Probability for Data Science - Formula Sheet 0

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Math handbook

Exponential Properties:

$e^0 = 1, \quad e^{a+b} = e^a \cdot e^b$

$e^{a-b} = \frac{e^a}{e^b}, \quad (e^a)^b = e^{ab}$

$e^{-x} = \frac{1}{e^x}, \quad e^{\infty} = \infty, \quad e^{-\infty} = 0$

Logarithm Properties:

$\ln(ab) = \ln(a) + \ln(b), \quad \ln\left(\frac{a}{b}\right) = \ln(a) - \ln(b)$

$\ln(a^b) = b \cdot \ln(a), \quad \ln(1) = 0, \quad \ln(e) = 1$

Key Relationship:

$e^{\ln(x)} = x, \quad \ln(e^x) = x, \quad x > 0$

Approximations:

$e^x \approx 1 + x + \frac{x^2}{2} + O(x^3) \quad$ (for small x)

$\ln(1+x) \approx x - \frac{x^2}{2} + O(x^3) \quad$ (for small x)

Useful Integrals:

$\int e^x \, dx = e^x + C$

$\int e^{kx} \, dx = \frac{1}{k} e^{kx} + C$

$\int \ln(x) \, dx = x\ln(x) - x + C$

$\int_0^\infty e^{-\lambda x} \, dx = \frac{1}{\lambda} \quad$ (Useful for Poisson, Exp.)

$\int_0^\infty x e^{-x} \, dx = 1$

$\int_0^\infty x^2 e^{-x} \, dx = 2$

Key Derivatives:

$\frac{d}{dx} e^x = e^x \quad \frac{d}{dx} e^{kx} = k e^{kx}$

$\frac{d}{dx} \ln(x) = \frac{1}{x}$

Variable Transformations:

If $Y = g(X)$, then $f_Y(y) = f_X(g^{-1}(y)) \cdot \left| \frac{d}{dy} g^{-1}(y) \right|$

Integration Formulas

$\int c \, dx = cx + C$

$\int x^n \, dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$

$\int_a^b k \cdot f(x) \, dx = k \cdot \int_a^b f(x) \, dx$

$\int \sin(x) \, dx = -\cos(x) + C$

$\int \cos(x) \, dx = \sin(x) + C$

$\int \ln(x) \, dx = x\ln(x) - x + C$

$\int e^x \, dx = e^x + C$

$\int e^{kx} \, dx = \frac{1}{k} e^{kx}, \quad e^0 = 1, \, e^{-\infty} = 0$

$\int e^{g(x)} \, dx = \frac{1}{g\prime(x)} e^{g(x)}$

$\int (f(x) + g(x)) \, dx = \int f(x) \, dx + \int g(x) \, dx$

$\int u(x)v'(x) \, dx = u(x)v(x) - \int v(x)u'(x) \, dx$

$\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$

$\int a \cdot (b + c) = \int (a \cdot b) + \int (a \cdot c)$

$\int_0^{+\infty} x \cdot e^{-kx} \, dx = \frac{1}{k^2}, \quad k > 0$

Differentiation Formulas

$\frac{d}{dx}[c] = 0$

$\frac{d}{dx}[x^n] = n \cdot x^{n-1}, \quad n \in \mathbb{R}$

$\frac{d}{dx}[\sin(x)] = \cos(x)$

$\frac{d}{dx}[\cos(x)] = -\sin(x)$

$\frac{d}{dx}[\tan(x)] = \sec^2(x), \quad x \neq \frac{\pi}{2} + n\pi$

$\frac{d}{dx}[\ln(x)] = \frac{1}{x}, \quad x > 0$

$\frac{d}{dx}[\log_a(x)] = \frac{1}{x\ln(a)}$

$\frac{d}{dx}[e^x] = e^x$

$\frac{d}{dx}[e^{g(x)}] = g'(x) \cdot e^{g(x)}$

$\frac{d}{dx}[f(x) + g(x)] = f'(x) + g'(x)$

$\frac{d}{dx}[f(x) - g(x)] = f'(x) - g'(x)$

$\frac{d}{dx}[f(x) \cdot g(x)] = f'(x)g(x) + f(x)g'(x)$

$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}, \quad g(x) \neq 0$

$\frac{d}{dx}[f(g(x))] = f'(g(x)) \cdot g'(x)$

Change of Variables in PDFs (Continuous Case)

$$f_Y(y) = f_X(g^{-1}(y)) \cdot \left| \frac{d}{dy} g^{-1}(y) \right|$$

- **Linear transformation:** $Y = X - n \quad f_Y(y) = f_X(y+n)$

  $$X = Y + n, \quad \frac{d}{dy} g^{-1}(y) = 1$$

- **Scaling:** $Y = nX$, with $n > 0 \quad f_Y(y) = \frac{1}{n} f_X\left(\frac{y}{n}\right)$

  $$X = \frac{Y}{n}, \quad \frac{d}{dy} g^{-1}(y) = \frac{1}{n}$$

- **Logarithm:** $Y = \log(X) \quad f_Y(y) = e^y f_X(e^y)$

  $$X = e^Y, \quad \frac{d}{dy} g^{-1}(y) = e^Y$$

- **Absolute value:** $Y = |X|$

  $$f_Y(y) = \begin{cases} f_X(y) + f_X(-y), & y \geq 0 \\ 0, & y < 0 \end{cases}$$

- **Reciprocal (inversion):** $Y = \frac{1}{X} \quad f_Y(y) = \frac{1}{|y^2|} f_X\left(\frac{1}{y}\right)$

  $$X = \frac{1}{Y}, \quad \frac{d}{dy} g^{-1}(y) = -\frac{1}{y^2}$$

- **Square root:** $Y = X^2$, for $X \geq 0 \quad f_Y(y) = \frac{1}{2\sqrt{y}} f_X(\sqrt{y})$

  $$X = \sqrt{Y}, \quad \frac{d}{dy} g^{-1}(y) = \frac{1}{2\sqrt{y}}$$

- **Power function:** $Y = X^k$, with $k > 0$

  $$X = Y^{1/k}, \quad \frac{d}{dy} g^{-1}(y) = \frac{1}{k} y^{\frac{1}{k}-1}$$

  $$f_Y(y) = \frac{1}{k} y^{\frac{1}{k}-1} f_X(y^{1/k})$$

Change of Variables in PMFs (Discrete Case)

If $Y = g(X)$ is a discrete transformation, then:

$$P(Y = y) = P(X = g^{-1}(y))$$

- **Linear transformation:** $Y = X - n \rightarrow P(Y = y) = P(X = y + n)$
- **Scaling:** $Y = nX$ with integer $n \rightarrow P(Y = y) = P\left(X = \frac{y}{n}\right)$ only if y/n is an integer
- **Absolute value:** $Y = |X|$

  $$P(Y = y) = \begin{cases} P(X = y) + P(X = -y), & y \geq 0 \\ 0, & y < 0 \end{cases}$$

- **Reciprocal (special case for integers):** $Y = \frac{1}{X}$

  $$P(Y = y) = P\left(X = \frac{1}{y}\right) \quad \text{(only if } 1/y \text{ is an integer)}$$

- **Square transformation (for non-negative integers):** $Y = X^2$

  $$P(Y = y) = \begin{cases} P(X = \sqrt{y}) + P(X = -\sqrt{y}), & y \text{ is a perfect square} \\ 0, & \text{otherwise} \end{cases}$$