

Lesson 1 of module 1

H_0 : coin fair

H_A : coin isn't fair

Prior
0.66
0.34

Data: 5 Heads

likelihood:

$$P(HHHHH | H_0) = \left(\frac{1}{2}\right)^5 = \frac{1}{32} = 0.03125$$

$$P(HHHHH | H_A) = \int_0^1 p^5 dp = \left[\frac{p^6}{6}\right]_0^1 = \frac{1}{6}$$

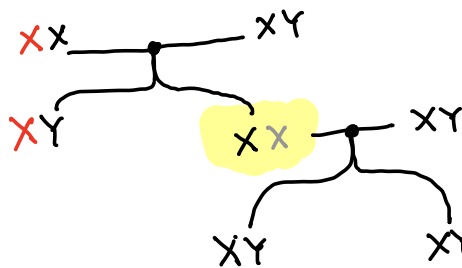
Marginal:

$$\begin{aligned} P(HHHHH) &= P(\text{data} | H_0) P(H_0) + P(\text{data} | H_A) P(H_A) \\ &= (0.03125)(0.66) + \left(\frac{1}{6}\right)(0.34) \\ &\approx 0.078 \end{aligned}$$

$$P(H_0 | \text{data}) = \frac{P(\text{data} | H_0) P(H_0)}{P(\text{data})} = \frac{(0.03125)(0.66)}{(0.078)} \approx 0.268$$

Hemophilia Example .

X-linked trait



Prior
50/50

Prior: 0.5

$\theta = 0$ (not a carrier)

Data: $\vec{y} = (0, 0)$

Prior: 0.5

$\theta = 1$ (a carrier)

← Evidence

likelihood:

$$p(\vec{y} | \theta=1) = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$$

$$p(\vec{y} | \theta=0) = 1 \cdot 1 = 1$$

marginal:

$$p(\vec{y}) = p(\vec{y} | \theta=1) p(\theta=1) + p(\vec{y} | \theta=0) p(\theta=0)$$

$$= \frac{1}{4} \cdot \frac{1}{2} + 1 \cdot \frac{1}{2} = \frac{5}{8}$$

posterior:

$$p(\theta=1 | \vec{y}) = \frac{p(\vec{y} | \theta=1) p(\theta=1)}{p(\vec{y})} = \frac{\frac{1}{4} \cdot \frac{1}{2}}{\frac{5}{8}} = \frac{\frac{1}{8}}{\frac{5}{8}} = \frac{1}{5} = 0.2 \text{ or } 20\%$$

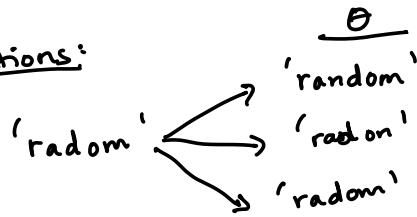
What if there is third child
who is also XY and not afflicted.

$y = (0)$

$$p(\theta=1 | y) = \frac{p(y | \theta=1) p(\theta=1)}{p(y)} = \frac{\left(\frac{1}{2}\right) \left(\frac{1}{5}\right)}{\frac{1}{2} \cdot \frac{1}{5} + 1 \cdot \frac{4}{5}}$$

$$= \frac{1}{9} \approx 11\%$$

Spelling Corrections:



data: $y = \text{radom}$

$$p(\theta|y) \propto p(\theta)p(y|\theta)$$

Prior:

| θ | relative freq. | prob. |
|----------|-----------------------|-------|
| random | 760×10^{-7} | 0.923 |
| radon | 60.5×10^{-7} | 0.073 |
| radom | 3.12×10^{-7} | 0.004 |

$$\frac{760}{760 + 60.5 + 3.12}$$

Likelihood:

| θ | $p(\text{'radom'} \theta)$ |
|----------|----------------------------|
| random | 0.00143 |
| radon | 0.000143 |
| radom | 0.975 |

↑
E
↑
4
↑
radom
↓
d f

Posterior:

| θ | $p(\theta)p(\text{'radom'} \theta)$ | $p(\theta \text{'radom'})$ |
|----------|-------------------------------------|----------------------------|
| random | 1470×10^{-10} | ~ 0.325 |
| radon | 8.65×10^{-10} | ~ 0.002 |
| radom | 3040×10^{-10} | ~ 0.673 |