HW #3

1.

Likelihood Table

color	stolen	not stolen	
Yellow	2	3	5/10
Red	3	2	5/10
all	5	5	
	5/10	5/10	

a) p(stolen)

$p(\text{yellow}) = \frac{1}{2}$

$p(\text{stolen}) = \frac{1}{2}$

$p(\text{stolen} \mid \text{yellow}) = 2/5$

$$p(\text{yellow} \mid \text{stolen}) = \frac{2/5 \times \frac{1}{2}}{\frac{1}{2}} = \frac{2}{5} = .4$$

b) p(red | stolen)

$p(\text{red}) = \frac{1}{2}$

$p(\text{stolen}) = \frac{1}{2}$

$p(\text{stolen} \mid \text{red}) = 3/5$

$$p(\text{red} \mid \text{stolen}) = \frac{3/5 \times \frac{1}{2}}{\frac{1}{2}} = \frac{3}{5} = .6$$

2. Eigen Vector

$Av = \lambda v$

$$\begin{bmatrix} 3 & 5 \\ 3 & 1 \end{bmatrix}$$

$$\det\left(\begin{pmatrix} 3 & 5 \\ 3 & 1 \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\right) \Rightarrow \lambda^2 - 4\lambda - 12$$

solve equation $\lambda^2 - 4\lambda - 12$

$\lambda = 6 \quad \lambda = -2$

$$\lambda = 6 \quad \begin{bmatrix} 3-6 & 5 \\ 3 & 1-6 \end{bmatrix} = \begin{bmatrix} -3 & 5 \\ 3 & -5 \end{bmatrix}$$

$$\begin{bmatrix} -3 & 5 \\ 3 & -5 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \end{bmatrix} \quad \text{assume } v_2 = t$$

$$\Leftrightarrow v_1 = \frac{5t}{3} \quad v_2 = t$$

$$v = \begin{bmatrix} \frac{5t}{3} \\ t \end{bmatrix} = \begin{bmatrix} \frac{5}{3} \\ 1 \end{bmatrix}$$

$$\lambda = -2 \quad \begin{bmatrix} 3+2 & 5 \\ 3 & 1+2 \end{bmatrix} = \begin{bmatrix} 5 & 5 \\ 3 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 5 \\ 3 & 3 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \quad \text{assume } v_2 = t$$

$v_1 = -t$

$v_2 = t$

$$v = \begin{bmatrix} -t \\ t \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$