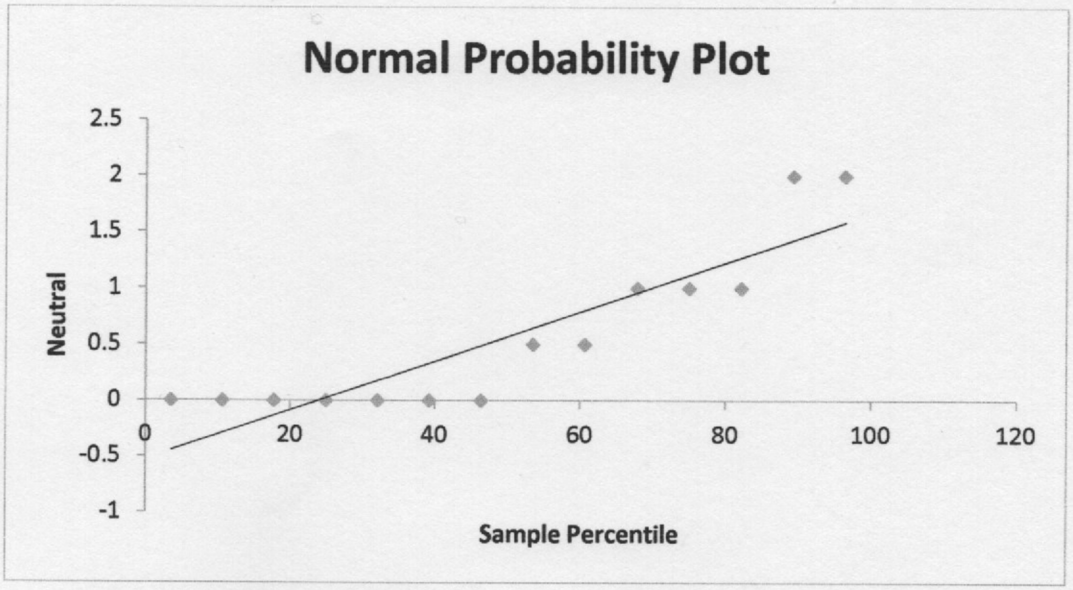
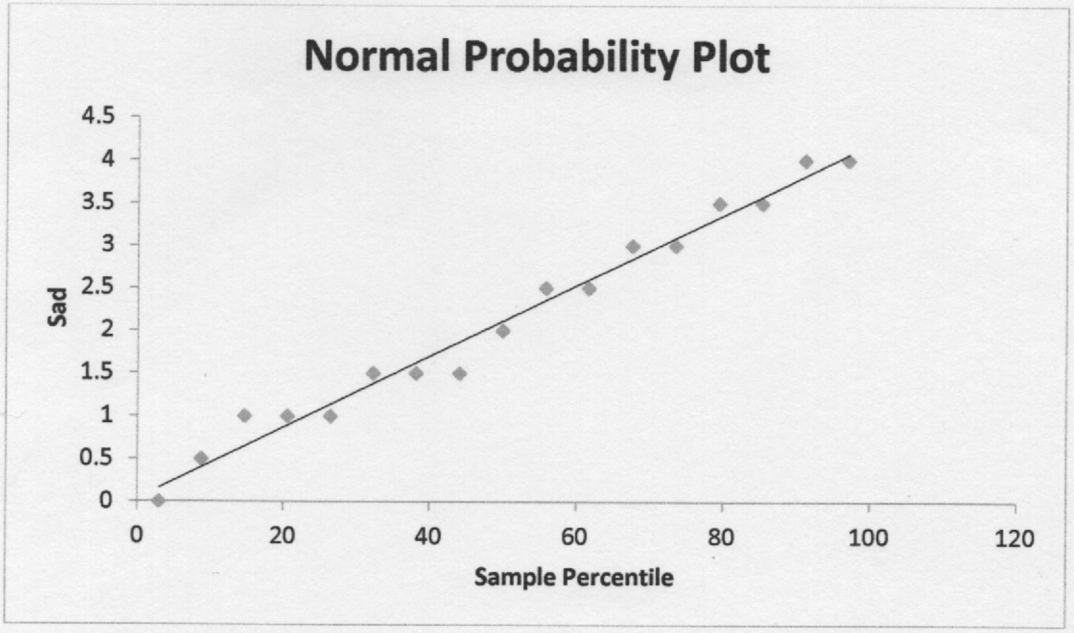
April Ullrich

Homework #4

Problem 7.71

a)





- The t-procedure is appropriate in this case as both groups have relatively small sample sizes and would follow a t-distribution as opposed to a z-distribution. In addition, we can use the t approach as the population standard deviations are unknown.

b)

Neutral Prices	
Sample Size	14
Mean	0.57
Standard Deviation	0.734

Sad Prices	
Sample Size	17
Mean	2.12
Standard Deviation	1.24

c)

Null Hypothesis: (H_0: $\mu_1 = \mu_2$) There is no sufficient significant difference of the mean price of purchasing insulated water bottles for the Neutral group and Sad group.

Alternative Hypothesis: (H_A: $\mu_1 \neq \mu_2$) There is sufficient significant difference of the mean price of purchasing insulated water bottles for the Neutral group and Sad group.

d) $\bar{X}_1$ = average price for Neutral group

x_2 = average price for Sad group

$\alpha = 0.05$

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{0.57 - 2.12}{\sqrt{\frac{(0.73)^2}{14} - \frac{(1.24)^2}{17}}} = -4.3056$$

$\underline{df}$

$v = \min\{n_1 - 1, n_2 - 1\}$

$= \min\{14-1, 17-1\}$

13 degrees of freedom

$P_{value} = 2P(t > -4.3056) = 1 - 2P(t \leq -4.3056) = 0.000$

Since the p-value is very small compared to the significance level of 0.05, we reject the null hypothesis. Therefore, we can conclude that there is a sufficient significant difference of the mean price of purchasing insulated water bottles for the Neutral group and the Sad group.

e) 95% Confidence Interval

$$(\bar{x}_1-\bar{x}_2) \pm t^* \sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}} = (0.571-2.118) \pm (2.16)\sqrt{\frac{(0.73)^2}{14} - \frac{(1.24)^2}{17}}$$

$$= -1.547 \pm 0.776$$

95% CI: $(-2.323, -0.771)$

Problem 7.89

a) $H_0: (\mu_{Breastfed} = \mu_{formula})$ - The hemoglobin level among breast-fed babies is the same as the hemoglobin level among formula babies.

$H_A: (\mu_{Breast\text{-}fed} > \mu_{formula})$ - The hemoglobin level among breast-fed babies is higher than the hemoglobin level among formula babies.

$$t = \frac{13.3-12.4}{\sqrt{\frac{1.7^2}{23}+\frac{1.8^2}{19}}} = 1.654$$

P-value = 0.053

Since, the P-value is greater than the significance level, $\alpha = 0.05$ (assumed), we fail to reject the null hypothesis. Thus, we can conclude there is no significant evidence to support the claim that the mean hemoglobin level is higher among breast-fed babies.

b) 95% Confidence Interval

$$(\bar{x}_1-\bar{x}_2) \pm t^* \sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}} = (13.3-12.4) \pm 2.01\sqrt{\frac{1.7^2}{23}+\frac{1.8^2}{19}}$$

95% C.I.: $(-0.2434, 2.0434)$

c) The key assumption that makes parts a and b valid is that both the samples are 2 independent samples from a normal population.

Problem 7.102

A. Ullrich

a) $n_1 = 11$ $s_1^2 = 3.5$ $\alpha = 0.05$
$n_2 = 16$ $s_2^2 = 9.1$

$H_0: \sigma_1 = \sigma_2$ (The two population standard deviations are equal)

$H_A: \sigma_1 \neq \sigma_2$ (The two population standard deviations are not equal)

$F = \frac{\text{Larger variance}}{\text{Smaller variance}}$ $\sigma_2 > \sigma_1$

$F = \frac{s_2^2}{s_1^2} = \frac{9.1}{3.5}$ $\boxed{F = 2.6}$ — F-test statistic value

b) $F_{(n_2-1, n_1-1), \alpha} = F_{(16-1, 11-1), 0.05}$

$= F(0.05, 15, 10)$

$= \boxed{2.85}$ — calculated in Excel by FINV function

c) Since $2.6 < 2.85$, that means that the test statistic value is less than the critical value. Thus, we fail to reject the null hypothesis and it is concluded that the 2 populations have equal standard deviations.

Problem 7.122

Assuming $\alpha = 0.05$

a) Table of Calculated Data for Groups 1 & 2

	N	Mean	Standard Dev.	SE Mean
Group 1	10	49.69	2.32	0.73
Group 2	10	50.55	1.92	0.61

T-Value = −0.90

P-Value = 0.383

DF = 17

$Var_1 = (2.32)^2 = 5.38$

$Var_2 = (1.92)^2 = 3.686$

Since the P-Value is greater than the assumed level of significance 0.05, we fail to reject the null hypothesis. Thus, we can conclude that there is not sufficient evidence to indicate that the group 1 mean is different from group 2's mean.

b) $H_0: \mu_d = 0$ Assume $\alpha = 0.05$

$H_A: \mu_d \neq 0$

95% CI: $(-1.76087, 0.054874)$

T-Value: -2.13

P-Value: 0.062

Table for Calculated Data for Groups 1 & 2

	N	Mean	St. Dev	SE Mean
Group 1	10	49.692	2.318	0.733
Group 2	10	50.545	1.924	0.609
Difference	10	-0.853	1.269	0.401

$Var_{diff} = (1.2691)^2 = 1.611$

Since the p-value is greater than the significance level of 0.05, we fail to reject the null hypothesis. Thus, we can conclude that there is not sufficient evidence to indicate that the group 1 and group 2 mean is different from 0.

c) The two tests, the two-sample t test and the paired t-test, show no difference between the 2 population means.

8.71 $n_1 = 60$ $X_1 = 48$

a) $\hat{p}_1 = \frac{X_1}{n_1} = \frac{48}{60} = 0.80$ $SE(\hat{p_1}) = \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1}} = \sqrt{\frac{0.80(1-0.80)}{60}} = 0.05164$

$n_2 = 132$ $X_2 = 291$

$\hat{p}_2 = \frac{X_2}{n_2} = \frac{52}{132} = 0.3939$ $SE(\hat{p_2}) = \sqrt{\frac{\hat{p_2}(1-\hat{p_2})}{n_2}} = \sqrt{\frac{(0.3939)(1-0.3939)}{132}} = 0.042529$

b)

95% C.I.: $\left((0.80-0.3939) \pm 1.96\sqrt{\frac{0.80(0.20)}{60} + \frac{0.3939(0.6061)}{132}}\right) = (0.2750, 0.5372)$

c) $H_0: p_1 = p_2$

$H_A: p_1 \neq p_2$ $\hat{p} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{48+52}{60+132} = 0.52083$

$$Z = \frac{(\hat{p}_1 - \hat{p}_2) - (p_1 - p_2)}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}} = \frac{(0.80-0.3939)-(0)}{\sqrt{0.52083(1-0.52083)\left(\frac{1}{60}+\frac{1}{32}\right)}} \approx \boxed{5.22}$$

Since the test statistic falls within the rejection region, we reject the null hypothesis. Thus, we can conclude that there is a difference in the proportion of female references that are girls and male references that are boys.