SOLUTIONS: HW #3

① Let B_i = the event "bearing came from supplier i, for $i = 1, 2, 3$

Let A = the event "a chosen bearing from inventory is defective"

Then $P(B_1) = .5 \quad P(B_2) = .3 \quad P(B_3) = .2$

$$P(A) = P(B_1)P(A|B_1) + P(B_2)P(A|B_2) + P(B_3)P(A|B_3)$$
$$= .5(.02) + .3(.03) + .2(.04)$$
$$= .027$$

∴ about 2.7% of the stock is defective

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② Let:

A = chip drawn from urn II is red

B_1 = chip drawn from urn I is red

B_2 = " " " " I is white

$$P(A) = P(B_1)P(A|B_1) + P(B_2)P(A|B_2)$$
$$= \left(\tfrac{2}{6}\right)\left(\tfrac{4}{5}\right) + \left(\tfrac{4}{6}\right)\left(\tfrac{3}{5}\right) = 2/3$$

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③ Let

B_1 = "red chip transferred from urn I"

B_2 = "white " " " " " "

A = "red chip is drawn from urn II"

We seek:

$P(B_2|A) = P($white chip was transferred from urn I given that a red chip was drawn from urn 2)

$$= \frac{P(AB_2)}{P(A)} = \frac{P(A|B_2)P(B_2)}{P(A|B_1)P(B_1) + P(A|B_2)P(B_2)}$$

$$= \frac{\left(\tfrac{2}{4}\right)\left(\tfrac{2}{3}\right)}{\left(\tfrac{3}{4}\right)\left(\tfrac{1}{3}\right) + \left(\tfrac{2}{4}\right)\left(\tfrac{2}{3}\right)} = \frac{4}{7}$$

(4) Let
B_1 = a "0" was sent
B_2 = a "1" was sent
A = a "1" is received

We seek: $P(B_1|A)$

$$P(B_1|A) = \frac{P(AB_1)}{P(A)} = \frac{P(B_1)P(A|B_1)}{P(B_1)P(A|B_1) + P(B_2)P(A|B_2)}$$

$$= \frac{(.7)(.2)}{(.7)(.2) + (.3)(.8)} = .37$$

(5.)
$$P(C|B) = \frac{P(B|C)P(C)}{P(B|C)P(C) + P(B|C^c)P(C^c)}$$

$$= \frac{(.9)(.0001)}{(.9)(.0001) + (.001)(.9999)}$$

$$= \underline{\underline{.08}}$$

That is, only 8% of those women identified as having the disease actually do have it!

The practicality of large-scale screening programs directed at diseases with low prevalence is open to question.