CS306: Introduction to IT Security

Fall 2020

Lecture 7: Public-key Cryptography

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STEVENS INSTITUTE of TECHNOLOGY
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7.0 Announcements

CS306: Announcements

- HW2 did not come – too much in view of next week's midterm exam
- Road ahead
  - ~~no lecture on October 13 (next week, classes will run on Monday schedule)~~
  - regular lecture on October 20
  - midterm exam on October 27
    - online exam, quiz format
    - accommodations to be provided as needed
    - covers all materials discussed so far: lectures 1-7, labs 1-7, HW1
    - Lab 7 will offer a general revision on most important topics
    - exact list of topics to be provided tomorrow

CS306: Tentative Syllabus

Week	Date	Topics	Reading	Assignment
1	**Sep 1**	**Introduction**	**Lecture 1**	**-**
2	**Sep 8**	**Symmetric-key encryption**	**Lecture 2**	**Lab 1**
3	**Sep 15**	**Perfect secrecy**	**Lecture 3**	**Lab 2, HW 1**
4	**Sep 22**	**Ciphers in practice I**	**Lecture 4**	**Lab 3, HW 1**
5	**Sep 29**	**Ciphers in practice II**	**Lecture 5**	**Lab 4**
6	**Oct 6**	**MACs & hashing**	**Lecture 6**	**Lab 5**
-	**Oct 13**	**No class (Monday schedule)**		**Lab 6**
7	Oct 20	Public-key cryptography	Lecture 7	Lab 7, HW2

CS306: Tentative Syllabus (continued)

Week	Date	Topics	Reading	Assignment
8	Oct 27	**Midterm**	All materials covered	
9	Nov 3	Network/Web security		
10	Nov 10	Software/Database security		
11	Nov 17	Cloud security		
12	Nov 24	AC/Authentication/Privacy		
13	Dec 1	Economics		
14	Dec 8	Legal & ethical issues		
15	Dec 10 (or later)	**Final** (closed "books")	All materials covered*	

* w/ focus on what covered after midterm

Two weeks ago

- Message authentication
  - MACs
  - Replay attacks
  - Constructions
- Cryptographic hashing
  - Hash functions
  - Constructions
- Demo
  - Hash functions in practice

Today

- Revision on message authentication & cryptographic hashing
  - Practical applications
    - authenticated encryption, hash functions security strength, HMAC
- Public-key (PK) cryptography
  - Motivation, PK Infrastructure, PK encryption, digital signatures
  - Discrete log problem, DH key agreement, hybrid encryption
- Demo
  - The length-extension attack…

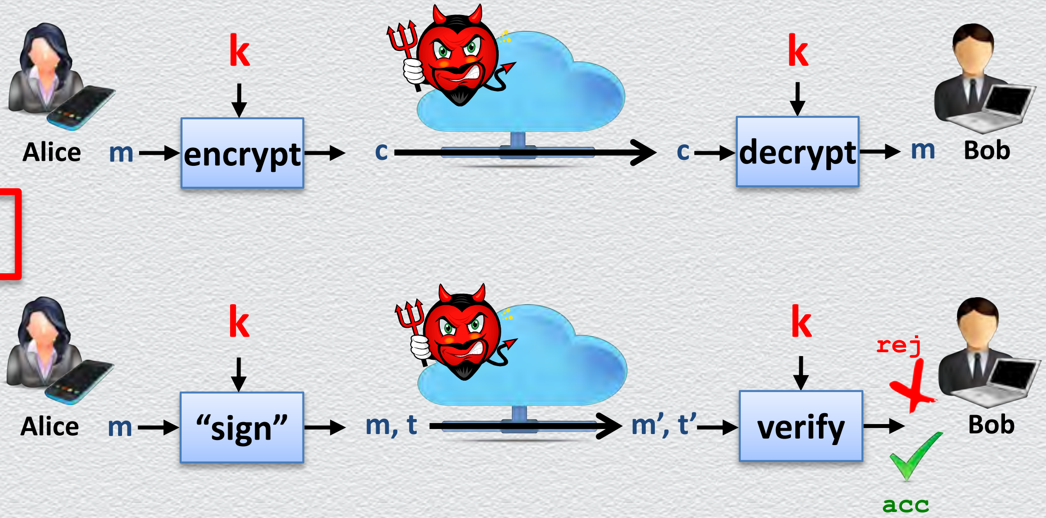
7.1 Public-key encryption & digital signatures

Recall: Principles of modern cryptography

(A) security definitions, **(B) precise assumptions**, (C) formal proofs

For **symmetric-key** message encryption/authentication

- adversary
  - types of attacks
- trusted set-up
  - secret key is distributed securely
  - secret key remains secret
- trust basis
  - underlying primitives are secure
  - **PRG, PRF, hashing, ...**
    - **e.g., block ciphers, AES, SHA-2, etc.**



On “secret key is distributed securely”

Alice & Bob (or 2 individuals) must **securely obtain** a **shared secret key**

- “securely obtain” ⇒ **strong assumption** to accept
  - need of a secure channel
- “shared secret key” ⇒ **challenging problem** to manage
  - too many keys

⇒ **Public-key cryptography to the rescue…**

On “secret key is distributed securely”

Alice & Bob (or 2 individuals) must **securely obtain** a **shared secret key**

- “securely obtain” → **(A) strong assumption** to accept
  - requires secure channel for key distribution (chicken & egg situation)
  - seems impossible for two parties having no prior trust relationship
  - not easily justifiable to hold a priori
- “shared secret key” → **(B) challenging problem** to manage
  - requires too many keys, namely $O(n^2)$ keys for n parties to communicate
  - imposes too much risk to protect all such secret keys
  - entails additional complexities in dynamic settings (e.g., user revocation)

Alternative approaches?

Need to securely distribute, protect & manage many **session-based** secret keys

- (A) for secure distribution, just "make another assumption…"
  - employ **"designated" secure channels**
    - physically protected channel (e.g., meet in a "sound-proof" room)
  - employ **"trusted" party**
    - entities authorized to distribute keys (e.g., key distribution centers (KDCs))
- (B) for secure management, just 'live with it!"

Public-key cryptography to the rescue…

Public-key (or asymmetric) cryptography

disclaimer on names
private = secret

Goal: devise a cryptosystem where key setup is "more" manageable

Main idea: **user-specific** keys (that come in pairs)

- user U generates two keys (U_{pk}, U_{sk})
  - **U_{pk} is public** – it can safely be known by everyone (even by the adversary)
  - **U_{sk} is private** – it must remain secret (even from other users)

Usage

- employ **public** key U_{pk} for certain "**public**" tasks (performed by **other users**)
- employ **private** key U_{sk} for certain "**sensitive**" tasks (performed by **user U**)

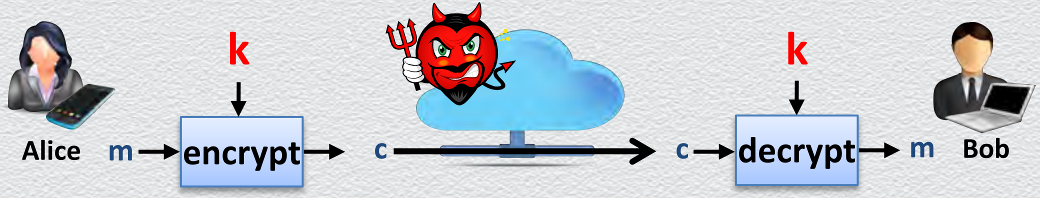
Assumption

- **public-key infrastructure (PKI)**: public keys become **securely** available to users

From symmetric to asymmetric encryption

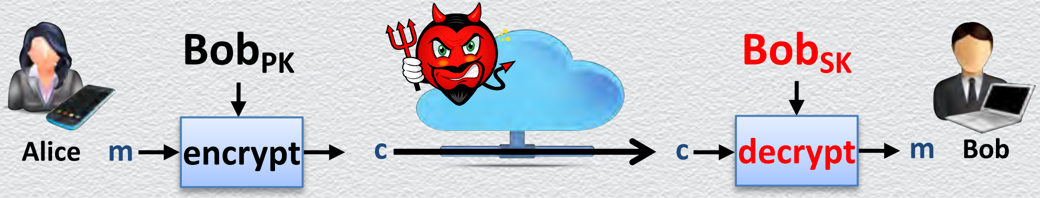
secret-key encryption

- main limitation
  - session-specific keys



public-key encryption

- main flexibility
  - user-specific keys



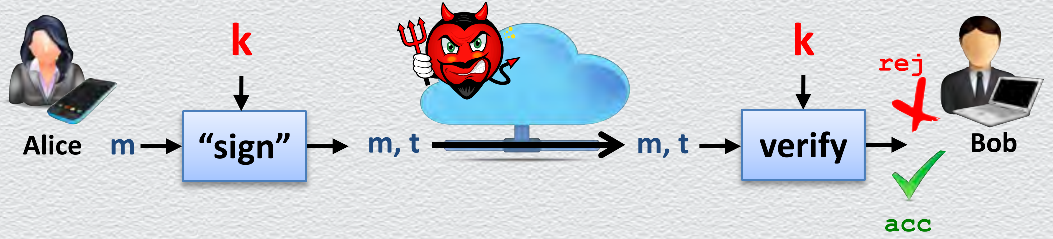
“sensitive” task

- messages encrypted by receiver’s PK can (only) be decrypted by receiver’s SK

From symmetric to asymmetric message authentication

secret-key message authentication (or MAC)

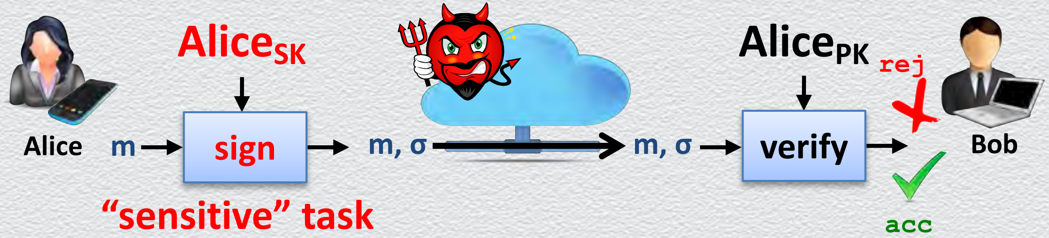
- main limitation
  - **session-specific** keys



public-key message authentication

(or **digital signatures**)

- main flexibility
  - **user-specific** keys



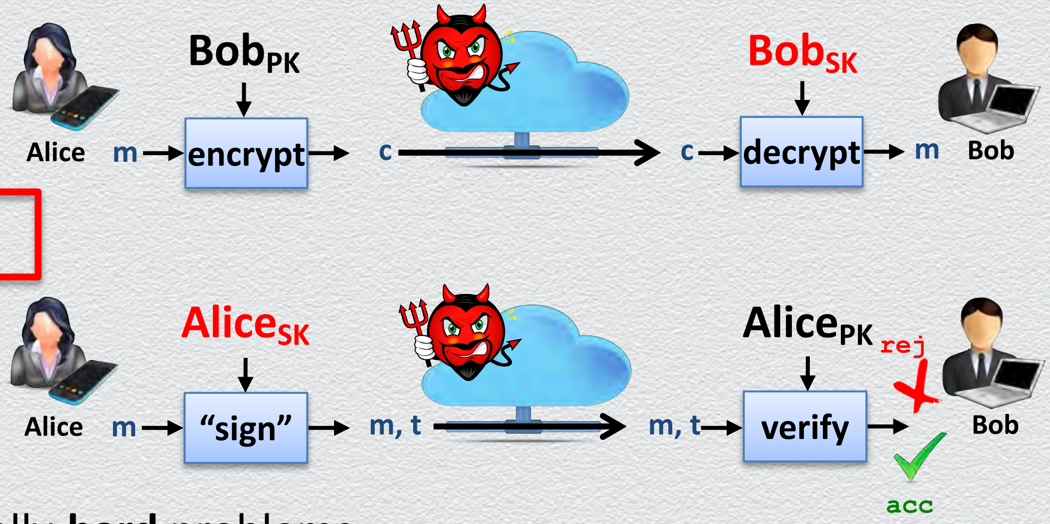
- (only) messages signed by sender's SK can be verified by sender's PK

Thus: Principles of modern cryptography

(A) security definitions, **(B) precise assumptions**, (C) formal proofs

For **asymmetric-key** message encryption/authentication

- adversary
  - types of attacks
- trusted set-up
  - PKI is needed
  - secret keys remain secret
- trust basis
  - underlying primitives are secure
  - typically, **algebraic** computationally-**hard** problems
    - e.g., **discrete log**, **factoring**, etc.



General comparison

Symmetric crypto

- key management
  - less scalable & riskier
- assumptions
  - secret & authentic communication
  - secure storage
- primitives
  - generic assumptions
  - more efficiently in practice

Asymmetric crypto

- key management
  - more scalable & simpler
- assumptions
  - authenticity (PKI)
  - secure storage
- primitives
  - math assumptions
  - less efficiently in practice (2-3 o.o.m.)

Public-key infrastructure (PKI)

A mechanism for securely managing, in a dynamic multi-user setting,
user-specific public-key pairs (to be used by some public-key cryptosystem)

- **dynamic, multi-user**
  - the system is open to anyone; users can join & leave
- **user-specific public-key pairs**
  - each user U in the system is assigned a unique key pair (U_{pk}, U_{sk})
- **secure management** (e.g., authenticated public keys)
  - public keys are authenticated: current U_{pk} of user U is publicly known to everyone

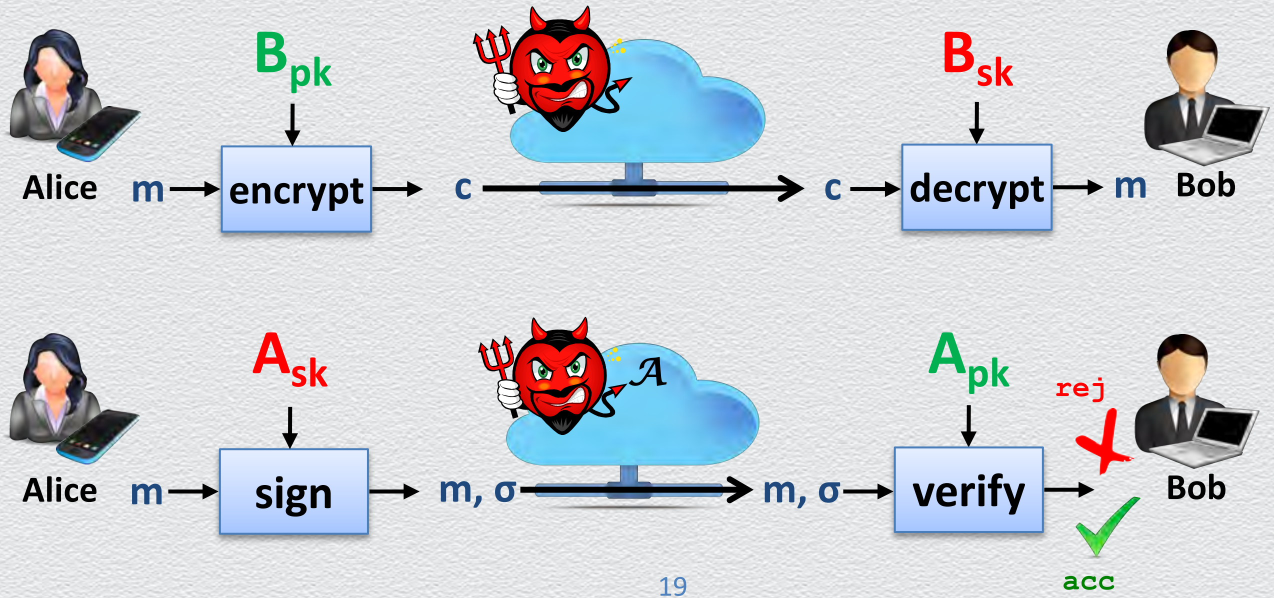
Very challenging to realize

- currently using **digital certificates**; ongoing research towards a better approach…

Overall: Public-key encryption & signatures

Assume a trusted set-up

- public keys are securely available (PKI) & secret keys remain secret



Secret-key vs. public-key encryption

	Secret Key (Symmetric)	Public Key (Asymmetric)
Number of keys	1	2
Key size (bits)	56–112 (DES), 128–256 (AES)	Unlimited; typically no less than 256; 1000 to 2000 currently considered desirable for most uses
Protection of key	Must be kept secret	One key must be kept secret; the other can be freely exposed
Best uses	Cryptographic workhorse. Secrecy and integrity of data, from single characters to blocks of data, messages and files	Key exchange, authentication, signing
Key distribution	Must be out-of-band	Public key can be used to distribute other keys
Speed	Fast	Slow, typically by a factor of up to 10,000 times slower than symmetric algorithms

Public-key cryptography: Early history

Proposed by Diffie & Hellman

- documented in “New Directions in Cryptography” (1976)
- solution concepts of public-key encryption schemes & digital signatures
- key-distribution systems
  - Diffie-Hellman key-agreement protocol
    - “reduces” symmetric crypto to asymmetric crypto

Public-key encryption was earlier (and independently) proposed by James Ellis

- classified paper (1970)
- published by the British Governmental Communications Headquarters (1997)
- concept of digital signature is still originally due to Diffie & Hellman

7.2 Public-key certificates

How to set up a PKI?

- How are public keys stored? How to obtain a user's public key?
- How does Bob know or 'trust' that A_{PK} is Alice's public key?
- How A_{PK} (a bit-string) is securely bound to an entity (user/identity)?

public key: A_{PK}
secret key: A_{SK}

public key: B_{PK}
secret key: B_{SK}

Achieving a PKI…

How can we maintain the invariant that at all times

- any given **user** U is **assigned** a **unique** public-private key pair; and
- any other user known U's **current** public key?
  - secret keys can be lost, stolen or they should be revoked

entails binding users/identities to public keys

Recall

- PK cryptosystems come with a Gen algorithm which is run by U
  - on input a security-strength parameter, it outputs a random valid key pair for U
- public keys can be made publicly available
  - e.g., sent by email, published on web page, added into a public directory, etc.

Distribution of public keys

Public announcement

- users distribute public keys to recipients or broadcast to community at large

Publicly available directory

- can obtain greater security by registering keys with a public directory

Both approaches have problems and are vulnerable to forgeries

Do you trust your public key?

- Impostor claims to be a true party
  - true party has a public and private key
  - impostor also has a public and private key
- Impostor sends impostor’s own public key to the verifier
  - says, “This is the true party’s public key”
  - this is the critical step in the deception

Certificates: Trustable identities & public keys

Certificate

- a public key & an identity **bound** together
- in a document **signed by** a certificate authority

Certificate authority (CA)

- an authority that users **trust** to securely bind identity to public keys
  - CA **verifies identities** before generating certificates for these identities
  - secure binding via **digital signatures**
    - **ASSUMPTION**: The authority's PK CA_{PK} is authentic

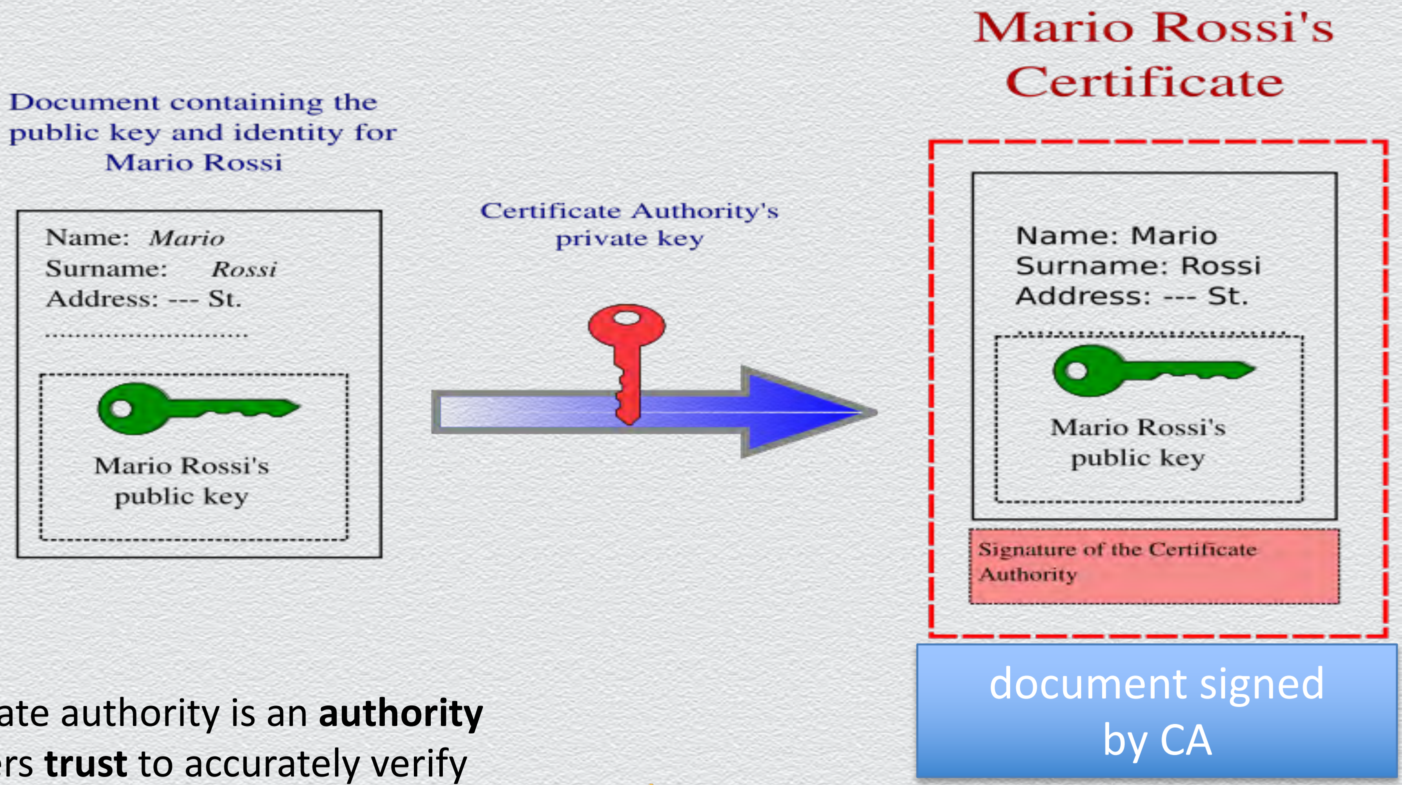
Public-key certificates in practice

Current (imperfect) practice for achieving trustable identities & public keys

- everybody trusts a Certificate Authority (CA)
  - everybody knows PK_{CA} & trusts that CA knows the corresponding secret key CA_{SK}
- a certificate binds identities to public keys in a CA-signed statement
  - e.g., Alice obtains a signature on the statement “Alice’s public key is 1032xD”
- users query CA for public keys of intended recipients or signers
  - e.g., when Bob wants to send an encrypted message to Alice
    - he first **obtains & verifies** a certificate of Alice’s public key
  - e.g., when Alice wants to verify the latest software update by Company
    - she first **obtains & verifies** a certificate of Company’s public key

Example

a certificate is a public key and an identity bound together and signed by a certificate authority (CA)



a certificate authority is an **authority** that users **trust** to accurately verify identities before generating certificates that bind those identities to keys

Symantec

Certificate hierarchy

Single CA certifying every public key is impractical

Instead, use trusted root certificate authorities

- root CA signs certificates for intermediate CAs, they sign certificates for lower-level CAs, etc.
  - certificate "chain of trust"
    - $\text{sig}_{\text{Symantec}}$("Stevens", $\text{PK}_{\text{Stevens}}$)
    - sig_{UMD}("faculty", $\text{PK}_{\text{faculty}}$)
    - $\text{sig}_{\text{faculty}}$("Nikos", PK_{Nikos})

Example 1: Certificate signing & hierarchy

To create Diana's certificate:

Diana creates and delivers to Edward:

Name: Diana Position: Division Manager Public key: 17EF83CA ...

Edward adds:

Name: Diana Position: Division Manager Public key: 17EF83CA ...	hash value 128C4

Edward signs with his private key:

Name: Diana Position: Division Manager Public key: 17EF83CA ...	hash value 128C4

Which is Diana's certificate.

To create Delwyn's certificate:

Delwyn creates and delivers to Diana:

Name: Delwyn Position: Dept Manager Public key: 3AB3882C ...

Diana adds:

Name: Delwyn Position: Dept Manager Public key: 3AB3882C ...	hash value 48CFA

Diana signs with her private key:

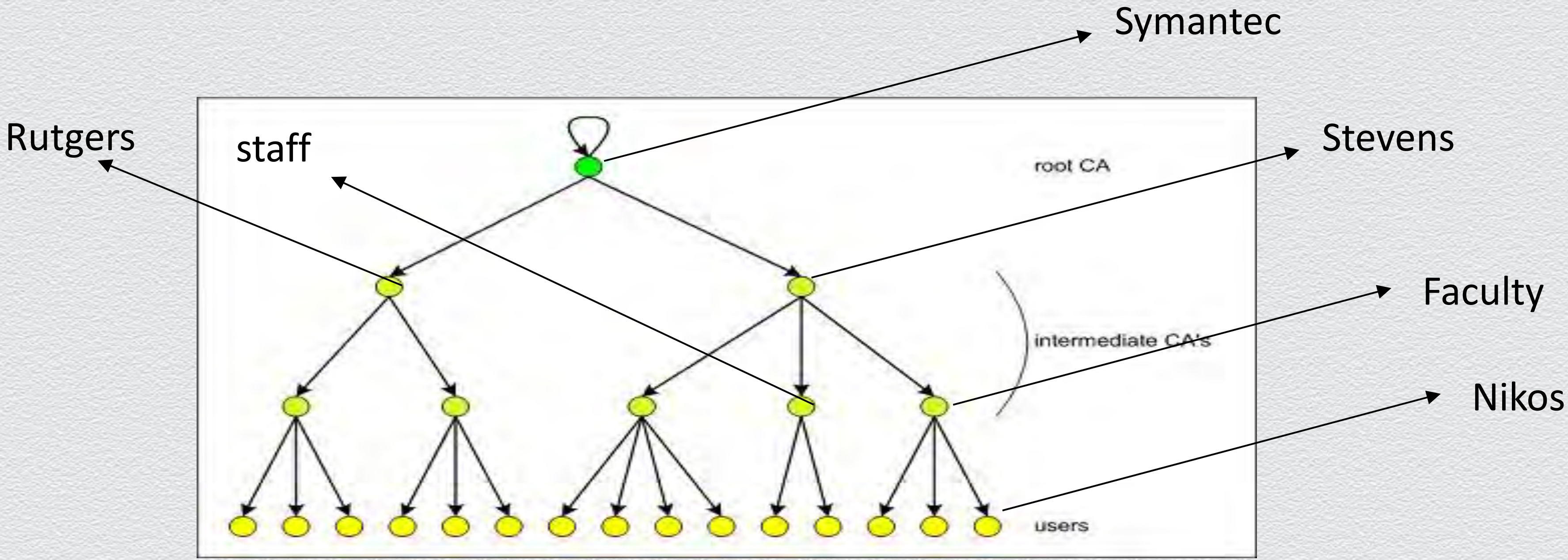
Name: Delwyn Position: Dept Manager Public key: 3AB3882C ...	hash value 48CFA

And appends her certificate:

Name: Delwyn Position: Dept Manager Public key: 3AB3882C ...	hash value 48CFA
Name: Diana Position: Division Manager Public key: 17EF83CA ...	hash value 128C4

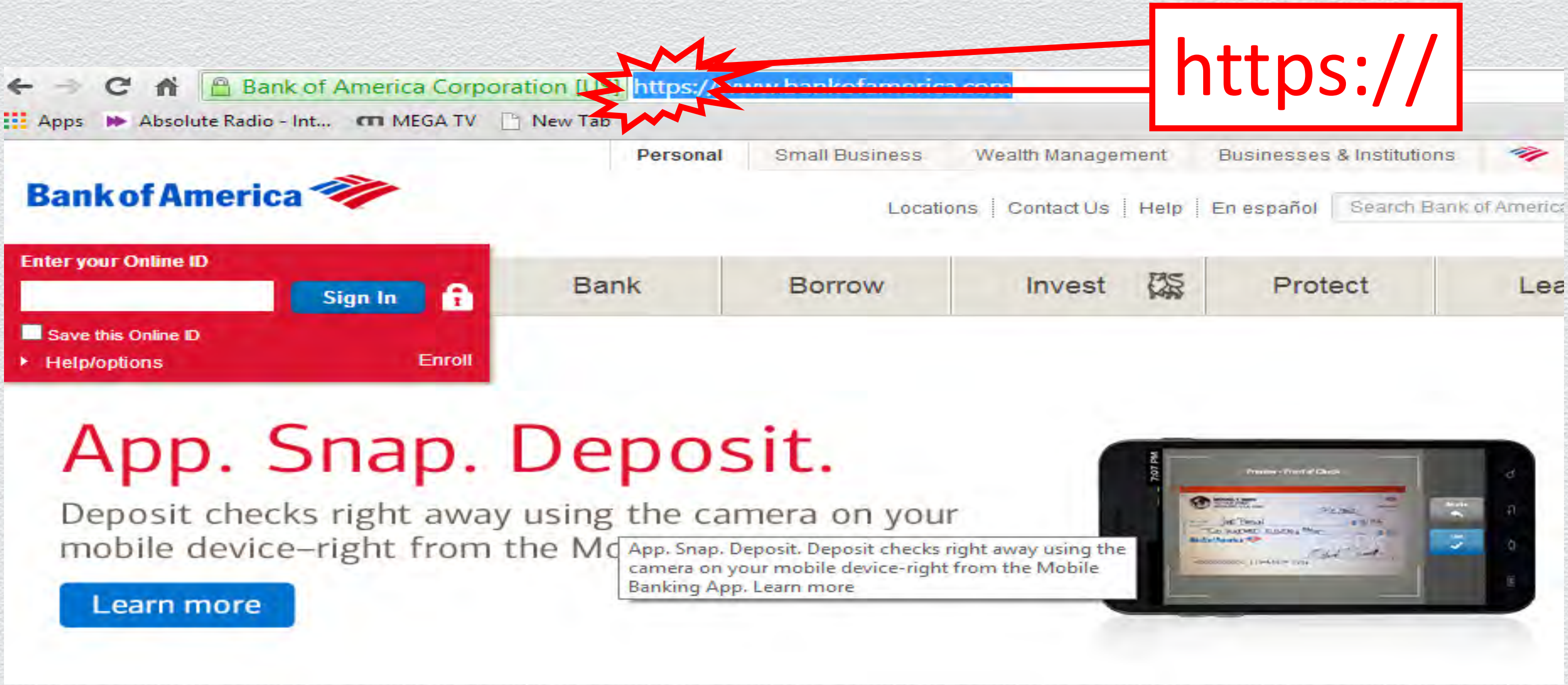
Which is Delwyn's certificate.

Example 2



What bad things can happen if the root CA system is compromised?

Secure communication over the Internet



What cryptographic keys are used to protect communication?

X.509 certificates

Defines framework for authentication services

- defines that public keys stored as certificates in a public directory
- certificates are issued and signed by a CA

Used by numerous applications: SSL

Example: see certificates accepted by your browser

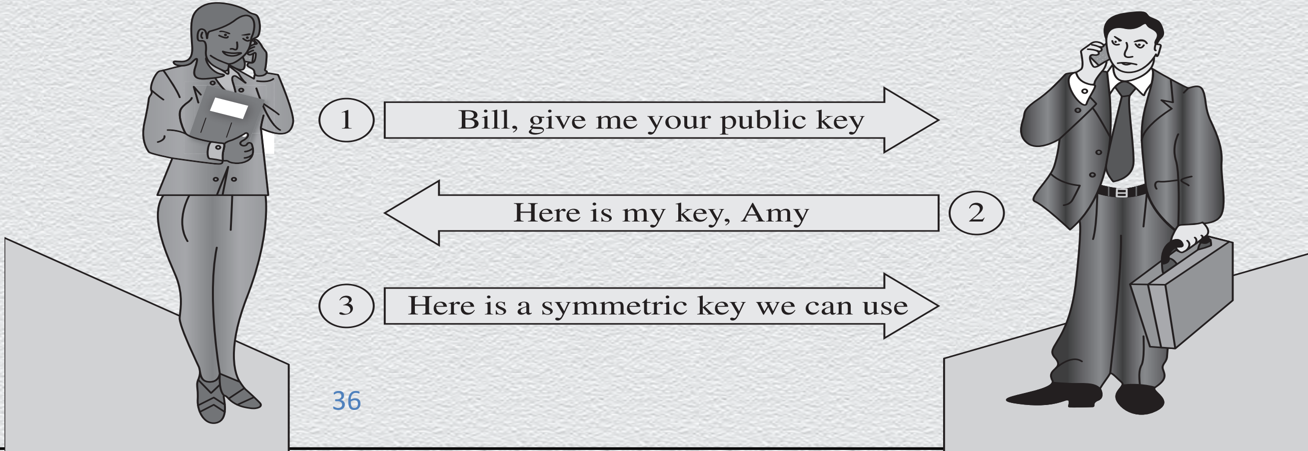
7.3 Hybrid encryption

Secret-key cryptography is “reduced” to public-key

PK encryption can be used “on-the-fly” to securely distribute session keys

Main idea: Leverage PK encryption to securely distribute session keys

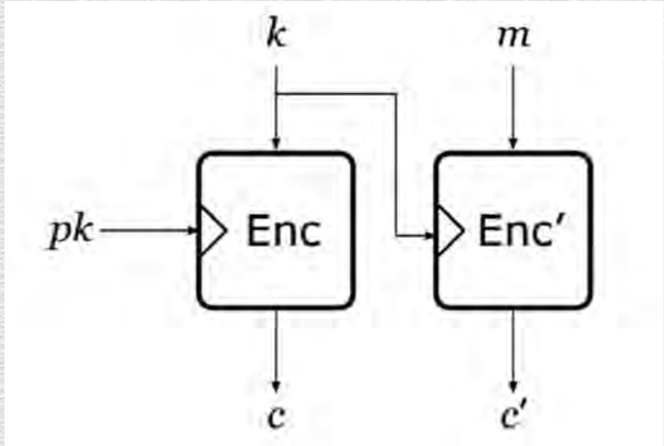
- sender generates a fresh session-specific secret key k and **learns** receiver’s public key R_{pk}
- session key k is sent to receiver encrypted under key R_{pk}
- session key k is employed to run symmetric-key crypto
  - e.g., how **not** to run above protocol



Hybrid encryption

"Reduces" secret-key crypto to public-key crypto

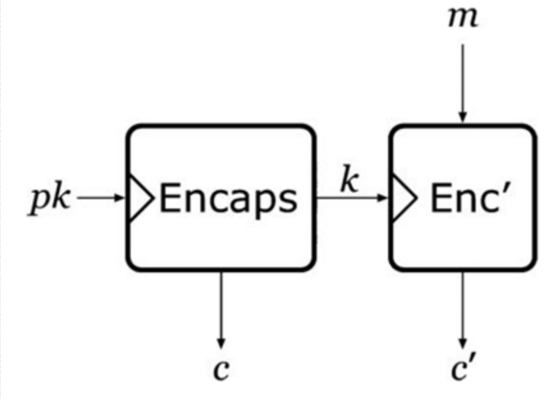
- better performance than block-based public-key CPA-encryption
- main idea
  - apply PK encryption on random key k
  - use k for secret-key encryption of m



Hybrid encryption using the KEM/DEM approach

"Reduces" secret-key crypto to public-key crypto

- main idea
  - **encapsulate** secret key k into c
  - use k for secret-key encryption of m
  - KEM: key-encapsulation mechanism - Encaps
  - DEM: data encapsulation mechanism - Enc'



- KEM/DEM scheme
  - CPA-secure if KEM is CPA-secure and Enc' EAV-secure
  - CCA-secure if KEM and Enc' are CCA-secure

7.4 The Discrete Log problem & its applications

The discrete logarithm problem

Setting

- if p be an odd prime, then $G = (Z_p^*, \cdot)$ is a cyclic group of order $p - 1$
  - $Z_p^* = \{1, 2, 3, \ldots, p-1\}$, generated by some g in Z_p^*
    - for $i = 0, 1, 2, \ldots, p-2$, the process **$g^i \bmod p$** produces all elements in Z_p^*
  - for any x in the group , we have that **$g^k \bmod p = x$**, for some integer k
  - k is called the **discrete logarithm** (or log) of x (mod p)

Example

- $(Z_{17}^*, \cdot)$ is a cyclic group G with order 16, 3 is the generator of G and $3^{16} = 1 \bmod 17$
- let $k = 4$, $3^4 = 13 \bmod 17$ (which is easy to compute)
- the inverse problem: if $3^k = 13 \bmod 17$, what is k? what about **large p**?

Computational assumption

Discrete-log setting

- cyclic G = (Z_p^*, $\cdot$) of order p – 1 generated by g, prime p of length t ($|p|$=t)

Problem

- given G, g, p and x in Z_p^*, compute the discrete log k of x (mod p)
- we know that $x = g^k \bmod p$ for some unique k in {0, 1, …, p-2}… but

Discrete log assumption

- for groups of specific structure, **solving the discrete log problem is infeasible**
- any efficient algorithm finds discrete logs negligibly often (prob = $2^{-t/2}$)

Brute force attack

- cleverly enumerate and **check $O(2^{t/2})$ solutions**

ElGamal encryption

Assumes discrete-log setting (cyclic $G = (Z_p^*, \cdot) = \langle g \rangle$, prime p, message space Z_p)

Gen

- secret key: random number $x \in Z^*_p$ public key: $A = g^x \bmod p$, along w/ G, g, p

Enc

- pick a fresh random $r \in Z^*_p$ and set $R = A^r$ ($= g^{xr}$)
- send ciphertext $\mathbf{Enc_{PK}(m) = (c_1, c_2)}$ where $\mathbf{c_1 = g^r}$, $\mathbf{c_2 = m \cdot R \bmod p}$

Dec

- $\mathbf{Dec_{SK}(c_1,c_2) = c_2\, (1/c_1^x) \bmod p}$ where $\mathbf{c_1^x = g^{xr}}$

Security is based on **Computational Diffie-Hellman** (CDH) assumption

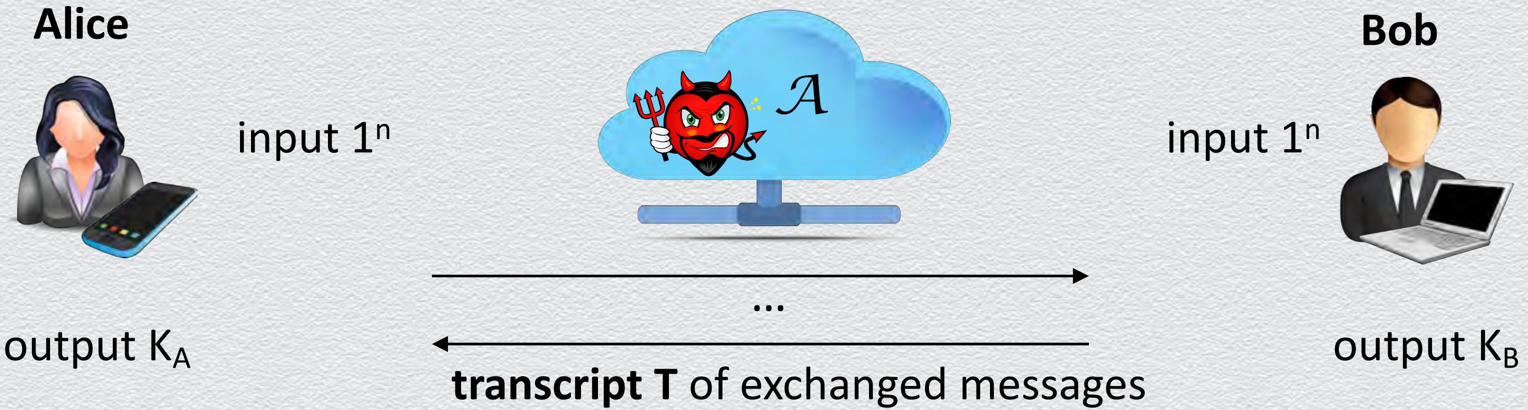
- given (g, g^a, g^b) it is hard to compute g^{ab}

A signature scheme can be also derived based on above discussion

Application: Key-agreement (KA) scheme

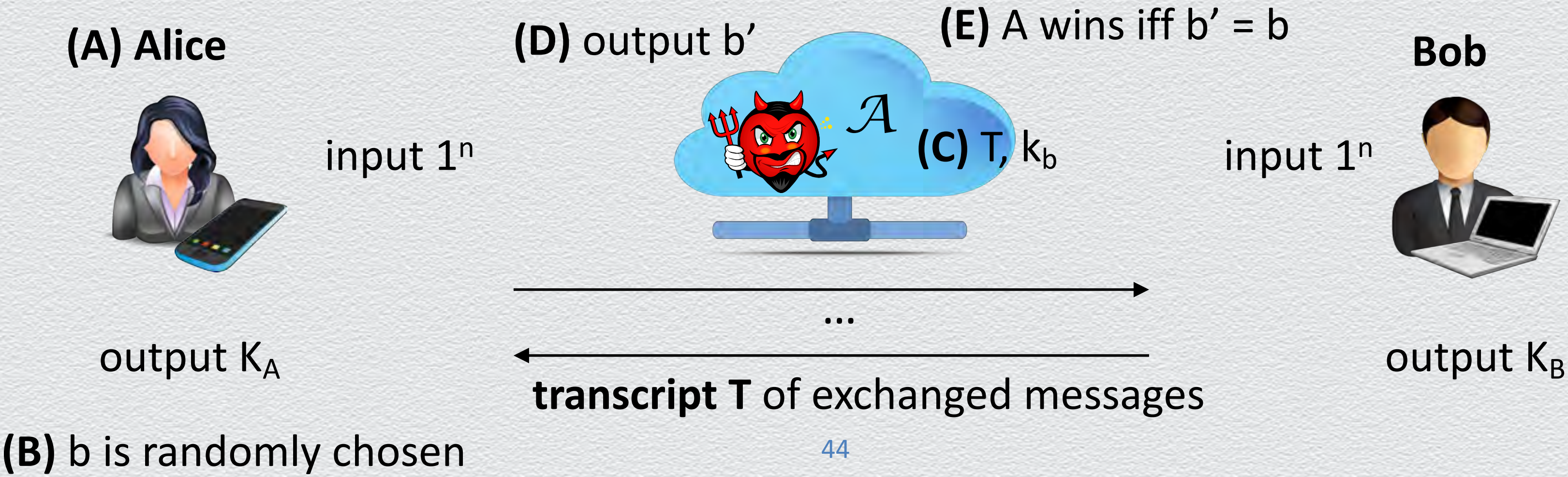
Alice and Bob want to securely establish a **shared key** for secure chatting over an **insecure** line

- instead of meeting in person in a secret place, they want to use the insecure line…
- KA scheme: they run a key-agreement protocol Π to contribute to a shared key K
- correctness: $K_A = K_B$
- security: no PPT adversary $\mathcal{A}$, given T, can distinguish K from a trully random one



Key agreement: Game-based security definition

- scheme $\Pi(1^n)$ runs to generate $K = K_A = K_B$ and transcript T; random bit b is chosen
- adversary $\mathcal{A}$ is given T and k_b; if b = 1, then $k_b = K$, else k_b is random (both n-bit long)
- $\mathcal{A}$ outputs bit b' and wins if b' = b
- then: **Π is secure if no PPT A wins non-negligibly often**



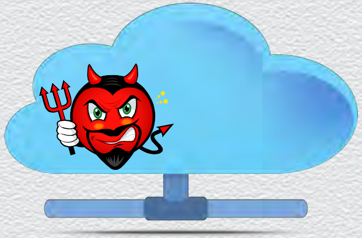
The Diffie-Hellman key-agreement protocol

Alice and Bob want to securely establish a **shared key** for secure chatting over an **insecure** line

- DH KA scheme Π
  - discrete log setting: p, g public, where <g> = Z^*_p and p prime

Alice

input 1^n



Bob

input 1^n

(1) randomly pick secret a

(2) randomly pick secret b

(3) send g^a mod p →

← **(4)** send g^b mod p

(5) set **K = g^{ab} mod p** = $(g^b \bmod p)^a \bmod p$

(6) set **K = g^{ab} mod p** = $(g^a \bmod p)^b \bmod p$

Security

- discrete log assumption is necessary but not sufficient
- decisional DH assumption
  - given g, g^a and g^b, g^{ab} is computationally indistinguishable from uniform

Authenticated Diffie-Hellman