SOLUTIONS: HW 6B

(1.) For a "fair" game, $E(\cdot) = 0$

So: Let X = winnings on a play

and let c = the amount she pays to play

then

X	$5-c$	$3-c$	$-c$
$P(x)$	$2/13$	$2/13$	$9/13$

and $E(X) = \frac{2}{13}(5-c) + \frac{2}{13}(3-c) + \frac{9}{13}(-c) = 0$

$\Rightarrow c = \$1.23$

(2.)

$$E(Z) = E(e^x) = \int_1^3 e^x \cdot \tfrac{1}{2}\, dx = \frac{e^3 - e}{2}$$

(3.) Let X = the number of games it takes to win the series.

$P(X=4) = 2(\tfrac{1}{2})^4 = 1/8$

$P(X=5) = 2\binom{4}{3}(\tfrac{1}{2})^3(\tfrac{1}{2})^1[\tfrac{1}{2}] = 1/4$

$P(X=6) = 2\binom{5}{3}(\tfrac{1}{2})^3(\tfrac{1}{2})^2[\tfrac{1}{2}] = 5/16$

$P(X=7) = 2\binom{6}{3}(\tfrac{1}{2})^3(\tfrac{1}{2})^3[\tfrac{1}{2}] = 5/16$

$\Sigma = 1$ ✓

and

$$E(X) = \sum_{x=4}^{7} x P(X=x) = 5\tfrac{13}{16} = 5.8125 \text{ games}$$

(4.) Long way:

Let $Y = X^3$ and find $f_Y(y)$ and then $E(Y) = \int y\, f_Y(y)\, dy$

$$f(y) = f(x)\left|\frac{dx}{dy}\right| = \frac{1}{2}\cdot\frac{1}{3}\, y^{-2/3} \quad \text{for } 27 < y < 125$$

and $E(Y) = \int_{27}^{125} y \cdot \frac{1}{2}\cdot\frac{1}{3}\, y^{-2/3}\, dy = 68$

Short way:

$$E(X^3) = \int_3^5 x^3 f(x)\, dx = \int_3^5 x^3 \cdot \frac{1}{2}\, dx = 68$$

(5.) a) Let X = the number of boys

$E(X) = 1.P(X=1) + 2P(X=2) + 3P(X=3)$

$= 1 \cdot P(BG) + 2P(BBG) + 3P(BBB)$

$= 1 \cdot \frac{1}{4} + 2(\frac{1}{8}) + 3(\frac{1}{8}) = 7/8$

b) Let Y = the number of girls

then Y is either 0 or 1 so

$E(Y) = 0 \cdot P(Y=0) + 1(P(Y=1))$

$= P(G \text{ or } BG \text{ or } BBG) = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} = \frac{7}{8}$

(6.) a)

X	P(X)
12	.2
15	.5
20	.3

Y	P(Y)
12	.10
15	.35
20	.55

b) .25

c) NO, X and Y are not independent

d) $E(X+Y) = E(X) + E(Y) = \$15.9 + \$17.45 = \$33.35$

7. Let X be the number of wives seated next to their husbands. Let W_i = wife i ; $i=1,2,\ldots,10$.

Let $X_i = \begin{cases} 1 & \text{if } W_i \text{ is next to her husband} \\ 0 & \text{otherwise} \end{cases}$ for $i=1,2,\ldots,10$

Then $X = X_1+X_2+\ldots+X_{10}$

To find $P(W_i$ is next to her husband$)$, put W_i anywhere on the circle. Then there are 19 other seats, of which 2 are next to W_i.

So $P(W_i$ next to her husband$) = 2/19$

and $E(X) = E(X_1)+E(X_2)+\ldots+E(X_{10}) = 10\cdot(2/19) = \frac{20}{19}$

8. Let X be the number of empty boxes.

Let $X_i = \begin{cases} 1 & \text{if the } i^{th} \text{ box is empty} \\ 0 & \text{otherwise} \end{cases}$ $i=1,2,\ldots,50$

Then $X = X_1+X_2+\ldots+X_{50}$

$$E(X_i) = 1\cdot P(i^{th} \text{ box is empty}) = \left(\frac{49}{50}\right)^{100}$$

So $E(X) = E(X_1)+E(X_2)+\ldots+E(X_{50}) = 50\left(\frac{49}{50}\right)^{100} \doteq 6.63$