HW #6A - SOLUTIONS

1.

Y \ X	1	2	3	
1	$\frac{1}{12}$	$\frac{1}{6}$	0	.25
2	0	$\frac{1}{9}$	$\frac{1}{5}$	.31
3	$\frac{1}{18}$	$\frac{1}{4}$	$\frac{2}{15}$	.44
	.139	.528	.333	

marginal distributions:

$P(X=1) = .139$ | $P(Y=1) = .25$
$P(X=2) = .528$ | $P(Y=2) = .31$
$P(X=3) = .333$ | $P(Y=3) = .44$

conditional distributions:

$P(X \mid Y=1)$
$P(X=1 \mid Y=1) = \frac{1}{3}$
$P(X=2 \mid Y=1) = \frac{2}{3}$
$P(X=3 \mid Y=1) = 0$

$P(X \mid Y=2)$
$P(X=1 \mid Y=2) = 0$
$P(X=2 \mid Y=2) = .358$
$P(X=3 \mid Y=2) = .645$

$P(X \mid Y=3)$
$P(X=1 \mid Y=3) = .126$
$P(X=2 \mid Y=3) = .568$
$P(X=3 \mid Y=3) = .303$

$P(Y \mid X=1)$
$P(Y=1 \mid X=1) = .6$
$P(Y=2 \mid X=1) = 0$
$P(Y=3 \mid X=1) = .4$

$P(Y \mid X=2)$
$P(Y=1 \mid X=2) = .316$
$P(Y=2 \mid X=2) = .210$
$P(Y=3 \mid X=2) = .473$

$P(Y \mid X=3)$
$P(Y=1 \mid X=3) = 0$
$P(Y=2 \mid X=3) = .6$
$P(Y=3 \mid X=3) = .4$

X and Y are __not__ independent

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2. a) $\int_0^1\int_0^1 K(3x+9y)\,dx\,dy = 1 \Rightarrow K = \frac{1}{6}$ $\therefore f(x,y) = \frac{1}{6}(3x+9y)\ \begin{cases} 0 \le x \le 1 \\ 0 \le y \le 1 \end{cases}$

b) $\int_{1/2}^{1}\int_0^{1/2} \frac{1}{6}(3x+9y)\,dx\,dy = \frac{5}{16}$

c) $g(x) = \int_0^1 \frac{1}{6}(3x+9y)\,dy = \frac{2x+3}{4}$; $0 \le x \le 1$

$h(y) = \int_0^1 \frac{1}{6}(3x+9y)\,dx = \frac{6y+1}{4}$; $0 \le y \le 1$

d) No, X and Y are NOT independent, since $g(x)h(y) \ne f(x,y)$. Independence requires that the joint p.d.f. factors into the product of the marginal p.d.f.s.

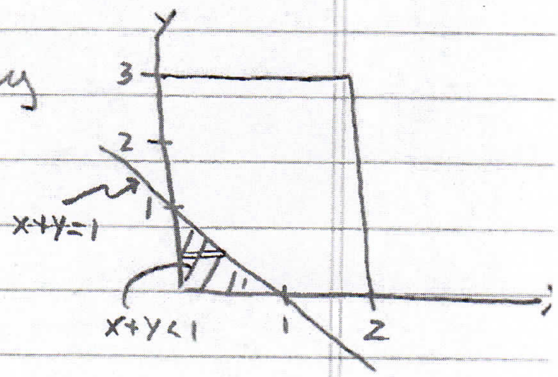
(3.) $X \sim U(0,2) \Rightarrow f_X(x) = \frac{1}{2} \quad ; \quad 0 \le x \le 2$

$Y \sim U(0,3) \Rightarrow g_Y(y) = \frac{1}{3} \quad ; \quad 0 \le y \le 3$

$\therefore$ The JOINT P.D.F. of X and Y is $h(x,y) = \frac{1}{6}$ for $0 \le x \le 2$, $0 \le y \le 3$

(since X and Y are independent, $h(x,y) = f_X(x) \cdot g_Y(y)$)

$$P(X+Y<1) = \iint\limits_{\substack{x,y \\ \text{such that} \\ x+y<1}} h(x,y)\,dx\,dy$$



$$= \int_0^1 \int_0^{1-y} \frac{1}{6}\,dx\,dy$$

$$= \int_0^1 \left[\frac{x}{6} \Big|_0^{1-y} \right] dy$$

$$= \int_0^1 \frac{1-y}{6}\,dy = \frac{1}{12}$$

4. a) $P(X<2, Y>2) = \int_0^2 \int_2^\infty 6e^{-2x-3y}\,dy\,dx$ (other order equally ok)

$$= 6\int_0^2 e^{-2x}\left[\int_2^\infty e^{-3y}dy\right]dx = 6\int_0^2 e^{-2x}\left[\frac{e^{-3y}}{-3}\Big|_{y=2}^{y=\infty}\right]dx$$

$$= 6\left(\frac{e^{-6}}{3}\right)\int_0^2 e^{-2x}dx = 2e^{-6}\left[\frac{e^{-2x}}{-2}\right]_{x=0}^{x=2}$$

$$= 2e^{-6}\left[\frac{e^{-4}-1}{-2}\right]$$

$$= (1-e^{-4})(e^{-6}) = .0025$$

b) marginal pdf for $X = g(x) = \int_0^\infty 6e^{-2x-3y}dy$

$$= 6e^{-2x}\left[\frac{e^{-3y}}{-3}\Big|_{y=0}^{\infty}\right]$$

$$= 6e^{-2x}\left[\frac{1}{3}\right]$$

$$= 2e^{-2x} \qquad x>0$$

c) marginal pdf for $Y = h(y) = \int_{x=0}^\infty 6e^{-2x-3y}dx$

$$= 6e^{-3y}\int_0^\infty e^{-2x}dx$$

$$= 6e^{-3y}\left[\frac{e^{-2x}}{-2}\right]_{x=0}^{x=\infty}$$

$$= 6e^{-3y}\left(\frac{1}{2}\right)$$

$$= 3e^{-3y}$$

d) Yes, since $g(x)h(y) = 2e^{-2x}\cdot 3e^{-3y} = 6e^{-2x-3y} = f(x,y)$