HW#5 : SOLUTIONS

1. $P(N=0) = P(N=0 \mid p=.3)P(p=.3)$
$+P(N=0 \mid p=.5)P(p=.5)$
$+ P(N=0 \mid p=.7)P(p=.7)$
$= \left[\binom{10}{0}(.3)^0(.7)^{10}\right]\cdot\frac{1}{3} + \left[\binom{10}{0}(.5)^0(.5)^{10}\right]\cdot\frac{1}{3} + \left[\binom{10}{0}(.7)^0(.3)^{10}\right]\cdot\frac{1}{3}$
$=$ .0097 or .01

2. Let X = # people you question until some born in December

$p = P(\text{an individual is born in December}) = \frac{1}{12}$

a) $P(X=5) = q^4 p = \left(\frac{11}{12}\right)^4\left(\frac{1}{12}\right)$
$=$ .059 or .06

b) $E(X) = \frac{1}{p} = \frac{1}{1/12} = 12$

3. Let # of errors per page $\sim$ Poisson w/ $\lambda = 3$ errors/page

X = # errors in t pages

$P(X=k \text{ in } t \text{ periods (pages)}) = \frac{e^{-\lambda t}(\lambda t)^k}{k!}$

a) $t=1$ $\quad P(X=2) = \frac{e^{-3(1)}(3\cdot 1)^2}{2!} = \frac{(.0497)(9)}{2} = .22$

b) $t=3$ $\quad P(X=6 \text{ in } 3 \text{ pages}) = \frac{e^{-3(3)}(3\cdot 3)^6}{6!} = .09$

4. $n = 200 \quad p = .01 \quad np \to \lambda = (200)(.01) = 2$

Let X = # of defectives in 200 items $\sim bin(n, p)$, $n = 200$, $p = .01$

Using Poisson approximation with $\lambda = 2$:

$P(X \geq 2) = 1 - P(0) - P(1)$
$= 1 - \frac{e^{-2}2^0}{0!} - \frac{e^{-2}2^1}{1!} = .594$

EXTRA CREDIT PROBLEM:

Let T = the number of rolls required to get a "6"

$p = P(\text{a "6"}) = \frac{1}{6}$

$q = 5/6$

$$P(\text{Mary wins}) = P(T \text{ is an } \underline{\text{even}} \text{ number})$$

$$= P(T = 2 \text{ or } 4 \text{ or } 6 \text{ or} \ldots)$$

$$= P(T=2) + P(T=4) + P(T=6) + \ldots$$

$$= qp + q^3 p + q^5 p + \ldots$$

$$= qp(1 + q^2 + q^4 + \ldots)$$

$$= \frac{qp}{1-q^2}$$

$$= \frac{\frac{5}{6}\cdot\frac{1}{6}}{1-\left(\frac{5}{6}\right)^2} = \frac{5}{11} \doteq .45$$