

Equivalence Testing

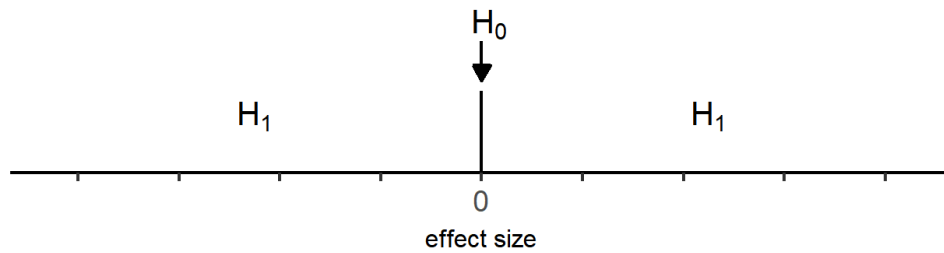
Most information in this exercise comes from [“Equivalence Testing for Psychological Research: A Tutorial”](#) by Lakens, Scheel, and Isager, and my article [“Equivalence Tests: A Practical Primer for t Tests, Correlations, and Meta-Analyses”](#). See these two articles for more examples and information on equivalence testing.

Scientists should be able to provide support for the null hypothesis. A limitation of null-hypothesis significance testing is that the null-hypothesis can be rejected, but not accepted. When you perform a statistical test, and the outcome is a p -value larger than the alpha level (α), the only formally correct conclusion is that the data are not surprising, assuming the null hypothesis is true. It is not possible to conclude that there is *no* effect – our test might simply have lacked the statistical power to detect it. So how can you ever falsify a prediction? In this assignment, we will examine how to provide support for the lack of a meaningful effect.

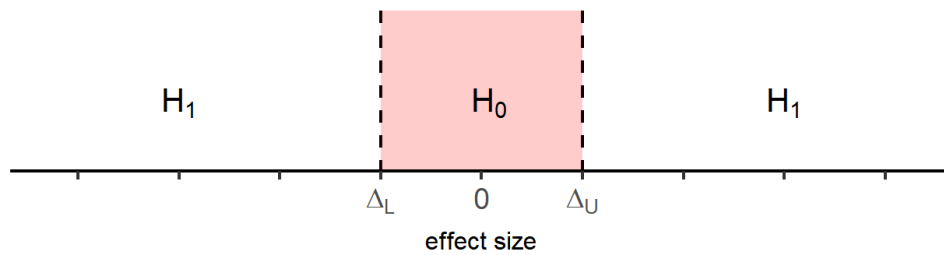
In Frequentist statistics, we can make a statement about the absence of a meaningful effect using **equivalence testing**. Equivalence testing can be used to test whether an observed effect is surprisingly small, assuming a meaningful effect exists in the population. The test is a simple variation of widely used null hypothesis significance tests (NHST). To understand the idea behind equivalence tests, it is useful to realize the null hypothesis we test against can be any numerical value. When we compare two groups, we often test whether we can reject that the difference between these groups is zero. If you look at Figure 1A on the next page, you can see a point null hypothesis at an effect size of 0, and the two-sided alternative hypothesis of an effect that is different from 0.

Sometimes we want to reject values other than zero. Imagine a researcher who is interested in voluntary participation in a national program to train young infants' motor skills. The researcher wants to test whether more boys than girls are brought into the program by their parents. Because the human sex ratio of newborn boys to girls is not exactly 1:1, we should not expect 50% of participants to be boys. On average, 103 boys are born for every 100 girls (United States & Central Intelligence Agency, 2016), so approximately 50.74% of applicants should be boys, and 49.26% should be girls. If boys and girls were exactly equally likely to be brought into the program by their parents, we would not expect a difference of zero, but $50.74\% - 49.26\% = 1.5\%$ more boys. Rather than testing against a null hypothesis of 0 difference, the researcher tests against a null hypothesis of 0.015.

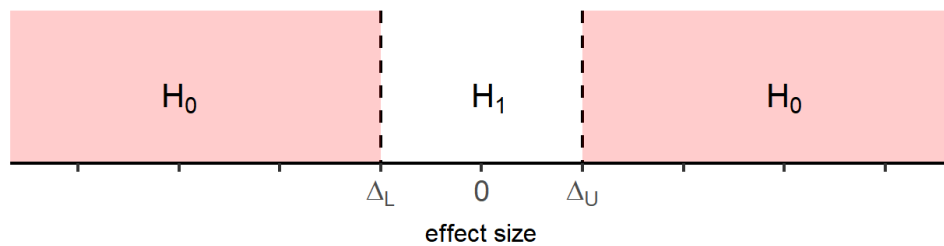
A: Classic NHST (two-sided)



B: Minimal effects test



C: Equivalence test



D: Inferiority test

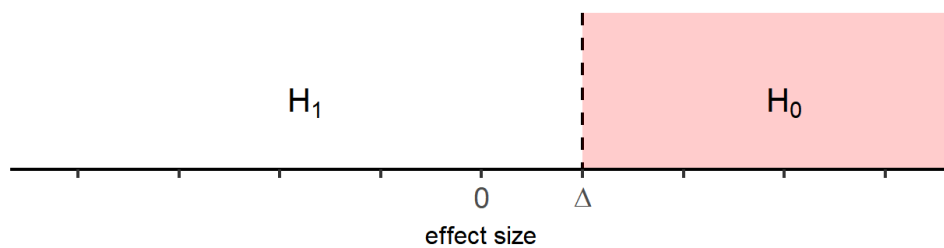


Figure 1: Illustration of null hypotheses (H_0) and alternative hypotheses (H_1) for different types of significance tests. A) NHST: Tests if the hypothesis (H_0) that an effect is equal to 0 can be rejected. B) Minimal effects test: Tests if the hypothesis (H_0) that an effect is larger than Δ_L and smaller than Δ_U can be rejected. C) Equivalence test: Tests if the hypothesis (H_0) that an effect is smaller than Δ_L or larger than Δ_U can be rejected. D) Inferiority test: Tests if the hypothesis (H_0) that an effect is larger than Δ can be rejected.

Alternatively, the researcher could decide that even if the true ratio in the population is not exactly 0.015, the null hypothesis should consist of a **range of values** around the difference in proportions of 0.015 that can be considered trivially small. The researcher could for example test if the difference is smaller than -0.005, or larger than 0.035. This test against two bounds, with H_0 being a range rather than one value (see Figure 1B), is known as a two-sided **minimal effects test** (Murphy & Myers, 1999).

Equivalence tests can be seen as the opposite of minimal effects tests: They examine whether the presence of effects that are large enough to be considered meaningful can be rejected (see Figure 1C). An equivalence test consists of two one-sided tests (and is therefore also called the **TOST** procedure). We test whether the observed effect is statistically larger than the lower equivalence bound (indicated by Δ_L in Figure 1) and statistically smaller than the upper equivalence bound (indicated by Δ_U in Figure 1). In our example, the researcher can perform an equivalence test to examine whether the gender difference in participants is not as large or larger than the **smallest effect size of interest (SESOI)**. After an extensive discussion with experts, the researcher decides that as long as the difference in proportions is not more extreme than 6%, the gender difference is too small to care about. Given an expected true difference in the population of 0.015, the researcher will test if the observed difference falls outside the boundary values (or **equivalence bounds**) of -0.045 and 0.075 by performing a one-sided test against -0.045 and another one-sided test against 0.075. If differences more extreme than these boundary values can be rejected in two one-sided tests (TOST), the researcher will conclude statistical equivalence, the gender difference will be considered trivially small, and no money will be spent on addressing a gender difference in participation. Both one-sided tests need to be statistically significant to conclude equivalence. It is custom to report only the one-sided test with the largest p -value (because if this is significant, so is the other one-sided test) when reporting an equivalence test.

Equivalence testing originates from the field of pharmacokinetics where researchers might want to show that a new, cheaper, drug works just as well as an existing drug. A very simple equivalence testing approach is the 'two-one-sided t-tests' (TOST) procedure (Schuirmann, 1987), which tests whether the observed effect is larger than the lower equivalence bound, and smaller than the upper equivalence bound.

In equivalence tests the null hypothesis is that there *is* a true effect more extreme than a smallest effect size of interest (SESOI), as defined by the upper and lower equivalence bounds (indicated by Δ_U and Δ_L in Figure 1) That's right – the null-hypothesis is now that there *is* an effect, and we are going to try to reject it (with a $p < 0.05$). The alternative hypothesis is that the effect is anywhere in the *equivalence range*. The equivalence range is a range of effect sizes that are deemed equivalent to the absence of an effect that is

worthwhile to examine (e.g., $\Delta_L = -0.3$ to $\Delta_U = 0.3$, where Δ is a difference that can be defined by either standardized differences such as δ , or raw differences such as 1 scale point or 50 milliseconds). If the p -value for both tests indicates the observed data is surprising, assuming $-\Delta_L$ or Δ_U are true, we can follow a Neyman-Pearson approach to statistical inferences and reject effect sizes more extreme than the equivalence bounds.

If we have a clear directional hypothesis we might only be interested in whether an observed effect is not larger than some effect we deem meaningful, which means we can test for inferiority (see Figure 1D), where we test whether the observed effect size is statistically lower than the SESOI (indicated by Δ). In an **inferiority test** we only care about effects is surprisingly smaller than one equivalence bound.

When null-hypothesis significance testing and equivalence tests are used, there are four possible outcomes of a study. These four cases are illustrated in the figure below (adapted from [Lakens, 2017](#)). A mean difference of Cohen's $d = 0.5$ (either positive or negative) is specified as a smallest effect size of interest in an independent t -test (see the vertical dashed lines at -0.5 and 0.5). Data is collected, and one of four possible outcomes is observed (squares are the observed effect size, thick lines the 90% CI, and thin lines the 95% CI). A 90% confidence interval ($1-2\alpha$) is used instead of a 95% confidence interval ($1-\alpha$) because two one-sided tests (each with an alpha of 5%) are performed.

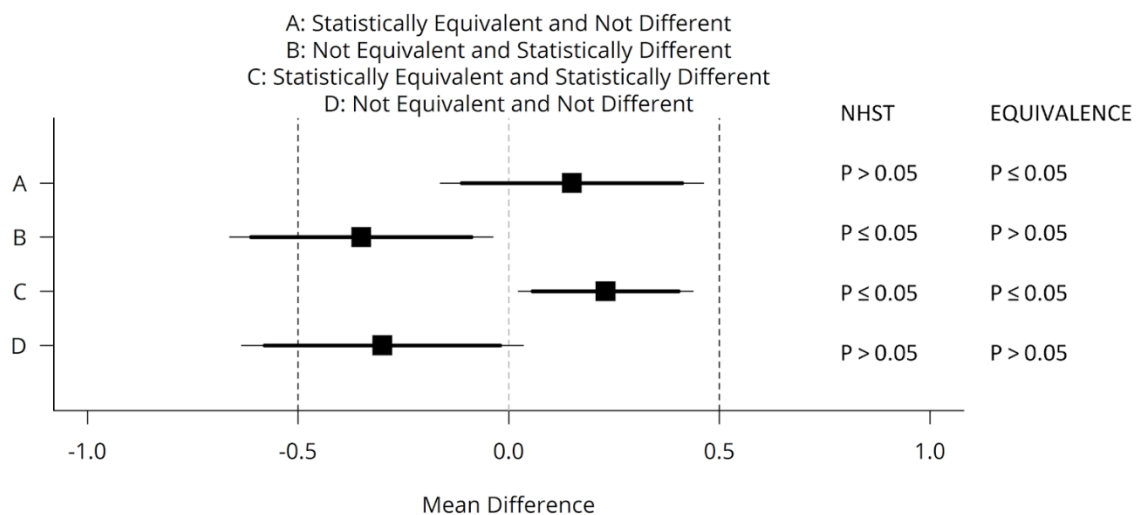


Figure 2. Four possible outcomes when combining NHST and equivalence tests.

We can conclude statistical equivalence if we find the pattern indicated by A: The confidence interval overlaps with 0, but not with the smallest effect size of interest. The p -value from the traditional NHST is not significant ($p > 0.05$), and the p -value for the

equivalence test is significant ($p \leq 0.05$). Thus, we conclude the effect is statistically equivalent: It does not differ from 0, and it is too small to be considered meaningful. We can also conclude the effect is significant, and that the possibility that the effect is large enough to matter cannot be rejected, under pattern B, which means we can reject the null, and the effect might be large enough to care about. Using equivalence tests, we can also observe pattern C: An effect is statistically significant, but also smaller than anything we care about. In other words, we can conclude there is an effect, but it lacks **practical significance** – it is too small to matter in practice. Finally, we can observe pattern D, where we cannot reject an effect of 0, nor an effect that is large enough to care about. We thus remain **undecided** – we need more data to draw a conclusion.

It is worth pointing out how equivalence tests prevent 2 common misinterpretations of p -values. First, equivalence tests allow one to distinguish between the **absence of a meaningful effect** after finding $p > .05$ in NHST, and **inconclusive** results (patterns A vs D), thus illustrating why one can't conclude there is no effect after observing $p > .05$. Second, equivalence test allows one to identify **statistically significant but practically insignificant** results (pattern C) when both the NHST and equivalence test yield $p < .05$.

Testing for equivalence is just as simple as performing the normal statistical tests you already use today. You don't have to learn any new statistical theory. Given how easy it is to use equivalence tests, and how much they improve your statistical inferences, it is surprising how little they are used, but I'm confident that will change in the future. The TOST procedure entails performing two one-sided tests to examine whether the observed data is surprisingly larger than a lower equivalence boundary (Δ_L), or surprisingly smaller than an upper equivalence boundary (Δ_U):

$$t_L = \frac{\bar{M}_1 - \bar{M}_2 - \Delta_L}{\sigma \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \text{ and } t_U = \frac{\bar{M}_1 - \bar{M}_2 - \Delta_U}{\sigma \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \quad (1)$$

Where M indicates the means of each sample, n is the sample size, and σ is the pooled standard deviation:

$$\sigma = \sqrt{\frac{(n_1 - 1)SD_1^2 + (n_2 - 1)SD_2^2}{n_1 + n_2 - 2}} \quad (2)$$

The formulas are highly similar to the normal formula for the t -statistic. The difference between a NHST t -test and the TOST procedure is that the lower equivalence boundary Δ_L and the upper equivalence boundary Δ_U are subtracted from the mean difference between groups (in a normal t -test, we compare the mean difference against 0, and thus the delta drops out of the formula because it is 0).

The TOST procedure for a t -test is identical to calculating a 90% CI around the effect, and checking whether this 90% CI falls completely within the equivalence range (see Figure 2 above).

In this assignment we will perform equivalence tests using the TOSTER package in R - if you want to follow along below, you can install TOSTER in R using: `install.packages('TOSTER')`. It works by entering summary statistics into a function, and providing all the information you need to perform and report an equivalence test. **You can also download a spreadsheet** to perform these calculations: <https://osf.io/qzjaj/>

Let's assume we performed a study where we have two independent groups, a control and experimental condition, and we are interested in the average score of participants on some validated questionnaire, measured using a 7-point scale. We want to know if there is a difference between means. Before we collect the data, we considered an effect size of $d = 0.4$ the smallest effect size of interest, and therefore set the lower equivalence bound to $d = -0.4$ and the upper equivalence bound to $d = 0.4$. After collecting the data, we have 130 participants in the control group and 134 participants in the experimental group. The control group score was $M = 4.2$, $SD = 1.1$, and the experimental group score was $M = 4.3$, $SD = 1.2$.

We now have all the information we need to perform an equivalence test. In R, the function will look like:

```
TOSTtwo(m1 = 4.2, m2 = 4.3, sd1 = 1.1, sd2 = 1.2, n1 = 130, n2 = 134,
low_eqbound_d = -0.4, high_eqbound_d = 0.4)
```

TOSTtwo is the function name for the TOST procedure for two independent groups. It needs as input the means ($m1$ and $m2$), the standard deviations ($sd1$ and $sd2$), the sample sizes in each group ($n1$ and $n2$) and you need to specify the equivalence bounds, which for the TOSTtwo function should be in standardized scores (Cohen's d - you can use the TOSTtwo.raw function to enter equivalence bounds in unstandardized mean differences). Note that you need to consistently fill in the information for one group at $m1$, $sd1$, and $n1$, and for the other group as $m2$, $sd2$, $n2$. Also note that the lower equivalence bound is entered as a negative value, because we are testing against $d = -0.4$.

When you run this single line of code, you will receive written output, and a figure:

TOST results:

t-value lower bound: 2.55 p-value lower bound: 0.006
t-value upper bound: -3.96 p-value upper bound: 0.00005
degrees of freedom : 261.17

Equivalence bounds (Cohen's d):

low eqbound: -0.4

high eqbound: 0.4

Equivalence bounds (raw scores):

low eqbound: -0.4604

high eqbound: 0.4604

TOST confidence interval:

lower bound 90% CI: -0.334

upper bound 90% CI: 0.134

NHST confidence interval:

lower bound 95% CI: -0.379

upper bound 95% CI: 0.179

Equivalence Test Result:

The equivalence test was significant, $t(261.17) = 2.545$, $p = 0.00575$, given equivalence bounds of -0.460 and 0.460 (on a raw scale) and an alpha of 0.05.

Null Hypothesis Test Result:

The null hypothesis test was non-significant, $t(261.17) = -0.706$, $p = 0.481$, given an alpha of 0.05.

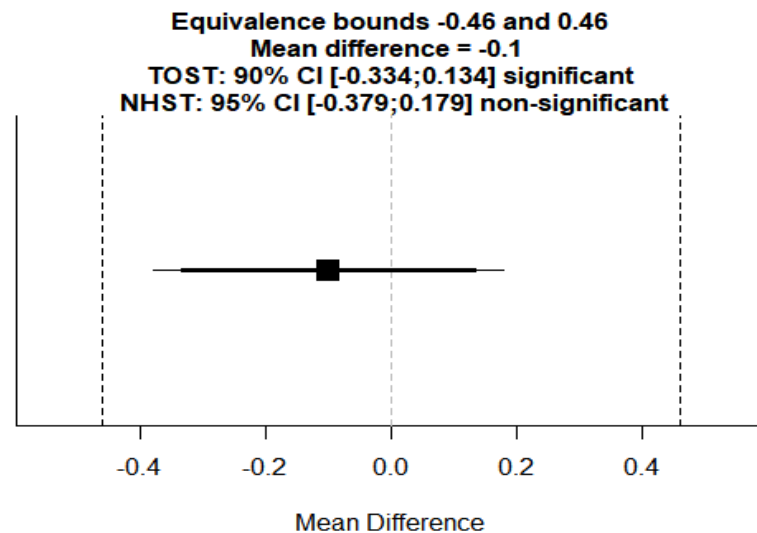
Based on the equivalence test and the null-hypothesis test combined, we can conclude that the observed effect is statistically not different from zero and statistically equivalent to zero.

Just like the default t -test in R, the TOSTtwo function will by default calculate Welch's t -test (instead of student's t -test). This is actually the recommended default (see [Delacre, Lakens, and Leys, 2017](#)), but you can request Student's t -test by adding `var.equal=TRUE` as an argument to the TOSTtwo function.

The TOST results provide the two one-sided tests, one against the lower bound of $d = -0.4$ which yields a t -value of 2.545 and a p -value of 0.00575, and the test against the upper bound of $d = 0.4$ yields a t -value of -3.957536 and a p -value of 0.00005. The output also provides a written summary (remember that it is custom to only report the largest p -value) which after rounding reads: Using alpha = 0.05 the equivalence test based on Welch's t -test was significant, $t(261.167) = 2.55$, $p = 0.006$. Because the equivalence test is significant, we can reject effects $d < -0.4$ and $d > 0.4$, and therefore reject effects we found large enough to be meaningful when we designed the study.

TOST is performed against equivalence bounds that are considered the smallest effect size of interest. The SESOI can sometimes be determined objectively, for example based on just noticeable differences which are clear quantifiable theoretical predictions. In lieu of objective justifications, the SESOI should ideally be based on a cost-benefit analysis (for example, which effect sizes are large enough to be worth studying). Since both costs and benefits are necessarily subjective, the SESOI will vary across researchers, fields, and time. The goal of setting a SESOI is to clearly justify why designing a study that has a high probability of rejecting effects more extreme than the specified equivalence bounds contributes to our knowledge base (see Lakens, 2017, Lakens, Scheel, & Isager, 2018). A SESOI should be chosen such that inferences based on it answer a meaningful question. In the study above, we can conclude equivalence using bounds of $d = -0.4$ and $d = 0.4$ – but someone else is free to consider an effect size of $d = 0.2$ meaningful, and design a much larger study to examine such a relatively small effect size.

The picture provided with the output provides the raw equivalence bounds (both raw and standardized equivalence bounds are provided in the output as well) which in this case translate to mean differences of -0.46 and 0.46. It is good to always look at, and think about, the raw equivalence bounds. These should make sense, even when you chose to use standardized equivalence bounds. We also see the observed mean difference (in this case, -0.1), and the 90% confidence interval (related to the TOST procedure) and the 95% confidence interval (related to the NHST test).



Q1) When the 90% CI around an effect size falls within the equivalence range, the observed effect is statistically smaller than the smallest effect size of interest. Based on your knowledge about confidence intervals, and looking at the picture above, when you lower the equivalence range from -0.4 – 0.4 to -0.3 – 0.3, what is needed for the equivalence test to be significant (assuming the effect size estimate and standard deviation remains the same)?

- A) A larger effect size.
- B) A lower alpha level.
- C) A larger sample size.
- D) Lower statistical power.

To answer questions 2 through 9, open the EquivalenceTesting.R script. The script will allow you to perform the TOST procedure for the questions below by filling in the summary statistics. To help you along, I have already copied the function and all arguments – you just need to fill in the correct values for each function, and interpret the output.

Q2) Researchers often manipulate something they are interested in. To ensure their manipulation does not inadvertently alter participants' moods, they assess positive and negative emotions using the PANAS. Let's assume in one specific experiment, positive mood in one condition ($M_1 = 4.55$, $SD_1 = 1.05$, $n_1 = 15$) did not differ from the mood in the other condition ($M_2 = 4.87$, $SD_2 = 1.11$, $n_2 = 15$). The researchers conclude: "*Mood did not differ between conditions, $t = 0.81$, $p = .42$* ". Let's assume we consider any effect larger than $d = -0.5$ and smaller than $d = 0.5$ equivalent (even though $d = 0.5$ is actually a medium effect size!). Use the R code for the independent t -test, and fill in the 8 numbers (note: you need to fill in -0.5, not 0.5, as the lower equivalence bound). Were the authors correct

in concluding mood did not differ between conditions, given the equivalence range of -0.5 to 0.5?

Use 0.05 as the alpha level for all questions in this assignment.

A) Yes

B) No

Q3) If we increase the sample size in the above example to 150 participants in each condition, and assuming the means and standard deviations remain the same, which conclusion would we draw?

A) Equivalent: The difference in mood is not statistically significant, and it is statistically equivalent.

B) Undetermined: The difference in mood is not statistically significant, and it is not statistically equivalent.

C) Not zero, and not meaningful: The difference in mood is statistically significant, and it is statistically equivalent.

D) Not zero, and meaningful: The difference in mood is statistically significant, and it is not statistically equivalent.

When should you consider an effect too small to be meaningful? This is clearly a subjective choice, and what you consider meaningful can change over time. Ideally, you can determine the equivalence range either based on practical considerations or theoretical considerations. For example, if an advertisement campaign increases sales, but not enough to earn back the cost of the advertising campaign, you might decide that the advertising campaign has an effect smaller than your smallest effect size of interest. Alternatively, you might believe, based on available theories, that a delay in response selection in a Stroop experiment where people respond verbally is larger than 20 milliseconds.

The choice for a smallest effect size of interest is also relevant when performing a power analysis. Because it is often unclear which effect size you can expect (after all, if you already knew the effect size, you would not need to do the experiment!) researchers sometimes make sure they have sufficient power for the smallest effect size they find interesting. Although it might be difficult to decide upon a SESOI, you might find it much easier to decide upon a maximum sample size you are willing to collect, and look at the effect size you can reject in an equivalence test. This can be examined using power analyses. Just as with a normal t-test, you can perform a power analysis for an equivalence test, which is very useful when designing a study.

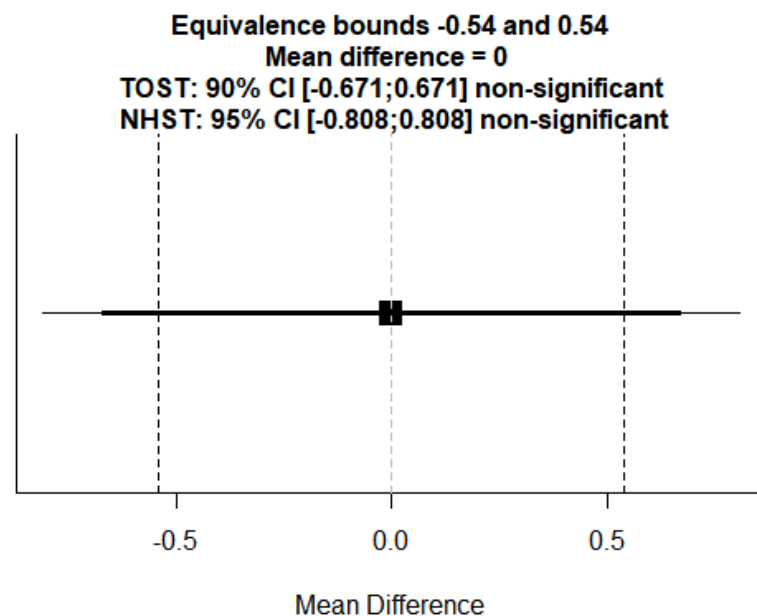
We used an equivalence range of $d \neq -0.5$ to $d = 0.5$ in question 2, even with a sample size of 15 participants in each condition. One might wonder if that sample size actually gave us sufficient statistical power to detect equivalence. The TOSTER package also has functions for power analysis. The powerTOSTtwo function can be used to calculate the required sample size, the observed power, or the required equivalence bounds to achieve a desired power given a sample size. If we run the function:

```
powerTOSTtwo(alpha = 0.05, N = 15, low_eqbound_d = -0.5,
high_eqbound_d = 0.5)
```

we are calculating the power we have when we have 15 participants in each group (N = 15), bounds of -0.5 and 0.5, and we use an alpha of 0.05. The function returns the following information:

The statistical power is 0 % for equivalence bounds of -0.5 and 0.5.

This might look like a mistake. How can we have 0 power? The answer becomes obvious if we look at what would happen in the study described in question 2 when we would have observed exactly the same means in both groups, so a mean difference of 0 (but the same sample sizes in each group, and the same standard deviations). This hypothetical scenario is plotted in the figure below:



We see the 90% confidence interval is so wide (due to the small sample size and given the standard deviations) that it will ALWAYS include either the lower or the upper equivalence bound. The 90% confidence interval is simply so wide that it will never fall within the equivalence bounds. Thus, there is 0 probability that we can conclude equivalence. The sample size is simply too small. We can see power analysis is very important in general, but especially when performing equivalence tests.

Q4) We might wonder how wide our bounds would need to be, to have decent power to conclude equivalence. Let's aim for 90% power, and use an alpha of 0.01 for our test. The powerTOSTtwo function can be used to calculate how wide our equivalence bounds would need to be, to have 90% power with 15 participants in each condition and an alpha of 0.01 (this power analysis assumes the true effect size is 0 – more advanced power analyses can be performed with PASS software). Which equivalence bounds should a researcher use if they want 90% power with 15 participants and an alpha of 0.01? Round the answer to two digits after the decimal.

- A) $d = -1.07$ and $d = 1.07$
- B) $d = -1.20$ and $d = 1.20$
- C) $d = -1.32$ and $d = 1.32$
- D) $d = -1.45$ and $d = 1.45$

You can see that with such a small sample, **we can only reject effect sizes that are very large ($d > 1$)**. Is it interesting to perform a study where you can only reject effects that are very large? It depends. Most effect sizes studied in for example psychology, but also many other social sciences, are much smaller. Asking 'can we reject very large effects' is therefore not very interesting (unless a theory explicitly predicts only very large effects, obviously!)

Q5) The most common use of power analysis is to determine the sample size needed to design a study with high power to detect a significant effect. If we want to have 90% power, use an alpha of 0.01, and use equivalence bounds of $d = -0.5$ and $d = 0.5$, how many participants **in each group** or condition should we collect? Use the code in the R file.

- A) 87
- B) 105
- C) 127
- D) 254

Q6) Change the equivalence range to -0.1 and 0.1. To be able to reject effects outside such a very narrow equivalence range, you'll need a large sample size. With an alpha of 0.01, and a desired power of 0.9 (or 90%), how many participants would you need in each group?

- A) 2165
- B) 2604
- C) 3155

D) 6310

You can see it takes a very large sample size to have high power to reliably reject very small effects. This should not be surprising – it also requires a very large sample size to *detect* small effects!

Q7) You can do equivalence tests for all tests. The TOSTER package has functions for t-tests, correlations, differences between proportions, and meta-analyses. Let's do an equivalence test for a meta-analysis. Hyde, Lindberg, Linn, Ellis, and Williams (2008) report that effect sizes for gender differences in mathematics tests across the 7 million students in the US represent trivial differences, where a trivial difference is specified as an effect size smaller than $d = 0.1$. The table with Cohen's d and se is reproduced below:

Grades	$d + se$
Grade 2	0.06 +/- 0.003
Grade 3	0.04 +/- 0.002
Grade 4	-0.01 +/- 0.002
Grade 5	-0.01 +/- 0.002
Grade 6	-0.01 +/- 0.002
Grade 7	-0.02 +/- 0.002
Grade 8	-0.02 +/- 0.002
Grade 9	-0.01 +/- 0.003
Grade 10	0.04 +/- 0.003
Grade 11	0.06 +/- 0.003

For grade 2, when we perform an equivalence test with boundaries of $d = -0.1$ and $d = 0.1$, using an alpha of 0.01, which conclusion can we draw? Use the TOSTER function TOSTmeta, and enter the alpha, effect size (ES), standard error (se), and equivalence bounds.

A) Equivalent: The difference in test scores is not statistically significant, and it is statistically equivalent.

B) Undetermined: The difference in test scores is not statistically significant, and it is not statistically equivalent.

C) Not zero, and not meaningful: The difference in test scores is statistically significant, and it is statistically equivalent.

D) Not zero, and meaningful: The difference in test scores is statistically significant, and it is not statistically equivalent.

Q8) Olson, Fazio, and Hermann (2007) reported correlations between implicit and explicit measures of self-esteem, such as the IAT, Rosenberg's self-esteem scale, a feeling thermometer, and trait ratings. In Study 1 71 participants completed the self-esteem measures. Because no equivalence bounds are mentioned, we can see which equivalence bounds the researchers would have 50% power to detect (the bounds a study has 50% power for, is related to the bounds that can just be detected with $p < .05$, see Lakens, Scheel, and Isager, 2018 or this [blog post](#)). Use the powerTOSTr function (which is used for correlations, and when the equivalence bounds are set based on r). Which boundaries do we have 50% power for, with an alpha of 0.05, and 71 participants? Round the bounds to 2 digits after the decimal.

A) $r = -0.19$ and $r = 0.19$

B) $r = -0.21$ and $r = 0.21$

C) $r = -0.27$ and $r = 0.27$

D) $r = -0.32$ and $r = 0.32$

Q9) The correlations observed by Olson et al (2007), Study 1, are presented in the table below (significant correlations are flagged by an asterisk).

Measure	IAT	Rosenberg	Feeling thermometer	Trait ratings
IAT	-	-.12	-.09	-.06
Rosenberg		-	.62*	.09
Feeling thermometer			-	.29*
Trait ratings				-

We can test each correlation for equivalence, for example the correlation between the IAT and the Rosenberg self-esteem scale of -0.12, given 71 participants. When you test all 4 non-significant correlations (-0.12, -0.09, -0.06, and 0.09) for equivalence, using an alpha of 0.05 and equivalence bounds of $r = -0.2$ and $r = 0.2$, how many are statistically equivalent?

- A) 0
- B) 1
- C) 2
- D) 3

Conclusion

It is important to be able to provide support for the null-hypothesis, if you want to be able to test theories that predict no effect, or when you want to be able to falsify theory that predicts an effect. Allowing the possibility for your prediction to be wrong is very important. According to Lakatos, even though we rarely outright reject our hypotheses, we enter a degenerative research line when our predictions do not work out. Finally, being able to conclude an effect is too small to be worthwhile to examine enables us to improve our statistical inferences.

You will always have to make assumptions about the alternative hypothesis, either by specifying an equivalence region consisting of a range of effect sizes you find meaningful, or by specifying a prior distribution. You can use either Frequentist or Bayesian approaches to test absence of evidence (or even both!). Don't simply conclude that a $p > 0.05$ means there is no effect – instead, provide quantitative arguments for the conclusion that there is no effect, for example by using equivalence tests.

References

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