11/03/2017 **Quiz02 Math331** Section A **Student** Brianna Trollo

- **Write out each formula, procedure, and then present the desired numbers.**
- **Any answer without the desired formula and details gets no credit.**
- **α=0. 05, pf(100,1,12)=1−0.000001, pf(21.81, 2,12)=1−0.0001, pf(1.34,2,12)=1−0.298.**

An evaluation of a new coating applied to three different materials was conducted at two different laboratories. Each laboratory tested 3 samples from each of the treated materials. The recorded results are listed as below. We want to check the significance of materials, laboratories and interaction.

	Materials (B)		
LABS (A)	1	2	3
1	4.1	3.1	3.5
	3.9	2.8	3.2
	4.3	3.3	3.6
2	2.7	1.9	2.7
	3.1	2.2	2.3
	2.6	2.3	2.5

1. Evaluate overall mean, column means, row means and cell means. ----------20pts

overall → $\bar{X}_{\cdots} = \frac{1}{n}\sum_{i=1}^{r}\sum_{j=1}^{c}\sum_{k=1}^{n_{ij}} X_{ijk} = \frac{1}{18}[4.1+3.9+4.3+2.7+3.1+2.6+1.9+2.2+2.3+3.1+2.8+3.3+3.5+3.2+3.6+2.7+2.3+2.5] = 3$

cell → $\bar{X}_{11} = \frac{1}{n_{11}}\sum_{k=1}^{n_{ij}} X_{ijk} = \frac{1}{3}[4.1+3.9+4.3] = \boxed{4.1}$

row → $\bar{X}_{i\cdot\cdot} = \frac{1}{n_{i\cdot}}\sum_{j=1}^{c}\sum_{k=1}^{n_{ij}} X_{ijk} = \frac{1}{9}[4.1+3.9+4.3+3.1+2.8+3.3+3.5+3.2+3.6] = \boxed{3.53}$

lumn → $\bar{X}_{\cdot j\cdot} = \frac{1}{n_{\cdot j}}\sum_{i=1}^{r}\sum_{k=1}^{n_{ij}} X_{ijk} = \frac{1}{6}[4.1+3.9+4.3+2.7+3.1+2.6] = \boxed{3.45}$

B

A	1	2	3	Avg
1	4.1	3.07	3.43	3.53
2	2.8	2.13	2.5	2.48
Avg	3.45	2.6	2.97	3.01

2. Evaluate Sum of Squares due to two factors, interaction and error. ---------- 40pts

$SSA = c\sum_{i=1}^{r} n_{i\cdot}(\bar{X}_{i\cdot\cdot} - \bar{X}_{\cdots})^2 = 3\cdot 3[(3.53-3.01)^2 + (2.48-3.01)^2] = \boxed{4.9617}$

$SSB = r\sum_{j=1}^{c} n_{\cdot j}(\bar{X}_{\cdot j\cdot} - \bar{X}_{\cdots})^2 = 2\cdot 3[(3.45-3.01)^2 + (2.6-3.01)^2 + (2.97-3.01)^2] = \boxed{2.1798}$

$$SSAB = \sum_{i=1}^{r}\sum_{j=1}^{c} n_{ij}(\bar{x}_{ij\cdot} - \bar{x}_{i\cdot\cdot} - \bar{x}_{\cdot j\cdot} + \bar{x}_{\cdots})^2$$

$= 3[(4.1-3.53-3.45+3.01)^2 + (3.07-3.53-2.6+3.01)^2 + (3.43-3.53-2.97+3.01)^2$
$+ (2.8-2.48-3.45+3.01)^2 + (2.13-2.48-2.6+3.01)^2 + (2.5-2.48-2.97+3.01)^2] = \boxed{0.1338}$

$$SSE = \sum_{i=1}^{r}\sum_{j=1}^{c}\sum_{k=1}^{n_{ij}} (x_{ijk} - \bar{x}_{ij\cdot})^2$$

$= (4.1-4.1)^2 + (3.9-4.1)^2 + (4.3-4.1)^2$
$+ (3.1-3.07)^2 + (2.8-3.07)^2 + (3.3-3.07)^2$
$+ (3.5-3.43)^2 + (3.2-3.43)^2 + (3.6-3.43)^2$
$+ (2.7-2.8)^2 + (3.1-2.8)^2 + (2.6-2.8)^2$
$+ (1.9-2.13)^2 + (2.2-2.13)^2 + (2.3-2.13)^2$
$+ (2.7-2.5)^2 + (2.3-2.5)^2 + (2.5-2.5)^2 = \boxed{0.6001}$

8/100

$SST = SSA + SSB + SSAB + SSE$

$= 4.9617 + 2.1798 + 0.1338 + 0.6001 = 7.8754$

3. Determine the degree of freedom for all Sum of Squares. ---------- 10pts

$df_A = r-1 = 2-1 = \boxed{1}$

$df_B = c-1 = 3-1 = \boxed{2}$

$df_{AB} = (r-1)(c-1) = (2-1)(3-1) = \boxed{2}$

$df_{Error} = n-cr = 18-3\cdot2 = 18-6 = \boxed{12}$

$df_{Total} = n-1 = 18-1 = \boxed{17}$

4. Produce the two ANOVA table based on the results obtained in the above. ---------- 15pts

Source	Degree of Freedom	SS	MS	F	p-value
A	1	4.9617	$4.9617 = \frac{SSA}{r-1}$	$\frac{MSA}{MSE} = 99.234$	$1-0.000001 = 0.999999$
B	2	2.1798	$1.0899 = \frac{SSB}{c-1}$	$\frac{MSB}{MSE} = 21.798$	$1-0.0001 = 0.9999$
AxB	2	0.1338	$0.0669 = \frac{SSAB}{(r-1)(c-1)}$	$\frac{MSAB}{MSE} = 1.338$	$1-0.298 = 0.702$
Error	12	0.6001	$0.0500 = \frac{SSE}{n-cr}$		
Total	17	7.8754			

5 Conclude your study. ---------- 15pts

H_{o_1}: all rows have a common population mean

H_{o_2}: all columns have a common population mean

H_{o_3}: all cells have a common population mean.

p-value of A $= 0.999999 > \alpha = 0.05$, therefore we do not reject H_{o_1}

p-value of B $= 0.9999 > \alpha = 0.05$, therefore we do not reject H_{o_2}.

p-value of AxB $= 0.702 > \alpha = 0.05$, therefore we do not reject H_{o_3}.