GECCO 2015 Tutorial:

Cartesian Genetic Programming

Evolved picture

Julian F. Miller, Andrew J. Turner
Dept of Electronics
University of York, UK
julian.miller@york.ac.uk
andrew.turner@york.ac.uk

Evolved picture

GECCO'15 Companion, July 11–15, 2015, Madrid, Spain.
ACM 978-1-4503-3488-4/15/07.
http://dx.doi.org/10.1145/2739482.2756571

Abstract

- Cartesian Genetic Programming (CGP) is a well-known form of Genetic Programming developed by Julian Miller in 1999-2000. In its classic form, it uses a very simple integer address-based genetic representation of a program in the form of a directed graph. Graphs are very useful program representations and can be applied to many domains (e.g. electronic circuits, neural networks). It can handle cyclic or acyclic graphs.
- In a number of studies, CGP has been shown to be comparatively efficient to other GP techniques. It is also very simple to program. The classical form of CGP has undergone a number of developments which have made it more useful, efficient and flexible in various ways. These include self-modifying CGP (SMCGP), cyclic connections (recurrent-CGP), encoding artificial neural networks and automatically defined functions (modular CGP).
- SMCGP uses functions that cause the evolved programs to change themselves as a function of time. This makes it possible to find general solutions to classes of problems and mathematical algorithms (e.g. arbitrary parity, n-bit binary addition, sequences that provably compute pi and e to arbitrary precision, and so on).
- Recurrent-CGP allows evolution to create programs which contain cyclic, as well as acyclic, connections. This enables application to tasks which require internal states or memory. It also allows CGP to create recursive equations.
- CGP encoded artificial neural networks represent a powerful training method for neural networks. This is because CGP is able to simultaneously evolve the networks connections weights, topology and neuron transfer functions. It is also compatible with Recurrent-CGP enabling the evolution of recurrent neural networks.
- The tutorial will cover the basic technique, advanced developments and applications to a variety of problem domains. It will present a live demo of how the open source cgplibrary can be used.

Contents

Genetic Programming

- The automatic evolution of computer programs
  - Tree-based, Koza 1992
  - Stack-based, Perkis 1994, Spector 1996 onwards (push-pop GP)
  - Linear GP, Nordin and Banzhaf 1996
  - **Cartesian GP,** Miller 1999
  - Parallel Distributed GP, Poli 1996 (related to CGP)
  - Grammatical Evolution, Ryan 1998
  - Lots of others…

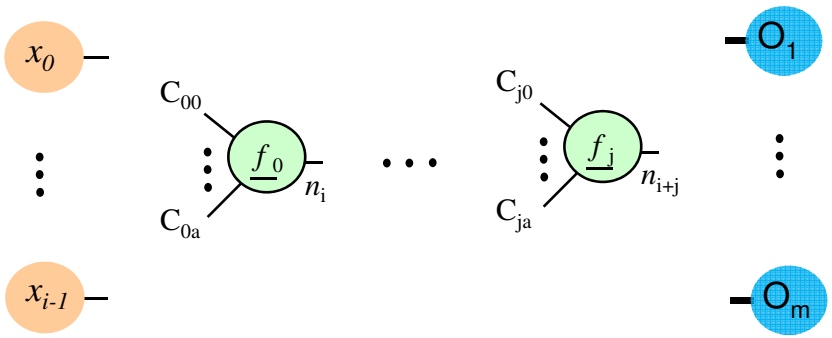
Origins

- Grew out of work in the evolution of digital circuits, Miller and Thomson 1997.
  - First actual mention of the term *Cartesian Genetic Programming* appeared at GECCO in 1999.
- Originally, represents programs or circuits as a two dimensional grid of program primitives.
  - Hence *Cartesian* GP
- This is loosely inspired by the architecture of digital circuits called FPGAs (field programmable gate arrays)

Why use CGP?

- Its performance is highly competitive with other forms of GP
- It is naturally suited to multiple input multiple-output (MIMO) tasks
- It is simple and only requires a simple evolutionary algorithm
- It encodes cyclic or acyclic graphs which are highly flexible for many applications
  - It allows internally calculated values to be reused multiple times
  - It can naturally encode artificial neural networks
- It benefits from explicit neutral genetic drift
- It does not suffer from program bloat. In fact, it naturally compresses solutions.

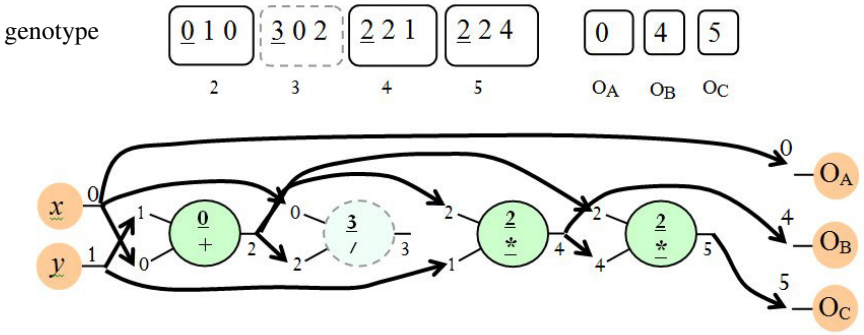
General form of CGP



All functions have as many inputs as the *maximum* function arity

Unused connections are ignored

Example

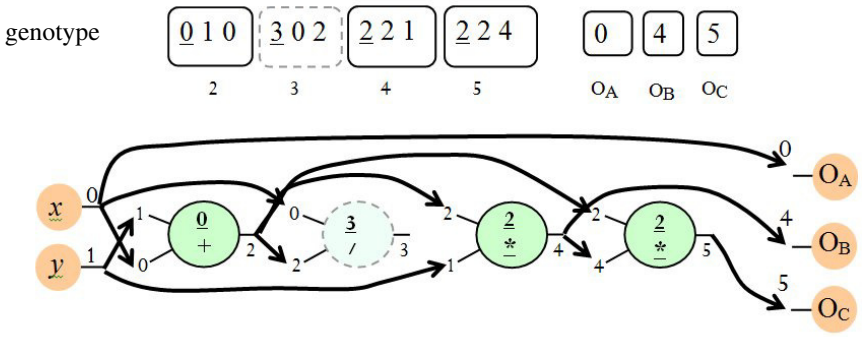


Function genes (are addresses in user defined lookup table)

0	+ Add 1	- Subtract	
2	* Multiply3	/ Divide (protected)	

So what does the graph represent?

genotype



phenotype

$O_A = x$

$O_B = (y + x)\, y$

$O_C = (y + x)^2\, y$

The CGP genotype-phenotype map

- When you decode a CGP genotype many nodes and their genes can be ignored because they are not referenced in the path from inputs to outputs
- These genes can be altered and make no difference to the *phenotype*, they are non-coding
- Clearly there is a many-to-one genotype to phenotype map
- How redundant is the mapping?
  - Massively redundant.
  - For example, a nine node genotype has 118,124 possible 3 node phenotypes (assuming one program input and all nodes have arity equal to one).

Decoding CGP chromosomes is *easy and fast*

```
// L  = MaxGraph.Length
// I  = Number of program inputs
// N = Number of program outputs
bool          ToEvaluate[L]
double        NodeOutput[L+I]
int           NodesUsed[M]
```

1

```
// identify initial nodes that need to be evaluated (once)
p = 0
do
  ToEvaluate[OutputGene[p]] = true
  p = p + 1
while (p < N)
// determine nodes needed
p = L-1
q=0
do
  if (ToEvaluate[p])
    x = Node[p].Connection1
    y = Node[p].Connection2
    ToEvaluate[x] = true
    ToEvaluate[y] = true
    q=q+1
    NodesUsed[q]=p
  endif
  p = p - 1
while ( p >= 0)
```

2

```
// load input data values
p = 0
do
   NodeOutput[p] = InputData[p]
   p = p + 1
while (p < I)
```

3

```
//Execute graph
for  p = I to p < q+I
     x = Node[NodesUsed[p]].Connection1
     y = Node[NodesUsed[p]].Connection2
     z = Node[NodesUsed[p]].Function
     NodeOutput[p] = ComputeNode(NodeOutput[x], NodeOutput[y],z)
endfor
```

4

Why decoding the CGP genotype map is extremely fast

- Some people wrongly think that CGP is slow compared to other forms of GP because it has a genotype-phenotype mapping stage.
- First one determines which nodes are used. This is one pass and is only done once.
  - Time is insignificant compared with fitness evaluation
- To determine the output from CGP only requires looking up a few entries in an array
- Phenotypes in CGP are usually a fraction of the genotype size

Point mutation

- Most CGP implementations only use mutation.
- Carrying out mutation is very *simple*. It consists of the following steps. The genes must be chosen to be valid alleles
  - Of course, it can be also be done probabilistically

```
//Decide how many genes to change:num_mutations
while (mutation_counter < num_mutations)
{
        get random gene to change
        if (gene is a function gene)
          change gene to randomly chosen new valid function
        else if (gene is a connection gene)
          change gene to a randomly chosen new valid connection
        else
          change gene to a new valid output connection
}
```

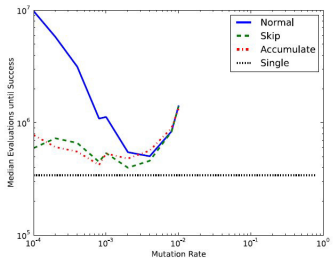
Advice on CGP parameters and experiments

- Only allow a small number of mutations
- Choose a fairly large number of nodes (100 to 1000)
- Use a 1+ 4 evolutionary strategy
- Only calculate fitness of genotypes that have different phenotypes
  - Check whether an offspring has exactly the same active nodes as the parent, if so do not calculate its fitness and do not increment the evaluation counter (Skip method – see next slide)
- Only evaluate active nodes
- When making comparisons with other GP methods either give them the same budget in processed nodes or the same number of fitness evaluations.

Single, a new mutation method?

- *Single*: Keep mutating until an active gene is mutated (*Goldman and Punch 2013*)
- Exactly one active gene is mutated for all offspring.
- It looks like no mutation rate is required, however its effective mutation rate is determined by the length of the genotype
  - Remains to be investigated

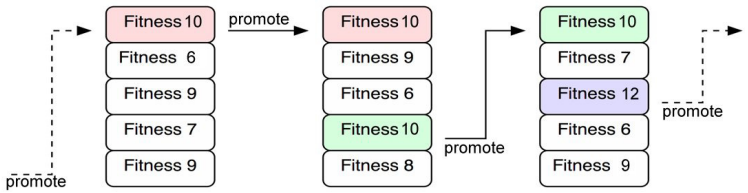
Single active gene mutation strategy: results (*Goldman and Punch 2013*)



- 3bit parallel multiplier
  - Multiples two three-bit numbers in parallel
  - Hard problem
- On other problems the best performance of *single* is slightly worse than other mutation schemes
- *Normal* = standard CGP
- *Skip*: set offspring's fitness to parent if identical
- *Single*: mutate until one active gene is changed
  - 29% less computation than Normal!

Evolutionary Strategy

- CGP often uses a variant of a simple algorithm called (1 + 4) Evolutionary Strategy
  - However, an offspring is always chosen if it *is equally as fit* or has better fitness than the parent



CGP Library

- **Andrew Turner** – developer/maintainer
- **cgplibrary.co.uk**
- **Cross Platform** – Linux, Windows, Mac
- **Open Source** – GNU LGPL
- **Fully Documented API**
- **Many Tutorials** – beginner and advance
- **Very Simple**
- **Written in C** – fast, no dependencies
- **Language Bindings** – via SWIG

CGP Library – Overview

- **Cartesian Genetic Programing (CGP)**
- **Recurrent CGP**
- **CGP Artificial Neural Networks**
- **Recurrent CGP Artificial Neural Networks**
- **What more could you want ;)**
- **Designed for:**
  - **Teaching**
  - **Research**
  - **Applications**

Live Demo

- **Tutorials: cgplibrary.co.uk**
- **andrew.turner@york.ac.uk**
- **Please as questions!**
- **Or talk to me later :)**

Additional Features

- Save, store and load chromosomes
- Embed CGP in other applications
- Highly Customisable
  - Fitness functions
  - Node Functions
  - Selection Scheme
  - Mutation Scheme
  - Reproduction Scheme
- Additional Language bindings

Final Words

- Much more info @ cgplibrary.co.uk
- Hosted with GitHub:

Github.com/AndrewJamesTurner/CGP-Library

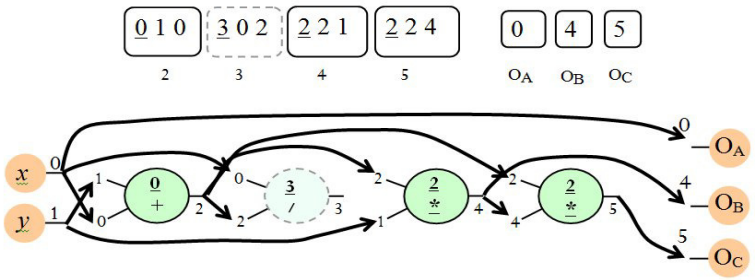
- Citable using:

A. J. Turner, J. F. Miller, ***Introducing a cross platform open source Cartesian Genetic Programming library***, GPEM, 2014, 83-91

Crossover or not?

- Recombination doesn't seem to add anything *(Miller 1999, "An empirical study…")*
- However if there are multiple chromosomes with independent fitness assessment then it helps a LOT (*Walker, Miller, Cavill 2006, Walker, Völk, Smith, Miller, 2009)*
- Some work using a floating point representation of CGP shows that crossover is useful for symbolic regression (*Clegg, Walker, Miller 2007, Meier, Gonter, Kruse 2013)*
- Cone-based crossover (Kaufmann and Platzner 2007, 2008)

CGP samples the solution space in a very interesting way



One gene mutation can create a wide variety of offspring, since it can activate or deactivate large number of nodes

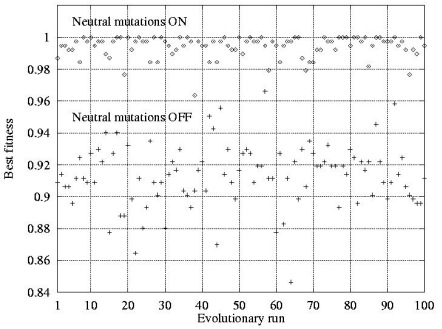
Neutral search is fundamental to success of CGP

- A number of studies have been carried out to indicate the importance to neutral search
  - *Miller and Thomson 2000, Vassilev and Miller 2000, Yu and Miller 2001, Miller and Smith 2006*)

Neutral search and the three bit multiplier problem (*Vassilev and Miller 2000*)

Importance of neutral search can be demonstrated by looking at the success rate in evolving a correct three-bit digital parallel multiplier circuit

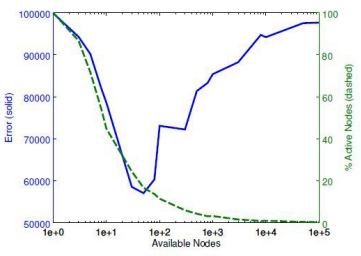
Graph shows final fitness obtained in each of 100 runs of 10 million generations with neutral mutations enabled compared with disabled neutral mutations.



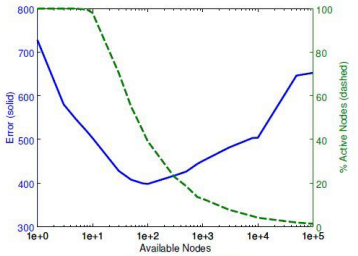
How big should the genotype be?

- Miller and Smith investigated the efficiency of CGP search as a function of the number of available nodes and mutation rate on a couple of small problems (*Miller and Smith 2006*). It was found that
  - Least number of evaluations required for success when 95% of the genes were inactive!
  - Efficiency of the search appeared to continuously improve as the number of nodes increased...
- Turner and Miller have recently investigated neutral drift in a recent paper (*Turner and Miller in press*). Using 3% probabilistic mutation and up to 100,000 nodes, the following results were obtained:

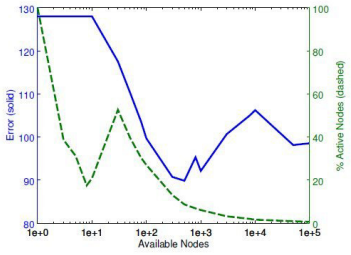
Efficiency of search v. max number of nodes



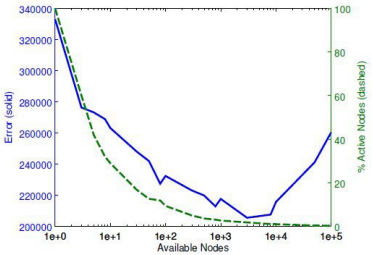
Double Pole



Multiplier 4 bit



Parity8Bit



Tower Problem

Modular/Embedded CGP (*Walker, Miller 2004, 2008, Kaufmann and Platzner 2007, 2008*)

- So far have described a form of CGP (classic) that does not have an equivalent of Automatically Defined Functions (ADFs)
- Modular CGP allows the use of modules (ADFs)
  - Modules are dynamically created and destroyed
  - Modules can be evolved
  - Modules can be re-used

Creating, Destroying, Replicating and Evolving Modules

- Created by the compress operator
  - Randomly acquires sections of the genotype into a module
    - Sections cannot contain modules
- Destroyed by the expand operator
  - Converts the original module back into a section of the genotype
- Replicated by mutation operator
  - Mutation can change a primitive function node to a module, or change a module to a primitive function node
- Evolution of Module
  - All instances of a module in the genotype can be changed by a mutation to a module

Module Survival

- Twice the probability of a module being destroyed than created
- Modules have to replicate to improve their chance of survival
  - This reduces the probability of their extinction
- Modules must also be associated with a high fitness genotype in order to survive
  - Offspring inherit the modules of the fittest parent

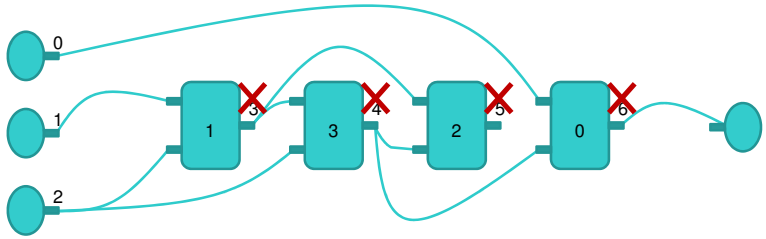
Evolving Modules

- Structural mutation
  - Add input
  - Remove input
  - Add output
  - Remove output
- Module point-mutation operator
  - Restricted version of genotype point-mutation operator
    - Uses only primitive functions

Self-modifying Cartesian Genetic programming (Harding, Miller, Banzhaf 2007 onwards)

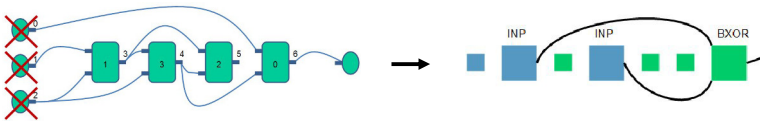
- A developmental form of CGP
  - Includes self modification functions in addition to computational functions
  - ‘General purpose’ GP system
  - Phenotype can vary over time (with iteration)
  - Can switch off its own self-modification
- Some representational changes from classic CGP…

Changes to CGP: relative addressing



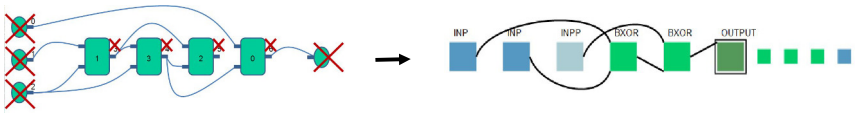
- Replaced direct node addressing with relative addressing
  - Always use 1 row (not rectangular)
  - Connection genes say how many nodes back

Changes to CGP: Inputs



- Replace input calls with a function.
  - We call these functions INP, INPP, SKIPINP
- Pointer keeps track of ‘current input’.
  - Call to INP returns the current input, and moves the pointer to the next input.
- Connections beyond graph are assigned value 0.

Changes to CGP: Outputs



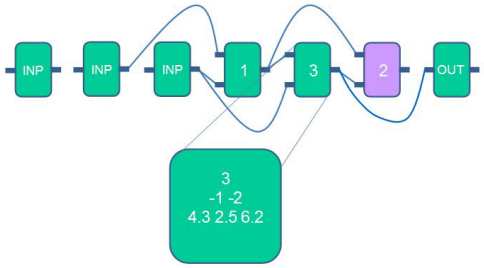
- Removed output nodes.
- Genotype specifies which nodes are outputs.
- If no OUTPUT function then last active node is used
  - Other defaults are used in situations where the number of outputs does not match the number required

Changes to CGP: Arguments

- Nodes also contain a number of 'arguments'.
  - 3 floating point numbers
  - Used in various self-modification instructions
  - Cast to integers when required

SMCGP Nodes: summary

- Each node contains:
  - Function type
  - Connections as relative addresses
  - 3 floating point numbers



SMCGP: Functions

- Two types of functions:
  - Computational
    - Usual GP computational functions
  - Self-modifying
    - Passive computational role (see later)

Some Self-Modification Functions

Operator	Parameters: use node address and the three node arguments	Function
MOVE	Start, End, Insert	Moves each of the nodes between Start and End into the position specified by Insert
DUP	Start, End, Insert	Inserts copies of the nodes between Start and End into the position specified by Insert
DELETE	Start, End	Deletes the nodes between Start and End indexes
CHF	Node, New Function	Changes the function of a specified node to the specified function
CHC	Node, Connection1, Connection2	Changes the connections in the specified node

SMCGP Execution

- Important first step:
  - Genotype is duplicated to phenotype.
  - Phenotypes are executed:
    - Self modifications are only made to the phenotype.

Self Modification Process: The To Do list

- Programs are iterated.
- If *triggered*, self modification instruction is added to a To Do list.
- At the end of each iteration, the instructions on this list are processed.
- The maximum size of the To Do list can be predetermined

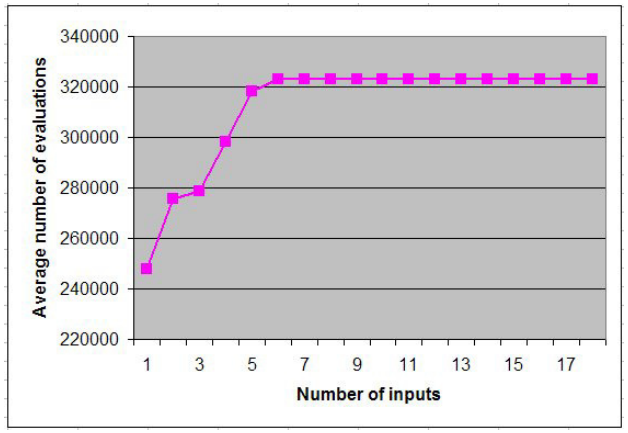
Computation of a SM node

- Functions can be appended to the To Do list under a variety of conditions (triggered)
  - If active
  - If value(first input) > value(the second input)
    - Data dependent self-modification
- And:
  - The To Do list isn't too big.

Example: Evolving Even-Parity

- Each iteration of program should produce the next parity circuit.
  - On the first iteration the program has to solve 2 bit parity. On the next iteration, 3 bit ... up to 22 parity
  - Fitness is the cumulative sum of *incorrect* bits
- Aim to find *general* solution
  - Solutions can be proved to general
    - See GPEM 2010 paper
- CGP or GP cannot solve this problem as they have a finite set of inputs (terminals)

Scaling behaviour of SMCGP on even-parity



Evolving pi

- Iterate a maximum of 10 times
- If program output does not get closer to pi at the next iteration, the program is stopped and large fitness penalty applied
- Fitness at iteration, *i*, is absolute difference of output at iteration *i* and pi
- One input: the numeric constant 1.

Evolving pi: an evolved solution

- An evolved solution

$$f(i) = \begin{cases} cos(sin(cos(sin(0)))) & i = 0 \\ f(i-1) + sin(f(i-1)) & i > 0 \end{cases}$$

- f(10) is correct to the first 2048 digits of pi
- It can be proved that *f*(*i*) rapidly converges to pi in the limit as *i* tends to infinity

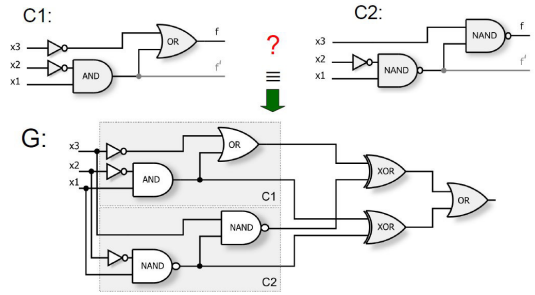
Further results

- Other mathematically provable results found so far:
  - Evolved a program that can carry out the bitwise addition of an arbitrary number of inputs
  - Evolved a sequence that converges to e
- Other results
  - Evolved a sequence function that generates the first 10 Fibonacci numbers (probably general)
  - Evolved a power function x^n
  - Bioinformatics classification problem (finite inputs)
    - SMCGP performed no worse than CGP

Application 1: Digital circuit synthesis with CGP

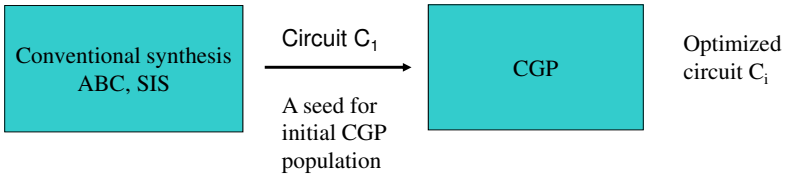
- Digital Circuits with hundreds of variables can be optimized using CGP (*Vassicek and Sekanina* 2011)
  - Won the $3000 silver award in human competitive workshop at GECCO 2011
- The method employs a SAT solver to identify whether two circuits are logically equivalent
  - In many cases this can be done in polynomial time

Circuit equivalence checking and SAT



- If C1 (reference) and C2 (evolved) are not functionally equivalent then there is at least one assignment of the inputs for which the output of G is 1.

CGP for optimizing conventionally synthesized circuits

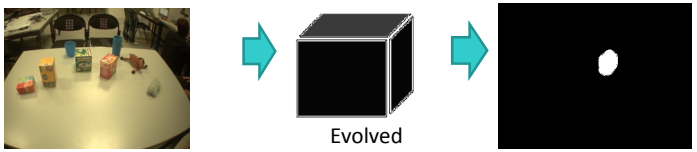


The seed for CGP is provided by using the logic synthesis package, ABC (http://www.eecs.berkeley.edu/~alanmi/abc/)

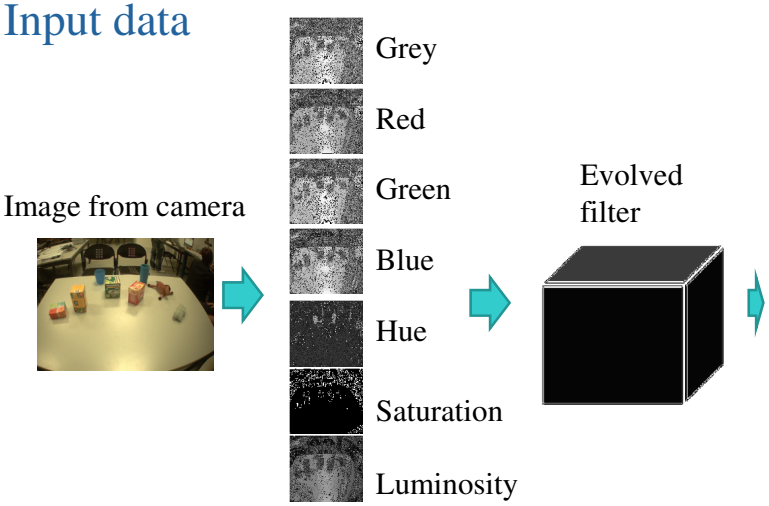
The fitness function is as follows:

- Use a **SAT solver** to decide whether candidate circuit C_i and reference circuit C_1 are functionally equivalent.
  - If so, then fitness(C_i) = the number of nodes – number of gates in C_i;
  - Otherwise: fitness(C_i) = 0.
- The method is now much faster by using a circuit simulator prior to SAT solver to disprove the equivalence between a candidate solution and its parent (*Vassicek 2015*)

Application 2: Evolving Image Filters with CGP (*Harding, Leitner, Schmidhüber 2013*)

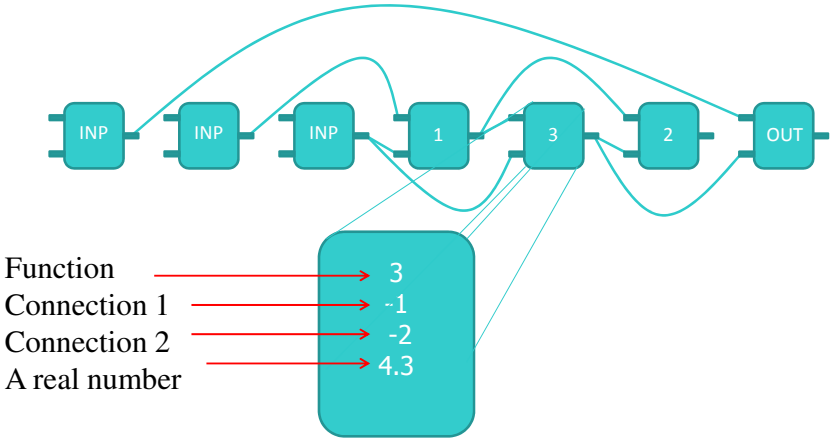


Input data



Split colour image is used as inputs

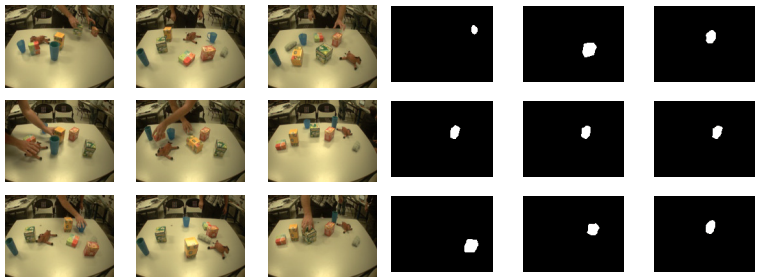
Genotype representation (like SMCGP but no SM functions)



Large Function Set

NOP	LOG	*TRIANGLES*
INP	MAX	*LINES*
INPP	MIN	SHIFTDOWN
SKIP	EQ	SHIFTUP
ADD	*GAMMA*	SHIFTLEFT
SUB	GAUSS	SHIFTRIGHT
CONST	SOBELX	*SIFTa*
MUL	SOBELY	GABOR
ADDC	AVG	NORMALIZE
SUBC	UNSHARPEN	RESCALE
MULC	THRESHOLD	*GRABCUT*
ABSDIFF	THRESHOLDBW	MINVALUE
CANNY	SMOOTHMEDIAN	MAXVALUE
DILATE	*GOODFEATURESTOTRACK*	AVGVALUE
ERODE	*SQUARES*	RESCALE
LAPLACE	*CIRCLES*	RESIZETHENGABOR

Fitness

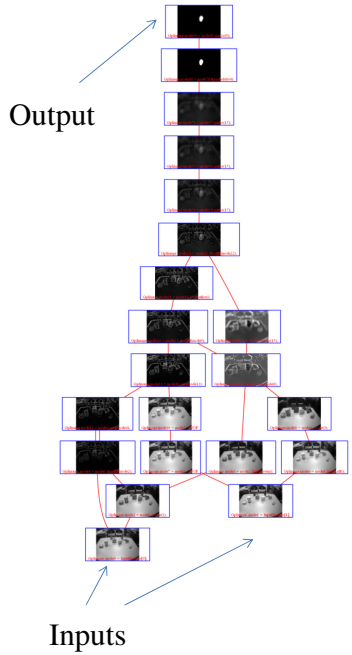


•Fitness = sum of mean square error of pixel values between each input/target

Evolved Filter code

```
public class MyEvolvedFilter : GpImageFilterRunner
{
public override GpImage RunFilter()  {
    GpImage  node0 = InputImages[0];
    GpImage  node1 = InputImages[1];
    GpImage node2 = node0.erode(1);
    GpImage node3 = node2.ShiftDown();
    GpImage  node4 = node0.absdiff(node2);
    GpImage node5 = node0.avg(node2);
    GpImage node7 = node5; //NOP
    GpImage node8 = node1.erode(3);
    GpImage node9 = node3.sub(node8);
    GpImage node12 = node4.add(node4);
    GpImage node13 = node7.min(node12);
    GpImage  node16 = node13.absdiff(node9);
    GpImage node24 = node16.sub(node9);
    GpImage node50 = node24.gauss(15);
    GpImage node78 = node50.gauss(15);
    GpImage node89 = node78.threshold(64);
    GpImage node99 = node89.gauss(13);
    return node99;
    }
public override void SetUsedInputs() {
    this.UsedInputs.Add(1);
    this.UsedInputs.Add(0);
}
}
```

Evolved Filter Dataflow



Tea-box filter: demonstration

Application 3: CGP encoded Artificial Neural Networks (CGPANN)

- CGP has been used to encode both feed-forward ANNs and recursive ANNs. The nodes genes consist of:
  - Connection genes (as usual)
  - Function genes
    - Sigmoid, hyperbolic tangent. Gaussian
  - Weights
    - Each connection gene carries a real-numbered weight
- Pole balancing, Arm Throwing, Classification
  - Very competitive results with other TWEANN methods (*Khan, Khan and Miller 2010, Turner and Miller 2013*)
- Breast cancer detection (*Ahmad et al 2012, Turner and Miller 2013*)

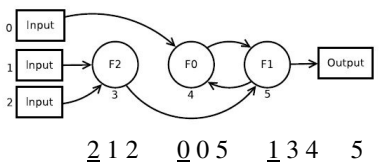
Cyclic CGP

- When outputs are allowed to connect to inputs through a clocked delay (flip-flop) it is possible to allow CGP to include feedback.
- By feeding back outputs generated by CGP to an input, it is possible to get CGP to generate sequences
  - In this way iteration is possible
- There are a couple of publications using iteration in CGP (*Khan, Khan and Miller 2010, Walker, Liu, Tempesti,Tyrrell 2010, Minarik, Sekanina 2011*)

Recurrent CGP

- By allowing nodes to receive inputs from the right CGP can be easily extended to encode recursive computational structures
- Recurrent CGP Artificial Neural Networks can be explored in this framework
- Only just begun to be explored (*Turner and Miller 2014*)

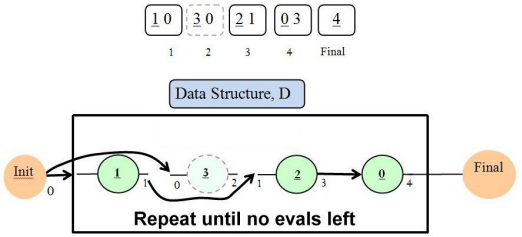
Recurrent CGP: Details



- Probability of recursive links controlled by a user-defined parameter recurrent connection probability (rcp)
- Decoding
  1. set all active nodes to output zero
  2. apply the next set of program inputs
  3. update **all** active nodes **once** from program inputs to program outputs
  4. read the program outputs
  5. repeat from 2 until all program input sets have been applied

CGP encoded algorithms (Ryser-Welch, Miller, Asta 2015)

- Algorithms can be encoded in CGP
- Here is one way
  1. Assume all nodes have an arity of one
  2. Assume a single input
  3. Assume no data passed through graph
  4. CGP defines a variable length ordered set of instructions



Algorithm

D ← Init(D)
do
{
D ← Op1(D)
D ← Op2(D)
D ← Op0(D)
D ← Op0(D)
}
while (evals left)
D ← Final(D)

CGP acceleration (*Vassicek and Slany 2012*)

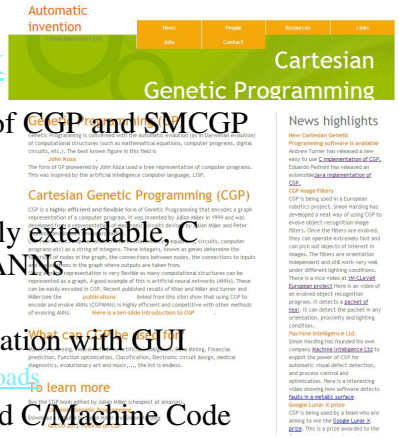
- CGP decoding step is replaced with native machine code that directly calculates response for a single training vector.
- Requires little knowledge of assembly language or target machine code.
- Integration of the machine code compiler requires modifying only a few lines of code
- Achieves 5 times speedup over standard implementation

Some Applications of CGP (incomplete)

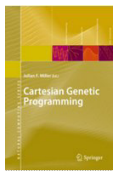
- Circuit Design
  - ALU, parallel multipliers, digital filters, analogue circuits, circuit synthesis and optimization
- Machine Learning
  - Classification
- Mathematical functions
  - Prime generating polynomials
- Control systems
  - Maintaining control with faulty sensors, helicopter control, general control, simulated robot controller
- Image processing
  - Image filters, Mammary Tumour classification, object recognition
- Robotics
  - gait
- Bio-informatics
  - Molecular Post-docking filters
- Artificial Neural Networks
- Developmental Neural Architectures
  - Wumpus world, checkers, maze solving
- Evolutionary Art
- Artificial Life
  - Regenerating 'organisms'
- Optimization problems
  - Applying CGP to solve GA problems

CGP Resources I:

http://www.cartesiangp.co.uk



- **Julian Miller**: C implementations of CGP and SMCGP available at
  http://www.cartesiangp.co.uk
- **Andrew Turner**: Easy to use, highly extendable, C implementation that includes CGPANNs
  http://www.cgplibrary.co.uk/
- **Eduardo Pedroni**: Java implementation with GUI
  https://bitbucket.org/epedroni/jcgp/downloads
- **Zdenek Vassicek**: Highly optimised C/Machine Code implementation
  http://www.fit.vutbr.cz/~vasicek/cgp/
- Cartesian Genetic Programming book
  - Published in 2011 by Springer



CGP Resources II:

- **David Oranchak** has implemented CGP in Java. Documentation is available at
  http://oranchak.com/cgp/doc/
- **Brian Goldman** has implemented CGP in Python
  https://github.com/brianwgoldman/ReducingWastedEvaluationsCGP
- **Jordan Pollack** has implemented symbolic regression in CGP with Matlab
  - See CGP web site
- **Lawrence Ashmore** has implemented a Java evolutionary art package using CGP
  - See CGP web site

Conclusions

- Cartesian Genetic Programming is a graph based GP method capable of representing many computational structures
  - programs, circuits, neural networks, systems of equations, algorithms…
- Genetic encoding is compact, fast, simple and easy to implement and can handle multiple outputs easily.
- The unique form of genetic redundancy in CGP makes mutational search highly effective
- The effectiveness of CGP has been compared with many other GP methods and it is highly competitive

Bibliography(incomplete) and references

Ahmad A. M., Khan G. M., Mahmud, S. A., Miller J. F. Breast Cancer Detection Using Cartesian Genetic Programming evolved Artificial Neural Networks. Proceedings of the Genetic and Evolutionary Computation Conference (GECCO), (2012) 1031—1038.

Ashmore L. An investigation into cartesian genetic programming within the field of evolutionary art. http://www.emoware.org/evolutionary_art.asp, Department of Computer Science, University of Birmingham (2000)

Clegg J., Walker J. A., Miller J. F. A New Crossover Technique for Cartesian Genetic Programming. Proceedings of Genetic and Evolutionary Computation Conference, ACM Press (2007) 1580-1587.

DiPaola S., Gabora L. Incorporating characteristics of human creativity into an evolutionary art algorithm, Genetic Programming and Evolvable Machines (2009) Vol. 10. For further info see: http://dipaola.org/evolve/

DiPaolo S. Evolving Creative Portrait Painter Programs using Darwinian Techniques with an Automatic Fitness Function. Electronic Visualizationa and the Arts Conference (2005)

Gajda, Z., Sekanina, L.. Gate-Level Optimization of Polymorphic Circuits Using Cartesian Genetic Programming, Proceedings of Congress on Evolutionary Computation. IEEE Press (2009)

Gajda Z., Sekanina, L.. Reducing the Number of Transistors in Digital Circuits Using Gate-Level Evolutionary Design, Proceedings of Genetic and Evolutionary Computation Conference. ACM, (2007) 245-252.

Garmendia-Doval B., Miller J.F., Morley S.D. Post Docking Filtering using Cartesian Genetic Programming. *Genetic Programming Theory and Practice II.* O'Reilly U-M., Yu T., Riolo R., Worzel B. (Eds.). University of Michigan Illinois USA. Springer (2004).

Glette K., Torresen J., Paul Kaufmann P., Platzner., M. A Comparison of Evolvable Hardware Architectures for Classification Tasks. In Proceedings of the 8th International Conference on Evolvable Systems: From Biology to Hardware, Springer LNCS 5216 (2008) 22-33.

Goldman, B. W., Punch, W. F. Reducing Wasted Evaluations in Cartesian Genetic Programming, Proceedings of European Conference on Genetic Programming, Springer LNCS 7831 (2013) pp. 61–72.

Goldman, B. W., Punch, W. F. Analysis of Cartesian Genetic Programming's Evolutionary Mechanisms, IEEE Transactions on Evolutionary Computation, (In press)

Goldman, B. W., Punch, W. F. Length bias and search limitations in cartesian genetic programming. In Proceedings of the 15th annual conference on Genetic and evolutionary computation, ACM, (2013) 933-940.

Harding S. L., Leitner, J., Schmidhuber, J.. Cartesian Genetic Programming for Image Processing, Genetic Programming Theory and Practice, University of Michigan Illinois USA. Springer. 2012

Harding, S., Graziano, V., Leitner, J., Schmidhuber. J. MT-CGP: Mixed Type Cartesian Genetic Programming, Proceedings of the Genetic and Evolutionary Computation Conference (2011) pp 751-758.

Harding, S., Miller, J. F., Banzhaf, W. SMCGP2: Self Modifying Cartesian Genetic Programming in Two Dimensions, Proceedings of the Genetic and Evolutionary Computation Conference (2011) pp 1491-1498.

Harding S. L., Miller J. F. Banzhaf W. Developments in Cartesian Genetic Programming: Self-modifying CGP. Genetic Programming and Evolvable Machines, Vol. 11 (3/4) (2010) pp 397-439.

Harding S. L., Miller J. F. Banzhaf W. Self Modifying Cartesian Genetic Programming: Finding algorithms that calculate pi and e to arbitrary precision, Proceedings of the Genetic and Evolutionary Computation Conference, 2010.

Harding S. L., Miller J. F., Banzhaf W. A Survey of Self-Modifying CGP. Genetic Programming Theory and Practice, Riolo R., (Eds.). University of Michigan Illinois USA. Springer. 2010

Harding S. L., Miller J. F. Banzhaf W. Self Modifying Cartesian Genetic Programming: Parity. Proceedings of Congress on Evolutionary Computation, IEEE Press (2009) 285-292

Harding S. L., Miller J. F. Banzhaf W. Self Modifying Cartesian Genetic Programming: Fibonacci, Squares, Regression and Summing, Proceedings of the 10th European Conference on Genetic Programming, Springer LNCS (2009) 133-144

Harding S. L., Miller J. F., Banzhaf W. Self-Modifying Cartesian Genetic Programming, Proceedings of Genetic and Evolutionary Computation Conference, ACM Press, (2007) 1021-1028.

Harding S., Banzhaf W. Fast Genetic Programming on GPUs. Proceedings of 10th European Conference on Genetic Programming, Springer LNCS 4445 (2007) 90-101

Harding S. L., Miller J. F. Evolution of Robot Controller Using Cartesian Proceedings of the 6th European Conference on Genetic Programming, Springer LNCS 3447 (2005) 62-72.

Hirayama Y., Clarke T, Miller J. F. Fault Tolerant Control Using Cartesian Genetic Programming, Proceedings of Genetic and Evolutionary Computation Conference, ACM Press, (2008) 1523-1530 .

Hrbacek, R., Sekanina, S. Towards highly optimized Cartesian genetic programming: from sequential via SIMD and thread to massive parallel implementation. Proceedings of the 2014 conference on Genetic and evolutionary computation, ACM (2014) 1015-1022

Kalganova T., Miller J. F., Evolving More Efficient Digital Circuits by Allowing Circuit Layout Evolution and Multi-Objective Fitness. Proceedings of the First NASA/DOD Workshop on Evolvable Hardware, IEEE Computer Society (1999) 54-63.

Kalganova T., Miller J. F., Fogarty T. C. Some Aspects of an Evolvable Hardware Approach for Multiple-Valued Combinational Circuit Design Proceedings of the 2nd International Conference on Evolvable Systems: From Biology to Hardware. Springer LNCS 1478 (1998) 78-89.

Kaufmann P., Platzner M. Advanced Techniques for the Creation and Propagation of Modules in Cartesian Genetic Programming. Proceedings of the Genetic and Evolutionary Computation Conference, ACM Press, (2008) 1219-1226.

Kaufmann P., Platzner M. MOVES: A Modular Framework for Hardware Evolution. In Proceedings of the NASA/ESA Conference on Adaptive Hardware and Systems, IEEE Computer Society Press (2007) 447-454

Kaufmann P., Platzner M. Toward Self-adaptive Embedded Systems: Multiobjective Hardware Evolution. In Proceedings of the 20th International Conference on Architecture of Computing Systems, Springer, LNCS 4415 (2007) 119-208.

Khan, G. M., Miller, J. F., Halliday, D. M. Evolution of Cartesian Genetic Programs for Development of Learning Neural Architecture, Evolutionary Computation, Vol. 19, No. 3 (2011) pp 469-523

Khan, M. M., Khan, G. M., J. F. Miller, J. F. "Efficient representation of recurrent neural networks for markovian/non-markovian non-linear control problems," in Proceedings of the 10th International Conference on Intelligent Systems Design and Applications (ISDA2010) (2010) 615–620

Khan, G. M., Miller J. F., Khan, M. M. Evolution of Optimal ANNs for Non-Linear Control Problems Using Cartesian Genetic Programming. Proceedings of International Conference on Artificial Intelligence (ICAI 2010)

Khan, G. M., Halliday, D. M., Miller, J. F.,Intelligent agents capable of developing memory of their environment, Angelo Loula A., Queiroz, J. (Eds.) Advances in Modelling Adaptive and Cognitive Systems, Editora UEFS (2010)

Khan G. M., Halliday D. M., Miller J. F. In Search of Intelligent Genes: The Cartesian Genetic Programming Neuron. Proceedings of Congress on Evolutionary Computation, IEEE Press (2009)

Khan G. M., Halliday D. M., Miller J. F. Breaking the synaptic dogma: evolving a neuro-inspired developmental network. Proceedings of 7th International Conference on Simulated Evolution and Learning, LNCS, 5361 (2008) 11-20

Khan G. M., Halliday D. M., Miller J. F. Coevolution of neuro-developmental programs that play checkers. Evolvable Systems: From Biology to Hardware. Springer LNCS 5216 (2008) 352 - 361.

Khan G. M., Halliday D. M., Miller J. F. Coevolution of Intelligent Agents using Cartesian Genetic Programming. Proceedings of Genetic and Evolutionary Computation Conference, ACM Press, (2007) 269-276.

Kuyucu T., Trefzer M. A., Miller J. F., Tyrrell. A. M. On the Properties of Artificial Development and Its Use in Evolvable Hardware. Proceedings of Symposium on Artificial Life , Part of IEEE Symposium on Computational Intelligence, IEEE Press (2009).

Liu H., Miller J. F., Tyrrell A. M. , Intrinsic evolvable hardware implementation of a robust biological development model for digital systems, Proceedings of the NASA/DOD Evolvable Hardware Conference, IEEE Computer Society (2005) 87-92.

Liu H., Miller J. F., Tyrrell A. M. A Biological Development Model for the Design of Robust Multiplier. Applications of Evolutionary Computing: EvoHot 2005, Springer LNCS 3449 (2005) 195-204

Liu H., Miller J. F., Tyrrell A. M. An Intrinsic Robust Transient Fault-Tolerant Developmental Model for Digital Systems. Workshop on Regeneration and Learning in Developmental Systems, Genetic and Evolutionary Computation Conference (2004).

Meier, A., Gonter, M., Kruse, R.. Accelerating convergence in cartesian genetic programming by using a new genetic operator. In Proceeding of the fifteenth annual conference on Genetic and evolutionary computation conference, pages 981–988. ACM, 2013.

Miller J. F. Cartesian Genetic Programming, Springer 2011.

Miller J.F., Smith S.L. Redundancy and Computational Efficiency in Cartesian Genetic Programming. *IEEE Transactions on Evolutionary Computation*, 10 (2006) 167-174.

Miller J. F. Evolving a self-repairing, self-regulating, French flag organism. Proceedings of Genetic and Evolutionary Computation Conference, Springer LNCS 3102 (2004) 129-139.

Miller J. F., Thomson P. Beyond the Complexity Ceiling: Evolution, Emergence and Regeneration. Workshop on Regeneration and Learning in Developmental Systems, Genetic and Evolutionary Computation Conference (2004).

Miller J.F., Banzhaf W., Evolving the Program for a Cell From French Flags to Boolean Circuits. Kumar S., Bentley P. *On Growth, Form and Computers*. Elsevier Academic Press (2003).

Miller J. F., Thomson P. A Developmental Method for Growing Graphs and Circuits. Proceedings of the 5th International Conference on Evolvable Systems: From Biology to Hardware, Springer LNCS 2606 (2003) 93-104.

Miller J. F. Evolving developmental programs for adaptation, morphogenesis, and self-repair. Proceedings of the 7th European Conference on Artificial Life, Springer LNAI 2801 (2003) 256-265.

Miller J. F. What bloat? Cartesian Genetic Programming on Boolean problems. Genetic and Evolutionary Computation Conference, Late breaking paper (2001) 295 - 302.

Miller J. F., Hartmann M. Evolving *messy* gates for fault tolerance: some preliminary findings. Proceedings of the 3rd NASA/DOD Workshop on Evolvable Hardware. IEEE Computer Society (2001) 116-123.

Miller J. F., Hartmann M. Untidy evolution: Evolving *messy* gates for fault tolerance. Proceedings of the 4th International Conference on Evolvable Systems: From Biology to Hardware. Springer LNCS 2210 (2001) 14-25.

Miller J.F., Kalganova T., Lipnitskaya N., Job D. The Genetic Algorithm as a Discovery Engine: Strange Circuits and New Principles. *Creative Evolutionary Systems*. Morgan Kaufmann (2001).

Miller J.F., Job D., Vassilev V.K. Principles in the Evolutionary Design of Digital Circuits - Part I. *Journal of Genetic Programming and Evolvable Machines*, 1 (2000) 8-35.

Miller J.F., Job D., Vassilev V.K. Principles in the Evolutionary Design of Digital Circuits - Part II. *Journal of Genetic Programming and Evolvable Machines*, 3 (2000) 259-288.

Miller J. F., Thomson P. Cartesian Genetic Programming. Proceedings of the 3rd European Conference on Genetic Programming. Springer LNCS 1802 (2000) 121-132.

Miller J. F. On the filtering properties of evolved gate arrays. Proceedings of the First NASA/DOD Workshop on Evolvable Hardware. IEEE Computer Society (1999) 2-11.

Miller J. F. Digital Filter Design at Gate-level using Evolutionary Algorithms. Proceedings of the 1st Genetic and Evolutionary Computation Conference. Morgan Kaufmann (1999) 1127-1134.

Miller J. F. An empirical study of the efficiency of learning boolean functions using a Cartesian Genetic Programming Approach. Proceedings of the 1st Genetic and Evolutionary Computation Conference. Morgan Kaufmann (1999) 1135-1142.

Miller J. F. Evolution of Digital Filters using a Gate Array Model. Proceedings of the First Workshop on Image Analysis and Signal Processing. Springer LNCS 1596 (1999) 17-30.

Miller J. F., Kalganova T., Lipnitskaya N., Job D. The Genetic Algorithm as a Discovery Engine: Strange Circuits and New Principles. Proceedings of the workshop on the AISB Symposium on Creative Evolutionary Systems. AISB (1999) 65-74.

Miller J. F., Thomson P. Aspects of Digital Evolution: Evolvability and Architecture. Proceedings of The Fifth International Conference on Parallel Problem Solving from Nature. Springer LNCS 1498 (1998) 927-936.

Miller J. F., Thomson P. Aspects of Digital Evolution: Geometry and Learning. Proceedings of the 2nd International Conference on Evolvable Systems: From Biology to Hardware. Springer LNCS 1478 (1998) 25-25.

Miller J. F., Thomson P. Evolving Digital Electronic Circuits for Real-Valued Function Generation using a Genetic Algorithm . Proceedings of the 3rd Conference on Genetic Programming. Morgan Kaufmann (1998) 863-868

.

Miller J.F., Thomson P., Fogarty T.C. Designing Electronic Circuits Using Evolutionary Algorithms: Arithmetic Circuits: A Case Study. Genetic Algorithms and Evolution Strategies in Engineering and Computer Science: Recent Advancements and Industrial Applications. Quagliarella, D., Periaux J., Poloni C., Winter G. (Eds.). Wiley (1997)

Minarik, M., Sekanina, L. Evolution of Iterative Formulas Using Cartesian Genetic Programming.Proceedings of the 15th International Conference on Knowledge-Based and Intelligent Information and Engineering Systems (KES 2011) Part I, LNCS volume 6881 (2011) 11-20

Payne, A. J., Stepney, S.. Representation and Structural biases in CGP, Proceedings of Congress on Evolutionary Computation, IEEE Press (2009)

Rothermich J., Wang F., Miller J. F. Adaptivity in Cell Based Optimization for Information Ecosystems. Proceedings of the Congress on Evolutionary Computation. IEEE Press (2003) 490-497.

Rothermich J., Miller J. F. Studying the Emergence of Multicellularity with Cartesian Genetic Programming in Artificial Life. Proceedings of the 2002 U.K. Workshop on Computational Intelligence (2002).

Ryser-Welch, P, Miller, J. F., Asta, S. Generating human-readable algorithms for the Travelling Salesman Problem using Hyper-Heuristics, In proceedings of 5th Workshop on Evolutionary Computation for the Automated Design of Algorithms (ECADA), GECCO 2015 (to appear)

Seaton, T, Miller, J. F. , Clarke, T. Semantic Bias in Program Coevolution. Proceedings of the European Conference on Genetic Programming, (Krawiec, K et al. (Eds.) pp. 193-204, Springer, LNCS Vol. 7831, 2013.

Seaton, T, Miller, J. F. , Clarke, T. An Ecological Approach to Measuring Locality in Linear Genotype to Phenotype Maps. Proceedings of the European Conference on Genetic Programming, Springer, LNCS Vol. 7244 (2012) 170-181.

Seaton, T., Brown G., Miller J. F.., Analytic Solutions to Differential Equations under Graph-based Genetic Programming. Proceedings of the 13th European Conference on Genetic Programming. Springer LNCS 6021 (2010) 232-243

Sekanina, L. Evolvable Components - From Theory to Hardware Implementations, Springer (2003)

Sekanina, L. Image Filter Design with Evolvable Hardware, Proceedings of Evolutionary Image Analysis and Signal Processing, Springer LNCS 2279 (2002) 255-266.

Sekanina, L, Vašíček Z. On the Practical Limits of the Evolutionary Digital Filter Design at the Gate Level, Proceedings of EvoHOT, Springer, LNCS 3907 (2006) 344-355.

Sekanina, L., Harding, S. L., Banzhaf, W., Kowaliw, T. Image Processing and CGP in Miller, J.F. (Ed.) Cartesian Genetic Programming, Springer 2011.

.

Turner, A. J., Miller, J. F. Cartesian Genetic Programming encoded Artificial Neural Networks: A Comparison using Three Benchmarks. Proceedings of the Genetic and Evolutionary Computation Conference (GECCO), pp. 1005--1012, ACM, 2013

Turner, A. J., Miller, J. F. Introducing A Cross Platform Open Source Cartesian Genetic Programming Library, Genetic Programming and Evolvable Machines Vol. 16, pp. 83-91 (2015)

Turner, A. J., Miller, J. F. Recurrent Cartesian Genetic Programming, Proceedings of the 13th International Conference on Parallel Problem Solving from Nature (PPSN). T. Bartz-Beielstein et al. (Eds.): PPSN XIII 2014, Springer LNCS 8672 (2014) 476-486.

Turner, A. J., Miller, J. F. Recurrent Cartesian Genetic Programming Applied to Famous Mathematical Sequences, York Doctoral symposium. Technical Report of Dept. Computer Science, Univ.of York (2014)

Turner, A. J., Miller, J. F. Neutral Genetic Drift: An Investigation using Cartesian Genetic ProgrammingGenetic Programming and Evolvable Machines (in press)

Kisung Seo, K., Hyun, S. Toward Automatic Gait Generation for Quadruped Robots Using Cartesian Genetic Programming. EvoApplications 2013, LNCS Vol. 7835, pp. 599–605.

Vašíček Z, Sekanina L. Hardware Accelerators for Cartesian Genetic Programming, Proc. Eleventh European Conference on Genetic Programming, Springer LNCS Vol. 4971 (2008) 230-241

Vašíček, Z. Sekanina, L.. Formal verification of candidate solutions for post-synthesis evolutionary optimization in evolvable hardware. Genetic Programming and Evolvable Machines, 12(3) (2011) 305-327, 2011.

Vašíček Z. Slany, K. Efficient Phenotype Evaluation in Cartesian Genetic Programming. Proceedings of the 15th European Conference on Genetic Programming, Springer LNCS Vol. 7244 (2012) 266-278.

Vašíček Z. Cartesian GP in Optimization of Combinational Circuits with Hundreds of Inputs and Thousands of Gates. Proceedings of the 15th European Conference on Genetic Programming, Springer LNCS Vol. 9025 (2015) 139-150.

Vassilev V. K., Miller J. F. Scalability Problems of Digital Circuit Evolution. Proceedings of the 2nd NASA/DOD Workshop on Evolvable Hardware. IEEE Computer Society (2000) 55-64.

Vassilev V. K., Miller J. F. The Advantages of Landscape Neutrality in Digital Circuit Evolution. Proceedings of the 3rd International Conference on Evolvable Systems: From Biology to Hardware. Springer LNCS 1801 (2000) 252-263.

Vassilev V. K., Miller J. F. Towards the Automatic Design of More Efficient Digital Circuits. Proceedings of the 2nd NASA/DOD Workshop on Evolvable Hardware. IEEE Computer Society (2000) 151-160.

Vassilev V. K., Miller J. F., Fogarty T. C. Digital Circuit Evolution and Fitness Landscapes. Proceedings of the Congress on Evolutionary Computation. IEEE Press (1999) 1299-1306.

Vassilev V. K., Miller J. F., Fogarty T. C. On the Nature of Two-Bit Multiplier Landscapes. Proceedings of the First NASA/DOD Workshop on Evolvable Hardware. IEEE Computer Society (1999) 36-45.

Völk K., Miller J. F., Smith, S. L. Multiple Networks CGP for the Classification of Mammograms. Proceedings of the 11th European Workshop on Image Analysis and Signal Processing (EvoIASP), Springer LNCS (2009).

Voss M. S. Social programming using functional swarm optimization. In Proceedings of IEEE Swarm Intelligence Symposium (2003)

Voss M. S., Howland, J. C. III.Financial modelling using social programming. Financial Engineering and Applications (2003)

Walker J. A., Liu Y., Tempesti G., Tyrrell A. M., "Automatic Code Generation on a MOVE Processor Using Cartesian Genetic Programming," in Proceedings of the International Conference on Evolvable Systems: From Biology to Hardware, Springer LNCS vol. 6274 (2010) 238–249

Walker J.A., Völk, K. , Smith, S. L., Miller, J. F. Parallel evolution using multi-chromosome cartesian genetic programming, Genetic Programming and Evolvable Machines, 10 (4), (2009) pp 417-445

Walker J. A., Hilder, J. A., Tyrrell. A. M. Towards Evolving Industry-feasible Intrinsic Variability Tolerant CMOS Designs, Proceedings of Congress on Evolutionary Computation, IEEE Press (2009)

Walker J.A., Miller J.F. The Automatic Acquisition, Evolution and Re-use of Modules in Cartesian Genetic Programming. *IEEE Transactions on Evolutionary Computation*, 12 (2008) 397-417.

Walker J. A. Modular Cartesian Genetic Programming. PhD thesis, University of York, 2008.

Walker J. A., Miller J. F. Solving Real-valued Optimisation Problems using Cartesian Genetic Programming. Proceedings of Genetic and Evolutionary Computation Conference, ACM Press (2007) 1724-1730.

Walker J. A., Miller J. F. Changing the Genospace: Solving GA Problems using Cartesian Genetic Programming, Proceedings of 10th European Conference on Genetic Programming, Springer LNCS 4445 (2007) 261-270.

Walker J. A., Miller J. F. Predicting Prime Numbers using Cartesian Genetic Programming, Proceedings of 10th European Conference on Genetic Programming. Springer LNCS 4445, (2007) 205-216

Walker J. A., Miller J. F., Cavill R. A Multi-chromosome Approach to Standard and Embedded Cartesian Genetic Programming, Proceedings of the 2006 Genetic and Evolutionary Computation Conference. ACM Press, (2006) 903-910.

Walker J. A., Miller J. F. Embedded Cartesian Genetic Programming and the Lawnmower and Hierarchical-if-and-only-if Problems, Proceedings of the 2006 Genetic and Evolutionary Computation Conference. ACM Press, (2006) 911-918.

Walker J. A., Miller J. F. Improving the Evolvability of Digital Multipliers Using Embedded Cartesian Genetic Programming and Product Reduction. Proceedings of 6th International Conference in Evolvable Systems. Springer, LNCS 3637 (2005) 131-142.

Walker J. A., Miller J. F. Investigating the performance of module acquisition in Cartesian Genetic Programming, Proceedings of the 2005 conference on Genetic and Evolutionary Computation. ACM Press (2005) 1649-1656.

Walker J. A., Miller J. F. Evolution and Acquisition of Modules in Cartesian Genetic Programming. Proceedings of the 7th European Conference on Genetic Programming. Springer LNCS 3003 (2004) 187-197.

Yu T., Miller J.F., Through the Interaction of Neutral and Adaptive Mutations Evolutionary Search Finds a Way. *Artificial Life*, 12 (2006) 525-551.

Yu T., Miller J. F. Finding Needles in Haystacks Is Not Hard with Neutrality. Proceedings of the 5th European Conference on Genetic Programming. Springer LNCS 2278 (2002) 13-25.

Yu T., Miller J. F. Neutrality and Evolvability of a Boolean Function Landscape, Proceedings of the 4th European Conference on Genetic Programming. Springer LNCS, 2038, (2001) 204-217.

Zhan S., J.F. Miller, A. M., Tyrrell. An evolutionary system using development and artificial Genetic Regulatory Networks for electronic circuit design, Biosystems, 96 (3) (2009) pp 176-192

Zhan S., Miller J. F., Tyrrell A. M. Obtaining System Robustness by Mimicking Natural Mechanisms . Proceedings of Congress on Evolutionary Computation. IEEE Press (2009)

Zhan S., Miller J. F., Tyrrell A. M. A Development Gene Regulation Network For Constructing Electronic Circuits . Evolvable Systems: From Biology to Hardware. LNCS 5216 (2008) 177 – 188

Zhan S., Miller J. F., Tyrrell A. M. An Evolutionary System using Development and Artificial Genetic Regulatory Networks Proceedings of 9th IEEE World Congress on Computational Intelligence. Congress on Evolutionary Computation. IEEE Press (2008) 815-822.