

Conditional Probability $P(A|B)$

$$A = \{5\}$$

$$B = \{1, 3, 5\}$$



$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \boxed{\frac{1}{3}}$$

$$P(A) = \frac{1}{6} \leftarrow \begin{array}{l} \swarrow \{5\} \\ \nwarrow \{1, 2, 3, 4, 5, 6\} \end{array}$$

$$C = \{2, 4, 6\}$$

$$P(A|C) = \frac{P(A \cap C)}{P(C)} = \boxed{\frac{0}{3}}$$

Example: Medical Testing

Event A = Having the disease

Event B = Testing positive

Prior: $P(A) = 0.01$

Likelihood: $P(B|A) = 0.99$

False positive rate 0.05

Marginal:

$$\begin{aligned} P(B) &= P(B|A) \cdot P(A) + P(B|A^c) \cdot P(A^c) \\ &= (0.99)(0.01) + (0.05) \cdot (0.99) \\ &\approx 0.0594 \end{aligned}$$

Posterior:

$$P(A|B) = \frac{P(B|A) P(A)}{P(B)} = \frac{(0.99)(0.01)}{(0.0594)} \approx 0.167!$$

Example: A fair coin?



	<u>Prior</u>
H_0 : Coin is fair	0.5
H_A : Coin is biased	0.5

Data: TTT

$$P(H_0 | TTT) = \frac{P(TTT | H_0) P(H_0)}{P(TTT)} = \frac{(1/2)^3 (1/2)}{3/16} = \frac{1/16}{3/16} = \boxed{\frac{1}{3}}$$

$$P(TTT) = P(TTT | H_0) P(H_0) + P(TTT | H_A) P(H_A)$$

$$P(TTT | H_A) = \int_0^1 p^3 dp = \left. \frac{p^4}{4} \right|_0^1 = \frac{1}{4}$$

$$= \left(\frac{1}{2}\right)^3 \cdot \left(\frac{1}{2}\right) + \left(\frac{1}{4}\right) \cdot \frac{1}{2} = \frac{1}{16} + \frac{1}{8} = \frac{3}{16}$$