Advanced Macroeconomics: Het.

Assignment 1

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1 Setup

First I define the **stationary equilibrium** for the model:

For a given Γ_{ss}, ϕ^0_{ss}, and ϕ^1_{ss} the steady state is

1. Quantities K_{ss}, L^0_{ss}, and L^1_{ss},
2. Prices r_{ss}, w^0_{ss}, and w^1_{ss},
3. A distribution D_{ss} over z_{t-1} and a_{t-1}
4. And policy functions $a^*_{ss}(z_t, a_{t-1})$, $\ell^{0*}_{ss}(z_t, a_{t-1})$, $\ell^{1*}_{ss}(z_t, a_{t-1})$, and $c^*_{ss}(z_t, a_{t-1})$

Such that

1. Firms maximize profits:

$$
\begin{aligned}
r_{ss} &= \alpha \Gamma_{ss} K_{ss}^{\alpha-1} (L^0_{ss})^{\frac{1-\alpha}{2}} (L^1_{ss})^{\frac{1-\alpha}{2}} - \delta \\
w^0_{ss} &= \frac{1-\alpha}{2} \Gamma_{ss} K^\alpha_{ss} (L^0_{ss})^{-\frac{1+\alpha}{2}} (L^1_{ss})^{\frac{1-\alpha}{2}} \\
w^1_{ss} &= \frac{1-\alpha}{2} \Gamma_{ss} K^\alpha_{ss} (L^0_{ss})^{\frac{1-\alpha}{2}} (L^1_{ss})^{-\frac{1+\alpha}{2}}
\end{aligned}
$$

2. $a^*_{ss}(\cdot)$, $\ell^{0*}_{ss}(\cdot)$, $\ell^{1*}_{ss}(\cdot)$, and $c^*_{ss}(\cdot)$ solves the household problem with $\{r_{ss}, w^0_{ss}, w^1_{ss}\}_{t=0}^\infty$
3. $\mathbf{D_{ss}} = \mathbf{\Lambda'_{ss}} \mathbf{\Pi'_{ss}} \mathbf{D_{ss}}$ is the invariant distribution implied by the household problem
4. Mutual fund balance sheet is satisfies with $A_{ss} = K_{ss}$
5. The capital market clears $A_{ss} = \int a^*_{ss}(\beta_i, \phi^0, \phi^1, z_t, a_{t-1}) d\mathbf{D_{ss}}$
6. Labor market 0 clears $L^0_{ss} = \int \ell^{0*}_{ss}(\beta_i, \phi^0, z_t, a_{t-1}) d\mathbf{D_{ss}}$
7. Labor market 1 clears $L^1_{ss} = \int \ell^{1*}_{ss}(\beta_i, \phi^1, z_t, a_{t-1}) d\mathbf{D_{ss}}$
8. The goods market clears $Y_{ss} = \int c^*_{ss}(\beta_i, \phi^0, \phi^1, z_t, a_{t-1}) d\mathbf{D_{ss}}$

Next I define the **transition path** for the model:

1. Shocks $\mathbf{Z} = \{\mathbf{\Gamma}, \boldsymbol{,} \boldsymbol{\phi^0}, \boldsymbol{\phi^1}\}$
2. Unknowns $\mathbf{U} = \{\mathbf{K}, \mathbf{L^0}, \mathbf{L^1}\}$
3. Targets: $\left\{\mathbf{A_t} - \mathbf{A^{hh}_t}\right\}$ (asset market clearing), $\left\{\mathbf{L^0_t} - \mathbf{L^{0,hh}_t}\right\}$, and $\left\{\mathbf{L^1_t} - \mathbf{L^{1,hh}_t}\right\}$ (clearing of both labor markets).
4. Aggregate variables: $\mathbf{X} = \left\{\mathbf{\Gamma}, \mathbf{K}, \mathbf{r}, \mathbf{w^0}, \mathbf{w^1}, \mathbf{L^0}, \mathbf{L^1}, \mathbf{C}, \mathbf{Y}, \mathbf{A}, \mathbf{A^{hh}}, \mathbf{C^{hh}}, \mathbf{L^{0,hh}}, \mathbf{L^{1,hh}}\right\}$

5. Household inputs: $\mathbf{X_t^{hh}} = \{\mathbf{r}, \mathbf{w^0}, \mathbf{w^1}, \boldsymbol{\phi^0}, \boldsymbol{\phi^1}\}$

6. Household outputs $\mathbf{Y_t^{hh}} = \left\{\boldsymbol{A^{hh}}, \boldsymbol{C^{hh}}, \boldsymbol{L^{0,hh}}, \boldsymbol{L^{1,hh}}\right\}$

Giving the equation system

$$\begin{bmatrix} A_t - A_t^{hh} \\ L_t^0 - L_t^{0,hh} \\ L_t^1 - L_t^{1,hh} \end{bmatrix} = \begin{bmatrix} \mathbf{0} \end{bmatrix} \forall t \in \{0, 1, ...T-1\} \tag{1}$$

and

$$\begin{aligned}
r_t &= \alpha \Gamma_t K_{t-1}^{\alpha-1} (L_t^0)^{\frac{1-\alpha}{2}} (L_t^1)^{\frac{1-\alpha}{2}} - \delta \\
w_t^0 &= \frac{1-\alpha}{2} \Gamma_t K_{t-1}^{\alpha} (L_t^0)^{-\frac{1+\alpha}{2}} (L_t^1)^{\frac{1-\alpha}{2}} \\
w_t^1 &= \frac{1-\alpha}{2} \Gamma_t K_{t-1}^{\alpha} (L_t^0)^{\frac{1-\alpha}{2}} (L_t^1)^{-\frac{1+\alpha}{2}} \\
A_t &= K_t \\
A_t^{hh} &= \mathbf{a_t^{*'}} \mathbf{D_t} \\
L_t^{0,hh} &= \ell^{\mathbf{0},*'} \mathbf{D_t} \\
L_t^{1,hh} &= \ell^{\mathbf{1},*'} \mathbf{D_t} \\
\mathbf{D_t} &= \Pi_z^{'} \underline{\mathbf{D}}_{\mathbf{t}} \\
\underline{\mathbf{D}}_{\mathbf{t+1}} &= \Lambda_t \mathbf{D_t} \\
&\underline{\mathbf{D}}_{\mathbf{0}} \textit{is given}
\end{aligned}$$

Figure 1 below, shows the DAG for the model.

Figure 1: Directed Acyclic Graph

Notes: Shows the Directed Acyclic Graph for the model. "phi_low" and "phi_high" is ϕ_t^0 and ϕ_t^1, respectively.

2 Solve for the stationary equilibrium

Below I show some of the values of interest as well as two figures that show what affects wealth inequality. Table 1 implies that the model was not solved correctly. I still continue with the figures relevant for this part of the question, as the behavior seems reasonable.

Table 1: Some variables of interest

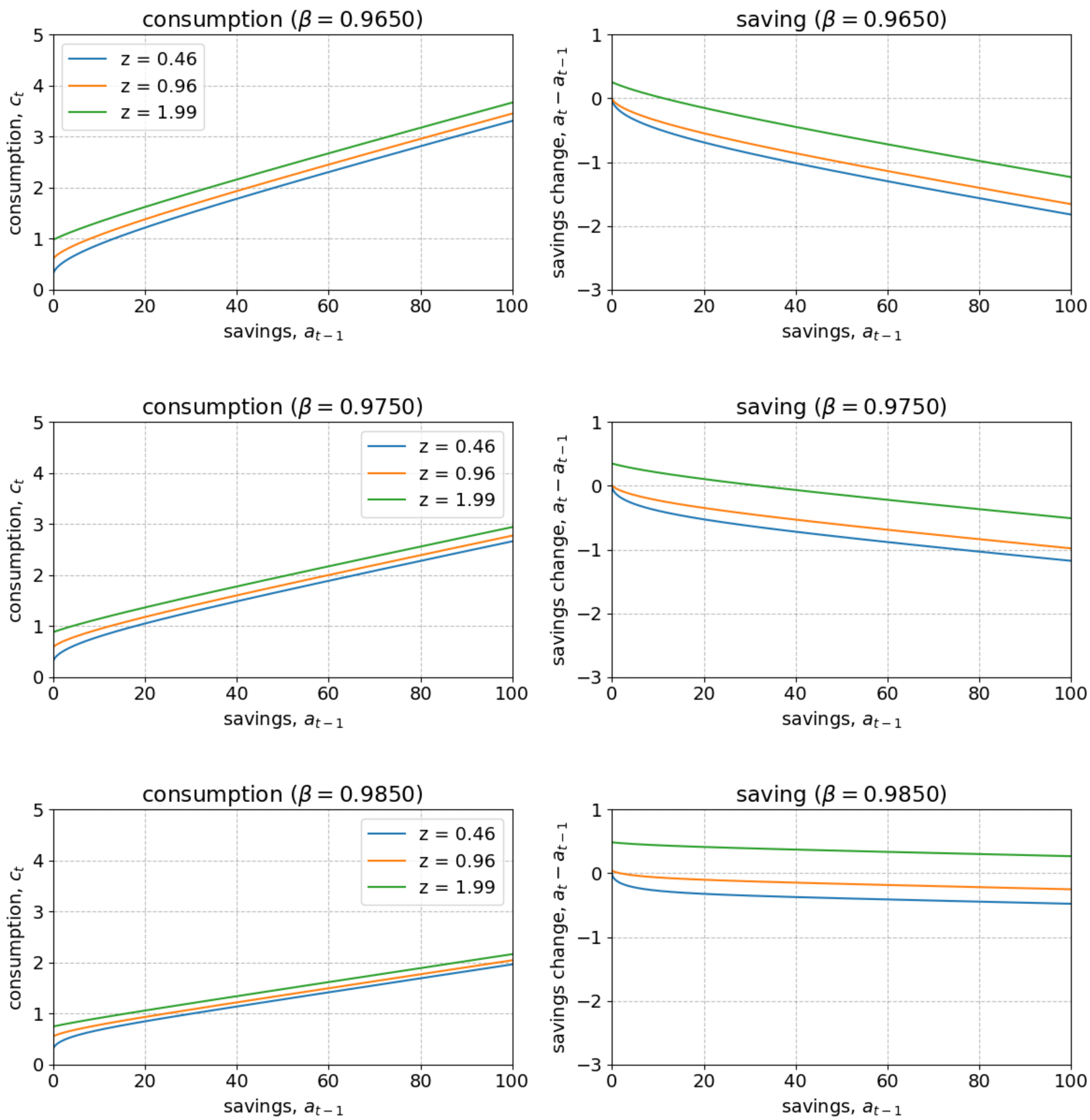
clearing A_{ss}	Clearing L^0_{ss}	Clearing L^1_{ss}	Clearing Y_{ss}
0.0000	0.1667	-0.3333	-0.1028

Figure 2 shows consumption and savings for different income shocks for a given beta. It is apparent that for any given beta, a positive income shock implies higher savings and higher consumption. For a high beta, of say 0.9850, and a high income shock of 1.99, the change in savings is positive for all savings last period (or for all the plotted savings at least). This is because a high beta, means that the future is not discounted as high, as with a low beta. The opposite is true for a low beta and a low income shock (except for the special case where savings are zero). Overall this means that stochastic income and different discount rates are drivers of wealth inequality.

Figure 3[1] shows the distribution of savings by discount factor. We see that low discount factor,

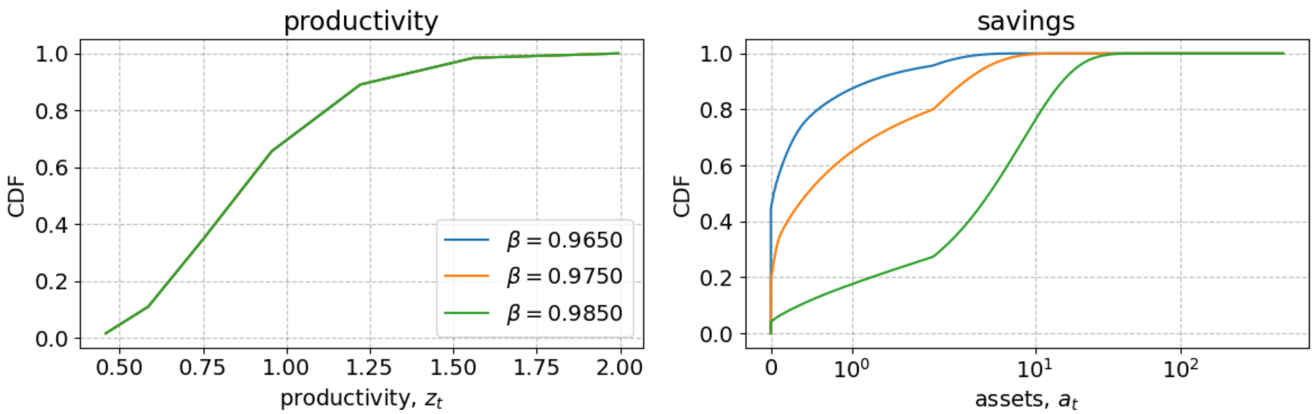
[1] One should probably add the productivity of labor to this figure

Figure 2: Consumption and savings for given betas



meaning that the future is not highly valued, means lower savings. For example, about 50% of households with a discount factor of 0.9650 have no savings. Conversely, less than about 10% of households with a discount factor of 0.9850, have no savings.

Figure 3: Distribution of savings



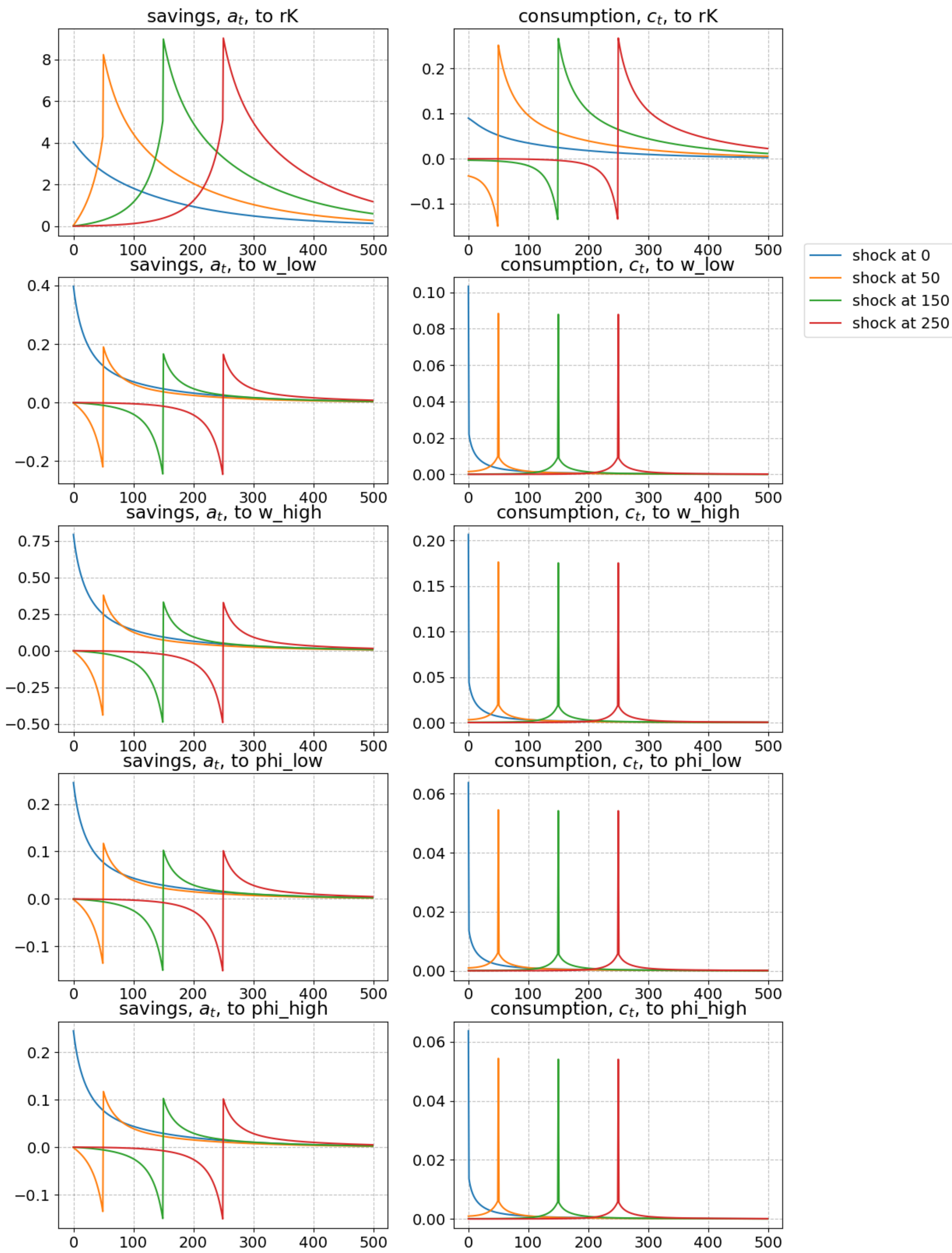
3 Computing Jacobians

(I presumme we need to add some explanation for the exam for full points here?)

Figure 4 shows the Jacobians of the household, where "phi_low" and phi_high is ϕ^0 and ϕ^1, respectively.

3.1 Transition path for temporary shock to labor productivity

Figure 4: Jacobians



4 Transition path

(Since the steady state wasn't solved properly, i don't continue with the transition-path)

4.1 Transition path for temporary shock to labor productivity

4.2 Transition path for permanent shock to productivity