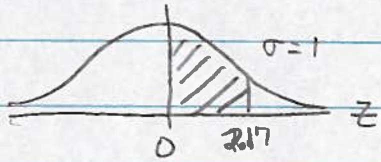
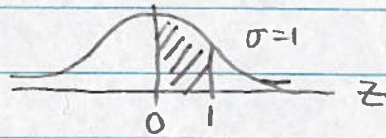
HW#7 - SOLUTIONS

1. a)



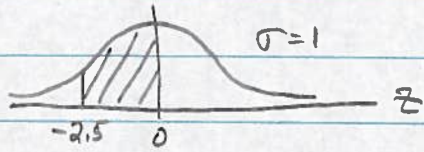
$P(0 < z \le 2.17) = .4850$

b)



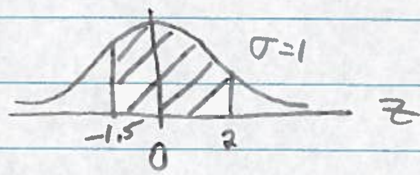
$P(0 < z < 1) = .3413$

c)



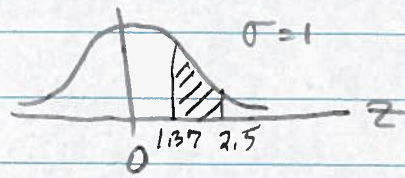
$P(-2.5 < z < 0) = .4938$

d)



$P(-1.5 \le z \le 2) = .9104$

e)



$P(1.3 \le z \le 2.5) = .0791$

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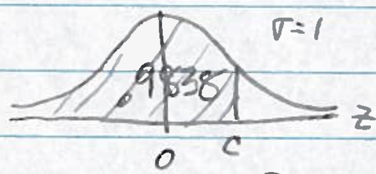
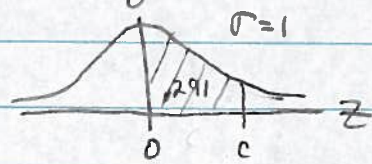
2. a)



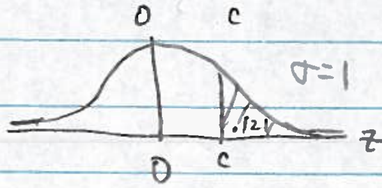
$c = 2.14$

b)



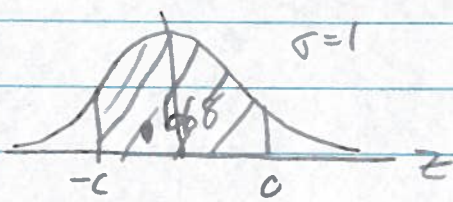
$c = .81$

c)



$c = 1.17$

d)

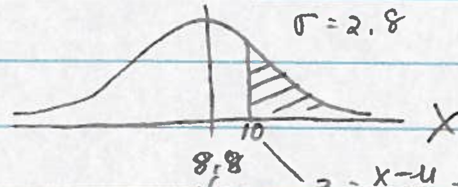


$c = .97$

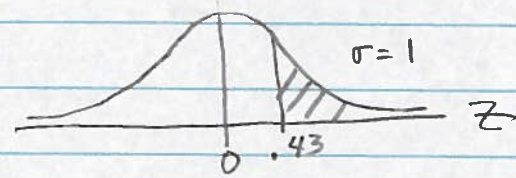
$\mu \quad \sigma$

3. $X \sim N(8.8, 2.8)$

a)

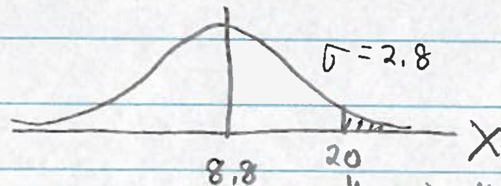


$z = 0$ $\quad z = \frac{x - \mu}{\sigma} = \frac{10 - 8.8}{2.8} = .43$

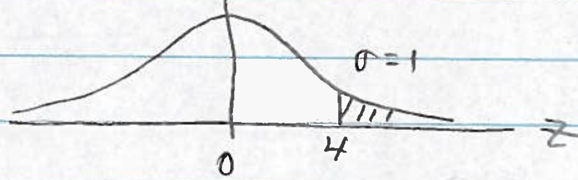


$P(X \geq 10) = P(Z \geq .43) = .3336$

b)

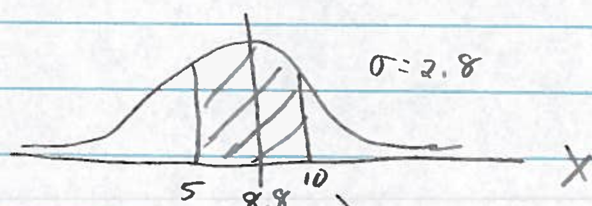


$z = 0$ $\quad z = \frac{x - \mu}{\sigma} = \frac{20 - 8.8}{2.8} = 4$



$P(X \geq 20) = P(Z \geq 4) \doteq 0$

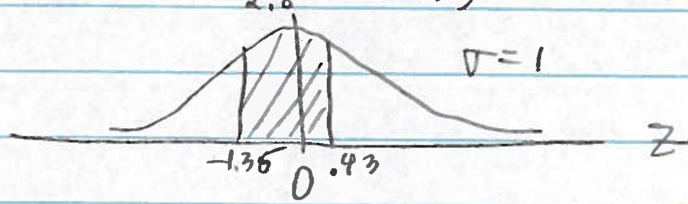
c)



$z = \frac{x - \mu}{\sigma} = \frac{10 - 8.8}{2.8} = .43$

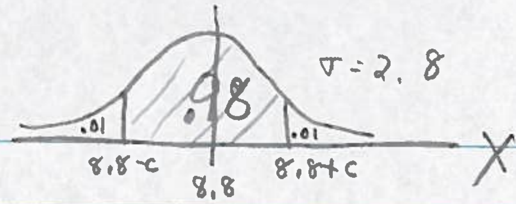
$z = \frac{x - \mu}{\sigma}$

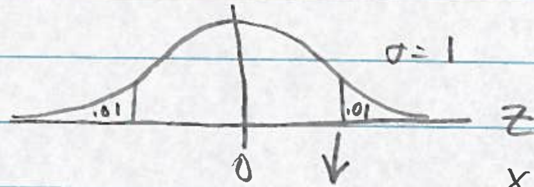
$= \frac{5 - 8.8}{2.8} = -1.35$



$P(5 \leq X \leq 10) = P(-1.36 < Z < .43) = .5792$

d)

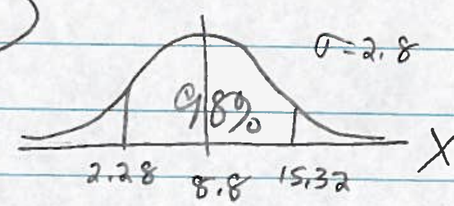




$$z = 2.33 = \frac{x-\mu}{\sigma} = \frac{(8.8+c)-(8.8)}{2.8} = \frac{c}{2.8} = 2.33$$

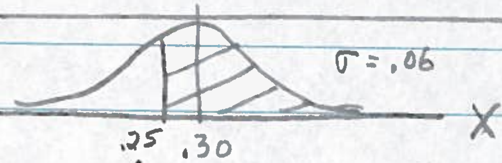
$\Rightarrow c = 6.52$

so $8.8 + 6.52 = 15.32$
$8.8 - 6.52 = 2.28$

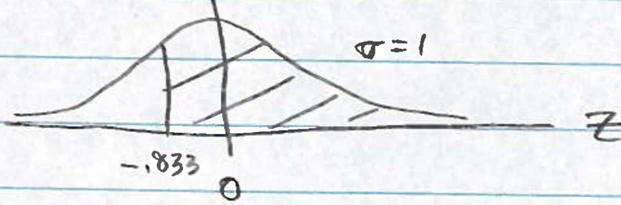


e) P(at least one has diameter > 10 inches)
= 1 − P(none has diameter > 10 inches)
$= 1 - (1 - .3326)^4 = 1 - .198 = .802$

4. a)

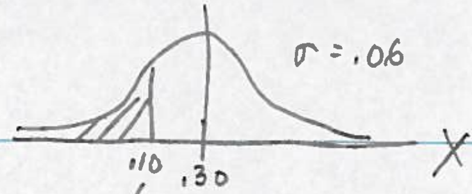


$$z = \frac{x-\mu}{\sigma} = \frac{.25-.30}{.06} = -.833$$

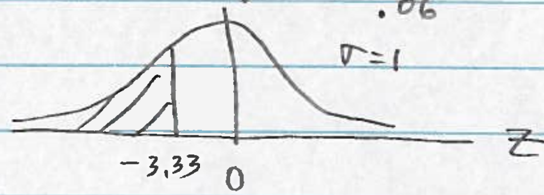


$P(X > .25) = P(Z > -.833) = .7967$

b)

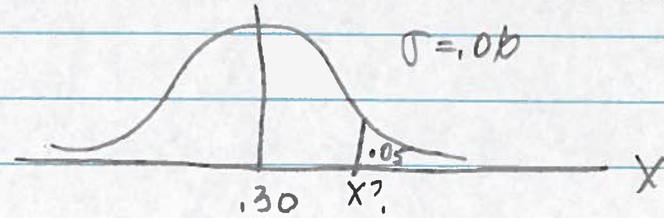


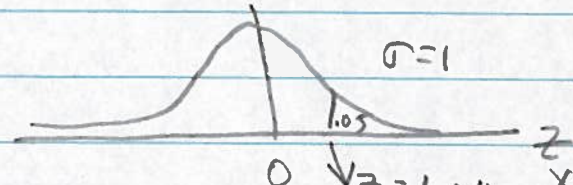
$$z = \frac{x-\mu}{\sigma} = \frac{.10-.30}{.06} = -3.33$$



$$P(X < .10) = P(Z < -3.33) = .0004$$

c)





$$z = 1.64 = \frac{x-\mu}{\sigma} = \frac{x - .30}{.06}$$

$$\Rightarrow x = 1.64(.06) + .30 = .398$$

Largest 5% of all values are > .398
(95th percentile)

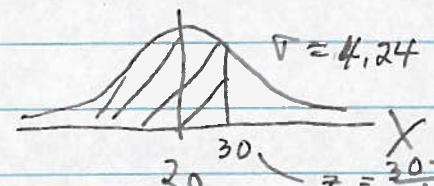
(5.) Let X = the r.v.: the number out of 200 that need to be scrapped.

$X \sim$ binomial with $n = 200$ and $p = .10$

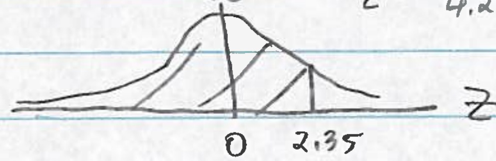
Use Normal Approximation to Binomial with

$\mu = np = 20$ and $\sigma = \sqrt{npq} = 4.24$

a) $P(X \leq 30)$:

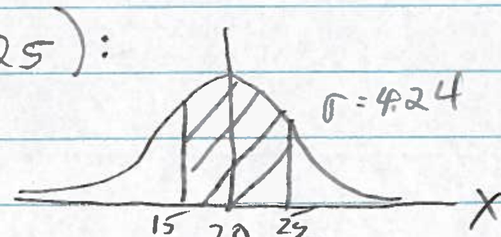


$z = \frac{30-20}{4.24} = 2.35$



So: $P(X \leq 30) = P(Z \leq 2.35) = .9906$

b) $P(15 \leq X \leq 25)$:



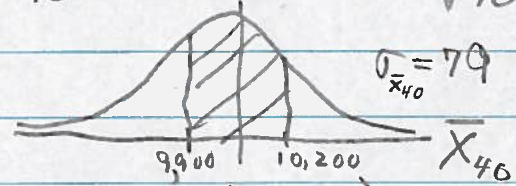
$z = \frac{15-20}{4.24} = -1.18$ $\quad z = \frac{25-20}{4.24} = 1.18$

So: $P(15 \leq X \leq 25) = P(-1.18 \leq Z \leq 1.18) = .762$

6. $X \sim ??$ with mean 10,000 and st. dev. 500

So, since $n > 30$,

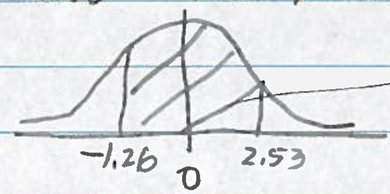
$$\bar{X}_{40} \sim N\left(10{,}000, \frac{500}{\sqrt{40}}\right) = N(10{,}000, 79)$$



$$z = \frac{10{,}200 - 10{,}000}{79} = 2.53$$

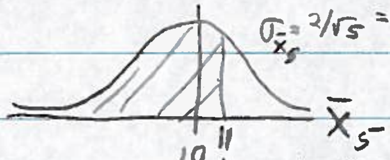
$$z = \frac{9900 - 10{,}000}{79} = -1.26$$

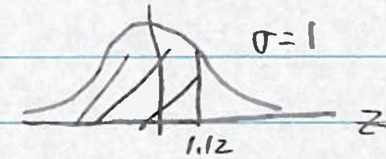
So: $P(9{,}900 \le \bar{X}_{40} \le 10{,}200) = P(-1.26 \le z \le 2.53) = .8905$



7. $X \sim N(10, 2) \Rightarrow \bar{X}_5 \sim N(10, 2/\sqrt{5})$ and $\bar{X}_6 \sim N(10, 2/\sqrt{6})$

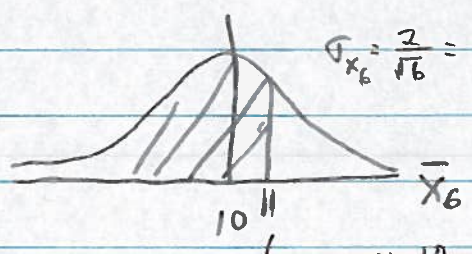
since $X \sim N$ .... otherwise you need $n > 30$

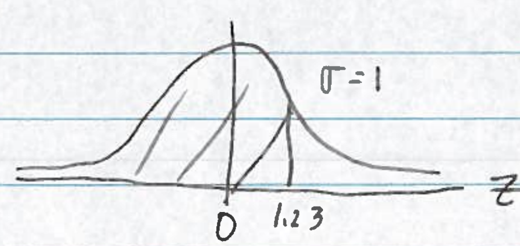




$$z = \frac{11-10}{.89} = 1.12$$

So: $P(\bar{X}_5 \le 11) = .8686$ DAY 1





$$z = \frac{11-10}{.81} = 1.23$$

So: $P(\bar{X}_6 \le 11) = .8907$ DAY 2

So: $P(\text{BOTH DAYS} \le 11) = (.8686)(.8907) = .77$