

Faulty Caliper Problem:

$$\theta = 1: y \sim N(1, 1)$$

$$\theta = -1: y \sim N(-1, 1)$$

Prior

0.5

0.5

likelihood

$$p(y=0.5 | \theta=1) = \frac{1}{\sqrt{2\pi}} e^{-\left(\frac{(y-1)^2}{2}\right)} \approx 0.1405$$

marginal

$$\begin{aligned} p(y=0.5) &= p(y=0.5 | \theta=1) p(\theta=1) + p(y=0.5 | \theta=-1) p(\theta=-1) \\ &= 0.09605 \end{aligned}$$

Posterior

$$p(\theta=1 | y=0.5) = \frac{0.07022}{0.09605} \approx 0.73$$