

Supplementary Material for Lesson 11

1 Fisher Information

The Fisher information (for one parameter) is defined as

$$\mathcal{I}(\theta) = E \left[\left(\frac{d}{d\theta} \log(f(X|\theta)) \right)^2 \right]$$

where the expectation is taken with respect to X which has PDF $f(x|\theta)$. This quantity is useful in obtaining estimators for θ with good properties, such as low variance. It is also the basis for the Jeffreys prior.

Example: Let $X | \theta \sim N(\theta, 1)$. Then we have

$$\begin{aligned} f(x|\theta) &= \frac{1}{\sqrt{2\pi}} \exp \left[-\frac{1}{2}(x - \theta)^2 \right] \\ \log(f(x|\theta)) &= -\frac{1}{2} \log(2\pi) - \frac{1}{2}(x - \theta)^2 \\ \frac{d}{d\theta} \log(f(x|\theta)) &= -\frac{2}{2}(x - \theta)(-1) = x - \theta \\ \left(\frac{d}{d\theta} \log(f(x|\theta)) \right)^2 &= (x - \theta)^2 \end{aligned}$$

and so $\mathcal{I}(\theta) = E[(X - \theta)^2] = \text{Var}(X) = 1$.