HW #9 SOLUTIONS

1. $\hat{p} \pm z_{\frac{\alpha}{2}} \sqrt{\frac{p(1-p)}{n}}$ (use $\hat{p}$ for p)

$.175 \pm 1.64 \sqrt{\frac{.175(.825)}{200}} \Rightarrow .175 \pm .0442$

$(.1308, .2192)$ or $P(.1308 < p < .2192) = .90$

2. $\hat{p} \pm z_{\alpha/2} \sqrt{\frac{p(1-p)}{n}}$

$.68 \pm 1.96 \sqrt{\frac{.68(.32)}{500}} \Rightarrow .68 \pm .04$

$(.64, .72)$ or $P(.64 < p < .72) = .95$

3. $E = .05$ $\quad z_{\alpha/2} = z_{.025} = 1.96$ $\quad \sigma_x = 0.3$ $\quad n = ?$

$E = z_{\alpha/2} \frac{\sigma_x}{\sqrt{n}}$ $\quad .05 = 1.96 \frac{.3}{\sqrt{n}} \Rightarrow n = 139$

4. $E = .02$

a) use $p = .68$

$E = z_{\alpha/2} \sqrt{\frac{p(1-p)}{n}}$

$.02 = 1.96 \sqrt{\frac{.68(.32)}{n}}$

square both sides: $.0004 = 3.84 \left(\frac{(.68)(.32)}{n}\right) \Rightarrow n = 2089$

b) use $p = .5$

$.02 = 1.96 \sqrt{\frac{(.5)(.5)}{n}}$

squaring both sides: $.0004 = 3.84 \left(\frac{(.5)(.5)}{n}\right) = n = 2400$

5. $n = 20 \quad \bar{x} = 1584 \quad s_x = 607$ USE t-dist.

$$\bar{x} \pm t_{n-1, \alpha/2}\left(\frac{s_x}{\sqrt{n}}\right) = 1584 \pm t_{19, .005}\left(\frac{607}{\sqrt{20}}\right)$$

$$= 1584 \pm 2.861\left(\frac{607}{\sqrt{20}}\right)$$

$$= 1584 \pm 388.3$$

SO: $P(1195.7 < \mu < 1972.3) = .99$

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6. $n = 20 \quad \bar{x} = .9255 \quad s_x = .0809$

$$\bar{x} \pm t_{n-1, \alpha/2}\left(\frac{s_x}{\sqrt{n}}\right) = .9255 \pm t_{19, .025}\left(\frac{.0809}{\sqrt{20}}\right)$$

$$= .9255 \pm 2.093(.0181)$$

$$= .9255 \pm .0379$$

SO: $P(.8876 < \mu < .9634) = .95$

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7. A STATISTICAL HYPOTHESIS is an assertion concerning one or more POPULATION PARAMETERS

a) H_0: the claim about the population parameter(s) that is initially assumed to be true: the "prior belief" or "standard" or "claim" or "hypothesis of no change"

b) H_1: The assertion that is contradictory to Ho

c) test statistic: a function of the sample data on which the decision (reject Ho or do not reject Ho) is made

d) Critical/rejection region: the set of all test statistic values for which H_0 will be rejected

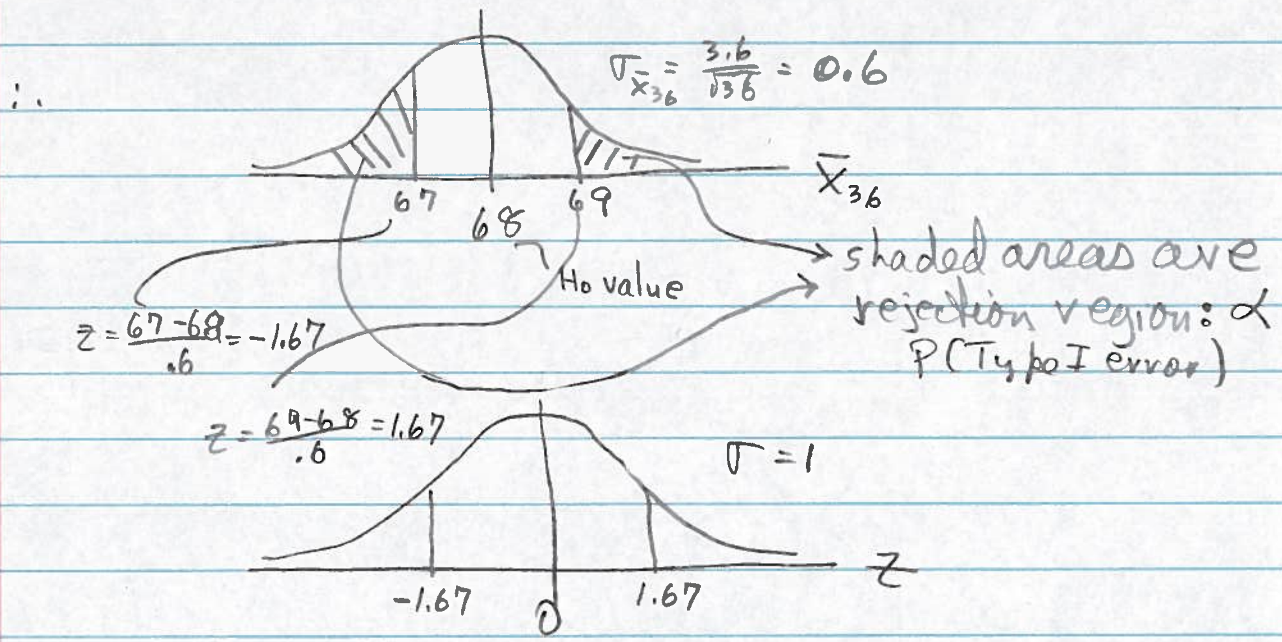
e) Type I error: rej H_0 when H_0 is true

f) Type II error: fail to reject H_0 when H_1 is true

g) Significance level of the test: α where $\alpha = P(\text{Type I error})$

8\. a) $\sigma_x = 3.6 \quad n = 36 \quad \Rightarrow \sigma_{\bar{x}_{36}} = \frac{3.6}{\sqrt{36}} = 0.6$

$\therefore$



$\therefore \alpha = P(\text{Type I error}) = P(\bar{x}_{36} \text{ is in rejection region when } H_0 \text{ is true})$

$= P(\bar{x}_{36} > 69) + P(\bar{x}_{36} < 67)$

$= P(z > 1.67) + P(z < -1.67)$

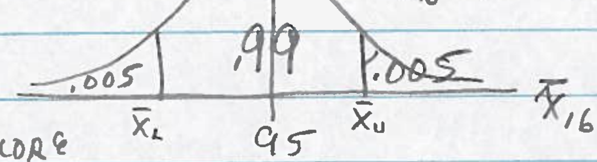
$= .0950$

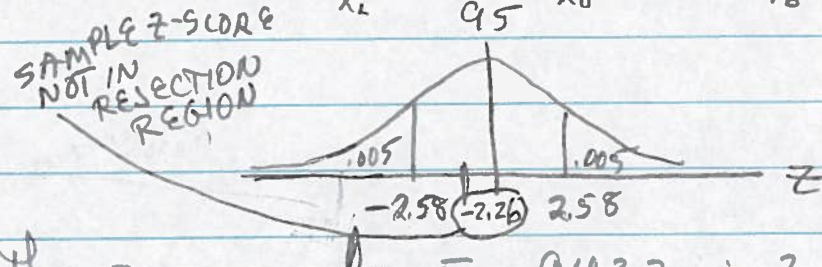
b) similar to part (a) with $n = 64 \Rightarrow \alpha = .0264$

9. a) $H_0: \mu = 95$

$H_1: \mu \neq 95$

$\sigma_{\bar{x}_{16}} = \frac{1.2}{\sqrt{16}} = .3$





The z-score for $\bar{x}_{16} = 94.32$ is $z = \frac{94.32 - 95}{.3} = -2.26$

which is <u>IN</u> the rejection region

so conclusion: The sample data do NOT lead us to reject the null hypothesis that the mean is still 95 degrees, ACCEPT H_0

b) CRITICAL REGION:

FOR Z: $Z < -2.58$ or $Z > 2.58$

In terms of $\bar{x}_{16}$:

$$\frac{\bar{x}_L - 95}{.3} = -2.58 \Rightarrow \bar{x}_L = 94.226$$

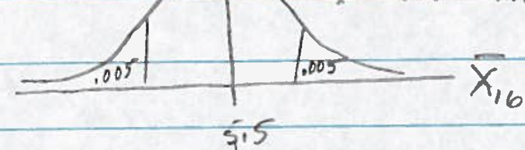
$$\frac{\bar{x}_U - 95}{.3} = 2.58 \Rightarrow 95.774$$

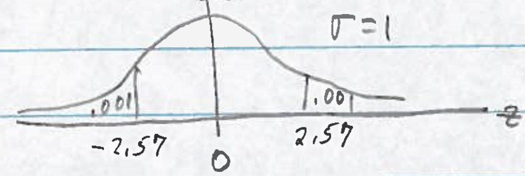
FOR $\bar{x}_{16}$: $\bar{x}_{16} < 94.226$ or $\bar{x}_{16} > 95.774$

10. a) $H_0: \mu = 5.5$ $\qquad \bar{X}_{16} = 5.25 \qquad n = 16$

$H_1: \mu \neq 5.5$

$\sigma_{\bar{x}} = \frac{\sigma_x}{\sqrt{n}} = \frac{.3}{\sqrt{16}} = .075$





CRITICAL (REJECTION) REGION is $Z_{sample} < -2.57$ or > 2.57

Our sample z is

$$Z_{sample} = \frac{5.25 - 5.5}{.075} = -3.33$$

CONCLUSION: The sample data lead us to reject the hypothesis that the population mean is still 5.5. Reject Ho.

b) CRITICAL REGIONS:

$Z < -2.57$ or $Z > 2.57$

for $\bar{X}$: $\quad -2.57 = \frac{\bar{X}_L - 5.5}{.075} \Rightarrow \bar{X}_L = 5.31$

$2.57 = \frac{\bar{X}_U - 5.5}{.075} \Rightarrow \bar{X}_U = 5.69$

SO: CRITICAL REGION for $\bar{X}_{sample}$ is

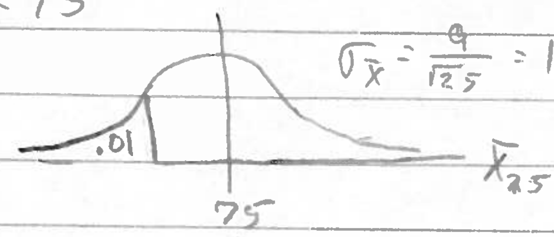
$\bar{X}_{sample} < 5.31$ or $\bar{X}_{sample} > 5.69$

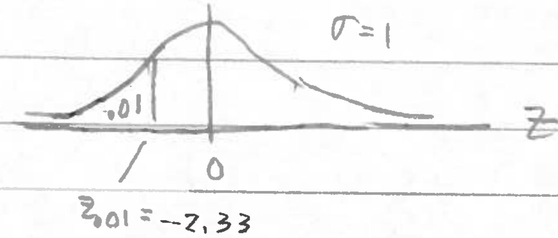
(11) $H_0 = 75$ $\alpha = .01$ $\sigma = 9.0$ $n = 25$ $\bar{X}_{25} = 72.3$

$H_1 < 75$

$\sigma_{\bar{X}} = \frac{9}{\sqrt{25}} = 1.8$

a)





Critical region for Z_{sample}: $Z_{sample} < -2.33$

Critical region for $\bar{X}_{sample}$: $\bar{X}_{sample} < 70.81$

SINCE: $-2.33 = \frac{\bar{X}_L - 75}{1.8} \Rightarrow \bar{X}_L = 70.81$

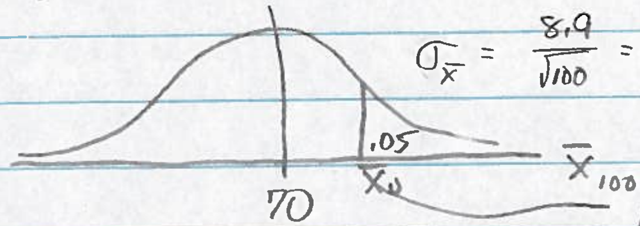
b) Since 72.3 is not in critical region,
ACCEPT H_0: The sample data lead us to believe that the mean has not changed from 75.

OR: in terms of Z: $Z_{sample} = \frac{72.3 - 75}{1.8} = -1.5$

NOT IN CRITICAL REGION

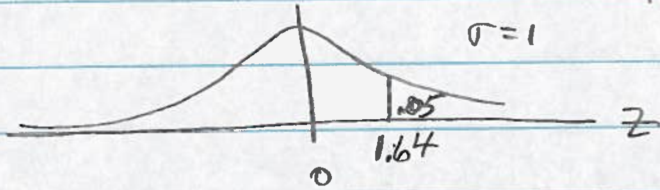
(12.) $H_0: \mu = 70$ $\qquad n = 100 \quad \sigma = 8.9 \quad \alpha = .05$

$H_1: \mu > 70$ $\qquad \bar{X}_{100} = 71.8$



$\sigma_{\bar{x}} = \frac{8.9}{\sqrt{100}} = .89$

$1.64 = \frac{x_0 - 70}{.89} \Rightarrow \bar{x}_0 = 71.46$



CRITICAL REGION: $Z_{sample} > 1.64$

or $\bar{X}_{100}\, sample > 71.46$

OUR $Z_{sample} = \frac{71.8 - 70}{.89} = 2.02$ $\quad$ IN CRITICAL REGION

So: the data lead us to REJ Ho: that is, the mean life is no longer 70 years; it is larger