails, or online meetings at either 8:30 PM or 9:00 AM, Dr. Wang has been unbelievably helpful this whole process.

I'd like to thank Dr. Ann Stapleton for facilitating my learning of R, Github, UNIX, among so many other technical skills. Dr. Stapleton also help me learn other best practices for large software based projects. Without already having learned these skills working for Dr. Stapleton I would not have been able to widen the scope of this project to what it is.

I'd like to thank my family and friends for supporting me through this project. Without you guys, I don't know if I could have done it.

I'd like to thank the Texas Advanced Computing Center (TACC) at The University of Texas at Austin for providing HPC resources that have contributed to the research results reported within this paper. URL: http://www.tacc.utexas.edu