HW#4 SOLUTIONS

(1.) P(at least one has the characteristic) $= 1 - P(\text{none has the characteristic})$

$= 1 - \left(\frac{2099}{2100}\right)^{3000} = .7604$

(2.) $E(\text{profit}) = \frac{1}{12}(7) + \frac{1}{12}(9) + \frac{1}{4}(11) + \frac{1}{4}(13) + \frac{1}{6}(15) + \frac{1}{6}(17)$

$= \$12.67$

(3.) Let X = the # of tails in 2 tosses $P(H) = 3/4$ $P(T) = \frac{1}{4}$

$P(X=0) = P(HH) = \frac{3}{4} \cdot \frac{3}{4} = \frac{9}{16}$

$P(X=1) = P(HT \text{ or } TH) = \frac{3}{4} \cdot \frac{1}{4} + \frac{1}{4} \cdot \frac{3}{4} = 3/8$

$P(X=2) = P(TT) = \frac{1}{4} \cdot \frac{1}{4} = 1/16$

note: $\Sigma = 1$ ✓

$\therefore$

X	0	1	2
P(X)	9/16	3/8	1/16

and $E(X) = (0)\left(\frac{9}{16}\right) + 1(3/8) + 2\left(\frac{1}{16}\right) = \frac{1}{2}$

(4.) For a "fair" game, $E(\cdot) = 0$

So let: X = winnings on a play

and c = the amount she pays to play

then

X	5 − c	3 − c	−c
P(X)	2/13	2/13	9/13

and $E(X) = \frac{2}{13}(5-c) + \frac{2}{13}(3-c) + \frac{9}{13}(-c) = 0 \Rightarrow \boxed{c = \$1.23}$

EASIER WAY:

$E(X) = (5)\left(\frac{8}{52}\right) + (3)\left(\frac{8}{52}\right) + 0\left(\frac{36}{52}\right) = \$1.23$

so she should pay \$1.23 to make E(game) = 0

5. Let X = # of defectives out of 15 $\left(X \sim bin(15, .05) \right)$
$(n = 15, p = .05)$

$P(X \geq 2) = 1 - P(X < 2)$

$= 1 - P(X=0) - P(X=1)$

$= 1 - \binom{15}{0}(.05)^0(.95)^{15} - \binom{15}{1}(.05)^1(.95)^{14}$

$= .171$

6. Let X = the number of "seconds" in a sample of 6

$X \sim bin(6, .1)$ $n = 6$ $p = .1$

a) $P(X=1) = \binom{6}{1}(.1)^1(.9)^5 = .3543$

b) $E(X) = np = (6)(.1) = .6$ "seconds"

c) $P(X \geq 2) = 1 - P(X=0) - P(X=1)$

$= 1 - \binom{6}{0}(.1^0)(.9^6) - \binom{6}{1}(.1)^1(.9)^5$

$= .1143$

d)
P(at most 5 selected to get four good ones)
= P(first 4 are good OR one bad one in first 4 and fifth one is good)

$= \binom{4}{0}(.1)^0(.9)^4 + \left[\binom{4}{1}(.1)^1(.9)^4\right](.9)$

$= .918$