Theodore Jagodits / MA 331 / HW#3

I pledge my honor that I have abided by the Stevens Honor System

Problem #1

Moment Estimation

Let $X \sim U[0,\theta]$ $\quad f(x) = \frac{1}{\theta}$ for $0 \le x \le \theta$

$$\mu_1 = E[X] = \frac{1}{\theta}\int_0^\theta x\,dx = \frac{\theta}{2}$$

$$\theta = 2\mu_1$$

Maximum Likelihood Estimator

$$f(x|\theta) = \begin{cases} \frac{1}{\theta}, & 0 \le x \le \theta \\ 0, & \text{otherwise} \end{cases}$$

$$L(\theta) = \prod_{i=1}^{n} f(x_i|\theta) = \frac{1}{\theta^n}(x_1, \ldots x_n \in [0,\theta])$$

$$= \frac{1}{\theta^n} I(\max(x_1, \ldots x_n) \le \theta)$$

$$h(\theta) = 0 \text{ if } \theta < \max(x_1, \ldots, x_n)$$

$$L(\theta) = \frac{1}{\theta^n} \text{ if } \theta \ge \max(x_1, \ldots, x_n)$$

$$\text{MLE}: \hat{\theta} = \max(x_1, \ldots, x_n)$$

6.17 $n = 340$ $\mu = 5.4$ $\theta_{pop} = 2.3$

$m = z \cdot \frac{\theta}{\sqrt{n}}$

a) $m = 1.96 \times \frac{2.3}{\sqrt{340}} = \underline{.244}$

95% ci $(5.155, 5.6 \cancel{44} 445)$

b) $2.576 \cdot \frac{2.3}{\sqrt{340}} = \underline{.3213}$

99% CI $(5.0787, 5.7213)$

6.27 $n = 1200$ $\bar{x} = 11.5$ $\theta = 8.3$

a) $\bar{x} \pm 1.96 \cdot \frac{8.3}{\sqrt{1200}} = .4696$

95% CI : $(11.03, 11.97)$

b) No, it means that we are 95% confident that the average time is in this interval, not 95% of students are listening in this interval.

c) the sample size is pretty large $(n = 1200)$ so we could say that it approximates the normal distribution

6.28

a) $\bar{x} = 690$ min $\quad \theta = 498$ min

b) $\bar{x} \pm 1.96 \cdot \frac{498}{\sqrt{1200}} = 28.177$

95% CI (6.61.82, 718.18)

c) you could get ~~th~~ this from 6.27 by multiplying the answer by 60.

6.58 $\quad z = 1.77 \quad h_0 : \mu = \mu_0$

a) $\quad h_a : \mu > \mu_0$

$p(Z \geq z) = p(z \geq 1.77) = 1 - .9616 = .0384$

b) $h_0 : \mu = \mu_0$, $h_a : \mu \leq \mu_0$

$p(z \leq 1.77) = .9616$

c) $h_0 : \mu = \mu_0$, $h_a : \mu \neq \mu_0$

$2P(Z \geq 1.77) = 2(1 - .9616) = .0768$

6.59

$h_0 : \mu = \mu_0 \quad z = -1.69$

a) $h_a : \mu > \mu_0$

$P(Z \geq -1.69) = .\cancel{0455}\ 1 - .0455 = .9545$

b) $h_a : \mu < \mu_0$

$P(Z \leq -1.69) = 0.0455$

c) $h_a : \mu \neq \mu_0$

$2(P(Z \leq 1.69)) = .01659$

6.71 $\mu = 115$ $n = 25$ $\sigma = 30$ $\bar{x} = 127.8$

a) $z = \frac{127.8 - 115}{30} = 2.13$ $h_a: \mu > 115$

$p(z \geq 2.13) = 1 - .9836 = 0.01659$

b) assumption: SRS was ~~normal~~ follows normal distribution
no skews or outliers

6.73 a) $h_0: \mu = 0$ mpg
$h_a: \mu \neq 0$ mpg

b) $\bar{x} = \frac{1}{n}\sum x$ $\bar{x} = 2.73$

$z = \frac{2.73 - 0}{\frac{3}{\sqrt{20}}} = 4.07$ $p = 2P(z \geq 4.07) = .0000$

Since the P value is so small, we reject the null hypothesis. There is sufficient evidence to conclude there is a significant difference between computers and drivers calculations.

6.99
$z = \frac{2453.7 - 2403.7}{\frac{420}{\sqrt{100}}} = .57$

$h_a: \mu > 2403.7$ $P(z \geq .57) = 1 - .7157 = .0$

7.22

a) d.f. = n − 1 = 15 $h_0 : \mu = 0$ $n = 16$

b) based on the t $h_a : \mu > 0$ $t = 2.15$
distribution table, the critical values.
[2.131, 2.249]

c) based on the previous answer, the Pvalues are [.025 and 0.02)

d) - at 5% significance level, the Pvalue is between (0.02, 0.025) which will be less than .05, therefore we reject the

- at 1% significance level, with Pvalue (0.02, 0.025) which will be more than 0.01. since the Pvalue is greater than the significance level, we fail to reject the h_0 at the 1% level.

e) The exact Pvalue can be calculated using the t-distribution command on the calculator
tdist(2.15, 15, 1) = .024137

7.23

$t = 2.01 \quad n = 27$

a) d.f. $= n-1$
$= 27 - 1 = 26$

b) based on the t-distribution table at $t=2.01$, the critical values are 1.706, 2.056

c) the one sided t-test fails between (0.05 & .025)
However, this is a 2-sided t-test, the probabilities are 2(.05) & 2(.025), the Pvalue is between 0.05 and .10

d) At 5% significance level, the critical value for 0.05 and 26 d.f. is ± 2.056. Since $t = 2.01 < 2.056$ we fail to reject the null hypothesis as there is no significance.

At 1% significance, the critical value for 0.01 and 26 d.f. is ± 2.779. Since $2.01 < 2.0779$, we fail to reject the H_0 at 1% significance level.

e) the exact Pvalue was calculated

tdist (2.01, 26, 2)

Pvalue = 0.0549